Mechanics (1)
BMS012

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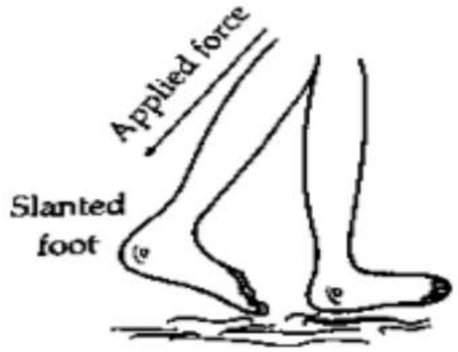
Contains

Chapter 1

1.1 vectors

Vectors are used in many branches of science such as mathematics, physics, and engineering. Vectors play an important role in engineering ; the velocity , acceleration of a moving object and the forces , reactions and couples that acting on it can all be described with vectors.



1.1.1 Definition of vector

A vector is an object that has both a magnitude and a direction. Geometrically, vector is a directed line segment, whose length is its magnitude and its direction from its tail to its head.

In physics, Vectors can be classified as free, sliding, or fixed, such that

Free vector

A free vector is a vector that's its action is not confined to or associated with a unique line in space. For example, a Force F is applied to the center of a box, pushing it forward(case 1). And if we change the position of the force's influence such that, it is applied to the bottom of the box which also results in it being pushed forward(case 2). In both cases, the point of application (position) of the force vector was different, but the magnitude and direction remained the same. Therefore, the force vector can be labeled a free vector.

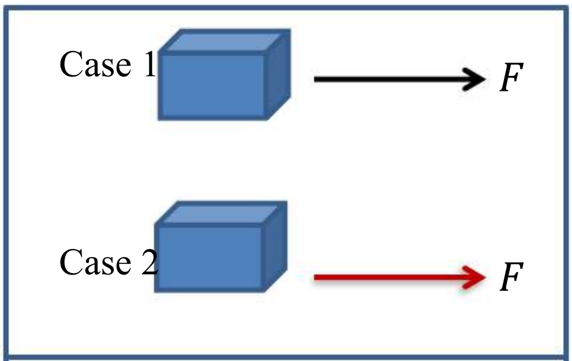


Fig.1: Free vector

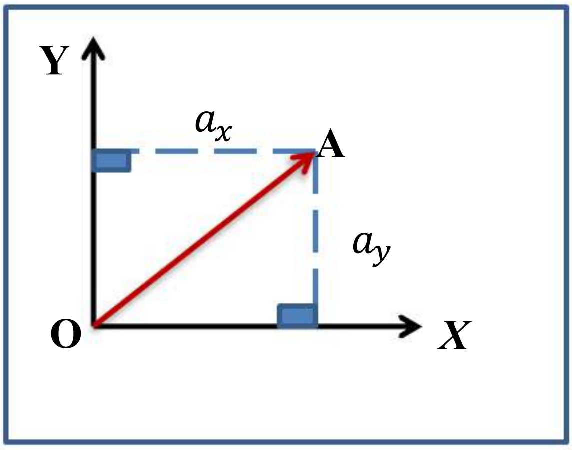
Sliding vector
A sliding vector has a unique line of action in space but not a unique point of application. For example, when an external force acts on
a rigid body, the force can be applied at any point along its line of action
without changing its effect on the body as a whole, and thus it is a sliding vector.

Fixed vector
A fixed vector is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be
specified by a fixed vector at the point of application of the force. In this
instance the forces and deformations within the body depend on the
point of application of the force, as well as on its magnitude and line of
action.

We will denote to vectors by capital Letters with the symbol → on it; $\vec{A}, \vec{B}. \vec{C}, \dots \dots$

A unit vector is a vector of length one unit and it is often denoted by a lowercase letter with a circumflex, or "hat", as in $\hat{u}$ and we will pronounce to this vector as u-hat.

1.1.2 Representation of vectors



In Cartesian Plane : A vector in the Cartesian plane, showing the position of a point A with coordinates (a_x, a_y) and it has the form $\overrightarrow{oA}$, the point O is the origin point (0,0),

Let vector $\overrightarrow{oA}$ makes an angle θ with the direction of OX axis , the component of the vector $\overrightarrow{oA}$ in the direction of OX axis is a_x and its component in direction of Oy axis is a_y . Simply, we can write the vector $\overrightarrow{oA}$ as

$$\overrightarrow{oA} = (a_x, a_y)$$

Or

$$\overrightarrow{oA} = a_x\hat{\imath} + a_y\hat{\jmath}$$

Where: $\hat{\imath}$ is the unit vector in the direction of OX axis and $\hat{\jmath}$ is the unit vector in the direction of OY axis.

The components $a_x$and a_y are denoted by the angle θ as

$$a_x = \|\overrightarrow{oA}\| \cos\theta$$

$$a_y = \|\overrightarrow{oA}\| \sin\theta$$

Where $\|\overrightarrow{oA}\|$ is the length of the vector $\overrightarrow{oA}$.

From equations (1.3) and (1.4), we get

$$\tan\theta = \frac{a_y}{a_x}$$

Length of the vector

Length of the vector is the square root of sum of squares of its compounds, there for

$$\|\overrightarrow{oA}\| = \sqrt{a_x^2 + a_y^2}$$

Unit vector

The unit vector is the quotient of the vector divided by its length. So, if $\vec{u}$ any vector, the unit vector in the direction of vector $\vec{u}$ is given by

$$\hat{u} = \frac{\vec{u}}{\|\vec{u}\|}$$

In three dimensional space: Vectors are identified with triples of scalar components, a_x, a_y and a_z, i.e.

$$\overrightarrow{oA} = (a_x, a_y, a_z)$$

Or

$$\overrightarrow{oA} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

Where $\hat{k}$ is the unit vector in the direction of OZ axis

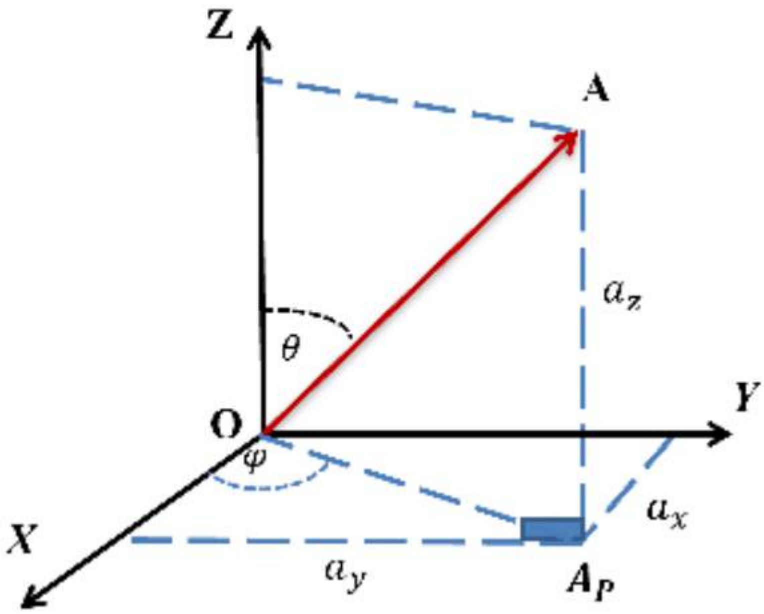


Fig.2: vector in 3D space

As shown in figure 2, the angle between vector $\overrightarrow{OA}$ and z-axis is , the projection of point A is A_P , OA_P makes an angle with x-axis. We denoted to this angle by symbol φ, there for the components of vector $\overrightarrow{OA}$ are

$$a_z = \|\overrightarrow{OA}\| \cos\theta$$

$$a_x = \|\overrightarrow{OA}\| \sin\theta \cos\varphi$$

$$a_y = \|\overrightarrow{OA}\| \sin\theta \sin\varphi$$

The coordinate direction angles of vector, from figure 3,

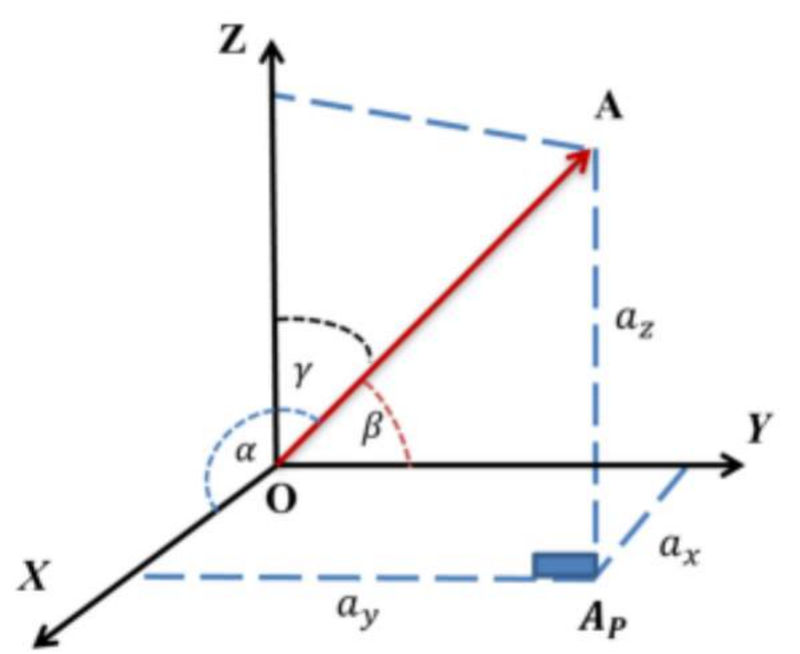


Fig.3: Coordinate direction angles of vector

α is the angle between the vector $\overrightarrow{OA}$ and the x-axis

β is the angle between the vector $\overrightarrow{OA}$ and the y-axis

γ is the angle between the vector $\overrightarrow{OA}$ and the z-axis

α, β and γ are called the coordinate angles of vector, vector $\overrightarrow{OA}$ is written by these angles as

$$\overrightarrow{oA} = \|\overrightarrow{OA}\| \cos\alpha\, \hat{\imath} + \|\overrightarrow{OA}\| \cos\beta\, \hat{\jmath} + \|\overrightarrow{OA}\| \cos\gamma\, \hat{k}$$

The unit vector in the direction of vector $\overrightarrow{OA}$ is

$$\widehat{OA} = \cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k}$$

The relation between angles α, β and γ is

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

The length of $\overrightarrow{OA}$ is

$$\left\|\overrightarrow{OA}\right\| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

α, β and γ are given also from

$$\cos\alpha = \frac{a_x}{\left\|\overrightarrow{OA}\right\|}$$

$$\cos\beta = \frac{a_y}{\left\|\overrightarrow{OA}\right\|}$$

$$\cos\gamma = \frac{a_z}{\left\|\overrightarrow{OA}\right\|}$$

1.1.3 Cartesian position vector

The position of a point $P = (P_x, P_y, P_z)$ with respect to origin point $O = (0,0,0)$ is $\overrightarrow{OP}$, we write it in the form

$$\vec{r}_{op} = \overrightarrow{OP} = (P_x, P_y, P_z) - (0,0,0)$$

In general, the position vector of any point , B, with respect to another point , A, is $\overrightarrow{AB}$ or $\vec{r}_{AB}$, where

$$\vec{r}_{AB} = \vec{B} - \vec{A} = \vec{r}_B - \vec{r}_A$$

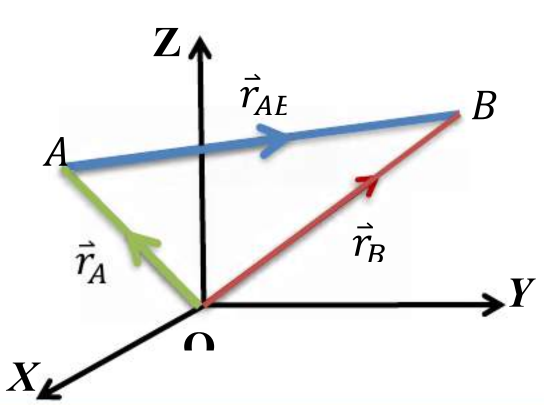


Fig.4: position vector

1.1.4 Operations on vectors

Addition: Let, we have two vectors $\vec{A}$ and $\vec{B}$, such that

$$\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{B} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

The addition of $\vec{A}$and $\vec{B}$ is

$$\vec{A} + \vec{B} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}$$

1.1.5 Product of two vectors

Scalar product or dot product: The Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is a real number, writing as $\vec{A} \cdot \vec{B}$, and it is given by

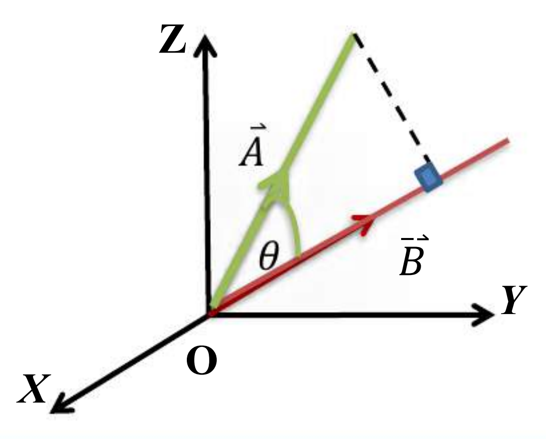


Fig.5: Scalar product vector product of

$$\vec{A} \cdot \vec{B} = (a_x \times b_x) + (a_y \times b_y) + (a_z \times b_z)$$

Or

$$\vec{A} \cdot \vec{B} = \|\vec{A}\|\|\vec{B}\| \cos \boldsymbol{\theta}$$

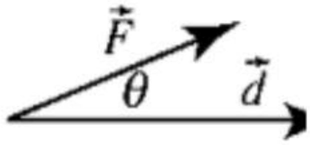
Where θ is the angle between $\vec{A}$ and $\vec{B}$, from the above equation

$$\theta = cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|\|\vec{B}\|}\right)$$

The projection of vector $\vec{A}$ along the direction of vector $\vec{B}$ is

$$\|\vec{A}\|\cos\boldsymbol{\theta} \;=\; \frac{\vec{A}\cdot\vec{B}}{\|\vec{B}\|}$$

One important physical application of the scalar product is the calculation of work, such that



$$W = \vec{F}\cdot\vec{d} = \|\vec{F}\|\cos\boldsymbol{\theta}\;\|\vec{d}\|$$

The scalar product is also used for the expression of magnetic potential energy and the potential of an electric dipole.

Notes:

- $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- If λ is a scalar and $\vec{A}$ is a vector, then
  $\lambda\vec{A} = \lambda\left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) = \lambda A_x\hat{\imath} + \lambda A_y\hat{\jmath} + \lambda\, A_z\hat{k}$
- The dot product of unit vectors are

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\jmath}\cdot\hat{k} = \hat{\imath}\cdot\hat{k} = 0$$

- $\lambda\left(\vec{A}\cdot\vec{B}\right) = \left(\lambda\vec{A}\right)\cdot\vec{B} = \vec{A}\cdot\left(\lambda\vec{B}\right) = \left(\vec{A}\cdot\vec{B}\right)\lambda$
- $\vec{A}\cdot\left(\vec{B}\pm\vec{C}\right) = \vec{A}\cdot\vec{B}\pm\vec{A}\cdot\vec{C}$
- The scalar product is negative when $90^o < \theta < 180^o$
- If vector $\vec{A}$ is perpendicular to vector $\vec{B}$,then $\vec{A}\cdot\vec{B} = 0$

Vector product (cross product): The vector product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A} \times \vec{B}$ and is often referred to as a cross product**.** The vector product is a vector that has its direction perpendicular to both vectors $\vec{A}$ and $\vec{B}$ and it is denoted by $\hat{n}$. In other words, vector $\vec{A} \times \vec{B}$ is perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$. The magnitude of the vector product is defined as

$$\vec{A} \times \vec{B} = \|\vec{A}\|\|\vec{B}\| \sin\theta \ \hat{n}$$

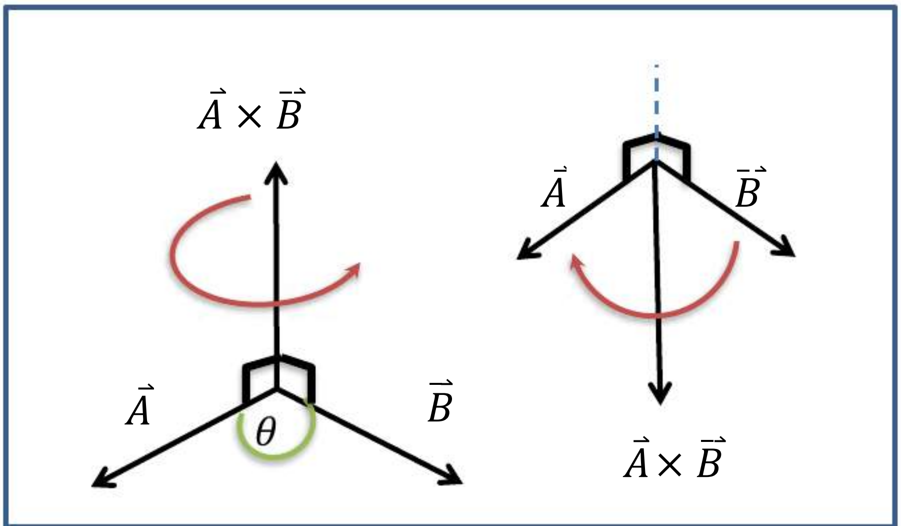


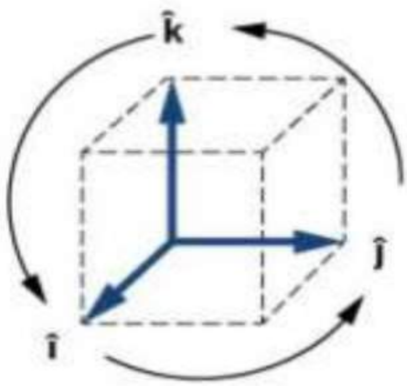
Fig.6: vector product of two vectors

The cross product of unit vectors are

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \vec{0}$$

$\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$

$\hat{k} \times \hat{\imath} = \hat{\jmath}$, $\hat{\jmath} \times \hat{k} = -\hat{\jmath}$, ...



We can find the cross product $\vec{A} \times \vec{B}$ for vectors $\vec{A}$ and $\vec{B}$ as the following

$$\vec{A} \times \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\ \right)$$

$$\vec{A} \times \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\ \right)$$

$$= A_x\hat{\imath} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\ \right) + A_y\hat{\jmath} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\ \right) + A_z\hat{k} \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\ \right)$$

$$= \left(A_yB_z - A_zB_y\right)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

There for , the cross product in the (formal) determinant is

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Notes:

- $\vec{A} \times \vec{A} = \vec{0}$
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- If $\vec{A}$is parallel to $\vec{B}$, then $\vec{A} \times \vec{B} = \vec{0}$
- If $\hat{n}$ is a unit vector perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, then $\hat{n} = \frac{\vec{A}\times\vec{B}}{\|\vec{A}\|\|\vec{B}\| \sin\theta}$
- If $\vec{A}$and $\vec{B}$ are parallel, then $\vec{A} = \lambda\vec{B}$
- The quantity $\|\vec{A}\|\|\vec{B}\| \sin\theta$ is the area of the parallelogram with sides $\vec{A}$ and $\vec{B}$. The cross product only makes sense in R^3

1.1.6 The scalar triple product

Let $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$, $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$ and $\vec{C} = C_x\hat{\imath} + C_y\hat{\jmath} + C_z\hat{k}$. Then the scalar triple product is given by the formula

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

In Geometry: The absolute value of $\vec{A} \cdot (\vec{B} \times \vec{C})$ is the volume of the parallelepiped constructed on the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as it is shown in the figure (1.7)

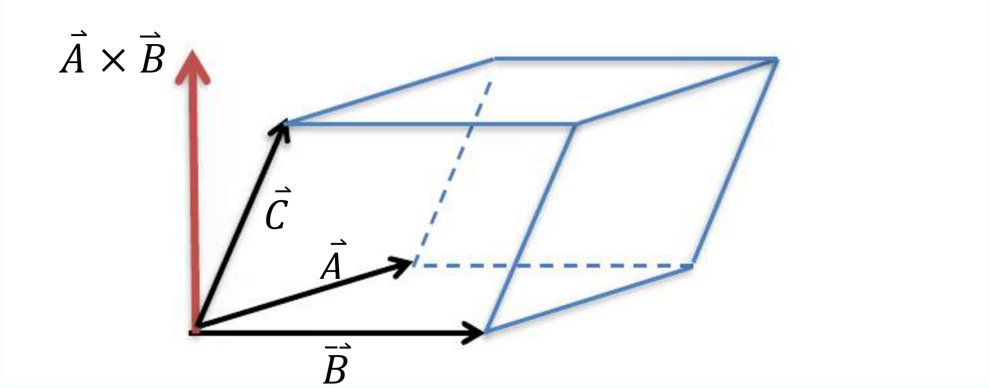


Fig.7: Geometric Interpretation of scalar triple product

1.1.7 Vector triple product

The second type of triple product is the vector triple product of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, it has the form $\vec{A} \times (\vec{B} \times \vec{C})$.

The vector triple product, $\vec{A} \times (\vec{B} \times \vec{C})$ is a vector, is normal to $\vec{A}$ and normal to $\vec{B} \times \vec{C}$ which means it is in the plane of $\vec{B}$ and $\vec{C}$. The vector triple product can express as a linear combination of the vectors $\vec{B}$ and $\vec{C}$, i.e.,

$$\vec{A} \times (\vec{B} \times \vec{C}) = \alpha\vec{B} + \beta\vec{C}$$

Where α and β are scalar.

The vector triple product can be written in the form

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

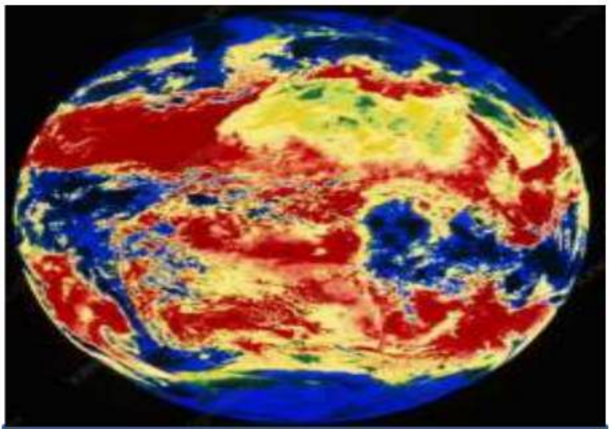


Fig.8: The heat of the Earth. Color-coded infrared view of

1.1.8 Scalar field

A scalar field is a function which assigns to every point of space a scalar value, this value is a real number or a physical quantity.

A good example of a scalar field would be the temperature in room, Also the temperature of the earth.

Another example of scalar field, Potential fields, such as the Newtonian gravitational potential, or the electric potential in electrostatics, are scalar fields which describe the more familiar forces.

1.18 Vector field

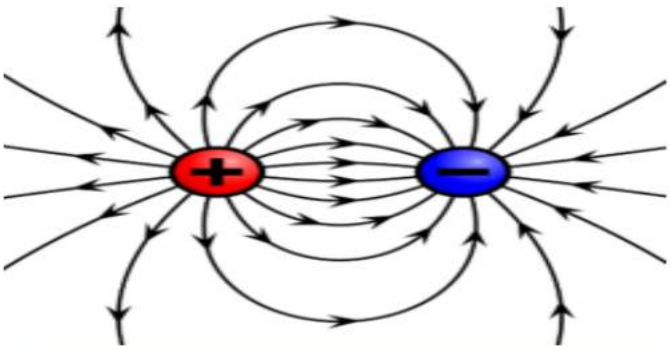


Fig.9: Vector field, the electric field lines of two charges .

A vector field is a function that assigns a vector to every point in space. For example, the velocity of the air is considered a vector field . Also, the electric field and magnetic field are considered a vector fields.

1.19 Gradient of scalar field

Suppose that, we have a scalar field $F(x, y, z)$, what is the change that occurs to the function F when the variables x, y and z change an infinitesimal change dx, dy and dz ?

If we denote to this change by dF , then

$$dF = \frac{\partial F}{\partial x}\,dx + \frac{\partial F}{\partial y}\,dy + \frac{\partial F}{\partial z}\,dz$$

The previous equation can be written on the form

$$dF = \left(\frac{\partial F}{\partial x}\,\hat{\imath} + \frac{\partial F}{\partial y}\,\hat{\jmath} + \frac{\partial F}{\partial z}\,\hat{k}\right)\cdot\left(\,dx\hat{\imath} + \,dy\hat{\jmath} + dz\hat{k}\right)$$

$$dF = \left(\frac{\partial}{\partial x}\,\hat{\imath} + \frac{\partial}{\partial y}\,\hat{\jmath} + \frac{\partial}{\partial z}\,\hat{k}\right)F\cdot\left(\,dx\hat{\imath} + \,dy\hat{\jmath} + dz\hat{k}\right)$$

In the above equation the bracket $\left(\frac{\partial}{\partial \mathrm{x}}\,\hat{\mathrm{i}} + \frac{\partial}{\partial \mathrm{y}}\,\hat{\mathrm{j}} + \frac{\partial}{\partial \mathrm{z}}\,\hat{\mathrm{k}}\right)$ is called Nabla operator or Del operator and it is symbolized by the symbol $\vec{\nabla}$, such that

$$\vec{\nabla} = \frac{\partial}{\partial \mathrm{x}}\,\hat{\mathrm{i}} + \frac{\partial}{\partial \mathrm{y}}\,\hat{\mathrm{j}} + \frac{\partial}{\partial \mathrm{z}}\,\hat{\mathrm{k}}$$

So

$$dF = \vec{\nabla}F\cdot\left(\,dx\hat{\imath} + \,dy\hat{\jmath} + dz\hat{k}\right)$$

The quantity $\vec{\nabla}F$ is called the vector derivative of scalar field F or gradient of F, where

$$grad\ F = \vec{\nabla}\mathrm{F} = \frac{\partial F}{\partial x}\,\hat{\imath} + \frac{\partial F}{\partial y}\,\hat{\jmath} + \frac{\partial F}{\partial z}\,\hat{k}$$

The rate of change of a function of several variables in the direction of any arbitrary vector $\vec{\mathrm{u}}$ is called the directional derivative in the direction $\vec{u}$. Let $D_u F$ is the directional derivative of $F(x, y, z)$ in the direction $\vec{u}$, then

$$D_u F = \vec{\nabla} F \cdot \hat{u}$$

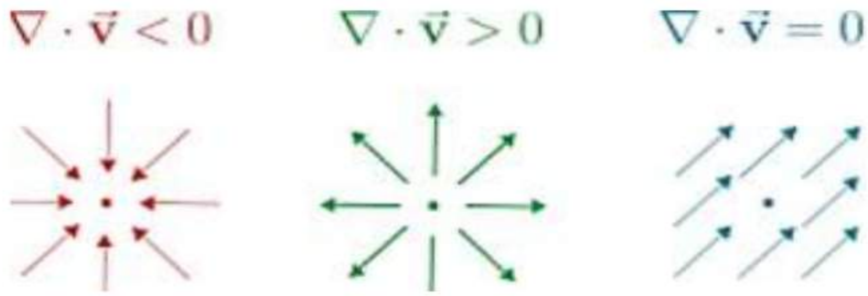
1.1.2 Divergence of a vector field

Let $\vec{A}(x, y, z)$ be a vector field, where $\vec{A} = A_1\hat{\imath} + A_2\hat{\jmath} + A_3\hat{k}$, then the divergence of $\vec{A}(x, y, z)$ is

$$\operatorname{div} \vec{A} = \nabla \cdot \vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

A good example on divergence of vector field , when air is heated in a region, it will locally expand, causing a positive divergence in the area of expansion

The divergence operator works on a vector field and produces a scalar field as a result

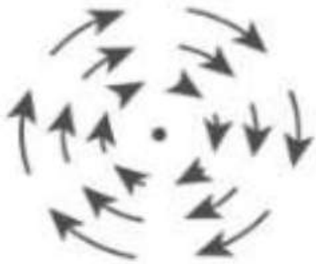


1.1.1 The curl of the vector field

The vector product of the operator nabla ∇ and a vector function $\vec{A}(x, y, z)$ is known as the curl of the vector field $\vec{A}(x, y, z)$

$$\operatorname{curl}\vec{A} = \nabla \times \vec{A} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{bmatrix}$$

There are a lot of physical examples on the curl of the vector field. For example, if the air is circulating in a particular region, then the curl in that region will represent the axis of rotation. The magnitude of the curl is twice the angular velocity of the vector field.



1.1.2 Laplacian operator

Laplacian operator is a measure of the second derivative of a scalar or vector field

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Just as in 1D where the second derivative relates to the curvature of a function, the Laplacian relates to the curvature of a field.

Solved examples

Example1.

Let $\vec{A} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k})m$ and $\vec{B} = (5\hat{\imath} + 6\hat{\jmath} + 4\hat{k})m$ be two vectos. Find

(i) $\|\vec{A} + 2\vec{B}\|$

(ii) The unit vector in the direction of $\vec{A} + 2\vec{B}$

(iii) The angle between vectors $\vec{A}\ and\ \vec{B}$

(iv) $\vec{A} \times \vec{B}$

Solution

(i) $\vec{A} + 2\vec{B} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}) + 2(5\hat{\imath} + 6\hat{\jmath} + 4\hat{k})$

$$\vec{A} + 2\vec{B} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}) + (10\hat{\imath} + 12\hat{\jmath} + 8\hat{k})$$

$$\vec{A} + 2\vec{B} = (12\hat{\imath} + 13\hat{\jmath} + 11\hat{k})$$

$$\|\vec{A} + 2\vec{B}\| = \sqrt{12^2 + 13^2 + 11^2} = 20.833m$$

(ii) The unit vector in the direction of $\vec{A} + 2\vec{B} = \frac{\vec{A}+2\vec{B}}{\|\vec{A}+2\vec{B}\|}$

$$= \frac{(12\hat{\imath} + 13\hat{\jmath} + 11\hat{k})}{20.833}$$

$$= \frac{12}{20.833}\hat{\imath} + \frac{13}{20.833}\hat{\jmath} + \frac{11}{20.833}\hat{k}$$

$$= 0.576\hat{\imath} + 0.624\hat{\jmath} + 0.528\hat{k}$$

(iii) The angle between vectors $\vec{A}\ and\ \vec{B}$ is θ,where

$$\theta = cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|.\|\vec{B}\|}\right)$$

$$\theta = cos^{-1}\left(\frac{10 + 6 + 12}{\sqrt{14}.\sqrt{77}}\right)$$

$$\theta = cos^{-1}\left(\frac{28}{\sqrt{14}.\sqrt{77}}\right)$$

$\theta = cos^{-1}$ (0.8528)

$$\theta = 31.482^o$$

(iv) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 5 & 6 & 4 \end{vmatrix}$

$$\vec{A} \times \vec{B} = (4 - 18)\hat{\imath} - (8 - 15)\hat{\jmath} + (12 - 5)\hat{k}$$

$$\vec{A} \times \vec{B} = -14\hat{\imath} + 7\hat{\jmath} + 7\hat{k}$$

Example2

Let $f(x, y, z)$ be a function such that $f(x, y, z) = e^{xy} + 2xy + z^2$. calculate ∇f , $\nabla^2 f$ at the point (1,0,1).

Solution

$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$$

$$\nabla f = (ye^{xy} + 2y + 0)\hat{\imath} + (xe^{xy} + 2x + 0)\hat{\jmath} + 2z\hat{k}$$

$$\nabla f\,(1{,}0{,}1) = (0 + 0 + 0)\hat{\imath} + (1 + 2 + 0)\hat{\jmath} + 2\hat{k}$$

$$\nabla f(1{,}0{,}1) = 3\hat{\jmath} + 2\hat{k}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla^2 f = (y^2 e^{xy}) + (x^2 e^{xy}) + 2$$

$$\nabla^2 f(1{,}0{,}1) = 1 + 2 = 3$$

Exaample 3.

Let $f(x, y, z)$ be a function such that $f(x, y, z) = x^3 + 5xy + 3z^2$. Find the directional derivative of $F(x, y, z)$ in the direction $\vec{\mathrm{u}}$; $\vec{u} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ at point (0,1,0).

Solution

$$\nabla f = (3x^2 + 5y + 0)\hat{\imath} + (0 + 5x + 0)\hat{\jmath} + (0 + 0 + 6z^2)\hat{k}$$

$$\hat{u} = \frac{3\hat{\imath} + 2\hat{\jmath} - \hat{k}}{\sqrt{14}}$$

$$\mathrm{D}_u f = \nabla f \cdot \hat{u}$$

$$\mathrm{D}_u f = \left((3x^2 + 5y)\hat{\imath} + 5x\hat{\jmath} + 6z^2\hat{k}\right) \cdot \frac{3\hat{\imath} + 2\hat{\jmath} - \hat{k}}{\sqrt{14}}$$

$$\mathrm{D}_u f = \frac{3(3x^2 + 5y)\hat{\imath} + 10x\hat{\jmath} - 6z^2\,\hat{k}}{\sqrt{14}}$$

$$\mathrm{D}_u f(0,1,0) = \frac{15}{\sqrt{14}}\hat{\imath}$$

Example 4.

