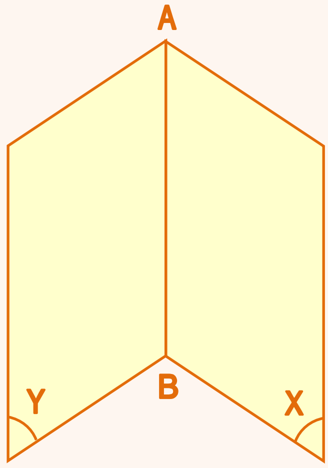
Chapter -6-

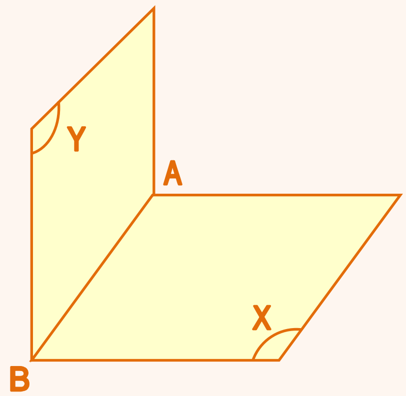
Space Geometry

Dihedral Angles & Perpendicular Planes :

If (X) is a half plane and (Y) is a half plane, then they meet in a common line $\overleftrightarrow{AB}$. So $(X)\cup(Y)$ is called a (dihedral angle) and the intersect line $\overleftrightarrow{AB}$ is called its (edge)

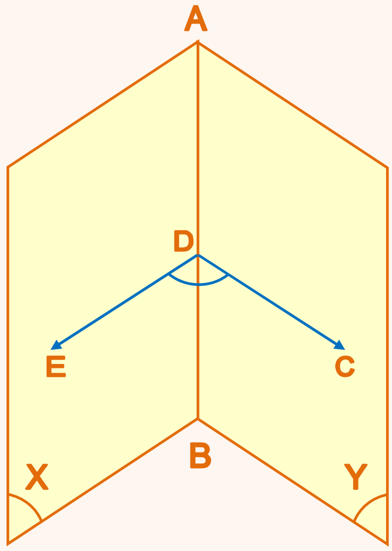
A dihedral angle $(X)-\overleftrightarrow{AB}-(Y)$ is a figure formed by two half planes meeting in a common line. The common line is called the edge and half planes are called the faces of dihedral angle.





The Plane Angle of A Dihedral Angle :

is an angle whose sides are perpendicular to the edge of the dihedral angle from a point which belong to it and each of them lie in one of the two faces of the dihedral angle.



1) $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$
2) $\overleftrightarrow{ED} \perp \overleftrightarrow{AB}$
3) $\overleftrightarrow{CD} \subset (X)$
4) $\overleftrightarrow{ED} \subset (Y)$

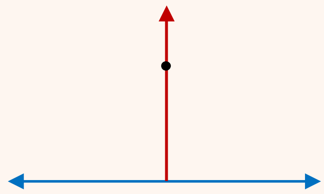
$\sphericalangle$ CDE is a Plane Angle of dihedral angle $(X) - \overleftrightarrow{AB} - (Y)$

Notes :

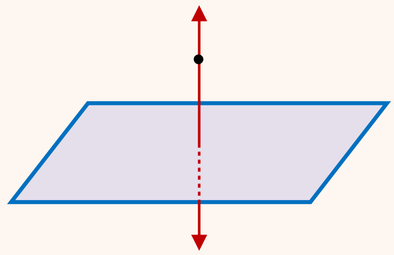
- *Measure of a dihedral angle is equal to measure of its plane angle and vice versa.*
- *Two planes are orthogonal to each other if they form a right dihedral angle and vice versa.*

Postulates

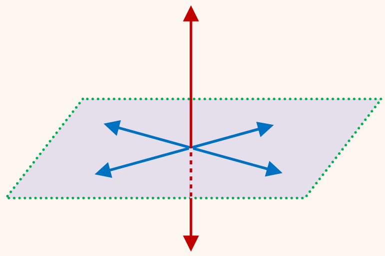
1. In a plane , there exists a unique line perpendicular on given line through the given point



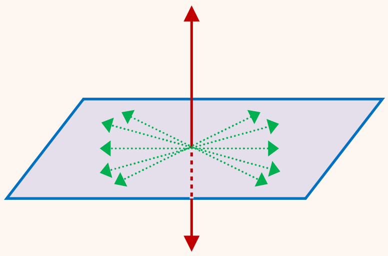
2. There exists a unique line perpendicular on given plane through the given point



