Building Economic Tools with Real World Pricing Policies

A Simple Regulated Energy Market Model Formulated as a Mixed Complementarity Problem and Implemented using the Extended Mathematical Programming tool

Conventional methods used in bottom up economic models are well established tools in market research, but often fall short of representing reality. The underlying methods (linear optimization) are based on assumptions that are well founded in economic theory, but fail to capture the actual rules enforced in the market. KAPSARC has developed a growing body of literate on the application of new methods to model energy economies where such rules are incremental (Saudi Arabia, China, GCC). These papers demonstrate of how large bottom up models of integrated energy sectors, typically solved as standard optimization problems, can be generalized as equilibrium problems to represent regulation that distorts competitive prices.

One approach is to convert linear optimization models into a Mixed Complementarity Problem (MCP). A more flexible equilibrium problem is defined that can intuitively capture pricing rules that must not conform to the perfectly competitive market assumption. In this tutorial we do not explain the theory behind how an MCP is formulated, instead provide simple example of how it can be used to capture price distortions introduced by market regulation, such as fixed prices, average cost pricing, and price caps. We also show how the Extended Mathematical Programming (EMP) tool available in the General Algebraic Modeling System (GAMS) can be used to automate this process, alleviating a time consuming process that is prone to human coding error.

MCP of a simple integrated fuel and power supply market

First we will introduce a simple linear optimization model of integrated fuel and power supply. From here we derive the optimality conditions of the equivalent Linear Complementarily Problem (LCP). These conditions define the price of fuel transferred between the supplier and power producers in a perfectly competitive market. The corresponding condition is modified to capture a different pricing policy observed/enforced in the real world.

In this simple example we show how to represent a fixed or administered pricing rule. We postulate an upstream fuel supply sector selling fuel an open export market, at international prices, and to a power sector supplying a fixed demand of electricity. The power sector can invest in two types of technologies; low investment cost with higher fuel consumption and variable cost (low efficiency), and high investment cost with better efficiency and lower fixed cost.

First we define the indices and primal variables related to the supply of power and fuel.

a	index of technologies supplied by the power sector; (e.g. single or combined cycle turbines)
f	fuel supply variable
e	fuel export variable
q_a	power generation variable for technology *a*
i_a	variable for the investment in technology *a*

Next we define the fixed coefficients used to calibrate fuel and power supply activities

C	is the marginal fuel production cost
K_a	is the unit capital cost of purchasing additional a
H_a	is the heat rate of each technology (fuel consumed per unit of power produced)
D	is the fixed power demand
S	is the total available fuel
P	is the international fuel price

Our linear optimization problem is presented in equation block (1). The variable *z* represent the objective value, the total systems costs, that should be minimized to find an optimal supply of fuel and power, assuming a perfectly competitive fuel market.

Constraint (1.1) represents the fixed demand for power. (1.2) limits the production of electricity to the total available capacity. (1.3) is the fuel supply constraint. (1.4) is the fuel demand balance, the total demand for fuel from export and consumption by the power sector. In (1.4) we also introduce the dual variable π (in parentheses). We will describe the variable in more detail later, for unfamiliar readers, it represents the marginal value on the constraint, in (1.4) the value supplying an additional unit of fuel, or the competitive market price, when the market has reached equilibrium. Finally we define all the primal variables to be positive.

$$\min z(f, i_a, e) = fC + \sum_a i_a k_a - eP \quad \text{s.t.} \tag{1}$$

$$D - \sum_a q_a \leq 0 \tag{1.1}$$

$$q_a - i_a \leq 0 \tag{1.2}$$

$$f \leq S \tag{1.3}$$

$$\sum_a q_a H_a + e - f \leq 0 \qquad (\pi \geq 0) \tag{1.4}$$

$$q_a \geq 0, i_a \geq 0, e \geq 0$$

Equation block (1) is a standard optimization problem. To make it clear that we are solving the problem of two different sectors, we split (1) into two separate problems for the fuel (2) and power (3) sectors. First we define a new free variable p representing the price at which fuel is purchased by the power sector from the supply sector.

$$\min z_{fuel}(f,e) \quad \text{s.t.} \tag{2}$$

$$z_{fuel}(f,e) = fC - eP - \sum_a q_a H_a p \tag{2.1}$$

$$f - S \leq 0 \tag{2.2}$$

$$\sum_a q_a H_a + e - f \leq 0 \qquad (\pi \geq 0) \tag{2.3}$$

$$e \geq 0, f \geq 0$$

In (2) the fuel sector minimizes the objective value $z_{fuel}(f,e)$ (2.1) subject to the supply constraint and fuel demand balance, (2.2) and (2.3), respectively. The objective value includes the cost of supplying fuel fC less the revenues from exports eP as well as revenues from selling to the power sector $\sum_a q_a H_a p$. Note that the last term is exogenous to the fuel supplier's problem, since $q_a$is not controlled by the power sector.

