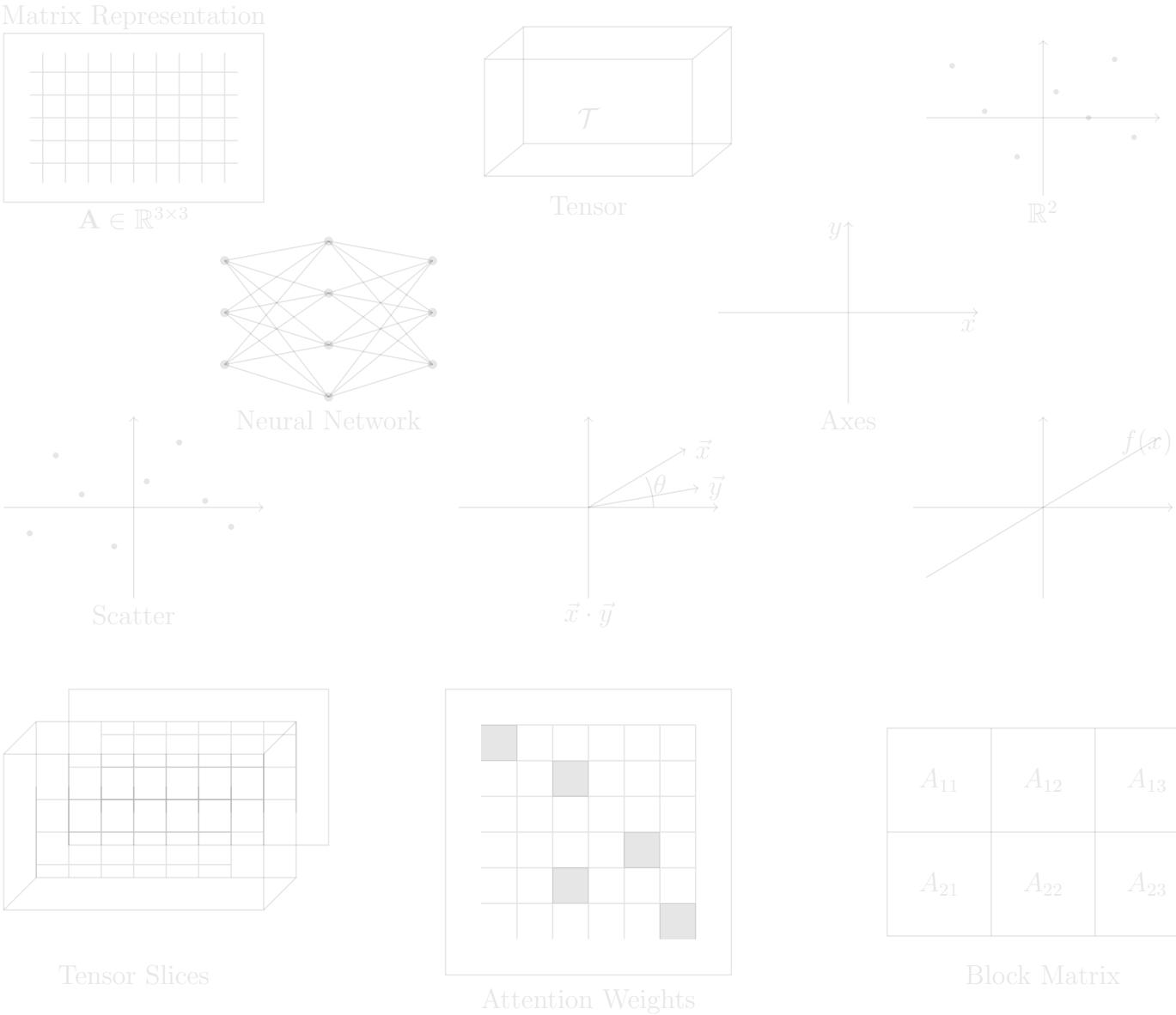
All You Need

Everything You Need to Become an Artificial Intelligence Researcher



Abdulla Ali Khamis Alshamsi

Contents

Introduction of the Volume of Books

0.1 Why am I writing this volume of books

One day I decided to learn artificial intelligence programming. What happened was strange; I discovered that the more I delved, the more I realized there was still so much to learn. The deeper I explored this world, the more I understood that I hadn't learned anything, or that what I had learned was just a drop in the ocean. That's why I decided to write down everything I learned on my journey to learning artificial intelligence programming.

What I saw in this science confirmed that we in the UAE need a lot of learning. Therefore, I will use this book to teach future generations in the UAE how to program artificial intelligence, based on the principle of self-learning, just as I implemented it at School 42 Abu Dhabi. What we need is to organize our ideas and begin.

0.2 We must be at the forefront

The competition between Saudi Arabia and the UAE to acquire artificial intelligence is a drop in the ocean compared to what companies like OpenAI, Google, and other major American firms are doing. We must accelerate development because the capabilities these companies possess rival those of most countries in the world. As Emiratis and residents of the UAE, we must expedite the development of this community in programming and advancing advanced artificial intelligence.

It's true we have educational and infrastructure projects, but we lack a huge number of educated personnel. That's why I decided to write this book for them—a solid foundation upon which to build this national workforce, and even for residents of the UAE.

0.3 What will this volume of books contain

Book 1: Tensor Mathematics and Neural Networks Guide

You will learn the fundamentals of mathematics and how to program it using Pytorch. While the first challenge will be easy, the second chapter will be quite difficult if you're new to this field. This isn't because it's impossible to grasp, but because you need four things to overcome the difficulties: first, understand the equation; then, write it using Pytorch; then, ask yourself what its application is in artificial intelligence; and finally, practice creating equations using it. Pay attention: you must complete all four of these points together in every chapter. Don't skip or skip any challenge without completing these four points, because the learning system is

built upon what you've learned previously.

Book 1

Tensor Mathematics and Neural Networks Guide

1.1 Introduction

1.1.1 What will the book teach you

Before venturing into the world of AI programming, you must learn the fundamentals of Mathematicals and programming using PyTorch. You will start from the very basics and progress to the most complex mathematical equations. This book will lay the foundation for what's to come, so make sure you understand every lesson and every challenge in this book before moving on to other books.

To emphasize, the book will consist of many exercises, each including a question and a method for finding the solution. The solution will not be provided; you must create it yourself. The book will guide you, as finding the solution is your responsibility. The approach may seem unconventional at times, but this is the foundation of intelligent learning and problem-solving.

1.1.2 The Chapters (T00 - T08) included in this book

- T00 - Tensor Algebra Laws
- T01 - Vector Laws (1D)
- T02 - Matrix Laws (2D)
- T03 - Higher-Dimension Tensor Laws
- T04 - Tensor Shape Transformation Laws
- T05 - Differential Calculus Laws
- T06 - Optimization Laws
- T07 - Statistics Probability Laws
- T08 - Neural Network Transformation Laws

1.2 T00 - Tensor Algebra Laws

1.2.1 Exercise 00 - Tensor Addition Law

Part 1 - why this matters

Tensor addition is the first operation used to combine numerical information in artificial intelligence systems. It appears in almost every model, from adding bias terms to accumulating gradients during training. Understanding this operation is essential before moving to more complex tensor transformations.

Part 2 - Mathematical form

Mathematical Definition

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor (matrix) addition is a binary operator defined as:

$$+ : \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} + \mathbf{Y}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the operation is defined component-wise by:

$$Z_{i,j} = X_{i,j} + Y_{i,j} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

Tensor addition in PyTorch is performed using the built-in function `torch.add(x, y)`. This operation applies element-wise addition and requires both tensors to have the same shape.

Before performing any tensor operation, ensure that the inputs are PyTorch tensors created using `torch.tensor()`. Understanding how tensors are constructed is essential before manipulating them.

PyTorch tensors can exist on different devices:

- CPU (default)
- GPU (CUDA-enabled devices)

When working with multiple tensors, both tensors must be located on the same device. Mixing CPU and GPU tensors will result in a runtime error.