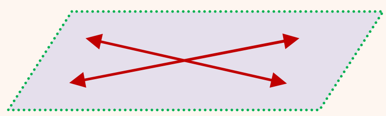
3. A perpendicular line on two intersect lines at intersect point is perpendicular on their plane



4. A perpendicular line on a plane is perpendicular on all lines drawn from its trace within that plane



5. For each two intersect lines, there is one unique plane containing them



Theorem (7):

If two planes are orthogonal, the line drawn in one of these planes perpendicular to the intercept line of two planes will be perpendicular to the other plane too.

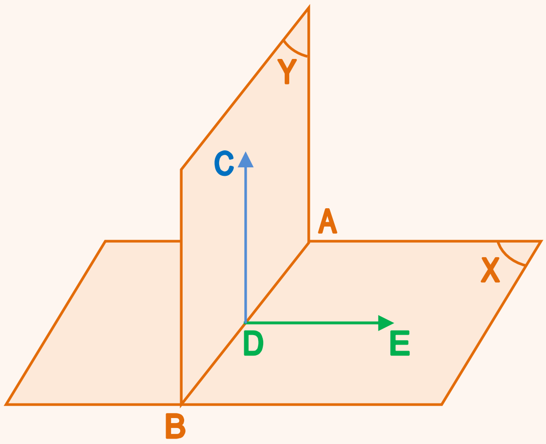
Given :

$(Y) \perp (X)$,

$(Y) \cap (X) = \overleftrightarrow{AB}$

$\overleftrightarrow{CD} \subset (Y)$

$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point D



Required :

$\overleftrightarrow{CD} \perp (X)$

Proof :

In (X) draw $\overleftrightarrow{DE} \perp \overleftrightarrow{AB}$ (In a plane , there exists a unique line perpendicular on given line through the given point)

$\overleftrightarrow{CD} \subset (Y)$, $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$ (Given)

$\therefore \sphericalangle CDE$ is a plane angle of dihedral angle $(X) - \overleftrightarrow{AB} - (Y)$ (Definition of plane angle)

$\therefore$ m $\sphericalangle CDE = 90^\circ$ (Measure of a dihedral angle is equal to measure of its plane angle and vice versa)

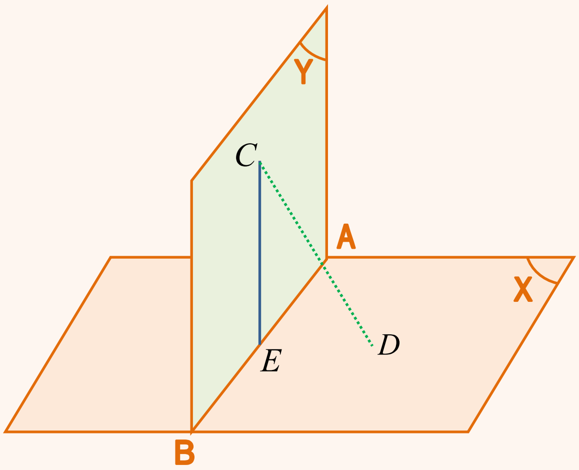
$\overleftrightarrow{CD} \perp \overleftrightarrow{DE}$ (If a measure between two lines 90°, then the lines are orthogonal)

$\overleftrightarrow{CD} \perp (X)$ (A perpendicular line on two intersect lines at intersect point is perpendicular on their plane)

(Q.E.D)

Conclusion of Theorem (7) :

If two planes are orthogonal, and a line is drawn from any point in first plane perpendicular to the other plane, then the first plane must contain that line.



