Suez National University

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Properties of matter and heat

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Introduction

Physics is based on experimental observations and quantitative measurements. The main objective of physics is to find the limited number of fundamental laws that govern natural phenomena and to use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

When a discrepancy between theory and experiment arises, new theories must be formulated to remove the discrepancy. Many times a theory is satisfactory only under limited conditions; a more general theory might be satisfactory without such limitations. For example, the laws of motion discovered by Isaac Newton (1642–1727) in the 17th century accurately describe the motion of objects moving at normal speeds but do not apply to objects moving at speeds comparable with the speed of light. In contrast,

the special theory of relativity developed by Albert Einstein (1879–1955) in the early 1900s gives the same results as Newton's laws at low speeds but also correctly describes motion at speeds approaching the speed of light. Hence, Einstein's special theory of relativity is a more general theory of motion.

Classical physics **includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, optics, and electromagnetism developed before 1900. Important contributions to classical physics**

were provided by Newton, who developed classical mechanics as a systematic theory and was one of the originators of calculus as a mathematical tool. Major developments in mechanics continued in the 18th century, but the fields of thermodynamics and electricity and magnetism were not developed until the latter part of the 19th century, principally because before that time the apparatus for controlled experiments was either too crude or unavailable.

A major revolution in physics, usually referred to as *modern physics,* began near the end of the 19th century. Modern physics developed mainly because of the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity not only correctly described the motion of objects moving at speeds comparable to the speed of light but also completely revolutionized the traditional concepts of space, time, and energy. The theory of relativity also shows that the speed of light is the upper limit of the speed of an object and that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas there is a great deal of overlap among physics, chemistry, and biology. Evidence for this overlap is seen in the names of some subspecialties in science—biophysics, biochemistry, chemical physics,

biotechnology, and so on. Numerous technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable developments in the latter half of the 20th century were (1) unmanned planetary explorations and manned moon landings, (2) microcircuitry and high-speed computers, (4) several remarkable results in genetic engineering. The impacts of such developments and discoveries on our society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

Units of Measurement

1. Standards of Length, mass and time

In nature, each measurement is associated with a physical quantity, such as the length of an object. The laws of physics are expressed as mathematical relationships among physical quantities. In mechanics, the three fundamental quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these quantities. In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (System International), and its fundamental units of length, mass, and time are the *meter, kilogram,* and *second,* respectively. Other standards for SI fundamental units established by the committee are those for temperature (the *kelvin*), electric current (the *ampere*), luminous intensity (the *candela*), and the amount of substance (the *mole*).

The units of the fundamental units in the SI are

	SI	Cgs	Brit

Length	**Meter**	**Centimeter**	**Foot**
Time	**Second**	**Second**	**Second**
Mass	**Kilogram**	**Gram**	**Slug**

(a) Length

The length is defined as the distance between two points in space. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. Current requirements of science and technology, however, necessitate more accuracy than that with which the separation between the lines on the bar can be determined. In the 1960s and 1970s, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. In October 1983, however, the meter was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second. In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. This definition of the meter is valid throughout the Universe based on our assumption that light is the same everywhere.

(b) Mass

The SI fundamental unit of mass, the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France. This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A

duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland.

(c) <u>Time</u>

In 1967, the second was defined to take advantage of the high precision attainable in a device known as an *atomic clock*, which measures vibrations of cesium atoms. One second is now defined as 9 192 631 770 times the period of vibration of radiation from the cesium-133 atom.

System of units

There are three systems of units commonly used, which are usually named according to the initials of the three basic units,

(a) C. G. S (Centimeter-Gram-Second) system.
(b) F. P. S (Foot-Pound-Second) system.
(c) M. K. S (Meter-Kilogram-Second) system.

(a) <u>C. G. S system</u>

The CGS system (French system) of units is based on centimeter as the unit of length, the gram as the unit of mass and the second as the unit of time. This system of units, widely used in science, is also known as Gaussian system. However, many of the derived units in this system are inconveniently small.

(b) <u>F. P. S system</u>

In this system of units which is known as the British system, the basic units of length, force (instead of mass) and time as the fundamental quantities are foot, pound and second respectively.

(c) <u>M. K. S system</u>

The system of units based on the fundamental units of the meter, the kilogram and the second is called the meter-kilogram-second or MKS.

<u>The basic and derived units are</u>:

<u>Basic Units (SI)</u>

The basic units are shown in Table. 1.

Quantity	Unit	SI symbol
Length	Meter	M
Mass	Kilogram	Kg
Time	Second	S
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of substance	mole	Mol
Luminous Intensity	Candela	Cd.

<u>Derived Units</u>

Multiplication of physical quantities creates new units. When you calculate the area, the unit becomes multiplied by itself to become, m^2. The unit of area is an example of a derived unit. Other derived units occur so often they are named after illustrious scientists, in honor of their work. Table 2, list derived units and their special names.

Quantity	Name	Symbol	Expression in terms of other units
Frequency	Hertz	Hz	1/s

Force, Weight	**Newton**	**N**	**m.kg.s^{-2}**
Work, energy	**Joule**	**J**	**N.m**
Power, Radiant flux	**Watt**	**W**	**J/s**
Pressure, Stress	**Pascal**	**Pa**	**N/m^2**
Electric charge	**Coulomb**	**C**	**Coulomb**
Electrical potential difference	**Volt**	**V**	**Joule/ Coulomb**
Electric resistance	**Ohm**	**Ω**	**V/A**
Electric capacitance	**Farad**	**F**	**Coulomb /Volt**
Magnetic flux	**Weber**	**Wb**	**T.** m^2
Magnetic field	**Tesla**	**T**	**Wbm^{-2}**
Inductance	**Henry**	**H**	**V.s/A=Wb/A**
temperature	**Degree Celsius**	**°C**	**°C**

Prefixes and Magnitudes

To make sense of the vast range over which physical quantities are measured, prefixes are used as a short-cut to writing the magnitude using scientific notation.

Prefix	Magnitude	Prefix	Magnitude
Deca Da	10^{1}	**deci d**	10^{-1}
Hecto h	10^{2}	**centi c**	10^{-2}
Kilo k	10^{3}	**mili m**	10^{-3}
Mega M	10^{6}	**Micro µ**	10^{-6}
Giga G	10^{9}	**nano n**	10^{-9}
Tera T	10^{12}	**pico p**	10^{-12}
Peta P	10^{15}	**Femto f**	10^{-15}
Exa E	10^{18}	**atto a**	10^{-18}

Dimensions and dimensional analysis

(a) Dimensions

In physics, the word of dimension denotes to the physical nature of the quantity. For example, the dimension of the distance is length. In dimensional analysis we specify the dimension of length, mass and time by L, M and T respectively. We also use the brackets [] to denote the dimension of a physical quantity, for example, velocity $[v] = L/T$ **and area** $[A] = L^2$**. The dimensional analysis can be used to**

- **Derive an equation**
- **Check the correctness of equation**

Dimensions of any physical quantity are the powers to which the fundamental units be raised in order to represent that quantity. Therefore, all the quantities listed in table below which possess dimension of length (L), mass (M) and time (T). All other quantities which are derivable from fundamental quantities according to their defining relation may have multiple basic dimensions (M^a L^b T^c) in accordance with the exact manner of their dependence on the fundamental quantities. The brackets [] means that it tells only the nature of the quantity and not its magnitude.

Physical quantity	Relation	Dimensional formula
Area	Length x width	$[LxL]=[L^2]$
Volume	Length x width x thickness	$[LxLxL]=[L^3]$
Velocity	Distance/time	$[L] / [T]=[LT^{-1}]$
Acceleration	Velocity/time	$[LT^{-1}] / [T]= [LT^{-2}]$
Momentum	Mass x Velocity	$[M] [LT^{-1}]=[MLT^{-1}]$
Force	Mass x acceleration	$[M][LT^{-2}]=[MLT^{-2}]$
Work	Force x distance	$[MLT^{-2}][L]= [ML^2T^{-2}]$
Energy	Force x distance	$[MLT^{-2}][L]= [ML^2T^{-2}]$

Pressure	**Force / area**	**$[MLT^{-2}]/[L^2]= [ML^{-1}T^{-2}]$**
Impulse	**Force x time**	**$[MLT^{-2}][T]= [MLT^{-1}]$**
Power	**Work / time**	**$[ML^2T^{-2}]/[T]= [ML^2T^{-3}]$**

Example 1

Derive an expression for the position x of the car at time t if the car starts from rest and moves with constant acceleration a.

$$x \propto a^m t^n$$

$$[x] = L \qquad [a^m t^n] = \left(\frac{L}{T^2}\right)^m (T)^n$$

$$L^1 = \left(\frac{L}{T^2}\right)^m (T)^n = L^m T^{n-2m}$$ **Equating power of both sides**

$$m = 1 \,,\; n - 2m = 0 \quad n = 2$$

$$x \propto at^2 = kat^2$$ **where k is constant**

Example 2

Show the correctness of the expression where v is the velocity, a is the acceleration and t is the time.

$$[vt] = \frac{L}{T}T = L \qquad [a] = \left(\frac{L}{T^2}\right) = \frac{L^2}{T^2} \qquad [vt] \neq a$$

Not correct

Example 3

Show the correctness of the expression $v = at$ **where** v **is the velocity, a is the acceleration and t the time.**

$$[at] = \frac{L}{T^2}T = \frac{L}{T} \qquad [v] = \frac{L}{T} \qquad\qquad v = at$$

correct

Example 4

Shows that the following equation is dimensionally correct or not v=ax

$$[v] = \frac{L}{T} \qquad [ax] = \left(\frac{L}{T^2}\right)(L) = \frac{L^2}{T^2}$$

Not correct

Example 5

Derive the acceleration of a particle moving with uniform speed v in circle of radius r

$$a \propto r^n v^m$$

$$[a] = \frac{L}{T^2} \qquad [r^n v^m] = (L)^n \left(\frac{L}{T}\right)^m$$

$$\frac{L}{T^2} = (L)^n \left(\frac{L}{T}\right)^m \qquad \Rightarrow LT^{-2} = L^{n+m}T^{-m}$$ **Equating power of both sides**

$$m = 2 \;,\; n + m = 1 \;\; n = -1$$

$$a \propto \frac{v^2}{r}$$

Estimates and order-of-magnitude calculations

It is useful to compute an approximation answer to a given physical problem even when a little information is available. This approximation can be modified if a greater precision is needed. The order of magnitude of a certain quantity is the power of ten of the number that describes the quantity. For example,

- $0.0086 \sim 10^{-2}$ **since** $8 \rangle 5$

  $0.0086 \sim 0.01 \sim 1 \times 10^{-2}$
- $0.0021 \sim 10^{-3}$ **since** $2 \langle 5$

  $0.0021 \sim 0.001 \sim 1 \times 10^{-3}$
- $720 \sim 10^{+3}$
- $5 \times 10^{10} \sim 10^{+11}$

Converting Units

Use the units as a guide to doing conversions. For example, Convert 25.0 pounds/square inch into Newton/square meter.

1. 1 pound = 4.45 Newton

2. 1 inch = 2.54 cm

3. 100 cm = 1 m

2. Set up the units, remembering to square where needed

$$\frac{pounds}{inch^2} \times \frac{newton}{pound} \times \left(\frac{inch}{cm}\right)^2 \times \left(\frac{cm}{m}\right)^2 = \frac{newton}{m^2}$$

3. Put in the numbers and calculate

$$25.0\frac{pounds}{inch^2}\times\frac{4.45\ newton}{1\ pound}\times\left(\frac{1\ inch}{2.54\ cm}\right)^2\times\left(\frac{100\ cm}{1\ m}\right)^2=172438\ \frac{newton}{m^2}$$

Example 6

Using dimensional analysis and applying the principles of homogeneity we can find the numerical value of a physical quantity in any system if its value in some other system is known. Let us convert a work of 1 Joule into erg.

M.K.S	**work**	**C.G.S**
System 1	**$M^1 L^2 T^{-2}$**	**system 2**
M_1=1Kg	**a=1**	**M_2=1g**
L_1=1m	**b=2**	**L_2=1cm**
T_1=1s	**c=-2**	**T_2=1s**
n_1		**n_2=?**

$n_1 (M_1^a\ L_1^b\ T_1^c) = n_2\ (M_2^a\ L_2^b\ T_2^c)$

$$n_2=n_1\left(\frac{M_1^aL_1^bT_1^c}{M_2^aL_2^bT_2^c}\right)=n_1\left[\frac{M_1^a}{M_2^a}\right]\left[\frac{L_1^b}{L_2^b}\right]\left[\frac{T_1^c}{T_2^c}\right]=n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$n_2=n_1\left[\frac{1Kg}{1g}\right]^1\left[\frac{1m}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}$$

$$n_2=n_1\left[\frac{1000g}{1g}\right]^1\left[\frac{100cm}{1cm}\right]^2\left[\frac{1s}{1s}\right]^{-2}=n_1x1000x10000=10^7n_1$$

1 Joule=10^7 erg

Example 7:

Convert an atmospheric pressure of 76 cm of mercury column in M.K.S units. Density of mercury equal $13.6\,gm.cm^{-3}$ **and gravity of acceleration equal** $980 cms^{-2}$ **(density of mercury is 13.6 g/cm^3). Hint p_1=13.6 x76 x 980.**

C.G.S	**Pressure**	**M.K.S**
System 1	$M^1 L^{-1} T^{-2}$	**system 2**
M_1=1g	a=1	M_2=1Kg
L_1=1cm	b=-1	L_2=1m
T_1=1s	c=-2	T_2=1s
P_1		P_2=?

$P_2\,(M_1^a\,L_1^b\,T_1^c) = P_1\,(M_2^a\,L_2^b\,T_2^c)$

$$P_2 = P_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

$$P_2 = 76x13.6x980\left[\frac{1g}{1Kg}\right]^1\left[\frac{1cm}{1m}\right]^{-1}\left[\frac{1s}{1s}\right]^{-2}$$

$$P_2 = 76x13.6x980\left[\frac{1g}{1000g}\right]^1\left[\frac{1cm}{100cm}\right]^{-1}\left[\frac{1s}{1s}\right]^{-2} = 1.0129\times10^5\,Nm^{-2}$$

Problems

1. Check the correctness of the following equations

(i) $x = a^m t^n$ **with m=1, n=2.** **(ii)** $v^2 = v_0^2 + 2at$ **(iii)** $t = 2\pi\sqrt{\frac{g}{\ell}}$

2. Which of the following equations are dimensionally correct?

(a) $v_f = v_i + ax$ **(b) y=2(m) cos(kx), where** $k = 2m^{-1}$

3. Newton's law of universal gravitation is represented by $F = \frac{GMm}{r^2}$

Where F is the magnitude of the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units $Kg.m/s^2$**. What are the SI units of the proportionality constant G?**

4. Kinetic energy $(K = \frac{1}{2}.mv^2)$ **has dimensions** $Kg.m^2/s^2$**.It can be written in terms of the momentum P with mass m as** $P^2/2m$

(a) Determine the proper units for momentum using dimensional analysis. (b) The unit of force is the newton N, where $1N = 1Kg.m/s^2$**. What are the units of momentum p in terms of a Newton and another fundamental SI**
unit?

5. Assume the equation $x = At^3 + Bt$ **describes the motion of a particular object, with x having the dimension of length and t having the dimension of time. Determine the dimensions of the constants A and B.**
(b) Determine the
dimensions of the derivative $\frac{dx}{dt} = 3At^2 + B$ **.**

6. How many meters are there in each one

(i) giga meter **(ii) Femtometer**

(iii) nanometer **(iv) picometer**

(v) hectometer

7. How many grams are there in each one

(i) teragram **(ii) attogram**

(iii) decagram **(iv) microgram**

(v) Petagram

8. Write the dimensional formula for the following quantities:

(i) Pressure **(ii) Density**

(iii) Frequency **(iv) Force**

(v) Work **(vi) Power**

9. Name the quantities which have the following dimensional formula

(i) $[M^{1}L^{1}T^{-1}]$ **(ii)** $[M^{1}L^{2}T^{-2}]$ **(iii)** $[M^{1}L^{-1}T^{-2}]$

10. Height of a liquid in a capillary tube is given by $h = \dfrac{r\rho g}{2T\cos\theta}$ **where T is the surface tension force,** ρ **is the density of the liquid,** θ **is the angle of contact and g is the gravity of acceleration. Check whether the relation is correct.**

11. Velocity v of sound depends upon the coefficient of elasticity E of the medium and the density ρ **of the medium. Obtain an expression for v.**

12. Time period of a simple pendulum depends upon its length and acceleration due to gravity at that place. Obtain an expression for T using the method of dimensional analysis.

13. The SI units include the fundamental units,

(A) Length, mass and time **(B) Velocity, mass and time**

(C) mass, pressure and time **(D) mass, Temperature and time**

14. The unit of force is

(A) Kg. m/S **(B) Kg. m/S^2** **(C) Kg. m^2/S** **(D) Kg. m/S^3**

15. The unit of power is

(A) Kg. m^2/S **(B) Kg. m/S^2** **(C) Kg. m^2/S^3** **(D) None of them**

16. The unit of pressure is

(A) Kg. m^{-1}/S **(B) Kg. m^2/S** **(C) Kg. m. S** **(D) Kg/m.S^2**

17. The dimension of acceleration is

(A) $[MLT^{-1}]$ **(B)** $[M^0 LT^{-2}]$ **(C)** $[LT^{-1}]$ **(D)** $[MT^{-1}]$

18. The dimension of work is

(B) $[MLT^{-2}]$ **(B)** $[ML^2T^{-2}]$ **(C)** $[M^2LT^{-1}]$ **(D)** $[ML^2T^{-2}]$

Elastic properties of materials

In reality, all objects are deformed as a result of applying external forces but internal forces tend to resist the deformation. By removing external forces, body may return to its original shape and becomes elastic or may be deformed and becomes inelastic. *Elasticity* is a property of a certain material which returns to its original shape after removing the external forces that acting upon it. We shall discuss the deformation of solids in terms of the concepts of stress and strain

Stress

Stress is defined as the external force acting on an object per unit cross section area.

Strain

Strain is defined as the measure of the deformation degree of the object.

Elastic modulus

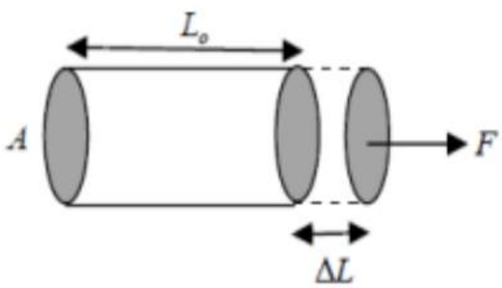
Is the ratio between stress and strain which depends on the material which deformed and on the nature of the deformation,

$$\text{Elastic modulus} = \frac{stress}{strain}$$

There are three types of deformation.

Young's modulus: Elasticity in length

Consider a long rod of initial length L_o and cross sectional area A. When the rod is subjected to perpendicular external force F, it

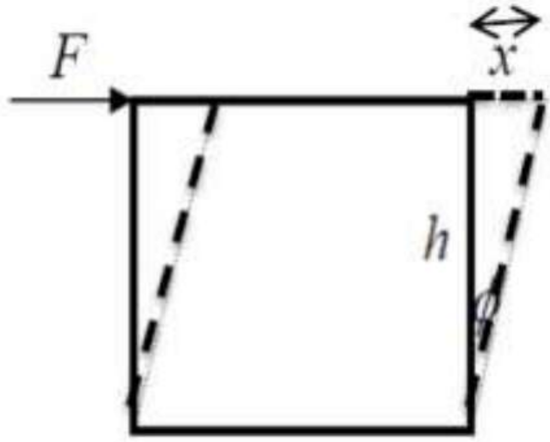


will be stretched by ΔL, which is the difference between L and L_o ($\Delta L = L - L_o$). We define the *tensile stress* as the ratio between the external force F and area A and the tensile strain as the ratio of the change in length ΔL and original length L_o. Young's modulus is defined as the ratio between tensile stress and tensile strain

$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F/A}{\Delta L/L_o}$$

$$Y = \frac{F/A}{\Delta L/L_o}$$

N/m^2



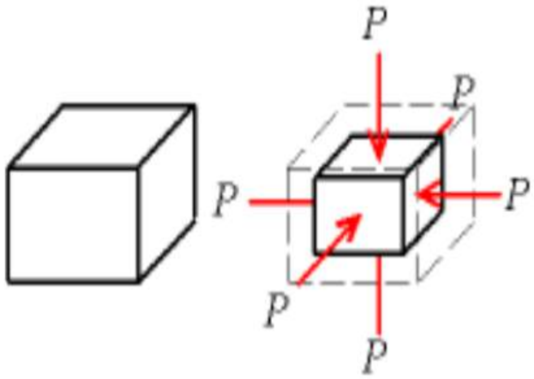
<u>Shear modulus: Elasticity of shape</u>

When a force F is applies tangentially to upper object face while the other force –F is applied in opposite direction for the down face. The tangential force per unit area of the upper face is called *shear stress*. The *shear strain* is the ratio between Δx and h where Δx is the horizontal distance that the sheared force moves and h is the height of the object. The shear modulus is defined as

$$S = \frac{F/A}{\Delta x/h} \qquad N/m^2$$

Bulk modulus B: Volume elasticity

The bulk modulus (B) of a substance measures the substance's resistance to uniform compression.



When object of volume V_o is subjected to a uniform force over its entire surface (as object immersed in fluid), the object is subjected to a change in its volume not in its shape. The volume stress (or the pressure P) is defined as the ratio of the total force F exerted on a surface to the area of the surface A. If the pressure changes by an amount $\Delta P = \Delta F / A$, the object will experience volume change ΔV.

The bulk modulus B is defined as

$$B = -\frac{\Delta F / A}{\Delta V / V_o} = -\frac{\Delta P}{\Delta V / V_o}$$

Negative sign is inserted because the increase in pressure $(+\Delta P)$ causes a decrease in volume (-ve. value) and vice versa. The inverse of the bulk modulus is called substance's compressibility

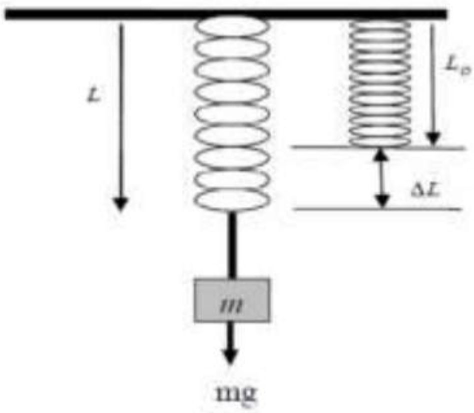
$$K = 1 / B = -\frac{\Delta V / V_o}{\Delta P}.$$

Example:

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1\times10^{5}\ N/m^{2}$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure $2\times10^{7}\ N/m^{2}$. The volume of the sphere in air is 0.50 m^{3}. By how much does this volume change once the sphere is submerged?

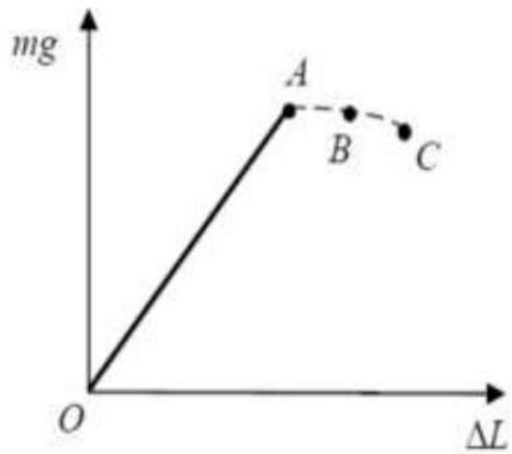
$$B = -\frac{\Delta P}{\Delta V / V_o}$$

$$\Delta V = -\frac{\Delta P}{B} V_o = -\frac{0.5\times(2\times10^{7}-1\times10^{5})}{6.1\times10^{10}} = -1.6\times10^{-4} m^{3}$$



Hook's law

Hooke's law of elasticity states that the extension of a spring is in direct proportion with the load applied to it. When a load $F = mg$ is hung from a spring of initial length L_o, the length of the spring extends to $L = L_o + \Delta L$. If the load is increased, the extension increases also as,

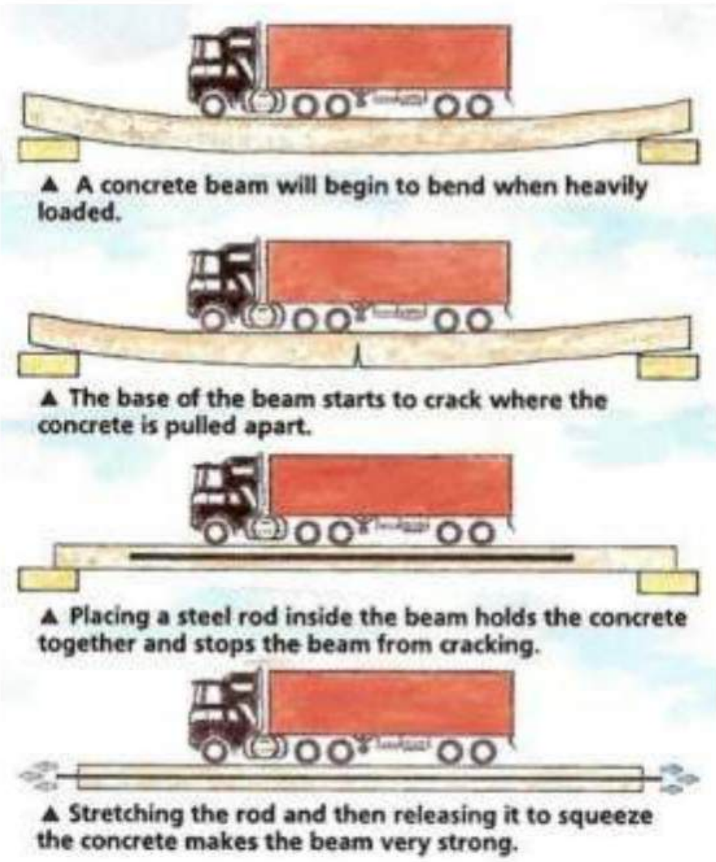


$$F = k\Delta L$$

Where k is a constant called the *rate* or *spring constant* (in SI units:

N/m or kg/s^2). For relatively small stresses, the rod will returns to its original shape when the force F is removed and the rod obeys to Hook's law as represented by straight line (OA) in Figure. Increasing load up to maximum value –point A- (elastic limit) which after it the rod deformed and do not returns to its original shape. The spring is said to have undergone plastic deformation; the load applied has exceeded the elastic limit. Further increase in stress, the material ultimately breaks at point C.