To deepen your understanding, explore:

- How to create tensors using `torch.tensor()`
- How to check the device of a tensor
- How to move tensors between CPU and GPU

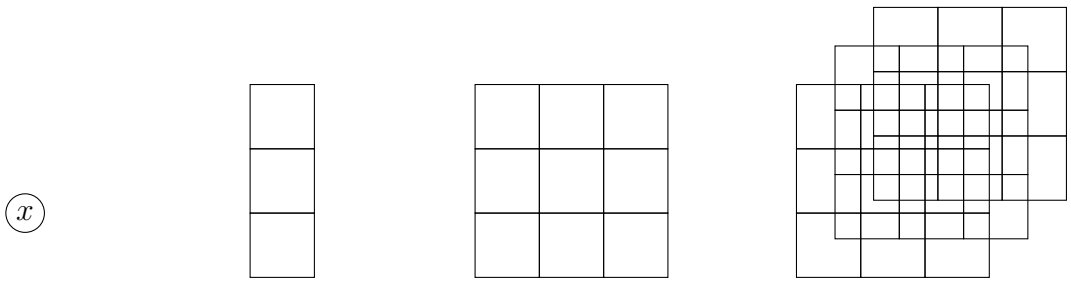
This exercise assumes CPU-based tensors, but the same logic applies to GPU tensors when explicitly moved to a CUDA device.

Part 4 - Python code skeleton

```
import torch

def E00(x,y):
    z = torch...
    return z

def main():
    x = torch.tensor...
    y = torch.tensor...
    print(E00(...))

if __name__ == "__main__":
    main()
```

1.2.2 Exercise 01 - Tensor subtraction Law

Part 0 - Before we begin



Scalar **Vector** **Matrix** **Tensor**

All numerical data in artificial intelligence is ultimately represented using the same underlying structure; the difference lies only in how many dimensions the data has.

- **Scalar**: A single numerical value with no dimensions.
- **Vector**: A one-dimensional collection of ordered values.
- **Matrix**: A two-dimensional grid of values arranged in rows and columns.
- **Tensor**: A generalization to three or more dimensions, used to represent complex data such as images, sequences, or batches.

Part 1 - why this matters

This operation has the same importance as the previous exercise. The only difference is that subtraction is used instead of addition. Understanding this change is essential before moving to more advanced tensor operations.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor (matrix) subtraction is a binary operator defined as:

$$- : \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} - \mathbf{Y}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the operation is defined component-wise by:

$$Z_{i,j} = X_{i,j} - Y_{i,j} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, tensor subtraction is performed using the built-in function `torch.sub(x, y)`. The reader is encouraged to search for this function and understand how it applies element-wise subtraction between tensors.

Part 4 - Python code skeleton

```
import torch

def E01(x,y):
    z = torch...
    return z

def main():
    x = torch.tensor...
    y = torch.tensor...
    print(E01(...))

if __name__ == "__main__":
    main()
```

1.2.3 Exercise 02 - Tensor Hadamard Multiplication Law

Part 1 - why this matters

This exercise continues the logical progression of tensor operations. After learning how tensors are combined through addition and how differences are expressed through subtraction, Hadamard multiplication introduces a new concept: controlling how information is scaled rather than simply accumulated or removed.

Unlike addition and subtraction, element-wise multiplication directly modifies the strength of each individual value. Each component can be amplified, reduced, or completely suppressed depending on the corresponding value in the second tensor. This changes the behavior of the data itself, not just its numerical total.

Understanding this operation is essential because it represents a foundational mechanism used later in more advanced tensor transformations. Mastering this concept at an early stage makes the transition to complex model structures more intuitive and coherent.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Hadamard multiplication (element-wise multiplication) is a binary operator defined as:

$$\odot : \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \odot \mathbf{Y}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the operation is defined component-wise by:

$$Z_{i,j} = X_{i,j} \cdot Y_{i,j} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, Hadamard (element-wise) multiplication can be performed using the built-in operator `x * y`.

The reader is encouraged to search for a function called `torch.mul()` and understand how it performs element-wise multiplication between tensors.

Part 4 - Python code skeleton

```
import torch

def E02(x,y):
    z = torch...
    return z

def main():
    x = torch.tensor...
    y = torch.tensor...
    print(E02(...))

if __name__ == "__main__":
    main()
```

1.2.4 Exercise 03 - Tensor Division Law

Part 0 - The weight

In artificial intelligence, a weight represents the strength of influence that an input has on the model's internal computation and final decision. Each input signal is multiplied by a weight, allowing the model to emphasize important information and suppress irrelevant or noisy signals. Learning in neural networks is therefore the process of continuously adjusting these weights to reflect how much the model should trust each input.

$$2000 \times 2000 = 4{,}000{,}000$$

$$0.2 \times 0.2 = 0.04$$

The large numerical gap produced by unscaled weights leads to unstable matrix computations, where values grow too fast and dominate subsequent operations. Keeping weights within a controlled and close numerical range ensures stable tensor operations, preserves relative relationships between features, and allows learning algorithms to update weights smoothly without numerical explosion or loss of precision.

Part 1 - why this matters

Tensor Division Law is required because repeated multiplication and accumulation in neural networks cause values to either explode or vanish, and division is used to rescale tensors into a numerical range where learning remains stable.

In normalization, division is not applied merely to modify data values, but to decouple magnitude from structure, allowing the model to learn patterns rather than be dominated by raw numerical scale.

When working with ratios, probabilities, and feature weights, division transforms absolute values into relative relationships, enabling meaningful comparison and scale-independent decision making.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Division Law (element-wise):

$$\oslash : \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \oslash \mathbf{Y}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \frac{X_{i,j}}{Y_{i,j}} \quad \text{for } i,j \in \{1,2,3\}, \quad Y_{i,j} \neq 0.$$

Part 3 - Hints and Helper

In PyTorch, tensor division (element-wise division) can be performed using the built-in operator `x / y`.

The reader is encouraged to search for a function called `torch.div()` and understand how it performs element-wise division between tensors.

Part 4 - Python code skeleton

```
import torch

def E03(x,y):
    z = torch...
    return z

def main():
    x = torch.tensor...
    y = torch.tensor...
    print(E03(...))

if __name__ == "__main__":
    main()
```

1.2.5 Exercise 04 - Tensor Power Law

Part 0 - The Magnitude

In artificial intelligence, knowing whether a value is positive or negative is often less important than understanding its magnitude relative to zero. Magnitude represents the size or intensity of an error or deviation without regard to direction, since a model must measure how large an error is rather than where it lies.

For this reason, power-based operations are used to represent magnitude in a numerically stable form, enabling reliable measurement and optimization of error and deviation.

Part 1 - why this matters

In artificial intelligence, it is essential to distinguish between the influence of a signal controlled by weights and the magnitude of a value measured independently of its direction.

The Tensor Power Law is used to convert values into measurable magnitudes, enabling stable numerical evaluation of error and deviation.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Power Law (element-wise) is defined as:

$$\circledast : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \circledast 2$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = (X_{i,j})^2 \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, tensor power (element-wise exponentiation) can be performed using the built-in operator `x ** 2`.

The reader is encouraged to search for a function called `torch.pow()` and understand how it applies element-wise power operations between tensors or between a tensor and a scalar.

Part 4 - Python code skeleton

```
import torch

def E04(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E04(...))

if __name__ == "__main__":
    main()
```

1.2.6 Exercise 05 - Tensor Square Root Law

Part 0 - The element-wise

What "element-wise" means. An **element-wise** operation applies the same rule to each entry of a tensor *independently*, so the value at one position is computed only from the value(s) at the *same position*. As a result, the output typically has the **same shape** as the input (or the broadcasted shape if broadcasting is used).

Shape requirement (matrix × matrix). For two matrices, an element-wise binary law is defined only when the shapes match:

$$\odot : \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}, \qquad Z_{i,j} = X_{i,j} \odot Y_{i,j}.$$

Here, each output entry $Z_{i,j}$ depends only on $X_{i,j}$ and $Y_{i,j}$.