Given : $(Y) \perp (X)$, $(X) \cap (Y) = \overleftrightarrow{AB}$, $C \in (Y)$, $\overleftrightarrow{CD} \perp (X)$

Required : $\overleftrightarrow{CD} \subset (Y)$

Proof :

In (Y) draw $\overleftrightarrow{CE} \perp \overleftrightarrow{AB}$ (In a plane , there exists a unique line perpendicular on given line through the given point)

$\because (Y) \perp (X)$ (Given)

$\therefore \overleftrightarrow{CE} \perp (X)$ (Theorem 7)

$\overleftrightarrow{CD} \perp (X)$ (Given)

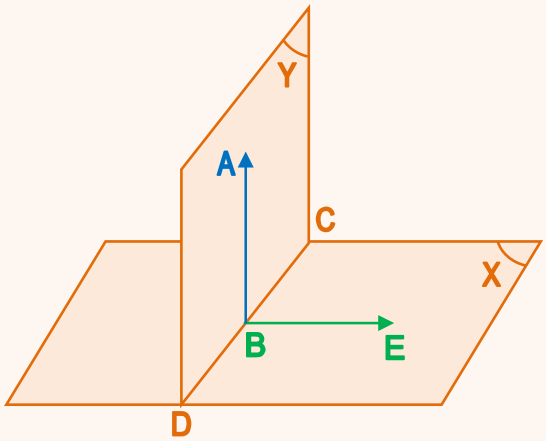
$\therefore \overleftrightarrow{CE} = \overleftrightarrow{CD}$ (There exists a unique line perpendicular on given plane through the given point)

$\therefore \overleftrightarrow{CD} \subset (Y)$

(Q.E.D)

Theorem (8): If a line is perpendicular to a plane, then any plane containing that line is perpendicular to the given plane

Two planes are orthogonal if one plane has a line perpendicular on the other



Given : $\overleftrightarrow{AB} \perp (X)$, $\overleftrightarrow{AB} \subset (Y)$

$(X) \cap (Y) = \overleftrightarrow{CD}$

Required : $(Y) \perp (X)$

Proof :

$B \in \overleftrightarrow{CD}$ (The intersection plane contains the common points)

In (X) draw $\overleftrightarrow{BE} \perp \overleftrightarrow{CD}$ (In a plane , there exists a unique line perpendicular on given line through the given point)

$\because \overleftrightarrow{AB} \perp (X)$ (Given)

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{CD} , \overleftrightarrow{BE}$ (A perpendicular line on a plane is perpendicular on all lines drawn from its trace within that plane)

$\because \overleftrightarrow{AB} \subset (Y)$ (Given)

$\therefore \sphericalangle ABE$ is a plane angle of dihedral angle $(Y) - \overleftrightarrow{CD} - (X)$

$\therefore m \sphericalangle ABE = 90^\circ$ ($\overleftrightarrow{AB} \perp \overleftrightarrow{BE}$)

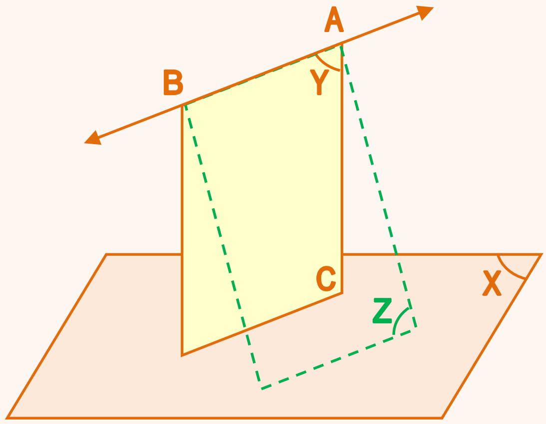
$\Rightarrow$ Measure of the dihedral angle $(Y) - \overleftrightarrow{CD} - (X) = 90^\circ$
(Measure of the dihedral angle is equal to measure of its plane angle)

$\therefore (Y) \perp (X)$ (If the measurement of the dihedral angle = 90° , then the two planes are orthogonal and vice versa)

(Q.E.D)

Theorem (9):

Through a given external line not perpendicular to a given plane there is only one unique plane perpendicular to the given plane.



Given : $\overleftrightarrow{AB}$ is not perpendicular on (X)

Required :

Find a unique plane that contains $\overleftrightarrow{AB}$ and perpendicular on (X)

Proof :

from the point (A) draw $\overleftrightarrow{AC} \perp$ (X) (There exists a unique line perpendicular on given plane through the given point)

$\because$ $\overleftrightarrow{AB}$ intersect $\overleftrightarrow{AC}$

$\exists$ unique plane like (Y) containing them (For each two intersect lines, there is a unique plane containing them)

$\because$ (Y) $\perp$ (X) (Theorem 8)

To prove uniqueness :

Let (Z) another plane contains $\overleftrightarrow{AB}$ and perpendicular on (X)

$\because$ $\overleftrightarrow{AC} \perp$ (X) (By proof)