Find $curl\vec{A}$ and $div\ \vec{A}$ at point (0,0,1),where

$$\vec{A} = (3x^2 + xy)\hat{\imath} + 3x\hat{\jmath} + z^2\,\hat{k}$$

Solution

$$curl\vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ (3x^2 + xy) & 3x & z^2 \end{vmatrix}$$

$$curl\vec{A} = \left(\frac{\partial}{\partial y}z^2 - \frac{\partial}{\partial z}3x\right)\hat{\imath} - \left(\frac{\partial}{\partial x}z^2 - \frac{\partial}{\partial z}(3x^2 + xy)\right)\hat{\jmath}$$
$$+\left(\frac{\partial}{\partial x}3x - \frac{\partial}{\partial y}(3x^2 + xy)\right)\hat{k}$$

$$curl\vec{A} = (0)\hat{\imath} - (0 - 0)\hat{\jmath} + (3 - x)\hat{k} = (3 - x)\hat{k}$$

$$curl\vec{A}(1,1,0) = 2\hat{k}$$

$$div\ \vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

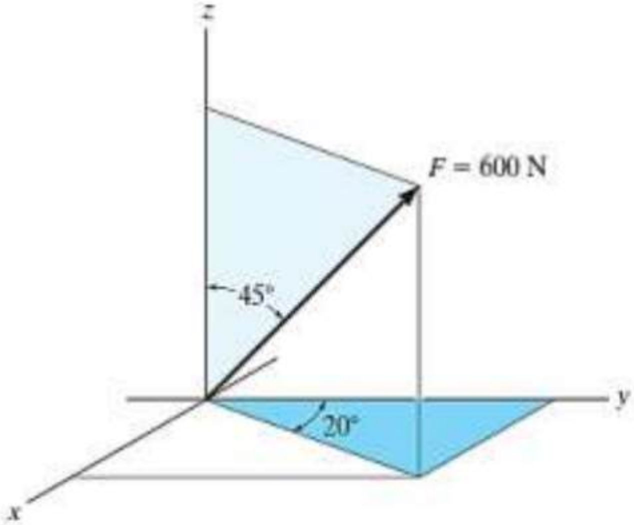
$$div\ \vec{A} = \frac{\partial\,(3x^2 + xy)}{\partial x} + \frac{\partial\,(3x)}{\partial y} + \frac{\partial\,(z^2)}{\partial z}$$

$$div\ \vec{A} = (6x + y) + 0 + 2z$$

$$div\ \vec{A}(1,1,0) = 7$$

Example 5.

Write the given force F in the following figure as vector



solution

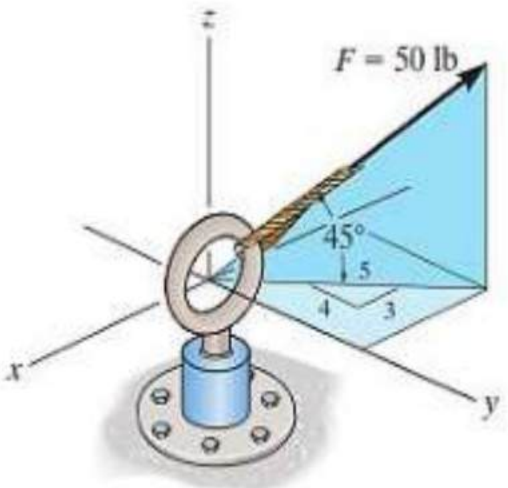
$$\vec{F} = Fsin45sin20\hat{\imath} + Fsin45cos20\hat{\jmath} + Fcos45\hat{k}$$

$$\vec{F} = 600sin45sin20\hat{\imath} + 600sin45cos20\hat{\jmath} + 600cos45\hat{k}$$

$$\vec{F} = 145.17\hat{\imath} + 398.68\hat{\jmath} + 563.82\hat{k}$$

Example 6.

Express the force as a Cartesian vector.



Solution

$$\vec{F} = -50cos45\left(\frac{3}{5}\right)\hat{\imath} + 50cos45\left(\frac{4}{5}\right)\hat{\jmath} + 50\sin 45\,\hat{k}$$

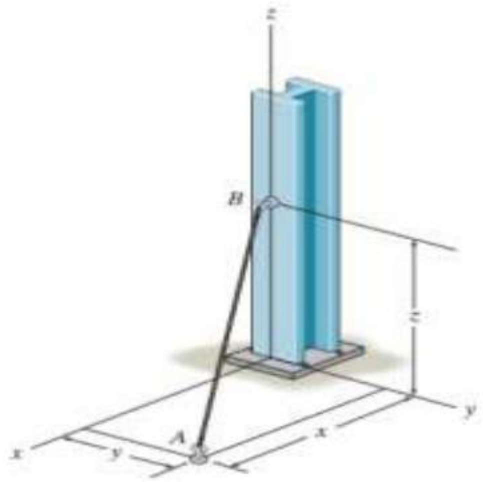
$$\vec{F} = -21.21\hat{\imath} + 28.28\hat{\jmath} + 35.36\hat{k}$$

Problems

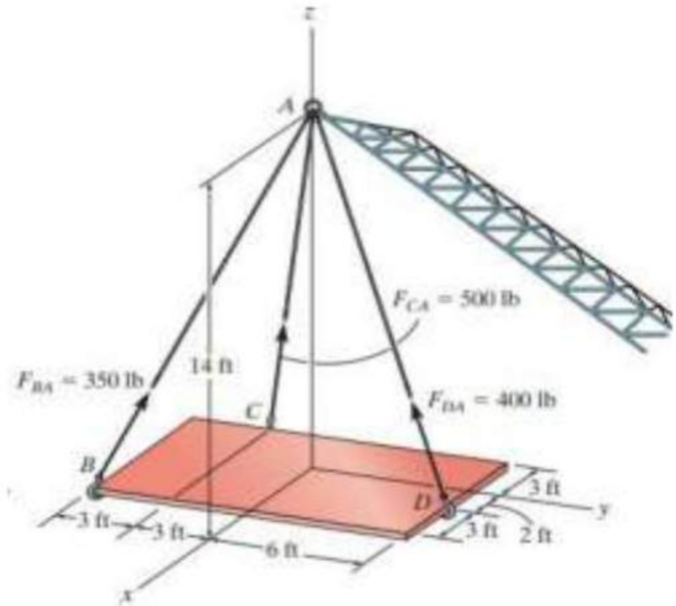
1- Given the vectors: $\vec{A} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{B} = 5\hat{\imath} + 5\hat{\jmath}$. Determine:

a. Their magnitude.
b. The direction of $\vec{B}$.
c. $\vec{A} + \vec{B}$
d. $\vec{A} - 2\vec{B}$
e. A unit vector parallel to $\vec{A}$.
f. A vector of magnitude 2 and opposite to $\vec{B}$.

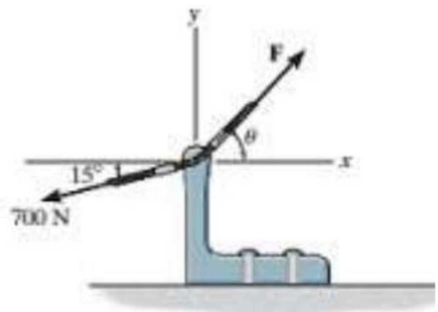
2- Cable AB of length 8m is anchored to the ground at A, as shown . If z = 5 m, determine the location + x, + y of point A. Choose a value such that x = y



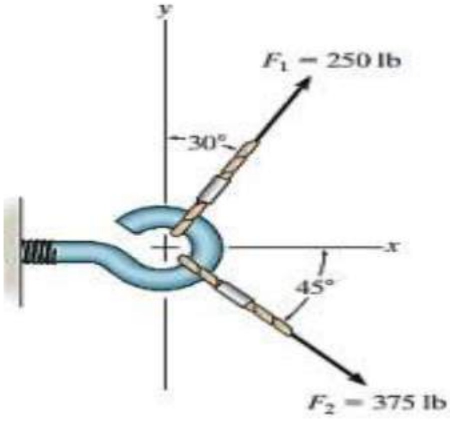
3- The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



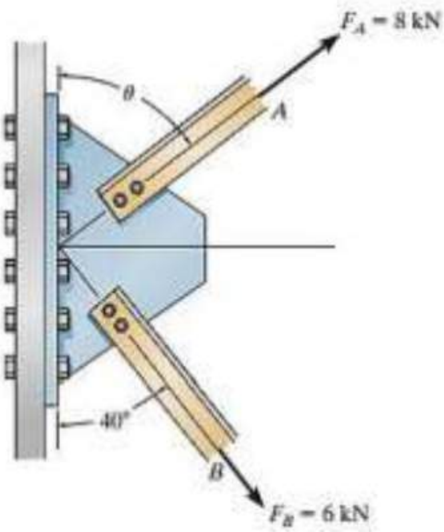
4- As showin the the figure, If $\theta = 30^o$ and $= 500N$, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



5- Determine the magnitude of the resultant force of the forces F_1, F_1 and its direction, measured counterclockwise from the positive x axis.

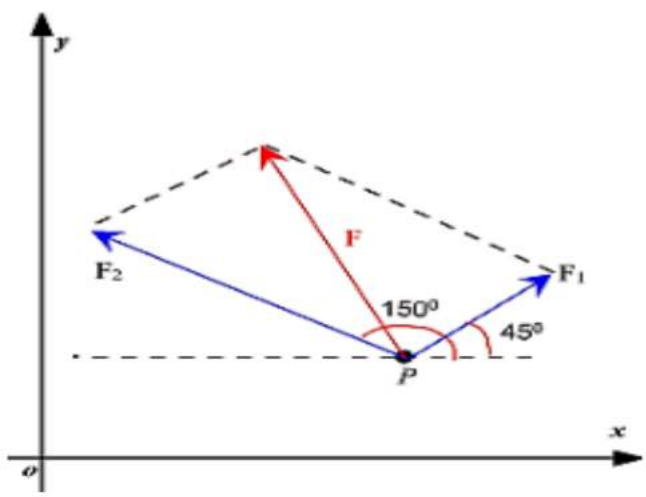


6- The plate is subjected to the two forces F_A, F_B as shown. If $= 30^o$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



7- A ball is thrown with an initial velocity of 60 feet per second, it makes an angle of 30° with the horizontal. Find the vertical and horizontal components of the velocity.

8- Two forces F_1 and F_2 with magnitudes 20 and 30 lb, respectively, act on an object at a point P as shown. Find the resultant forces acting at P.

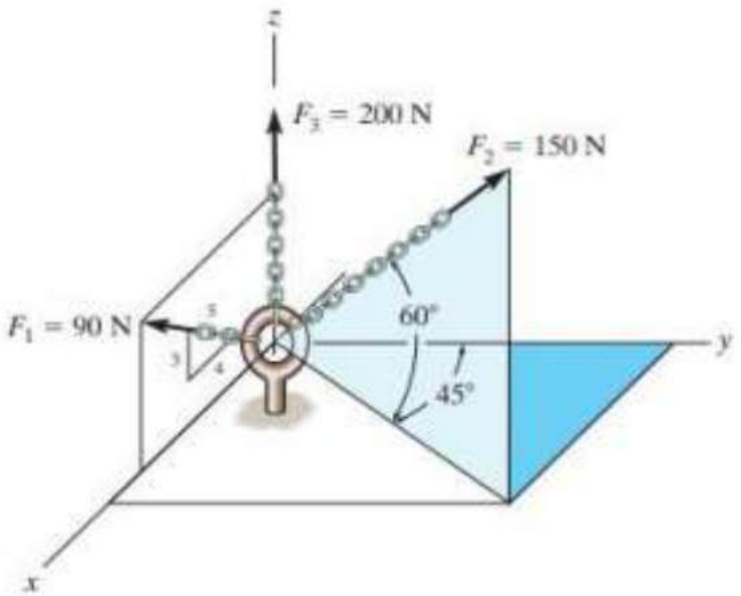


9- A force is given by the vector $\vec{F} = 2\hat{\imath} + 5\hat{\jmath}$ and moves an object from the point (1,2) to the point (2,6). Find the work done.

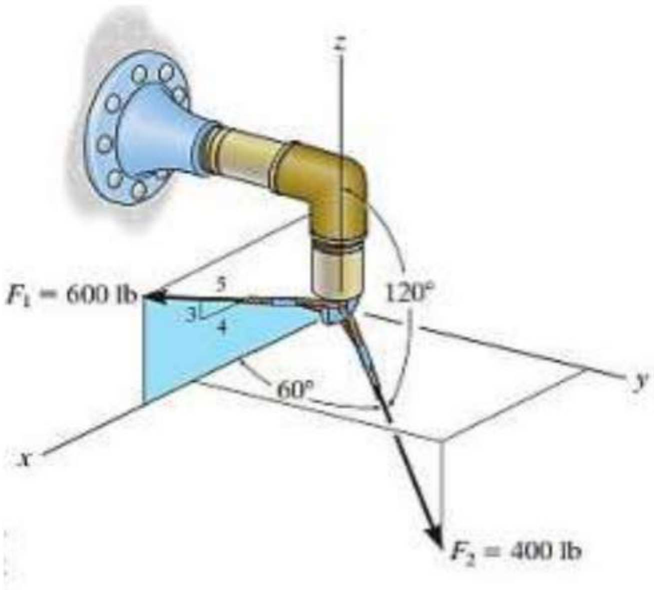
10- An airplane can fly at 200mph in still air and wishes to fly due west. There is a wind of 25mph blowing from the south west (exactly half way between south and west). Determine the direction the pilot must point the plane in order to fly in the desired direction and determine the resultant speed of the plane.

11- A car travels from A to B along 30 km north, then 60 km east, then 110 km south. Determine the displacement of the car from A to D.

12- As shown in the figure, express each force in Cartesian vector form.



13- Determine the magnitude and direction of the resultant force acting on the pipe assembly



14- If vector $\vec{V}$ is irrotational, then $curl\,\vec{V} = \vec{0}$

a) Ture
b) False

15- Which of the following can be a direction angle of a vector

a) 30°, 45°, 60°
b) 30°, 45°, 90°
c) 45°, 45°, 60°
d) 60°, 45°, 60°

16- Let α be a scalar and $\vec{A}$,$\vec{B}$ be a two vectors , if the $\vec{A} = \alpha\vec{B}$, then $\vec{A}$ and $\vec{B}$ are

a) Collinear vector
b) Along the same line
c) Parallel vectors
d) All of these

17- The maximum value of the directional derivative takes place in the direction of ∇f and its magnitude is

a) $|\nabla f|$
b) $|\sqrt{\nabla f}|$

c) $|\nabla^2 f|$
d) ∇f

18- The unit normal to the surface $x^3 - xyz + z^3 = 1$ at (1,1,0) is…

a) $3i - 2k$
b) $3j - 2k$
c) $5i + k$
d) $none\ of\ these$

20- The volume of the parallelepiped formed by the coterminous edges a,b,c is represented by

a) $\vec{a} \cdot \vec{b} \cdot \vec{c}$
b) $\vec{a} \times \vec{b} \times \vec{c}$
c) $\vec{a} \cdot (\vec{b} \times \vec{c})$
d) $\vec{a} \times (\vec{b} \cdot \vec{c})$

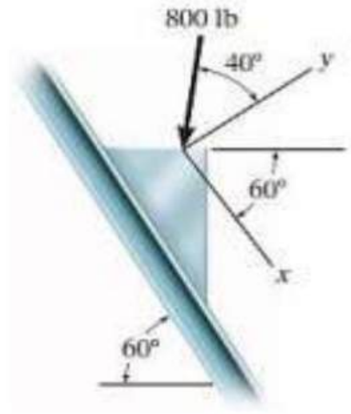
21- When two vectors are perpendicular, their
a) Dot product is zero
b) Cross product is zero
c) Both are zero
d) Both are not necessarily zero

23-If $\vec{A} \times \vec{B} = \vec{R}$ and $A + B = R$, then what is the angle between $\vec{A}\ and\ \vec{B}$
a) 0
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) π

24- What is the maximum rate of change of the function f at the point (2, 3, 6)?
a) 7
b) 19
c) 13
d) $none\ of\ these$

25-The x and y components of the 800-lb force is

a) 514,-613
b) 589,613
c) -514,613
d) 300,-200



26- Find the angle α, for the vector making an angle by y and z axis as 60° and 45° respectively. It makes an angle of α with x-axis. The magnitude of the force is 200N.

a) 60°
b) 120°
c) 45°
d) 90°

27. Mcq questions

1. Which of the following is a scalar quantity?
A) Force
B) Velocity
C) Work
D) Displacement

2. If a vector $\vec{A}$ is multiplied by a scalar -2, the resulting vector will be:

A) Half in magnitude and same in direction.
B) Twice in magnitude and same in direction.
C) Twice in magnitude and opposite in direction.
D) Half in magnitude and opposite in direction.

3. The dot product of two perpendicular vectors is always:
A) 1
B) 0
C) -1
D) Equal to the product of their magnitudes.

4. The cross product of two parallel vectors is always:
A) A vector of magnitude 1.
B) A vector with maximum magnitude.
C) A zero vector.
D) Equal to the dot product of the vectors.

5. If | $\vec{A} \times \vec{B}$ |= $\vec{A} \cdot \vec{B}$, then the angle between vectors $\vec{A}$ and $\vec{B}$ is:
A) $0°$
B) $45°$
C) $90°$
D) $180°$

6. Which of the following operations is not meaningful?
A) Adding a scalar to a vector.
B) Taking the dot product of two vectors.
C) Taking the cross product of two vectors.
D) Multiplying a vector by a scalar.

7. For two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A} + \vec{B} = \vec{A} - \vec{B}$, then:
A) $\vec{A}$ is a null vector.
B) $\vec{B}$ is a null vector.
C) $\vec{A}$ is parallel to $\vec{B}$.
D) $\vec{A}$ is perpendicular to $\vec{B}$.

8. The magnitude of the scalar product of two vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ and $4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is:
A) 28

B) 26
C) 24
D) 29

9. The direction of the vector resulting from the cross product $\vec{A} \times \vec{B}$ is given by:

A) The Right-Hand Thumb Rule.
B) The Left-Hand Thumb Rule.
C) The magnitude of $\vec{A}$.
D) The magnitude of $\vec{B}$.

10. If the magnitude of the cross product and dot product of two vectors are equal, the angle between them is:
A) $30°$
B) $45°$
C) $60°$
D) $90°$

11. The direction angles α, β, and γ of a vector in 3D space are the angles the vector makes with the:

A) x, y, and z-axes respectively.
B) y, z, and x-axes respectively.
C) The coordinate planes.
D) The origin.

12. For any vector $\vec{A} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$, the cosine of its direction angles are given by:
A) $\cos\,\alpha = \frac{a_x}{|\vec{A}|}, \cos\,\beta = \frac{a_y}{|\vec{A}|}, \cos\,\gamma = \frac{a_z}{|\vec{A}|}$
B) $\cos\,\alpha = \frac{|\vec{A}|}{a_x}, \cos\,\beta = \frac{|\vec{A}|}{a_y}, \cos\,\gamma = \frac{|\vec{A}|}{a_z}$
C) $\tan\,\alpha = \frac{a_x}{|\vec{A}|}, \tan\,\beta = \frac{a_y}{|\vec{A}|}, \tan\,\gamma = \frac{a_z}{|\vec{A}|}$
D) $\sin\,\alpha = \frac{a_x}{|\vec{A}|}, \sin\,\beta = \frac{a_y}{|\vec{A}|}, \sin\,\gamma = \frac{a_z}{|\vec{A}|}$

13. What is the relationship between the direction cosines l, m, n of a vector?

A) $l + m + n = 1$

B) $l^2 + m^2 + n^2 = 0$

C) $l^2 + m^2 + n^2 = 1$

D) $lm + mn + nl = 1$

14. A vector has direction angles $\alpha = 60°$ and $\beta = 45°$. What is the angle γ it makes with the z-axis?

A) $30°$

B) $45°$

C) $60°$

D) $90°$

15. If a vector makes equal angles with all three coordinate axes, what is the measure of each angle?

A) $0°$

B) $45°$

C) $\cos^{-1}(\frac{1}{\sqrt{2}})$

D) $\cos^{-1}(\frac{1}{\sqrt{3}})$

16. The direction ratios of a vector are:

A) The same as its direction cosines.

B) Proportional to its direction cosines.

C) Inversely proportional to its direction cosines.

D) Unrelated to its direction cosines.

17. A vector is given by $\vec{r} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$. What is the direction cosine corresponding to the y-axis?

A) $\frac{2}{3}$

B) $\frac{-2}{3}$

C) $\frac{1}{3}$

D) $\frac{-1}{3}$

18. A unit vector has:

A) Direction ratios that sum to 1.

B) Direction cosines that are equal to its components.

C) Magnitude equal to 1.

D) All direction angles equal to 90°.

19. If the direction cosines of a vector are $(\frac{1}{2}, \frac{1}{2}, k)$, what is the value of k?

A) $\frac{1}{\sqrt{2}}$

B) $\frac{1}{2}$

C) $\frac{\sqrt{2}}{2}$

D) $\frac{\sqrt{3}}{2}$

20. A vector lies in the x-y plane. What is the angle γ it makes with the z-axis?

A) $0°$

B) $45°$

C) $90°$

D) $180°$

Answer Key & Detailed Explanations

1. C) Work

- *Explanation:* Scalar quantities have only magnitude. Work is a scalar quantity as it is the dot product of Force and Displacement vectors. Force, Velocity, and Displacement are vector quantities as they have both magnitude and direction.

2. C) Twice in magnitude and opposite in direction.

- *Explanation:* Multiplying a vector by a scalar changes its magnitude. A positive scalar only changes magnitude, but a negative scalar also reverses the direction. Here, -2 doubles the magnitude and makes the direction opposite.

3. B) 0

- *Explanation:* The dot product formula is $\vec{A} \cdot \vec{B} = |A||B| \cos \theta$. When $\theta = 90°$, $\cos 90° = 0$, so the dot product is zero.

4. C) A zero vector.

- *Explanation:* The magnitude of the cross product is $|\vec{A} \times \vec{B}| = |A||B| \sin \theta$. For parallel vectors, $\theta = 0°$ and $\sin 0° = 0$. Therefore, the magnitude is zero, resulting in a zero vector.

5. B) $45°$

- *Explanation:* We are given $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$.
  - $|\vec{A} \times \vec{B}| = |A||B| \sin \theta$
  - $\vec{A} \cdot \vec{B} = |A||B| \cos \theta$

Equating them: $|A||B| \sin \theta = |A||B| \cos \theta$
This simplifies to $\tan \theta = 1$, so $\theta = 45°$.

6. A) Adding a scalar to a vector.

- *Explanation:* Adding a scalar (which has no direction) to a vector (which has direction) is a physically meaningless operation. It is not defined in vector algebra.

7. B) $\vec{B}$ is a null vector.

- *Explanation:* If $\vec{A} + \vec{B} = \vec{A} - \vec{B}$, then subtracting $\vec{A}$ from both sides gives $\vec{B} = -\vec{B}$. This is only possible if $2\vec{B} = \vec{0}$, which means $\vec{B}$ is a null (zero) vector.

8. A) 28

- *Explanation:* The scalar (dot) product is calculated as: $(2)(4) + (3)(2) + (4)(3) = 8 + 6 + 12 = 26$.

(Note: There seems to be a discrepancy. The calculation gives 26, which is option B. If the intended answer is A) 28, the vectors in the question might have been different. The correct solution for the given vectors is 26). **Correction:** Let's re-check the calculation: (2*4=8) + (3*2=6) + (4*3=12) = 26. Therefore, the correct answer is **B) 26**.

9. A) The Right-Hand Thumb Rule.

- *Explanation:* The Right-Hand Thumb Rule is used to determine the direction of the cross product $\vec{A} \times \vec{B}$. If you point your fingers in the direction of $\vec{A}$ and curl them towards $\vec{B}$, your thumb points in the direction of the resultant vector.

10. B) $45°$

Explanation: This is the same as question 5. The condition

$|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$ leads to $|A||B| \sin\theta = |A||B| \cos\theta$, which simplifies to $\tan\theta = 1$, so $\theta = 45°$.

11. A) x, y, and z-axes respectively.

- *Explanation:* By definition, α is the angle with the positive x-axis, β with the positive y-axis, and γ with the positive z-axis.

12. A) $\cos\alpha = \frac{a_x}{|\vec{A}|}, \cos\beta = \frac{a_y}{|\vec{A}|}, \cos\gamma = \frac{a_z}{|\vec{A}|}$

- *Explanation:* This is the fundamental definition of direction cosines. The component of a vector along an axis is the projection of the vector onto that axis, which is $|\vec{A}| \cos\theta$.

13. C) $l^2 + m^2 + n^2 = 1$

. *Explanation:* If $l = \cos\alpha = \frac{a_x}{|\vec{A}|}$, $m = \cos\beta = \frac{a_y}{|\vec{A}|}$, $n = \cos\gamma = \frac{a_z}{|\vec{A}|}$, then $l^2 + m^2 + n^2 = \frac{a_x^2 + a_y^2 + a_z^2}{|\vec{A}|^2} = \frac{|\vec{A}|^2}{|\vec{A}|^2} = 1$.

14. C) 60°

Explanation: Using the identity $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$:

$$\cos^2(60^\circ) + \cos^2(45^\circ) + \cos^2\gamma = 1$$
$$(\frac{1}{2})^2 + (\frac{1}{\sqrt{2}})^2 + \cos^2\gamma = 1$$
$$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$$
$$\frac{3}{4} + \cos^2\gamma = 1$$
$$\cos^2\gamma = \frac{1}{4}$$

$\cos\gamma = \frac{1}{2}$ (Taking the positive root as it's a direction angle)

Therefore, $\gamma = 60^\circ$.

15. D) $\cos^{-1}(\frac{1}{\sqrt{3}})$

Explanation: If the angles are equal, $\alpha = \beta = \gamma$. So, $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$.

$$3\cos^2\alpha = 1$$
$$\cos^2\alpha = \frac{1}{3}$$
$$\cos\alpha = \frac{1}{\sqrt{3}}$$

Therefore, $\alpha = \cos^{-1}(\frac{1}{\sqrt{3}})$.

16. B) Proportional to its direction cosines.

- *Explanation:* Direction ratios (a, b, c) are numbers that are proportional to the direction cosines (l, m, n). That is, $l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}$.

17. B) $\frac{-2}{3}$

- *Explanation:* First, find the magnitude: $|\vec{r}| = \sqrt{(2)^2 + (-2)^2 + (1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$.

The direction cosine for the y-axis (m) is $\frac{a_y}{|\vec{r}|} = \frac{-2}{3}$.

18. C) Magnitude equal to 1.

- *Explanation:* This is the definition of a unit vector. Its direction cosines are simply its components along the x, y, and z axes.

19. B) $\frac{1}{2}$

Explanation: Using $l^2 + m^2 + n^2 = 1$:

$$(\frac{1}{2})^2 + (\frac{1}{2})^2 + k^2 = 1$$

$$\frac{1}{4} + \frac{1}{4} + k^2 = 1$$

$$\frac{1}{2} + k^2 = 1$$

$$k^2 = \frac{1}{2}$$

$$k = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

(Note: $\frac{\sqrt{2}}{2}$ is the simplified form of $\frac{1}{\sqrt{2}}$, which is equivalent to option C). The value $\frac{1}{2}$ from the question's pattern is incorrect for k. The correct calculation gives k=1/√2).

Correction: The correct answer is **C)** $\frac{\sqrt{2}}{2}$.

20. C) 90°

Explanation: If a vector lies in the x-y plane, it has no z-component. Therefore, it is perpendicular to the z-axis. The angle between the vector and the z-axis, γ, is 90°, and $\cos\,\gamma = 0$.

Chapter 2

Forces

A force is any action that is acting upon an object as a result of its interaction with another object, it is generally characterized by its point of application, its direction and its magnitude. The force is measured in the SI unit of newtons $(N = \frac{kg \cdot m}{sec^2})$ and represented by the symbol F. A force can cause an object with mass to change its velocity (which includes to begin moving from a state of rest), i.e., to accelerate. Force can also be described intuitively as a push or a pull.

Units of force					
	newton (SI unit)	dyne	kilogram-force, kilopond	pound-force	poundal
1 N	$\equiv 1\ kg{\cdot}m/s^2$	$= 10^5$ dyn	≈ 0.10197 kp	≈ 0.22481 lbf	≈ 7.2330 pdl
1 dyn	$= 10^{-5}$ N	$\equiv 1\ g{\cdot}cm/s^2$	$\approx 1.0197 \times 10^{-6}$ kp	$\approx 2.2481 \times 10^{-6}$ lbf	$\approx 7.2330 \times 10^{-5}$ pdl
1 kp	$= 9.80665$ N	$= 980665$ dyn	$\equiv g_n \cdot (1\ kg)$	≈ 2.2046 lbf	≈ 70.932 pdl
1 lbf	≈ 4.448222 N	≈ 444822 dyn	≈ 0.45359 kp	$\equiv g_n \cdot (1\ lb)$	≈ 32.174 pdl
1 pdl	≈ 0.138255 N	≈ 13825 dyn	≈ 0.014098 kp	≈ 0.031081 lbf	$\equiv 1\ lb{\cdot}ft/s^2$
The value of g_n as used in the official definition of the kilogram-force is used here for all gravitational units.					

Types of forces

Forces can be divided into primarily into two types of forces:

Contact Forces: Any types of forces that require being in contact with another object come under 'Contact Force'. All mechanical forces are contact forces. Contact forces further divide into following types of forces:

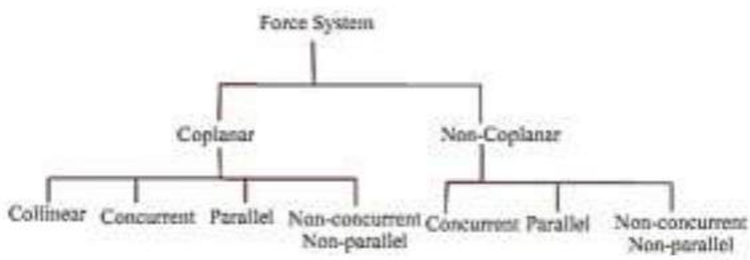
- **Muscular force :** Muscles functions to produce a resulting force which is known as 'muscular force'. Muscular force exists only when it is in contact with an object. We apply muscular force during the basic day to day work of our life such as breathing, digestion, lifting a bucket, pulling or pushing some object. Muscular force comes in handy to simply our work
- **Frictional Force:** friction force is the force exerted by a surface as an object moves across it or makes an effort to move across it. There are at least two types of friction force - sliding and static friction. Though it is not always the case, the friction force often opposes the motion of an object.
- **Tension Force:** Tension is the force applied by a fully stretched cable or wire anchored on to an object. This causes a 'tension force' that pulls equally in both directions and exerts equal pressure.
- **Normal Force:** The normal force is the force that surfaces exert to prevent solid objects from passing through each other. Normal force is a contact force. If two surfaces are not in contact, they can't exert a normal force on each other.
- **Air Resistance Force:** Air resisting forces are types of forces wherein objects experience a frictional force when moving through the air. These forces are resistive in nature.
- **Applied Force:** A force that is applied to a person or object, i.e. When someone pushes a table across the room, he applies a force that acts when it comes in contact with another object
- **Spring Force**: spring force is the force exerted by a compressed or stretched spring upon any object that is attached to it. An object that

compresses or stretches a spring is always acted upon by a force that restores the object to its rest or equilibrium position. For most springs (specifically, for those that are said to obey "Hooke's Law"), the magnitude of the force is directly proportional to the amount of stretch or compression of the spring.

Non-contact Forces: A non-contact force is a force applied to an object by another body that is not in direct contact with it. Non-contact forces come into play when objects do not have physical contact between them or when a force is applied without any interaction. For example:

- **Magnetic force:** The ***magnetic force*** is a consequence of the electromagnetic ***force***, one of the four fundamental ***forces*** of nature, and is caused by the motion of charges. Two objects containing charge with the same direction of motion have a ***magnetic*** attraction ***force*** between them
- **Electrostatics force:** The electrostatic force is also known as the Coulomb force or Coulomb interaction. It's the attractive or repulsive force between two electrically charged objects. Like charges repel each other while unlike charges attract each other. Coulomb's law is used to calculate the strength of the force between two charges
- **Gravitational force**: The gravitational force is a force that attracts any two objects with mass. We call the gravitational force attractive because it always tries to pull masses together, it never pushes them apart.

Classification of force,



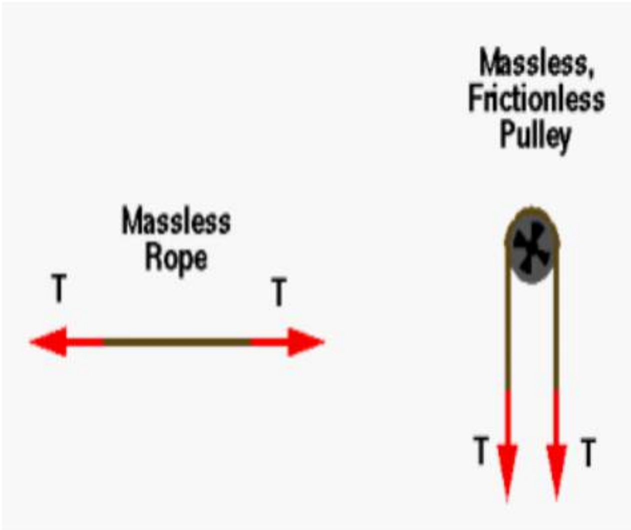
In mechanics, when a system has more than one force acting, it is known as a force system or system of force. The following systems of forces are important from the subject point of view:

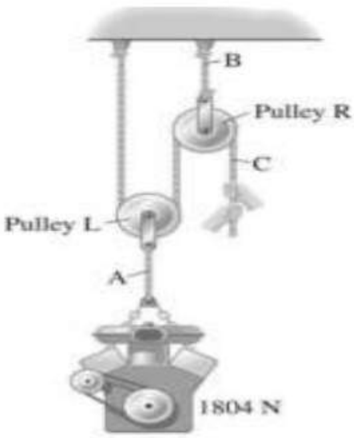
- **Collinear Force System:** When the lines of action of all the forces of a system act along the same line, this force system is called collinear force system.
- **Parallel Forces:** When the lines of action of a set of forces are parallel, the system is is called parallel force system.
- **Coplanar Force System**: When the lines of action of a set of forces lie in a single plane is called coplanar force system.
- **Coplanar and non-concurrent force system:** These forces do not meet at a common point; however, they lie in a single plane
- **Non-coplanar and concurrent force system**: In this system, the forces lie in a different planes but pass through a single point.
- **Non-coplanar and non-concurrent force system:** The forces which do not lie in a single plane and do not pass through a single point are known as non-coplanar and non-concurrent forces
- **Non-Coplanar Force System:** When the line of action of all the forces do not lie in one plane, is called Non-coplanar force system
- **Concurrent Force System:** The forces when extended pass through a single point and the point is called point of concurrency. The lines of actions of all forces meet at the point of concurrency. Concurrent forces may or may not be coplanar
- **Non-concurrent Force System:** When the forces of a system do not meet at a common point of concurrency, this type of force system is called non-concurrent force system. Parallel forces are the example of this type of force system. Non-concurrent forces may be coplanar or non-coplanar.
- **Coplanar and concurrent force system:** A force system in which all the forces lie in a single plane and meet at one point.