Why we use element-wise laws in AI. Element-wise laws are used to transform or scale data *without mixing information* across positions. They are essential for operations such as normalization, magnitude measurement (e.g., squaring), and pointwise non-linear transformations, where we want to adjust values while preserving the tensor structure.

Why $\mathbb{R}^{3\times3}$ and $\mathbb{R}^{3\times2}$ is not element-wise. If $\mathbf{X} \in \mathbb{R}^{3\times3}$ and $\mathbf{Y} \in \mathbb{R}^{3\times2}$, then an element-wise binary operation is **not defined** because there is no one-to-one alignment between entries:

$$\mathbb{R}^{3\times3} \not\odot \mathbb{R}^{3\times2} \quad \text{(shape mismatch)}.$$

So your intuition is correct: this is **not** an element-wise operation. However, this does **not** automatically mean it is a different "type of operation" by itself; it is simply a **shape mismatch** for element-wise binary laws. (Only if a broadcasting rule applies can some mismatched shapes still work, but 3×3 and 3×2 do not broadcast to a common shape.)

Part 1 - why this matters

In artificial intelligence, operations like squaring are used to measure intensity or error without being affected by sign, but they also change the measurement unit and push values into a much larger numerical range than the original scale. The Tensor Square Root Law restores values back to their natural scale after magnitude-based computations, making results interpretable and comparable by returning them to the same "unit" as the original signal rather than leaving them in squared units. Additionally, the square root compresses the numerical range and mitigates the growth caused by squaring and accumulation, improving tensor-level numerical stability and reducing the risk of explosion or precision loss during training.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$ with $X_{i,j} \geq 0$.

Tensor Square Root Law (element-wise) is defined as:

$$\sqrt{\cdot} : \mathbb{R}^{3\times3}_{\geq 0} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \sqrt{\mathbf{X}}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \sqrt{X_{i,j}} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, tensor square root (element-wise square root) can be performed using the built-in function `torch.sqrt()`.

The reader is encouraged to explore how `torch.sqrt()` applies the square root operation element-wise on tensors and to understand its role in restoring scale after magnitude-based operations.

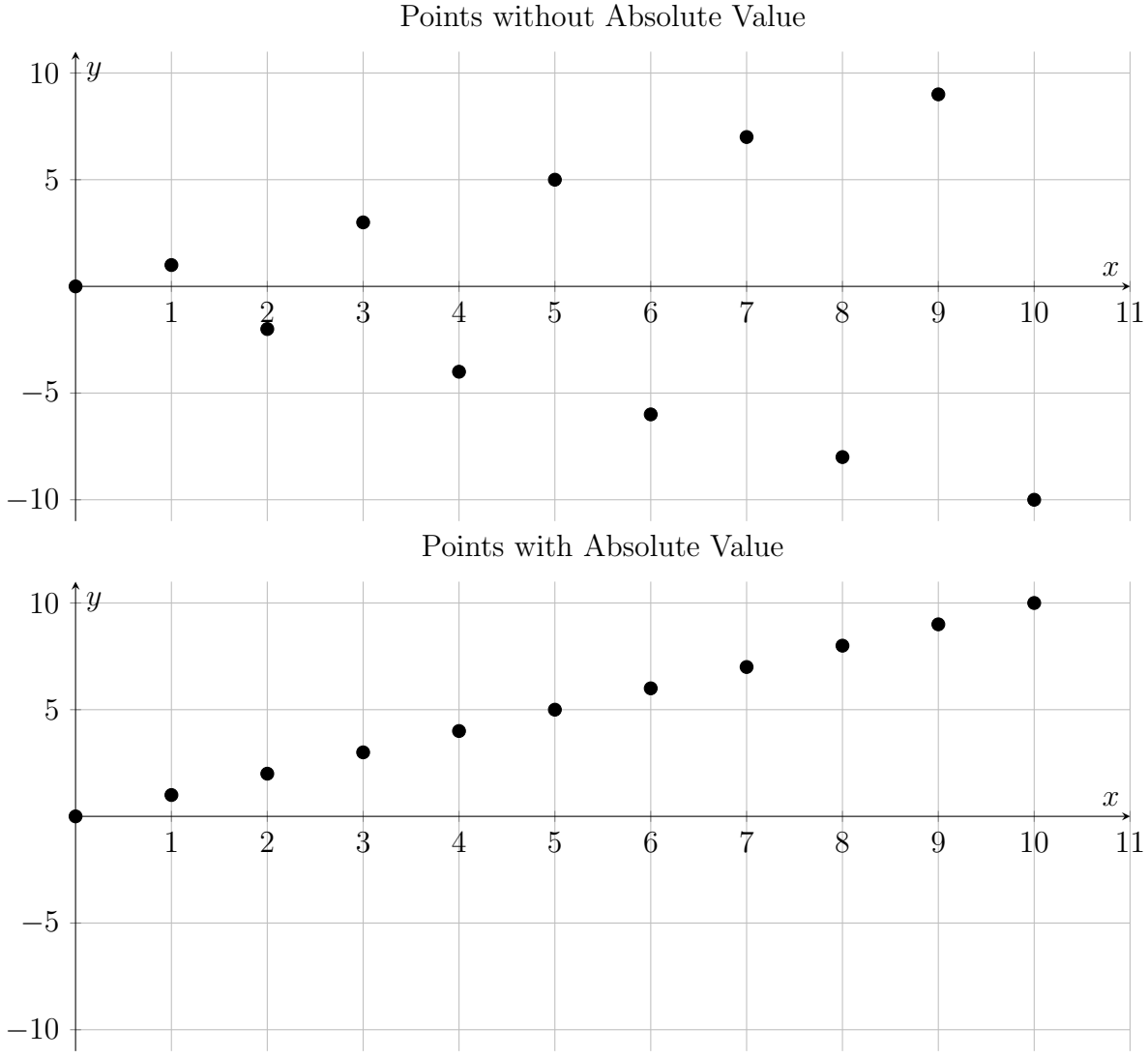
Part 4 - Python code skeleton

```
import torch

def E05(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E05(...))

if __name__ == "__main__":
    main()
```

1.2.7 Exercise 06 - Tensor Absolute Value Law

Part 1 - why this matters

In artificial intelligence equations, the goal is not always to determine the direction of an error or the sign of a value, but rather to measure its true magnitude regardless of whether it is positive or negative. Positive and negative values represent different directions around a reference point, but when evaluating model performance or computing deviation, the sign can be misleading because it allows errors to cancel each other out during aggregation. For this reason, it becomes necessary to remove the sign and convert values to non-negative quantities, so that every deviation is counted as an independent contribution reflecting its actual intensity rather than its direction. This transformation ensures that no error disappears simply because an opposite error exists.



The absolute value provides a precise and straightforward way to achieve this goal without altering the numerical scale of values or artificially amplifying them. Unlike squaring, which changes the measurement unit and exaggerates large differences, the absolute value preserves linear relationships between values and enables direct, numerically stable measurement of deviation. This is why it is widely used in artificial intelligence equations whenever a fair and

balanced assessment of error or distance is required, especially in scenarios where excessive influence from large values must be avoided or clear interpretability of results is desired. In this sense, removing the sign is not a mathematical simplification, but a deliberate choice to separate measurement from direction within the model.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Absolute Value Law (element-wise) is defined as:

$$|\cdot| : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = |\mathbf{X}|$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = |X_{i,j}| \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor absolute value (element-wise absolute value) can be computed using the built-in function `torch.abs()`.

The reader is encouraged to explore how `torch.abs()` removes the sign of each element independently and how it is commonly used in error measurement and magnitude-related computations.

Part 4 - Python code skeleton

```
import torch

def E06(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E06(...))

if __name__ == "__main__":
    main()
```

1.2.8 Exercise 07 - Tensor Negation Law

Part 0.1 - Optimization and Gradients

In artificial intelligence, the term Optimization is not used in its general linguistic sense. Instead, it refers to the controlled adjustment of a model's numerical parameters within a high-dimensional mathematical space. Researchers treat the model as a purely numerical system

governed by a single objective function, and optimization defines how this system evolves over time toward configurations that reduce error according to a predefined criterion.

