Yineleme Çözüm Yöntemleri

1. Yerine koyma metodu ✗
2. Master metodu ✓
3. Yineleme ağacı metodu ✓

$T(n) = 2T(n/3) + \Theta(n)$
$T(n) = 3T(n/4) + \Theta(\sqrt{n})$
$T(n) = T(n-1) + \Theta(1)$
$T(n) = T(n/2) + T(n/3) + n$

1. Master metodu

$$T(n) = aT(n/b) + f(n)$$

$a \geq 1$, $b > 1$ ve f asimptotik pozitif ise

$f(n)$'i $n^{\log_b a}$ ile karşılaştırın.

Durum 1: Eğer $f(n) = O(n^{\log_b a - \varepsilon})$, $\varepsilon > 0$
yani $f(n)$ $n^{\log_b a}$'dan daha yavaş büyür
ise Çözüm: $T(n) = \Theta(n^{\log_b a})$

Örnek: $T(n) = 4T(n/2) + n = \Theta(n^2)$

a = 4, b = 2, f = n

n ile $n^{\log_2 4} = n^2$ 'yi karşılaştır.

$n = O(n^{2-\varepsilon})$ $\varepsilon = 1$ için

O zaman Durum 1, $T(n) = \Theta(n^2)$

Durum 2: Eğer $f(n) = \Theta(n^{\log_b a})$, yani f ile $n^{\log_b a}$ aynı oranda büyür ise

Çözüm: $T(n) = \Theta(n^{\log_b a} \cdot \lg n)$

Örnek: $T(n) = 4T(n/2) + n^2$

$\underset{a}{4}$, $\underset{b}{2}$, $\underset{f}{n^2}$

n^2 ile $n^{\log_2 4} = n^2$ yi karşılaştır.

$n^2 = \Theta(n^2)$ dir.

Durum 2 dir.

Çözüm: $T(n) = \Theta(n^2 \lg n)$

Durum 3: $f(n) = \Omega(n^{\log_b a + \varepsilon})$, $\varepsilon > 0$ sabit ve $f(n)$ düzenlilik koşulunu sağlıyor ise

Çözüm: $T(n) = \Theta(f(n))$

Düzenlilik koşulu: $a \cdot f(n/b) \le c \cdot f(n)$ için $c < 1$ olan bir sabittir.

Durum 3 de $f(n)$ $n^{\log_b a}$ dan daha hızlı büyür.

Örnek: $T(n) = 4T(n/2) + n^3 \rightarrow f(n)$

n^3 ile $n^{\log_2 4} = n^2$ yi karşılaştır.

$n^3 = \Omega(n^{2+\varepsilon})$ $\varepsilon = 1$ için

düzenlilik koşulu

$$\begin{cases} a \cdot f(n/b) \le c \cdot f(n) \\ 4 \cdot f(n/2) \le c \cdot f(n) \\ 4 \cdot \frac{\cancel{n^3}}{8} \le c \cdot \cancel{n^3} \\ \frac{1}{2} \le c \end{cases}$$

buradan $c = \frac{1}{2} < 1$ seçilebilir.

Durum 3 olur. O zaman $T(n) = \Theta(n^3)$

$T(n) = 4T(n/2) + n = \Theta(n^2)$ durum 1

$T(n) = 4T(n/2) + n^2 = \Theta(n^2 \lg n)$ durum 2

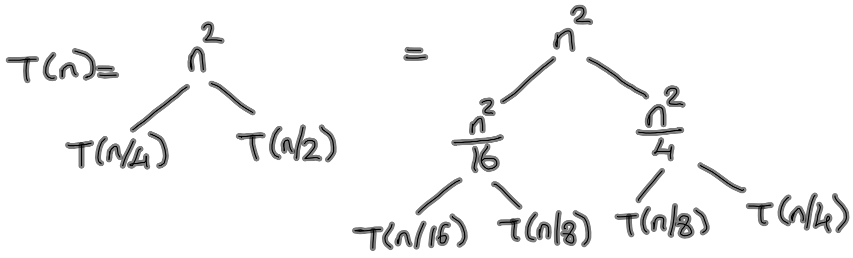
$T(n) = 4T(n/2) + n^3 = \Theta(n^3)$ durum 3