<u>Prestressed Concrete</u>



If the stress on a solid object exceeds a certain value, the object fractures. The maximum stress that can be applied before fracture occurs—called the *tensile strength, compressive strength,* or *shear strength*—depends on the nature of the material and on the type of applied stress. For example, concrete has a tensile strength of about $2\times10^6 N/m^2$, a compressive strength of $20\times10^6 N/m^2$, and a shear strength of

$2\times10^{6}\,N/m^{2}$. If the applied stress exceeds these values, the concrete fractures. It is common practice to use large safety factors to prevent failure in concrete structures.

Concrete is normally very brittle when it is cast in thin sections. Therefore, concrete slabs tend to sag and crack at unsupported areas as shown in Figure. The slab can be strengthened by the use of steel rods to reinforce the concrete as illustrated in Figure. Because concrete is much stronger under compression (squeezing) than under tension (stretching) or shear, vertical columns of concrete can support very heavy loads, whereas horizontal beams of concrete tend to sag and crack. A significant increase in shear strength is achieved, however, if the reinforced concrete is prestressed as shown in Figure. As the concrete is being poured, the steel rods are held under tension by external forces. The external forces are released after the concrete cures; the result is a permanent tension in the steel and hence a compressive stress on the concrete. The concrete slab can now support a much heavier load.

Example:

A $300gm.$ load is hung on a wire having a length $5m$, cross-sectional area $5\times10^{-4}m^{2}$ and Young's modulus $8\times10^{10}\,N/m^{2}$. What is its increase in length?

$\mathrm{m}=300\mathrm{gm.}=0.3\mathrm{kg}$ $\quad L=5m$ $\quad A=5\times10^{-4}m^{2}$

$Y=8\times10^{10}\,N/m^{2}$ $\quad \Delta L=?$ $\quad \mathrm{F}=\mathrm{mg}=0.3\times10=3\mathrm{N}$

$$\mathrm{Y}=\mathrm{stress/strain}=\frac{\mathrm{FL_i}}{\mathrm{A\Delta L}}=\frac{3\times5}{5\times10^{-4}\,\Delta\mathrm{L}}=8\times10^{10}$$

$$\Delta L = \frac{3\times 5}{5\times 10^{-4}\times 8\times 10^{10}} = 0.3752\times 10^{-6}\,m$$

Example:

A segment of steel railroad track has a length of $25m$ when the temperature is 0^oC (a) what is its length when temperature is 50^oC (linear expansion coefficient of the steal is $11\times 10^{-6}\,{}^oC^{-1}$). (b) Suppose that the ends of the rail are rigidly clamped at 0^oC so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 60^oC. (Young's modulus is $20\times 10^{10}\,N/m^2$).

(i) $L_i = 25m$**,** $T_i = 0^oC$ **,** $T_f = 50^oC$**,** $L_f = ?$

$$\alpha = 11\times 10^{-6}\,{}^oC$$

$$\Delta L = L_f - 25 = 11\times 10^{-6}(25)(50-0)$$

$$L_f = 11\times 10^{-6}(1250) + 25 = .0132 + 30 = 25.014m$$

(ii) $Stress = F/A = ?,$ $\Delta L = .014m$ $L_i = 25m$

$$Y = 20\times 10^{10}\,N/m^2$$

$$Y = stress/strain = Stress\,\frac{L_i}{\Delta L}$$

$$stress = Y\frac{\Delta L}{L_i} = 20\times 10^{10}\times\frac{.014}{25} = 11.2\times 10^{7}\,N/m^2$$

Example:

A steal wire of diameter 1 mm can support a tension of $0.2\times 10^{3}\,\mathrm{N}$. A cable to support a tension $20\times 10^{3}\,\mathrm{N}$ should have diameter of what order of magnitude?

$r_1 = 0.5\times 10^{-3}\,\mathrm{m}$ $\qquad F_1 = 0.2\times 10^{3}\,\mathrm{N}$

$r_2 = ?$ $\qquad F_1 = 20\times 10^{3}\,\mathrm{N}$

$S_1 = \dfrac{F_1}{A_1} = \dfrac{F_1}{\pi r_1^2}$ $\qquad S_2 = \dfrac{F_2}{A_2} = \dfrac{F_2}{\pi r_2^2}$ $\qquad S_1 = S_2$

$\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $\qquad \dfrac{F_1}{r_1^2} = \dfrac{F_1}{r_1^2}$

$\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$ $\qquad \dfrac{0.2\times 10^{3}}{(0.5\times 10^{-3})^2} = \dfrac{20\times 10^{3}}{r_2^2}$

$$r_2^2 = \frac{20\times 10^{3}\times (0.5\times 10^{-3})^2}{0.2\times 10^{3}} = 0.25\times 10^{-4}$$

$r_2 = 0.5\times 10^{-2}\,\mathrm{m}$

Example:

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is $1\times 10^{5}\,N/m^{2}$ (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is $2\times 10^{7}\,N/m^{2}$. The volume of the sphere in air is $0.5m^{3}$. By how much does this volume change once the sphere is submerged? (The bulk modulus of the brass is $6.1\times 10^{10}\,N/m^{2}$)

$$B = -\frac{\Delta P}{\Delta V / V_o}$$

$$\Delta V = -\frac{\Delta P V_o}{B}$$

$$\Delta V = -\frac{0.5m^3(2\times10^7 N/m^2 - 1\times10^5 N/m^2)}{6.1\times10^{10} N/m^2} = -1.6\times10^{-4}\, m^3$$

Problems:

1. **A spring with spring constant 0.4 dyne/cm has a force of 40 dynes applied to it (stretching it). How much does the spring stretch?**
2. **A force of 600 N will compress a spring 0.5 m. What is the spring constant of the spring?**
3. **A spring has spring constant 0.1 N/m. What force is necessary to stretch the spring by 2 m?**
4. **A force of 40 N will stretch a spring 0.1 m. How far will a force of 80 N stretch it?**
5. **According to Hooke's Law, the force needed to stretch a spring is proportional to the amount the spring is stretched. If fifty N of force stretches a spring five cm, how much will the spring be stretched by a force of 120 N?**
6. **A mine elevator is supported by a single steel cable 2.5cm in diameter. The total mass of the elevator and occupants is 670kg. By how much does the cable stretch when the elevator hangs by a) 12m of cable at surface? b) 362m of cable at mine shaft's bottom? (Neglect mass of cable)**

7. **A solid sphere of radius 10 cm is subjected to a uniform pressure of $5\times10^{8}\,\mathrm{Nm^{-2}}$. Determine the consequent change in volume. Bulk modulus of the material of the sphere is $3.14\times10^{11}\,\mathrm{Nm^{-2}}$.**
8. **A wire 10m long has a cross-sectional area $1.25\times10^{-4}\,\mathrm{m^2}$. It is subjected to a load of 5Kg. If Y for the material is $4\times10^{10}\,\mathrm{Nm^{-2}}$. Calculate the elongation produced in the wire.**
9. **A steel cable 3.00 cm^2 in cross-sectional area has a mass of 2.40 kg per meter of length. If 500 m of the cable is hung over a vertical cliff, how much does the cable stretch under its own weight? Take $Y_{steel} = 2\times10^{11}\,Nm^{-2}$.**
10. **A wire of length *L*, Young's modulus *Y*, and cross-sectional area *A* is stretched elastically by an amount ΔL. By Hooke's law, the restoring force is $-k\Delta L$. (a) Show that $k = YA/L_o$. (b) Show that the work done in stretching the wire by an amount ΔL is $\Delta L = 0.5YA(\Delta L)^2/L$.**
11. **A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area $2\times10^{-4}\,m^2$, and Young's modulus $8\times10^{10}\,Nm^{-2}$. What is its increase in length?**
12. **Assume that Young's modulus is $1.5\times10^{10}\,Nm^{-2}$ for bone and that the bone will fracture if stress greater than $1.8\times10^{8}\,Nm^{-2}$ is imposed on it. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?**

13. **A steel wire of diameter 1 mm can support a tension of 0.2 kN. A cable to support a tension of 20 kN should have diameter of what order of magnitude?**

14. **If the shear stress in steel exceeds $4\times10^{8}\,Nm^{-2}$, the steel ruptures. Determine the shearing force necessary to (a) shear a steel bolt 1.00 cm in diameter and (b) punch a 1.00-cm-diameter hole in a steel plate 0.500 cm thick.**

Fluid statics

Fluid

is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container that contain them.

Fluid static

Mechanics of the fluid at rest

Fluid dynamics

Mechanics of the fluid in motion

Density (ρ)

is the ratio between mass (m) and volume (V) of an object

$$\rho = \frac{m}{V} \qquad Kg/m^3 \qquad \textbf{(4.1)}$$

Relative density (ρ_r)

is the ratio between weight (mg) and volume (V) of an object

$$\rho_r = \frac{mg}{V} \qquad N/m^3 \qquad \textbf{(4.2)}$$

Pressure (P) inside fluid

The forces exerted on an object by static fluid are perpendicular to the surfaces of the object. The pressure (P) is defined as, the force (F) exerted on the surface area (A) of the object

$$P = \frac{F}{A} \qquad Pascal = N/m^2 \qquad \textbf{(4.3)}$$

Example:

The mattress of a water bed is $2m$ **long by** $2m$ **wide and** $30cm$ **depth (the density of water is** $1000Kg/m^3$**).**

(a) Find the weight of the water in the mattress.

(b) Find the pressure by the water on the floor of the bed

$A = 2\times 2 = 4m^2 \qquad h = 30cm = 0.3m$

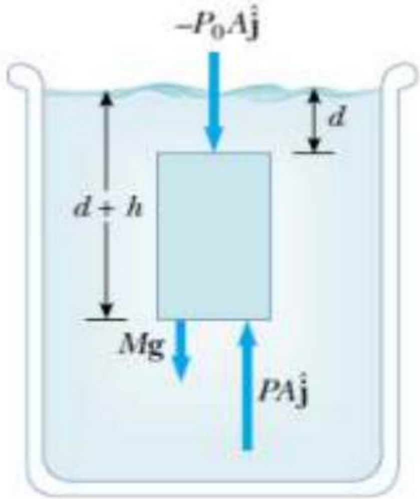
$V = Ah = 4\times 0.3 = 1.2m^3 \qquad \rho = 1000Kg./m^3$

(a)Weight $= W = mg = \rho Vg = 1000\times 1.2\times 9.8 = 1.18\times 10^4$

(b) $P = F/A = \dfrac{1.18\times 10^4}{4} = 2.95\times 10^3 \qquad Pascal = N/m^2$

Variation of pressure with depth

We will show how the pressure in the fluid increases with depth. Consider an imaginary liquid cylinder of cross-sectional area A **and length** h **at depth** d **from outer liquid surface inside liquid of uniform density** ρ**. The pressure exerting on the top of the surface of the liquid is** P_o**, while the pressure exerting on the bottom of the surface of the liquid is** P **. Then, the upward force exerted by the outside fluid on the bottom of the cylinder is** PA**, the downward force exerted on the top is**



$-P_o A$ **.The mass of the cylinder inside the liquid is**

$$m = \rho V = \rho A h \tag{4. 4}$$

The weight of the cylinder is

$$mg = \rho g A h$$

In equilibrium we get

$$PA - P_o A - mg = 0 \qquad PA = P_o A + mg$$

$$P = P_o + \rho g h \tag{4.5}$$

Where P_o **is the atmospheric pressure (** $P_o = 1.013 \times 10^5\ Pa$ **) at the surface of the liquid. Equation (4.5) means that the pressure P at a depth h below a point in the liquid is P_o greater by an amount** $\rho g h$**.**

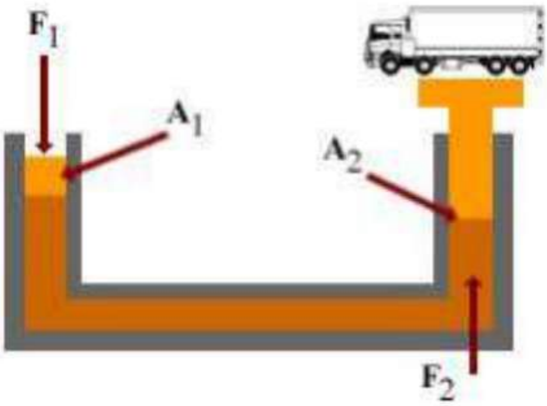
Example:

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5.0 m deep?

$$P = P_o + \rho g h \qquad \rho = 1 \times 10^3\ \mathrm{kg/m^3} \qquad \mathrm{h = 5m} \quad \mathrm{P - P_o = ?}$$

$$\mathrm{P - P_o = \rho g h = 1 \times 10^3 \times 9.8 \times 5 = 4.9 \times 10^4} \qquad \textbf{Pa}$$

Pascal's law



Since the pressure in a fluid depends on the depth and P_o, any increase in pressure at the surface is transmitted to every point in the fluid. Pascal's law states that "a change in the pressure applied to a fluid is transmitted to every point of the fluid and to the walls of the container"

One important application of Pascal's law is the hydraulic press which includes:

1- Force F_1 is applied to a small

piston of area A_1

2- The pressure is transmitted through the liquid to a larger piston of area A_2.

3- Pressure is the same on both sides of the system,

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = F_1 \frac{A_2}{A_1} \qquad \Rightarrow F_2 \rangle F_1 \qquad \textbf{(4.6)}$$

Example:

In a car lift used in service station, compressed air exerts a force on a small piston that has a circular cross section and a radius 5cm. The

pressure is transmitted by a liquid to a piston that has a radius of 15cm **. What force must be compressed air exert to lift a car weighting** 13300 N **. What air pressure produces this force?**

$r_1 = 5cm = .05m$ $\qquad r_2 = 15cm = 0.15m$

$F_1 = ?$ $\qquad F_2 = 13300\,N$

$F_2 = F_1 \frac{A_2}{A_1}$ $\qquad \Rightarrow F_1 = F_2 \frac{A_1}{A_2}$

$$F_1 = 13300 \frac{\pi (0.05)^2}{\pi (0.15)^2} = 1478N$$

Example:

Estimate the force exerted on your eardrum of surface area 1cm^2 **due to the water above when you are swimming at the bottom of the pool that is** 0.5m **deep**

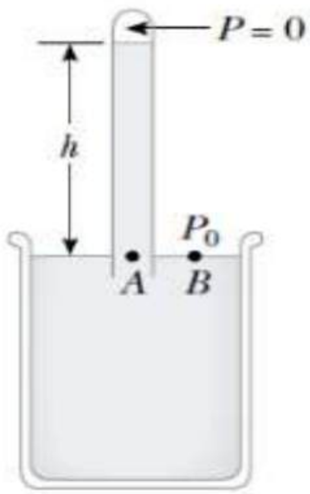
$\text{h} = 0.5\text{m},$ $\qquad \text{A} = 1\text{cm}^2 = 1\times10^{-4}\,\text{m}^2$

$$\text{P} - \text{P}_o = \rho\text{gh} = 1\times10^3 \times 9.8 \times 5 = 4.9\times10^4\,\text{pa}$$

$$\text{F} = (\text{P} - \text{P}_o)\text{A} = 4.9\times10^4 \times 10^{-4} = 4.9\text{N}$$

Pressure measurement

Barometer **is a device measures the atmospheric pressure which consists of long tube closed at one end filled with mercury and then inverted into a dish of mercury. The closed end of the tube is nearly a vacuum so**



the pressure at the top of the mercury column can be taken as zero. $P_A = P_B = P_o$ **is the atmospheric pressure which is given as**

$$P_o = \rho g h \qquad \textbf{(4.6)}$$

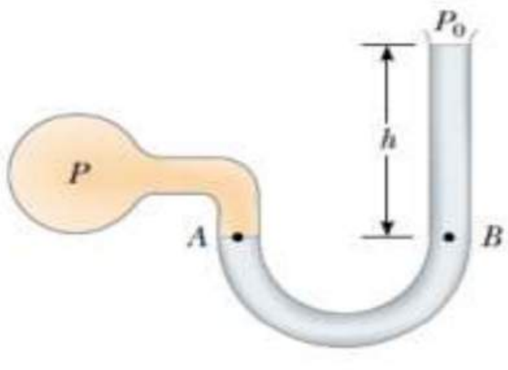
Where ρ **is the density of the mercury.**

($\rho = 13.6\times10^3 Kg/m^3$), $P_o = 1.013\times10^5 Pa$

Then the height of the mercury column is

$$h = \frac{P_o}{g\rho} = \frac{1.013\times10^5}{9.8\times13.6\times10^3} = 0.76m$$

Then ***one atmospheric pressure*** **is defined to be the pressure equivalent of a column of mercury that is** 0.76m **in height at** 0^oC**.**

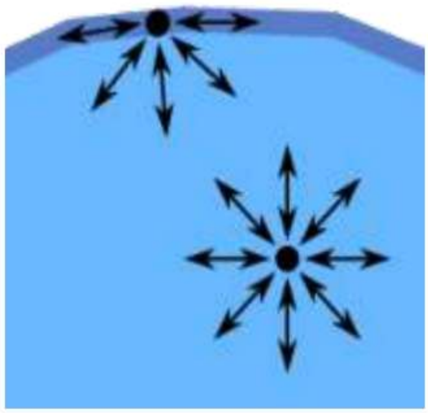


Manometer **is a device that measures the pressure of the gas contained in a vessel. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The pressure at points A and B must be the same. Then the pressure at point B is** $P = P_o + \rho g h$**. The pressure P is called absolute pressure while the difference** $\text{P} - \text{P}_o = \rho\text{gh}$ **is called gauge pressure.**

Surface tension

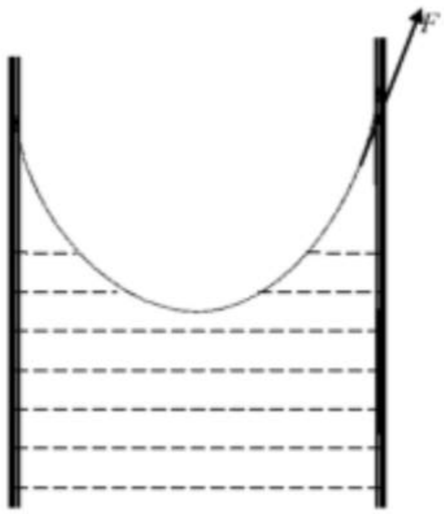
Surface tension is a property of the surface of a liquid that allows it to resist an external force. It is revealed, for example, in floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects to run on the water surface. This property is caused by cohesion of like molecules.

Origin of surface tension



The cohesive forces among the liquid molecules are responsible for this phenomenon of surface tension. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards. This creates some internal pressure and forces liquid surfaces to contract to the minimal area.

Surface tension is responsible for the shape of liquid droplets in which droplets of water tend to be pulled into a spherical shape due to the surface tension. In the absence of other forces, including gravity, drops of virtually all liquids would be perfectly spherical. The spherical shape minimizes the necessary "wall tension" of the

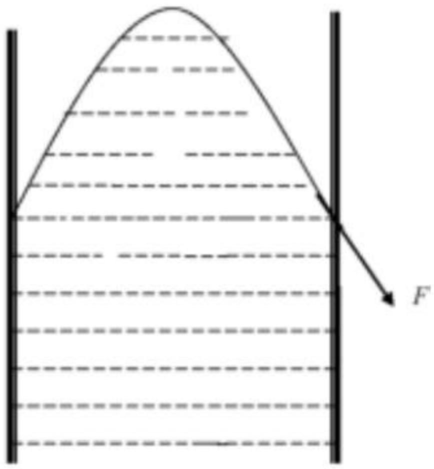


surface layer according to Laplace's law. Surface tension coefficient can be defined as the force per unit length.

$\gamma = F / \lambda$ **N/m**

F **is the tension force result from cohesive force between molecules**

λ**, is the circumference of the liquid film.**

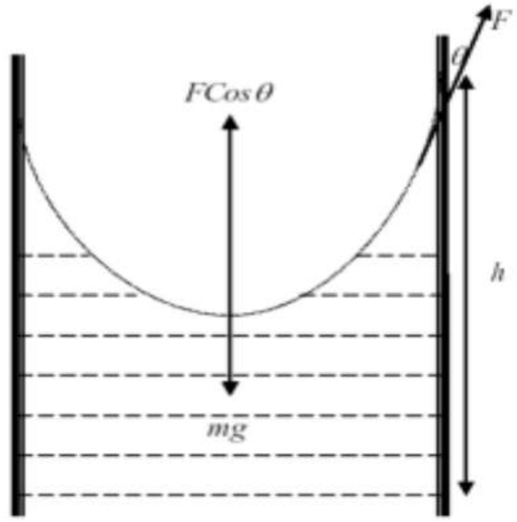


If a liquid is placed in a tube, two forces exist: *(i)* adhesive force, F_a between the liquid and the wall of the tube, *(ii)* cohesive force F_c between the molecules of the liquid. If $F_a \rangle F_c$, the liquid rises in the tube and concave liquid is formed (for example water). Otherwise, $F_a \langle F_c$, the liquid has a convex surface (for example, mercury). The relation between the two forces explains why water droplets wet the surface while the mercury droplets have spherical shapes.

Contact angle

The surface of any liquid is an interface between that liquid and some other medium. If a liquid is in a container, then besides the liquid-air interface at its top surface, there is also an interface

between the liquid and the walls of the container. When the two surfaces meet, they form a contact angle, θ, which is the angle the tangent to the surface makes with the solid surface.



Liquid in a vertical tube Capillarity

Capillarity is the rising of liquids in tubes against gravity. The liquid is raised in a tube of radius r to a height h when the surface tension force balance with the gravitational force (weight of the liquid) as

$$mg = FCos\theta \tag{4. 7}$$

Where, mg is the weight of the liquid θ is the contact angle and F is the tension force which is given as

$$F = \gamma\lambda = \gamma(2\pi r) \tag{4. 8}$$

$\lambda = 2\pi r$ **is the circumference of the film formed inside tube, r is the radius of the tube. From (4. 7) and (4. 8) we get**

$$mg = V\rho g = 2\pi r\gamma\ Cos\theta \tag{4. 9}$$

Where ρ is the density of the liquid, V is the volume of the liquid in the tube which is given as $V = \pi r^2 h$ is used in (4. 9) to get

$$\pi r^2 h\rho g = 2\pi r\,\gamma\, Cos\,\theta \qquad \Rightarrow rh\rho g = 2\gamma\, Cos\,\theta$$

$$h = \frac{2\gamma\, Cos\,\theta}{r\rho g} \tag{4. 10}$$

Then the height of the liquid h is proportional inversely with the radius $(1/r)$ and the liquid density $(1/\rho)$ and directly with the surface tension coefficient γ. If the angle is $\theta \rangle 90^o$, the liquid will descend and h becomes negative (as mercury), the liquid will rise for $\theta \langle 90^o$ (as water) and h becomes positive.

<u>Surface Tension Examples</u>

1- Walking on water

Small insects such as the water strider can walk on water because their weight is not enough to penetrate the surface.

2- Floating a needle

If carefully placed on the surface, a small needle can be made to float on the surface of water even though it is several times as dense as water. If the surface is agitated to break up the surface tension, then needle will quickly sink.

3- Don't touch the tent

Common tent materials are somewhat rainproof in that the surface tension of water will bridge the pores in the finely woven material. But if you touch the tent material with your finger, you break the surface tension and the rain will drip through.

4- Soaps and detergents

Soaps help the cleaning of clothes by lowering the surface tension of the water so that it more readily soaks into pores and soiled areas.

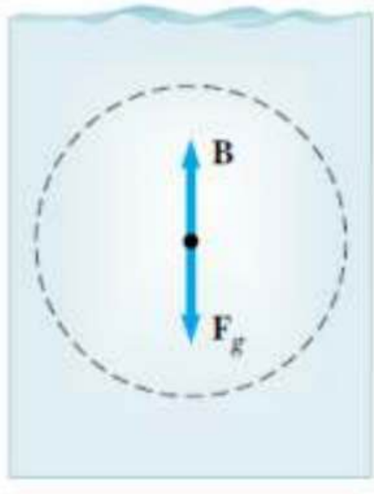
5- Washing with cold water

The major reason for using hot water for washing is that its surface tension is lower and it is a better wetting agent. But if the detergent lowers the surface tension, the heating may be unnecessary.