$$\min z_{power}(i,q_a) \text{ s.t.} \tag{3}$$

$$z_{power}(i,q_a) = \sum_a q_a H_a p + \sum_a i_a K_a \tag{3.1}$$

$$D - \sum_a q_a \leq 0 \tag{3.2}$$

$$q_a - i_a \leq 0 \tag{3.3}$$

$$p = ? \tag{3.4}$$

$$q_a \geq 0, i_a \geq 0$$

The power sector's optimization problem minimizes the objective value $z_{power}(i,q_a)$ (3.1) including fuel purchase costs $\sum_a q_a H_a p$ plus investment costs, subject to the power demand and capacity limit, (3.2) and (3.3). We also need to define the price variable in (3.4), but will leave the identity blank for now set it later.

We introduce the price equation in the sector purchasing the commodity, to reflect the policy impacting the sector. Note we could also define the price in the supplier's problem, or as a separate regulator's that could have its own objective function. Where the price is declared is a matter of convenience, since it is simply an identity that defines given policy, and does not constrain any sectors primal variable.

As expected combining (2) and (3) such that $z(f,e,i_a) = z_{fuel}(f,e) + z_{power}(q_a,i_a)$ results in the same problem as (1).Rather than solving (1) as a standard optimization (try this as an exercise), we would like to represent the equivalent Linear Complementarity Problem (LCP) by deriving the optimality conditions for (2) and (3);

1. Defining Lagrange multipliers, or dual variables, associated with inequality constraints in (2) and (3)
2. express the Lagrangian of each sectors optimization problem
3. write the stationarity, or KKT conditions associated with (2) and (3)

In complementarity theory dual variables are independent variables orthogonal to the inequality constraints in an optimization problem. For example we define a dual variable λ for (3.2)

$$D - \sum_a q_a \leq 0 \qquad \perp \qquad \lambda \geq 0,$$

that can only be positive value when the constraint is binding. An economic interpretation of this expression is that when the supply of electricity equals demand $\sum_a q_a = D$ the marginal value, or price, of the electricity is positive, $\lambda \geq 0$. If the market is oversupplied $\sum_a q_a > D$, then the marginal price is zero, $\lambda = 0$.

Additional reading is recommended on duality/complementarity to understand the use of duality theory in equilibrium model

To express the Lagrangian of (2) and (3) we define;

μ the dual on the fuel demand constraint (2.2), the marginal value of supplying fuel to other sectors (the power sector), and the price of fuel in a competitive market. This will play an important role in our formulation of the MCP.

π the dual variable for the fuel supply constraint (2.3).

λ the dual variable associated with the demand constraint (3.2) . The economic interpretation is the marginal value associated with the supply of electricity, or in a competitive electricity market, the price of electricity.

η_a the dual variable on the capacity limit (3.3). Interpreted as the scarcity premium on power supply technologies.

$\alpha_a, \beta_a, \delta, \varepsilon$ are the duals associated with the lower bounds (non-negativity) of q_a, i_a, e, and f respectively.

Next we define the Lagrangian associated with (2) and (3) in equation (4) and (5) respectively, and derive the optimality or stationarity conditions of the original linear program.

$$\Lambda_{fuel} = fC - eP - \sum_a q_a H_a \tau + (\sum_a q_a H_a + e - f)\mu + (f - S)\pi - e\delta - f\varepsilon \tag{4}$$

$$\Lambda_{power} = \sum_a q_a H_a \tau + \sum_a i_a K_a + (D - \sum_a q_a)\lambda + \sum_a (q_a - i_a)\eta_a - \sum_a q_a \alpha_a - \sum_a i_a \beta_a \qquad (5)$$

Recall when formulating the Lagrangian, inequality constraints must conform to the direction of the optimization (max/ min) when defining the complementarity pairs. In a minimization express each constraint on the left hand side of the upper bound inequality multiplied by its dual, e.x. $-\delta \leq 0 \Rightarrow -\delta e$.