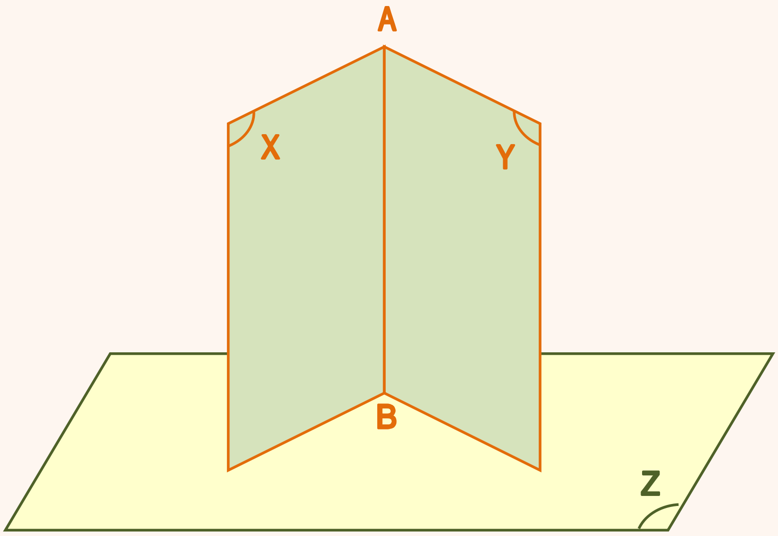
$\therefore$ $\overleftrightarrow{AC} \subset$ (Z) (Conclusion of Theorem 7)

$\therefore$ (Y) = (Z) (For each two intersect lines, there is a unique plane containing them)

(Q.E.D)

Conclusion of Theorem (9) :

If two intersect planes are each perpendicular to a third plane, then intersect line is perpendicular on the third plane



Given :

$$(X) \cap (Y) = \overleftrightarrow{AB}$$
$$(X)\ ,\ (Y) \perp (Z)$$

Required :

$$\overleftrightarrow{AB} \perp (Z)$$

Proof :

If $\overleftrightarrow{AB}$ not perpendicular on (Z) then, there is only one unique plane contains $\overleftrightarrow{AB}$ and perpendicular on (Z) (Theorem 9)

$$\therefore\ \overleftrightarrow{AB} \perp (Z)$$

(Q.E.D)

Exercises (6 -1)

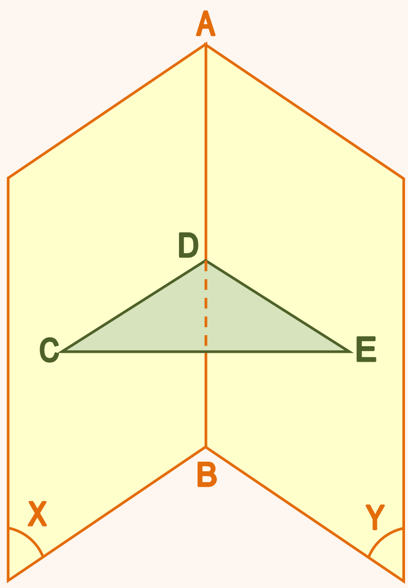
Q1) *Prove that plane of plane angle which belongs to dihedral angle is perpendicular on its edge*

Given :

$\measuredangle CDE$ *is plane angle of dihedral angle* $(X)-\overleftrightarrow{AB}-(Y)$

Required :

$(CDE) \perp \overleftrightarrow{AB}$



Proof :

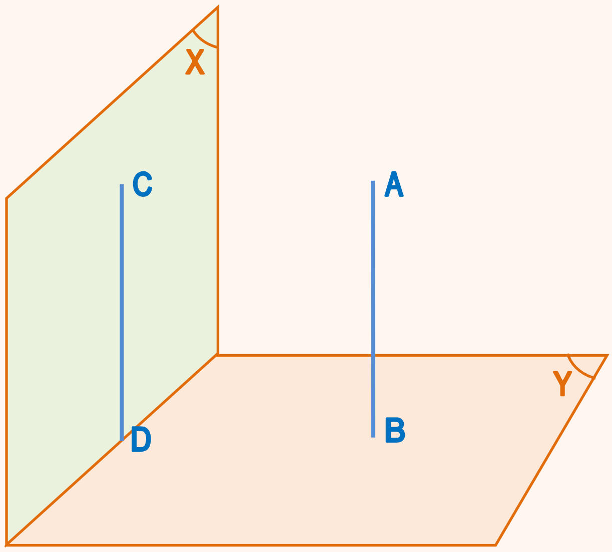
$$\left.\begin{array}{l} \overleftrightarrow{CD} \perp \overleftrightarrow{AB} \\ \overleftrightarrow{ED} \perp \overleftrightarrow{AB} \end{array}\right\}$$ *(Definition of plane angle)*