6- Surface tension disinfectants

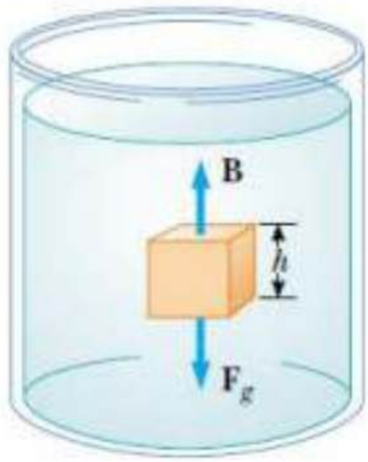
Disinfectants are usually solutions of low surface tension. This allows them to spread out on the cell walls of bacteria.

Buoyant forces



Buoyant force (B) is the upward force exerted by the fluid on any immersed object. When an object, of mass m, immersed in water as in figure, at equilibrium the upward buoyant force B must balance with the downward gravitational force $F_g = mg$**. This buoyant force is the resultant force due to all forces applied by the water surrounding the object.**

Archimedes' principle



This principle states that "the magnitude of the buoyant force B equals the weight of the fluid displaced by the object".

Under the water, the buoyant force B that equals the weight of the displaced water is much greater than the weight of the object $(B \rangle mg)$**. Then there is a large net upward force which cannot hold the object under water and finally floats.**

Consider a cube of volume $V_o = Ah$ **immersed in water as in figure, the pressure** P_b **at the bottom of the cube is greater than the pressure at the top** P_t **by an amount** $\rho_f gh$ **as**

$$\Delta P = P_b - P_t = \rho_f gh \qquad \textbf{(4.11)}$$

The buoyant force is given as

$$B = \Delta PA = A(P_b - P_t) = \rho_f ghA = \rho_f gV_o = Mg \qquad M = \rho_f V_o \qquad \textbf{(4.12)}$$

Where, $M = \rho_f V_o$ **mass of the displaced fluid.**

ρ_f **is the density of the fluid.**

V_o **is the volume of the cube.**

Now we discuss two situations:

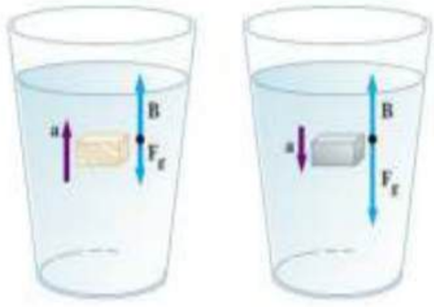
(a) <u>Total submerged object</u>

The volume of the object is equal to the volume of the displaced fluid ($V_o = V_f$)**.**

$$B = \rho_f gV_o \qquad F_g = Mg = \rho_o gV_o \qquad \textbf{(4.13)}$$

The net force is given as

$$B - F = (\rho_f - \rho_o)gV_o$$

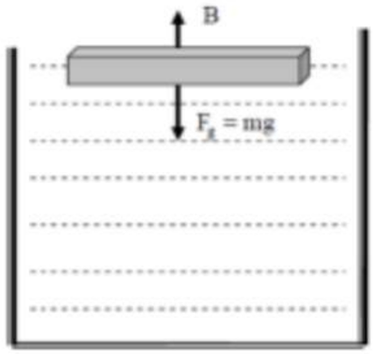


If $\rho_o \langle \rho_f$**, the object accelerates upwards and floats.**

If $\rho_o \rangle \rho_f$**, the object accelerates**

downwards and sinks.

If $\rho_o = \rho_f$ **, the object in equilibrium.**



(b) <u>Floating objects</u>

In the case of floating objects on the surface of the fluid where $\rho_o \langle \rho_f$ **(object is only partially submerged),**

$B = F_g$ **,** $V_f \neq V_o$

$B = \rho_f g V_f$ $F_g = Mg = \rho_o g V_o$

$B = F_g$ $\rho_f V_f = \rho_o V_o$

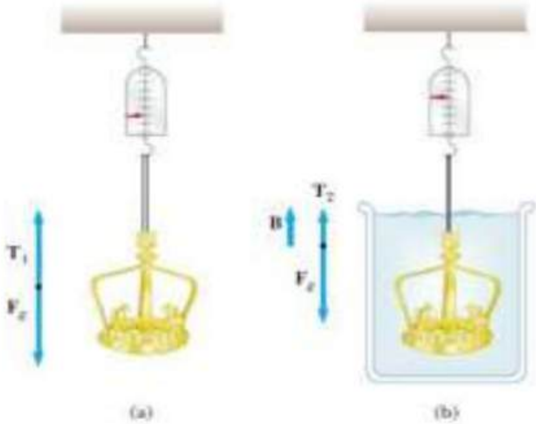
$$\frac{V_f}{\rho_o} = \frac{V_o}{\rho_f} \qquad \textbf{(4.14)}$$

Apparent weight in a fluid

If the weight of the body in air is W_{air} **(actual weight) and its weight inside water is** W_{water}**(apparent weight), the buoyant force B is given as**

$$B = \text{Actual weight} - \text{Apparent weight} = W_{air} - W_{water} = \rho_w g V_o \qquad \textbf{(4.15)}$$

Example:



Archimedes was asked to determine whether a crown made for the king consisted of pure gold. In air crown weighted $7.84\,\text{N}$ **and in water weights, determine the** $6.84\,\text{N}$ **density of the crown** [$\rho_w = 10^3\,\text{Kg/m}^3$]**.**

$W_{air} = 7.84\,\text{N}$, $W_{water} = 6.84\,\text{N}$, $\rho_w = 10^3\,\text{Kg/m}^3$

$\rho_o = ?$

$$B = \rho_w g V_o = W_{air} - W_{water} = 7.84 - 6.84 = 1\text{N}$$

$9.8\times 10^3 \times V_o = 1\text{N}$ $V_o = \dfrac{1}{9.8\times10^3} = 1.02\times10^{-4}\,\text{m}^3$

$$W_{air} = 7.84\text{N} = \rho_o g V_o = 9.8\times1.02\times10^{-4}\,\rho_o$$

$\rho_o = \dfrac{7.84\text{N}}{9.8\times1.02\times10^{-4}} = 7.84\times10^3\,\text{kg/m}^3$ **, the crown is not purely gold because the density of gold is** $9.3\times10^3\,\text{kg/m}^3$ **.**

Example:

A ping-pong ball has a diameter of $3.8\,\text{cm}$ **and average density of** $0.084\,\text{g/cm}^3$**. What force is required to hold it completely submerged under water?**

$2r = 3.8cm = 3.8\times10^{-2}\,m$ $\qquad r = 1.9\times10^{-2}\,m$

$\rho_o = 0.084 g/cm^3 = 0.084\times\frac{10^{-3}}{10^{-6}} kg/m^3 = .084\times10^3 kg/m^3$

Inside the liquid

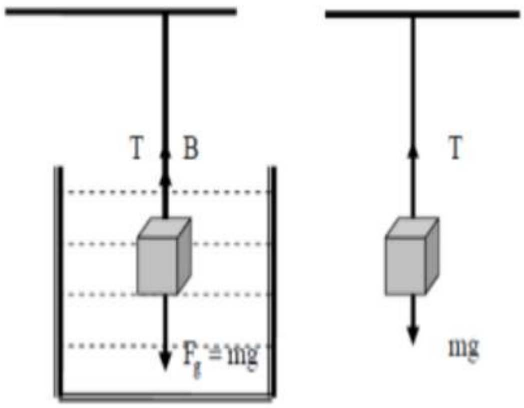
$B = \rho_f g V_o$ $\qquad F_g = Mg = \rho_o g V_o$

$B - F = (\rho_f - \rho_o) g V_o$ $\qquad V_o = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.14)(1.9\times10^{-2})^3 = 28.7135\times10^{-6}\,m^3$

$B - F = (1\times10^3 - .084\times10^3)\times9.8\times28.7135\times10^{-6} = 257.7\times10^{-3}\,N$

Example:

A piece of aluminum with mass $1Kg$ and density $2700\,Kg/m^3$ is suspended from a string and then completely immersed in a container of water. Calculate the tension in the string (a) before and (b) after the metal is immersed.



$\rho_w = 10^3 Kg/m^3$ $\qquad m = 1Kg$ $\qquad \rho_o = 2700\,Kg/m^3$

(a) $T_1 = ?$

$T_1 = mg = \rho_o V_o g = 1\times 9.8 = 9.8N$

(b) $T_2 = ?$ $V_o = \dfrac{m}{\rho_o} = \dfrac{1}{2700} = .37\times 10^{-3} m^3$

$B + T_2 = mg$ $T_2 = mg - B = \rho_o g V_o - \rho_f g V_o = (\rho_o - \rho_f) g V_o$

$T_2 = (2700 - 1000)\times 9.8 \times 0.37 \times 10^{-3} = 6.17N$

Example:

A cube of wood having an edge dimension of $20cm$ **and a density of** $650 Kg/m^3$ **floats on water: (a) What is the distance from the horizontal top surface of the cube to the water level (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?**

$V_o = (20\times 10^{-2})^3 = (0.20)^3 = 8\times 10^{-3} m^3$

$\rho_o = 650 Kg/m^3$

(b) $h = ?$

$\rho_f V_f = \rho_o V_o$ $V_f \times 10^3 = 650\times 8\times 10^{-3}$

$V_f = \dfrac{650\times 8\times 10^{-3}}{10^3} = 5.2\times 10^{-3} m^3$

$V = V_o - V_f = 8\times 10^{-3} - 5.2\times 10^{-3} = 2.8\times 10^{-3} m^3$

$h = 0.2 - \left(2.8\times 10^{-3}\right)^{\frac{1}{3}} 0.2 - 0.14 = 0.6m$

(b) $\rho_{lead} = 11.3\times10^3\,kg/m^3$

$\rho_f V_f = \rho_o V_o + \rho_{lead} V_{lead}$ $\qquad\qquad$ $\rho_{lead} V_{lead} = \rho_f V_f - \rho_o V_o$

$$V_{lead} = \frac{V_o(\rho_f - \rho_o)}{\rho_{lead}} = \frac{8\times10^{-3}(10^3 - 0.65\times10^3)}{11.3\times10^3} = 2.8m^3$$

Example:

A plastic sphere floats in water with 50 percent of its volume submerged. This same sphere floats in glycerin with 40 percent of its volume submerged. Determine the densities of the glycerin and the sphere?

$V_o = 0.5V_P$ $\qquad\qquad$ $\frac{V_w}{V_P} = 50\%$

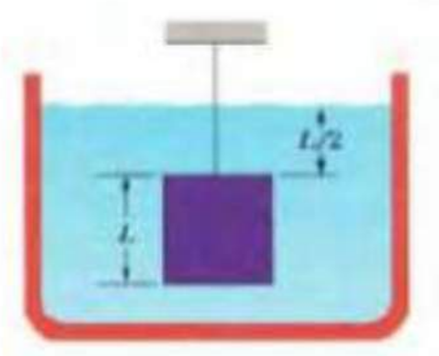
$V_o = 0.4V_P$ $\qquad\qquad$ $\frac{V_G}{V_P} = 40\%$ $\qquad\qquad$ $\frac{V_o}{V_w} = \frac{\rho_w}{\rho_P}$

$$\rho_P = \rho_w \frac{V_w}{V_p} = 0.5\times10^3 = 500Kg/m^3$$

$\frac{V_P}{V_G} = \frac{\rho_G}{\rho_P}$ $\qquad\qquad$ $\rho_G = \rho_P \frac{V_P}{V_G} = 500\times\frac{1}{0.4} = 1250Kg/m^3$

Problems

1. A cube of length $L-0.6\text{m}$ **and a mass 450Kg is suspended by a rope in an open tank in a liquid of density** $1030\,\text{Kg/m}^3$ **. Find (a) the magnitude of the downward force on the top of the cube from the liquid and the atmosphere, assuming the atmospheric pressure is 1 atm. (b) the magnitude of the upward force on the bottom of the cube. (c) Calculate the magnitude of the buoyant force using Archimedes principle.**



2. An iron anchor of density $7870\,\text{Kg/m}^3$ **appears 200N lighter in water than in air. (a) What is the volume of the anchor? (b) How much does it weight in air?**

3. A boat floating in fresh water displaces water weighting 35.6KN. (a) What is the weight of the water this boat displaces when floating in salt water of density $1.1\times10^3\,\text{Kg/m}^3$ **?. (b) What is the difference between the volume of the fresh water displaced and the volume of salt water displaced?**

4. A 5Kg object is released from rest while fully submerged in a liquid. The liquid displaced by the submerged object has a mass of 3Kg. How far and in what direction does the object move in 0.2s assuming that it moves freely and that the drag force on it from the liquid is negligible?

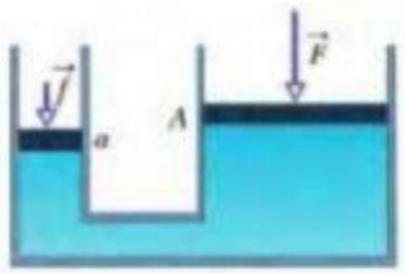
5. **A block of wood floats in fresh water with two-thirds of its volume V submerged and in oil with 0.6 V submerged. Find the density of (a) the wood and (b) the oil?**

6. **A 50Kg women balances on one heel of a pair of high-heeled shoes. If the heel is circular and has a radius of 0.5 cm. What the pressure does she exert on the floor?**

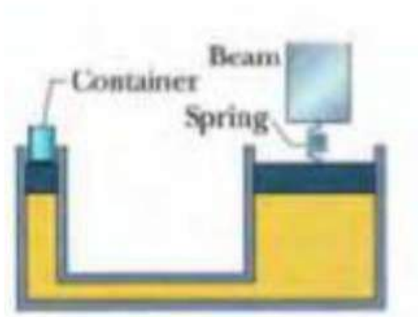
7. **A swimming pool has dimensions** $30\text{m} \times 10\text{m}$ **and a flat bottom. When the pool is filled to a depth** 2m **with a fresh water what is the force caused by the water on the bottom? On each end? On each side?**

8. **Normal atmospheric pressure is** $1.013 \times 10^5\ \text{Pa}$**. The approach of a storm causes the height of a mercury barometer to drop by 20mm from the normal height. What is the atmospheric pressure? (the density of the mercury is** $13.59\,\text{gm}/\text{cm}^3$ **).**

9. **A piston of cross-sectional area (a) is used in a hydraulic press to exert a small force of magnitude f on the enclosed liquid. A connecting pipe leads to a larger piston of cross-sectional area A. (a) What force magnitude F will the larger piston sustain without moving? (b) If the piston diameters are 380cm and 530cm. what is the force magnitude on the small piston will balance a 20 KN force on the large piston?**



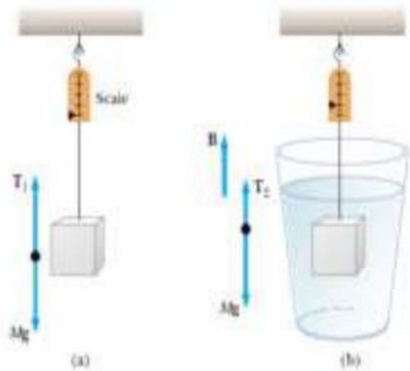
10. In Figure, a spring of constant $3\times10^{4}\,N/m$ is between a rigid beam and the output piston of a hydraulic lever. An empty container with negligible mass sits on the input piston. The input piston has area (A) and the output piston has area 18A. Initially the spring is at its rest length. How many kilograms of sand must be slowly poured into the container to compress the spring by 5 cm?



11. An office window has dimensions 3.4m by 2.1m. As a result of the passage of a storm, the outside air pressure drops to 0.96 atm., but inside the pressure held at 1 atm. What net force pushes out on the window?

12. A Ping-Pong ball has a diameter of 3.80 cm and average density of 0.084 0 g/cm3. What force is required to hold it completely submerged under water?

13. A piece of aluminum with mass 1.00 kg and density 2 700 kg/m^3 is suspended from a string and then completely immersed in a container of water (Figure). Calculate the tension in the string (a) before and (b) after the metal is immersed.



14. A cube of wood having an edge dimension of 20.0 cm and a density of 650 kg/m^3 floats on water. (a) What is the distance from the horizontal top surface of the cube to the water level? (b) How much lead weight has to be placed on top of the cube so that its top is just level with the water?

16. A plastic sphere floats in water with 50.0 percent of its volume submerged. This same sphere floats in glycerin with 40.0 percent of its volume submerged. Determine the densities of the glycerin and the sphere.

Fluid dynamics

Steady flow (laminar flow)

If each particle of the fluid follows a smooth path such that the paths of different particles never cross each other, the flow is called steady flow. In this case the velocity of the fluid particles passing any point remains constant in time.

Turbulent flow

Turbulent flow is an irregular flow characterized by a small eddies.

Viscous fluid

Viscous fluid is associated with the resistance that two adjacent layers of fluid have to moving relative to each other.

Ideal fluid flow

Ideal fluid has the following properties

(i) Non-viscous fluid in which no resistance between fluid layers

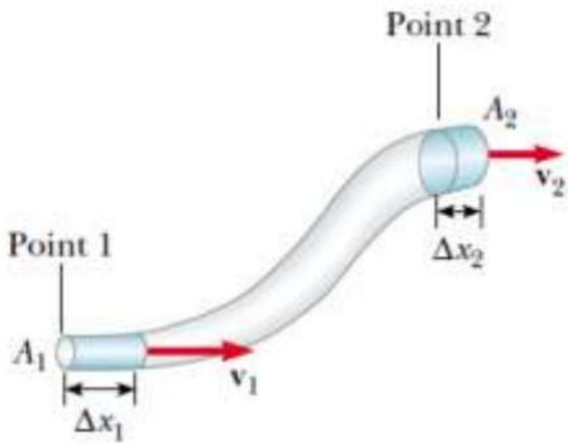
(ii) Incompressible fluid where the density of the fluid remains constant

(iii) Steady flow, where the velocity at each point remains constant

(iv) Irrotational fluid, where the fluid has no angular momentum about any point.

Continuity equation

Consider an ideal fluid flowing through a pipe of uniform size. In a time Δt , the fluid at the bottom of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$ through an area A_1 with mass



$$m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$$

Also, the fluid at the top of the pipe upper pipe moves a distance $\Delta x_2 = v_2 \Delta t$ through an area A_2 with mass

$$m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$$

Because the fluid is incompressible and the flow is steady, then

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

The continuity equation is given as

$$A_1 v_1 = A_2 v_2 \qquad \Rightarrow (\mathit{flow\ rate}) Av = \text{constant} \qquad \textbf{(5.1)}$$

Then, the continuity equation states that, "the product of the area and the fluid speed at all points along the pipe is constant for incompressible fluid".

Example:

Each second $5525\,m^3$ of water flows over the $670\,m$ wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2m deep as it reaches the cliff. What is its speed at that instant?

Solution

$$flow\ rate = Av$$

$$flow\ rate = 5525\,m^3/s \quad v = ? \qquad A = 670 \times 2 = 1340\,m^2$$

$$5525 = 1340\,v \quad v = \frac{5525}{1430} = 4m/s$$

Example:

A water hose 2.5 cm in diameter is used by a gardener to fill a 30L bucket. The gardener notes that it takes 1 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.5 cm^2 is attached to the hose. The nozzle is held so that water is projected horizontally from a point 1mabove the ground. Calculate the velocity at the nozzle·

Solution

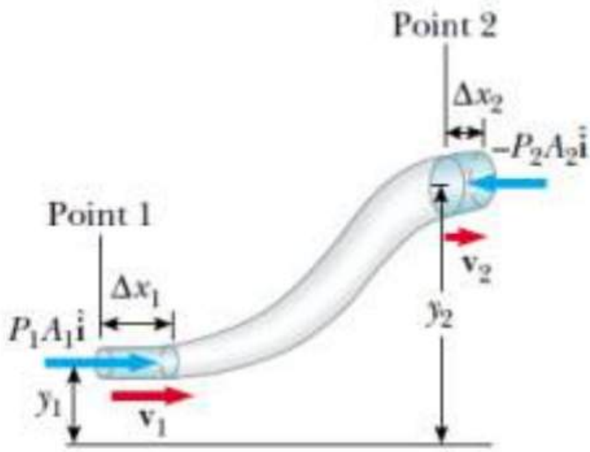
$$A_1 = 0.5cm^2 = 0.5 \times 10^{-4}\,m^2, \qquad A_2 = \pi(1.25 \times 10^{-2})^2 = 4.91 \times 10^{-4}\,m^2$$

$$A_1 v_1 = A_2 v_2 = 30L/\min = \frac{30 \times 10^3\,cm^3}{60s} = 0.5 \times 10^{-3}\,m^3/s$$

$$A_2 = \pi(1.25 \times 10^{-2})^2 = 4.91 \times 10^{-4}\,m^2$$

$$v_1 = \frac{0.5 \times 10^{-3}}{0.5 \times 10^{-4}} = 10m/s \qquad v_2 = \frac{0.5 \times 10^{-3}}{4.91 \times 10^{-4}} = 1.02m/s$$

Bernoulli's equation



The force on the lower end is $F_1 = P_1A_1$ and the force on the upper end is $F_2 = P_2A_2$. The work done by the fluid on the lower end to move the fluid distance Δx_1 with velocity v_1 is $W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$ .This work leads to

(i) Change in the kinetic energy of the fluid $\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

(ii) Change in the potential energy of the fluid $\Delta U = mgy_2 - mgy_1$

(iii) Work done by the fluid on the upper part $W_2 = F_2\Delta x_2 = -P_2A_2\Delta x_2$ $= P_2V$

Then the work done on the lower end is given as

$$P_1V = P_2V + \frac{1}{2}\rho V(v_2^2 - v_1^2) + \rho Vg(y_2 - y_1)$$

Where $V = \Delta x_1A_1 = \Delta x_2A_2$ **and** $m = \rho V$ **. Then,**

$$P_1 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

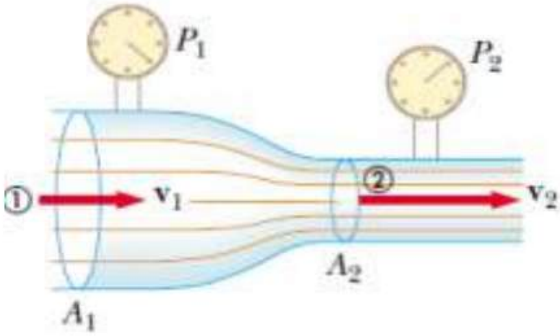
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 = \text{constant} \qquad \textbf{(5.2)}$$

Therefore, Bernoulli's equation shows that the pressure decreases with increasing velocity and height.

Applications on Bernoulli's equation

(1) Venturi tube

This tube is characterized by the following conditions $v_1 \langle v_2$, $P_1 \rangle P_2$ and $y_1 = y_2$. In this case Bernoulli's equation is



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$2(P_1 - P_2) = +\rho(v_2^2 - \rho v_1^2)$$

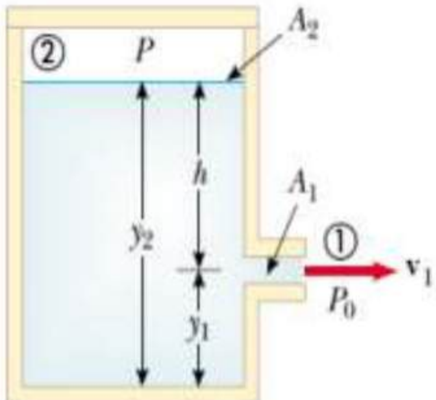
From continuity equation where $v_1 = \frac{A_2}{A_1} v_2$, then

$$2(P_1 - P_2) = \rho(v_2^2 - \frac{A_2^2}{A_1^2} v_2^2) = \rho v_2^2 (\frac{A_1^2 - A_2^2}{A_1^2})$$

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}} \qquad \textbf{(5.3)}$$

(2) Torricelli's law

In this case, Torricelli assumed $v_2 = 0$, $A_1 \langle A_2$ and $h = y_2 - y_1$, then



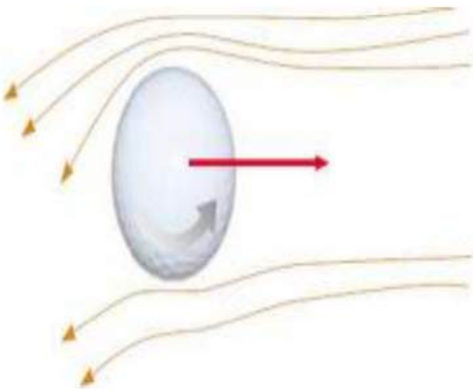
$$P_o = P + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g h$$

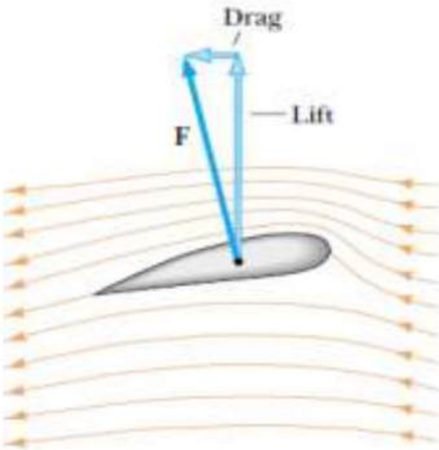
$$\rho v_1^2 = 2(P_o - P) + 2\rho g h$$

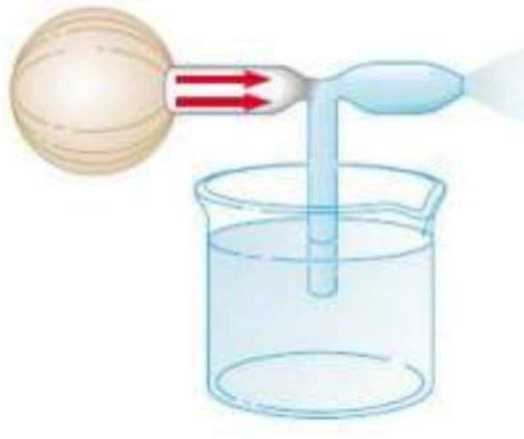
$$v_1 = \sqrt{\frac{2(P_o - P) + 2\rho g h}{\rho}} \qquad \textbf{(5.4)}$$

<u>Other Applications of Fluid Dynamics</u>

Consider the streamlines that flow around an airplane wing as shown in Figure. Let us assume that the airstream approaches the wing horizontally from the right with a velocity v_1. The tilt of the wing causes the airstream to be deflected downward with a velocity v_2. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream exerts a force F on the wing that is equal in magnitude and opposite in direction. This force has a vertical component called the lift (or aerodynamic lift) and a horizontal component called drag. The lift depends on several factors, such as the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal. The curvature of the wing surfaces causes the pressure above the wing to be lower than that below the wing, due to the Bernoulli effect. This assists with the lift on the wing. As the angle between the wing and the horizontal increases, turbulent flow can set in above the wing to reduce the lift.