From a research perspective, Optimization transforms learning into a process that can be precisely managed and analyzed. Rather than being an abstract or emergent behavior, learning becomes a sequence of small, measurable parameter updates. This framing allows researchers to design learning algorithms whose behavior can be studied, compared, and refined, making model training both predictable and interpretable.

Within artificial intelligence research, the Gradient is understood as a mathematical sensing mechanism rather than merely a derivative. It provides local information about how sensitive the model is to small changes in its internal parameters, revealing the directions in which the objective function changes most rapidly.

In practice, the Gradient serves as raw directional information rather than a decision rule. Researchers do not follow it blindly; instead, they manipulate its influence through additional operations such as negation, scaling, or constraint enforcement. Thus, the Gradient supplies the signal, while Optimization defines how that signal is used to guide the learning process.

Part 0.2 - Difference and Error Function

In artificial intelligence, Difference is not understood as a simple subtraction operation, but as a foundational concept that signals the existence of a gap between two states: what the model produces and what it is expected to produce. Researchers treat Difference as the first mathematical acknowledgment that the current system deviates from its objective, without yet explaining the cause of that deviation.

From a research standpoint, Difference is the most primitive representation of error before any further processing. Once the difference is formulated, it can be treated as an independent mathematical object that can be scaled, transformed, sign-inverted, or passed through additional functions. For this reason, Difference is not the end of computation but its starting point, forming the basis for all subsequent correction steps.

The Error Function represents the next conceptual layer beyond Difference. In the researcher's view, an Error Function is a deliberate definition of what constitutes error and how severely it should be penalized. It is not merely a computational formula, but a formal policy that encodes the learning objective and determines which deviations matter and which can be ignored.

At a deeper level, the Error Function governs the long-term behavior of the model. It shapes the optimization landscape by deciding how raw differences are aggregated and weighted, thereby influencing the direction and stability of learning. As a result, changing the Error Function can fundamentally alter the learning dynamics of a system, even when the data and model architecture remain unchanged.

Part 0.3 - Signal directions and Opposition

In artificial intelligence, the direction of a signal is as important as its numerical magnitude. Researchers treat each signal as a force that pushes the model in a specific direction within the

solution space. The same numerical value can be beneficial or harmful depending on its sign, which means that model behavior cannot be understood by magnitude alone without considering direction.

From a training perspective, signal direction determines whether a proposed change moves the model closer to its objective or farther away from it. For this reason, many learning procedures prioritize controlling the direction of change before adjusting its strength. An incorrect direction, even if small, can accumulate over time and lead to significant divergence during training.

The concept of Opposition in artificial intelligence does not imply rejection, but rather a deliberate decision to act against a given signal direction. Researchers recognize that certain signals may carry valid information about the system while still leading to undesirable behavior if followed directly. Opposition allows this information to be used while preventing harmful movement.

At a deeper research level, opposition represents the point at which control is imposed on model behavior. Through operations such as negation, the learning process can exploit directional information without being dominated by it. This idea underlies practices like reversing the gradient during optimization and explains why tensor-level operations such as negation appear early and repeatedly in learning algorithms.

Part 1 - why this matters

The primary purpose of the Tensor Negation Law is to provide a simple and consistent way to reverse the effect of values without altering the structure or layout of the data. In many artificial intelligence systems, we do not need to introduce new information; instead, we need to express that an existing contribution should act in the opposite direction. Negation achieves this by preserving the same tensor shape and positions while inverting the sign of each value.

Tensor negation is used extensively because many core operations are based on comparison and correction. When computing differences between model outputs and target values, or when updating parameters during training, negation is required to move against an undesired direction. Without negation, it would be impossible to formally represent error, opposition, or corrective movement in a clean and structured mathematical way.

Most importantly, Tensor Negation preserves the tensor's internal structure. The dimensionality, alignment, and positional meaning of the data remain unchanged, while only the sign is inverted. This makes negation a safe and essential operation in learning systems, as it modifies numerical behavior without breaking the integrity of tensor-based representations or mixing information across locations.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Negation Law (element-wise) is defined as:

$$-: \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = -\mathbf{X}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = -\,X_{i,j} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, tensor negation (element-wise sign inversion) can be performed using the built-in function `torch.neg()`, or equivalently by applying the unary minus operator `-x`.

The reader is encouraged to explore how unary negation acts independently on each tensor entry without changing the tensor shape, and to compare the explicit use of `torch.neg()` with the operator form to understand their equivalence at the tensor level.

Part 4 - Python code skeleton

```
import torch

def E07(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E07(...))

if __name__ == "__main__":
    main()
```

1.2.9 Exercise 08 - Tensor Sign Law

Part 1 - why this matters

In artificial intelligence systems, numerical values are constantly used to control decisions, updates, and internal signals. However, not every operation requires knowing the exact magnitude of a value. In many situations, what truly matters is the *direction* of influence rather than its size. Positive and negative values indicate opposite effects within the model, and identifying this direction alone is often sufficient to guide learning or adjustment processes.

Neural networks, in particular, repeatedly update their parameters based on signals that may vary widely in scale. Large values can cause unstable behavior, while very small values may become negligible during aggregation. By extracting only the sign of a tensor, the model can ignore harmful scale differences and focus solely on whether each component pushes the system forward, backward, or not at all. This leads to more stable and interpretable behavior in parts of the learning process where controlled directional movement is preferred over exact measurement.

Mathematically, the Tensor Sign Law allows the model to separate *decision* from *measurement*. Instead of treating numbers as precise quantities, the law converts them into directional indicators. This is especially useful in optimization and control-related operations, where consistent movement in the correct direction is more important than reacting strongly to large but possibly noisy values.

The Tensor Sign Law can be expressed in its simplest form as:

$$z = \text{sign}(x)$$

where:

- x represents a real-valued scalar or a single element of a tensor.
- $\text{sign}(x)$ extracts the direction of x.
- $z = 1$ if $x > 0$, indicating a positive direction.
- $z = -1$ if $x < 0$, indicating a negative direction.
- $z = 0$ if $x = 0$, indicating no directional influence.

When applied element-wise to tensors, this operation enables neural networks to reason about directional behavior independently of magnitude, forming a foundation for stable updates, directional analysis, and simplified control mechanisms within artificial intelligence models.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Sign Law (element-wise) is defined as:

$$\text{sign}(\cdot) : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \text{sign}(\mathbf{X})$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} > 0 \\ 0 & \text{if } X_{i,j} = 0 \\ -1 & \text{if } X_{i,j} < 0 \end{cases} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor sign operation (element-wise sign extraction) can be computed using the built-in function `torch.sign()`.

The reader is encouraged to explore how `torch.sign()` separates *direction* from *magnitude*, returning only the sign of each element, and how this operation is commonly paired with absolute value or norm based laws in optimization, gradient analysis, and control of update direction.

Part 4 - Python code skeleton

```
import torch

def E08(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E08(...))

if __name__ == "__main__":
    main()
```

1.2.10 Exercise 09 - Tensor Clamp Law

Part 0 - conceptual overview

In a neural network, all numerical behavior revolves around a small set of interconnected quantities: weights, activations, scores, and gradients. These elements do not exist independently; instead, they form a continuous feedback loop that drives learning and decision-making within the model.

Weights represent the internal parameters of the network and determine how input information is transformed. Activations are the intermediate signals produced when inputs are combined with weights and passed through nonlinear functions. Scores are the final numerical outputs of the network before interpretation, often representing confidence, preference, or likelihood. Together, weights and activations shape the scores produced by the model.

Gradients complete the loop by measuring how changes in weights affect the final scores and error. They flow backward through the network and guide how weights should be adjusted. Because activations, scores, and gradients can grow unbounded or become unstable, controlling their numerical range becomes essential. Understanding this relationship explains why mechanisms such as clamping are needed to keep all components operating within safe and meaningful limits.

Part 1 - why this matters

Artificial intelligence systems continuously generate numerical values while processing data, updating parameters, and producing outputs. These values are not inherently constrained and can easily grow too large or become too small during training or inference. When such uncontrolled values propagate through a neural network, they may cause numerical instability, saturation, or unreliable behavior that degrades the overall performance of the model.

