Name: Abdullah Ikram

PW Skills → Assignment - 12

Recurrence Relations

1.] Find the value of T(2) for the recurrence relation:

$T(n) = 3T(n-1) + 12n$, given that $T(0) = 5$

$T(1) = 3T(0) + 12 = 15 + 12 = 27$

$T(2) = 3T(1) + 24 = (3)(27) + 24 = 81 + 24 = 105$

$\Rightarrow \boxed{T(2) = 105}$ ans.

2.] Given a recurrence relation, solve it using the substitution method:

(a) $T(n) = T(n-1) + c$, if $n <= 1 \rightarrow T(n) = 1$

if $n > 1 \rightarrow T(n) = T(n-1) + c$

$T(n) = T(n-1) + c$

$T(n) = T(n-2) + 2c$

$T(n) = T(n-3) + 3c$

$T(n) = T(n-k) + kc$

$n - k = 1 \Rightarrow k = (n-1)$

$T(n) = T(1) + (n-1)c \Rightarrow \cancel{1 + (n-1)c} =$

$= 1 + (n-1)c \Rightarrow T(n) = O(n)$

(b) $T(n) = 2T(n/2) + n$

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T\left(\frac{n}{2}\right) + n & n > 1 \end{cases}$$

$\Rightarrow T(n) = 2T(n/2) + n$

$T(n) = 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$

$T(n) = 4T\left(\frac{n}{4}\right) + 2n$

$T(n) = 4\left(2T\left(\frac{n}{8}\right) + \frac{n}{4}\right) + 2n$

$$T(m) = 8T\left(\frac{m}{8}\right) + 3m$$

$$T(m) = 2^k\, T\left(\frac{m}{2^k}\right) + km$$

$$\frac{m}{2^k} = 1 \Rightarrow \cancel{m = 2k} \Rightarrow$$

$$\Rightarrow \cancel{m = 21} \Rightarrow m = 2^k \Rightarrow \log m = k \log 2$$

$$\Rightarrow k = \log_2^m$$

$$T(m) = 2^{\log_2^m}\, T(1) + (\log_2^m)(m) = m + m\log m$$

$$\Rightarrow \cancel{T=} \Rightarrow T(m) = O(m\log_2^m)$$

(c) $T(m) = 2T\left(\frac{m}{2}\right) + c$

$$T(m) = \begin{cases} 1 & m = 1 \\ 2T\left(\frac{m}{2}\right) + c & m > 1 \end{cases}$$

$$T(m) = 2T\left(\frac{m}{2}\right) + c$$

$$T(m) = 2\left(2T\left(\frac{m}{4}\right) + c\right) + c$$

$$T(m) = 4T\left(\frac{m}{4}\right) + 3c$$

$$T(m) = 4\left(2T\left(\frac{m}{8}\right) + c\right) + 3c$$

$$T(m) = 8T\left(\frac{m}{8}\right) + 7c$$

$$T(m) = 8\left(2T\left(\frac{m}{16}\right) + c\right) + 7c = 16T\left(\frac{m}{16}\right) + \cancel{7c}\ 15c$$

$$T(m) = 2^k\, T\left(\frac{m}{2^k}\right) + (2^k - 1)c$$

$$\frac{m}{2^K} = 1 \Rightarrow m = 2^K \Rightarrow K = \log_2^m$$

$$T(m) = 2^{\log_2^m} T(1) + (2^{\log_2^m} - 1) c$$

$$T(m) = m + cm - c \Rightarrow T(m) = m(c+1) - c$$

$$\Rightarrow T(m) = O(m)$$

(d) $T(m) = T\left(\frac{m}{2}\right) + c$

$$T(m) = \begin{cases} 1 & m = 1 \\ T\left(\frac{m}{2}\right) + c & m > 1 \end{cases}$$

$$T(m) = T\left(\frac{m}{2}\right) + c$$

$$T(m) = T\left(\frac{m}{4}\right) + 2c$$

$$T(m) = T\left(\frac{m}{8}\right) + 3c$$

$$\left.\begin{aligned} T(m) &= T\left(\frac{m}{2^k}\right) + kc \\ \frac{m}{2^k} &= 1 \Rightarrow k = \log_2^m \end{aligned}\right\} \Rightarrow T(m) = 1 + \log_2^m$$

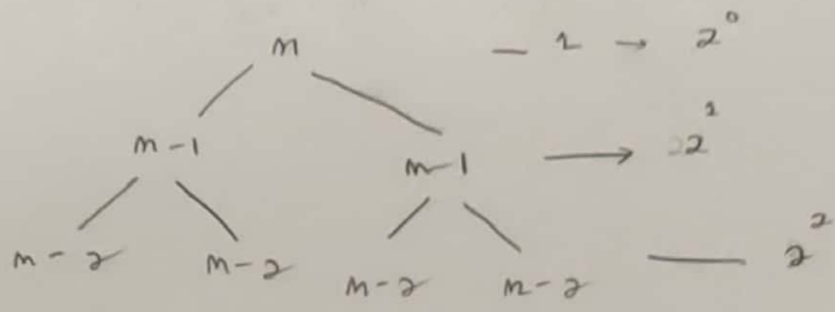
$$\Rightarrow T(m) = O(\log_2^m)$$

$\cancel{T(m) = T(\frac{m}{2}}$

3.) Given a recurrence relation, solve it using the recursive tree approach.

(a) $T(m) = 2T(m-1) + 1$

~~T(m)~~ $T(m) = \begin{cases} 1 & m = 0 \\ 2T(m-1) + 1 & m > 1 \end{cases}$



↓

K - levels

$m - K = 0 \Rightarrow K = m$

$\Rightarrow$ ~~1 + 2 + 3 ... (m-1) ⇒ O(m²)~~

$\Rightarrow$

$2^0 + 2^1 + 2^2 + \dots 2^m$ } Geometric series $\Rightarrow$ Common ratio $= r = \dfrac{2^1}{2^0}$

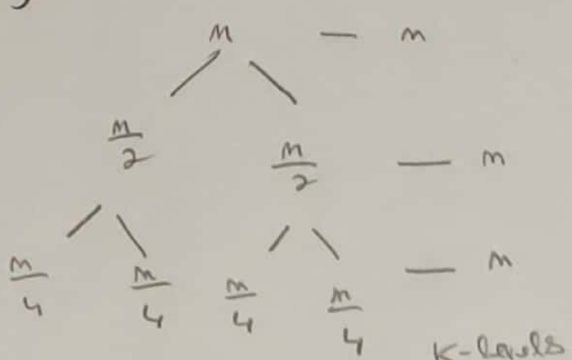
Sum of GP series : $\dfrac{a(r^m - 1)}{r-1}$, $r > 1$

$\Rightarrow \dfrac{(1)(2^m - 1)}{2-1} = O(2^m)$

$\Rightarrow T(m) = \underline{\underline{O(2^m)}}$

(b) $T(m) = 2T\left(\frac{m}{2}\right) + m$ $\qquad T(m) = \begin{cases} 2T\left(\frac{m}{2}\right) + m & m > 1 \\ 1 & m = 1 \end{cases}$

~~T(m) = 2T(~~



K-levels

$\frac{m}{2^k} \Rightarrow \frac{m}{2^k}$

$\frac{m}{2^k} = 1 \Rightarrow K = \log_2^m$

$T(m) = (m)(K) = m\log_2^m$

$\Rightarrow \boxed{T(m) = m\log_2^m}$ ans

④