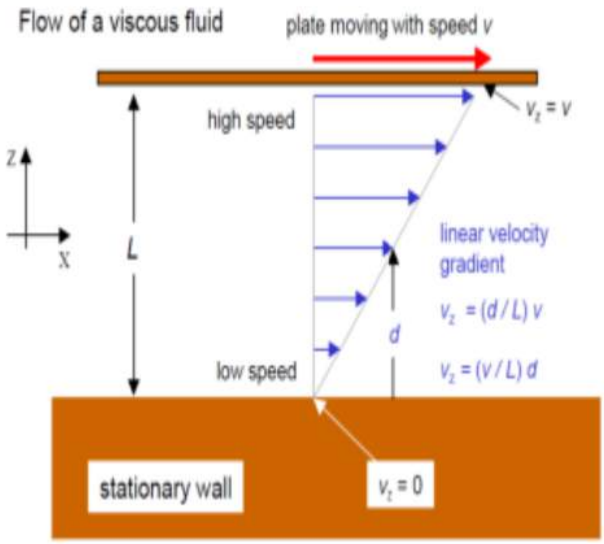
In general, an object moving through a fluid experiences lift as the result of any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence lift are the shape of the object, its orientation with respect to the fluid flow, any spinning motion it might have, and the texture of its surface. For example, a golf ball struck with a club is given a rapid backspin due to the slant of the club. The dimples on the ball increase the friction force between the ball and the air so that air

adheres to the ball's surface. This effect is most pronounced on the top half of the ball, where the ball's surface is moving in the same direction as the air flow. Figure shows air adhering to the ball and being deflected downward as a result. Because the ball pushes the air down, the air must push up on the ball. Without the dimples, the friction force is lower, and the golf ball does not travel as far. It may seem counterintuitive to increase the range by increasing the friction force, but the lift gained by spinning the ball more than compensates for the loss of range due to the effect of friction on the translational motion of the ball! For the same reason, a baseball's cover helps the spinning ball "grab" the air rushing by and helps to deflect it when a "curve ball" is thrown.

A number of devices operate by means of the pressure differentials that result from differences in a fluid's speed. For example, a stream

of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.

Viscosity



The viscosity of a fluid is defined as the resistance to flow the fluid. For example, honey has a higher viscosity than water. Viscosity is the property of a fluid which opposes the relative motion between two surfaces of the fluid that are moving at different velocities. In simple terms, viscosity means friction between the molecules of fluid. When the fluid is forced through a tube, the particles which compose the fluid generally move more quickly near the tube's axis and more slowly near its walls; therefore some stress (such as a pressure difference between the two ends of the tube) is needed to overcome the friction between particle layers to keep the fluid moving. For a given velocity pattern, the stress required is proportional to the fluid's viscosity.

When a fluid (e.g. air) flows past a stationary wall (e.g. table top), the fluid close to the wall does not move. However, away from the wall the flow speed is not zero. So a velocity gradient exists. This is due to adhesive, cohesive and frictional forces. We find that the magnitude of this gradient (how fast the speed changes with distance) is characteristic of the fluid. This is used to define the coefficient of viscosity

$$\eta = \frac{F}{A}\frac{L}{v} \qquad (N.Sec/m^2) \text{ Poise} \qquad (5.5)$$

Where, A is the area on which the force acts.

F is the shear force on the top of the fluid to move with a velocity v.

The coefficient of viscosity is defined as the tangential force per unit area required to maintain a unit velocity gradient.

Unit of viscosity is $N.Sec/m^2 = Pa.\sec$

$1 Poise = Dyne.Sec/cm^2$

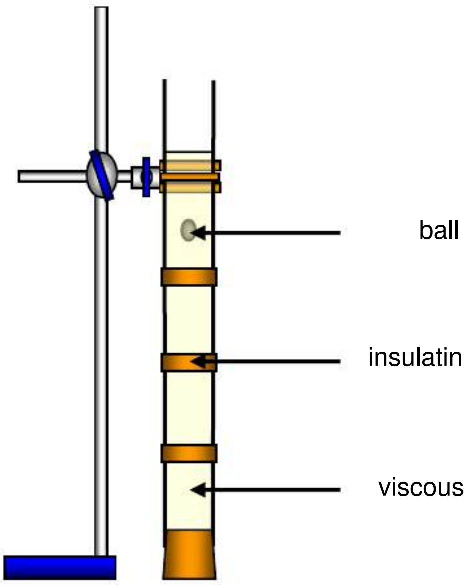
Note that:

1. If the speed of the top plate is low enough, then in steady state the fluid particles move parallel to it, and their speed varies from zero at the bottom to v at the top. Each layer of fluid moves faster than the one just below it, and friction between them gives rise to a force resisting their relative motion.
2. In many fluids, the flow velocity is observed to vary linearly from zero at the bottom to v at the top. Moreover, the magnitude *F* of the force acting on the top plate is found to be

proportional to the speed v and the area A of each plate, and inversely proportional to their separation L.

3. In solids, the viscous forces that arise during fluid flow must not be confused with the elastic forces that arise in a solid in response to shear, compression or extension stresses. While in the latter the stress is proportional to the *amount* of shear deformation, in a fluid it is proportional to the *rate* of deformation over time.

4. Amorphous solids, such as glass and many polymers, are actually liquids with a very high viscosity (greater than 10^{12} Pa·s).



5. Viscosity is measured with various types of viscometers and rheometers. A rheometer is used for those fluids that cannot be defined by a single value of viscosity and therefore require more parameters to be set and measured than is the case for a viscometer. Close temperature control of the fluid is essential to acquire accurate measurements, particularly in materials like lubricants, whose viscosity can double with a change of only 5 °C.

Stock's law or velocity sphere falling through a viscous liquid

When a steel ball is dropped into a viscous liquid, viscous forces are opposed to ball's motion. Viscous forces increases as the velocity of the ball increases. The resultant net force on the ball at equilibrium is zero. For a small sphere falling through a viscous fluid, the opposing force depends on:

1. **the terminal velocity v of the ball**
2. **the radius of the sphere**
3. **the coefficient of viscosity** η
4. **the viscous force** $F = 6\pi v r \eta$

If ρ **is the density of the ball and** ρ' **is the density of the liquid, then the downward force (weight of the ball),**

$$W = mg = \frac{4}{3}\pi r^3 \rho g \qquad \textbf{(5.6)}$$

The upward force (buoyant force) on the ball is

$$F_b = m'g = \frac{4}{3}\pi r^3 \rho' g \qquad \textbf{(5.7)}$$

The viscous force $F = 6\pi v r \eta$**. Then,**

$$W = F_b + F_{vis}$$

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \rho' g + 6\pi\eta r v$$

$$\frac{4}{3}\pi r^3 (\rho - \rho')g = 6\pi\eta r v$$

$$v = \frac{2g}{9\eta} r^2 (\rho - \rho') \tag{5.8}$$

Flow of a fluid through a pipe

Poiseuille's Law

The flow of fluid is affected greatly by the pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate Q is in the direction from high to low pressure. The greater the pressure difference between two points, the greater the flow rate. This flow rate is

$$Q = \frac{P_1 - P_2}{R} \tag{5.9}$$

Where P_1 and P_2 are the pressures at two points, such as at either end of a tube, and R is the resistance to flow. The resistance R includes everything, except pressure, that affects flow rate. For example, R is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of R. Turbulence greatly increases R, whereas increasing the diameter of a tube decreases.

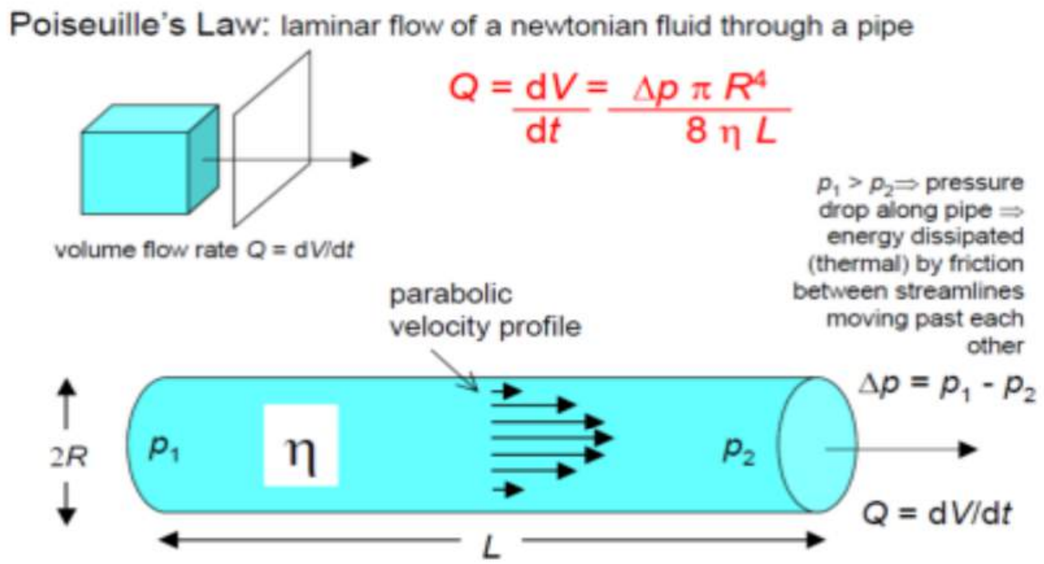
If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. We see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries.

The resistance R to laminar flow of an incompressible fluid having viscosity η through a horizontal tube of uniform radius r and length L, is given by

$$R = \frac{8\eta L}{\pi r^4} \qquad \textbf{(5.10)}$$

This equation is called Poiseuille's law for resistance. Taken together, Q and R give the following expression for flow rate:

$$Q = \frac{P_1 - P_2}{R} \ \textbf{(5.11)} \qquad \Rightarrow Q = \frac{\pi r^4 (P_1 - P_2)}{8\eta L} \qquad R = \frac{8\eta L}{\pi r^4} \&$$



Problems

1. A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16 m below the water level. If the rate of flow from the leak is equal to $2.5x10^{-3}$ m^3/min, determine (a) The speed at which the water leaves the hole (b) the diameter of the hole.

2. Water flows through a fire hose of diameter 6.35 cm at a rate of to 0.12 m^3/s. The fire hose ends in a nozzle of inner diameter 2.2 cm what is the speed at which the water exits the nozzle?

3. A garden hose with an internal diameter of 1.9 cm is connected to a stationary lawn sprinkler that consists of a container with 24 holes, each with 0.13 cm diameter. If the water in the hose has a speed 0.91m/s, at what speed does it leave the sprinkler holes?

4. Water is pumped steadily out of a flooded basement at a speed of 5m/s through a uniform hose of radius 1 cm. The hose passes out through a window 3 m above the waterline. What is the power of the pump?

5. Water is moving with a speed of 5m/s through a pipe with a cross-sectional area of $4cm^2$. The water gradually descends 10m as the pipe cross-sectional area increases to $8cm^2$. (a)What is the speed at the lower level (b) If the pressure at the upper level is $1.5\times10^5 Pa$ what is the pressure at the lower level?

6. In figure water flows through a horizontal pipe and then out into the atmosphere at a speed $v_1 = 15m/s$. The diameter of the left and the right sections of the pipe are 5 cm and 3 cm. (a) what volume of water flows into the atmosphere during a 10min period. In the left section of the pipe what are (a) the speed v_2 and (c) the gauge pressure?

7. Water flows through a fire hose of diameter 6.35 cm at a rate of $0.012\,m^3/s$. The fire hose ends in a nozzle of inner diameter 2.2 cm. what is the speed with which the water exits the nozzle?

8. A Venturi tube may be used as a fluid flow meter (Fig.). If the difference in pressure is $(P_1 - P_2) = 21\,KPa$, find the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is 1 cm, the radius of the inlet tube is 2 cm, and the fluid is gasoline ($\rho = 700\,Kg/m^3$).

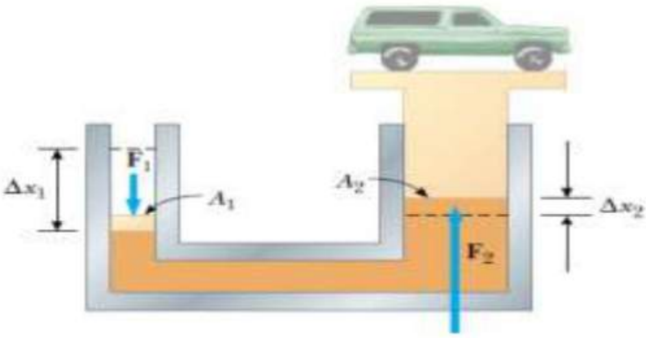
9. A venturimeter reads height $h_1 = 30cm$, $h_2 = 10cm$. Find the velocity flow in the pipe. $A_1 = 7.85\times10^{-3}\,m^2$, $A_2 = 1.26\times10^{-3}\,m^2$.

10. A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is 8×10^4 Pa and the pressure in the smaller pipe is 6×10^4 Pa, at what rate does water flow through the pipes?

11. Calculate the mass of a solid iron sphere that has a diameter of 3.00 cm.

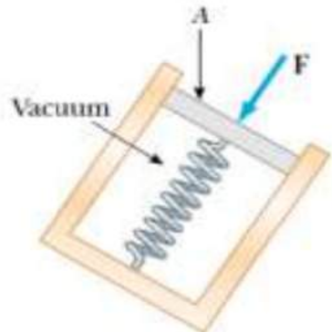
12. A 50.0-kg woman balances on one heel of a pair of high heeled shoes. If the heel is circular and has a radius of 0.500 cm, what pressure does she exert on the floor?

13. What is the total mass of the Earth's atmosphere? (The radius of the Earth is $6.37x10^6$ m, and atmospheric pressure at the surface is $1.013x10^5$ N/m^2.)



14. (a) Calculate the absolute pressure at an ocean depth of 1000 m. Assume the density of seawater is 1 024 kg/m^3 and that the air above exerts a pressure of 101.3 kPa. (b) At this depth, what force must the fram-e around a circular submarine porthole having a diameter of 30.0 cm exert to counterbalance the force exerted by the water?

15. The spring of the pressure gauge shown in Figure 14.2 has a force constant of 1 000 N/m, and the piston has a diameter of 2.00 cm. As the gauge is lowered into water, what change in depth causes the piston to move in by 0.500 cm?



16. The small piston of a hydraulic lift has a cross-sectional area of 3.00 cm^2, and its large piston has a cross-sectional area of 200 cm^2 (Figure). What force must be applied to the small piston for the lift to raise a load of 15.0 kN? (In service stations, this force is usually exerted by compressed air.)

17. What must be the contact area between a suction cup (completely exhausted) and a ceiling if the cup is to support the weight of an 80.0-kg student?

Oscillatory motion

Simple harmonic motion

Any particle that is initially displaced slightly from a stable equilibrium point will oscillate about its equilibrium position. It will, in general, experience a restoring force that depends linearly on the displacement *x* from equilibrium which make the particle to oscillates forth and back. A particle moving along x -axis make a simple harmonic motion described by

$$x(t) = A\cos(\omega t + \phi) \tag{1}$$

Where, x is the displacement in x-direction.

A **is the amplitude of oscillation (is the maximum displacement)**

ϕ **is the phase angle.**

$\omega = 2\pi f$ **is the angular frequency, f is the frequency of motion which is given by**

$$f = \frac{\omega}{2\pi} = \frac{1}{T} \tag{2}$$

T is the periodic time which is the time of complete oscillation. The velocity of the particle is given by

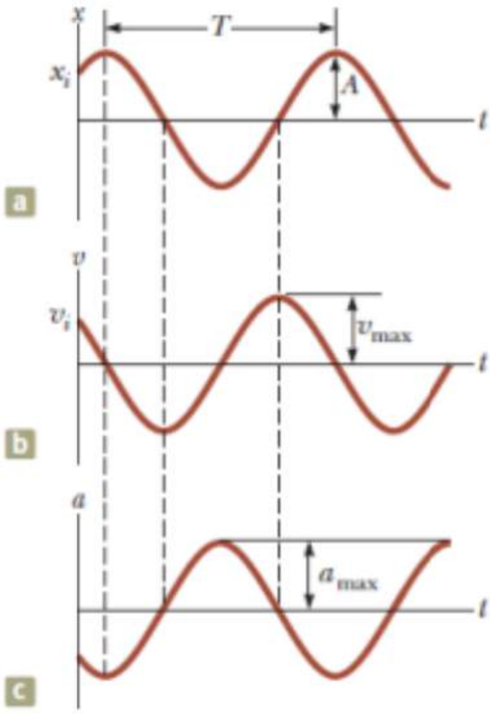
$$v = \frac{dx}{dt} = \frac{d}{dt}A\cos(\omega t + \phi)$$

$$= -A\,\omega \sin(\omega t + \phi) = -v_{max} \sin(\omega t + \phi) \qquad (3)$$

Where $v_{max} = A\omega$ is the maximum velocity. The acceleration of the particle is

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(-A\omega \sin(\omega t + \phi)\right)$$

$$= -A\,\omega^2 \cos(\omega t + \phi) = -\omega^2 x = -a_{max} \cos(\omega t + \phi) \qquad (4)$$



Where $a_{max} = A\omega^2$ is the maximum acceleration of the particle.

<u>Definitions</u>:

- **Amplitude (A): The maximum distance that an object moves from its equilibrium position. A simple harmonic oscillator moves back and forth between the two positions of maximum displacement, at $x = A$ and $x = -A$.**
- **Period (T): The time that it takes for an oscillator to execute one complete cycle of its motion. If it starts at $t = 0$ at $x = A$, then it gets back to $x = A$ after one full period at $t = T$.**
- **Frequency (f): The number of cycles (or oscillations) the object completes per unit time.**

<u>Example</u>

An object oscillates with simple harmonic motion along x-axis. Its displacement is given as $x = 4(m)\cos(\pi t + \frac{\pi}{4})$**. Determine**

(a) The amplitude, frequency, period, velocity and acceleration at time t**.**

(b) Displacement, velocity and acceleration at time at $t = 1\sec$

(c) The maximum speed and maximum acceleration

(d) Find the displacement of the object between $t = 0\sec$ **and** $t = 1\sec$**.**

Solution

$$x = A\cos(\omega t + \phi)$$

$$x = 4\cos(\pi t + \frac{\pi}{4})$$

(a) The amplitude is $A = 4$

The frequency is $f = \dfrac{\omega}{2\pi} = \dfrac{\pi}{2\pi} = 0.5\sec^{-1}$

The period is $T = \dfrac{1}{f} = \dfrac{1}{0.5} = 2\sec$

The velocity is $v = \dfrac{dx}{dt} = \dfrac{d}{dt}\left(4\cos(\pi t + \dfrac{\pi}{4})\right) = -4\pi\sin(\pi t + \dfrac{\pi}{4})m/\sec$

The acceleration is $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(-4\pi\sin(\pi t + \dfrac{\pi}{4})\right) = -4\pi^2\cos(\pi t + \dfrac{\pi}{4})m/\sec^2$

(b) $x(t = 1) = 4\cos(\pi \times 1 + \dfrac{\pi}{4}) = 4\cos(\dfrac{5\pi}{4}) = -2.83m$

$$v(t=1) = -4\pi \sin(\pi \times 1 + \frac{\pi}{4}) = -4\pi \sin(\frac{5\pi}{4}) = 8.89 m/\sec$$

$$a(t=1) = -4\pi^2 \cos(\pi \times 1 + \frac{\pi}{4}) = -4\pi^2 \cos(\frac{5\pi}{4}) = 27.9 m/\sec^2$$

(c) Maximum speed $v_{\max} = A\omega = 4\pi = 12.56 m/\sec$

Maximum acceleration $a_{\max} = 4\pi^2 = 39.48 m/\sec^2$

(d) displacement of the object between $t = 0\sec$ **and** $t = 1\sec$

$$x(t=1) - x(t=0) = 4\cos(\frac{5\pi}{4}) - 4\cos(\frac{\pi}{4}) = -2.83 - 2.83 = -5.66 m$$

<u>Example</u>

The maximum acceleration of a particle vibrating in S.H.M is (a) and maximum velocity is (b) Calculate the amplitude and period of oscillation.

Solution

(a) Maximum speed $v_{\max} = A\omega = b$

Maximum acceleration $a_{\max} = a = A\omega^2$

$$\frac{a_{\max}}{v_{\max}} = \frac{a}{b} = \frac{A\omega^2}{A\omega} = \omega \qquad \frac{a}{b} = \omega$$

(b) The amplitude is $A = \frac{b}{\omega} = \frac{bb}{a} = \frac{b^2}{a}$

Period of oscillations is $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi b}{a}$

Example

A particle oscillates with simple harmonic motion, so that its displacement varies according to the expression $x(t) = 5(m)\cos(2t + \pi/6)$ **where** x **is in meters and** t **is in seconds. At** $t = 0$ **find (a) the displacement of the particle (b) its velocity (c) its acceleration (d) Find the period and amplitude of the motion.**

Solution

Since the displacement is $x(t) = A\cos(\omega t + \phi)$ **Here,**

$A = 5m$**, and** $\phi = \pi/6$**,** $\omega = 2s^{-1}$**, then** $x(t) = 5(m)\cos(2t + \pi/6)$

(a) The displacement at $t = 0$ **is** $x(0) = 5\cos(\pi/6) = 4.33m$**.**

(b) The velocity at $t = 0$ **is.** $v(0) = -2 \times 5\sin(\pi/6) = -5m/s$

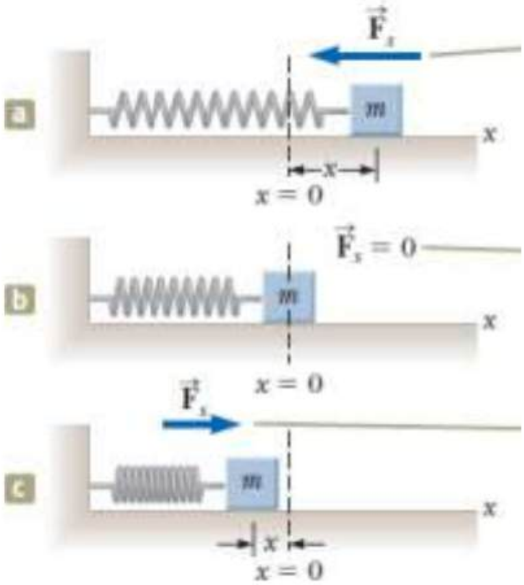
(c) The acceleration at $t = 0$ **is** $a(0) = -A\omega^2\cos(\pi/6) \quad = -17.3m/s^2$

(d) The period of the motion is $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi s$ **and the amplitude is 5 m.**

Applications of simple harmonic motion

(1) Mass attached to spring

Consider a mass m attached to a spring mounted on frictionless table. When the mass displaced from its equilibrium position there is a restoring force given by



$$F = -k\,x \qquad (5)$$

Where, k is the spring constant and x the displacement of the mass from its its equilibrium position. From Newton's second law

$$F = m\frac{d^2x}{dt^2} = -k\,x$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x$$

Where, $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of oscillating mass. Then the frequency of oscillation is given as

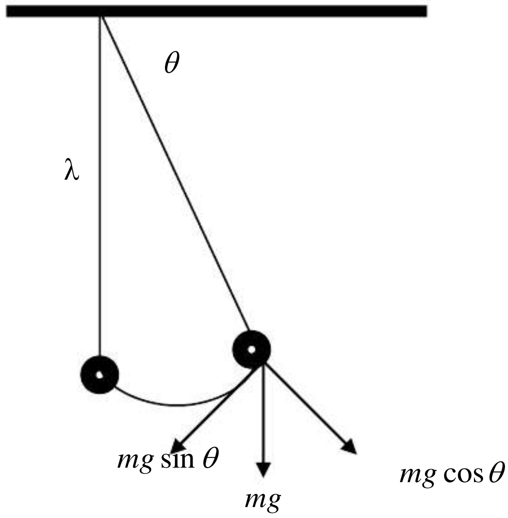
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (6)$$

The period of the oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \qquad (7)$$

(2) <u>The Simple Pendulum</u>

A simple pendulum consists of a small ball of mass m attached to a string of length λ. When this ball is displaced through a small angle θ., then the restoring force is



$$F = -mg\sin\theta$$

$$m\frac{d^2s}{dt^2} = -mg\sin\theta$$

For small θ, $\sin\theta \approx \theta$ and $s = \lambda\theta$ then

$$m\lambda\frac{d^2\theta}{dt^2} = -mg\theta \quad \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{g}{\lambda}\theta = -\omega^2\theta \qquad \textbf{(8)}$$

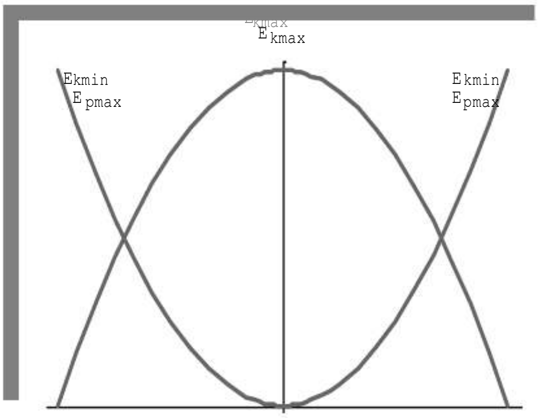
Where $\omega^2 = \frac{g}{\lambda}$ is the angular frequency of oscillating pendulum. The frequency is given as

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\lambda}} \qquad \textbf{(9)}$$

The period of the pendulum is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{\lambda}{g}} \qquad \textbf{(10)}$$

<u>Energy of the simple harmonic motion</u>



The total energy of the object E is the sum of kinetic energy K and potential energy U as

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}k\,x^2 \qquad \textbf{(11)}$$

Substituting equations (1) and (3) in (11) to get

$$E = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi) + \frac{1}{2}k\,A^2 \cos^2(\omega t + \phi)$$

Since, $\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2$ **then,**

$$E = \frac{1}{2}mA^2\omega^2 \sin^2(\omega t + \phi) + \frac{1}{2}mA^2\omega^2 \cos^2(\omega t + \phi) = \frac{1}{2}mA^2\omega^2 \qquad \textbf{(12)}$$

Then the total energy of the simple harmonic motion is constant of motion and is proportional to the square of the amplitude.