The Tensor Clamp Law introduces explicit boundaries that restrict values to a predefined and meaningful range. By enforcing lower and upper limits, the model is protected from extreme values that do not carry additional useful information. This controlled restriction allows learning processes to remain stable while still preserving the essential structure of the data. Instead of eliminating information, clamping preserves valid values and only corrects those that

exceed acceptable limits.

Within neural networks, clamping is commonly used to maintain safe operating ranges for activations, gradients, and outputs. It ensures that numerical values remain interpretable and physically or logically valid throughout computation. In this sense, the Tensor Clamp Law acts as a stabilizing mechanism that enables artificial intelligence systems to learn effectively without being disrupted by unbounded numerical behavior.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$ and let $a, b \in \mathbb{R}$ with $a \leq b$.
Tensor Clamp Law (element-wise) is defined as:

$$\text{clamp}_{[a,b]}(\cdot) : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \text{clamp}_{[a,b]}(\mathbf{X})$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} a & \text{if } X_{i,j} < a \\ X_{i,j} & \text{if } a \leq X_{i,j} \leq b \\ b & \text{if } X_{i,j} > b \end{cases} \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor clamp operation (element-wise value limiting) can be computed using the built-in function `torch.clamp()`.

The reader is encouraged to explore how `torch.clamp()` enforces lower and upper bounds on each tensor element independently, preventing values from exceeding predefined limits and ensuring numerical stability during computation.

Part 4 - Python code skeleton

```
import torch

def E09(x, m, M):
    z = torch...
    return z

def main():
    x = torch.tensor...
    m =...
    M =...
    print(E09(...))

if __name__ == "__main__":
    main()
```

1.2.11 Exercise 10 - Tensor Round Law

Part 1 - why this matters

Neural networks operate primarily in a continuous numerical space, where values such as weights, activations, and scores can take any real number. This continuous representation is essential for learning and optimization, but it does not always align with the requirements of decision-making or real-world execution. In many situations, a model must eventually produce discrete, integer-based outcomes rather than continuous values.

The Tensor Round Law provides a controlled way to convert continuous values into stable integer representations. By rounding values to the nearest integer, small numerical fluctuations are removed without introducing large distortions. This is particularly important when values are close to decision boundaries, where minor changes could otherwise cause inconsistent or unstable behavior.

Within artificial intelligence systems, rounding is commonly used when transitioning from internal numerical processing to final outputs, counters, or index-based operations. It enables models to move from soft, approximate representations to clear and interpretable decisions, ensuring that continuous learning processes can be safely integrated with discrete system requirements.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Round Law (element-wise) is defined as:

$$\text{round}(\cdot) : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \text{round}(\mathbf{X})$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \text{round}(X_{i,j}) \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor rounding operation (element-wise rounding to the nearest integer) can be computed using the built-in function `torch.round()`.

The reader is encouraged to explore how `torch.round()` converts continuous values into discrete ones by rounding each element independently, and how this operation is commonly used when transitioning from continuous representations to discrete decisions or stabilized numerical outputs.

Part 4 - Python code skeleton

```
import torch

def E10(x):
```

```
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E10(...))

if __name__ == "__main__":
    main()
```

1.2.12 Exercise 11 - Tensor Floor Law

Part 1 - why this matters

Artificial intelligence systems often work with continuous numerical values, yet many decisions and operations within these systems require discrete and strictly bounded outcomes. In such cases, fractional values cannot be interpreted directly, and a clear rule is needed to convert continuous quantities into valid integers without violating system constraints.

The Tensor Floor Law provides a conservative transformation by mapping each value to the largest integer that is less than or equal to it. This guarantees that the result never exceeds the original value, making it suitable for situations where upper limits must be respected. By consistently rounding downward, the operation prevents unintended overestimation that could lead to invalid decisions or resource misuse.

Within artificial intelligence pipelines, floor-based transformations are used when values represent counts, indices, or execution steps that must remain within safe and allowable bounds. The Tensor Floor Law ensures that continuous numerical outputs can be safely translated into discrete, actionable values while preserving logical consistency throughout the system.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Floor Law (element-wise) is defined as:

$$\lfloor \cdot \rfloor : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \lfloor \mathbf{X} \rfloor$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \lfloor X_{i,j} \rfloor \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor floor operation (element-wise rounding down to the nearest integer) can be computed using the built-in function `torch.floor()`.

The reader is encouraged to explore how `torch.floor()` consistently rounds values downward regardless of their fractional part, and how this operation is commonly used when enforcing conservative or lower-bound numerical decisions in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E11(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E11(...))

if __name__ == "__main__":
    main()
```

1.2.13 Exercise 12 - Tensor Ceil Law

Part 0 - Conservation and Safeguarding

Artificial intelligence systems are fundamentally numerical systems that rely on continuous values to represent uncertainty, estimates, and learned behavior. However, these systems do not operate in isolation; they must eventually interact with finite computational resources, discrete execution steps, and real-world constraints. As a result, numerical decisions in artificial intelligence are not only mathematical choices but also design decisions that reflect how the system handles risk, error, and failure.

Two fundamental design philosophies emerge when continuous values must be converted into discrete decisions: *conservation* and *safeguarding*. These philosophies define how an artificial intelligence system reacts when faced with uncertainty in numerical estimation. Rather than optimizing for mathematical elegance, they optimize for system reliability and practical correctness.

Conservation represents a cautious approach that prioritizes efficiency and resource control. Under this philosophy, the system avoids allocating or executing more than what is strictly justified by the current numerical estimate. The underlying assumption is that overestimation is more harmful than underestimation. Conservation is therefore used when excessive allocation, unnecessary computation, or inflated execution can degrade performance, increase cost, or destabilize the system.

Safeguarding represents a complementary but fundamentally different philosophy. Instead of minimizing usage, it prioritizes completeness and correctness. Under this approach, the system ensures that requirements are fully met, even if this means allocating slightly more resources or executing additional steps. The core assumption of safeguarding is that underestimation is more dangerous than overestimation, especially when missing data, incomplete processing, or early termination can lead to silent failures.

In artificial intelligence development, neither philosophy is universally superior. Each reflects a different tolerance for risk. Conservation accepts the possibility of doing less in exchange for efficiency, while safeguarding accepts controlled excess in exchange for reliability. The choice between them is not a mathematical preference but a system-level judgment about which type

of failure is unacceptable in a given context.

These philosophies directly influence how numerical transformations are applied within neural networks and AI pipelines. When values represent optional capacity, iterative refinement, or expandable processes, conservation-oriented decisions are often appropriate. When values represent mandatory coverage, irreversible decisions, or completeness requirements, safeguarding-oriented decisions become essential.

Understanding conservation and safeguarding as design principles allows AI developers to reason more clearly about numerical transformations such as floor and ceiling operations. Rather than viewing these operations as simple rounding rules, they can be understood as explicit expressions of system intent, encoding how the model chooses to balance efficiency against reliability within its computational logic.

Part 1 - why this matters

Artificial intelligence systems frequently work with continuous numerical values that represent estimated quantities such as required steps, resource usage, or data coverage. However, many execution-level decisions within these systems cannot operate on fractional values and must rely on integer-based quantities that fully satisfy the underlying requirement.

The Tensor Ceil Law provides a safe transformation by mapping each value to the smallest integer greater than or equal to it. This guarantees that the resulting value never underestimates the original quantity. By enforcing upward rounding, the model avoids incomplete processing, missed data, or insufficient allocation that could otherwise occur when fractional estimates are truncated.

Within artificial intelligence pipelines, ceiling-based transformations are used when full coverage and completeness are more important than minimizing numerical overhead. The Tensor Ceil Law ensures that continuous estimates are converted into discrete decisions that meet or exceed required thresholds, enabling reliable and robust system behavior.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$.

Tensor Ceil Law (element-wise) is defined as:

$$\lceil \cdot \rceil : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \lceil \mathbf{X} \rceil$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \lceil X_{i,j} \rceil \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor ceiling operation (element-wise rounding up to the nearest integer) can be computed using the built-in function `torch.ceil()`.

The reader is encouraged to explore how `torch.ceil()` consistently maps each value to the smallest integer greater than or equal to it, and how this operation is used when enforcing upward-safe or minimum-satisfying numerical decisions in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E12(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E12(...))

if __name__ == "__main__":
    main()
```

1.2.14 Exercise 13 - Tensor Reciprocal Law

Part 1 - why this matters

Artificial intelligence systems frequently rely on numerical values that represent relative importance, distance, confidence, or influence. In many such cases, a larger numerical value does not necessarily imply a stronger or more desirable effect. Instead, certain relationships require an inverse interpretation, where smaller values should exert greater influence and larger values should contribute less to the final outcome.