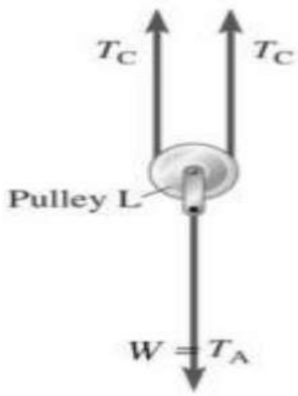
When two surfaces touch, they exert a forces on each other that are opposite in the direction and equal in magnitude, each force from these forces can be resolved into two perpendicular components, tangent force F_T and normal force F_N

Strings, ropes, wires and cables:

Strings, ropes, wires, cables etc. are used as a mechanism to transmit force, the tension in a rope is equal to the force being transmitted. Tension is always a "pulling" force. A rope cannot push an object. Therefore a rope under tension "pulls" in both directions.

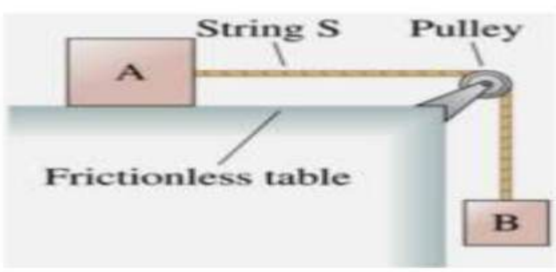






An ideal pulley is one that simply changes the direction of the tension:

Strings and ropes often pass over pulleys that change the direction of the tension. In principle, the friction and inertia in

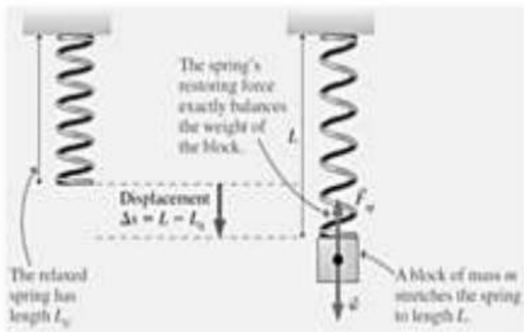


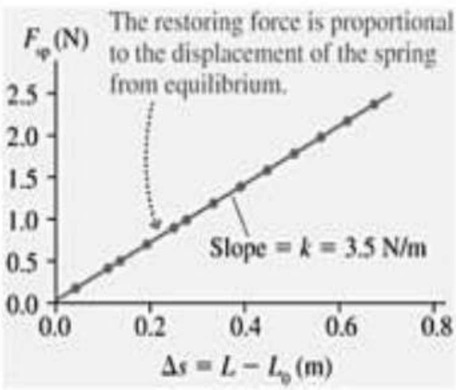
the pulley could modify the transmitted tension. Therefore, it is conventional to assume that such pulleys are massless and frictionless

Springs:

The unloaded spring has a length L_0. Hang a weight of mass m on it and it stretches to a new length L.

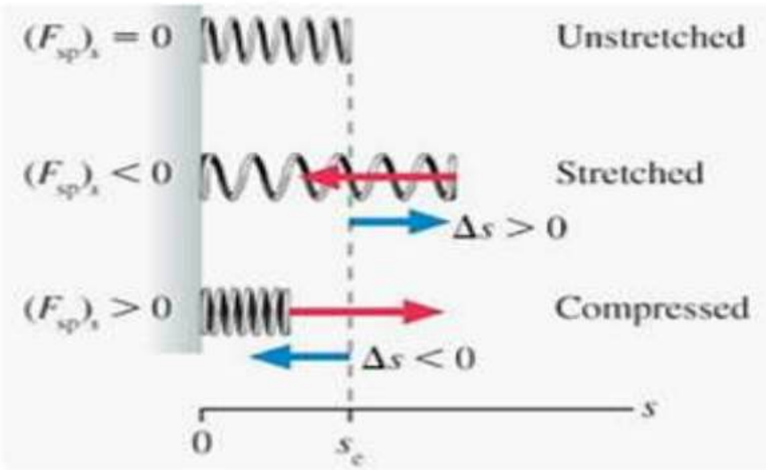
$\Delta s = L - L_0$ vs. the applied force $F_{sp} = mg$. We find that $F_{sp} = k\Delta s$, where k is the spring constant.





Hooke's Law for Springs force increases linearly with the amount the spring is stretched or compressed:

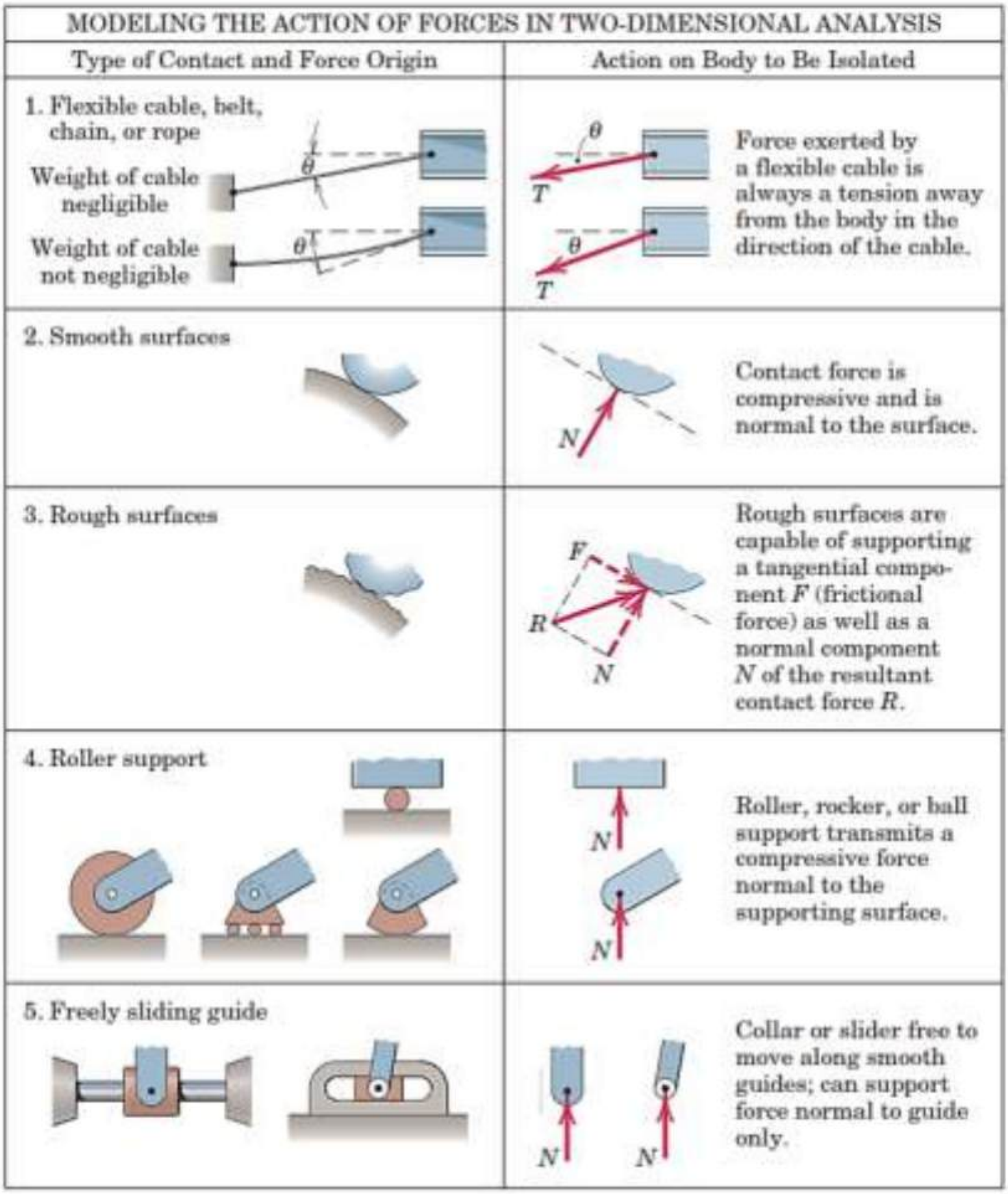
The linear proportionality between force and displacement is found to be valid whether the spring is stretched or compressed, and the force and displacement are always in opposite directions. Therefore, we write the force displacement relation as:



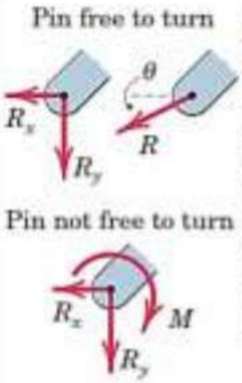
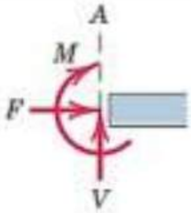
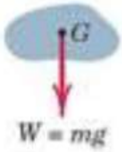
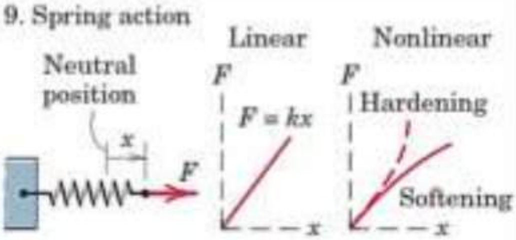
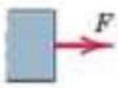
$$(F_{sp})_s = -k\Delta S$$

The force$\left(F_{sp}\right)_s$ always opposes the compression or extension of the spring.

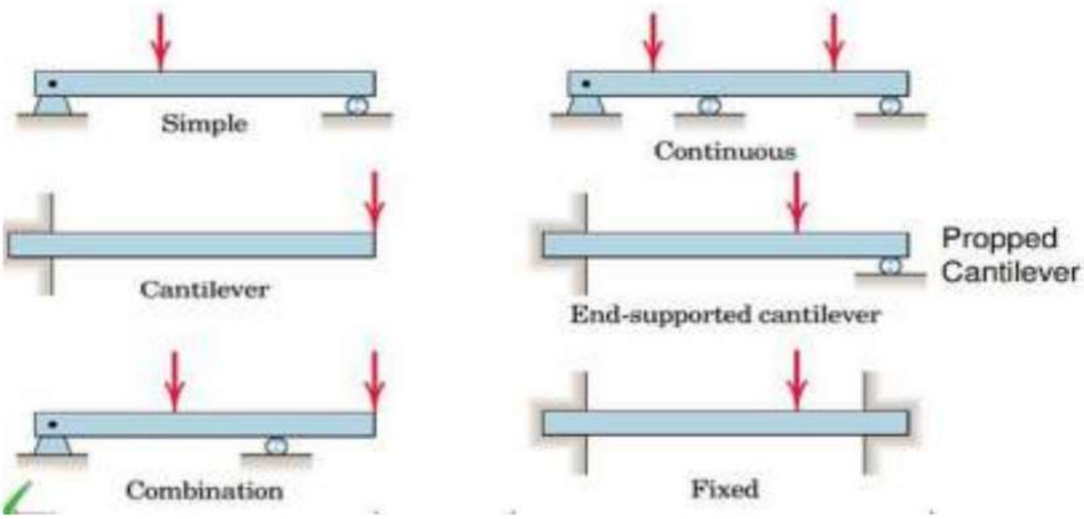
Support reactions



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Types of beams based on support conditions



Free Body Diagram: A free body diagram is a picture of the forces which act on an object and is the first (and perhaps the most important) step in solving force problems. The purpose of the free body diagram (FBD) is to help you identify and analyze the forces that act on a particular object or body.

Drawing a FBD: The steps (discussed in more detail below) for drawing a FBD are:

a. Isolate the body. This step requires that you separate the body (block, wheel, cart, etc.) that you are investigating from everything else and draw it.

b. Draw and label all forces acting on the body. This is the most difficult step. Here is a suggested method to use:

1) List the possible types of forces.

2) Answer the following questions about the types of forces:

a) Does the body have mass? If it does, weight must be included.

b) Is there an applied force described in the problem?

c) Is there a normal force? Does the body touch another surface? If so, there must be a normal force.

d) Is there a rope or string attached to the body? If so, there is a tension force directed in such a way as to pull the body.

e) Is there a friction force? If the surface is frictionless or smooth, then no friction force is present. Otherwise a friction force is present and is directed parallel to the surface and opposing motion or impending motion.

3) Determine where to draw the forces on the FBD.

The positions of the force vectors on the diagram will not affect our analysis. However, if we were to discuss rotational motion, position would become important. Use the following rules:

a) Identify the point of application of the force.

1) For a homogeneous mass, the weight acts at the geometric center of the object.

2) The point of application of applied and tension forces is normally specified in the problem statement.

3) Normal and friction forces involve areas of surfaces in contact. By convention we will choose the center of the surface in contact as the point of application of these two forces.

4) The normal force is drawn perpendicular to the surface of contact through the center of the body.

5) The friction force is drawn parallel to the contact surface.

b) The line of action for a force is defined as a straight line in the direction of the force and passing through the point of application.

c. Choose a coordinate axes (and direction of positive torque). Virtually any coordinate axes can be used in a problem; however, certain selections will make the application of Newton's Second Law much easier mathematically. Some rules to find a good set of axes:

1) Object experiencing acceleration. Choose one axis along the direction of acceleration. Choose the other axis perpendicular to the first.

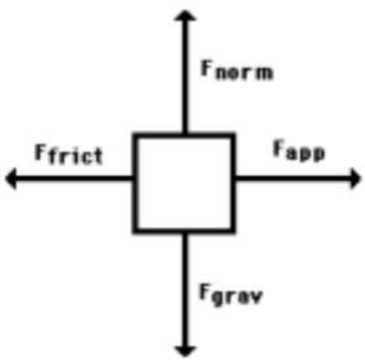
2) Object moving a constant velocity. Choose one axis along the direction of motion. Choose the other axis perpendicular to the first.

3) Object at rest. Choose one axis parallel to the surface upon which the object rests. Choose the other axis perpendicular to the first.

The direction of positive torque will be discussed later in the course when we cover rotational motion. At this stage in the course, we need not concern ourselves with torque.

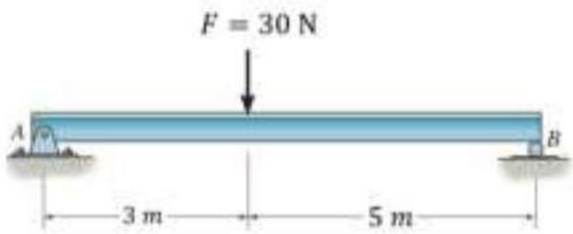
d. Include critical angles and dimensions. In many cases, it will be necessary to break force vectors into their components. Identifying critical angles allows the simple application of trigonometric relationships.

A simple example: A girl is suspended motionless from a bar which hangs from the ceiling by two ropes. The free-body diagram for the girl in this situation



Example:

A simple supported beam of weight 50 N as showing in the figure, draw FBD for the given beam

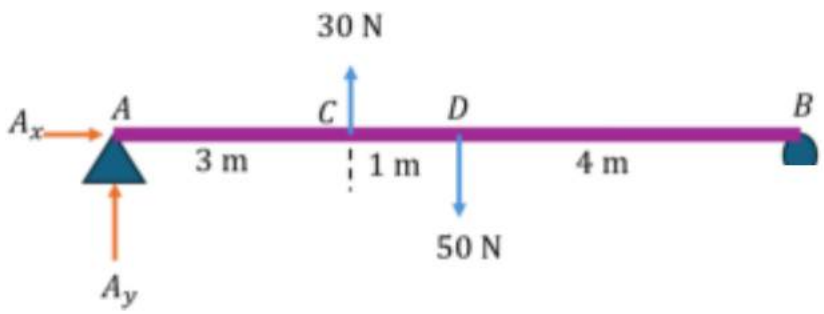


First, we name all the points at which all the forces and moments acting on the beam act.

Second, we name all the forces relative to their point of action and draw them as vectors

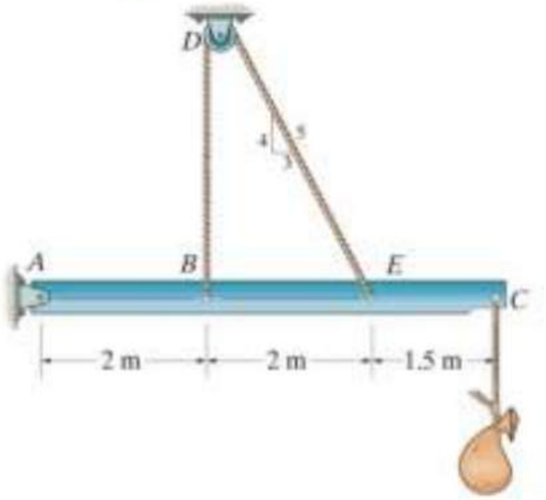
Remember that:

- **A roller support at A resists the vertical forces. Therefore, an upward vertical reaction force is developed at the support**
- **A pinned (or hinged) support at B resists both vertical and horizontal forces but allows rotation. Therefore, an upward vertical and horizontal reaction forces are developed at the support**
- **A weight of a regular beam is at the middle.**

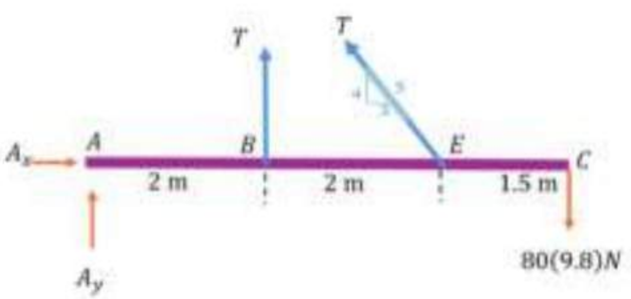


Example

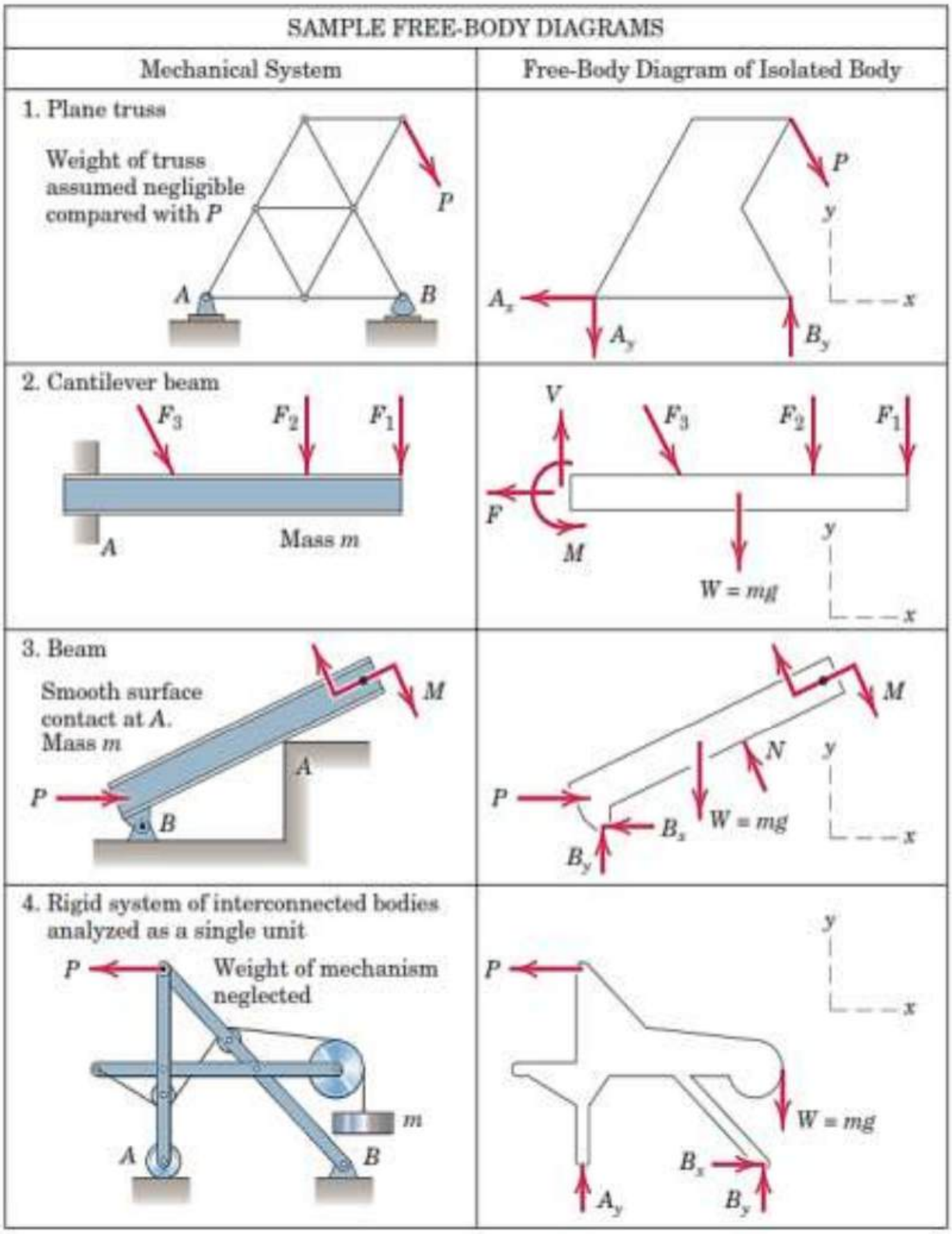
Draw the free-body diagram of the beam which supports the 80 kg load and is supported by the pin at A and a cable which wraps around the pulley at D.



As the same steps in the above example, we can draw FBD as the following



The following table shows FBD for some sample eaxamples



Resultant force

Let we have a forces $\vec{F}_1, \vec{F}_2, \ldots, \vec{F}_n$ acting on an object ,the resultant force $\vec{F}_R$ (also known as net force $\vec{F}_n$) of these forces is

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

$\vec{F}_R$ has the same effect of $\vec{F}_1, \vec{F}_2, \ldots, \vec{F}_n$ on the object.

To obtain the resultant force, we choose the appropriate coordinates that we use in solving the problem such xyz coordinates , then we resolve all the forces that affect the body in the directions of these cordinates to obtain

$R_x = \sum_1^n F_x$, $R_y = \sum_1^n F_y$ and $R_z = \sum_1^n F_z$

There for

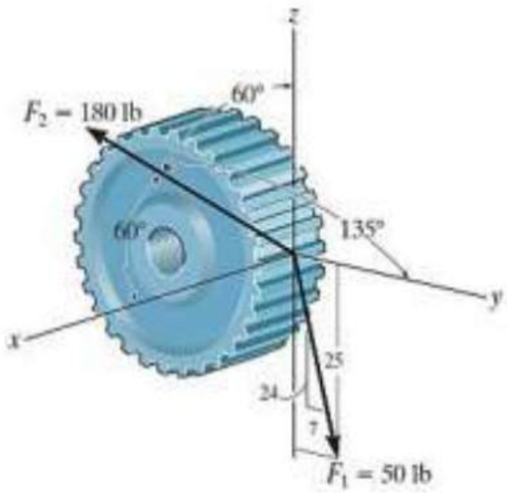
$$R = F_R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$$

The effect point of the resultant force R can be determined by using the fact: The effect of a given forces on an object has the same effect as resultant force R

Resultant of Coplanar Forces: When we are examining a system involving two or more forces, we are usually interested in finding the resultant force in terms of its magnitude as well as direction. The graphical, trigonometric, and vector approaches can be applied to problems involving *coplanar* (two-dimensional) forces.

Example1

The spur gear is subjected to the two forces caused by contact with other gears as shown in the following figure. Express each force as a Cartesian vector



Solution

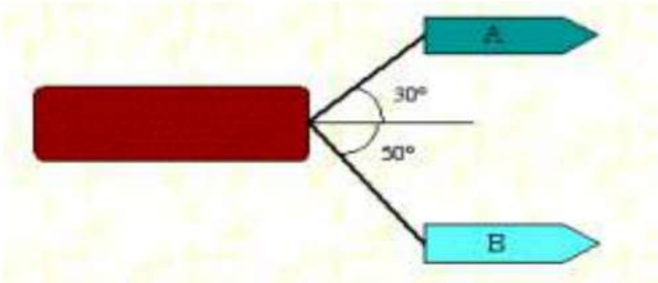
$$\vec{F}_1 = \left(\frac{7}{25} \times 50\hat{\jmath} - \frac{24}{25} \times 50\hat{k}\right) lb$$

$$\vec{F}_1 = (14\hat{\jmath} - 48\hat{k})lb$$

$$\vec{F}_2 = (180\cos 60^o\,\hat{\imath} + 180\cos 135^o\hat{\jmath} + 180\cos 60^o\,\hat{k})lb$$

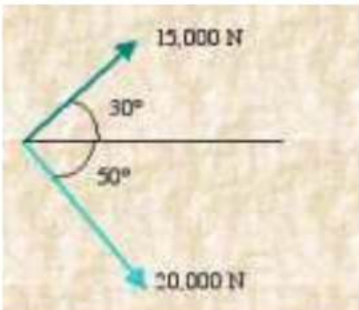
$$\vec{F}_2 = (90\hat{\imath} - 127\,\hat{\jmath} + 90\hat{k})lb$$

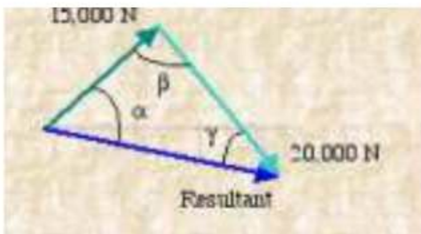
Example2.



Two tugboats are towing a cargo ship as shown below. Tugboat A exerts a force of 15,000 N at a 30° angle while tugboat B exerts a force of 20,000 N at a 50° angle. Determine the magnitude and direction of the resultant force acting on the cargo ship.

Solution: We begin the analysis by drawing the known force vectors. We then construct the force triangle by a head-to-tail connection of the two force component

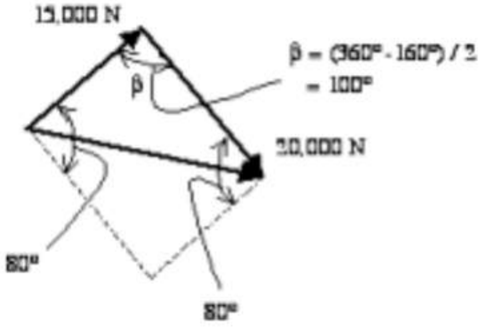




Graphical Approach: If the two known sides of the force triangle are drawn to scale, then we can simply measure the length of the resultant vector and multiply it times the scale factor, used for the other two sides, to find its magnitude. To find its direction, we can use a compass to measure its angle from the same reference line.

The accuracy of graphical approach depends on the accuracy in drawing the force triangle and the accuracy in measuring the length and angle of the resultant. Hence, it could be subject to a considerable error.

Trigonometric Approach: An alternative approach is to use the laws of sines and cosines to solve for the resultant. To do this, we need to first determine the angle β in the force triangle. With the help of the force parallelogram shown below, we determine the value of b knowing that the opposite corners of a parallelogram have equal angles.



With β known, we can use the law of cosines given as

$$\sum \vec{F} = \vec{0}$$

to solve for the magnitude of the resultant force

$$R = \sqrt{F_1{}^2 + F_2{}^2 - 2F_1F_1\cos\varphi}$$

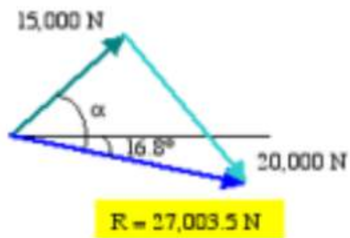
$$R = \sqrt{(15{,}000)^2 + (20{,}000)^2 - 2(15{,}000)(20{,}000)\cos 100^\circ}$$
$$= 27{,}003.5 \text{ N}$$

We then use the law of sines to solve for angle α

$$\frac{R}{\sin\beta} = \frac{b}{\sin\alpha}$$

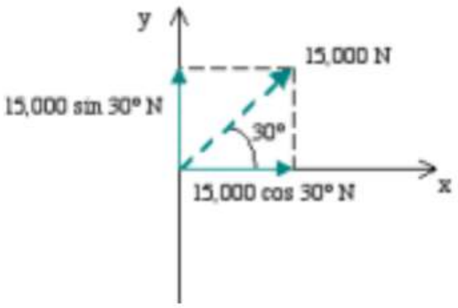
$$\sin\alpha = \frac{20{,}000}{27{,}003.5}\sin 100^\circ$$

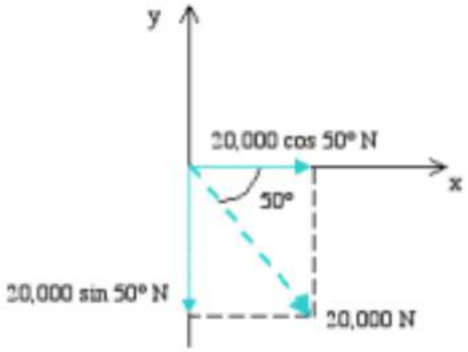
$$\alpha = 46.8^\circ$$



Therefore, the direction of the resultant force is 16.8° below the horizontal reference line.

Scalar Approach: In this approach, we resolve the force exerted by each tugboat into its x and y components as shown below





We then add the force components in the x direction together, and those in the y direction together to obtain the x and y components of the force resultant, respectively. In doing this, we must pay close attention to the sign convention on individual force components.

$$R_x = 15{,}000\cos 30^\circ + 20{,}000\cos 50^\circ$$
$$= 25{,}846.13 \text{ N} \rightarrow$$

$$R_y = 15{,}000\sin 30^\circ - 20{,}000\sin 50^\circ$$
$$= -7{,}820.9 \text{ N} \downarrow$$

With its components known, we can now solve for the magnitude of the force resultant as

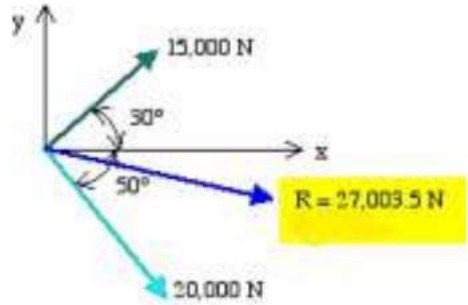
$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$R = 27003.5\ N$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-7820.9}{25846.13}$$

$$\theta = -16.8^o$$

Since θ is measured positive in the counter clockwise direction from x axis, the force resultant is, therefore, directed below the x axis as shown below.



Vector Approach: In this approach, each force is represented by its components in the rectangular Cartesian coordinates as

$$\vec{F}_A = F_{Ax}\hat{i} + F_{Ay}\hat{j}$$
$$= (15{,}000\cos 30°\hat{i} + 15{,}000\sin 30°\hat{j})\ \text{N}$$

$$\vec{F}_B = F_{Bx}\hat{i} + F_{By}\hat{j}$$
$$= (20{,}000\cos 50°\hat{i} - 20{,}000\sin 50°\hat{j})\ \text{N}$$

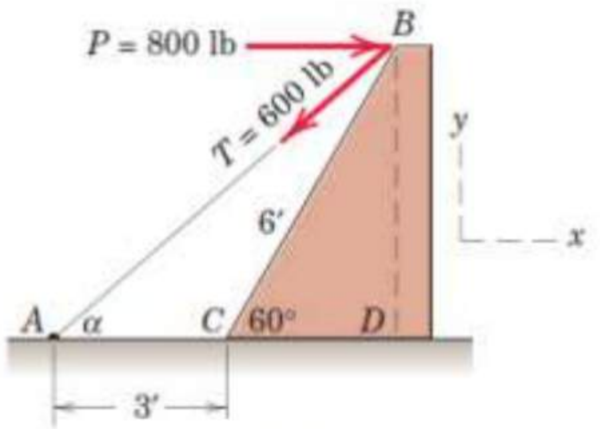
We can then solve for the force resultant by adding the two force vectors together.

$$\vec{R} = \vec{R}_{Ax} + \vec{R}_{Ay}$$
$$= (F_{Ax} + F_{Bx})\hat{i} + (F_{Ay} + F_{By})\hat{j}$$
$$= (15{,}000\cos 30° + 20{,}000\cos 50°)\hat{i} + (15{,}000\sin 30° - 20{,}000\sin 50°)\hat{j}$$

The magnitude and direction of the force resultant are then found in the same manner as that described in the scalar approach.

Having found the resultant force vector on the cargo ship, we know its direction of motion.

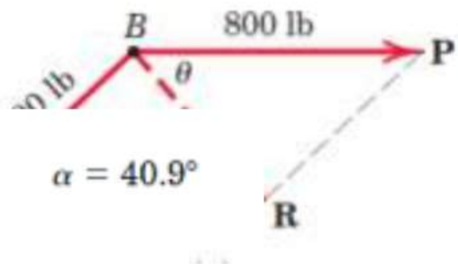
Example3.



Combine the two forces P and T, which act on the fixed structure at B, into a single equivalent force R.

Solution

Graphical solution: The parallelogram for the vector addition of forces T and P is constructed as shown in Fig.

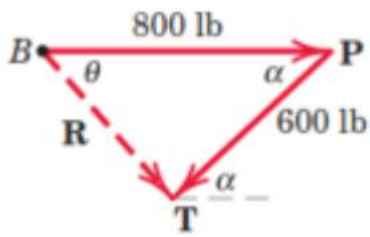


$$\tan \alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6 \sin 60^\circ}{3 + 6 \cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$

The scale used here is 1 in. = 800 lb; a scale of 1 in. = 200 lb would be more suitable for regular-size paper and would give greater accuracy. Note that the angle a must be determined prior to construction of the parallelogram. From the given figure Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525 \text{ lb} \qquad \theta = 49^\circ$$

Geometric solution: The triangle for the vector addition of **T** and **P** is shown in Fig. The angle α is calculated as above. The law of cosines gives



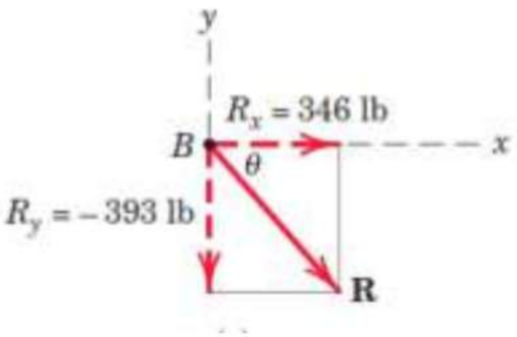
$$R^2 = (600)^2 + (800)^2 - 2(600)(800) \cos 40.9^\circ = 274{,}300$$

$$R = 524 \text{ lb}$$

From the law of sines, we may determine the angle θ which orients R. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9°} \qquad \sin\theta = 0.750 \qquad \theta = 48.6°$$

Algebraic solution : By using the *x*-*y* coordinate system on the given figure, we may write



$$R_x = \Sigma F_x = 800 - 600\cos 40.9° = 346 \text{ lb}$$

$$R_y = \Sigma F_y = -600\sin 40.9° = -393 \text{ lb}$$

The magnitude and direction of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ lb}$$

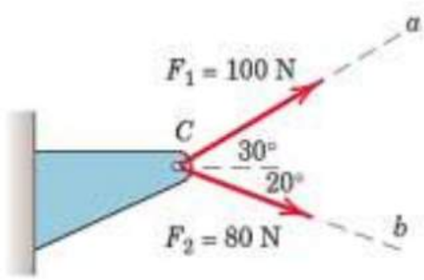
$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6°$$

The resultant **R** is written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ lb}$$

Example4

Forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ act on the bracket as shown. Determine the projection *F* of their resultant **R** onto the *b*-axis.