We note that

(i) When the potential energy reaches its maximum value ($U = \frac{1}{2}k\,A^2$), the kinetic energy is zero ($K = 0$).

(ii) When the potential energy reaches its minimum value ($U = 0$), the kinetic energy is maximum ($k = \frac{1}{2}k\,A^2$).

Example 4

A car of mass $1300\,kg$. is constructed using frame supported by four springs. Each spring has a force constant of $20000\,N/m$. If two people riding in the car have a combined mass of $160\,kg$. . Find the frequency of vibration of the car when it is driven over a path in the road.

Solution

$k = 20000\,N/m \qquad m = 1300 + 160 = 1460\,kg$

The mass of each spring is $m = \dfrac{1460}{4} = 365kg$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20000}{365}} = 1.18\,Hz$$

Example 5

A mass of $200\,g$. is connected to a light spring of force constant $5N/m$ and is free to oscillate on a horizontal frictionless track. If the mass is displaced $5cm$ from equilibrium and released from rest find

(i) The period of the motion

(ii) The maximum speed and maximum acceleration

(iii) The displacement, velocity and acceleration as a function of time.

Solution

(i) $m = 200\,gm = 0.2kg \qquad k = 5N/m \qquad A = 5cm = 0.05m$

$$T = 2\pi\sqrt{\frac{0.2}{5}} = 1.26\sec$$

(ii) $v_{\max} = A\omega \qquad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5}{0.2}}$

$$v_{\max} = A\omega = 0.05\times\sqrt{\frac{5}{.2}} = 0.25m/\sec$$

$$a_{\max} = A\omega^2 = 0.05\times\frac{5}{0.2} = 1.25\,m/\sec^2$$

(iii) $x = A\cos(\omega t) = 0.05\cos(5t)$

$$v = -A\omega\sin(\omega t) = -0.25\sin(5t)$$

$$a = -A\,\omega^2\cos(\,\omega t) = -1.25\cos(5t)$$

Example 6

A simple pendulum has a period of $2.5\sec$ **. (a)What is its length (b) what would its period on the moon where** $g_{moon} = 1.67\,m/\sec^2$ **.**

Solution

(a) $T = 2.5\sec \qquad \lambda = ?$

$$T = 2\pi\sqrt{\frac{\lambda}{g}} \qquad 2.5 = 2\pi\sqrt{\frac{\lambda}{9.8}} \qquad (2.5)^2 = 4\pi^2\frac{\lambda}{9.8}$$

$$\lambda = \frac{(2.5)^2\times 9.8}{4\pi^2} = 1.55m$$

(b) $g_{moon} = 1.67\, m/\sec^2$ $\lambda = 1.55 m$

$$T = 2\pi\sqrt{\frac{1.55}{1.67}} = 6.05\sec$$

Example 7

A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is $12\sec$. How tall the tower.

Solution

$\lambda = ?$ $T = 12\sec$

$$T = 2\pi\sqrt{\frac{\lambda}{g}} \qquad T^2 = 4\pi^2\frac{\lambda}{g}$$

$$\lambda = \frac{T^2 g}{4\pi^2} = \frac{(12)^2 \times 9.8}{4\pi^2} = 35.7\, m$$

Wave motion

Many of us experienced waves as children when we dropped a pebble into a pond. At the point the pebble hits the water's surface, circular waves are created. These waves move outward from the creation point in expanding circles until they reach the shore. If you were to examine carefully the motion of a small object floating on the disturbed water, you would see that the object moves vertically and horizontally about its original position but does not undergo any net displacement away from or toward the point at which the pebble hit the water. The small elements of water in contact with the object, as well as all the other water elements on the pond's surface, behave in the same way. That is, the water *wave* moves from the point of origin to the shore, but the water is not carried with it.

The world is full of waves, the two main types being *mechanical* waves and *electromagnetic* waves. In the case of mechanical waves, some physical medium is being disturbed; in our pebble example, elements of water are disturbed. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays. Here, in this part of the book, we study only mechanical waves.

Disturbance

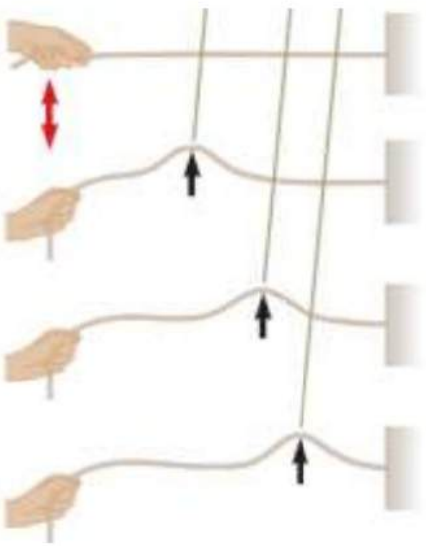
All mechanical waves require the following:

(1) some source of disturbance,

(2) a medium containing elements that can be disturbed, and

(3) some physical mechanism through which elements of the medium can influence each other.

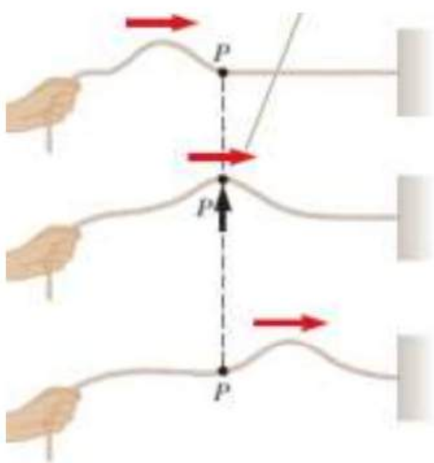
One way to demonstrate wave motion is to flick one end of a long string that is under tension and has its opposite end fixed as shown in Figure (1). In this manner, a single bump (called a *pulse*) is formed and travels along the string with a definite speed. Figure (1) represents four consecutive "snapshots" of the creation and propagation of the traveling



pulse. The hand is the source of the disturbance. The string is the medium through which the pulse travels—individual elements of the string are disturbed from their equilibrium position. Furthermore, the elements are connected together so they influence each other. The pulse has a definite height and a definite speed of propagation along the medium. The shape of the pulse changes very little as it travels along the string1. We shall first focus on a pulse traveling through a medium. Once we have explored the behavior of a pulse, we will then turn our attention to a *wave,* which is a *periodic* disturbance traveling through a medium. We create a pulse on our string by flicking the end of the string once as in Figure (1). If we were to move the end of the string up and down repeatedly, we would create a traveling wave, which has characteristics a pulse does not have.

As the pulse in Figure (1) travels, each disturbed element of the string moves in a direction *perpendicular* to the direction of propagation. Figure (2) illustrates this point for one particular element, labeled *P*. Notice that no part of the string ever moves in the direction of the propagation. A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a transverse wave.

Compare this wave with another type of pulse, one moving down a long, stretched spring as shown in Figure (3). The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring (to the right in Fig. 3). Notice that the direction of the displacement of the coils is *parallel* to the direction of propagation of the compressed region. A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a longitudinal wave.



Sound waves, are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travel through air. Some waves in nature exhibit a combination of

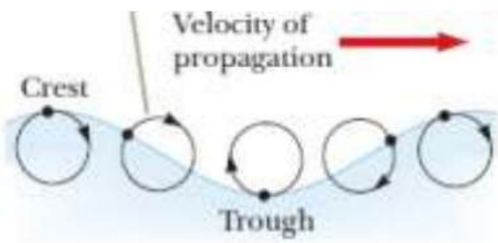
transverse and longitudinal displacements. Surface-water waves are a good example. When a water wave travels on the surface of deep water, elements of water at the surface move in nearly circular

paths as shown in Active Figure (4). The disturbance has both transverse and longitudinal components. The transverse displacements seen in Active Figure (4) represent the variations in vertical position of the water elements. The longitudinal

displacements represent elements of water moving back and forth in a horizontal direction.

The three-dimensional waves that travel out from a point under the Earth's surface at which an earthquake occurs are of both types, transverse and longitudinal. The longitudinal waves are the faster of the two, traveling at speeds in the range of 7 to 8 km/s near the surface. They are called P waves, with "P" standing for *primary,* because they travel faster than the transverse waves and arrive first at a seismograph (a device used to detect waves due to earthquakes). The slower transverse

waves, called S waves, with "S" standing for *secondary,* travel through the Earth at 4 to 5 km/s near the surface. By recording the time interval between the arrivals of these two types of waves at a seismograph, the distance from the seismograph to the point of origin of the waves can be determined. This distance is the radius of an imaginary sphere centered on the seismograph. The origin of



the waves is located somewhere on that sphere. The imaginary spheres from three or more monitoring stations located far apart from one another intersect at one region of the Earth and this region is where the earthquake occurred.

Consider a pulse traveling to the right on a long string as shown in Figure (5). Figure (5a) represents the shape and position of the pulse at time t=0. At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function that we will write as $y(x, 0)= f(x)$. This function describes the transverse position y of the element of the string located at each value of x at time t=0. Because the speed of the pulse is v, the pulse has traveled to the right a distance vt at the time t (Fig. 5b). We assume the shape of the pulse does not change with time. Therefore, at time t, the shape of the pulse is the same as it was at time t=0 as in Figure (5a). Consequently, an element of the string at x at this

time has the same y position as an element located at x-vt had at time t=0,

$$y(x,t) = y(x - vt, 0)$$

In general, then, we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x,t) = f(x - vt)$$

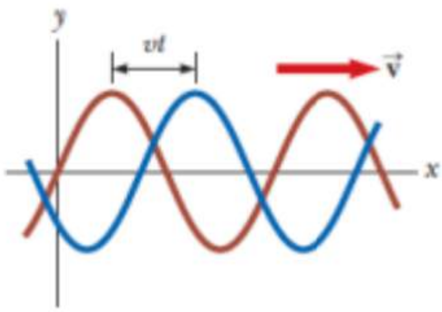
Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x,t) = f(x + vt)$$

The function *y*, sometimes called the wave function, depends on the two variables *x* and *t*. For this reason, it is often written *y*(*x*, *t*), which is read "*y* as a function of *x* and *t*."

Travelling waves

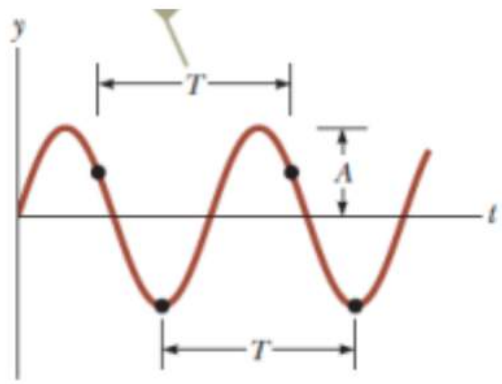
In this section, we introduce an important wave function whose shape is shown in active Figure (6). The wave represented by this curve is called a sinusoidal wave because the curve is the same as that of the function sin u plotted against u. A sinusoidal wave could be established on the rope in Figure (1) by shaking the end of the rope up and down in simple harmonic motion. The brown curve in active Figure (6) represents a snapshot of a traveling sinusoidal wave at *t*=0, and the blue curve represents a snapshot of the wave at some later time *t*. Imagine two types of motion that can occur. First, the entire waveform in Active Figure (6) moves to the right so that the brown curve moves toward the right and eventually reaches the position of the blue curve. This movement is the motion of the *wave*. If we focus on one element of the medium, such as the element at *x*=0, we see that



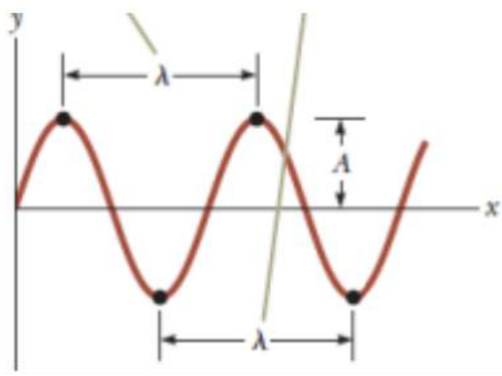
each element moves up and down along the *y* axis in simple harmonic motion. This movement is the motion of the *elements of the medium*. It is important to differentiate between the motion of the wave and the motion of the elements of the medium.

An ideal particle has zero size. We can build physical objects with nonzero size as combinations of particles. Therefore, the particle can be considered a basic building block. An ideal wave has a single frequency and is infinitely long; that is, the wave exists throughout the Universe.

In what follows, we will develop the principal features and mathematical representations of the analysis model of a traveling wave. This model is used in situations in which a wave moves through space without interacting with other waves or particles.



Active Figure (7a) shows a snapshot of a wave moving through a medium. Active Figure (7b) shows a graph of the position of one element of the medium as a function of time. A point in Active Figure (7a) at which the displacement of the element from its normal position is highest is called the crest of the wave. The lowest point is called the trough. The



distance from one crest to the next is called the wavelength λ (Greek letter lambda). More generally, the wavelength is the minimum distance between any two identical points on adjacent waves as in fig.(7).

If you count the number of seconds between the arrivals of two adjacent crests at a given point in space, you measure the period *T* of the waves. In general, the period is the time interval required for two identical points of adjacent waves to pass by a point as shown in Active Figure 7b. The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium. The same information is more often given by the inverse of the period, which is called the frequency *f*. In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that passes a given point in a unit time interval. The frequency of a sinusoidal wave is related to the period by the expression

$$f=1/T$$

The frequency of the wave is the same as the frequency of the simple harmonic oscillation of one element of the medium. The most common unit for frequency, is s^{-1}, or hertz (Hz). The corresponding unit for *T* is seconds.

The maximum position of an element of the medium relative to its equilibrium position is called the amplitude *A* of the wave as indicated in Active Figure 7. Waves travel with a specific speed, and

this speed depends on the properties of the medium being disturbed. For instance, sound waves travel through room temperature air with a speed of about 343 m/s (781 mi/h), whereas they travel through most solids with a speed greater than 343 m/s.

Consider the sinusoidal wave in Active Figure 16.8a, which shows the position of the wave at $t= 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Active Figure (7a). The next value of x for which y is zero is $x = \lambda /2$. Therefore,

$$y(x,0) = A\sin\left(\frac{a\lambda}{2}\right) = 0$$

For this equation to be true, we must have $a \lambda /2=\pi$, or $a=2\pi/\lambda$. Therefore, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written

$$y(x,0) = A\sin\left(\frac{2\pi}{\lambda}x\right)$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. Notice that the vertical position of an element of the medium is the same whenever x is increased by an integral multiple of λ. Based on our discussion of Equation 16.1, if the wave

moves to the right with a speed *v*, the wave function at some later time *t* is

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}x - vt\right)$$

If the wave were traveling to the left, the quantity *x-vt* would be replaced by *x+vt*. By definition, the wave travels through a displacement Δ*x* equal to one wavelength λ in a time interval Δ*t* of one period *T*. Therefore, the wave speed, wavelength, and period are related by the expression

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T}$$

Substituting this expression for *v* into y to get

$$y(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right)$$

This form of the wave function shows the *periodic* nature of *y*. Note that we will often use *y* rather than *y*(*x*, *t*) as a shorthand notation. At any given time *t*, *y* has the *same* value at the positions *x*, *x* +λ, *x*+2λ, and so on. Furthermore, at any given position *x*, the value of *y* is the same at times *t*, *t*+*T*, *t*+2*T*, and so on. We can express the wave function in a convenient form by defining two other quantities, the

angular wave number *k* (usually called simply the wave number) and the angular frequency v:

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Using these definitions, Equation 16.7 can be written in the more compact form

$$y(x,t) = A sin(kx - \omega t)$$

Then,

$v = \omega/k,\qquad v = \lambda f$

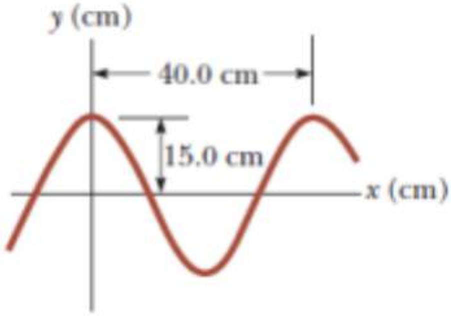
The wave function given by Equation 16.10 assumes the vertical position *y* of an element of the medium is zero at *x*= 0 and *t*=0. That need not be the case. If it is not, we generally express the wave function in the form

$$y(x,t) = A sin(kx - \omega t + \emptyset)$$

where $\emptyset$ is the phase constant. This constant can be determined from the initial conditions.

Example

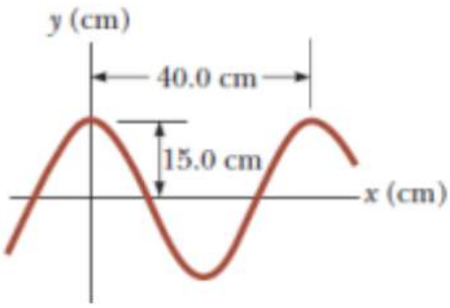
A sinusoidal wave traveling in the positive *x* direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at *t* 5 0 and *x* 5 0 is also



15.0 cm as shown in Figure (8).

(A) Find the wave number *k*, period *T*, angular frequency v, and speed *v* of the wave.

(B) Determine the phase constant f and write a general expression for the wave function.



Sinusoidal Waves on Strings

The wave function can be written as

$$y = A\sin(kx - \omega t)$$

We can use this expression to describe the motion of any element of the string. An element at point *P* (or any other element of the string) moves only vertically, and so its *x* coordinate remains constant. Therefore, the transverse speed v_y (not to be confused with the wave

speed v) and the transverse acceleration a_y of elements of the string are

$$v_y = \frac{\partial y}{\partial t} = -\omega A cos(kx - \omega t)$$

$$a_y = \frac{\partial v_y}{\partial t} = +\omega^2 A sin(kx - \omega t)$$

These expressions incorporate partial derivatives because y depends on both x and t. In the operation 'y/'t, for example, we take a derivative with respect to t while holding x constant. The maximum magnitudes of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

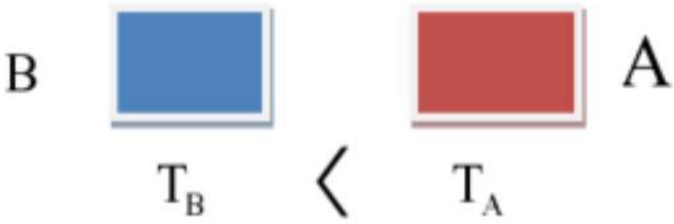
$$v_{ymax} = \omega A$$

$$a_{ymax} = +\omega^2 A$$

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (ωA) when y = 0, whereas the magnitude of the transverse acceleration reaches its maximum value ($+\omega^2 A$) when $y = \mp A$.

Temperature

Consider two objects A, B of different temperatures such that $T_A \rangle T_B$ placed in thermal contact. The two objects reach the same temperature (thermal equilibrium) as a result of heat transfer from hot object to cold object due to temperature difference. Temperature is a property that determines whether an object is in thermal equilibrium with other object.



Thermometer

Thermometer is a device that measures temperature. A thermometer consists of a mass of liquid (mercury) which expands in a glass capillary tube when heated.

Temperature scales

Temperature is measured with thermometers that may be calibrated to a variety of temperature scales.

i. Celsius scale $T(^\circ C)$

ii. Kelvin scale $T(^\circ K)$

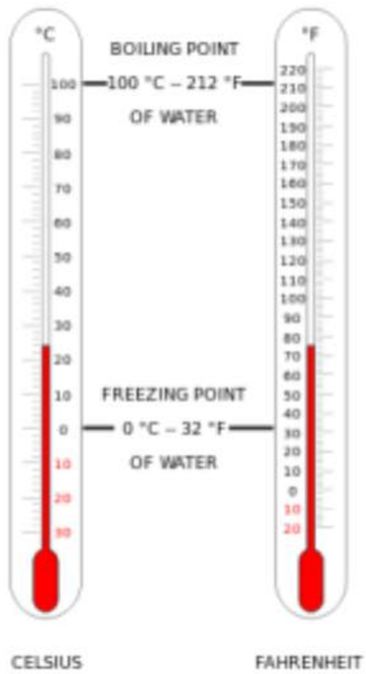
iii. Fahrenheit scale. $T(^\circ F)$

i. **The Celsius scale** ($T(^\circ C)$)

This system is a mixture of water and ice in thermal equilibrium at atmospheric pressure. The ice has a temperature of zero Celsius (0°C) which is called ice point of water. The boiling water has temperature (100°C). By dividing the scale between (0°C) and (100°C) into one hundred equal segments, this creates Celsius scale.

ii. **Absolute Kelvin scale** ($T(^\circ K)$)

This Kelvin scale based on the variation of pressure with temperature at constant volume. The pressure of the gas is zero at $T = -273^\circ C$**. This temperature value is called absolute zero or** $T = 0^\circ K = -273^\circ C$**. *The relation between* Celsius scale and Kelvin scale is**



$$T(^\circ K) = T(^\circ C) + 273$$

iii. **Fahrenheit scale** ($T(^\circ F)$)

A common temperature scale used in USA is the Fahrenheit scale. In this scale, the ice point is $32^\circ F$ and the boiling point is $212^\circ F$. The relation between Celsius and Fahrenheit scales is

$$T(^\circ F) = \frac{9}{5}T(^\circ C) + 32$$

$$T(^\circ C) = \frac{5}{9}\left(T(^\circ F) - 32\right)$$

Example: 10

Convert $50^\circ F$ into $^\circ C$ and $^\circ K$.

$$T(^\circ C) = \frac{5}{9}\left(T(^\circ F) - 32\right) = \frac{5}{9}(50 - 32) = \frac{5}{9}18 = 10^\circ C$$

$$T(^\circ K) = T(^\circ C) + 273 = 10 + 273 = 283^\circ K$$

Heat and the first law of thermodynamics

<u>Heat and Internal Energy</u>

At the outset, it is important that we make a major distinction between internal energy and heat. Internal energy is all the energy of a system that is associated with its microscopic components (atoms and molecules). Internal energy includes kinetic energy of random translational, rotational, and vibrational motion of molecules, potential energy within molecules, and potential energy between molecules.

Heat is defined as the transfer of energy across the boundary of a system due to a temperature difference between the system and its surroundings. When you *heat* a substance, you are transferring energy into it by placing it in contact with surroundings that have a higher temperature. This is the case, for example, when you place a

pan of cold water on a stove burner—the burner is at a higher temperature than the water, and so the water gains energy. We shall also use the term *heat* to represent the amount of energy transferred by this method.

As an analogy to the distinction between heat and internal energy, consider the distinction between work and mechanical energy discussed before. The work done on a system is a measure of the amount of energy transferred to the system from its surroundings, whereas the mechanical energy of the system (kinetic plus potential) is a consequence of the motion and configuration of the system. Thus, when a person does work on a system, energy is transferred from the person to the system. It makes no sense to talk about the work *of* a system—one can refer only to the work done *on* or *by* a system when some process has occurred in which energy has been transferred to or from the system. Likewise, it makes no sense to talk about the heat *of* a system—one can refer to *heat* only when energy has been transferred because of a temperature difference.

Both heat and work are ways of changing the energy of a system. It is also important to recognize that the internal energy of a system can be changed even when no energy is transferred by heat. For example, when a gas in an insulated container is compressed by a piston, the temperature of the gas and its internal energy increase, but no transfer of energy by heat from the surroundings to the gas has occurred. If the gas then expands rapidly, it cools and its internal energy decreases, but no transfer of energy by heat from it to the surroundings has taken place. The temperature changes in the gas are due not to a difference in temperature between the gas and its

surroundings but rather to the compression and the expansion. In each case, energy is transferred to or from the gas by *work*. The changes in internal energy in these examples are evidenced by corresponding changes in the temperature.

Units of Heat

We have an energy unit related to thermal processes, the calorie (cal), which is defined as the amount of energy transfer necessary to raise the temperature of 1 g of water from 14.5°C to 15.5°C.1 (Note that the "Calorie," written with a capital "C" and used in describing the energy content of foods, is actually a kilocalorie.).