The Tensor Reciprocal Law enables this inverse relationship by transforming each value into its multiplicative inverse. Through this transformation, large values are attenuated while small values are amplified, allowing the model to rebalance the contribution of different elements. This is particularly useful when values encode penalties, distances, or uncertainty measures, where dominance by large magnitudes would otherwise distort the model's behavior.

Within artificial intelligence pipelines, reciprocal-based transformations are used to regulate influence, normalize contributions, and prevent disproportionate effects from dominating learning or decision processes. By explicitly reversing the numerical relationship, the Tensor Reciprocal Law provides a principled way to express inverse dependence, enabling more stable and meaningful interactions between numerical components in neural networks and related systems.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$ such that $X_{i,j} \neq 0$ for all $i, j \in \{1, 2, 3\}$.
Tensor Reciprocal Law (element-wise) is defined as:

$$(\cdot)^{-1} : \mathbb{R}^{3\times3} \setminus \{0\} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X}^{-1}$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \frac{1}{X_{i,j}} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor reciprocal operation (element-wise inversion) can be computed using the built-in function `torch.reciprocal()`.

The reader is encouraged to explore how `torch.reciprocal()` computes the inverse of each element independently, and why special care must be taken to avoid zero values, as division by zero leads to undefined or unstable numerical behavior in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E13(x):
    z = torch...
    return z

def main():
    x = torch.tensor...
    print(E13(...))

if __name__ == "__main__":
    main()
```

1.2.15 Exercise 14 - Tensor Modulo Law

Part 1 - why this matters

Artificial intelligence systems often manipulate numerical values that grow unbounded through accumulation, iteration, or repeated updates. While such growth is mathematically valid, it is not always meaningful for the behavior the model is intended to represent. In many scenarios, the absolute size of a value is less important than its position within a fixed and repeating range.

The Tensor Modulo Law provides a principled way to map unbounded values back into a bounded, periodic domain. By computing the remainder of a division, the operation preserves the relative position of a value within a cycle while discarding the number of completed cycles. This allows models to reason about states, phases, or positions without being affected by uncontrolled numerical growth.

Within artificial intelligence pipelines, modulo-based transformations are used whenever periodic structure, bounded indexing, or cyclic behavior is required. They enable models to operate within finite domains, prevent numerical overflow, and encode repeating patterns directly into mathematical formulations. In this sense, the Tensor Modulo Law is not merely a

numerical convenience, but a formal mechanism for expressing periodicity and controlled value spaces in artificial intelligence systems.

Part 2 - Mathematical form

Let $\mathbf{X} \in \mathbb{R}^{3\times3}$ and let $m \in \mathbb{R}$ with $m \neq 0$.
Tensor Modulo Law (element-wise) is defined as:

$$\mathrm{mod}_m(\cdot) : \mathbb{R}^{3\times3} \to \mathbb{R}^{3\times3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \bmod m$$

where $\mathbf{Z} \in \mathbb{R}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = X_{i,j} \bmod m \quad \text{for } i, j \in \{1, 2, 3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor modulo operation (element-wise remainder computation) can be performed using the built-in function `torch.remainder()`.

The reader is encouraged to explore how `torch.remainder()` computes the remainder of each tensor element with respect to a given modulus, and how this operation is commonly used in periodic behavior, index wrapping, and bounded numerical representations within artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E14(x, m):
    z = torch...
    return z

def main():
    x = torch.tensor...
    m =...
    print(E14(...))

if __name__ == "__main__":
    main()
```

1.2.16 Exercise 15 - Tensor Logical AND Law

Part 1 - why this matters

Artificial intelligence systems frequently make decisions based on multiple conditions rather than a single numerical value. In many scenarios, an action, update, or computation should only occur when several requirements are satisfied simultaneously. This necessity cannot be

expressed through standard arithmetic operations alone and requires an explicit logical mechanism.

The Tensor Logical AND Law provides a formal way to encode simultaneous validity within mathematical expressions. By combining multiple boolean conditions into a single logical outcome, the operation ensures that a result is considered valid only when all contributing conditions are true. This allows decision rules to be represented directly inside tensor-based computations, rather than being handled externally through procedural logic.

Within artificial intelligence pipelines, logical AND operations are widely used to construct masks, enforce constraints, and control conditional execution. They enable models to selectively activate or deactivate elements based on multiple criteria at once, ensuring consistency and correctness in complex decision processes. In this sense, the Tensor Logical AND Law serves as a foundational tool for integrating logical reasoning into numerical and tensor-based AI systems.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \{0,1\}^{3\times 3}$.

Tensor Logical AND Law (element-wise) is defined as:

$$\wedge : \{0,1\}^{3\times 3} \times \{0,1\}^{3\times 3} \to \{0,1\}^{3\times 3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \wedge \mathbf{Y}$$

where $\mathbf{Z} \in \{0,1\}^{3\times 3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} = 1 \text{ and } Y_{i,j} = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor logical AND operation (element-wise logical conjunction) can be computed using the built-in function `torch.logical_and()`.

The reader is encouraged to explore how `torch.logical_and()` evaluates each pair of tensor elements independently, and how logical conjunction is used to enforce simultaneous conditions, masks, and rule-based constraints in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E15(X, Y):
    z = torch...
    return z

def main():
    X = torch.tensor...
    Y = torch.tenosr...
```

```
    print(E15(...))

if __name__ == "__main__":
    main()
```

1.2.17 Exercise 16 - Tensor Logical OR Law

Part 1 - why this matters

Artificial intelligence systems often operate in environments where information is incomplete, uncertain, or noisy. In such contexts, requiring all conditions to be satisfied simultaneously can make decision processes overly rigid and fragile. Instead, many AI systems must allow actions or conclusions to be validated through alternative acceptable conditions.

The Tensor Logical OR Law provides a formal mechanism for expressing such flexibility within mathematical formulations. By evaluating multiple conditions and accepting a result when at least one condition is true, the operation enables models to continue functioning even when some inputs or criteria fail. This supports robustness and adaptability in complex or imperfect environments.

Within artificial intelligence pipelines, logical OR operations are commonly used to construct fallback rules, permissive masks, and alternative decision paths. They allow models to represent "either–or" logic directly within tensor-based computations, ensuring that decision-making remains resilient without sacrificing logical clarity.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \{0,1\}^{3\times 3}$.

Tensor Logical OR Law (element-wise) is defined as:

$$\vee : \{0,1\}^{3\times 3} \times \{0,1\}^{3\times 3} \to \{0,1\}^{3\times 3}$$

Define:

$$\mathbf{Z} = \mathbf{X} \vee \mathbf{Y}$$

where $\mathbf{Z} \in \{0,1\}^{3\times 3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} = 1 \text{ or } Y_{i,j} = 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor logical OR operation (element-wise logical disjunction) can be computed using the built-in function `torch.logical_or()`.

The reader is encouraged to explore how `torch.logical_or()` evaluates each pair of tensor elements independently, and how logical disjunction is used to represent alternative conditions, fallback rules, and flexible masking in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E16(X, Y):
    z = torch...
    return z

def main():
    X = torch.tensor...
    Y = torch.tenosr...
    print(E16(...))

if __name__ == "__main__":
    main()
```

1.2.18 Exercise 17 - Tensor Logical NOT Law

Part 0 - combined logical reasoning

Artificial intelligence systems rarely make decisions based on a single condition. Instead, real-world decision-making requires evaluating multiple constraints, allowing alternatives, and explicitly excluding invalid states. Logical operators such as AND, OR, and NOT each capture a distinct aspect of this process, but none of them alone is sufficient to express complete decision logic.