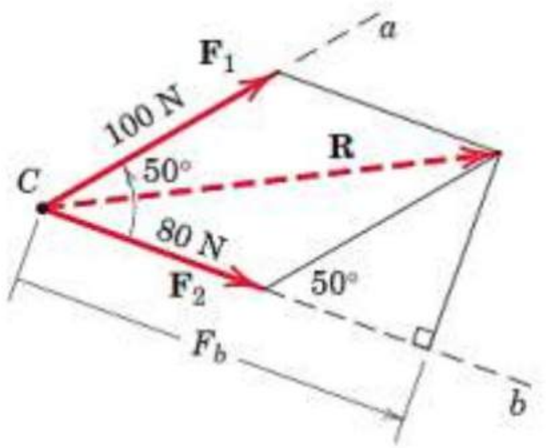
Solution

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130°$$

$R = 163.4$ N

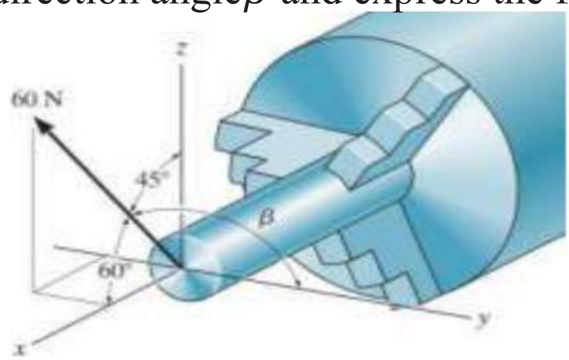
The figure also shows the orthogonal projection $\mathbf{F}_b$ of $\mathbf{R}$ onto the b-axis. Its length is

$F_b = 80 + 100 \cos 50° = 144.3$ N



Example 4

The stock mounted on the lathe is subjected to a force of 60N. Determine the coordinate direction angleβ and express the force as a Cartesian vector.



Solution

We have the relation

$$cos^2\,\alpha + cos^2\,\beta + cos^2\,\gamma = 1$$

$$cos^2\,60^o + cos^2\,\beta + cos^2\,45^o = 1$$

Solve for β, we get

$$\beta = 60^o\,,120^o$$

From the given figure

$$\beta = 120^o$$

The force as Cartesian vector

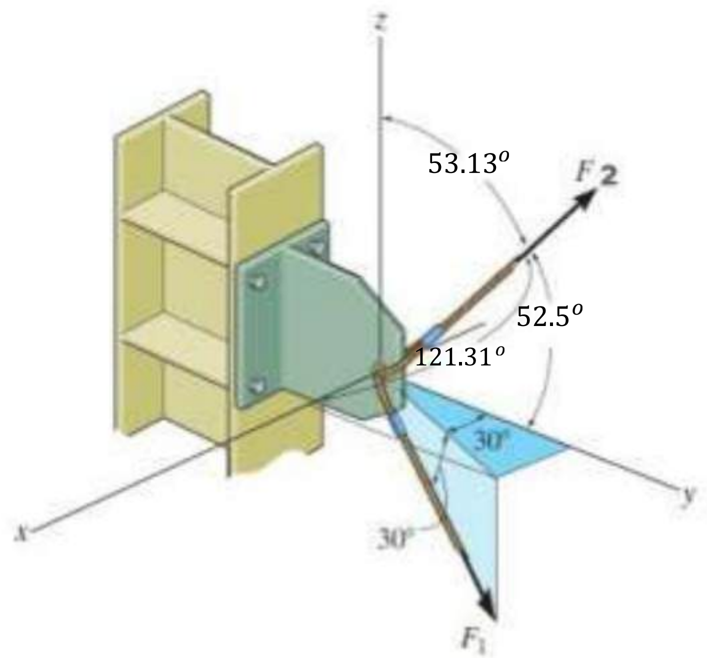
$$\vec{F} = F(\cos\alpha\,\hat{\imath} + \cos\beta\,\hat{\jmath} + \cos\gamma\,\hat{k})$$

$$\vec{F} = 60(\cos\ 60^o\,\hat{\imath} + \cos\ 120^o\,\hat{\jmath} + \cos\ 45^o\,\hat{k})$$

$$\vec{F} = (30\hat{\imath} - 30\hat{\jmath} + 42.4\hat{k})N$$

Example 5.

Two forces F_1 and F_2 are shown in the following figure. If $F_1 = F_2 = 300N$, determine the magnitude of the resultant force F_R and the coordinate direction angles of F_R.



Solution

To get on resultant force F_R, we have

$$\vec{F}_1 = 300(\cos 30^o \, sin30^o\hat{\imath} + \cos 30^o \, cos30^o\hat{\jmath} - sin30^o\hat{k})$$
$$\vec{F}_1 = (129.904\hat{\imath} + 225\hat{\jmath} - 150\hat{k})N$$
$$\vec{F}_2 = 300(\cos 121.31^o \, \hat{\imath} + \cos 52.5^o \, \hat{\jmath} + cos53.13^o \, \hat{k})$$
$$\vec{F}_2 = (-155.9\hat{\imath} + 182.63\hat{\jmath} + 180\hat{k})N$$

Therefore

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_R = -25.996\hat{\imath} + 407.63\hat{\jmath} + 30\hat{k}$$

The magnitude of $\vec{F}_R$ is

$$F_R = \sqrt{25.996^2 + 407.63^2 + 30^2}$$
$$F_R = 409.558N$$

The coordinate direction angles of F_R

$$\alpha = cos^{-1}\left(\frac{-25.996}{409.558}\right)$$
$$\alpha = cos^{-1}(-86.361) = 93.64^o$$
$$\beta = cos^{-1}\left(\frac{407.63}{409.558}\right) = 5.562^o$$
$$\gamma = cos^{-1}\left(\frac{30}{409.558}\right) = 85.8^o$$

Newton’s Laws

Sir Isaac Newton’s laws of motion explain the relationship between a physical object and the forces acting upon it. Understanding this information provides us with the basis of modern physics.

Newton’s Three Laws of Motion. Formulated by Newton in the late seventeenth century.

FIRST LAW. If the resultant force acting on a particle is zero, the particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion)

$$\sum \vec{\boldsymbol{F}} = \mathbf{0} \ , \ \vec{\boldsymbol{a}} = \mathbf{0}$$

SECOND LAW. If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

$$\sum \vec{F} \propto m\vec{a}$$

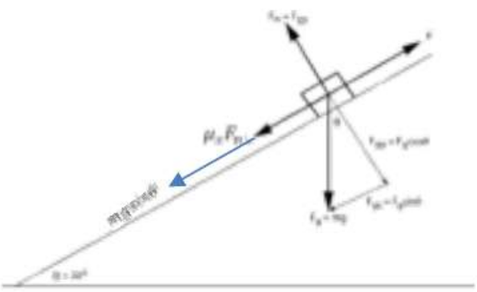
THIRD LAW. The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite direction

If a body is placed on a rough inclined plane inclined to the horizontal at an angle θ, and the resultant force is equal to zero in the direction of the plane and perpendicular to it, as shown in the diagram, then:

$$F_n - mgcos\theta = 0$$

$$or$$

$$F_n - F_{gy} = 0$$



Where:

$F_{gy} = mgcos\theta$ is the component of wight in the direction of normal to the inclined plane

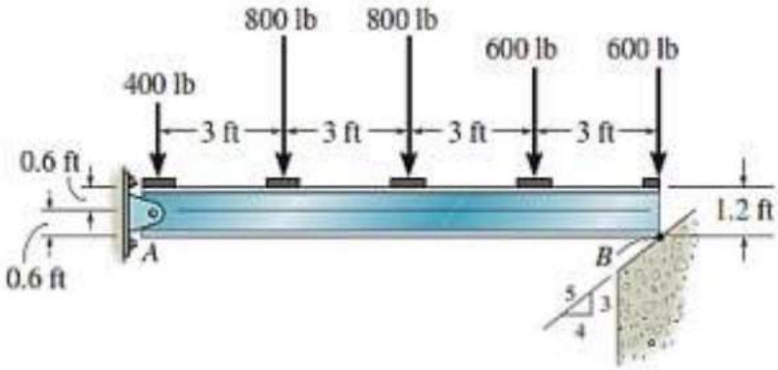
$$F - \mu_s F_n - mg sin\theta = o$$

Where

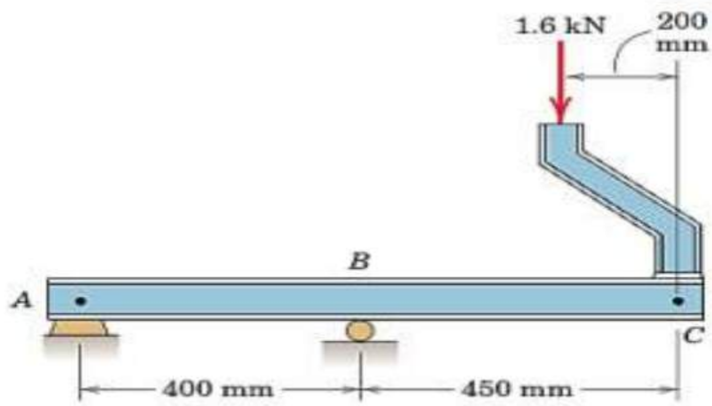
$mgsin\,\theta$ is the component of wight in the direction of the inclined plane, and μ_s is coefficient of friction between the body and the plane.

Problems

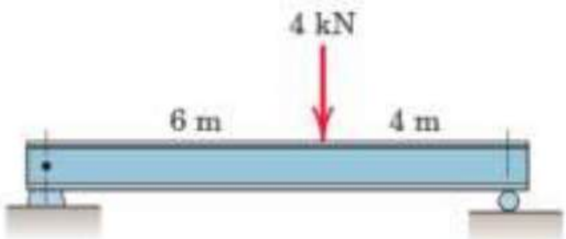
1- Draw the free-body diagram of the beam, which is pin supported at A and rests on the smooth incline at B.



2- Draw the free-body diagram of the following figure

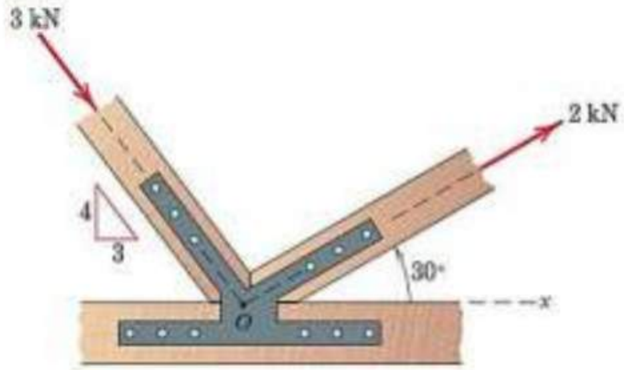


3- Draw the free-body diagram of the entire beam

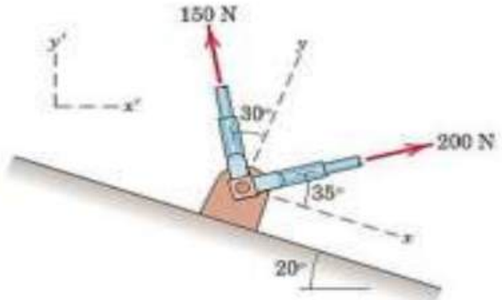


4- The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint *O*.

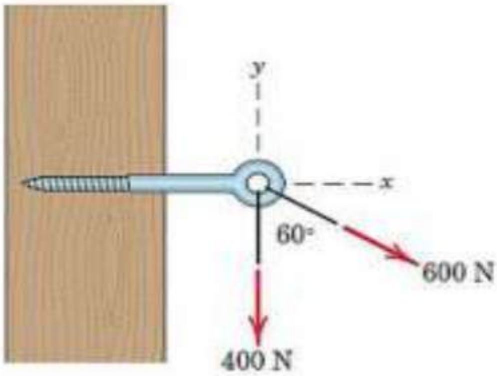
Determine the magnitude of the resultant **R** of the two forces and the angle θ which **R** makes with the positive x-axis.



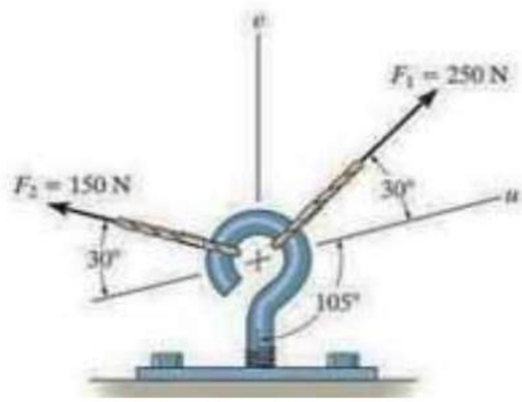
5- Determine the resultant **R** of the two forces applied to the bracket. Write **R** in terms of unit vectors along the x- and y-axes shown.



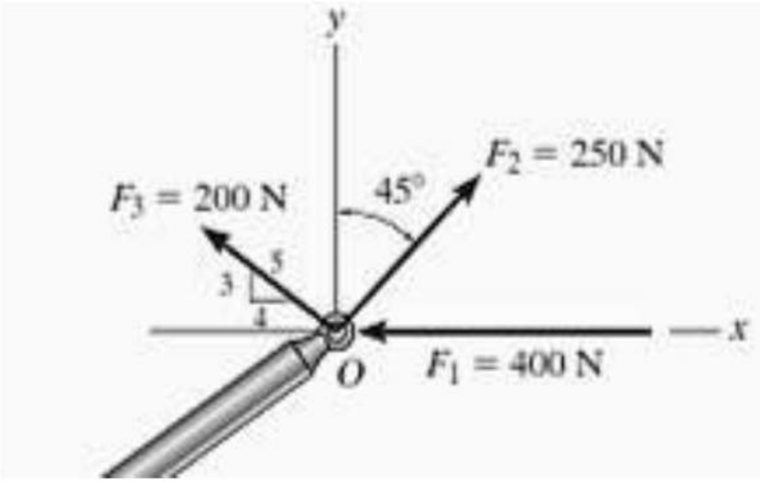
6-Determine the resultant **R** of the two forces shown by (a) applying the parallelogram rule for vector addition and (b) summing scalar components.



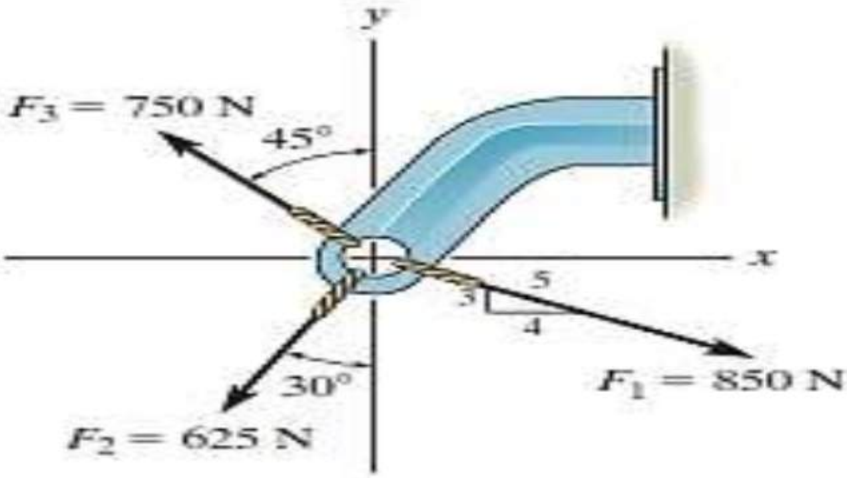
7- Resolve F_2 into components along the u and v axes and determine the magnitudes of these components.



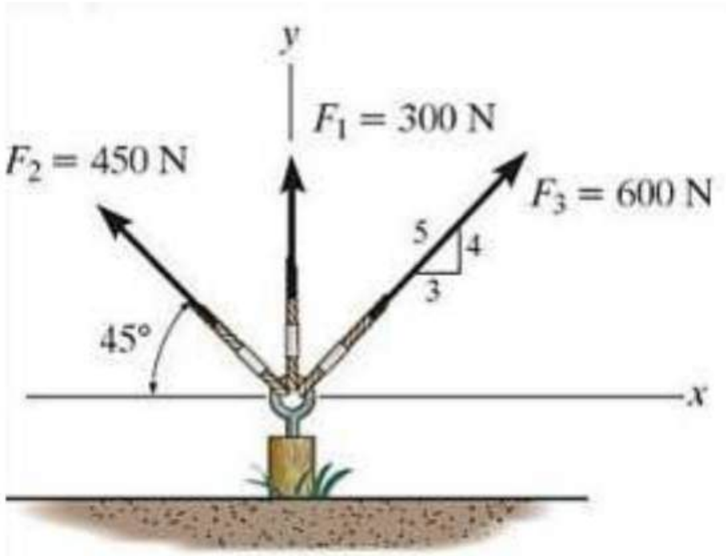
8- The end of the boom O is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



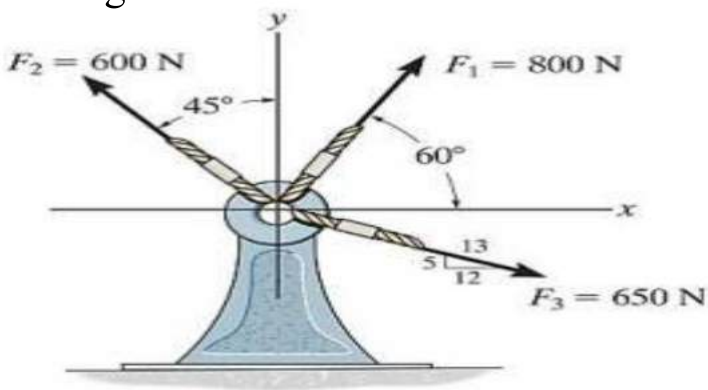
9- Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



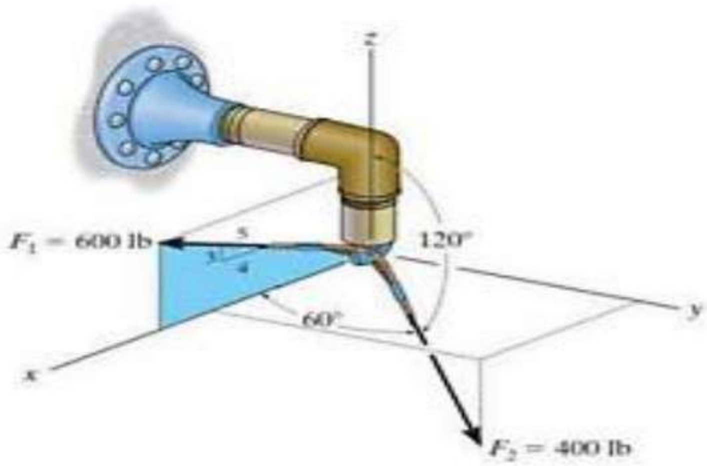
10- Three concurrent forces acting on a tent post. Find the magnitude and angle of the resultant force.



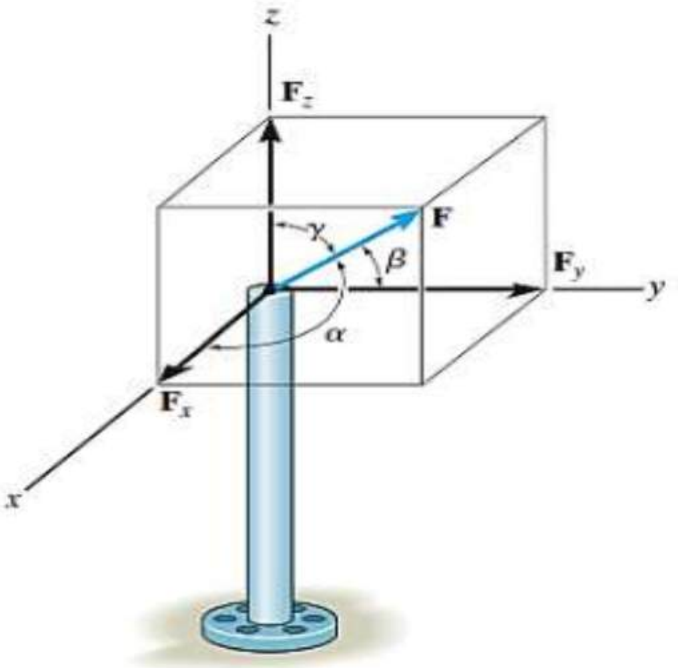
11- Three concurrent forces acting on a bracket, Find the magnitude and angle of the resultant force



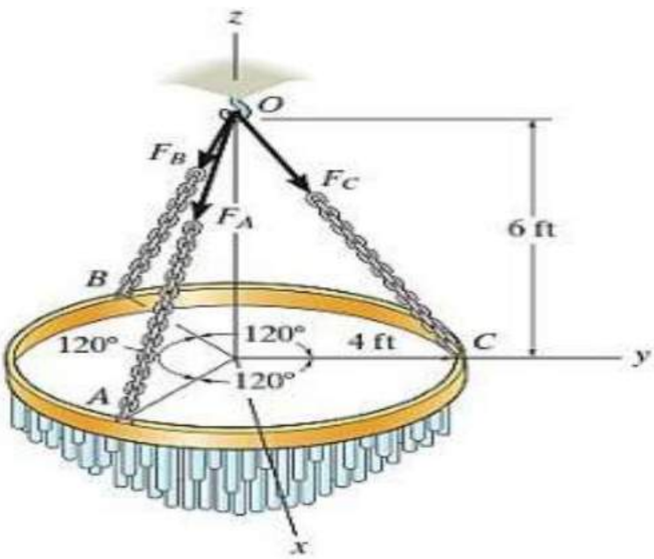
12- Determine the magnitude and direction of the resultant force acting on the pipe assembly



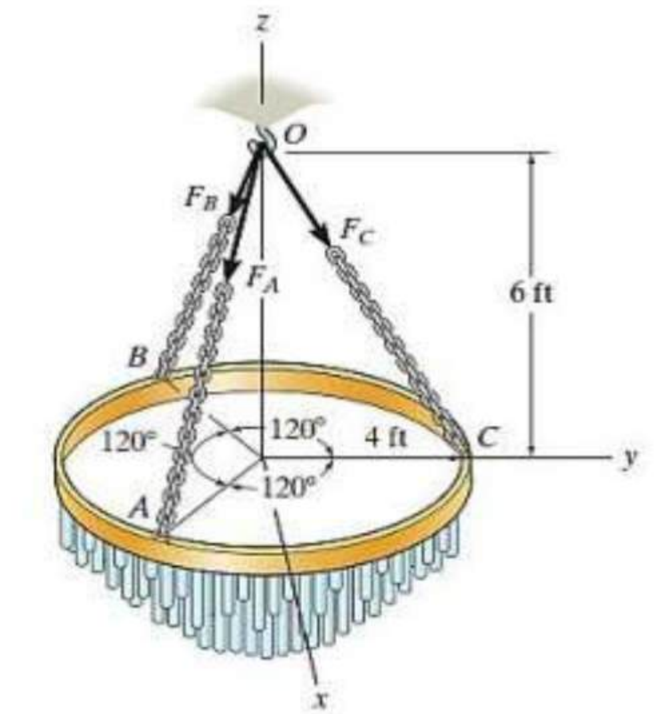
13- The pole is subjected to the force **F** which has components $F_x = 1.5\ kN$ and $F_z = 1.25\ kN$. If $\beta = 75^o$, determine the magnitudes of **F** and F_y



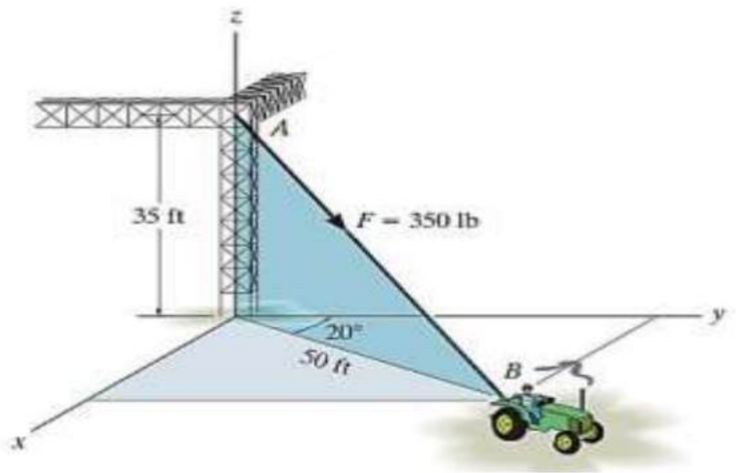
14- The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



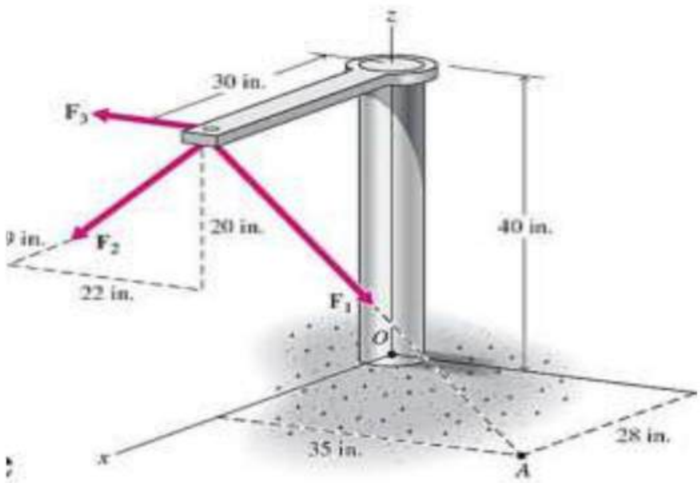
15- The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



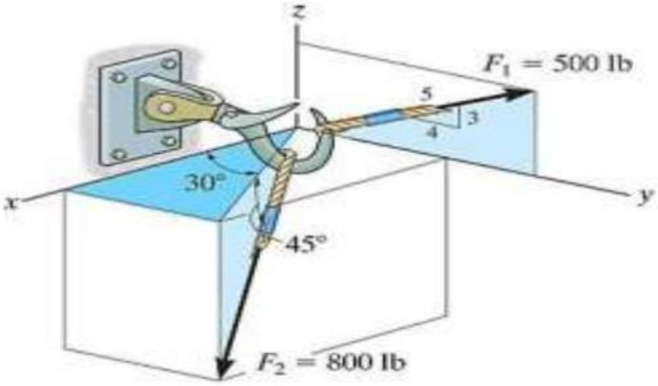
16- The cable attached to the tractor at *B* exerts a force of 350 lb on the framework. Express this force as a Cartesian vector.



17- Given: F_1 = 500 lb, F_2 = 300 lb, F_3 = 200 lb. Determine the resultant , express the resultant in the Cartesian format , find the angles formed by the resultant and the coordinate axes



18- Two forces F_1 and F_2 are applied to a hook. Find the resultant force in Cartesian vector form



19. **MCQs questions**

1. A book rests on a level table. According to Newton's First Law, which of the following statements is correct?

A) There are no forces acting on the book.
B) There is a net force on the book causing it to stay at rest.
C) The net force acting on the book is zero.
D) The force of gravity is the only force acting on the book.

2. A heavy chandelier is hung from the ceiling by a single chain. What is the reaction force to the tension in the chain pulling upward on the chandelier?

A) The weight of the chandelier pulling down.
B) The chandelier pulling down on the chain.
C) The ceiling pulling up on the chain.
D) The gravitational force of the chandelier on the Earth.

3. A block is at rest on an inclined ramp. How many forces are acting on the block? (Neglect air resistance)
A) 1
B) 2
C) 3
D) 4

4. Two people are playing a gentle game of tug-of-war with a rope. Both are stationary. If person A pulls with a force of 300 N to the left, and person B pulls with a force of 300 N to the right, what is the tension in the rope?
A) 0 N
B) 300 N
C) 600 N
D) It cannot be determined.

5. A computer monitor sits on a desk. The monitor exerts a downward force on the desk. The reaction force to this is:
A) The downward force of gravity on the monitor.
B) The upward force of the desk on the monitor.
C) The downward force of the monitor on the Earth.
D) The upward force of the desk on the Earth.

6. A sign is held in equilibrium by two cables, as shown in a diagram (imagine one cable to the left, one to the right, both at angles). The condition for equilibrium in the vertical direction is best described by:
A) The horizontal components of the tensions must be equal and opposite.
B) The sum of the vertical components of the tensions must equal the weight of the sign.
C) The tension in the left cable must equal the tension in the right cable.
D) The net force in all directions must be greater than zero.

7. According to Newton's Third Law, the forces of action and reaction:
A) Always act on the same object.
B) Are equal in magnitude but only if the object is not

accelerating.
C) Act on different objects and are equal in magnitude and opposite in direction.
D) Cancel each other out, resulting in a net force of zero on a system.

8. A person pushes against a solid, immovable wall with a force of 100 N. According to Newton's laws, what is the net force on the person?
A) 100 N away from the wall
B) 100 N towards the wall
C) 0 N
D) 50 N away from the wall

Answer Key & Explanations

1. **C) The net force acting on the book is zero.**

   Explanation: Newton's First Law states that an object at rest remains at rest if the net force on it is zero. The book has weight (gravity) pulling down and a normal force from the table pushing up. These two forces are balanced, resulting in a net force of zero.

2. **B) The chandelier pulling down on the chain.**

   Explanation: Newton's Third Law states that for every action, there is an equal and opposite reaction. The "action" is the chain pulling *up* on the chandelier. The "reaction" is the chandelier pulling *down* on the chain with an equal force.

3. **C) 3**

   ***Explanation**:* The three forces are: 1. Weight (gravity) acting straight down. 2. The Normal Force from the ramp, acting perpendicular to the surface. 3. The Static Friction force, acting parallel to the surface up the ramp (preventing the block from sliding down).

4. **B) 300 N**

Explanation: Since both are stationary, the net force on any segment of the rope must be zero. The tension throughout the rope is the same. If one end is pulled with 300 N, the other must also be pulled with 300 N to achieve equilibrium. The tension is 300 N.

5. **B) The upward force of the desk on the monitor.**

   Explanation: This is a classic Newton's Third Law pair. The action-reaction pair is: Force of Monitor on Desk (down) and Force of Desk on Monitor (up). They are equal and opposite and act on *different* objects.

6. **B) The sum of the vertical components of the tensions must equal the weight of the sign.**

   Explanation: For the sign to be in static equilibrium, the forces in the vertical (-y) direction must balance. The weight acts downward, so the sum of the upward vertical components from both cables must be equal to the weight.

7. **C) Act on different objects and are equal in magnitude and opposite in direction.**

   Explanation: This is the precise definition of Newton's Third Law. A common mistake is to think these forces cancel; they don't cancel for the purpose of calculating the motion of a single object because they act on *different* objects.

8. **C) 0 N**

   Explanation: The person is stationary (pushing but not moving). Since they are at rest, by Newton's First Law, the net force on them must be zero. The wall pushes back on the person with a force of 100 N (Newton's Third Law), which balances the force they are applying.

Chapter 3

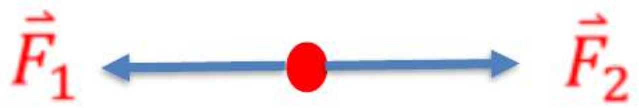
Equilibrium

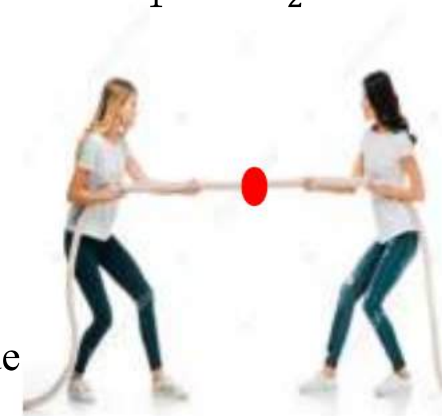
In engineering, the concept of Equilibrium is not merely a theoretical physics topic; it is the fundamental bedrock upon which all structural and mechanical design is built. It is the state that ensures bridges don't collapse, machines operate smoothly, and buildings stand firm.

At its core, equilibrium in engineering is the condition where a body or structure experiences no net force and no net moment, resulting in zero acceleration. An object in equilibrium is either at rest (static) or moving with constant velocity (dynamic). For most civil and mechanical engineering design purposes, Static Equilibrium—the state of being at rest—is the primary concern.

Equilibrium of a particle

- Equilibrium of a particle under two forces $\vec{F}_1$ *and* $\vec{F}_2$





$\vec{F}_1 + \vec{F}_2 = \vec{0}$ or $\vec{F}_1 = -\vec{F}_2$ and $F_1 = F_2$

- The two forces are equal in magnitude
- Opposite in direction
- Act along the same line of action

True or False :

1. When a particle is acted upon by only two forces and remains in equilibrium, the two forces must be exactly opposite and equal (True)

2. If two forces $F_1 = 30$ Nand $F_2 = 30$ N act in opposite directions on a particle, the resultant force is 60 N (False)

Complete the following :

- If two forces act on a particle in opposite directions. If one force is 25 N and the particle remains in equilibrium, the other force is …25 N
- If a particle is in equilibrium under two forces $F_1$and F_2. If $F_1 = 50$ N acts at an angle of 30°above the horizontal, the components of $F_2$are $F_{2x} = \cdots 25\sqrt{3}$ N and $F_{2y} = \cdots -25$ N

Equilibrium of a particle under Forces System

A particle under the effect of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ is in equilibrium if

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 \dots + \vec{F}_n = \vec{0}$$

$$\vec{R} = \sum_{i=1}^{n} \vec{F}_i = \vec{0}$$

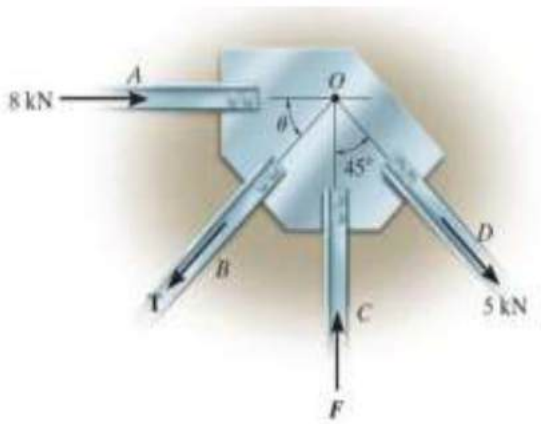
$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

$$R_z = \sum F_z = 0$$

Examples:

1-The gusset plate is subjected to the **forces of** four members. Determine the force in member B and its proper orientation for equilibrium. The forces are concurrent at point O. Take F = 12 kN.