Scientists are increasingly using the SI unit of energy, the *joule,* when describing thermal processes. In this book, heat, work, and internal energy are usually measured in joules.

The Mechanical Equivalent of Heat

We found that whenever friction is present in a mechanical system, some mechanical energy is lost. This lost mechanical energy does not simply disappear but is transformed into internal energy. Joule found that the loss in mechanical energy is proportional to the increase in temperature ΔT . The proportionality constant was found to be approximately 4.18 J/g °C. Hence, 4.18 J of mechanical energy raises the temperature of 1 g of water by 1°C. Then

$$1 Cal = 4.186 Joule$$

This equality is known, as the mechanical equivalent of heat.

Specific Heat and Calorimetry

When energy is added to a system and there is no change in the kinetic or potential energy of the system, the temperature of the system usually rises. (An exception when a system undergoes a change of state called a *phase transition*). If the system consists of a sample of a substance, we find that the quantity of energy required to raise the temperature of a given mass of the substance by some amount varies from one substance to another. The heat capacity *C* of a particular sample of a substance is defined as the amount of energy needed to raise the temperature of that sample by 1°C. From this definition, we see that if energy *Q* produces a change ΔT in the temperature of a sample, then

$$Q = C\Delta T \qquad \textbf{(7.1)}$$

The specific heat c of a substance is the heat capacity per unit mass. Thus, if energy *Q* transfers to a sample of a substance with mass *m* and the temperature of the sample changes by ΔT, then the specific heat of the substance is

$$c = \frac{Q}{m\Delta T} \qquad \textbf{(7.2)}$$

Specific heat is essentially a measure of how thermally insensitive a substance is to the addition of energy. The greater a material's specific heat, the more energy must be added to a given mass of the material to cause a particular temperature change. Then

$$Q = mc\Delta T \qquad \textbf{(7.3)}$$

Note

1. **The energy required to raise the temperature of 0.500 kg of water by 3.00°C is (0.500 kg)(4 186 J/kg .°C)(3.00°C)= 6.28 x 10^3 J. When the temperature increases, *Q* and ΔT are taken to be positive, and energy transfers into the system. When the temperature decreases, *Q* and ΔT are negative, and energy transfers out of the system.**
2. **Specific heat varies with temperature. However, if temperature intervals are not too large, the temperature variation can be ignored and *c* can be treated as a constant. For example, the specific heat of water varies by only about 1% from 0°C to 100°C at atmospheric pressure. Unless stated otherwise, we shall neglect such variations.**

Conservation of Energy: Calorimetry

One technique for measuring specific heat involves heating a sample to some known temperature T_x, placing it in a vessel containing water of known mass and temperature $T_w \langle T_x$, and measuring the temperature of the water after equilibrium has been reached. This technique is called calorimetry, and devices in which this energy transfer occurs are called calorimeters. If the system of the sample and the water is isolated, the law of the conservation of energy requires that the amount of energy that leaves the sample (of unknown specific heat) equal the amount of energy that enters the water.

Conservation of energy allows us to write the mathematical representation of this energy statement as

$$Q_{cold} = -Q_{hot} \tag{7.4}$$

Suppose m_x is the mass of a sample of some substance whose specific heat we wish to determine. Let us call its specific heat c_x and its initial temperature T_x. Likewise, let, m_w, c_w, and T_w represent corresponding values for the water. If T_f is the final equilibrium temperature after everything is mixed, we find that the energy transfer for the water is $m_w c_w (T_f - T_w)$ which is positive because $T_f \rangle T_w$ and that the energy transfer for the sample of unknown specific heat is $m_x c_x (T_f - T_x)$, which is negative. Then,

$$m_w c_w (T_f - T_w) = -m_x c_x (T_f - T_x) \tag{7.5}$$

Solving for c_x gives

$$c_x = -\frac{m_w c_w (T_f - T_w)}{m_x (T_f - T_x)} \tag{7.6}$$

Example:

A 0.050 0-kg ingot of metal is heated to 200.0°C and then dropped into a beaker containing 0.400 kg of water initially at 20.0°C. If the final equilibrium temperature of the mixed system is 22.4°C, find the specific heat of the metal.

Solution

$$m_w c_w (T_f - T_w) = -m_x c_x (T_f - T_x)$$

(0.400 kg)(4186)(22.4 - 20.0)= -(0.05)(200.0) -)(22.4c_x

$$c_x = 453 J / Kg.^oC$$

Latent heat

Substance undergoes a change in temperature when energy transfers between it and its surroundings. In some situations, the transfer energy does not result in a change of temperature such as phase change. Two common phase changes:

(i) From solid to liquid (melting).

(ii) From liquid to gas (boiling).

***All phase changes involve a change in internal energy but no change in temperature (for example, the increase in internal energy in boiling is represented by breaking of bonds between molecules in liquid state which allows the molecules to move further apart in the gas state with a corresponding increase in intermolecular potential energy*). Then the substance responds to addition or removal of energy as it changes phase. If** Q **is the amount of energy transfer needed to change the phase of a mass** m **of substance, then, the latent heat (L) is given as**

$$L = \pm \frac{Q}{m} \quad \textbf{(7.7)}$$

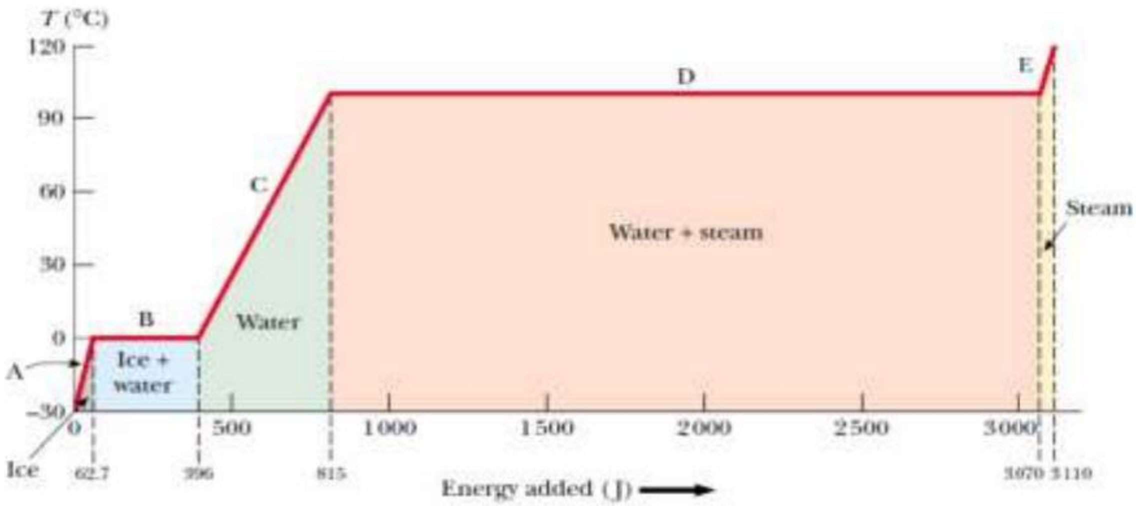
Positive (+ve.) sign means energy enters the system causing melting or vaporization.

Negative (-ve.) sign means energy leaving the system causing freezing or condensation.

To understand the role of latent heat in phase changes, consider the energy required to convert 1gm of ice at -30^oC *to steam at* 120^oC

Part A: T of ice changes from -30^oC ***to*** 0^oC **. The amount of energy added is given as** $Q = m_i c_i \Delta T = 1\times 10^{-3} \times 2090 \times 30 = 62.7J$ **(where** $c_i = 2090 J/Kg.^oC$ **).**

Part B: **When the temperature of the *ice re*aches** 0^oC **the ice-water mixture remains at this temperature until all the ice melts. The energy required to melt 0.1 gm of ice at** 0^oC **is**



$$Q = m_i L_f = 1\times10^{-3}\times3.33\times10^{5} = 333J$$

Part C: **Between 0^oC and 100^oC, no phase change occurs, so all energy added to the *water* is used to increase the temperature. The amount of energy required to increase the temperature rom 0^oC *to* 100^oC is**

$$Q = m_w c_w \Delta T = 1\times10^{-3}\times4.19\times10^{3}\times100 = 419J$$

Part D: **At 100.0°C, another phase change occurs as the water changes from water at 100.0°C to steam at 100.0°C. Similar to the ice–water mixture in part B, the water–steam mixture remains at 100.0°C (even though energy is being added) until all of the liquid has been converted to steam. The energy required to convert 1.00 g of water to steam at 100.0°C is**

$$Q = m_w L_w = 1\times10^{-3}\times2.26\times10^{6} = 2.26\times10^{3}J$$

Part E: **On this portion of the curve, as in parts A and C, no phase change occurs; thus, all energy added is used to increase the temperature of the steam. The energy that must be added to raise the temperature of the steam from 100.0°C to 120.0°C is**

$$Q = m_s c_s \Delta T = 1\times10^{-3}\times2.01\times10^{3}\times20 = 40.2J$$

We note that

(1)* L*arge energy transferred into water to vaporize it to steam

(2) **R***everse process large amount of energy transferred out of steam to condense it into water*

We can describe phase changes in terms of a rearrangement of molecules when energy is added to or removed from a substance. For the liquid-to-gas phase change, the molecules in a liquid are close together, and the forces between them are stronger than those between the more widely separated molecules of a gas. Therefore, work must be done on the liquid against these attractive molecular forces if the molecules are to separate. The latent heat of vaporization is the amount of energy per unit mass that must be added to the liquid to accomplish this separation.

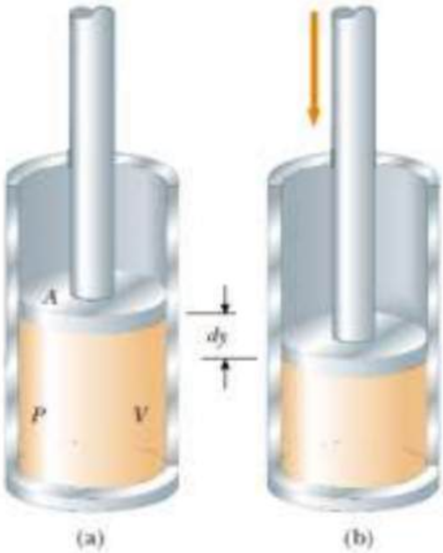
Similarly, for a solid, we imagine that the addition of energy causes the amplitude of vibration of the molecules about their equilibrium positions to become greater as the temperature increases. At the melting point of the solid, the amplitude is great enough to break the bonds between molecules and to allow molecules to move to new positions. The molecules in the liquid also are bound to each other, but less strongly than those in the solid phase. The latent heat of fusion is equal to the energy required per unit mass to transform the bonds among all molecules from the solid-type bond to the liquid-type bond.

<u>Work and Heat in Thermodynamic Processes</u>

The *state* of a system described by variables as pressure, volume, temperature, and internal energy are called state variables. It is important to note that a *macroscopic state* of an isolated system can be specified only if the system is in thermal equilibrium internally. In

the case of a gas in a container, internal thermal equilibrium requires that every part of the gas be at the same pressure and temperature.

A second category of variables in situations involving energy is transfer variables. These variables are zero *unless* a process occurs in which energy is transferred across the boundary of the system.

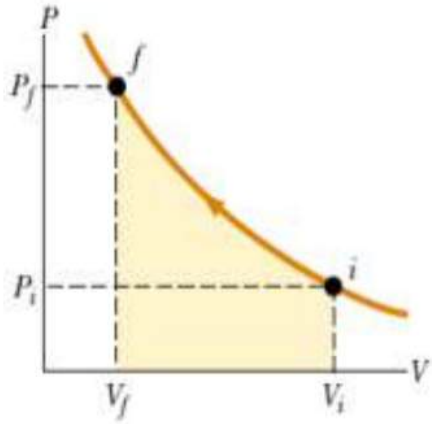


<u>Work done on the gas</u>

Here, we investigate the work done on a gas. Consider a gas contained in a cylinder fitted with a movable piston (figure). At equilibrium, the gas occupies a volume *V* and exerts a uniform pressure *P* on the cylinder's walls and on the piston. If the piston has a cross-sectional area *A*, the force exerted by the gas on the piston is *F*= *PA*. Now, let us assume that we push the piston inward and compress the gas quasi-statically, that is, slowly enough to allow the system to remain essentially in thermal equilibrium at all times. As the piston is pushed downward by an external force F through a displacement dy, the work done on the gas is

$$dW = F.dy = -Fdy = -PAdy$$

Because Ady is the change in volume of the gas dV, we can express the work done on the gas as



$$dW = -PdV$$

If the gas is compressed, dV is negative and the work done on the gas is positive. If the gas expands, dV is positive and the work done on the gas is negative. If the volume remains constant, the work done on the gas is zero.

The total work done on the gas as its volume changes from V_i to V_f is given by the integral

$$W = -\int_{V_i}^{V_f} PdV \qquad \textbf{(7. 8)}$$

To evaluate this integral, one must know how the pressure varies with volume during the process.

In general, the pressure is not constant during a process followed by a gas, but depends on the volume and temperature. If the pressure and volume are known at each step of the process, the state of the gas at each step can be plotted on a graph called a PV diagram, as in Figure. This type of diagram allows us to visualize a process through which a gas is progressing. The curve on a *PV* diagram is called the *path* taken between the initial and final states.

Note that the integral $W = -\int_{V_i}^{V_f} P dV$ is equal to the area under a curve on a *PV* diagram. Thus, we can identify an important use for *PV* diagrams:

The work done on a gas in a quasi-static process that takes the gas from an initial state to a final state is the negative of the area under the curve on a PV diagram, evaluated between the initial and final states.

The First Law of Thermodynamics

The first law of thermodynamics relates the *internal energy, work and amount of heat* providing the conservation of energy.

Suppose that a system undergoes a change from an initial state to a final state. During this change, energy transfer by heat *Q* to the system occurs, and work *W* is done on the system. As an example, suppose that the system is a gas in which the pressure and volume change from P_i and V_i to P_f and V_f . We conclude that the quantity *Q*+*W* is the change in the internal energy of the system. If we use the symbol E_{int} to represent the internal energy, then the *change* in internal energy ΔE_{int} can be expressed as

$$E_{\text{int}} = Q + W \qquad \textbf{(7. 9)}$$

This is known as the first law of thermodynamics.

Note:

1. **When a system undergoes an infinitesimal change in state in which a small amount of energy *dQ* is transferred by heat and a**

small amount of work dW is done, the internal energy changes by a small amount dE_{int}. Thus,

$$dE_{\text{int}} = dQ + dW \tag{7. 10}$$

2. **For *isolated system* (no interaction with its surroundings), no energy transfer by heat takes place and the work done on the system is zero; hence, the internal energy remains constant. That is, because *Q*= *W*= 0, it follows that $\Delta E_{\text{int}} = 0$. We conclude that the internal energy E_{int} of an isolated system remains constant.**
3. **Consider the case of a system (one not isolated from its surroundings) that is taken through a cyclic process—that is, a process that starts and ends at the same state. In this case, the change in the internal energy must again be zero $\Delta E_{\text{int}} = 0$. Then, the energy *Q* added to the system must equal the negative of the work *W* done on the system during the cycle. That is, in a cyclic process, $\Delta E_{\text{int}} = 0$ and Q=-W.**

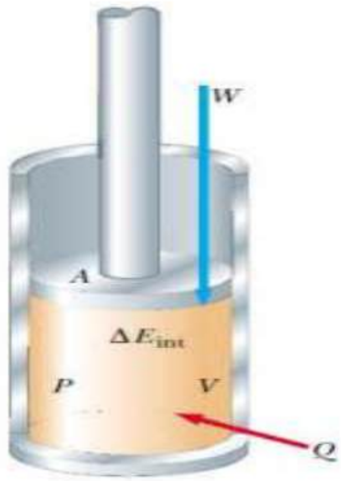
Some applications of the First Law of Thermodynamics

The first law of thermodynamics relates the changes in internal energy of system to transfers of energy by work or heat. In this section, we consider applications of the first law to processes through which a gas is taken. As a model, we consider the sample of gas contained in the piston–cylinder apparatus. This shows work being

done on the gas and energy transferring in by heat, so the internal energy of the gas is rising. We consider the following processes:

1. ***An adiabatic process*** **is one during which no energy enters or leaves the system by heat—that is, $Q= 0$. An adiabatic process can be achieved either by thermally insulating the walls of the system, such as the cylinder in Figure, or by performing the process rapidly, so that there is negligible time for energy to transfer by heat. Applying the first law of thermodynamics to an adiabatic process, we see that**

   $\Delta E_{int} = W$ **(adiabatic process)**

   From this result, we see that if a gas is compressed adiabatically such that W is positive, then ΔE_{int} is positive and the temperature of the gas increases. Conversely, the temperature of a gas decreases when the gas expands adiabatically.

   Adiabatic processes are very important in engineering practice. Some common examples are the expansion of hot gases in an internal combustion engine, the liquefaction of gases in a cooling system, and the compression stroke in a diesel engine.

2. **A process that occurs at constant pressure is called an *isobaric process*. An isobaric process could be established by allowing the piston to move freely so that it is always in equilibrium**

between the net force from the gas pushing upward and the weight of the piston plus the force due to atmospheric pressure pushing downward. The work done on the gas in an isobaric process is simply

$W = -P(V_f - V_i)$ (isobaric process)

where P is the constant pressure.

3. A process that takes place at constant volume is called an isovolumetric process. In Figure, clamping the piston at a fixed position would ensure an isovolumetric process. In such a process, the value of the work done is zero because the volume does not
change. Hence, from the first law we see that in an isovolumetric process, because $W = 0$,

$\Delta E_{\text{int}} = Q$ (isovolumetric process)

This expression specifies that if energy is added by heat to a system kept at constant volume, then all of the transferred energy remains in the system as an increase in its internal energy.

4. A process that occurs at constant temperature is called an isothermal process. The internal energy of an ideal gas is a function of temperature only. Hence, in an isothermal process involving an ideal gas, $\Delta E_{\text{int}} = 0$ and Q=-W.

Isothermal Expansion of an Ideal Gas

Suppose that an ideal gas is allowed to expand quasi-statically at constant temperature. Let us calculate the work done on the gas in the expansion from state *i* to state *f*. Because the gas is ideal and the process is quasi-static, we can use the expression *PV= nRT* for each point on the path.

Therefore, we have

$$W = -\int_{V_i}^{V_f} P dV = -\int_{V_i}^{V_f} \frac{nRT}{V} dV \qquad \textbf{(7. 11)}$$

Because *T* is constant in this case, it can be removed from the integral along with *n* and *R*:

$$W = -nRT\int_{V_i}^{V_f} \frac{dV}{V} = -nRT \ln V\Big|_{V_i}^{V_f} = nRT\ln\left(\frac{V_i}{V_f}\right) \qquad \textbf{(7. 12)}$$

Example:

A 1.0-mol sample of an ideal gas is kept at 0.0°C during an expansion from 3.0 L to 10.0 L. (a) How much work is done on the gas during the expansion? (b) How much energy transfer by heat occurs with the surroundings in this process? (c) If the gas is returned to the original volume by means of an isobaric process, how much work is done on the gas?

$$W = nRT\ln\left(\frac{V_i}{V_f}\right)$$

$$W = 1.\times 8.31\times 273\ln\left(\frac{3L}{10L}\right) = -2.7\times 10^3\, J$$

(b) $\Delta E_{\text{int}} = Q + W$

$0 = Q + W \qquad Q = -W = 2.7 \times 10^3 J$

(c) $W = -P(V_f - V_i) = -\frac{nRT_i}{V_i}(V_f - V_i)$ $\boldsymbol{V_i} = 10\boldsymbol{L}$ **,** $\boldsymbol{V_f} = 3\boldsymbol{L}$

$$W = -\frac{1 \times 8.31 \times 273}{10 \times 10^{-3}} \times (3 \times 10^{-3} - 10 \times 10^{-3}) = 1.6 \times 10^3\ J$$

Example:

Suppose 1.00 g of water vaporizes isobarically at atmospheric pressure ($1.013 \times 10^5 Pa$). Its volume in the liquid state is $V_t = V_{liquid} = 1cm^3$, and its volume in the vapor state is $V_f = V_{lvapor} = 1671cm^3$. Find the work done in the expansion and the change in internal energy of the system. Ignore any mixing of the steam and the surrounding air—imagine that the steam simply pushes the surrounding air out of the way.

$$W = -P(V_f - V_i) = -1.013 \times 10^5 (1671 \times 10^{-6} - 1 \times 10^{-6})$$

$$W = -169J$$

Example:

A 1.0-kg bar of copper is heated at atmospheric pressure. If its temperature increases from 20°C to 50°C, (a) what is the work done on the copper bar by the surrounding atmosphere? (b) What

quantity of energy is transferred to the copper bar by heat?(c) What is the increase in internal energy of the copper bar?

$$\Delta V = \beta V_i \Delta T = 5.1\times 10^{-5}(50-20)V_i = 1.5\times 10^{-3} V_i$$

$$V_i = m/\rho, \qquad \rho = 8.92\times 10^3\ Kg/m^3$$

$$\Delta V = 5.1\times 10^{-5}\left(\frac{1}{8.92\times 10^3}\right) = 1.7\times 10^{-7}\ m^3$$

$$W = -P\Delta V = -1.013\times 10^5 \times 1.7\times 10^{-7} = -1.7\times 10^{-2} J$$

$$(b)\ Q = mc\Delta T = 1\times 387\times 30 = 1.2\times 10^4 J$$

$$(c)\ \Delta E_{\text{int}} = QW = -12\times 10^4 + (-1.7\times 10^{-2} = 1.2\times 10^4 J$$

Heat transfer

There are three methods of heat transfer, by conduction, by convection and by radiation.

i. Thermal conduction

Thermal conduction is the process of energy transfer by heat. In this process, transfer can be represented by an exchange of kinetic energy between molecules, atoms and free electrons. To understand thermal conduction process, consider one end of a long rod is inserted into a flame. The molecules of the rod are vibrating about their equilibrium positions. As the flame heats the rod, the molecules near the flame begin to vibrate with greater and greater amplitudes and collide with

neighbors and transfer some of their energy. Thus heat energy transfers to the other end of the rod by the molecules of the rod.

If a rod of length L**, cross-sectional area** A **and has a temperature** T_h **at one end and** T_c **at the other end such that** $T_h \rangle T_c$ **. The heat energy transfers from the hot end to the cold end with a rate (or power P)**

$$P = \frac{\Delta Q}{\Delta t} \; \alpha \; \frac{A\,(T_h - T_c)}{L}$$

$$P = k\,\frac{A\,(T_h - T_c)}{L} \qquad \textbf{(Joule/sec)} \qquad \textbf{(7. 13)}$$

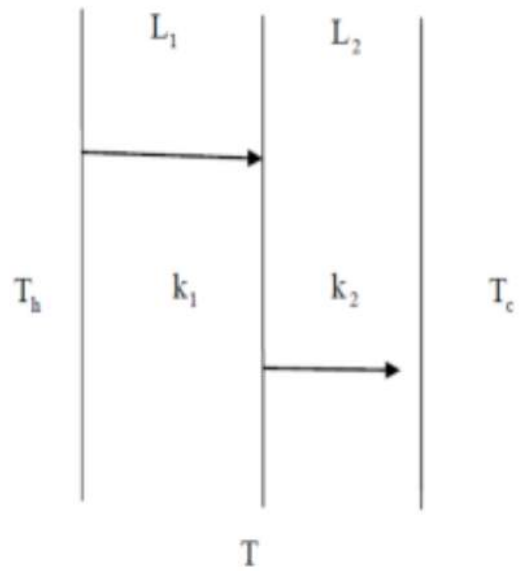
Where, k is the thermal conductivity of the rod. For a slab consists of different materials of thicknesses $L_1, L_2, ...$ **and thermal conductivities** $k_1, k_2, ...$**the total power is given as**

$$P = \frac{A\,(T_h - T_c)}{\sum_i (L_i / k_i)} \qquad \textbf{(7.14)}$$

Example:

Two slabs of thickness L_1 and L_2 and thermal conductivities k_1 and k_2 are in thermal contact with each other, as shown in Fig. 1. The temperatures of their outer surfaces are T_h and T_c, and $T_h \rangle T_c$. (a) Determine the temperature at the interface (b) The rate of energy transfer by conduction through the slabs in the steady state condition.

(a) The rate at which energy is transferred through slab 1 is



$$P_1 = k_1 \frac{A\,(T_h - T)}{L_1}$$

(7.15)

The rate at which energy is transferred through slab 1 is

$$P_2 = k_2 \frac{A\,(T - T_c)}{L_2}$$

(7.16)

In the steady state $P_1 = P_2 = P$ such that,

$$k_1 \frac{A\,(T_h - T)}{L_1} = k_2 \frac{A\,(T - T_c)}{L_2}$$

$$k_1\, L_2\, (T_h - T) = k_2\, L_1 (T - T_c)$$

$$(k_2 L_1 + k_1 L_2)T = k_2 L_1 T_c + k_1 L_2 T_h$$

Then the temperature at the interface is

$$T = \frac{k_2 L_1 T_c + k_1 L_2 T_h}{(k_2 L_1 + k_1 L_2)} \qquad \textbf{(7. 17)}$$

(b) The rate of energy transfer by conduction through the slabs in the steady state condition is obtained by substituting (7. 17) into (7. 14)

$$P = \frac{k_1 A}{L_1(k_2 L_1 + k_1 L_2)}\left(T_h k_2 L_1 + T_h k_1 L_2 - k_2 L_1 T_c - k_1 L_2 T_h\right)$$

$$P = \frac{k_1 A}{(k_2 L_1 + k_1 L_2)} k_2 \left(T_h - T_c\right) = \frac{A\left(T_h - T_c\right)}{(L_1/k_1 + L_2/k_2)} \qquad \textbf{(7. 18)}$$

ii. Convection

Convection means that heat energy transfers by the movement of warm substance. Hot air rises above because of its lower density while cold air descends down due to its greater density. Two types of convection

(i) Natural convection which results from the difference in density (example; boiling water, formation of clouds,…..).