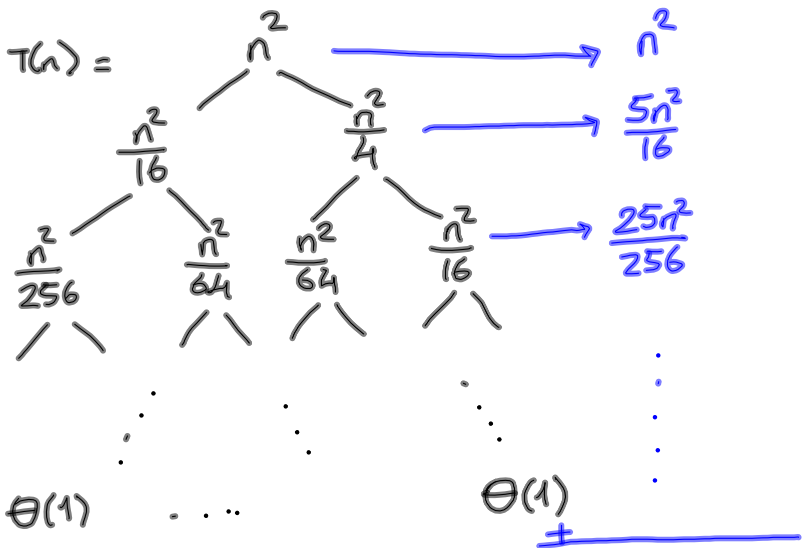
2. Yineleme Ağacı Metodu

$T(n) = T(n/4) + T(n/2) + n^2$ yinelemesini çözelim.

$T(n/4) = T(n/16) + T(n/8) + \frac{n^2}{16}$

$T(n/2) = T(n/8) + T(n/4) + \frac{n^2}{4}$





$$T(n) = n^2\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^k\right)$$

$$n^2\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^k\right) < n^2\underbrace{\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots\right)}_{\text{Geometrik seri}}$$

NOT: $1 + r + r^2 + \dots = \frac{1}{1-r}$ eğer $|r| < 1$

$$\underbrace{n^2\left(1 + \frac{5}{16} + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^k\right)}_{\downarrow} < n^2 \frac{1}{1-\frac{5}{16}} = \frac{16n^2}{11}$$

$$T(n) < \frac{16n^2}{11}$$

$$T(n) = O(n^2)$$

Exercises

4.5-1

Use the master method to give tight asymptotic bounds for the following recurrences.

a. $T(n) = 2T(n/4) + 1$.

b. $T(n) = 2T(n/4) + \sqrt{n}$.

c. $T(n) = 2T(n/4) + n$.

d. $T(n) = 2T(n/4) + n^2$.

1 ile $n^{\log_4^2} = n^{1/2}$ yi karşılaştır $1 = O(n^{1/2-\varepsilon})$ $\varepsilon = \frac{1}{2}$

Durum 1: $T(n) = \Theta(\sqrt{n})$

$\sqrt{n}$? $\sqrt{n} \to \sqrt{n} = \Theta(\sqrt{n})$ durum 2 $\to T(n) = \Theta(\sqrt{n}\lg n)$

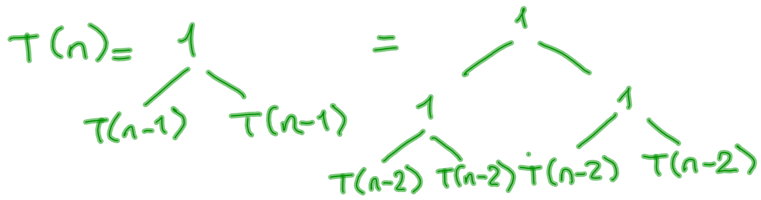
4.4-4

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$. ~~Use the substitution method to verify your answer~~.

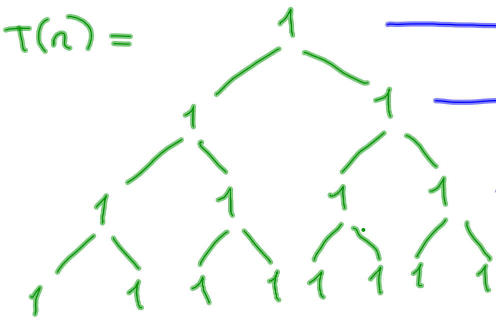
4.4-5

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T(n-1) + T(n/2) + n$. ~~Use the substitution method to verify your answer.~~

$T(n) =$



$T(n) =$



$\to 1$

$\to 2$

$\to 4 = 2^2$

$\to 8 = 2^3$

$h = n$

$\Theta(1)$ . . .

$\Theta(1) \to 2^n$

$+$

$T(n) = 1 + 2 + 2^2 + \ldots + 2^n$

Geometrik seri

$= \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$

$T(n) = \Theta(2^n)$

eksponersiyel çalışma zamanı

(üssel)

kötü bir algoritma