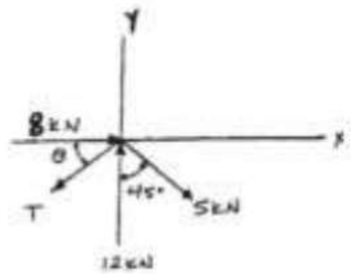
$\xrightarrow{+} \Sigma F_x = 0;\quad 8 - T\cos\theta + 5\sin 45^\circ = 0$

$+\uparrow \Sigma F_y = 0;\quad 12 - T\sin\theta - 5\cos 45^\circ = 0$

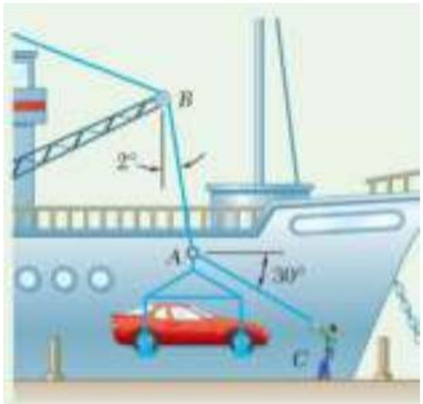
Solving,

$T = 14.3\text{ kN}$

$\theta = 36.3^\circ$



2.In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A worker ties a rope to the cable at A and pulls on it in order to center the automobile over its intended position on the dock. At the moment illustrated, the automobile is stationary, the angle between the cable and the vertical is 2°, and the angle between the rope and the hori- zontal is 30°. What are the tensions in the rope and cable?



$\sum F_x = 0$

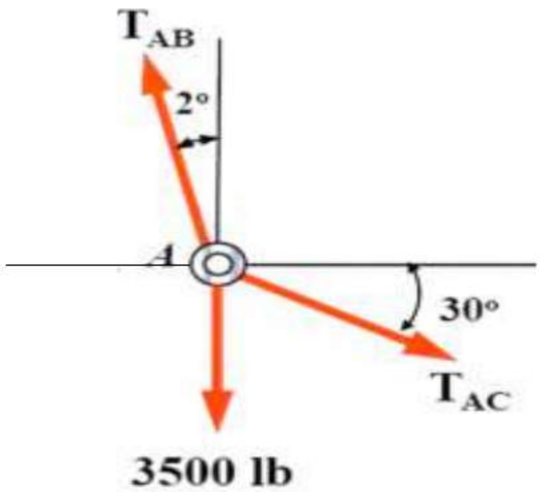
$\therefore T_{AC}\cos 30 - T_{AB}sin2 = 0$

$\sum F_y = 0$

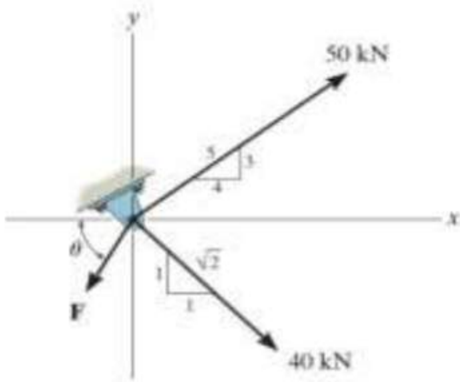
$\therefore\ T_{AB}cos2 - T_{AC}\sin 30 - 3500 = 0$

$$\frac{T_{AC}}{\sin\ 178} = \frac{T_{AB}}{\sin\ 60} = \frac{3500}{\sin 122}$$

$T_{AC} = 114\ Ib$ and $T_{AB} = 3574\ Ib$



3.If $\theta = 60^o$ and $F = 20$ kN determine the magnitude of the resultant force and its direction measured clockwise from the positive x axis.



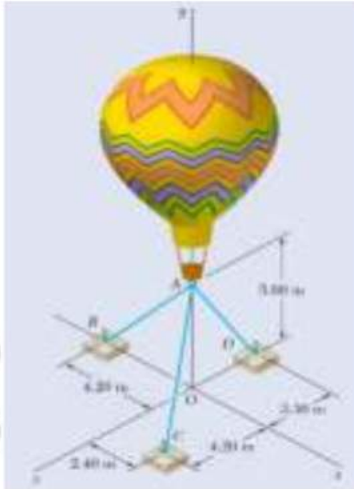
$\xrightarrow{+} F_{R_x} = \Sigma F_x;\quad F_{R_x} = 50\left(\frac{4}{5}\right) + \frac{1}{\sqrt{2}}(40) - 20\cos 60^\circ = 58.28$

$+\uparrow F_{R_y} = \Sigma F_y;\quad F_{R_y} = 50\left(\frac{3}{5}\right) - \frac{1}{\sqrt{2}}(40) - 20\sin 60^\circ = -15.60$

$F_R = \sqrt{(58.28)^2 + (-15.60)^2} = 60.3\text{ kN}$

$\phi = \tan^{-1}\left[\frac{15.60}{58.28}\right] = 15.0^\circ$

4. Three cables are used to tether a balloon as shown.
Determine the vertical force P exerted by the balloon
at A knowing that the tension in cable AC is 444 N



$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Substituting $T_{AC} = 444$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$
$$T_{AD} = 496.36 \text{ N}$$

$P = 956 \text{ N} \uparrow$ ◀

Equilibrium of rigid bodies

A rigid body is an extended area of material that includes all the points inside it, and which moves so that the distances and angles between all its points remain constant. Also, a rigid body is defined as a body on which the distance between two points never changes whatever be the force applied on it.

Consider we have a particle that is affected by a system of forces, then this particle is in equilibrium if

$$\sum \vec{F} = \vec{0}$$

This condition is not enough for equilibrium of rigid body.

Equilibrium of rigid bodies, the body is in static equilibrium, if both the net force and the net torque on it must be zero. Therefore the conditions of equilibrium are:

- **In two dimensions:**

$$\vec{R} = \sum \vec{F} = R_x\vec{\imath} + R_y\vec{\jmath} = \vec{0} \; and \; \sum \vec{M} = \sum M_x \, \vec{\imath} + \sum M_y \, \vec{\jmath} = \vec{0}$$

- **In three dimensions :**

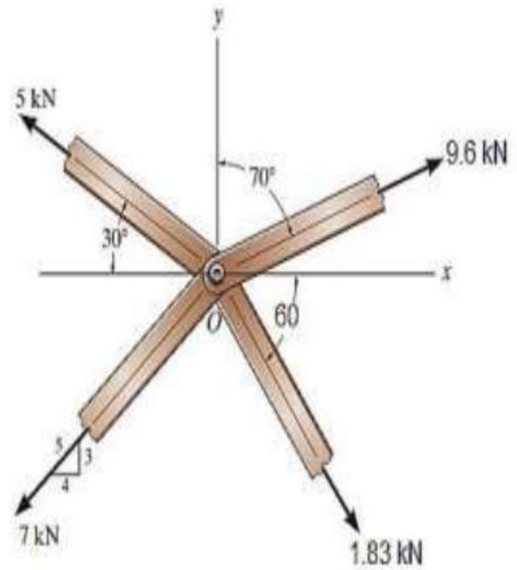
$$\vec{R} = \sum \vec{F} = R_x\vec{\imath} + R_y\vec{\jmath} + R_z\vec{k} = \vec{0}$$

$$\sum \vec{M} = \sum M_x \, \vec{\imath} + \sum M_y \, \vec{\jmath} + \sum M_z \, \vec{k} = \vec{0}$$

Solved Examples

Example1.

Consider the members of a truss are pin connected at joint O as shown in the figure. Show that this truss is in equilibrium.



Solution:

Resolve all forces in the direction of x-axis and y-axis, then calculate the summation of forces in theses direction;

We have all forces are Concurrent at a point O,

$$\sum F_x = 9.6 \sin 70 + 1.83 \cos 60 - 5\cos 30 = 0$$

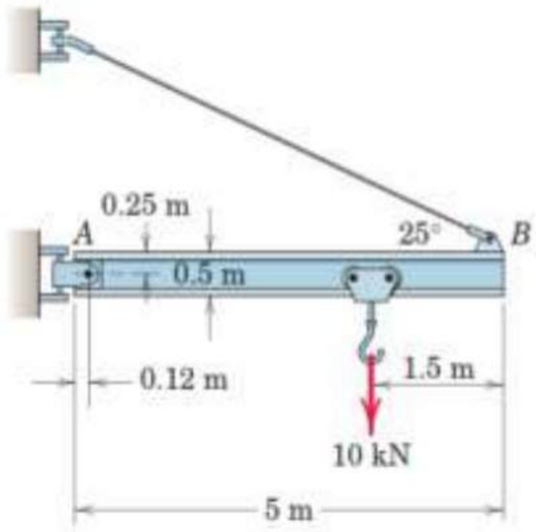
$$\sum F_y = 9.6 \cos 70 - 1.83 \sin 60 + 5\sin 30 = 0$$

Therefore

$$\sum \vec{F} = \vec{0}$$

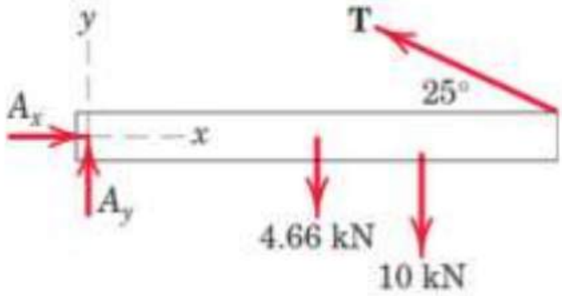
So, the truss is in equilibrium.

Example2.



Determine the magnitude T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard $0.5-m$ I-beam with a mass of $95\ kg$ per meter of length.

Solution



Let the reactions at A is A_x and A_y, the tension in the supporting cable is T .

Draw the free-body diagram as shown, therefor

$$\sum M_A = 0$$

$$(T\cos 25°)0.25 + (T\sin 25°)(5-0.12) - 10(5-1.5-0.12) - 4.66(2.5-0.12) = 0$$

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

Equating the sums of forces in the *x*- and *y*-directions to zero gives

$[\Sigma F_x = 0]$ $\quad A_x - 19.61\cos 25° = 0 \quad A_x = 17.77 \text{ kN}$

$[\Sigma F_y = 0]$ $\quad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37 \text{ kN}$

The magnitude of the force on the pin at A is

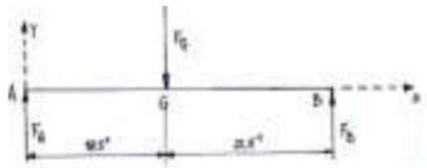
$[A = \sqrt{A_x^2 + A_y^2}]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$

Example 3.

The weight of the bicycle is 29 lb with center of gravity at *G*. Determine the normal forces at *A* and *B* when the bicycle is in equilibrium

Solution:

Draw the free-body diagram of the body as shown



We know that the weight of the bicycle is $F_G = 29\,\text{lb}$.

We will denote normal force at A as F_A, normal force at B as F_B and the moment around A as M_A.

The forces facing the positive direction of the axes and the clockwise moments will be taken as positive (+).

This kind of force system can be categorized as Category 3 which means we need one force equations (y axis) and one moments equation (z axis) to solve the problem.

$$\sum M_A = 0$$

$$F_G \cdot 18.5\,\text{in} - F_B \cdot (18.5\text{in} + 22.5\,\text{in}) = 0$$

$$F_G \cdot 18.5\text{in} - F_B \cdot 41\,\text{in} = 0$$

$$29\,\text{lb} \cdot 18.5\text{in} - F_B \cdot 41\,\text{in} = 0$$

$$F_B \cdot 41\,\text{in} = 29\,\text{lb} \cdot 18.5\text{in}$$

$$F_B = \frac{29 \cdot 18.5}{41}\,\text{lb}$$

$$F_B = \boxed{13.09\,\text{lb}}$$

Now we can use the second equation:

$$\sum F_y = 0$$

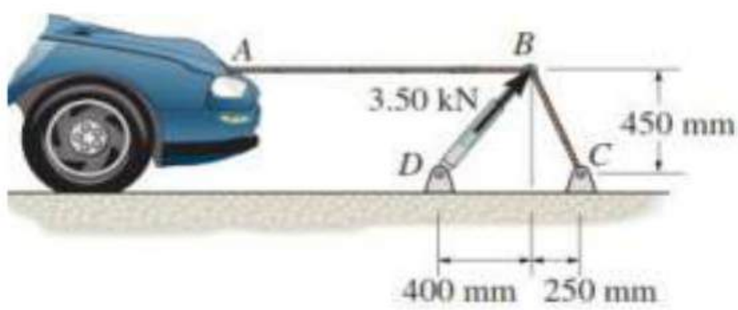
$$F_A - F_G + F_B = 0$$

$$F_A - 29\,\text{lb} + 13.09\,\text{lb} = 0$$

$$F_A - 15.91\,\text{lb} = 0$$

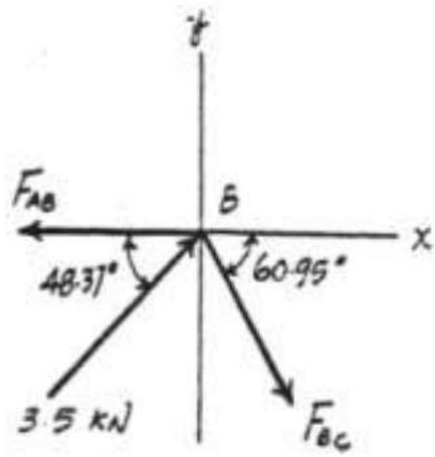
$$F_A = \boxed{15.91\,\text{lb}}$$

Example4.



The device shown is used to straighten the frames of wrecked autos. Determine the tension of each segment of the chain, i.e., AB and BC, if the force which the hydraulic cylinder DB exerts on point B is 3.50 kN, as shown.

Solution:



Draw the free-body diagram as shown.

From equations of Equilibrium, A direct solution for F_{BC} can be obtained by summing forces along the y axis.

$$+\uparrow \Sigma F_y = 0; \qquad 3.5 \sin 48.37° - F_{BC} \sin 60.95° = 0$$

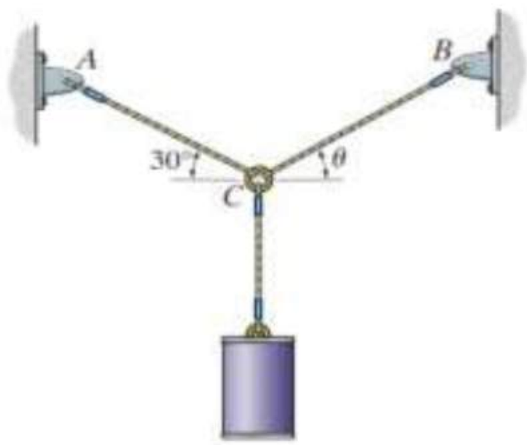
$$F_{BC} = 2.993 \text{ kN} = 2.99 \text{ kN}$$

Along x axis, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3.5 \cos 48.37° + 2.993 \cos 60.95° - F_{AB} = 0$$

$$F_{AB} = 3.78 \text{ kN}$$

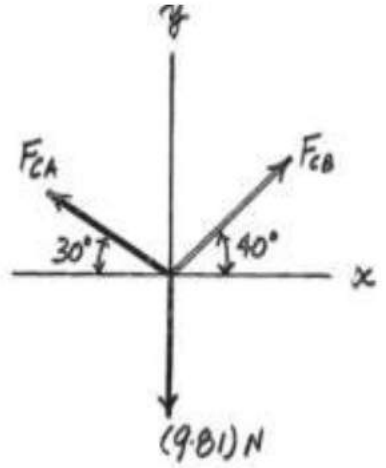
Example5.



Determine the tension developed in wires CB and CA required for equilibrium of the 10-kg cylinder. Take .$\theta = 40°$

Solution:

Applying the equations of equilibrium along the *x* and *y* axes to the free-body diagram shown



$\xrightarrow{+} \Sigma F_x = 0;$

$+\uparrow \Sigma F_y = 0;$

$F_{CB} \cos 40° - F_{CA} \cos 30° = 0$

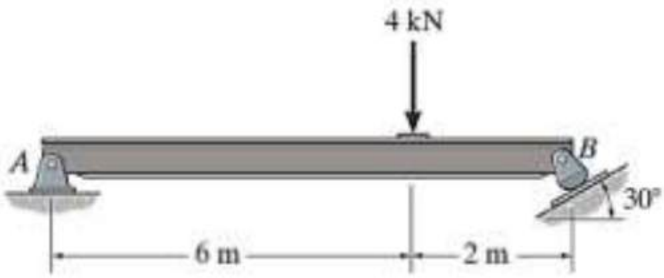
$F_{CB} \sin 40° + F_{CA} \sin 30° - 10(9.81) = 0$

Solving these equations for F_{CB} and F_{CA} , we get

$F_{CA} = 80\ N \qquad F_{CB} = 90.4\ N$

Example 6.

Determine the horizontal and vertical components of reaction at the pin *A* and the reaction of the rocker *B* on the beam.



Solution:

Draw the free-body diagram of the beam, to calculate the reaction at point B; N_B; we can write the moment equation of equilibrium about point A ,where $\sum M_A = 0$

$$N_B cos30 \times 8 - 4 \times 6 = 0 \Longrightarrow N_A = 3.464kN$$

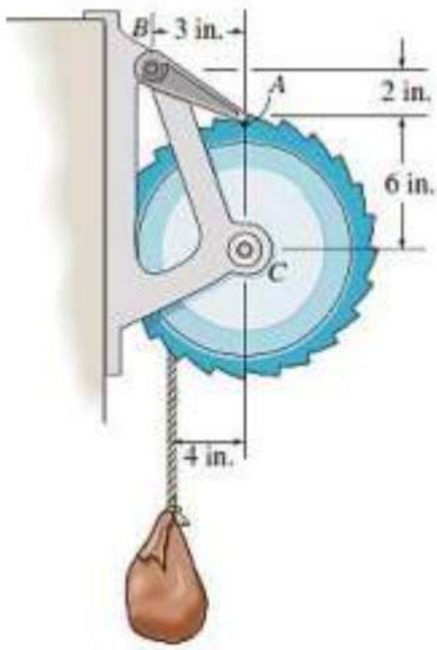
We can get on reactions at point *A* (A_x and A_y) , from equilibrium $\sum F_x = 0\ and\ \sum F_y = 0$

$$A_x - 3.464 \sin 30 = 0 \Longrightarrow A_x = 1.73kN$$

$$A_y + 3.464 \cos 30 - 4 = 0 \Longrightarrow A_y = 1kN$$

Example7.

The winch consists of a drum of radius 4 in., which is pin connected at its center *C*. At its outer rim is a ratchet gear having a mean radius of 6 in. The pawl *AB* serves as a two-force member (short link) and keeps the drum from rotating. If the suspended load is 500 lb, determine the horizontal and vertical components of reaction at the pin *C*.



Solution:

From equilibrium, $\sum M_C = 0$

$$500 \times 4 - F_{Ab} \times \frac{3}{\sqrt{13}} \times 6 = 0 \Rightarrow$$

$$F_{Ab} = 400.62\ Ib$$

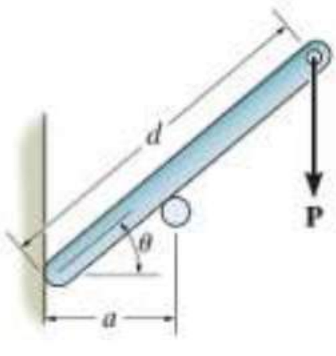
The reactions at point C are C_x and C_y , we have $\sum F_x = 0\ and\ \sum F_y = 0$

$$400.62 \times \frac{3}{\sqrt{13}} - C_x = 0 \Rightarrow C_x = 333Ib$$

$$C_x - 500 - 400.62 \times \frac{2}{\sqrt{13}} = 0 \Rightarrow C_y = 722Ib$$

Example 8.

Determine the distance *d* for placement of the load **P** for equilibrium of the smooth bar in the position θas shown. Neglect the weight of the bar.



Solution

We have $\sum F_y = 0$, therefore

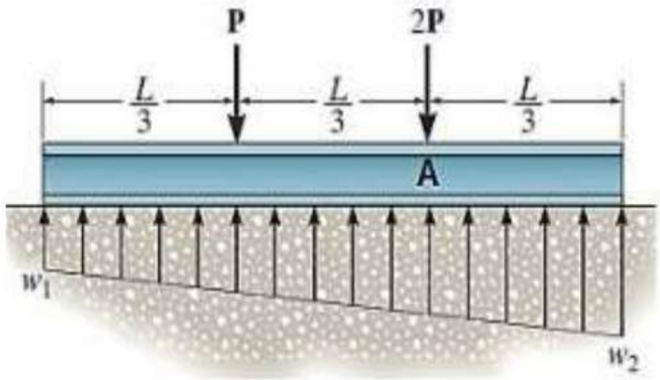
$$R\cos\theta - P = 0$$

$$\sum M_A = 0 \Rightarrow -P \times d\cos\theta + R \times \frac{a}{\cos\theta} = 0$$

So, $d = \frac{a}{\cos^3\theta}$

Example9.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium if $P = 500Ib$ and L $= \mathbf{12}$ ft



Solution:

From equilibrium $\sum \mathrm{M_A} = 0 \Rightarrow 500 \times 4 - \mathrm{w_1} \times 12 \times 2 = 0$

$$\mathrm{w_1} = 83.33\ \mathrm{Ib/ft}$$

Also, $\sum \mathrm{F_y} = 0$, therefore

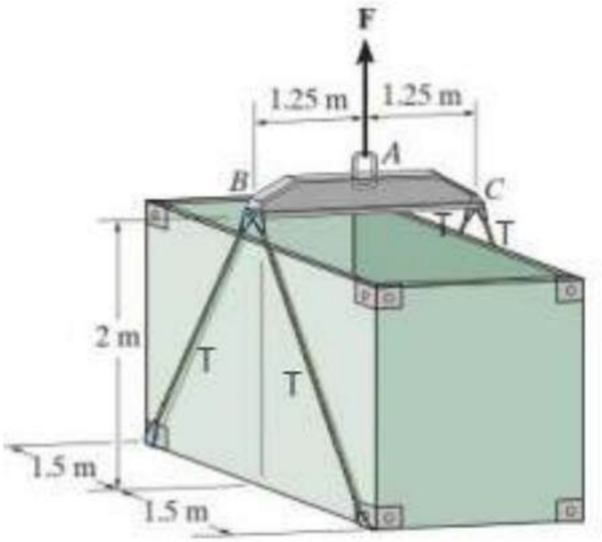
$$w_1 \times 12 + 0.5(w_2 - w_1) \times 12 - 500 - 1000 = 0$$

$$83.33 \times 12 + 0.5(w_2 - 83.33) \times 12 - 500 - 1000 = 0$$

$$w_2 = 166.67\ Ib/ft$$

Example10.

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at *A*.



Solution:

The tension in all wires is the same this is due to the symmetry. From equilibrium, we have$\sum F_z = 0$;

$$4T \times \frac{4}{5} - 600 \times 9.81 = 0$$
$$\frac{16}{5}T - 5886 = 0$$
$$T = 1839.375N$$

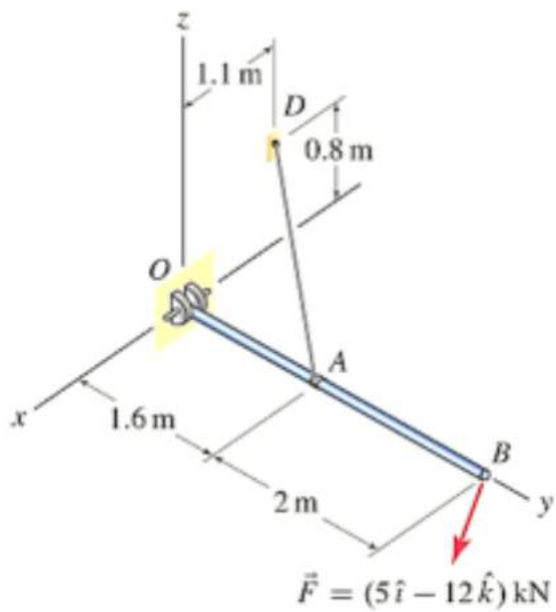
At sling A, the force **F** applied must support the weight of the load and strongback beam. i.e.,

$$F - 600 \times 9.81 - 30 \times 9.81 = 0$$

$$F = 6180.3N$$

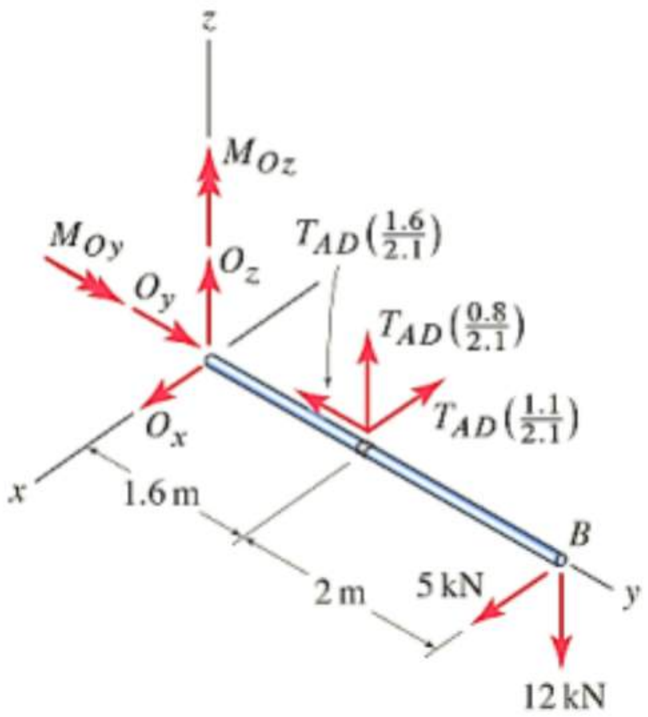
Example11.

Boom OAB is supported by a pin at point O and a cable. Determine the support reactions at O and the force supported by the cable.



Solution:

The support at O prevents translation of the boom in each of the x, y, and z directions; hence there must be reaction forces O_x, O_y *and* O_z in these directions. The pin prevents rotation of the boom about the y and z axes, hence there must be reaction moments M_{oy} and M_{oz} about these axes. The cable force T_{AD} is taken to be positive in tension.



We can resolve the cable force in the direction of coordinates. Therefore, the FBD as shown below.
Using equations of equilibrium, we get

$$\sum M_{Ox} = 0: \quad T_{AD}\left(\frac{0.8}{2.1}\right)(1.6\,\text{m}) - (12\,\text{kN})(3.6\,\text{m}) = 0,$$

$$\Rightarrow \boxed{T_{AD} = 70.9\,\text{kN}.}$$

$$\sum M_{Oy} = 0: \quad M_{Oy} = 0,$$

$$\Rightarrow \boxed{M_{Oy} = 0.}$$

$$\sum M_{Oz} = 0: \quad M_{Oz} + T_{AD}\left(\frac{1.1}{2.1}\right)(1.6\,\text{m}) - (5\,\text{kN})(3.6\,\text{m}) = 0,$$

$$\Rightarrow \boxed{M_{Oz} = -41.4\,\text{kN·m}.}$$

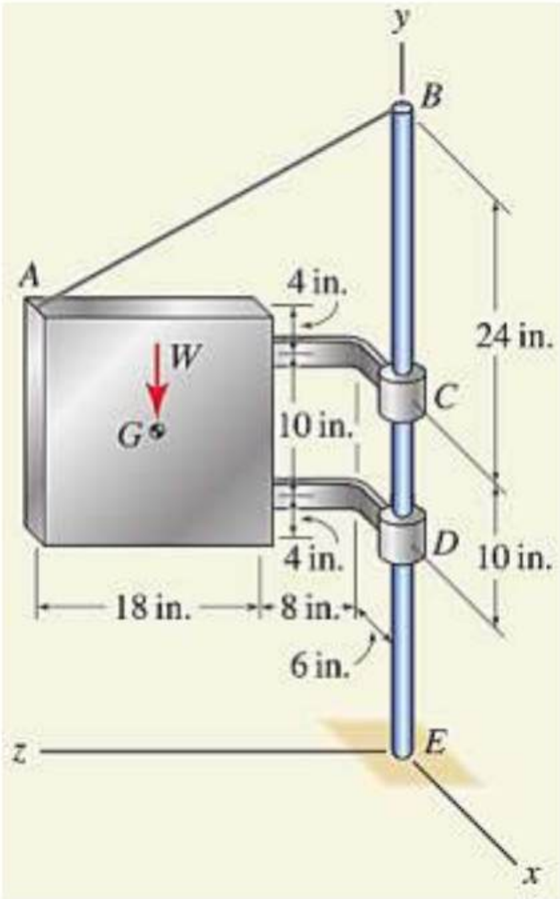
$$\sum F_x = 0: \quad O_x - T_{AD}\left(\frac{1.1}{2.1}\right) + 5\,\text{kN} = 0, \qquad \Rightarrow \boxed{O_x = 32.1\,\text{kN}.}$$

$$\sum F_y = 0: \quad O_y - T_{AD}\left(\frac{1.6}{2.1}\right) = 0, \qquad \Rightarrow \boxed{O_y = 54.0\,\text{kN}.}$$

$$\sum F_z = 0: \quad O_z + T_{AD}\left(\frac{0.8}{2.1}\right) - 12\,\text{kN} = 0, \qquad \Rightarrow \boxed{O_z = -15.0\,\text{kN}.}$$

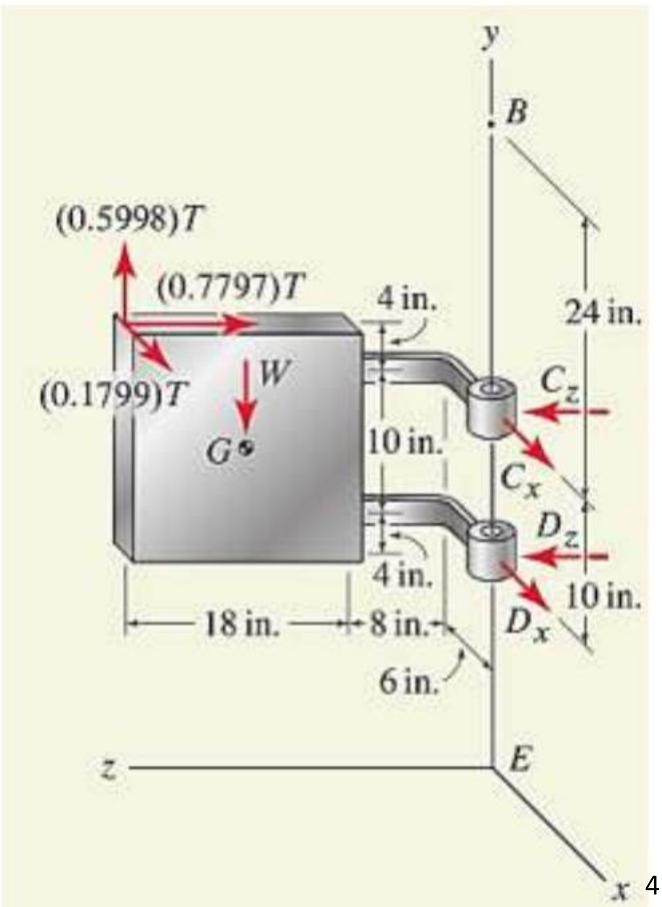
Example12.

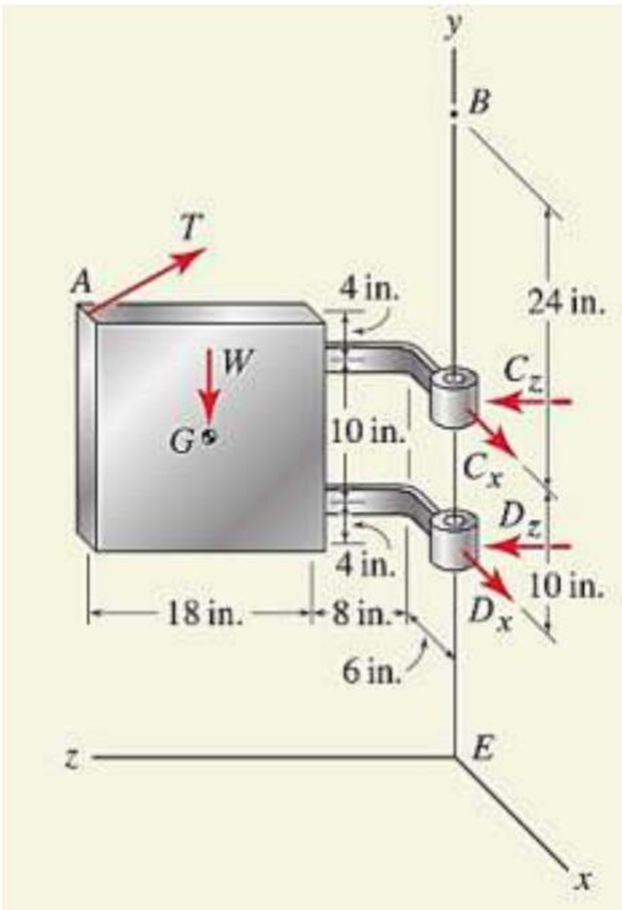
A heavy door seals a furnace used to heat-treat metal parts. The door's weight is $W = 200\ lb$ which acts through point G located at the center of the 18 in. by 18 in. door. Determine the force supported by cable AB and the reactions at the bearings at points C and D.