(ii) **Forced convection which results when heated substance is forced to move by fan or pump.**

iii. <u>Radiation</u>

This process of heat transfer is totally different from conduction and convection processes. All objects radiates energy in the form of electromagnetic waves which results from vibrations of object molecules. The thermal radiation has properties

(i) **it represents electromagnetic waves which propagates in vacuum (no need for medium).**

(ii) **it obeys laws of reflection and refraction**

(iii) **it obeys inverse square law(its intensity inversely proportional to the square of the distance from the source).**

(iv) **it obeys to the laws of diffraction, polarization and interference.**

<u>Black body radiation</u>

Any object emits thermal radiation from its surface. The thermal radiation consists of distribution of wavelengths of the electromagnetic spectrum.

Ideal absorber

Is the object that absorbs all energy incident on it $(e = 1)$ which is called black body.

Ideal reflector

Is the object that reflects all energy incidents on it ($e = 0$).

Black body

Is an ideal body that absorbs all radiation incidents on it. A black body can be approximated by, for example, an oven: a cavity surround by walls at temperature T and with a small opening through which light can enter and leave. As all light that enters is absorbed, so the spectrum of the light that comes out will be almost entirely a function of its temperature alone. A plot of the amount of energy inside the oven per unit volume per unit frequency interval versus frequency, at a temperature T, is called the blackbody curve.

Stefan's law

"The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature"

$P = \sigma A e T^4$

(Watt) (7. 19)

Where, P is the rate of energy radiated,

σ **is Stefan's constant** $\sigma = 5.6696 \times 10^{-8} Watt / m^{2\,o} k^4$

A **is the surface area of the object**

e **is the emissivity (ranges from 0 to 1)**

T **is the absolute temperature in** $^o k$

As an object radiates energy, it also absorbs electromagnetic radiation from surroundings. If the temperature of the surroundings is T_o, then

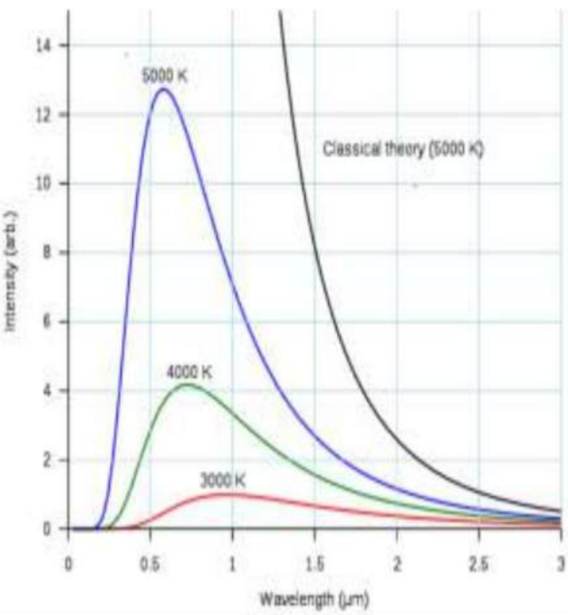
$$P = \sigma A e (T^4 - T_o^4)$$

Wien's displacement law

Wien's displacement law shows how the spectrum of black body radiation at any temperature is related to the spectrum at any other temperature. This law states that "the maximum wavelength (λ_{max}) at the peak intensity of the radiation produced by a black body is a function only of the temperature", then

$$\lambda_{max} = \frac{b}{T}$$

Where $b\,(= 2.898 \times 10^{-3}\,m.^{o}K)$ is known as Wien's displacement constant. Thus, the peak is displaced to shorter wavelengths as temperature increases.



Prevost's theory of heat exchange

Prevost stated that "all bodies emit radiant energy without any relation with the surrounding temperature".

According to this theory, it is noted that

(1) Every object emits radiant energy at all temperatures except at absolute zero temperature.

(2) The rate of emission increases with an increase in the temperature of the body and is independent of the temperature of the surroundings.

(3) Due to the exchange of heat energy between an object and its surroundings, there is a change in the temperature of the object.

Consider an isolated system of two objects maintained at two different temperatures. The object at high temperature loses more heat due to radiation and gains less heat due to absorption. The other object at lower temperature gains more heat due to absorption and loses lesser heat due to radiation. The exchange of heat energy between these two objects continues until the system attains an equilibrium temperature. The equilibrium state does not mean the stoppage of exchange but the system continues absorption and emission with equal magnitudes.

Thermal expansion of solids

Thermal expansion means that the volume of the substance increases as its temperature increases. It is important in engineering applications such as joints in buildings, brick walls, bridges ….etc.

Thermal expansion is the result of the change in the average separation between the atoms in an object. This understood as

follows; atoms in the object are separated from each other with distance about $10^{-10} m$. As the temperature of the solid increases the atoms oscillate with greater amplitudes; as a result, the average separation between the atoms increases. Consequently, the object expands. There are three types of expansion,

i. Linear expansion

If an object of initial length L_i and initial temperature T_i is heated to temperature T_f ,its length will increases to L_f . Then the object will elongated by an amount ΔL as a result of temperature increase by $\Delta T = (T_f - T_i)$ by

$$\Delta L = L_f - L_i = \alpha L_i (T_f - T_i) \qquad \textbf{(7. 20)}$$

Where, α $({}^oC^{-1})$ is the average coefficient of the linear expansion.

ii. Surface expansion

For an object of initial area A_i , the area will change by $\Delta A = (A_f - A_i)$ as temperature increases by $\Delta T = (T_f - T_i)$, then

$$\Delta A = A_f - A_i = \gamma A_i (T_f - T_i) \qquad \textbf{(7. 21)}$$

Where, γ $({}^oC^{-1})$ is the average coefficient of the surface expansion.

iii. Volume expansion

The initial volume of the object V_i increases by an amount $\Delta V = V_f - V_i$ as its temperature increases by $\Delta T = (T_f - T_i)$,

$$\Delta V = V_f - V_i = \beta V_i (T_f - T_i) \tag{7. 22}$$

Where, β $({}^oC^{-1})$ is the average coefficient of the volume expansion.

<u>Relation between Linear, Surface and</u>

<u>Volume expansion coefficients</u> (α, γ and β)

For isotropic cube of initial length L_i, initial area A_i and initial volume V_i material where temperature is propagated equally in all parts of the material, then we can get the relation between volume expansion coefficient β and linear expansion coefficient α as follows,

$$\Delta L = \alpha L_i \Delta T$$

$$\Delta A = \gamma A_i \Delta T$$

$$\Delta V = \beta V_i \Delta T$$

$$V_i + \Delta V = (L_i + \Delta L)(L_i + \Delta L)(L_i + \Delta L)$$

$$V_i + \Delta V = (L_i + \alpha L_i \Delta T)(L_i + \alpha L_i \Delta T)(L_i + \alpha L_i \Delta T)$$

$$V_i + \Delta V = L_i^3 (1 + \alpha \Delta T)(1 + \alpha \Delta T)(1 + \alpha \Delta T)$$

$$V_i + \Delta V = V_i(1 + 3\alpha^2 \Delta T^2 + 3\alpha\Delta T + \alpha^3 \Delta T^3)$$

$$\frac{\Delta V}{V_i} = 3\alpha^2 \Delta T^2 + 3\alpha\Delta T + \alpha^3 \Delta T^3$$

For $\alpha\Delta T \langle\langle 1$, we neglect terms $3\alpha^2\Delta T^2$ and $\alpha^3\Delta T^3$. Then

$$\frac{\Delta V}{V_i} = 3\alpha\Delta T \qquad \rightarrow \Delta V = 3\alpha V_i \Delta T$$

Since $\Delta V = \beta V_i \Delta T$ **then** $\beta = 3\alpha$ **(7.23)**

Try to prove that $\gamma = 2\alpha$ **?**

Example:

A segment of steel railroad track has a length of $30m$ when the temperature is $0^o C$ (a) what is its length when temperature is $40^o C$ (linear expansion coefficient of the steal is $11\times10^{-6}\,{}^oC^{-1}$). (b) Suppose that the ends of the rail are rigidly clamped at $0^o C$ so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to $40^o C$. (Young's modulus is $20\times10^{10} N/m^2$)

(a) $L_i = 30m$, $T_i = 0^o C$, $T_f = 40^o C$, $L_f = ?$

$$\alpha = 11\times10^{-6}\,{}^oC$$

$$\Delta L = L_f - 30 = 11\times10^{-6}(30)(40-0)$$

$$L_f = 11\times10^{-6}(1200)+30 = .0132+30 = 30.0132m$$

(b) Stress $Stress = F/A = ?$, $\quad \Delta L = .0132m \quad L_i = 30m$

$$Y = 20\times10^{10} N/m^2$$

$$Y = stress/strain = Stress\frac{L_i}{\Delta L}$$

$$stress = Y\frac{\Delta L}{L_i} = 20\times10^{10}\times\frac{.0132}{30} = 8.7\times10^{7} N/m^2$$

<u>Problems</u>

(1) A box with a total surface area of $1.2m^2$ and a wall of thickness of $4cm$ is made of an insulating material. A $10watt$ electric heater inside the box maintains the inside temperature at 15^oC above the inside temperature. Find the thermal conductivity of the insulating material.

(2) A glass window pan has an area of $3m^2$ and a thickness of $0.6cm$. If the temperature difference between its face is 25^oC, what is the rate of energy transfer by conduction through the window. $k = 0.8$

(3) A bar of gold is in thermal contact with a bar of silver of the same length and area. One end of the compound bar is maintained at 80^oC while the opposite end at 30^oC. When the energy transfer reaches steady state, what is the temperature at the junction. $k_G = 314$ **,** $k_S = 427$

(4) A thermal window with an area of $6m^2$ is constructed of two layers of glass, each $4mm$ thick, and separated from each other by an air space of $5mm$. If the inside surface is 20^oC and the outside is at -30^oC, what is the rate of energy transfer by conduction through the window. $k = 0.023$

(5) The surface of the sun has a temperature of about 5800^oK. The radius of the sun is $6.96\times10^8\, m$. Calculate the total energy radiated by the sun every second. Assume that the emissivity of the sun is 0.965. $\sigma = 5.67\times10^{-8}$

(6) The tungsten filament of a certain light bulb radiates $2W$ of light. The filament has a surface area $0.25mm^2$ and an emissivity of 0.95. Find the filament's temperature in Fahrenheit unit.

(7) The temperature of a silver bar rises by 10.0°C when it absorbs 1.23 kJ of energy by heat. The mass of the bar is 525 g. Determine the specific heat of silver.

(8) A 50.0-g sample of copper is at 25.0°C. If 1 200 J of energy is added to it by heat, what is the final temperature of the copper?

(9) A 1.50-kg iron horseshoe initially at 600°C is dropped into a bucket containing 20.0 kg of water at 25.0°C. What is the final temperature? (Ignore the heat capacity of the container, and assume that a negligible amount of water boils away.)

(10) A combination of 0.250 kg of water at 20.0°C, 0.400 kg of aluminum at 26.0°C, and 0.100 kg of copper at 100°C is mixed in an insulated container and allowed to come to thermal equilibrium.

Ignore any energy transfer to or from the container and determine the final temperature of the mixture.

(11) How much energy is required to change a 40.0-g ice cube from ice at &10.0°C to steam at 110°C?

(12). A 50.0-g copper calorimeter contains 250 g of water at 20.0°C. How much steam must be condensed into the water if the final temperature of the system is to reach 50.0°C?

(13) A 3.00-g lead bullet at 30.0°C is fired at a speed of 240 m/s into a large block of ice at 0°C, in which it becomes embedded. What quantity of ice melts?

(14). Steam at 100°C is added to ice at 0°C. (a) Find the amount of ice melted and the final temperature when the mass of steam is 10.0 g and the mass of ice is 50.0 g. (b) What If ? Repeat when the mass of steam is 1.00 g and the mass of ice is 50.0 g.

(15). A 1.00-kg block of copper at 20.0°C is dropped into a large vessel of liquid nitrogen at 77.3 K. How many kilograms of nitrogen boil away by the time the copper reaches 77.3 K? (The specific heat of copper is 0.092 0 cal/g.°C, latent heat of evaporation of nitrogen is 78Cal/g).

(16) A gas is compressed at a constant pressure of 0.800 atm from 9.00 L to 2.00 L. In the process, 400 J of energy leaves the gas by heat. (a) What is the work done on the gas? (b) What is the change in its internal energy?

(17) A thermodynamic system undergoes a process in which its internal energy decreases by 500 J. At the same time, 220 J of work is done on the system. Find the energy transferred to or from it by heat.

(18)A 2.00-mol sample of helium gas initially at 300 K and 0.400 atm is compressed isothermally to 1.20 atm. Noting that the helium behaves as an ideal gas, find (a) the final volume of the gas, (b) the work done on the gas, and (c) the energy transferred by heat.

(19) An ideal gas initially at 300 K undergoes an isobaric expansion at 2.50 kPa. If the volume increases from 1.00 m3 to 3.00 m3 and 12.5 kJ is transferred to the gas by heat, what are (a) the change in its internal energy and (b) its final temperature?

(20) A 1.00-kg block of aluminum is heated at atmospheric pressure so that its temperature increases from 22.0°C to 40.0°C. Find (a) the work done on the aluminum, (b) the energy added to it by heat, and (c) the change in its internal energy.

(21) How much work is done on the steam when 1.00 mol of water at 100°C boils and becomes 1.00 mol of steam at 100°C at 1.00 atm pressure? Assuming the steam to behave as an ideal gas, determine the change in internal energy of the material as it vaporizes.

(22) Liquid nitrogen has a boiling point of $-195.81^{\circ}\mathrm{C}$ **at atmospheric pressure. Express this temperature (a) in degrees Fahrenheit and (b) in Kelvins.**

23. Convert the following to equivalent temperatures on the Celsius and Kelvin scales: (a) the normal human body temperature, $98.6^{\circ}\mathrm{F}$**; (b) the air temperature on a cold day,** $-5^{\circ}\mathrm{F}$

24. The temperature difference between the inside and the outside of an automobile engine is $450^{\circ}\mathrm{C}$**. Express this temperature difference on (a) the Fahrenheit scale and (b) the Kelvin scale.**

25. The melting point of a gold is $1064\,^{\circ}C$, and the boiling point is $2660^{\circ}\,C$. (a) Express the temperature in Kelvins. (b) Compute the difference between these temperatures in Celsius and Kelvins.

26. The temperature of a silver bar rises by $10\,^{\circ}C$ when it absorbs $1.23\,KJ$ of energy by heat. The mass of the bar is 525gm. determine the specific heat of the silver.

27. A 50 gm sample of copper is at $25\,^{\circ}C$. If 1200J of energy is added to it by heat, what is the final temperature of the copper?

28. A 1.5 Kg iron horseshoe initially at $600\,^{\circ}C$ is dropped into a bucket containing 20 Kg of water at $25\,^{\circ}C$. What is the final temperature? (Ignore the heat capacity of the container, and assume that a negligible amount of water boils away).

29. Complete the following table

$^{\circ}C$	$^{\circ}F$	$^{\circ}K$
20		
	142	
		307
	-49	
		-255
49		

The Kinetic Theory of Gases

Molecular Model of an Ideal Gas

The ideal gas model shows that the pressure that a gas exerts on the walls of its container is a consequence of the collisions of the gas molecules with the walls. In developing this model, we make the following assumptions:

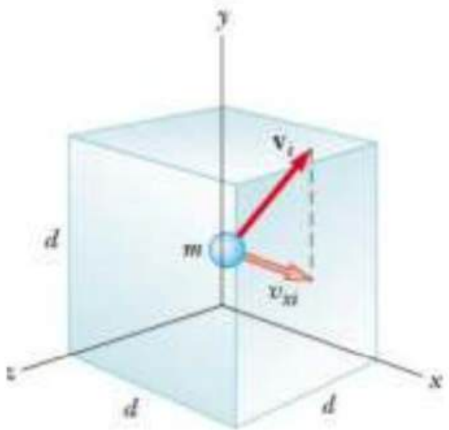
1. The number of molecules in the gas is large, and the average separation between them is large compared with their dimensions. This means that the molecules occupy a negligible volume in the container.

2. The molecules obey Newton's laws of motion, but as a whole they move randomly (each molecule can move in any direction with any speed). At any given moment, a certain percentage of molecules move at high speeds, and a certain percentage move at low speeds.

3. The molecules interact only by short-range forces during elastic collisions. This is consistent with the ideal gas model, in which the molecules exert no long-range forces on each other.

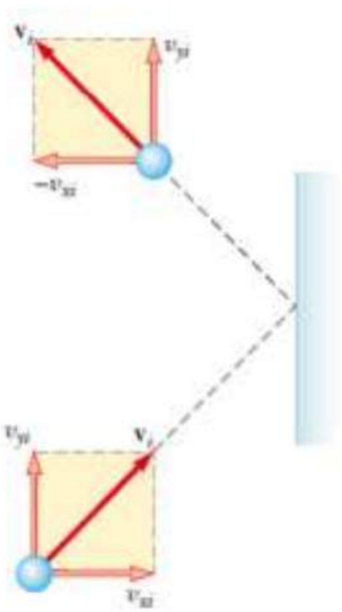
4. The molecules make elastic collisions with the walls.

5. The gas under consideration is a pure substance; that is, all molecules are identical.



Consider an ideal gas as consisting of single atoms, with no effect for molecular rotations or vibrations, on the average. For our first application of kinetic theory, *let us derive an*

***expression for the pressure of N molecules of an ideal gas in a container of volume V in terms of microscopic quantities*. The container is a cube with edges of length *d* (Fig). We shall first focus our attention on one of these molecules of mass *m*, and assume that it is moving so that its component of velocity in the *x* direction is v_{xi} as in Figure. (The subscript *i* here refers to the *i*th molecule, not to an initial value. We will combine the effects of all of the molecules shortly.) As the molecule collides elastically with any wall (assumption 4), its velocity component perpendicular to the wall is reversed because the mass of the wall is far greater than the mass of the molecule. Because the momentum component P_{xi} of the molecule is mv_{xi} before the collision and $-mv_{xi}$ after the collision, the change in the *x* component of the momentum of the molecule is**



$$\Delta P_{xi} = -mv_{xi} - mv_{xi} = -2mv_{xi}$$

Because the molecules obey Newton's laws (assumption 2), we can apply the impulse momentum theorem (Impulse=Ft) to the molecule to give us

$$\overline{F}_{i,onmolecule}\Delta t_{collision} = \Delta P_{xi} = -2mv_{xi} \quad \textbf{(7.1)}$$

Where $\overline{F}_{i,onmolecule}$ is the *x* component of the average force that the wall exerts on the molecule during the collision and $\Delta t_{collision}$ is the duration

of the collision. In order for the molecule to make another collision with the same wall after this first collision, it must travel a distance of *2d* in the *x* direction (across the container and back). Therefore, the time interval between two collisions with the same wall is

$$\Delta t = \frac{2d}{v_{xi}} \qquad \textbf{(7.2)}$$

The force that causes the change in momentum of the molecule in the collision with the wall occurs only during the collision. However, we can average the force over the time interval for the molecule to move across the cube and back. Sometime during this time interval, the collision occurs, so that the change in momentum for this time interval is the same as that for the short duration of the collision. Thus, we can rewrite the impulse-momentum theorem as

$$\overline{F}_i \Delta t = -2mv_{xi} \qquad \textbf{(7.3)}$$

where $\overline{F}_i$ **is the average force component over the time for the molecule to move across the cube and back. Because exactly one collision occurs for each such time interval, this is also the long-term average force on the molecule, over long time intervals containing any number of multiples of** Δt **.**

This equation and the preceding one enable us to express the *x* component of the long-term average force exerted by the wall on the molecule as

$$\overline{F}_i = \frac{-2mv_{xi}}{\Delta t} = \frac{-2mv^2_{xi}}{2d} = \frac{-mv^2_{xi}}{d} \qquad \textbf{(7.4)}$$

Now, by Newton's third law, the average x component of the force exerted by the molecule on the wall is equal in magnitude and opposite in direction:

$$\overline{F}_{i,on\,wall} = -\overline{F}_i = -\left(\frac{-mv^2{}_{xi}}{d}\right) = \frac{mv^2{}_{xi}}{d} \qquad \textbf{(7.5)}$$

The total average force $\overline{F}$ exerted by the gas on the wall is found by adding the average forces exerted by the individual molecules. We add terms such as that above for all molecules:

$$\overline{F} = \sum_{i=1}^{N}\frac{mv^2{}_{xi}}{d} = \frac{m}{d}\sum_{i=1}^{N}v^2{}_{xi} \qquad \textbf{(7.6)}$$

For a very large number of molecules, however, such as Avogadro's number, these variations in force (that appears for small number of molecules) are smoothed out, so that the average force given above is the same over *any* time interval. Thus, the *constant* force F on the wall due to the molecular collisions is

$$F = \frac{m}{d}\sum_{i=1}^{N}v^2{}_{xi} \qquad \textbf{(7.7)}$$

The average value of the square of the x component of the velocity for N molecules is

$$\overline{v_x^2} = \frac{\sum_{i=1}^{N}v^2{}_{xi}}{N} \qquad \textbf{(7.8)}$$

The total force on the wall can be written

$$F = \frac{m}{d} N \overline{v_x^2} \tag{7.9}$$

Now let us focus again on one molecule with velocity components v_{xi}, v_{yi}, and v_{zi}. *Then,*

$$v^2{}_i = v^2{}_{xi} + v^2{}_{yi} + v^2{}_{zi} \tag{7.10}$$

Hence, the average value of v^2 for all the molecules in the container is related to the average values of v_x^2, v_y^2 and v_z^2 according to the expression

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} \tag{7.11}$$

Because the motion is completely random (assumption 2), the average values $\overline{v_x^2}$, $\overline{v_y^2}$ andare equal to each other. $\overline{v_z^2}$

$$\overline{v^2} = 3\overline{v_x^2} \tag{7.12}$$

The total force exerted on the wall is

$$F = \frac{N}{3}\left(m\frac{\overline{v^2}}{d} \right) \tag{7.13}$$

Using this expression, we can find the total pressure exerted on the wall

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{N}{3}\left(m\frac{\overline{v^2}}{d^3} \right) = \frac{1}{3}\left(\frac{N}{V}\right) m\overline{v^2} = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right) \tag{7.14}$$

This result indicates that the pressure of a gas is proportional to the number of molecules per unit volume and to the average translational kinetic energy of the molecules $\frac{1}{2}m\overline{v^2}$ **.**

<u>Molecular Interpretation of Temperature</u>

We can gain some insight into the meaning of temperature by first writing Equation

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right)$$

$$PV = Nk_BT \qquad \textbf{(7.15)}$$

Where,

$$T = \frac{2}{3k_B}\left(\frac{1}{2}m\overline{v^2}\right)$$

This result tells us that temperature is a direct measure of average molecular kinetic energy. Then,

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT \qquad \textbf{(7.16)}$$

That is, the average translational kinetic energy per molecule is $\frac{3}{2}k_BT$ **. Because** $\overline{v_x^2} = \frac{1}{3}\overline{v^2}$ **, then**

$$\frac{1}{2}m\overline{v_x^2} = \frac{1}{2}k_BT \qquad \textbf{(7.17)}$$

Similarly,

$$\frac{1}{2}m\overline{v_y^2} = \frac{1}{2}k_BT \qquad \& \qquad \frac{1}{2}m\overline{v_z^2} = \frac{1}{2}k_BT \tag{7.18}$$

Thus, each translational degree of freedom contributes an equal amount of energy, $\frac{1}{2}k_BT$, refers to "degree of freedom" to the gas. In general, an independent means by which a molecule can possess energy. A generalization of this result, known as the theorem of equipartition of energy, states that:

"each degree of freedom contributes to the energy of a system $\frac{1}{2}k_BT$, where possible degrees of freedom in addition to those associated with translation arise from rotation and vibration of molecules."

The total translational kinetic energy of *N* molecules of gas is simply *N* times the average energy per molecule, which is given by Equation (8.16):

$$K_{tot\,trns} = N\left(\frac{1}{2}m\overline{v^2}\right) = \frac{3}{2}Nk_BT = \frac{3}{2}nRT \tag{7.19}$$

where $k_B = R/N_A$ for Boltzmann's constant and $n = N/N_A$ for the number of moles of gas. If we consider a gas in which molecules possess only translational kinetic energy, Equation (7.19) represents the internal energy of the gas. This result implies that *the internal energy of an ideal gas depends only on the temperature*.

The square root of $\overline{v^2}$ is called the ***root-mean-square*** (rms) ***speed*** of the molecules. It has the form,

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3}{m}k_B T} = \sqrt{\frac{3}{M}RT} \qquad \textbf{(7.20)}$$

Where, M is the molar mass in kilograms per mole and is equal to mN_A. This expression shows that, at a given temperature, lighter molecules move faster, on the average, than do heavier molecules.

Example:

A tank used for filling helium balloons has a volume of 0.300 m^3 and contains 2.00 mol of helium gas at 20.0°C. Assume that the helium behaves like an ideal gas. (a) What is the total translational kinetic energy of the gas molecules? (b) What is the average kinetic energy per molecule?