Combining logical AND, OR, and NOT into a single mathematical expression allows an AI system to represent full decision policies rather than isolated rules. Logical AND enforces simultaneous validity, ensuring that all required conditions are satisfied. Logical OR introduces flexibility by allowing multiple acceptable paths to a valid outcome. Logical NOT provides exclusion by explicitly rejecting states or conditions that must not occur. Together, these operators form a complete logical language for decision control.

The benefit of combining these operators lies in clarity and correctness. A single unified logical equation explicitly defines when an action is permitted, when it is rejected, and when alternative conditions are acceptable. This eliminates hidden assumptions, scattered conditional logic, and ambiguous decision pathways that often arise when logic is implemented procedurally rather than mathematically.

In artificial intelligence pipelines, such combined logical expressions are commonly used to construct complex masks, enforce hierarchical constraints, and control conditional execution within tensor-based computations. By embedding logical structure directly into mathematical formulations, models can apply decision rules efficiently and consistently across large-scale data structures.

Moreover, representing logic as a single equation enables analysis and verification. The combined expression can be simplified, tested for completeness, and examined for edge cases. This makes the decision process more transparent and robust, which is essential when AI systems operate in uncertain or safety-critical environments.

Ultimately, the combination of AND, OR, and NOT transforms logical reasoning from a set of disconnected checks into a coherent decision framework. Rather than asking isolated questions such as "is this condition true?", the system evaluates a complete policy that defines acceptance, rejection, and permissible alternatives in a unified and mathematically rigorous manner.

Part 1 - why this matters

Artificial intelligence systems frequently rely on boolean conditions to control execution, enforce constraints, and construct decision logic. In many situations, it is not sufficient to check whether a condition is true; the system must also reason about when a condition is explicitly false. This requires a formal way to invert logical states within mathematical expressions.

The Tensor Logical NOT Law provides a precise mechanism for logical negation, allowing models to reverse boolean values in a consistent and interpretable manner. By transforming true conditions into false ones and vice versa, the operation enables artificial intelligence systems to express exclusion, invalidation, and complementary conditions directly within tensor computations.

Within artificial intelligence pipelines, logical NOT operations are commonly used to deactivate masks, exclude invalid states, and construct complementary rules. They allow models to reason not only about what is permitted, but also about what must be explicitly rejected, ensuring clarity and completeness in logical decision-making processes.

Part 2 - Mathematical form

Let $\mathbf{X} \in \{0,1\}^{3\times 3}$.

Tensor Logical NOT Law (element-wise) is defined as:

$$\neg : \{0,1\}^{3\times 3} \to \{0,1\}^{3\times 3}$$

Define:

$$\mathbf{Z} = \neg \mathbf{X}$$

where $\mathbf{Z} \in \{0,1\}^{3\times 3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} = 0 \\ 0 & \text{if } X_{i,j} = 1 \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor logical NOT operation (element-wise logical negation) can be computed using the built-in function `torch.logical_not()`.

The reader is encouraged to explore how `torch.logical_not()` inverts each boolean tensor element independently, and how logical negation is used to exclude conditions, reverse masks, and express complementary constraints in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E17(X):
    z = torch...
    return z

def main():
    X = torch.tensor...
    print(E17(...))

if __name__ == "__main__":
    main()
```

1.2.19 Exercise 18 - Tensor Equality Law (Comparison)

Part 0 - comparison vs logical operations

Artificial intelligence systems rely on both numerical evaluation and logical decision-making. Although comparison operations and logical operations may appear similar at a surface level, they serve fundamentally different roles within AI pipelines. Understanding the distinction between them is essential for designing clear, correct, and interpretable decision mechanisms.

Comparison operations are concerned with evaluating relationships between values. They answer questions such as whether two values are equal, or whether one value is greater or smaller than another. The result of a comparison is a boolean outcome derived directly from numerical properties. In this sense, comparison acts as a bridge between continuous numerical representations and discrete decision boundaries.

Logical operations, on the other hand, do not evaluate numerical relationships. Instead, they operate on boolean conditions that already represent decisions, validity, or state membership. Logical AND, OR, and NOT combine, relax, or invert these boolean states to construct higher-level decision logic. Their purpose is not to measure or compare, but to control how conditions interact.

In artificial intelligence workflows, comparison operations typically appear first. Numerical outputs from models are compared against thresholds, targets, or reference values to produce boolean indicators. These indicators alone are often insufficient to represent complex decision policies, as real-world decisions rarely depend on a single condition.

Logical operations are then applied to the results of comparisons. They combine multiple boolean signals, enforce constraints, allow alternatives, or exclude invalid states. This separation of roles ensures clarity: comparisons define elementary conditions, while logical operators define how those conditions govern behavior.

Treating comparison and logical operations as distinct stages prevents conceptual confusion and improves system design. Comparisons define "what is true" at the numerical level, while logical operations define "what should be done" at the decision level. Together, they form a structured pathway from raw numerical outputs to coherent and controllable artificial intelli-

gence behavior.

Part 1 - why this matters

Artificial intelligence systems frequently need to determine whether two values represent the same state, outcome, or decision rather than measuring how far apart they are. In such situations, numerical distance is less relevant than exact agreement, and a clear boundary must be drawn between matching and non-matching values.

The Tensor Equality Law provides a formal mechanism for performing exact comparisons within tensor-based computations. By converting element-wise comparisons into boolean outcomes, the operation allows models to identify matching conditions, validate predictions, and construct decision masks that separate valid states from invalid ones.

Within artificial intelligence pipelines, equality comparisons are essential for evaluation, rule enforcement, and conditional execution. They enable systems to verify correctness, trigger specific actions, and control subsequent processing steps based on precise matching criteria. In this sense, the Tensor Equality Law serves as a foundational tool for transforming numerical outputs into explicit logical decisions.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Equality Law (element-wise comparison) is defined as:

$$=: \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \{0,1\}^{3\times3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} = \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} = Y_{i,j} \\ 0 & \text{if } X_{i,j} \neq Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor equality comparison (element-wise equality check) can be performed using the built-in function `torch.eq()`.

The reader is encouraged to explore how `torch.eq()` compares each pair of tensor elements independently and produces a boolean tensor that represents exact equality, which is commonly used for masking, condition checking, and rule-based logic in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E18(X, Y):
    z = torch...
```

```
    return z

def main():
    X = torch...
    Y = torch...
    print(E18(...))

if __name__ == "__main__":
    main()
```

1.2.20 Exercise 19 - Tensor Not Equal Law (Comparison)

Part 1 - why this matters

Artificial intelligence systems must not only confirm correct or expected states, but also detect when deviations occur. In many scenarios, meaningful actions are triggered not by agreement, but by difference. Identifying mismatches between values is therefore essential for monitoring, correction, and adaptive behavior.

The Tensor Not Equal Law provides a formal mechanism for detecting discrepancies within tensor-based computations. By producing a boolean outcome for each comparison, the operation highlights where values diverge from references, targets, or prior states. This allows models to isolate irregularities and focus attention on elements that require intervention.

Within artificial intelligence pipelines, not-equal comparisons are commonly used for error detection, change tracking, and conditional filtering. They enable systems to respond explicitly to unexpected outcomes rather than silently accepting them. In this sense, the Tensor Not Equal Law transforms numerical differences into actionable signals that guide correction and control within AI systems.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Not Equal Law (element-wise comparison) is defined as:

$$\neq: \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \{0,1\}^{3\times3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} \neq \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} \neq Y_{i,j} \\ 0 & \text{if } X_{i,j} = Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor not-equal comparison (element-wise inequality check) can be computed using the built-in function `torch.ne()`.