$\therefore\ (CDE) \perp \overleftrightarrow{AB}$ *(A perpendicular line on two intersect lines at intersect point is perpendicular on their plane)*

(Q.E.D)

Q2) ***Prove that ; if line is parallel to a plane and perpendicular to other plane then the two planes are orthogonal***

Given :



$\overleftrightarrow{AB} \parallel (X)$

$\overleftrightarrow{AB} \perp (Y)$

Required :

$(X) \perp (Y)$

Proof :

Let $C \in (X)$

Draw $\overleftrightarrow{CD} \perp (Y)$ *(There exists a unique line perpendicular on given plane through the given point)*

$\because \overleftrightarrow{AB} \perp (Y)$ *(Given)*

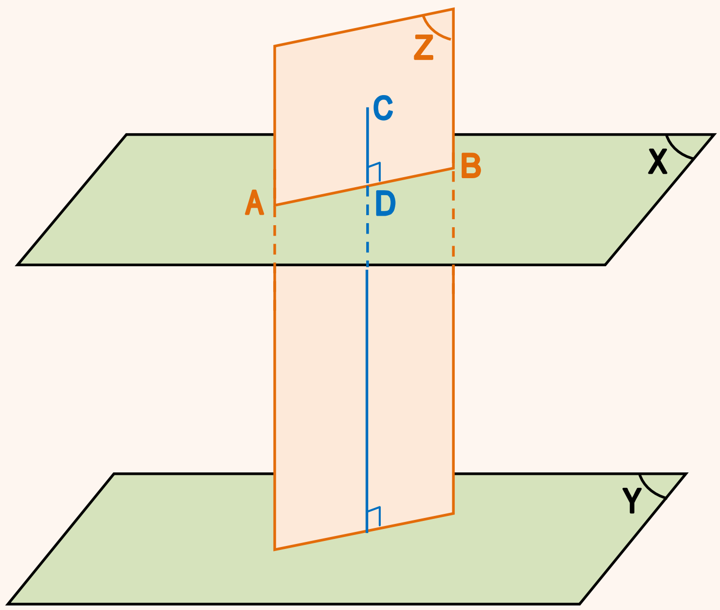
$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ *(If two lines are perpendicular to the same plane, they are parallel to each other)*

$\because C \in (X) \Rightarrow \overleftrightarrow{CD} \subset (X)$ *(If line parallel to a plane then the line which draw from point on that plane parallel to the other line will be subset on the plane)*

$\therefore (X) \perp (Y)$ *(Theorem 8)*

(Q.E.D)

Q3) Prove that if a plane perpendicular on one of two parallel planes is also perpendicular on the other



Given :

$(X)//(Y)$

$(Z)\perp(X)$

$(Z)\cap(X)=\overleftrightarrow{AB}$

Required :

$(Z)\perp(Y)$

Proof :

Let $C\in(Z)$ we draw $\overleftrightarrow{CD}\perp\overleftrightarrow{AB}$ in (Z) (In a plane , there exists a unique line perpendicular on given line through the given point)

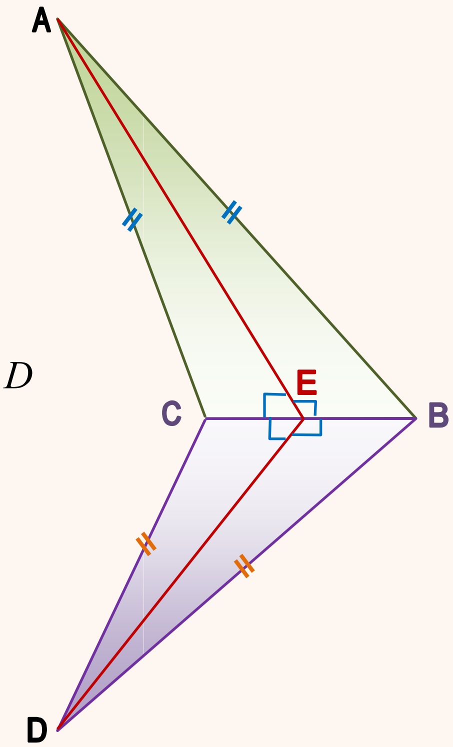
$\overleftrightarrow{CD}\perp(X)$ since $(Z)\perp(X)$ (Theorem 7)