Solution

(a) $K_{tot\,trns} = \frac{3}{2}nRT = \frac{3}{2}\times 2\times 8.31\times 293 = 7.3\times 10^{3}\, J$

(b) $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T = \frac{3}{2}\times 1.38\times 10^{-23}\times 293 = 6.07\times 10^{-21}\, J$

Molar Specific Heat of an Ideal Gas

Consider an ideal gas undergoing several processes such that the change in temperature is $\Delta T = T_f - T_i$ for all processes. From the

first law, $Q = \Delta E - W$ that the heat Q is different for each path because W (the negative of the area under the curves) is different for each path. Thus, the heat associated with a given change in temperature does *not* have a unique value.

We can address this difficulty by defining specific heats for two processes that frequently occur: changes at constant volume and changes at constant pressure. Because the number of moles is a convenient measure of the amount of gas, we define the molar specific heats associated with these processes with the following equations:

$$Q_{constant\ V} = nC_V \Delta T \qquad \text{(constant volume)} \qquad (7.21)$$

$$Q_{constant\ P} = nC_P \Delta T \qquad \text{(constant pressure)} \qquad (7.22)$$

where C_V is the molar specific heat at constant volume and C_P is the molar specific heat at constant pressure. When we add energy to a gas by heat at constant pressure, not only does the internal energy of the gas increase, but work is done on the gas because of the change in volume. Therefore, the heat $Q_{constant\ P}$ must account for both the increase in internal energy and the transfer of energy out of the system by work. For this reason, $Q_{constant\ P}$ is greater than $Q_{constant\ V}$ for given values of *n* and "*T*. Thus, C_P is greater than C_V.

Let us an ideal monatomic gas, that is, a gas containing one atom per molecule, such as helium, neon, or argon. When energy is added to a monatomic gas in a container of fixed volume, all of the added energy goes into increasing the translational kinetic energy of the atoms.

There is no other way to store the energy in a monatomic gas. Therefore, from Equation (8.19), we see that the internal energy E_{int} of *N* molecules (or *n* mol) of an ideal monatomic gas is

$$E_{\text{int}} = K_{tot\ trans} = \frac{3}{2}Nk_B T = \frac{3}{2}nRT \qquad \textbf{(7.23)}$$

Note that for a monatomic ideal gas, E_{int} is a function of *T* only.

Specific heat at constant volume

If energy is transferred by heat to a system at *constant volume*, then no work is done on the system. That is, for a constant-volume process. Hence, from the first law of thermodynamics, we see that

$$Q = \Delta E_{\text{int}} \qquad \textbf{(7.24)}$$

From (8.21), we get

$$\Delta E_{\text{int}} = nC_V \Delta T \qquad \textbf{(7.25)}$$

If the molar specific heat is constant, we can express the internal energy of a gas as

$$E_{\text{int}} = nC_V \Delta T \qquad \textbf{(7.26)}$$

This equation applies to all ideal gases—to gases having more than one atom per molecule as well as to monatomic ideal gases. In the limit of infinitesimal changes, we can use Equation 8.25 to express the molar specific heat at constant volume as

$$C_V = \frac{1}{n}\frac{dE_{\text{int}}}{dT} \qquad \textbf{(7.27)}$$

From Equation 8.23 into Equation 8.27, we find that

$$C_V = \frac{3}{2}\frac{1}{n}\frac{dnRT}{dT} = \frac{3}{2}R \qquad \textbf{(7.28)}$$

This expression predicts a value of for *all* monatomic gases.

Specific heat at constant pressure

Now suppose that the gas is at constant-pressure, the temperature again increases by ΔT. The energy that must be transferred by heat to the gas in this process is $Q = nC_P\Delta T$. Because the volume changes in this process, the work done on the gas is $W = -P\Delta V$ where *P* is the constant pressure at which the process occurs. Applying the first law of thermodynamics to this process, we have

$$\Delta E_{\text{int}} = Q + W = nC_P\Delta T - P\Delta V \qquad \textbf{(7.29)}$$

In this case, the energy added to the gas by heat is composed of: Part of it leaves the system by work (that is, the gas moves a piston through a displacement), and the remainder appears as an increase in the internal energy of the gas. In addition, because $PV = nRT$, we note that for a constant-pressure process, $P\Delta V = nR\Delta T$. Substituting this value for $P\Delta V$ into Equation 8.29 with $\Delta E_{\text{int}} = nC_V\Delta T$ (Eq. 8.25) gives

$$nC_V\Delta T = nC_P\Delta T - nR\Delta T$$

$$C_P - C_V = R \tag{7.30}$$

This expression applies to *any* ideal gas. Since $C_V = \frac{3}{2}R$ for ideal gas then $C_P = \frac{5}{2}R$ ($R = 8.31 J/mol.K$). Then,

$$\gamma = C_P / C_V = 5/3 = 1.67 \tag{7.31}$$

Example:

A cylinder contains 3.00 mol of helium gas at a temperature of 300 K. (a) If the gas is heated at constant volume, how much energy must be transferred by heat to the gas for its temperature to increase to 500 K? (b) How much energy must be transferred by heat to the gas at constant pressure to raise the temperature to 500 K? (specific heat of helium under constant volume is 12.5J/mol.K, specific heat of helium under constant volume is 20.8J/mol.K)

$$Q_1 = nC_V \Delta T = 3 \times 12.5 \times 200 = 7.5 \times 10^3 J$$

$$Q_2 = nC_P \Delta T = 3 \times 20.8 \times 200 = 12.5 \times 10^3 J$$

<u>Adiabatic Processes for an Ideal Gas</u>

An adiabatic process is one in which no energy is transferred by heat between a system and its surroundings. For example, if a gas is compressed (or expanded) very rapidly, very little energy is transferred out of (or into) the system by heat, and so the process is nearly adiabatic.

Suppose that an ideal gas undergoes an adiabatic expansion. At any time during the process, we assume that the gas is in an equilibrium

state, so that the equation of state $PV = nRT$ is valid. As we show below, the pressure and volume of an ideal gas at any time during an adiabatic process are related by the expression

$$PV^{\gamma} = \text{constant} \qquad \textbf{(7.32)}$$

Proof That $PV^{\gamma} = \text{constant}$ for an Adiabatic Process

When a gas is compressed adiabatically in a thermally insulated cylinder, no energy is transferred by heat between the gas and its surroundings; thus, *Q*=0. Let us imagine an infinitesimal change in volume *dV* and an accompanying infinitesimal change in temperature *dT*. The work done on the gas is PdV . Because the internal energy of an ideal gas depends only on temperature, the change in the internal energy in an adiabatic process is the same as that for an isovolumetric process between the same temperatures, $dE_{\text{int}} = nC_V dT$ (Eq. 7.25). Hence, the first law of thermodynamics,

$$dE_{\text{int}} = nC_V dT = -PdV$$

Since $PV = nRT$,

$$PdV + VdP = nRdT$$

Eliminating *dT* from these two equations, we find that

$$PdV + VdP = -\frac{R}{C_V} PdV$$

Substituting with $R = C_P - C_V$ **and dividing by PV, we obtain**

$$\frac{dV}{V} + \frac{dP}{P} = -\left(\frac{C_P - C_V}{C_V}\right)\frac{dV}{V} = (1-\gamma)\frac{dV}{V}$$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Integrating this expression, we have

$$\ln P + \gamma \ln V = \text{constant}$$

$$PV^{\gamma} = \text{constant}$$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

Since $PV = nRT$

$$\frac{nRT_1}{V_1} V_1^{\gamma} = \frac{nRT_2}{V_2} V_2^{\gamma} \qquad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

Example

Air at 20.0°C in the cylinder of a diesel engine is compressed from an initial pressure of 1.00 atm and volume of 800.0 cm³ to a volume of 60.0 cm³. Assume that air behaves as an ideal gas with ($\gamma = 1.4$ **and that the compression is adiabatic. Find the final pressure and temperature of the air.**

$P_1 = 1atm \qquad P_2 = ? \qquad T_1 = 20 + 273 = 293^o K$

$V_1 = 800cm^3 \qquad V_2 = 60cm^3 \quad T_2 = ?$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$$\Rightarrow P_2 = P_1\frac{V_1^{\gamma}}{V_2^{\gamma}} = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1\times\left(\frac{800}{60}\right)^{1.4} = 37.6atm$$

Since, $PV = nRT$ **,** $\frac{PV}{T} = nR = constant$ **(no escape for the gas from the cylinder)**

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \Rightarrow T_2 = \frac{P_2V_2}{P_1V_1}T_1 = \frac{37.6\times 60}{1\times 800}293 = 826^oK$$

<u>Degrees of freedom</u>

A *degree of freedom* is essentially a variable whose value may change. In the case of a physical system, the positions of the particles that comprise the system are degrees of freedom. For a single particle in 3-dimensional space, there are three degrees of freedom. Three coordinates are required to specify its location. We are particularly interested in variables that determine the energy of the system—the velocities determine the kinetic energy, the positions determine the potential energy, etc. In other words, we expect to associate some kinetic energy and some potential energy with each degree of freedom.

In effect, this text treats the kinetic and potential energies as degrees of freedom. An isolated single particle, having no internal structure, but able to move in three-dimensional space, has three degrees of freedom which may have energy associated with them: the three

components of its velocity. Since the particle is not interacting with any other particle, we do not count its position coordinates as degrees of freedom. On the other hand, a three-dimensional harmonic oscillator has potential energy as well as kinetic energy, so it has 6 degrees of freedom. Molecules in a gas have more degrees of freedom than simple spherical particles. A molecule can rotate as well as translate and its constituent parts can vibrate. A water molecule is comprised of three atoms, arranged in the shape of a triangle. The molecule can translate in three dimensions, and rotate around three different axes. That's 3+3 = 6 degrees of freedom for an isolated water molecule. Within the molecules, the atoms can vibrate relative to the center of mass in three distinct ways, or modes. That's another 2x3 = 6 degrees of freedom. Now, finally if the water molecule is interacting with other water molecules, then there is interaction between the molecules, and the degrees of freedom are 2x3+2x3+2x3 = 18. Notice that if we regard the molecule as three interacting atoms, not as a rigid shape, there are 3x2x3 = 18 degrees of freedom.

A system of N particles, such as a solid made of N harmonic oscillators, has 6 degrees of freedom per particle for a total of 6N degrees of freedom.

The idea is that each degree of freedom, as it were, contains some energy. The total internal energy of a system is the sum of all the energies of all the degrees of freedom. Conversely, the amount of energy that may be transferred into a system is affected by how many degrees of freedom the system has.

Example:

Calculate the work done in compressing (isothermally) 2 kg of dry air to one-tenth its initial volume at 15 ºC.

From the definition of work, W = ∫ pdV.

From the equation of state, p = nRT = (m/V)RT.

Then W = mRT ∫ dlnV = mRT ln(V_2/V_1) (remember the process is isothermal)

$$= (287\ \mathrm{J\ K^{-1}\ kg^{-1}})(288.15\ \mathrm{K})(2\ \mathrm{kg})(\ln 0.1)$$

$$= -3.81 \times 10^5\ \mathrm{J}.$$

Problems:

1. **In a 30.0-s interval, 500 hailstones strike a glass window of area 0.600 m^2 at an angle of 45.0° to the window surface. Each hailstone has a mass of 5.00 g and moves with a speed of 8.00 m/s. Assuming the collisions are elastic, find the average force and pressure on the window.**

2. **A sealed cubical container 20.0 cm on a side contains three times Avogadro's number of molecules at a temperature of 20.0°C. Find the force exerted by the gas on one of the walls of the container.**

3. **A spherical balloon of volume 4 000 cm^3 contains helium at an (inside) pressure of $1.2x10^5$ Pa. How many moles of helium are in the balloon if the average kinetic energy of the helium atoms is $3.6x10^{-22}$ J?**

4. **(a) How many atoms of helium gas fill a balloon having a diameter of 30.0 cm at 20.0°C and 1.00 atm? (b) What is the average kinetic energy of the helium atoms? (c) What is the root-mean-square speed of the helium atoms?**

5. **A cylinder contains a mixture of helium and argon gas in equilibrium at 150°C. (a) What is the average kinetic energy for each type of gas molecule? (b) What is the root-mean-square speed of each type of molecule?**

6. **A 5 L vessel contains nitrogen gas at 27.0°C and a pressure of 3.00 atm. Find (a) the total translational kinetic energy of the gas molecules and (b) the average kinetic energy per molecule.**

7. **(a) Show that 1 Pa =1 J/m^3. (b) Show that the density in space of the translational kinetic energy of an ideal gas is 3*P*/2.**

8. **Calculate the change in internal energy of 3.00 mol of helium gas when its temperature is increased by 2.00 K.**

9. A 1 mol sample of hydrogen gas is heated at constant pressure from 300 K to 420 K. Calculate (a) the energy transferred to the gas by heat, (b) the increase in its internal energy, and (c) the work done on the gas.

10. A 1 mol sample of air (a diatomic ideal gas) at 300 K, confined in a cylinder under a heavy piston, occupies a volume of 5.00 L. Determine the final volume of the gas after 4.40 kJ of energy is transferred to the air by heat.

11. In a constant-volume process, 209 J of energy is transferred by heat to 1 mol of an ideal monatomic gas initially at 300 K. Find (a) the increase in internal energy of the gas, (b) the work done on it, and (c) its final temperature.

12. A house has well-insulated walls. It contains a volume of 100 m3 of air at 300 K. (a) Calculate the energy required to increase the temperature of this diatomic ideal gas by 1.00°C. (b) What If? If this energy could be used to lift an object of mass *m* through a height of 2.00 m, what is the value of *m*?

13. An incandescent light bulb contains a volume V of argon at pressure P_1. The bulb is switched on and constant power σ is transferred to the argon for a time interval Δt. (a) Show that the pressure P_2 in the bulb at the end of this process is

$P_2 = P_1\left[1+\frac{\sigma \Delta t R}{P_1 V C_V}\right]$. **(b) Find the pressure in a spherical light bulb 10.0 cm in diameter 4.00 s after it is switched on, given that it has initial pressure 1atm and that 3.60 W of power is transferred to the gas.**

14. During the compression stroke of a certain gasoline engine, the pressure increases from 1 atm to 20 atm. If the process is adiabatic and the fuel–air mixture behaves as a diatomic ideal gas, (a) by what factor does the volume change and (b) by what factor does the temperature change? (c) Assuming that the compression starts with 0.016 mol of gas at 27.0°C, find the values of *Q*, *W*, and ΔE_{int} that characterize the process

15.A 2 mol sample of a diatomic ideal gas expands slowly and adiabatically from a pressure of 5 atm and a volume of 12 L to a final volume of 30 L. (a) What is the final pressure of the gas? (b) What are the initial and final temperatures? (c) Find *Q, W*, and ΔE_{int} .

Entropy and second law of thermodynamics

Introduction

The first law of thermodynamics states that a change in internal energy in a system can occur as a result of energy transfer by heat or by work, or by both. This law makes no distinction between the results of heat and the results of work (either heat or work can cause a change in internal energy). However, there is an important distinction between heat and work that is not evident from the first law. One manifestation of this distinction is that it is impossible to design a device that, operating in a cyclic fashion, takes in energy by heat and expels an *equal* amount of energy by work. A cyclic device that takes in energy by heat and expels a *fraction* of this energy by work is possible and is called a *heat engine*.

Although the first law of thermodynamics is very important, it makes no distinction between processes that occur spontaneously and those that do not. The *second law of thermodynamics*, the major topic in this chapter, establishes which processes do and which do not occur. The following are examples of processes that do not violate the principle of conservation of energy if they proceed in either direction, but are observed to proceed in only one direction, governed by the second law:

- **When two objects at different temperatures are placed in thermal contact with each other, the net transfer of energy by heat is always from the warmer object to the cooler object, never from the cooler to the warmer.**

• A rubber ball dropped to the ground bounces several times and eventually comes to rest, but a ball lying on the ground never gathers internal energy from the ground and begins bouncing on its own.

• An oscillating pendulum eventually comes to rest because of collisions with air molecules and friction at the point of suspension. The mechanical energy of the system is converted to internal energy in the air, the pendulum, and the suspension; the reverse conversion of energy never occurs.

All these processes are *irreversible*—that is, they are processes that occur naturally in one direction only. No irreversible process has ever been observed to run backward (if it were to do so, it would violate the second law of thermodynamics). From an engineering standpoint, perhaps the most important implication of the second law is the limited efficiency of heat engines. The second law states that a machine that operates in a cycle, taking in energy by heat and expelling an equal amount of energy by work, cannot be constructed.

Heat Engines and the Second Law of Thermodynamics

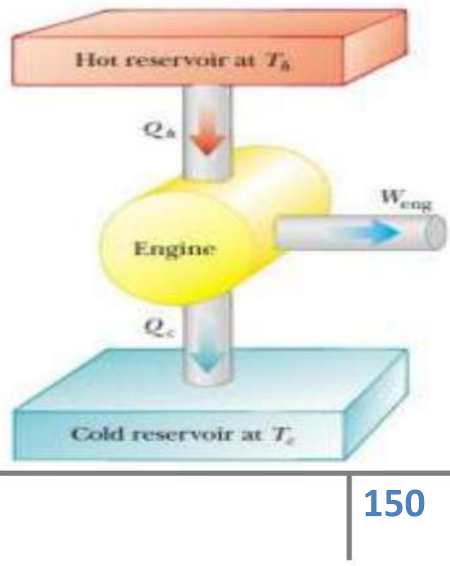
A heat engine is a device that takes in energy by heat and, operating in a cyclic process, expels a fraction of that energy by means of work. For a power plant produces electricity, coal or some other fuel is burned, and the high-temperature gases produced are used to convert liquid water to steam. This steam is directed at the blades of a turbine, setting it into rotation. The mechanical energy associated with this rotation is used to drive an electric generator. Another device that can be modeled as heat engine (the internal combustion engine in an automobile) uses energy from a burning fuel to perform work on pistons that results in the motion of the automobile.

A heat engine carries some working substance through a cyclic process during which

(1) the working substance absorbs energy by heat from a high-temperature energy reservoir,

(2) work is done by the engine, and

(3) energy is expelled by heat to a lower-temperature reservoir.



As an example, consider the operation of a steam engine, which uses water as the working substance. The water in a boiler absorbs energy from burning fuel and evaporates to steam, which then does work by

expanding against a piston. After the steam cools and condenses, the liquid water produced returns to the boiler and the cycle repeats. It is useful to represent a heat engine schematically as in Figure. The engine absorbs a quantity of energy $|Q_h|$ from the hot reservoir. For this discussion of heat engines, we will use absolute values to make all energy transfers positive and will indicate the direction of transfer with an explicit positive or negative sign. The engine does work W_{eng} (so that *negative* work $W = -W_{eng}$ is done *on* the engine), and then gives up a quantity of energy $|Q_c|$ to the cold reservoir. Because the working substance goes through a cycle, its initial and final internal energies are equal, and so $\Delta E_{\text{int}} = 0$.

Hence, from the first law of thermodynamics $\Delta E_{\text{int}} = Q + W = Q - W_{eng}$, and with no change in internal energy, the net work W_{eng} done by a heat engine is equal to the net energy Q_{net} transferred to it. As we can see from above Figure, $Q_{net} = |Q_h| - |Q_c|$; therefore

$$W_{net} = |Q_h| - |Q_c| \qquad \textbf{(8.1)}$$

If the working substance is a gas, the net work done in a cyclic process is the area enclosed by the curve representing the process on a PV diagram. This is shown for an arbitrary cyclic process in Figure. The thermal efficiency *e* of a heat engine is

defined as the ratio of the net work done by the engine during one cycle to the energy input at the higher temperature during the cycle:

$$e = \frac{W_{net}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} \qquad \textbf{(8.2)}$$

In practice, all heat engines expel only a fraction of the input energy *Q_h* by mechanical work and consequently their efficiency is always less than 100%. For example, a good automobile engine has an efficiency of about 20%, and diesel engines have efficiencies ranging from 35% to 40%. Equation (9.2) shows that a heat engine has 100% efficiency (e=1) only if $|Q_c| = 0$ —that is, if no energy is expelled to the cold reservoir. In other words, a heat engine with perfect efficiency would have to expel all of the input energy by work. The second law of thermodynamics states the following:

"It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than the input of energy by heat from a reservoir and the performance of an equal amount of work"

This statement of the second law means that, during the operation of a heat engine, W_{eng} can never be equal to $|Q_h|$, or, alternatively, that some energy $|Q_c|$ must be rejected to the environment.

Example:

An engine transfers $2x10^3$ J of energy from a hot reservoir during a cycle and transfers $1.5x10^3$ J as exhaust to a cold reservoir. (a) Find

the efficiency of the engine. (b) How much work does this engine do in one cycle?

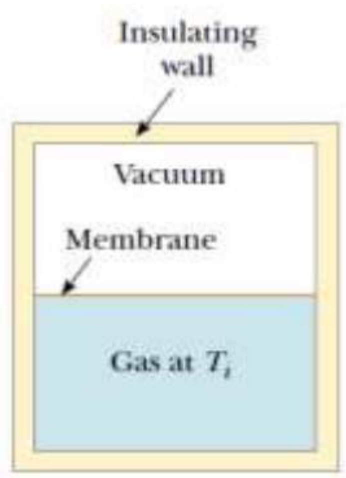
(a) $e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{1.5\times10^3}{2\times10^3} = 0.25 = 25\%$

(b) $W_{eng} = |Q_h| - |Q_c| = 2\times10^3 - 1.5\times10^3 = 0.5\times10^3\,J$

Reversible and Irreversible Processes

In a reversible process, the system undergoing the process can be returned to its initial conditions along the same path on a *PV* diagram, and every point along this path is an equilibrium state. A process that does not satisfy these requirements is irreversible.

All natural processes are known to be irreversible. As an example,



let us examine the adiabatic free expansion of a gas, which was already discussed in chapter 7, and show that it cannot be reversible. Consider a gas in a thermally insulated container, as shown in Figure. A membrane separates the gas from a vacuum. When the membrane is punctured, the gas expands freely into the vacuum. As a result of the puncture, the system has changed because it occupies a greater volume after the expansion. Because the gas does not exert a force through a displacement, it does no work on the surroundings as it expands. In addition, no energy is transferred to or from the gas by heat because the container is

insulated from its surroundings. Thus, in this adiabatic process, the system has changed but the surroundings have not. For this process to be reversible, we need to be able to return the gas to its original volume and temperature without changing the surroundings. Imagine that we try to reverse the process by compressing the gas to its original volume. To do so, we fit the container with a piston and use an engine to force the piston inward. During this process, the surroundings change because work is being done by an outside agent on the system. In addition, the system changes because the compression increases the temperature of the gas. We can lower the temperature of the gas by allowing it to come into contact with an external energy reservoir. Although this step returns the gas to its original conditions, the surroundings are again affected because energy is being added to the surroundings from the gas. If this energy could somehow be used to drive the engine that compressed the gas, then the net energy transfer to the surroundings would be zero. In this way, the system and its surroundings could be returned to their initial conditions, and we could identify the process as reversible. However, the second law specifies that the energy removed from the gas to return the temperature to its original value cannot be completely converted to mechanical energy in the form of the work done by the engine in compressing the gas. Thus, we must conclude that the process is irreversible. We could also argue that the adiabatic free expansion is irreversible by relying

on the portion of the definition of a reversible process that refers to equilibrium states.

Entropy

The first law involves the concept of internal energy. Temperature and internal energy are both state variables—that is, they can be used to describe the thermodynamic state of a system. Another state variable—this one related to the second law of thermodynamics—is entropy *S*. Entropy was originally formulated as a useful concept in thermodynamics. In statistical mechanics, the behavior of a substance is described in terms of the statistical behavior of its atoms and molecules. One of the main results of this treatment is that isolated systems tend toward disorder and that entropy is a measure of this disorder. For example, consider the molecules of a gas in the air in your room. If half of the gas molecules had velocity vectors of equal magnitude directed toward the left and the other half had velocity vectors of the same magnitude directed toward the right, the situation would be very ordered. However, such a situation is extremely unlikely. If you could actually view the molecules, you would see that they move haphazardly in all directions, bumping into one another, changing speed upon collision, some going fast and others going slowly. This situation is highly disordered.

The cause of the tendency of an isolated system toward disorder is easily explained. To do so, we distinguish between *microstates* and *macrostates* of a system. A microstate is a particular configuration of the individual constituents of the system. For example, the

description of the ordered velocity vectors of the air molecules in your room refers to a particular microstate, and the more likely haphazard motion is another microstate— one that represents disorder. A macrostate is a description of the conditions of the system from a macroscopic point of view and makes use of macroscopic variables such as pressure, density, and temperature for gases.

For any given macrostate of the system, a number of microstates are possible. For example, the macrostate of a four on a pair of dice can be formed from the possible microstates 1-3, 2-2, and 3-1. It is assumed that all microstates are equally probable. However, when all possible macrostates are examined, it is found that macrostates associated with disorder have far more possible microstates than those associated with order.

If we consider a system and its surroundings to include the entire universe, then the Universe is always moving toward a macrostate corresponding to greater disorder. Because entropy is a measure of disorder, an alternative way of stating this is the entropy of the Universe increases in all real processes. The original formulation of entropy in thermodynamics involves the transfer of energy by heat during a reversible process. Consider any infinitesimal process in which a system changes from one equilibrium state to another. If dQ_r is the amount of energy transferred by heat when the system follows a reversible path between the states, then the change in entropy dS is equal to this amount of energy for the reversible process divided by the absolute temperature of the system:

$$ds = \frac{dQ_r}{dT} \tag{8.3}$$

The change in entropy during a process depends only on the end points and therefore is independent of the actual path followed. Consequently, the entropy change for an irreversible process can be determined by calculating the entropy change for a reversible process that connects the same initial and final states.

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