Solution:

Draw F.B.D. as the following





From equilibrium, we get

$$\sum F_y = 0: \quad (0.5998)T - 200 \text{ lb} = 0 \quad \Rightarrow \quad \boxed{T = 333.4 \text{ lb.}}$$

$$\sum M_{Dx} = 0: \quad -(0.7797)T(14 \text{ in.}) - (0.5998)T(26 \text{ in.})$$
$$+ (200 \text{ lb})(17 \text{ in.}) + C_z(10 \text{ in.}) = 0$$
$$\Rightarrow \boxed{C_z = 544.0 \text{ lb.}}$$

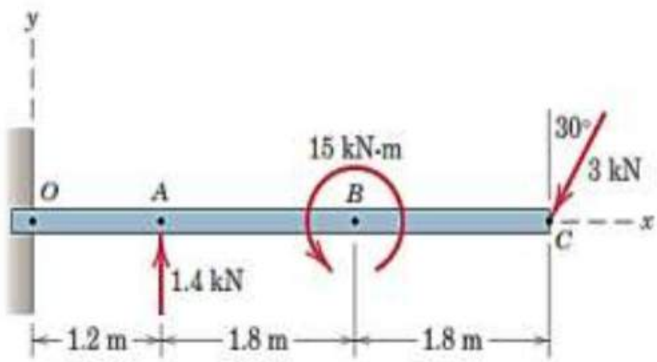
$$\sum M_{Dz} = 0: \quad -(0.5998)T(6 \text{ in.}) - (0.1799)T(14 \text{ in.})$$
$$+ (200 \text{ lb})(6 \text{ in.}) - C_x(10 \text{ in.}) = 0$$
$$\Rightarrow \boxed{C_x = -84.03 \text{ lb.}}$$

$$\sum F_x = 0: \quad (0.1799)T + C_x + D_x = 0 \quad \Rightarrow \quad \boxed{D_x = 24.01 \text{ lb.}}$$

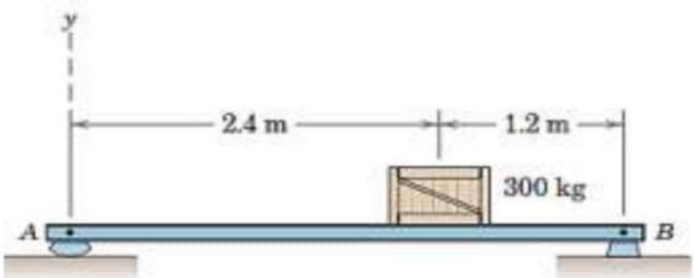
$$\sum F_z = 0: \quad -(0.7797)T + C_z + D_z = 0 \quad \Rightarrow \quad \boxed{D_z = -284.0 \text{ lb.}}$$

Problems

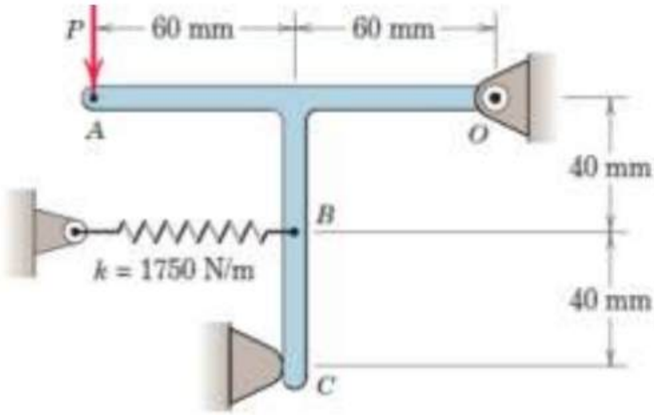
1- The 500-kg uniform beam is subjected to the three external loads shown. Compute the reactions at the support point *O*. The *x*-*y* plane is vertical.



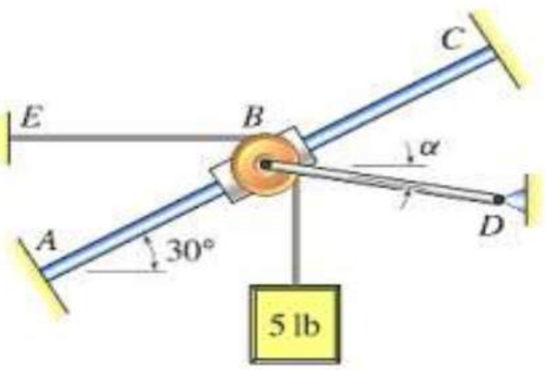
2- The uniform beam has a mass of 50 kg per meter of length. Determine the reactions at the supports.



3- When the 0.05-kg body is in the position shown, the linear spring is stretched 10 mm. Determine the force P required to break contact at C. Complete solutions for (a) including the effects of the weight and (b) neglecting the weight.



4- The structure consists of a collar at B that is free to slide along a straight fixed bar AC. Mounted on the collar is a frictionless pulley, around which a cable supporting a 5 lb weight is wrapped. The collar is further supported by a bar BD.

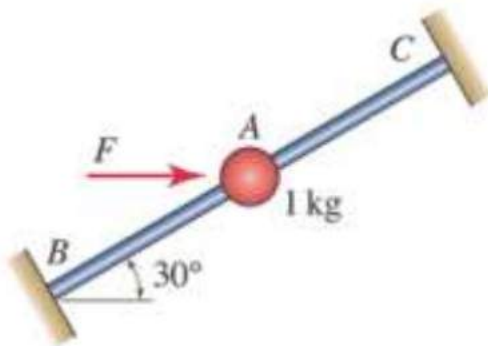


(a) If ، $\alpha = 0$, determine the force in bar BD needed to keep the system in equilibrium.

(b) Determine the value of α that will provide for the smallest force in bar BD,and determine the value of this force.

5- Bead A has 1 kg mass and slides without friction on bar BC.

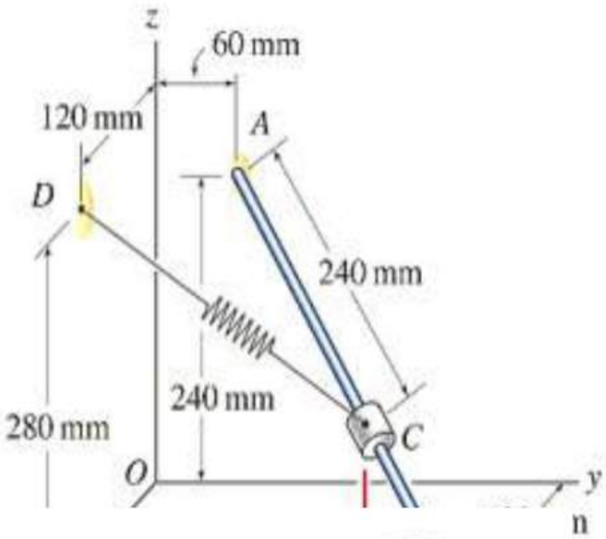
(a) Determine the force F needed to keep the bead in static equilibrium and the reaction force between the bead and bar.



(b) If the value of F determined in Part (a) is applied, will the bead move? Explain.

(c) If F is larger than the value determined in Part (a), describe what happens.

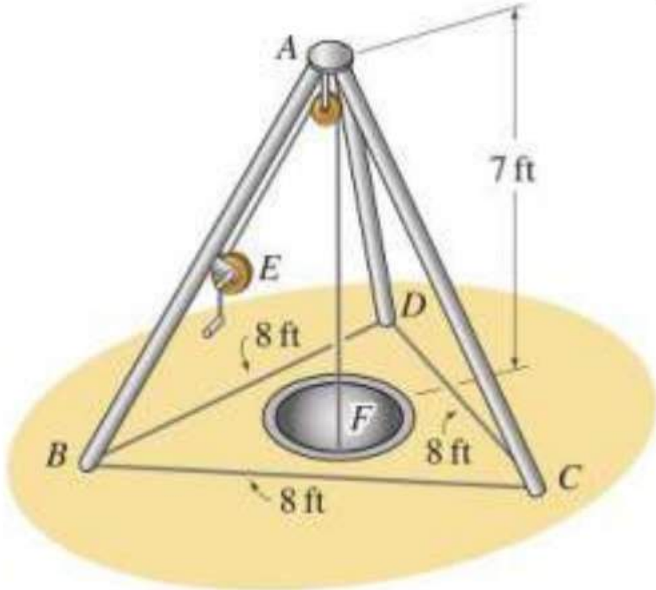
6- Bar AB is straight and is fixed in space. Spring *CD* has 3 N/mm stiffness and 200 mm unstretched length. If there is no friction between collar C and bar AB, determine



(a) The weight W of the collar that produces the equilibrium configuration shown.

(b) The reaction between the collar and bar AB.

7- A portable tripod hoist for moving objects in and out of a manhole is shown. The hoist consists of identical-length bars AB, AC, and AD that are connected by a socket at A and are supported by equal 8 ft length cables BC, BD, and CD to prevent ends B, C, and D of the bars from slipping. Cable *FAE* passes around a frictionless pulley at A and terminates at winch E, which is fixed to bar AB. Idealize points A and E to be particles where all bar and cable forces pass through these points. If the tripod is erected on level ground and is 7 ft high, and the object being lifted in the manhole weighs 300 lb,



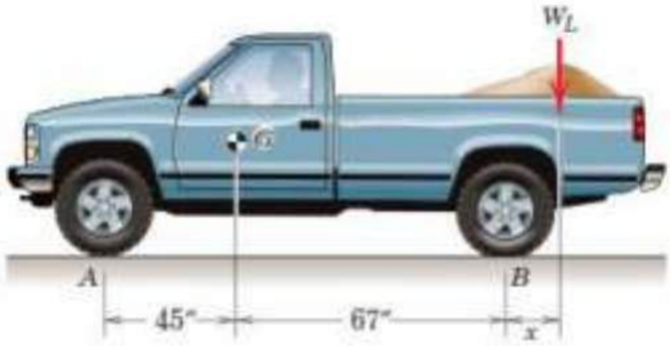
(a) Determine the forces in bars AC and AD and in portion AE of bar AB.

(b) Determine the force in portion EB of bar AB.

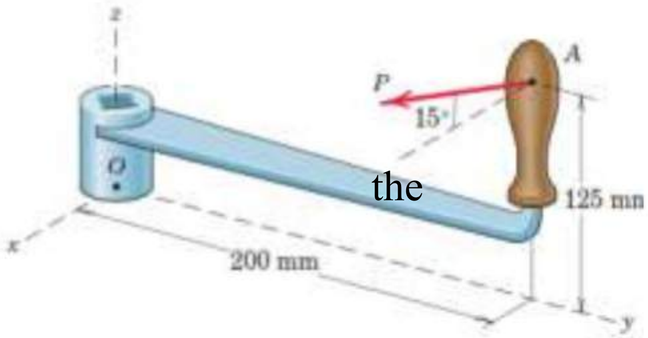
Hint: Define an xyz coordinate system where the x and y directions lie in the plane defined by points *B*, *C*, and D, and where the x or y direction coincides with one of cables BC, BD, or CD. Then determine the coordinates of points B, C, and D. The x and y coordinates of point A are then the averages of coordinates of points B, C, and D.

8- A former student of mechanics wishes to weigh himself but has access only to a scale *A* with capacity limited to 100 lb and a small 20-lb spring dynamometer *B*. With the rig shown he discovers that when he exerts a pull on the rope so that *B* registers 19 lb, the scale *A* reads 67 lb. What is his correct weight?

9- The indicated location of the center of gravity of the 3600-lb pickup truck is for the unladen condition. If a load whose center of gravity is $x = 16$ in. behind the rear axle is added to the truck, determine the load weight W_L for which the normal forces under the front and rear wheels are equal.

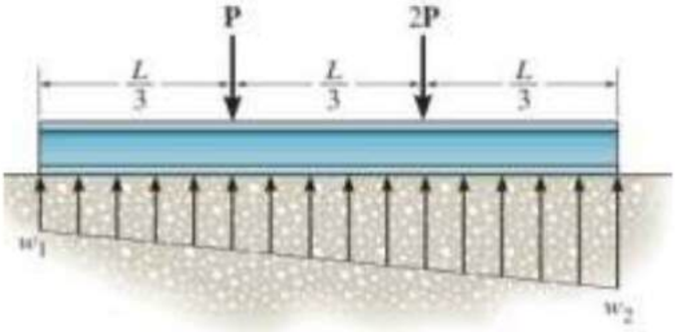


10- A force of magnitude P=180 N is applied to the stationary machine handle as shown. Write the force and moment the

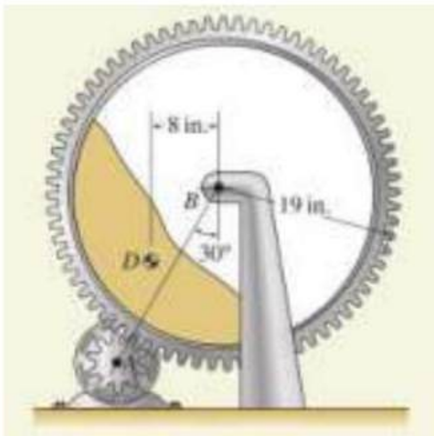


reactions at O as vectors. Neglect the weight of the handle assembly.

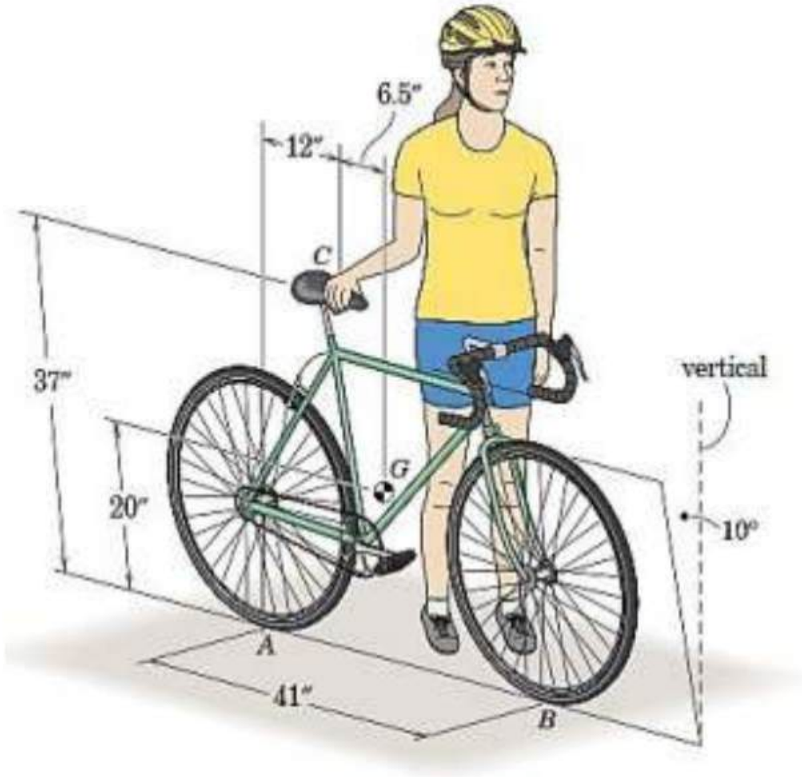
11- The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium in terms of the parameters shown.



12- A drum for mixing material rotates clockwise under the power of a geared motor at A. The drum weighs 320 lb and is supported by a bearing at point B, and the material being mixed weighs 140 lb with center of gravity at point D. If gear A is a loose fit with the gear on the drum, determine the reactions at point B and the gear tooth force required to operate the machine. Assume the machine operates at steady speed and the material being mixed maintains the same shape and position as the drum rotates.

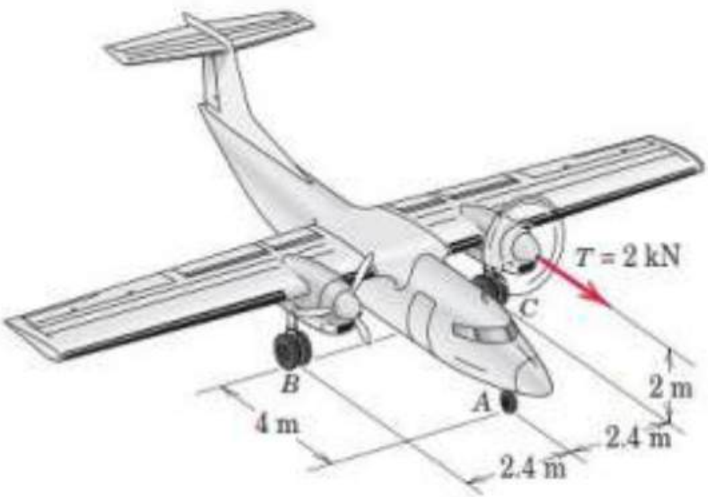


13- A rider holds her bicycle at the 10^o angle shown by exerting a force perpendicular to the plane of the bicycle frame. If friction at A and B is sufficient to prevent lateral slippage, determine the force exerted by the rider on the seat, the upward normal forces at A and B, and the lateral friction forces at A and B. Even though the bicycle is free to roll, assume that it does not. The bicycle weighs 29 lb with center of gravity at G.

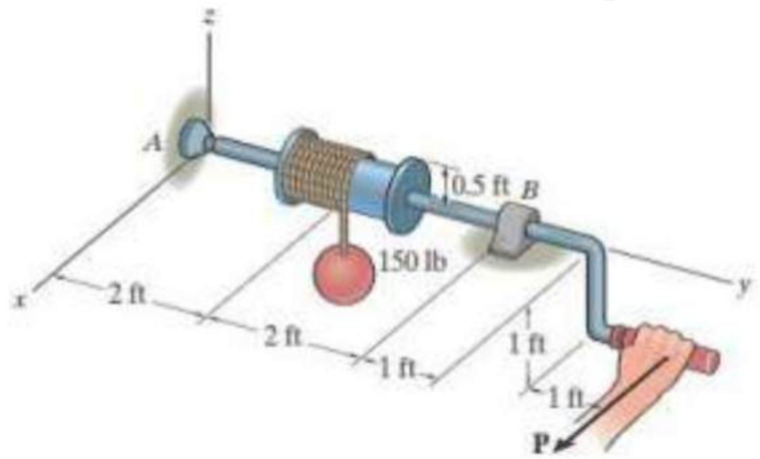


..

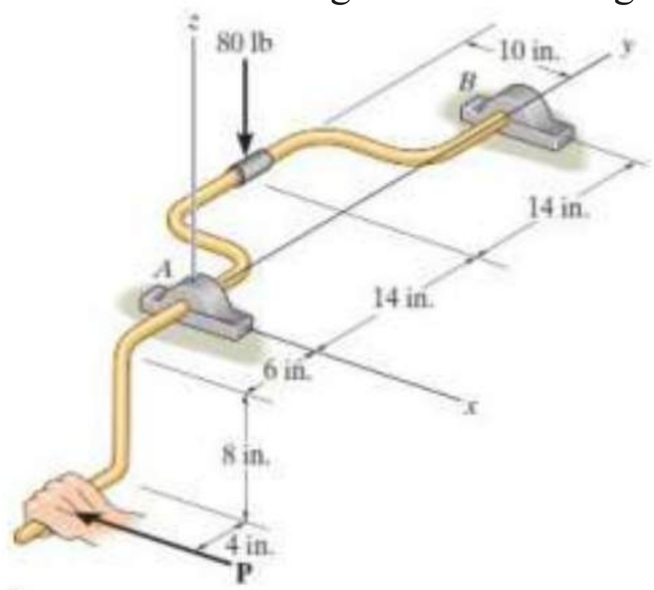
14- During a test, the left engine of the twin-engine airplane is revved up and a 2-kN thrust is generated. The main wheels at *B* and *C* are braked in order to prevent motion. Determine the change (compared with the nominal values with both engines off) in the normal reaction forces at *A*, *B*, and *C*.



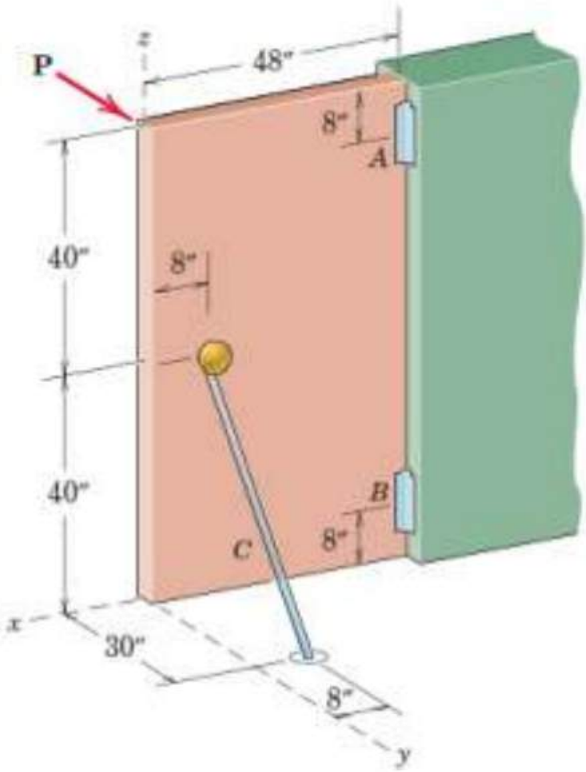
15- The windlass is subjected to a load of 150 lb. Determine the horizontal force **P** needed to hold the handle in the position shown, and the components of reaction at the ball-and-socket joint *A* and the smooth journal bearing *B*. The bearing at *B* is in proper alignment and exerts only force reactions on the windlass.



16- A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force P that must be applied to the handle and the x, y, z components of force at the smooth journal bearing A and the thrust bearing B. The bearings are properly aligned and exert only force reactions on the shaft.



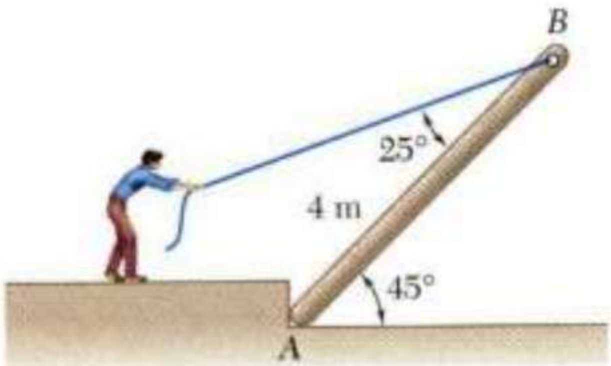
17- The uniform panel door weighs 60 lb and is prevented from opening by the strut C, which is a light two-force member whose upper end is secured under the door knob and whose lower end is attached to a rubber cup which does not slip on the floor. Of the door hinges A and B, only B can support force in the vertical z-direction. Calculate the compression C in the strut and the horizontal components of the forces supported by hinges A and B when a horizontal force $P = 50\ lb$ is applied normal to the plane of the door as shown.



18- The main condition for the rigid body is that the distance between various particles of the body does change.

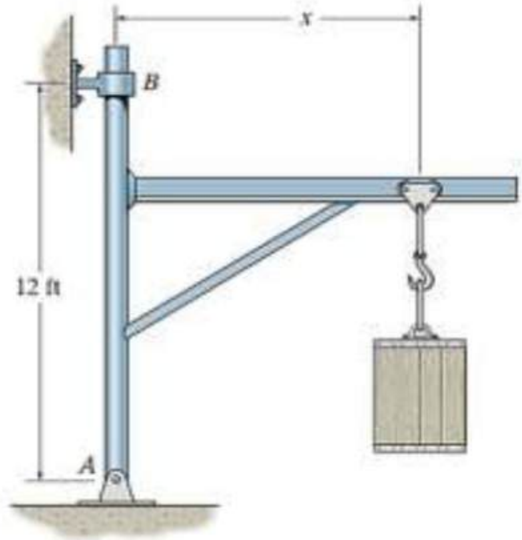
a) True
b) False

19- The force in cable, weight of AB and reaction force at A must be concurrent for static equilibrium and all its lines actions must pass through one intersection point.



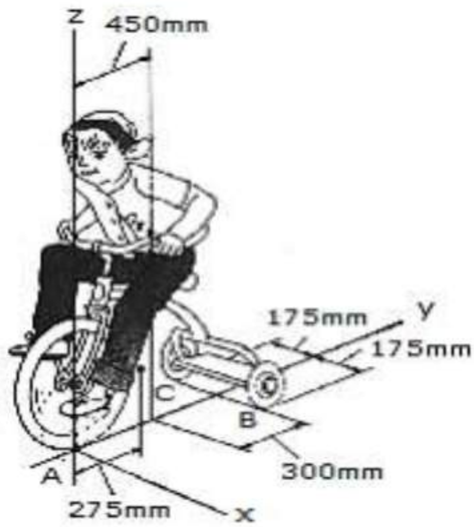
a) True
b) False

20- The jib crane is pin connected at A and supported by a smooth collar at B. If the load has a weight of 5000 lb, then the reactions on the jib crane at smooth collar B is`



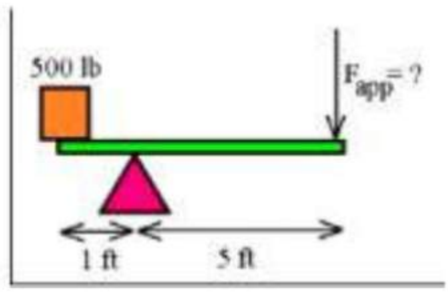
a) 5555lb
b) 3333.33lb
c) 2333.88lb
d) 5321.11lb

21- The girl has a mass of 17kg and mass center at Gg, and the tricycle has a mass of 10kg and mass center at Gt. Determine the normal reactions at each wheel for equilibrium.



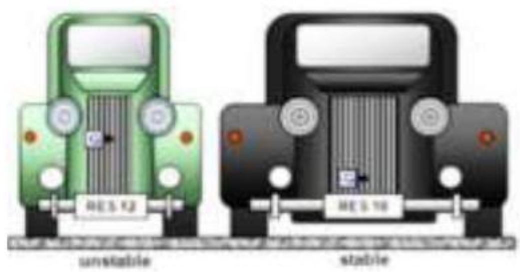
a) $N_A = 14.77$ N, $N_B = N_C = 6.12$ N
b) $N_A = 128.8$ N, $N_B = N_C = 68.0$ N
c) $N_A = 144.9$ N, $N_B = N_C = 60.0$ N
d) $N_A = 13.15$ N, $N_B = N_C = 6.93$ N

22- The force that must be applied to end of the class 1 lever shown below to lift the 500 lb load is



a) 105lb
b) 100lb
c) 200lb
d) 50lb

23 Why is the car on the left unstable?



a) Its center of mass is high

b) It has a high height
c) It has a narrow base
d) all of the above

24- When an object is experiencing a net moment,

a) It is in dynamic equilibrium.
b) It is rotating.
c) It is in static equilibrium.
d) It is translating

25- Moment is the product of a force exerted on an object times the distance from applied force to the rotational axis.

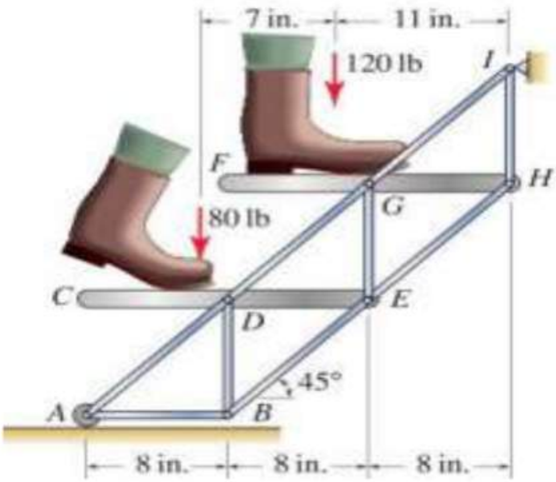
a) True
b) False

26- 11. The easiest way to open a heavy door is by applying the force
a) Near the hinges
b) At the edge of the door far from the hinges
c) In the middle of the door
d) At the top of the door

27- If the kid on the left weights 490 N, 0.5m away from the pivont point, then the second kid must have………. in order to keep it in static equilibrium

a) 340 N, 0.1 m away from the pivont point to the right
b) 490 N, 0.5m away from the pivont point to the right
c) 220 N, 0.2 m away from the pivont point to the left
d) none of these

28- The force exerted by a person on two steps of a short flight of stairs is shown in the figure. The vertical component of the support reaction at A is



a) 120 lb
b) 80 lb
c) 200 lb
d) None of these

29. **MCQs on Equilibrium of Forces**

1. An object is in equilibrium under the action of three forces along a straight line. Two of the forces are 10 N to the right and 5 N to the left. What is the magnitude and direction of the third force?

A) 5 N to the right
B) 5 N to the left
C) 15 N to the right
D) 15 N to the left

2. For a body to be in translational equilibrium in one dimension:

A) All forces must be equal.
B) The net force in the x-direction must be zero.
C) The net force in the y-direction must be zero.
D) The sum of all force magnitudes must be zero.

3. A 50 N weight is suspended by a single vertical cable. The tension in the cable is:

A) 0 N
B) 25 N

C) 50 N
D) 100 N

Part 2: Equilibrium in 2D (Coplanar Forces)

3. **The necessary and sufficient conditions for a particle to be in equilibrium under the action of coplanar forces are:**
   A) ΣFx = 0
   B) ΣFy = 0
   C) ΣFx = 0 and ΣFy = 0
   D) ΣFx = 0, ΣFy = 0, and ΣM = 0

5. Three coplanar forces acting on a particle keep it in equilibrium. The forces can be represented by:
A) Three sides of a triangle taken in order.
B) Three sides of a triangle taken in the opposite order.
C) The perpendicular bisectors of a triangle.
D) The medians of a triangle.

6. A sign of weight 100 N is suspended by two cables, each making a 45° angle with the horizontal. What is the tension in each cable? (Assume symmetry)

A) 50 N
B) 70.7 N
C) 100 N
D) 141.4 N

7. For a rigid body to be in complete equilibrium in a plane:
A) Only the net force must be zero.
B) Only the net moment about any point must be zero.
C) The net force must be zero, and the net moment about any point must be zero.
D) All forces must be concurrent.

Part 3: Equilibrium in 3D (Non-Coplanar Forces)

9. **The number of independent scalar equations for the equilibrium of a particle in three-dimensional space is:**
   A) 1
   B) 2
   C) 3
   D) 6

9. For a rigid body to be in complete equilibrium in three dimensions, the conditions that must be satisfied are:

A) $\Sigma Fx = 0$, $\Sigma Fy = 0$
B) $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Fz = 0$
C) $\Sigma Fx = 0$, $\Sigma Fy = 0$, $\Sigma Fz = 0$, $\Sigma Mx = 0$, $\Sigma My = 0$, $\Sigma Mz = 0$
D) $\Sigma Mx = 0$, $\Sigma My = 0$, $\Sigma Mz = 0$

10. A object is suspended by three cables in three-dimensional space. If the object is in equilibrium, the tension in each cable:

A) Must be equal.
B) Can be calculated by solving the three force equilibrium equations ($\Sigma Fx=0$, $\Sigma Fy=0$, $\Sigma Fz=0$).
C) Can be calculated by solving the six rigid body equilibrium equations.
D) Cannot be determined.

11. Lami's theorem, which relates forces and their opposite angles, is applicable for:
A) Any number of forces in 2D.
B) Two forces in equilibrium.
C) Three concurrent and coplanar forces in equilibrium.
D) Non-concurrent forces in 3D.

12. If a force system is in equilibrium, the resultant force is:

A) A couple.
B) A single force.
C) Zero.
D) A moment.

Answer Key & Detailed Explanations

1. B) 5 N to the left

Explanation: For 1D equilibrium, $\Sigma F = 0$. Let the third force be F. 10 N (right) + 5 N (left) + F = 0. Taking right as positive: $10 - 5 + F = 0 \Rightarrow F = -5$ N. The negative sign means the force is 5 N to the left.

2. B) The net force in the x-direction must be zero.

Explanation: In 1D, all forces act along a single line (say, the x-axis). Therefore, the only condition for translational equilibrium is that the net force along that line is zero ($\Sigma Fx = 0$).

3. C) 50 N

Explanation: This is a 1D equilibrium problem along the vertical (y) axis. The weight (50 N down) must be balanced by the tension (T up). $\Sigma Fy = 0 \Rightarrow T - 50 = 0$, so $T = 50$ N.

4. C) $\Sigma Fx = 0$ and $\Sigma Fy = 0$

Explanation: For a *particle* (point object) to be in equilibrium under coplanar forces, the vector sum of all forces must be zero. This gives two independent scalar equations: the sum of forces in the x-direction is zero, and the sum of forces in the y-direction is zero. Moments are not considered for a particle.

5. A) Three sides of a triangle taken in order.

Explanation: This is the **Triangle Law of Forces**. If three coplanar forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order.

6. B) 70.7 N

Explanation:

- Let T be the tension in each cable.
- The vertical component from each cable is T sin(45°).
- There are two cables, so total upward force = 2 * T sin(45°) = 2 * T * (√2/2) = T√2.
- For equilibrium: Total upward force = Weight => T√2 = 100 N.
- Therefore, T = 100 / √2 ≈ 70.7 N.

7. C) The net force must be zero, and the net moment about any point must be zero.

Explanation: For a *rigid body* to be in complete equilibrium (no translation and no rotation) in a plane, two conditions must be met:

1. **Translational Equilibrium:** The vector sum of all forces is zero ($\Sigma F_x = 0$ and $\Sigma F_y = 0$).
2. **Rotational Equilibrium:** The sum of the moments of all forces about *any* point is zero ($\Sigma M = 0$).

8. C) 3

Explanation: For a *particle* in 3D, the equilibrium condition is that the vector sum of forces is zero: $\Sigma \mathbf{F} = 0$. This vector equation can be broken down into three independent scalar equations: $\Sigma Fx = 0$, $\Sigma Fy = 0$, and $\Sigma Fz = 0$.

9. C) $\Sigma Fx = 0, \Sigma Fy = 0, \Sigma Fz = 0, \Sigma Mx = 0, \Sigma My = 0, \Sigma Mz = 0$

Explanation: For a *rigid body* in complete equilibrium in 3D, we must prevent both translation and rotation about any axis.