The reader is encouraged to explore how `torch.ne()` compares each pair of tensor elements independently and produces a boolean tensor that highlights mismatched values, which is commonly used for masking, filtering, and error detection in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E19(X, Y):
    z = torch...
    return z

def main():
    X = torch...
    Y = torch...
    print(E19(...))

if __name__ == "__main__":
    main()
```

1.2.21 Exercise 20 - Tensor Greater Than Law (Comparison)

Part 1 - why this matters

Artificial intelligence systems often rely on threshold-based decisions rather than exact matches. In many cases, the precise value of a quantity is less important than whether it exceeds a certain level. Such comparisons are fundamental when determining activation, eligibility, or dominance within a model.

The Tensor Greater Than Law enables artificial intelligence systems to identify values that surpass reference points or constraints. By converting numerical comparisons into boolean outcomes, the operation allows models to distinguish significant signals from insignificant ones and to trigger decisions based on exceeding predefined criteria.

Within artificial intelligence pipelines, greater-than comparisons are commonly used to enforce thresholds, activate conditional logic, and control flow based on magnitude. They provide a clear and interpretable boundary between acceptable and insufficient values, making them essential for decision-oriented numerical processing.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times 3}$.

Tensor Greater Than Law (element-wise comparison) is defined as:

$$>: \mathbb{R}^{3\times 3} \times \mathbb{R}^{3\times 3} \to \{0,1\}^{3\times 3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} > \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times 3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} > Y_{i,j} \\ 0 & \text{if } X_{i,j} \leq Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor greater-than comparison (element-wise comparison) can be computed using the built-in function `torch.gt()`.

The reader is encouraged to explore how `torch.gt()` compares tensor elements independently and produces a boolean tensor that identifies values exceeding a given reference, which is commonly used for thresholding, masking, and conditional execution in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E20(X, Y):
    z = torch...
    return z

def main():
    X = torch...
    Y = torch...
    print(E20(...))

if __name__ == "__main__":
    main()
```

1.2.22 Exercise 21 - Tensor Less Than Law (Comparison)

Part 1 - why this matters

Artificial intelligence systems frequently make decisions based on identifying insufficient, weak, or sub-threshold values rather than dominant ones. In many scenarios, recognizing when a value falls below a certain level is essential for filtering, suppression, or corrective action within a model.

The Tensor Less Than Law allows artificial intelligence systems to detect values that do not meet required criteria. By converting numerical comparisons into boolean outcomes, the operation enables models to isolate underperforming signals, enforce lower-bound constraints, and control behavior based on insufficiency rather than excess.

Within artificial intelligence pipelines, less-than comparisons are commonly used to deactivate elements, identify low-confidence regions, and regulate conditional execution. They provide a clear mechanism for detecting when values fail to reach necessary thresholds, supporting stable and controlled decision-making.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Less Than Law (element-wise comparison) is defined as:

$$<: \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \{0,1\}^{3\times3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} < \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} < Y_{i,j} \\ 0 & \text{if } X_{i,j} \geq Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor less-than comparison (element-wise comparison) can be computed using the built-in function `torch.lt()`.

The reader is encouraged to explore how `torch.lt()` evaluates each tensor element independently and produces a boolean tensor that highlights values below a given reference, which is commonly used for thresholding, masking, and conditional suppression in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E21(X, Y):
    z = torch...
    return z

def main():
    X = torch...
    Y = torch...
    print(E21(...))

if __name__ == "__main__":
    main()
```

1.2.23 Exercise 22 - Tensor Greater or Equal Law (Comparison)

Part 1 - why this matters

Artificial intelligence systems often rely on inclusive threshold decisions, where meeting a requirement is sufficient even if the value does not exceed it. In many practical scenarios, equality with a boundary condition is just as valid as surpassing it. Ignoring this distinction can lead to unnecessary rejection of valid states or premature suppression of meaningful signals.

The Tensor Greater or Equal Law allows artificial intelligence systems to express inclusive dominance by treating equality and superiority as acceptable outcomes. By converting numerical comparisons into boolean results, the operation ensures that values meeting or exceeding

a reference are treated consistently within decision logic.

Within artificial intelligence pipelines, greater-or-equal comparisons are commonly used to enforce minimum requirements, validate eligibility, and activate conditional execution when values reach acceptable thresholds. This inclusive comparison supports stable decision boundaries and prevents unintended exclusion at critical limits.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Greater or Equal Law (element-wise comparison) is defined as:

$$\geq: \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \{0,1\}^{3\times3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} \geq \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} \geq Y_{i,j} \\ 0 & \text{if } X_{i,j} < Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor greater-or-equal comparison (element-wise comparison) can be computed using the built-in function `torch.ge()`.

The reader is encouraged to explore how `torch.ge()` evaluates tensor elements independently and produces a boolean tensor that identifies values meeting or exceeding a given reference, which is commonly used for thresholding, masking, and inclusive condition enforcement in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E22(X, Y):
    z = torch...
    return z

def main():
    X = torch...
    Y = torch...
    print(E22(...))

if __name__ == "__main__":
    main()
```

1.2.24 Exercise 23 - Tensor Less or Equal Law (Comparison)

Part 1 - why this matters

Artificial intelligence systems often require inclusive lower-bound decisions, where values are considered acceptable not only when they are strictly smaller than a reference, but also when they exactly meet it. In many practical settings, reaching a boundary condition is sufficient and should not be treated as failure.

The Tensor Less or Equal Law allows artificial intelligence systems to express inclusive insufficiency by treating equality and inferiority as valid outcomes. By converting numerical comparisons into boolean results, the operation ensures that values below or equal to a reference are handled consistently within decision logic.

Within artificial intelligence pipelines, less-or-equal comparisons are commonly used to enforce upper limits, suppress excessive values, and validate constraints that allow boundary equality. This inclusive comparison supports stable control mechanisms and prevents unnecessary rejection at critical thresholds.

Part 2 - Mathematical form

Let $\mathbf{X}, \mathbf{Y} \in \mathbb{R}^{3\times3}$.

Tensor Less or Equal Law (element-wise comparison) is defined as:

$$\leq: \mathbb{R}^{3\times3} \times \mathbb{R}^{3\times3} \to \{0,1\}^{3\times3}$$

Define:

$$\mathbf{Z} = (\mathbf{X} \leq \mathbf{Y})$$

where $\mathbf{Z} \in \{0,1\}^{3\times3}$ and the law is defined component-wise by:

$$Z_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} \leq Y_{i,j} \\ 0 & \text{if } X_{i,j} > Y_{i,j} \end{cases} \quad \text{for } i,j \in \{1,2,3\}.$$

Part 3 - Hints and Helper

In PyTorch, the tensor less-or-equal comparison (element-wise comparison) can be computed using the built-in function `torch.le()`.

The reader is encouraged to explore how `torch.le()` evaluates tensor elements independently and produces a boolean tensor that identifies values below or equal to a given reference, which is commonly used for constraint enforcement, masking, and boundary-aware decision logic in artificial intelligence systems.

Part 4 - Python code skeleton

```
import torch

def E23(X, Y):
    z = torch...
```

```
        return z

    def main():
        X = torch...
        Y = torch...
        print(E23(...))

    if __name__ == "__main__":
        main()
```

1.3 T00 - Vector Laws (1D)

2

Conclusion of the Volume of Books

2.1 What next

The real goal is to delve deeper into artificial intelligence, machine learning, deep learning, and perhaps even self-programming in the future. Explore the philosophy of building mathematical models and move away from the notion that artificial intelligence will become uncontrollable. Research seeks solutions and development, not media attention. Focus on your research in artificial intelligence and create value in this world. What you've learned in these books is just a drop in the ocean. Artificial intelligence is like a black hole; once you get close to it, you'll be fascinated. And the strange thing is, you won't know if it has an end. Build your future with your own hands. Place yourself among the researchers. You are responsible for developing artificial intelligence. Be a role model for others.

2.2 Does this volume of books have an end

Yes, it has an end, and that's when the author dies. What I do is compile what I've learned throughout my life. Emirati society is my responsibility; I try to build a bright future for it, placing it among the ranks of the developed world. I believe we must expand our knowledge and compete with the rest of the world. The future must be bright, and bright, for the modern United Arab Emirates.