Next we derive the optimality conditions by taking the partial derivative of Λ_{fuel} and Λ_{power} with respect to the corresponding decision variables, which should be equal to zero in the optimal solution.

$$\frac{\partial \Lambda_{fuel}}{\partial e} = \mu - P - \delta = 0 \qquad (6.1)$$

$$e \geq 0 \perp \delta \geq 0$$

$$\frac{\partial \Lambda_{fuel}}{\partial f} = C - \mu + \pi - \varepsilon = 0 \qquad (6.2)$$

$$f \geq 0 \perp \varepsilon \geq 0$$

$$\frac{\partial \Lambda_{power}}{\partial q_a} = H_a p + \eta_a - \lambda - \alpha_a = 0 \qquad (6.3)$$

$$q_a \geq 0 \perp \alpha_a \geq 0$$

$$\frac{\partial \Lambda_{power}}{\partial i_a} = K_a - \eta_a - \beta_a = 0 \qquad (6.4)$$

$$i_a \geq 0 \perp \beta_a \geq 0$$

We can simplify the stationarity conditions by substituting the complementarity conditions for $e \geq 0$, $f \geq 0$ $q_a \geq 0, i_a \geq 0$ into equations (6.1) to (6.4) respectively. Substituting $e \geq 0 \perp \delta \geq 0$ into (6.1) we find that $\mu - P \geq 0 \perp e \geq 0$. Doing this for each stationarity condition gives us (7.1)-(7.4), complemented by the original primal variables, also known as the dual constraints in the Karush-Kuhn-Tucker (KKT) conditions. The KKT conditions are completed by (7.5) to (7.8), the original primal constraints complemented by their dual variables

$\mu \geq P$	$\perp e \geq 0$	(7.1)
$C - \mu + \pi \geq 0$	$\perp f \geq 0$	(7.2)
$H_a p \geq \lambda - \eta_a$	$\perp q_a \geq 0$	(7.3)
$K_a - \eta_a \geq 0$	$\perp i_a \geq 0$	(7.4)
$f \leq S$	$\perp \pi \geq 0$	(7.5)
$\sum_a q_a H_a + e - f \leq 0$	$\perp \mu \geq 0$	(7.6)
$\sum_a q_a \geq D$	$\perp \lambda \geq 0$	(7.7)
$i_a \geq q_a$	$\perp \eta_a \geq 0$	(7.8)

$$p = \begin{cases} \pi & \text{in the deregulated competitive market} \\ V & \text{in the market with administered prices} \end{cases} \quad (7.9)$$

Note that the system is not complete without defining the free variable p, which we left open when writing the power sectors problem statement. In (7.9) we define the price as either the fuel sectors marginal supply cost (competitive market assumption), or some coefficient V representing a fixed price set by a system regulator. Now the system of equations and variables is complete. To show that the competitive market price assumption is correct derive the KKT conditions for the integrated cost minimization problem (1) and show that they are equivalent to (7.1) – (7.8) when $p = \pi$. Notice that the fixed policy changes the structure of (7.3) governing the value associated with converting fuel into electricity.

If we construct the MCP such that $p = V < P$, power generators may be incentivized to use more expensive technology that compromises on fuel efficiency, since they pay below the market price for fuel, P. Alternatively charging consumers more than the market price (fuel tax) could drive investment in other expensive technologies that consume no fuel.

In larger models deriving and implementing all the optimality conditions and modifying how sectors are connected in order to capture a pricing rule is time consuming and prone to error. Fortunately the EMP tool developed for solving equilibrium problems using the Path solver in GAMS, can be used to compile the optimality conditions for equilibrium problems consisting of multiple integrated optimization problems. For reference we provide an example EMP code statement that will convert our two sector problem into the MCP represented by (7.1) to (7.9). Write each optimization problem using GAMS and call the models using the EMP statement provided below (see GAMS file *KEM_EMP_simple.gms*). For more info lease refer to the EMP.pdf manual.

The instructions for constructing the MCP using EMP in GAMS are as follows.

1. Code the linear optimization problem (objective function and primal constraints) for the separate integrated sectors (2) and (3).
2. Configure emp.info file used to compile the equilibrium conditions as follows. (Note must list each sectors optimization problem,s tarting with the objective variable (min or max), then list all the sectors variables and then all the equations in vector form.

```
* Generate emp.info file to construct the MCP
file myinfo /'%emp.info%'/;

put myinfo 'equilibrium';
```

```
* Fuel sectors objective problem in EMP.
put / 'min', z_fuel;
Put f; Put e;
Put EQ2_1; Put EQ2_2; Put EQ2_3/;
* Power sectors optimization problem in EMP
Put / 'min', z_power;
loop(a, put q(a); put i(a););
* Variables for fuel prices
put p;
put EQ3_1; put EQ3_2;
loop(a, Put EQ3_3(a) );
put EQ3_4;
* declare symbolic variable mu as dual on EQ7_8
put 'DualVar pi EQ2_3';
option MCP = path
solve KEM using EMP;
```