- **3 Equations for Translation:** $\Sigma Fx=0$, $\Sigma Fy=0$, $\Sigma Fz=0$.
- **3 Equations for Rotation:** $\Sigma Mx=0$ (sum of moments about the x-axis), $\Sigma My=0$, $\Sigma Mz=0$.

10. B) Can be calculated by solving the three force equilibrium equations ($\Sigma Fx=0$, $\Sigma Fy=0$, $\Sigma Fz=0$).
Explanation: The suspended object can be treated as a particle for this purpose because the cables are concurrent (they all meet at the object). Therefore, only the three force equilibrium equations are needed to find the three unknown tensions.

11. C) Three concurrent and coplanar forces in equilibrium.
Explanation: **Lami's theorem** states that if three coplanar forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces. It is a very specific and useful theorem for solving 3-force equilibrium problems in 2D.

12. C) Zero.

Explanation: By definition, if a system of forces is in equilibrium, the resultant (the single force that has the same effect as the system) is zero. This means it causes neither translation nor rotation.

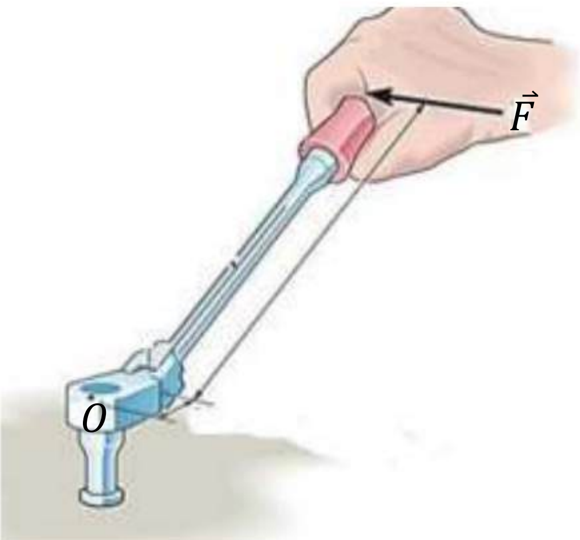
Chapter 4

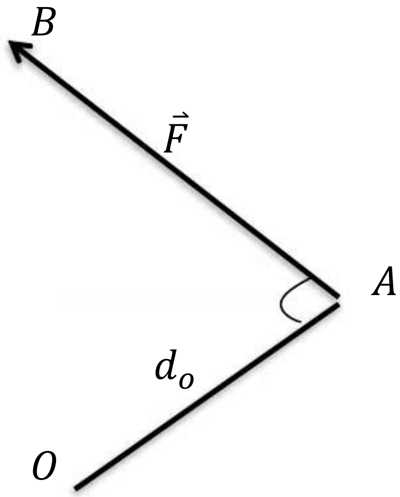
Moments and couples

In engineering, while a force is a push or pull that causes an object to move in a straight line (translation), we often need to describe its ability to cause an object to rotate. This rotational effect is quantified by two key concepts: the Moment of a Force and the Couple.

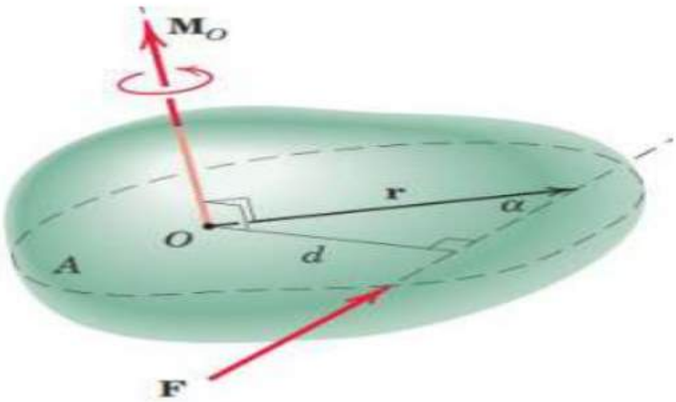
Moment of Force about any arbitrary point:

Let $\vec{F}$ is a vector force that has line of action , the moment of a scalar force F about any point O is $M_o = F\ d_o$ where d_o is the perpendicular distance from the point to the force's line of action.





Sense of Moment, it has minus sign (-) for counterclockwise moments and a plus sign (+) for clockwise moments .Sign consistency within a given problem is essential. The unit of moment is N.m



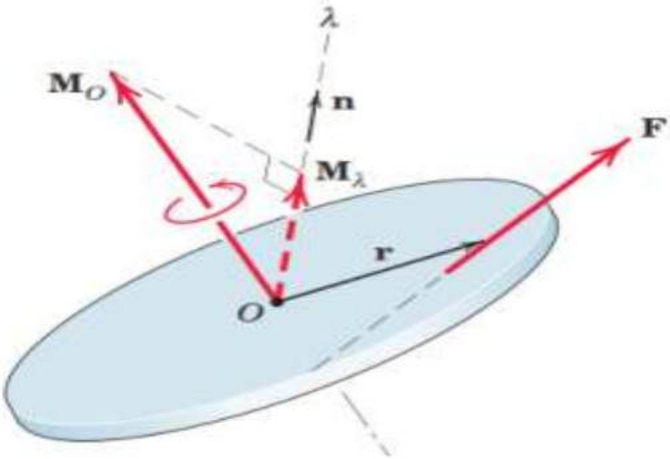
Let $\vec{F}$ is a vector force that

acting at a point A , the moment of force $\vec{F}$ about any point O is $\vec{M_o} = \overrightarrow{OA} \times \vec{F}$ where $\overrightarrow{OA} = \vec{r}$ is the position vector of point A with respect to the point O. so

$$\vec{M_o} = \vec{r} \times \vec{F}$$

Moment about arbitrary axis:

The component of M_o along the λ-axis using the dot product.



$$M_{o\lambda} = M_o \cdot \hat{u}_{o\lambda} = (r \times F) \cdot \hat{u}_{o\lambda}$$

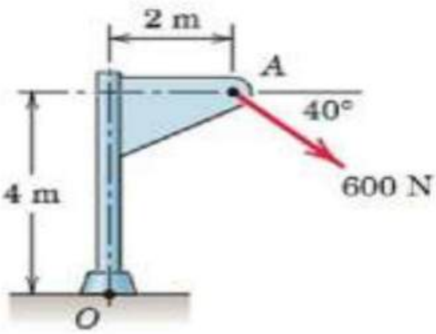
$$M_{o\lambda} = \begin{vmatrix} u_x & u_x & u_x \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

Where

$\hat{u}_{o\lambda}$ is the unite vector in the direction of the axis $o\lambda$, $\hat{u}_{o\lambda} = u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$.

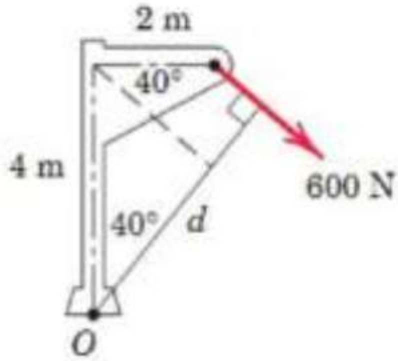
Example1

Calculate the magnitude of the moment about the base point O of the $600N$ force in different ways.



Solution

- By $M = F.d$ the moment is clockwise and has the magnitude M_o,the moment arm to the 600-N force is

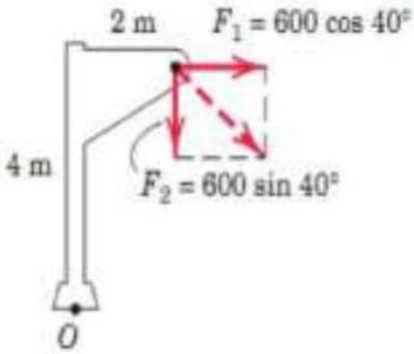


$d = 4\,cos\,40 + 2\,sin\,40 = 4.35\,m$

There for

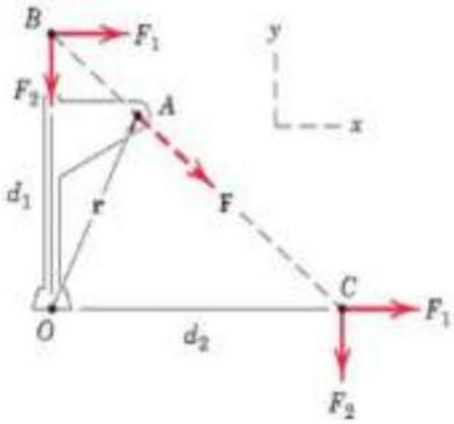
$$Mo = 600(4.35) = 2610\,N.m\ clockwise$$

- Resole the force into two components :



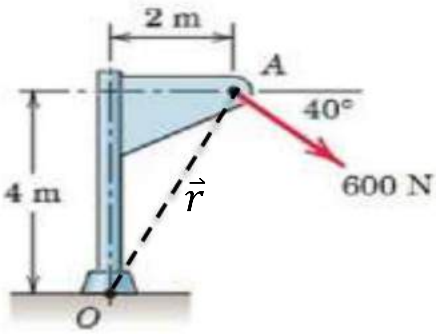
$F_1 = 600\,cos\,40 \approx 460\,N$
$F_2 = 600\,sin\,40 \approx 386\,N$
$M_o = 460(4) + 386(2)$
$= 2610\,N.m\ clockwise$

- By the principle of transmissibility move the 600N force along its line of action to point B, so the moment of F_2 vanishes and calculate the moment due to F_1 as



$d_1 = 4 + 2\,tan\,40 = 5.68\,m$
$M_o = 460(5.68)$
$= 2610\,N.m\ clockwise$

- By the vector expression for a moment



$\vec{F} = F_1\hat{\imath} + F_2\hat{\jmath}$

$\vec{F} = 600\cos 40\,\hat{\imath} - 600\sin 40\,\hat{\jmath}$

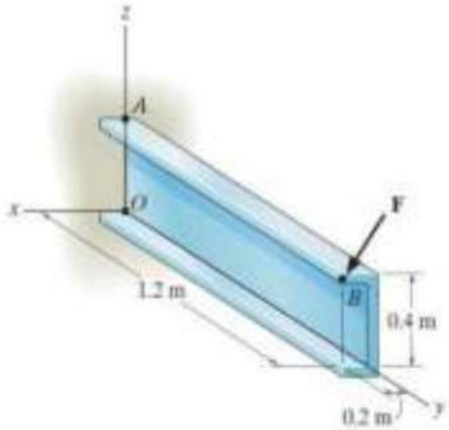
$\vec{F} = 460\hat{\imath} - 386\hat{\jmath}$

$\vec{r} = \overrightarrow{OA} = 2\hat{\imath} + 4\hat{\jmath}$

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 4 & 0 \\ 600\cos 40 & -600\sin 40 & 0 \end{vmatrix}$$

$$\vec{M}_o = \big((2 \times -386) - (4 \times 460)\big)\hat{k}$$

$\vec{M}_o = -2610\hat{k}$, since $M_o = 2610\ N.m\ clockwise$



Example2

The force $\vec{F} = \{600\hat{\imath} + 300\sin 40 - 600\hat{k}\} N$ acts at the end of the beam. Determine the moment of the force about point *A*.

Solution

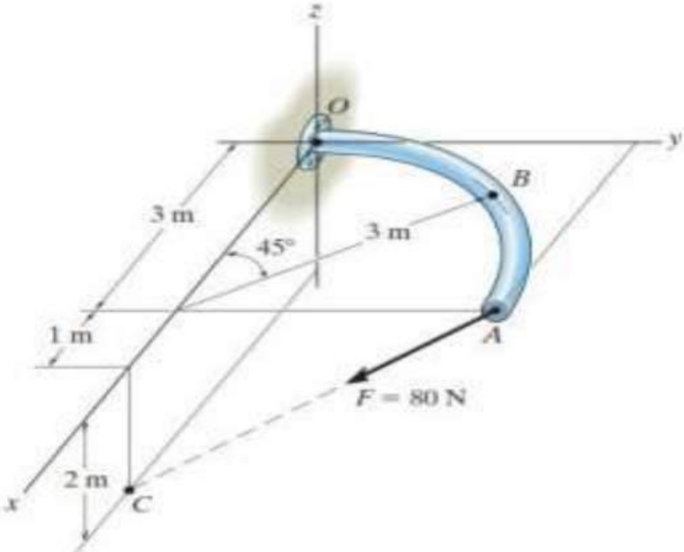
$$\vec{r} = \overrightarrow{OB} = 0.2\hat{\imath} + 1.2\hat{\jmath}$$

$$\vec{M}_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0.2 & 1.2 & 0 \\ 600 & 300 & -600 \end{vmatrix}$$

$$\vec{M}_o = -720\hat{\imath} + 120\hat{\jmath} - 660\hat{k}$$

$$M_o = \sqrt{(-720)^2 + (120)^2 + (-660)^2}$$

$M_o = 984.1 N.m$



Example3

The curved rod lies in the *x–y* plane and has a radius of 3 m. If a force of acts at its end as shown, determine the moment of this force about point *O*. F = 80 N

Solution

$$\widehat{AC} = 1\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$$

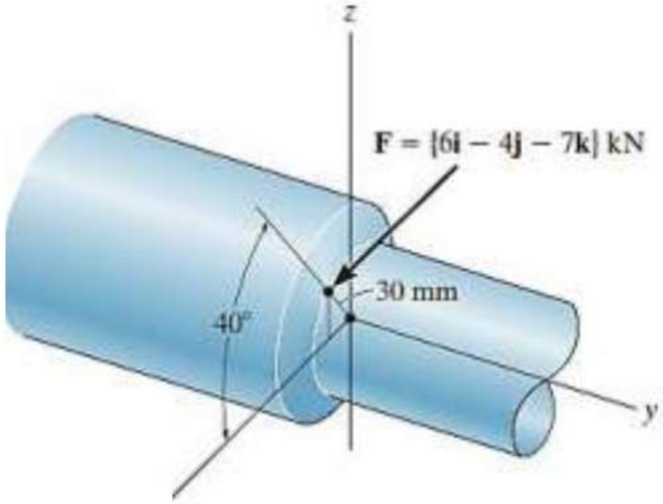
$$\overrightarrow{OC} = 4\hat{\imath} + 0\hat{\jmath} - 2\hat{k}$$

$$\vec{M}_o = \overrightarrow{OC} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 0 & -2 \\ \dfrac{80}{3.742} & \dfrac{-3 \times 80}{3.742} & \dfrac{-2 \times 80}{3.742} \end{vmatrix}$$

$$\vec{M}_o = -128\hat{\imath} + 128\hat{\jmath} - 257\hat{k}$$

Example4

The cutting tool on the lathe exerts a force **F** on the shaft as shown. Determine the moment of this force about the *y* axis of the shaft.



Solution

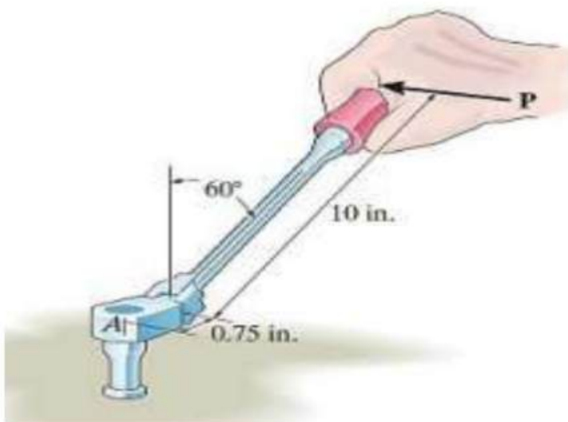
$$M_y = \mathbf{u}_y \cdot (\mathbf{r} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 30\cos 40^\circ & 0 & 30\sin 40^\circ \\ 6 & -4 & -7 \end{vmatrix}$$

$$M_y = 276.57\ \mathrm{N \cdot mm} = 0.277\ \mathrm{N \cdot m}$$

Example5

If a torque or moment of is required to loosen the 80 lb.in. bolt at *A*, determine the force *P* that must be applied perpendicular to the handle of the flex-headed ratchet wrench.



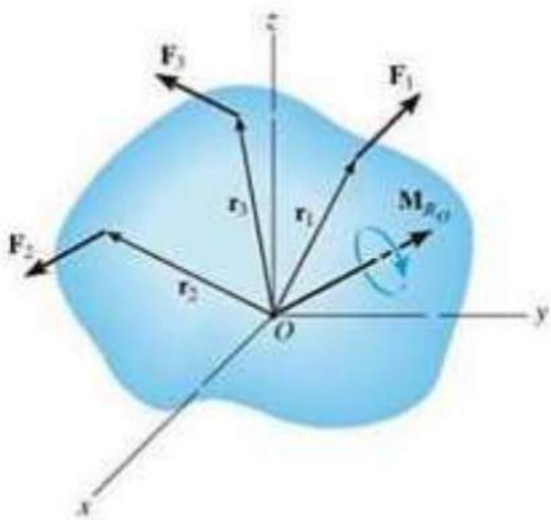
Solution

$$80 = P(0.75 + 10\sin 60^\circ)$$

$$P = \frac{80}{9.41} = 8.50 \text{ lb}$$

Resultant Moment

Let we have system that consists of forces $\vec{F}_1, \vec{F}_2, \ldots, \vec{F}_n$ with position vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ respectively, the resultant moment $\left(\vec{M}_R\right)_o$ about point O can be determined by finding the sum of The moments caused by all the forces in the system.



$$\left(\vec{M}_R\right)_o = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 \ldots + \vec{r}_n \times \vec{F}_n$$

If all forces acting at the same point, then

$$\left(\vec{M}_R\right)_o = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2 \ldots + \vec{r} \times \vec{F}_n$$

$$\left(\vec{M}_R\right)_o = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2 \ldots + \vec{F}_n\right)$$

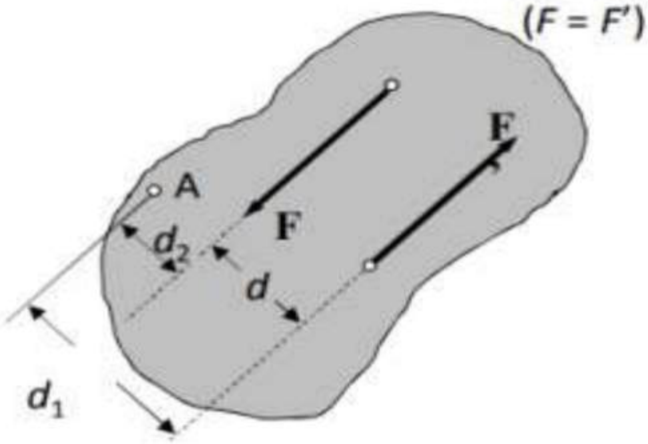
$$\left(\vec{M}_R\right)_o = \vec{r} \times \left(\sum_1^n \vec{F}_i\right)$$

$$\left(\vec{M}_R\right)_o = \vec{r} \times \left(\vec{F}_R\right)$$

Where $\vec{F}_R = \sum_1^n \vec{F}_i$

Moment of a couple

Two forces F and -F having the same magnitude, parallel lines of action, and opposite sense are said to form a couple



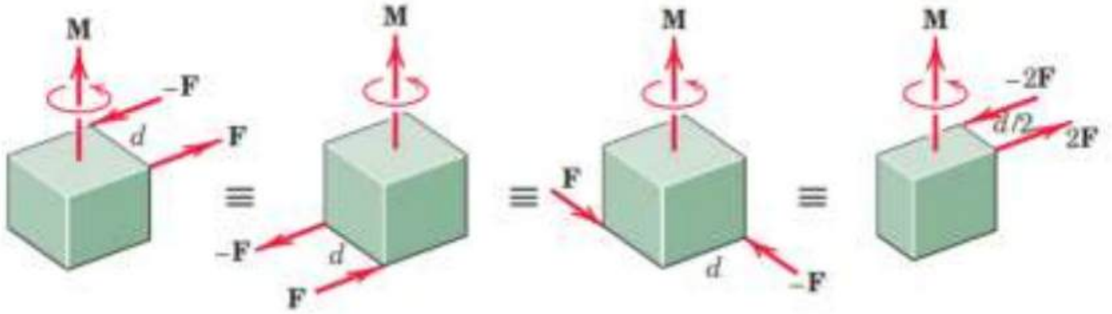
$$M = Fd_1 - Fd_2$$
$$= F(d_1 - d_2)$$
$$= Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated.

Note especially that the magnitude of the couple is independent of the distanced d_2 which locates the forces with respect to the moment center A. It follows that the moment of a couple has the same value for all moment centers.

Equivalent couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane.



The above figure shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

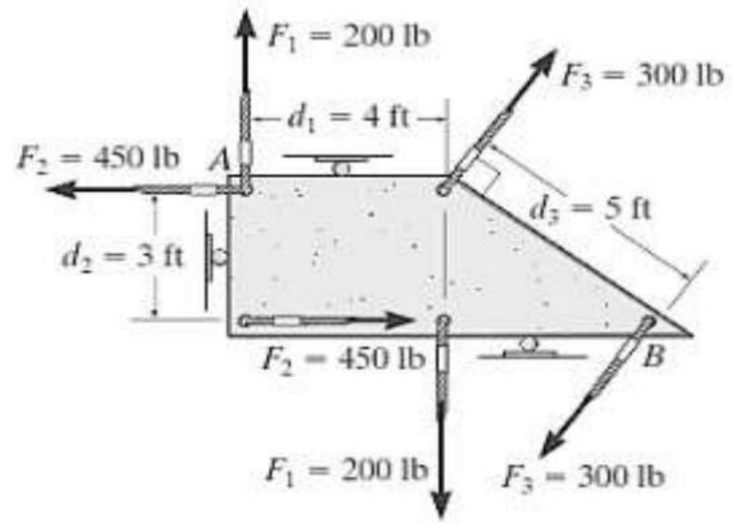
Resultant couple moment

Consider the couple moments $\vec{M}_1, \vec{M}_2 \dots, \vec{M}_n$ acting on a body, the resultant moment $\vec{M}_R$ is

$$\vec{M}_R = \vec{M}_1 + \vec{M}_2 \dots + \vec{M}_n$$

Example6

Determine the resultant couple moment of the three couples acting on the plate



Solution

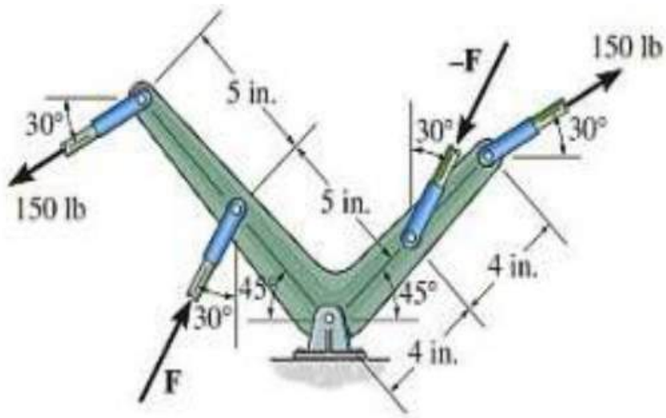
$$\circlearrowleft + M_R = \Sigma M;\; M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

$$= -950 \text{ lb} \cdot \text{ft} = 950 \text{ lb} \cdot \text{ft} \circlearrowright$$

Example7

Determine the magnitude of the couple force **F** so that the resultant couple moment on the crank is zero.



Solution

$$\circlearrowleft + (M_C)_R = \Sigma M_A;$$

$$0 = 150 \cos 15°(10) - F \cos 15°(5) - F \sin 15°(4) - 150 \sin 15°(8)$$

$$F = 194 \text{ lb}$$

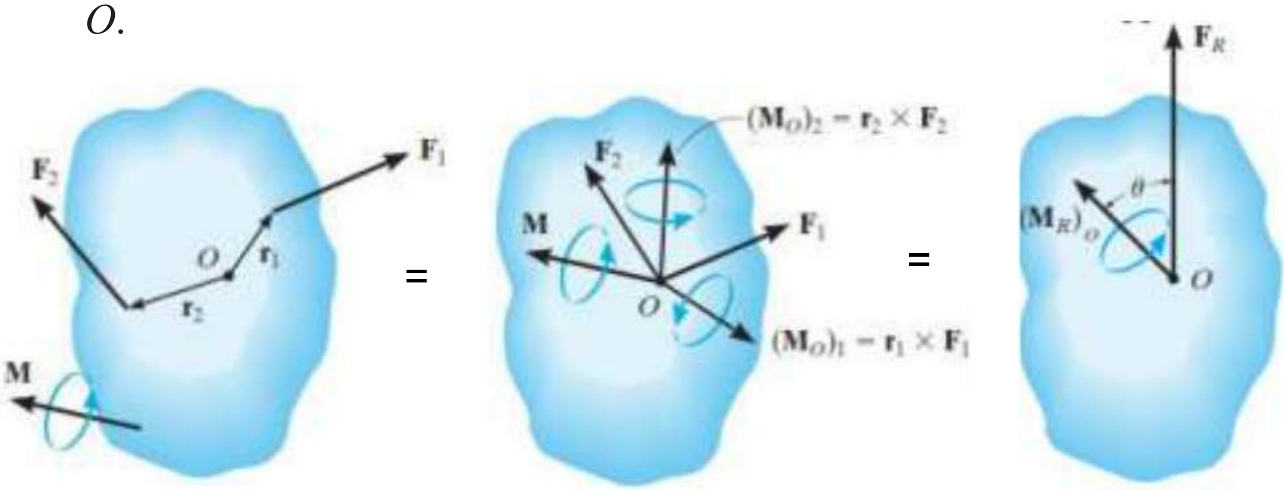
Force- couple systems

Let a system consists of forces and couple moments, we can reduce this system to an equivalent resultant force $\vec{F}_R$ acting at point O and a resultant couple moment $(\vec{M}_R)_O$ by using the following two equations.

$$\vec{F}_R = \sum_{i}^{n} \vec{F}_i$$

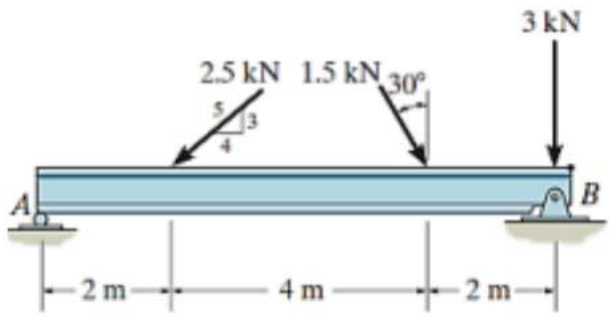
$$(\vec{M}_R)_O = \sum \vec{M}_o + \sum \vec{M}$$

Where the resultant force of the system $\vec{F}_R$ is equivalent to the sum of all the forces $\sum_i^n \vec{F}_i$, the resultant couple moment of the system $(\vec{M}_R)_O$ is equivalent to the sum of all the couple moments $\sum \vec{M}$ plus the moments of all the forces $\sum \vec{M}_o$ about point O.



Example8

Replace the force system acting on the beam by an equivalent force and couple moment at point A.



Solution

$$\xrightarrow{+} F_{R_x} = \Sigma F_x: \quad F_{R_x} = 1.5\sin 30^\circ - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25\text{ kN} = 1.25\text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y: \quad F_{R_y} = -1.5\cos 30^\circ - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799\text{ kN} = 5.799\text{ kN} \downarrow$$

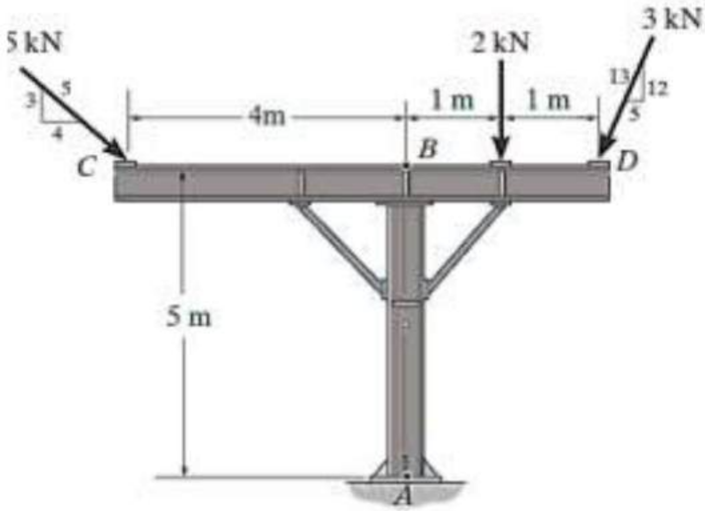
$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93\text{ kN}$$

$$\circlearrowleft + M_{R} = -34.8\text{ kN}\cdot\text{m} = 34.8\text{ kN}\cdot\text{m}\ \textbf{(Clockwise)} \quad)^\circ(6) - 3(8)$$

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8^\circ$$

Example8

Replace the force system acting on the frame by a resultant force and couple moment at point *A*.



Solution

Equivalent Resultant Force: Resolving F_1, F_2 and F_3into their *x* and *y* components, and summing these force components algebraically along the *x* and *y* axes, we have

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x;\qquad (F_R)_x = 5\left(\frac{4}{5}\right) - 3\left(\frac{5}{13}\right) = 2.846 \text{ kN} \rightarrow$

$+\uparrow (F_R)_y = \Sigma F_y;\qquad (F_R)_y = -5\left(\frac{3}{5}\right) - 2 - 3\left(\frac{12}{13}\right) = -7.769 \text{ kN} = 7.769 \text{ kN} \downarrow$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{2.846^2 + 7.769^2} = 8.274 \text{ kN} = 8.27 \text{ kN}$$

The angle θ of $\mathbf{F}_R$ is

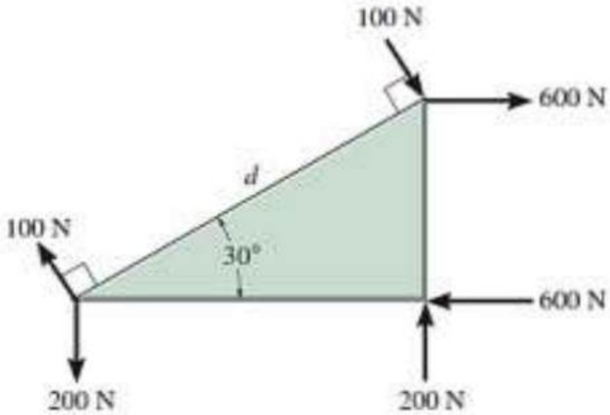
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{7.769}{2.846}\right] = 69.88° = 69.9°$$

$\circlearrowleft + (M_R)_A = \Sigma M_A;\qquad (M_R)_A = 5\left(\frac{3}{5}\right)(4) - 5\left(\frac{4}{5}\right)(5) - 2(1) - 3\left(\frac{12}{13}\right)(2) + 3\left(\frac{5}{13}\right)(5)$

$$= -9.768 \text{ kN}\cdot\text{m} = 9.77 \text{ kN}\cdot\text{m } \textbf{(Clockwise)}$$

Example 9

The ends of the triangular plate are subjected to three couples. Determine the plate dimension d so that the resultant couple is 350 N .m clockwise.



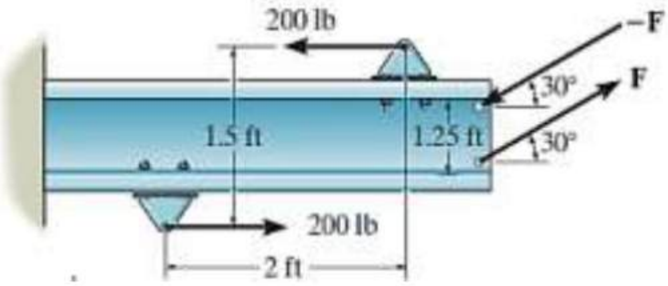
Solution

$\circlearrowleft + M_R = \Sigma M_A;\qquad -350 = 200(d \cos 30°) - 600(d \sin 30°) - 100d$

$$d = 1.54 \text{ m}$$

Example 10

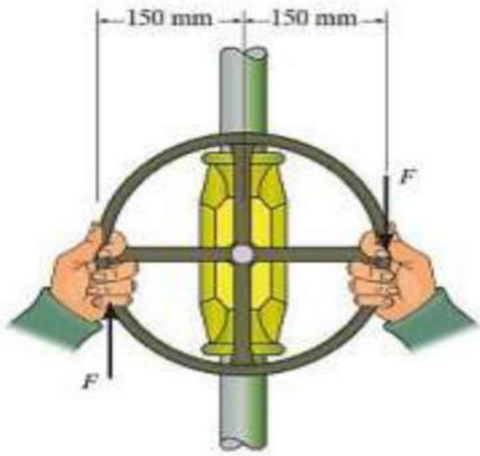
Two couples act on the beam. If $F = 125$ lb , determine the resultant couple moment.



Solution

$$\circlearrowleft + (M_R)_C = 200(1.5) + 125\cos 30^\circ(1.25)$$
$$= 435.32\ \text{lb}\cdot\text{ft} = 435\ \text{lb}\cdot\text{ft}\circlearrowleft$$

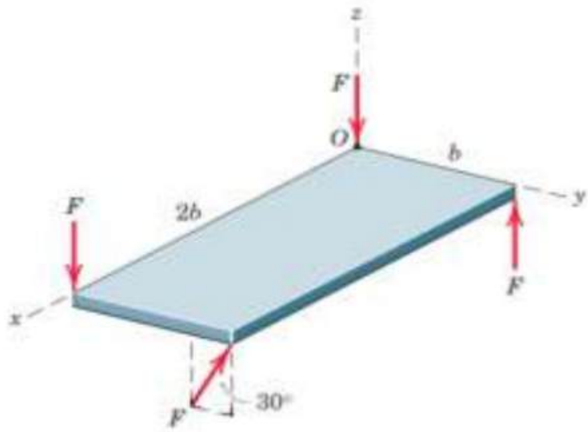
Example 11
The man tries to open the valve by applying the couple forces of F = 75 N to the wheel. Determine the couple moment produced.



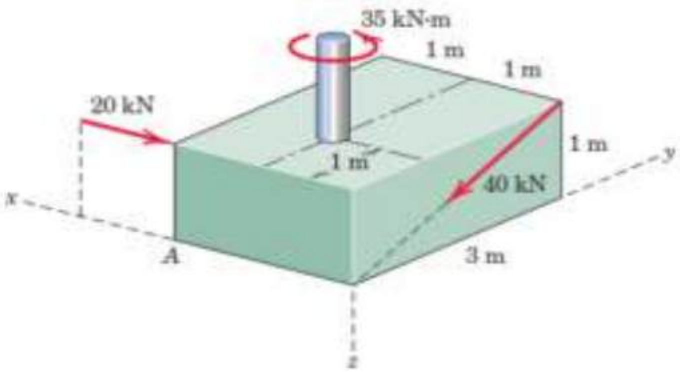
Solution

$$\circlearrowleft + M_c = \Sigma M; \qquad M_c = -75(0.15 + 0.15)$$
$$= -22.5\ \text{N}\cdot\text{m} = 22.5\ \text{N}$$

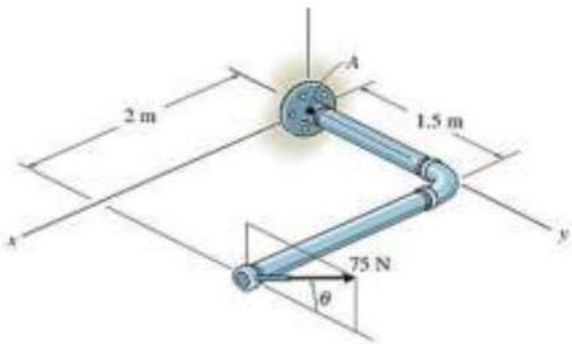
Problems
1- The thin rectangular plate is subjected to the four forces shown. Determine the equivalent force– couple system at O. Is **R** perpendicular to M_o?



2- Replace the two forces and single couple by an equivalent force–couple system at point A.

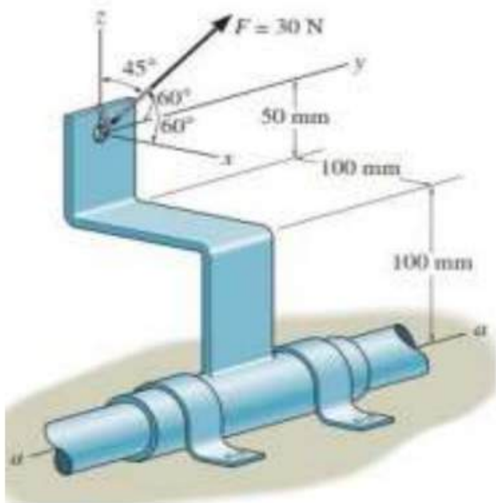


3- Using a ring collar the 75-N force can act in the vertical plane at various anglesθ.

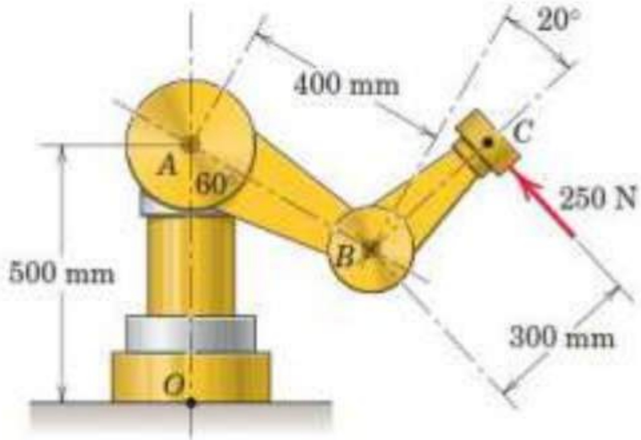


Determine the magnitude of the moment it produces about point A, plot the result of M (ordinate) versus θ (abscissa) for $0^{0} \leq \theta \leq 180^{0}$ and specify the angles that give the maximum and minimum moment.

4- The force of F = 30 N acts on the bracket as shown. Determine the moment of the force about the a-a axis of the pipe. Also, determine the coordinate direction angles of F in order to produce the maximum moment about the a-a axis. What is this moment?



5- Calculate the moment of the 250-N force about the base point O of the robot



6- …….. **is applied while trying to turn a key into a lock.**

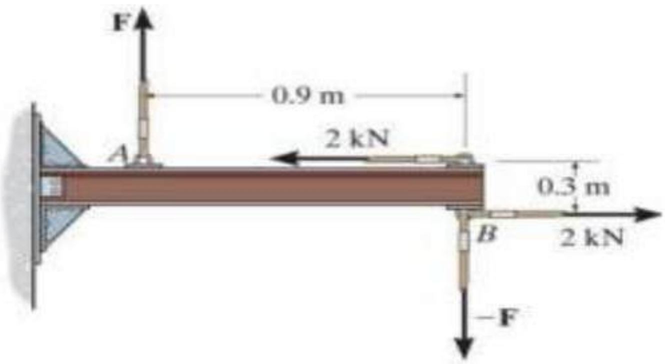
a) momentum

b) impulse

c) couple

d) moment

7- Two couples act on the beam with the geometry shown. The magnitude of F



is ….., so that the resultant couple moment is 1.5 kN·m clockwise

a) F = 20 kN
b) F = 2.33 kN
c) F = 30kN
d) F = 30.33 kN

8- The resultant force acting in the couple is …

a) Infinite
b) Zero
c) Twice the magnitude of the single force
d) Half the magnitude of the single force

9- A man is travelling in the car. He is driving the car. If he is taking a turn in the road. He is applying force to the steering wheel by holding the wheel with his both hands. The steering wheel is facing a moment of force.

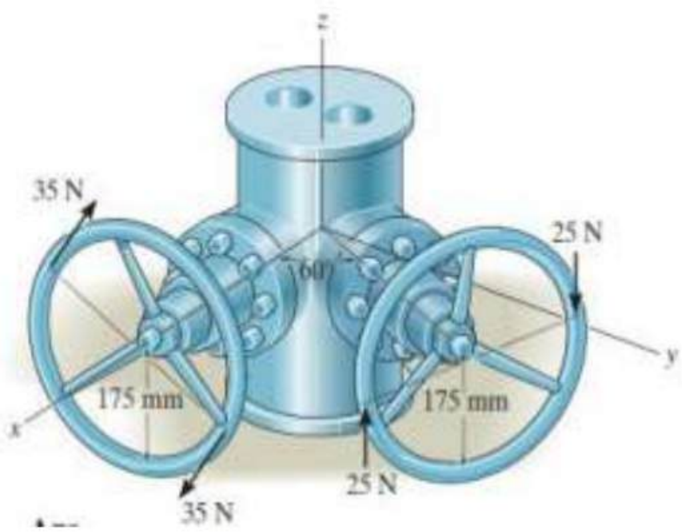
a) True

b) False

10- A single force and a couple acting in the same plane upon a rigid body

a) Balance each other
b) Cannot balance each other
c) Produce moment of a couple
d) Are equivalent

11- If a couple acts on each of the handles of the minidual valve, then the magnitude the



resultant couple moment is

a) 35 N.m

b)18.3N.m

c)10N.m

d) none of these

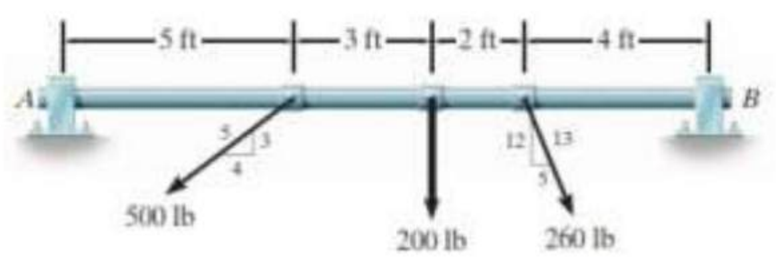
12- We can replace the three forces acting on the shaft by a single resultant force equals

a) 15lb

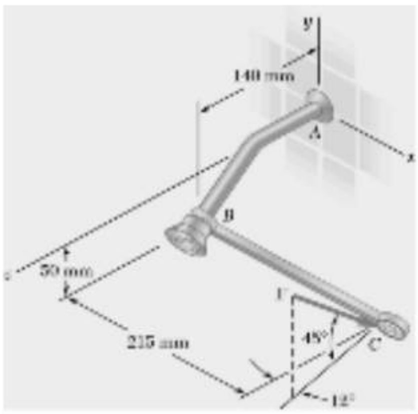
b) 798lb

c) 65 lb

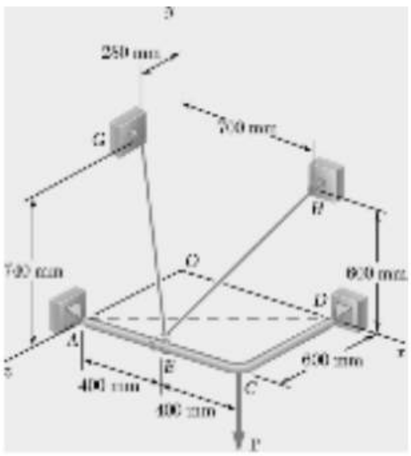
d) 30lb



14. A 36-N force is applied to a wrench to tighten a showerhead. Knowing that the centerline of the wrench is parallel to the x axis. Determine the moment of the force about A.



15. The frame ACD is hinged at A and D and is supported by a cable that passes through a ring at B and is attached to hooks at G and H. Knowing that the tension in the cable is 1125 N, determine the moment about the diagonal AD of the force exerted on the frame by portion BG of the cable.



13. **MCQs on Moments and Couples**

1. The moment of a force about a point is a measure of its:
A) Translational tendency
B) Rotational tendency
C) Magnitude only
D) Direction only

2. The S.I. unit of moment of force (torque) is:
A) Newton (N)
B) Joule (J)
C) Newton-meter (Nm)
D) Pascal (Pa)

3. The moment of a force is considered positive if it tends to cause:
A) Linear motion to the right
B) Linear motion to the left
C) Counter-clockwise rotation
D) Clockwise rotation

4. A couple consists of:
A) Two equal and parallel forces acting in the same direction
B) Two unequal and parallel forces acting in opposite directions
C) Two equal and parallel forces acting in opposite directions
D) Two equal forces acting perpendicular to each other

5. The moment of a couple is called:
A) Impulse
B) Torque
C) Force
D) Momentum

6. What is the main characteristic of the moment of a couple?
A) It depends on the position of the pivot point.
B) It is zero regardless of the pivot point.

C) It has the same value about any point in the plane.
D) It depends only on the distance between the forces.

7. Two forces, each of magnitude 10 N, form a couple. If the perpendicular distance between their lines of action is 2 m, what is the moment of the couple?

A) 5 Nm
B) 20 Nm
C) 200 Nm
D) 0.2 Nm

8. To produce maximum moment for a given force, the angle between the force vector and the position vector should be:

A) 0°
B) 45°
C) 90°
D) 180°

9. Which of the following statements about a couple is TRUE?

A) It produces translational motion.
B) It produces only rotational motion.
C) It produces both translational and rotational motion.
D) It produces no motion at all.

10. A force of 20 N is applied at the end of a 0.5 m long wrench. If the force is applied perpendicular to the wrench, what is the moment about the bolt?

A) 10 Nm
B) 40 Nm
C) 0.25 Nm
D) 100 Nm

11. The moment of a force about a point is calculated using the formula:

A) Force × Distance (perpendicular)
B) Force / Distance

C) Mass × Acceleration
D) Force × Velocity

12. If a force is applied directly through the pivot point, its moment about that pivot is:
A) Maximum
B) Minimum
C) Zero
D) Undefined

13. A couple can be balanced by:
A) A single force
B) Another couple of equal and opposite moment
C) A moment in the same direction
D) It cannot be balanced

14. In a seesaw balanced at its center, two children of different weights can balance by:
A) Sitting at equal distances from the pivot
B) The heavier child sitting closer to the pivot
C) The lighter child sitting closer to the pivot
D) Applying forces in the same direction

15. The moment of a force is a:
A) Scalar quantity
B) Vector quantity
C) Dimensionless quantity
D) Tensor quantity

Answer Key & Detailed Explanations

1. B) Rotational tendency

Explanation: The moment of a force (or torque) specifically measures how effective a force is at causing rotation about a pivot point.

2. C) Newton-meter (Nm)

Explanation: Moment = Force × Distance. Since force is in Newtons (N) and distance is in meters (m), the unit is Newton-meter (Nm).

3. C) Counter-clockwise rotation

Explanation: By convention, counter-clockwise moments are typically considered positive, and clockwise moments negative.

4. C) Two equal and parallel forces acting in opposite directions

Explanation: This is the definition of a couple. The forces are equal in magnitude, parallel, but act in opposite directions.

5. B) Torque

Explanation: The moment produced by a couple is indeed a torque, as it causes pure rotation.

6. C) It has the same value about any point in the plane.

Explanation: This is a key property of a couple. The moment (or torque) of a couple is independent of the pivot point. It is simply the product of one force and the perpendicular distance between them.

7. B) 20 Nm

Explanation: Moment of a couple = Force × Perpendicular distance between forces = 10 N × 2 m = 20 Nm.

8. C) 90°

Explanation: The moment is calculated as $M = F \times d \times \sin\theta$. The sine function is maximum ($\sin 90° = 1$) when $\theta = 90°$, meaning the force is applied perpendicular to the lever arm.

9. B) It produces only rotational motion.

Explanation: Since the two forces are equal and opposite, their net force is zero. Therefore, there is no translational acceleration, only a pure rotational effect.

10. A) 10 Nm

Explanation: Moment = Force × Perpendicular distance = 20 N × 0.5 m = 10 Nm.

11. A) Force × Distance (perpendicular)

Explanation: This is the fundamental formula for the magnitude of a moment. The distance must be the perpendicular distance from the pivot to the line of action of the force.

12. C) Zero

Explanation: If the force's line of action passes through the pivot point, the perpendicular distance (d) is zero. Therefore, Moment = F × 0 = 0.

13. B) Another couple of equal and opposite moment

Explanation: A couple produces only a rotational effect. To balance it (create static equilibrium), you need another couple that creates an equal and opposite rotational effect.

14. B) The heavier child sitting closer to the pivot

Explanation: For balance, the moments on each side must be equal ($M_1 = M_2$). Since Moment = Weight × Distance, a heavier weight (W) requires a smaller distance (d) to produce the same moment as a lighter weight at a greater distance.

15. B) Vector quantity

Explanation: Moment has both magnitude and direction (clockwise or counter-clockwise around an axis), making it a vector quantity. Its direction is given by the right-hand rule.

Chapter 5

Reduce the force and couple systems

To reduce a force and couple system, first choose a point to act as the origin, then find the resultant force ($\vec{F}_R$) by summing all force vectors and the resultant couple moment ($\vec{M}_R$) by summing the moments of all forces about the chosen point and adding any existing couples. The system can then be represented by this single resultant force acting at the origin and the single resultant couple moment.

1. Choose a point of reference

- Select an arbitrary point, often labeled 'O', to serve as the reference point for the equivalent force-couple system.

2. Determine the resultant force ($\vec{F}_R$)

- Sum all the force vectors in the system, including any couples that can be represented as a force.

$$F_R = \sum F_i$$

3. Determine the resultant couple moment ($\vec{M}_R$)

- Calculate the moment of each force about the chosen point 'O' by using the cross product:

$$\vec{M}_i = \vec{r}_i \times \vec{F}_i$$

- Sum these individual moments and add any pre-existing couple moments in the system.

$$\vec{M}_R = \sum(\vec{r}_i \times \vec{F}_i) + \sum\vec{M}_{couple,i}$$

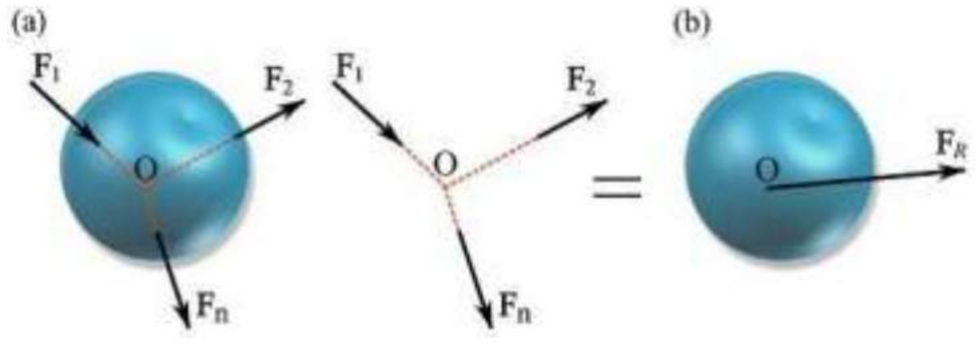
4. Combine into a single equivalent system

- The original system is now equivalent to a single resultant force ($\vec{F}_R$) acting at point 'O' and a single resultant couple moment ($\vec{M}_R$).
- For a further reduction to a single force, the resultant moment's line of action must be perpendicular to the resultant force, which is possible in specific cases like concurrent or coplanar systems.

Although all systems can be reduced to an equivalent system consisting of a force and a couple moment, there are systems that can be further reduced to a single resultant force or a wrench as presented below.

1-Concurrent force systems

In a concurrent force system as shown in the figure , the lines of action of the forces meet (intersect) at a common point O. Therefore, the forces produce no moment about O and the system can be reduced to an equivalent system consisting of one resultant force $\vec{F}_R$ as shown in the following Figure.



2- Coplanar force systems

In a coplanar force system , the lines of action of the forces are in the same plane, say x-y plane. The moments of the forces about any point on the plane are perpendicular to the plane, so are their resultant moment. This system can be initially reduced to a resultant force $\vec{F}_R = \sum \vec{F}_i$ located at any point O and a resultant couple moment, $\vec{M}_R = \sum \vec{r}_i \times \vec{F}_i$ of the forces about point O

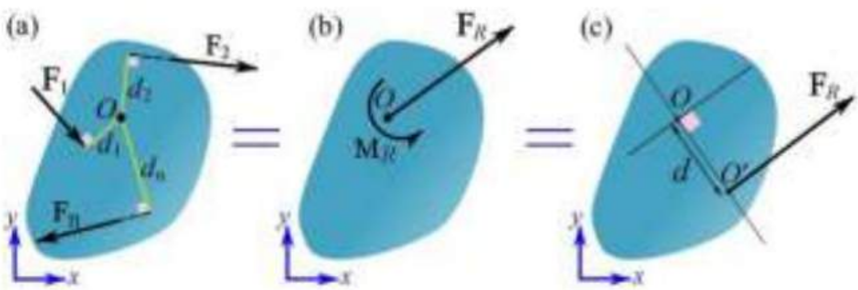
Then, the system can be further simplified to a system that contains only $\vec{F}_R$ as shown in Figure. This is achieved by offsetting in the plane $\vec{F}_R$ by distance d from O. Offsetting the force can replicate the same moment, $\vec{M}_R$, about point O . This leads to a further simplified system.

To find d, the resultant moments of the systems and must be equal. Choosing point O and using the scalar formulation, we write,

$$\underbrace{(M_R)_O}_{\text{System b}} = \underbrace{(M_R)_O}_{\text{System c}} \implies M_R = F_R d$$

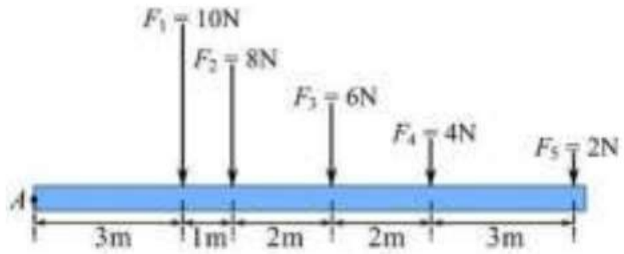
$$\therefore d = \frac{M_R}{F_R}$$

It should be noted that $\vec{F}_R$ should be offset in the correct side of O, in order to replicate the couple moment ($\vec{M}_R$). This means that $\vec{F}_R$ at its final location, O', should create a moment that is equal to $\vec{M}_R$ about O.



Example

Reduce (simplify) the system of coplanar forces to a system consisting of a single resultant force. Measured from point A, determine d, the location of the resultant force.



- Calculate $\vec{F}_R$

$$\vec{F}_R = -10 - 8 - 6 - 4 - 2 = -30 = 30 \downarrow$$

- Make the resultant moments of the original and the simplified systems equal.

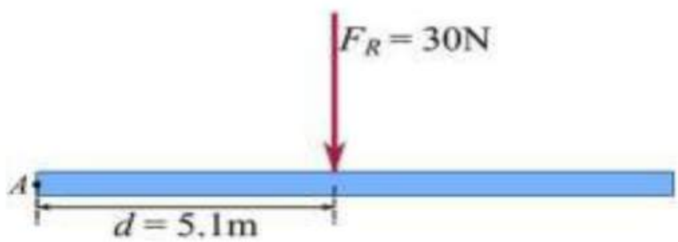
$$\text{Original system } (M_R)_A = \circlearrowleft + \sum Fd = -(10)(3) - (8)(4) - (6)(6) - (4)(8) - (2)(11)$$
$$= -152 \text{ N.m}$$

he negative sign of -152 N.m means the resultant moment is clockwise. Therefore, the downward resultant force should be right to the rotation point to replicate such a moment. Therefore,

$$\text{Original system } (M_R)_A = 152 \text{ N.m} \circlearrowright$$
$$\text{Simplified system } (M_R)_A = \circlearrowleft + F_R d = -(30)(d) \text{ N.m}$$
$$\Longrightarrow F_R d = (30)(d) \text{ N.m} \circlearrowright$$

Letting the original system $(M_R)_A =$ simplified system $(M_R)_A$leads to,

$$d = \frac{152 \text{ N.m}}{30 \text{ N}} = 5.1 \text{ m}$$



3- Parallel force systems (non-coplanar)

A parallel force system contains forces that are parallel to each other, say along the z axis. Parallel forces can slide to a common perpendicular plane like the *x-y* plane. A system of parallel forces can be initially simplified to a system consisting of a resultant force $\vec{F}_R = \sum \vec{F}_i$ applied at a point O in the *x-y* plane, and a resultant couple moment

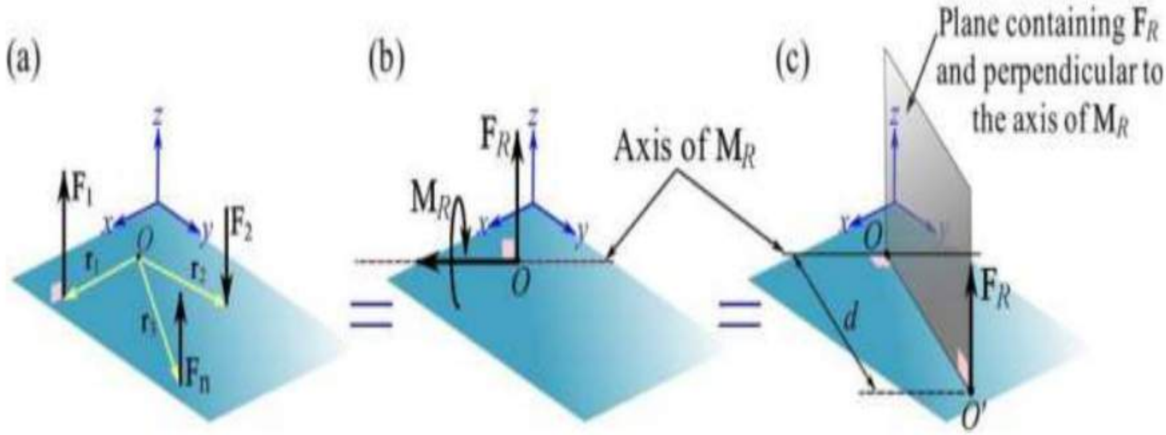
$\vec{M}_R = \sum \vec{r}_i \times \vec{F}_i$.

Since all forces are parallel, $\vec{F}_R$ is parallel to the z axis. Also, all moments of the forces are perpendicular to the axis z and therefore $\vec{M}_R$ is perpendicular to the resultant force.

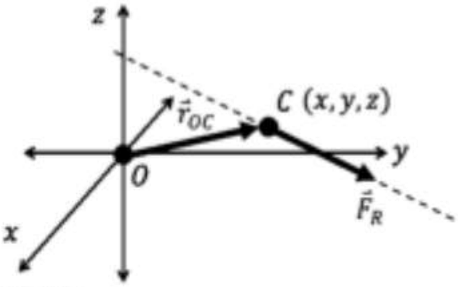
Similar to the further simplification of a coplanar system, where $\vec{F}_R$ and $\vec{M}_R$ are perpendicular, a system of parallel forces can be further reduced to a single resultant force . To achieve this, $\vec{F}_R$ should be offset (to point O') in the plane containing $\vec{F}_R$ and perpendicular to the axis of $\vec{M}_R$. The offset distance d from O is determined as

$$d = \frac{M_R}{F_R}$$

where $F_R = |\vec{F}_R|$ and $M_R = |\vec{M}_R|$. It should be noted that the should be offset in the correct side of O in order to replicate $\vec{M}_R$ about O.



Recall that the moment produced by a force is dependant on the location of the force. Once obtaining both a resultant force and moment, we can move the force such that it produces then resultant moment:



In this case, we know the resultant force ($\vec{F}$) and moment ($\vec{M}$) vectors and must solve for an unknown coordinate point:

- This is done using the Cross Product:

$$\vec{r}_{OC} \times \vec{F}_R = \vec{M}_R$$

$$\vec{r}_{OC} = r_C - r_O$$

Example:

$$\vec{F} = \{14i - 5j + 20k\}$$

$$\vec{M} = \{-10i - 40j - 3k\}$$

$$\vec{r} = r_B - r_A$$

$$\boxed{= (x, y, 0)} - (0,0,0) = \{xi + yj + 0k\}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ x & y & 0 \\ 14 & -5 & 20 \end{vmatrix} = \{(20y)i + (-20x)j + (-5x - 14y)k\}$$

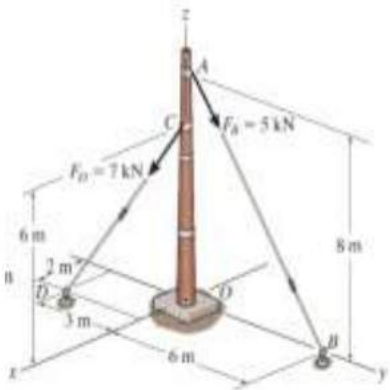
$$= -10 \qquad = -40 \qquad = -3$$

$$x = 2$$

$$y = -0.5$$

Solved Examples

1. Replace the two forces acting on the post by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



Sol.:

$$\mathbf{F}_B = F_B\mathbf{u}_{AB} = 5\left[\frac{(0-0)\mathbf{i} + (6-0)\mathbf{j} + (0-8)\mathbf{k}}{(0-0)^2 + (6-0)^2 + (0-8)^2}\right] = [3\mathbf{j} - 4\mathbf{k}]\text{ kN}$$

$$\mathbf{F}_D = F_D\mathbf{u}_{CD} = 7\left[\frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{(2-0)^2 + (-3-0)^2 + (0-6)^2}\right] = [2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}]$$

The resultant force $\mathbf{F}_R$ is given by

$$\mathbf{F}_R = \Sigma\mathbf{F};\ \mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_D$$

$$= (3\mathbf{j} - 4\mathbf{k}) + (2\mathbf{i} - 3\mathbf{j} - 6\mathbf{k})$$

$$= [2\mathbf{i} - 10\mathbf{k}]\text{ kN}$$

Equivalent Resultant Force: The position vectors $\mathbf{r}_{OB}$ and $\mathbf{r}_{OC}$ are

$$\mathbf{r}_{OB} = [6\mathbf{j}]\text{ m} \qquad \mathbf{r}_{OC} = [6\mathbf{k}]\text{ m}$$

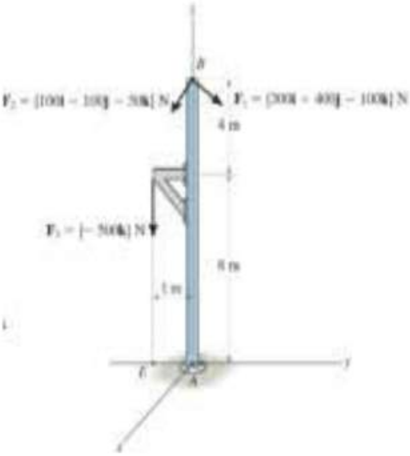
Thus, the resultant couple moment about point O is given by

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O; \qquad (\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_B + \mathbf{r}_{OC} \times \mathbf{F}_D$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 6 & 0 \\ 0 & 3 & -4 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 6 \\ 2 & -3 & -6 \end{vmatrix}$$

$$= [-6\mathbf{i} + 12\mathbf{j}]\text{ kN}\cdot\text{m}$$

2.Replace the force system by an equivalent force and couple moment at point A.



Sol.:

$\mathbf{F}_R = \Sigma\mathbf{F}$; $\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$= (300 + 100)\mathbf{i} + (400 - 100)\mathbf{j} + (-100 - 50 - 500)\mathbf{k}$$

$$= \{400\mathbf{i} + 300\mathbf{j} - 650\mathbf{k}\}\text{ N}$$

The position vectors are $\mathbf{r}_{AB} = \{12\mathbf{k}\}$ m and $\mathbf{r}_{AE} = \{-1\mathbf{j}\}$ m.

$\mathbf{M}_{R_A} = \Sigma\mathbf{M}_A$; $\quad \mathbf{M}_{R_A} = \mathbf{r}_{AB} \times \mathbf{F}_1 + \mathbf{r}_{AB} \times \mathbf{F}_2 + \mathbf{r}_{AE} \times \mathbf{F}_3$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 300 & 400 & -100 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -50 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix}$$

$$= \{-3100\mathbf{i} + 4800\mathbf{j}\}\text{ N}\cdot\text{m}$$

Problems

1. Three coplanar forces act on a rigid body:
   $F_1 = 100$ Nacting along the x-axis at point (0, 2 m)
   $F_2 = 80$ Nacting vertically upward at point (3, 0)
   $F_3 = 60$ Nacting at 45° below the x-axis at point (2, 3)
   a)Determine the equivalent single resultant force (magnitude and direction).
   b)Find the moment (couple) of the system about the origin.
   c)Represent the reduced equivalent system at the origin**.**

2. A body in the xy-plane is subjected to:
   $F_1 = 60$ N at 30° above the x-axis applied at (1, 0)
   $F_2 = 40$ N horizontally to the left applied at (0, 2)
   A couple moment $M = 50$ N\cdotpm (clockwise)
   Reduce this to a single resultant force and a single moment at the origin.

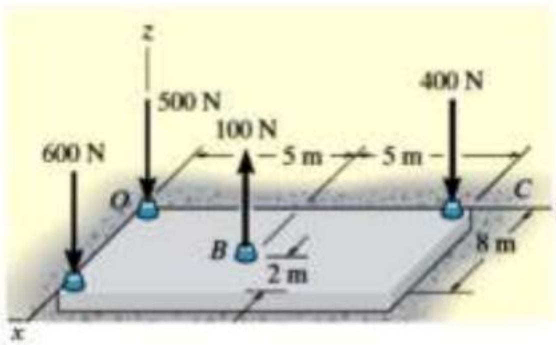
3. Two equal and opposite forces of magnitude 100 N act parallel to the
   x-axis: One at (0, 3), the other at (4, 0).

Find the magnitude and direction of the couple formed.
Show that the couple is independent of reference point.

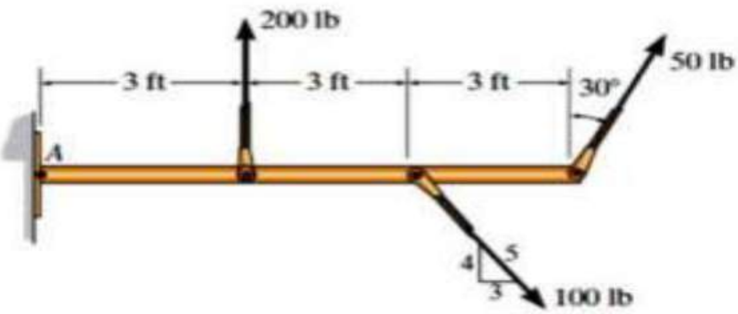
4. A rectangular plate (2 m × 3 m) has forces:

$F_1 = 120$ Nupward at the midpoint of the left edge
$F_2 = 150$ Ndownward at the midpoint of the right edge
A couple moment $M = 60$ N\cdotpm (counterclockwise)
Compute the resultant force.
Find the net moment about the bottom-left corner.
Represent the equivalent single force-couple system.

5. Replace the loading shown by an equivalent single resultant force and specify the x and y coordinates of its line of action.



6. Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the member measured from A .



MCQs questions

1. The process of replacing a given system of forces by a single force and a couple acting at a point is called:
A. Equilibrium of forces

B. Resolution of forces
C. Reduction of a force and couple system
D. Translation of forces

2. When several forces act on a rigid body, they can always be reduced to:
A. One force only
B. One couple only
C. A single resultant force and a single resultant couple
D. Two perpendicular forces

3. In reducing a force system to a single equivalent at the origin, the total moment about the origin is:
A. Zero
B. Equal to the algebraic sum of all forces
C. Sum of all moments and the moments of all forces about the origin
D. None of the above

4. Two equal and opposite parallel forces separated by a distance d form:
A. A resultant force of 2F
B. A couple of moment $F \times 2d$
C. A couple of moment $F \times d$
D. A resultant force of zero and no moment

5. A couple has:
A. Only moment, no resultant force
B. Only force, no moment
C. Both force and moment
D. Neither force nor moment

6. The equivalent system of a non-concurrent, non-parallel set of forces is generally:
A. One resultant force only

B. One couple only
C. A force and a couple at a chosen point
D. Two intersecting forces

7. The condition for a system to be equivalent to a single resultant force is:
A. $\sum\vec{F} = 0$
B. $\sum\vec{M}_O = 0$
C. $\sum\vec{F} \neq 0$
D. $\sum\vec{M}_O \neq 0$

8. If several forces act in space, the equivalent system at a point Oconsists of:

A. A resultant force only
B. A resultant force and a resultant couple
C. Two perpendicular forces
D. Equal and opposite couples

9. When a system of forces is reduced to an equivalent force-couple system at a point O, the couple moment is:

A. The same everywhere
B. Equal to the sum of all couples only
C. Sum of all couples and the moments of all forces about O
D. Always zero

10. If the resultant force is zero but the resultant couple is not zero, then the system is equivalent to:

A. A single force
B. A pure couple
C. A concurrent system
D. A parallel system

11. If a system of non-parallel forces lies in one plane, the reduced equivalent system will be:

A. Two perpendicular forces
B. A single force and a single couple in the same plane
C. One couple only
D. A resultant perpendicular to the plane

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