$\overleftrightarrow{CD}\perp(Y)$ since $(X)//(Y)$ (If line is perpendicular to one of two planes then it is perpendicular to the other)

$\therefore(Z)\perp(Y)$ (Theorem 8)

(Q.E.D)

Q4) A, B, C, D four points not in same plane where $AB = AC$, $E \in \overleftrightarrow{BC}$.
If $\sphericalangle\ AED$ is plane angle of dihedral angle $A - \overline{BC} - D$. Prove that $CD = BD$



Given :

A, B, C, D four points not in same plane

$AB = AC$, $E \in \overleftrightarrow{BC}$

$\sphericalangle\ AED$ is plane angle of dihedral angle $A - \overline{BC} - D$

Required :

$CD = BD$

Proof :

in $\Delta\ ABC$ $\quad AB = AC$ (Given)

$\because \overline{AE} \perp \overline{BC}$ (Definition of plane angle)

$CE = EB$ (The perpendicular line drown from vertex of isosceles triangle divides the base into half)

In $\Delta\ CED, BED$:

$\overline{DE}$ common

$CE = EB$ (proved)

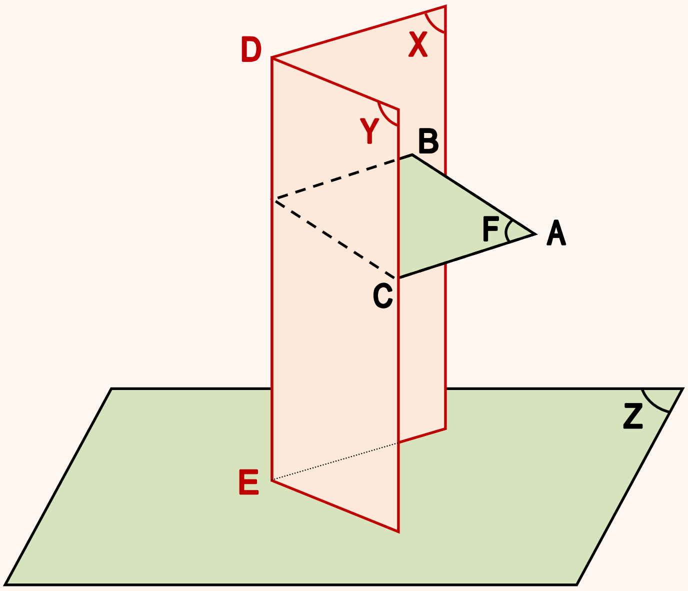
$m\sphericalangle\ BED = m\sphericalangle\ CED = 90^\circ$ (Definition of plane angle)

$\therefore\ \Delta CED \equiv \Delta BED$

From the congruence we conclude that $CD = BD$

(Q.E.D)

Q5) *If two intersecting lines are parallel on a given plane and they are each perpendicular on two intersecting planes, then the line of intersection of the two intersecting planes is perpendicular to the given plane*



Given :

$\overleftrightarrow{AB}, \overleftrightarrow{AC} \mathbin{//} (Z)$

$\overleftrightarrow{AB} \perp (X)$, $\overleftrightarrow{AC} \perp (Y)$

$(X) \cap (Y) = \overleftrightarrow{DE}$

Required :

$\overleftrightarrow{DE} \perp (Z)$

Proof :

Let $\overleftrightarrow{AB}, \overleftrightarrow{AC} \subset (F)$ *(For each two intersect lines, there is a unique plane containing them)*

$\therefore (F) \mathbin{//} (Z)$ *(If plane parallel to two intersecting lines then its parallel to their plane)*

$$\left.\begin{array}{l} \overleftrightarrow{AB} \perp (X) \text{ (given)} \Rightarrow (F) \perp (X) \\ \overleftrightarrow{AC} \perp (Y) \text{ (given)} \Rightarrow (F) \perp (Y) \end{array}\right\} \text{(Theorem 8)}$$

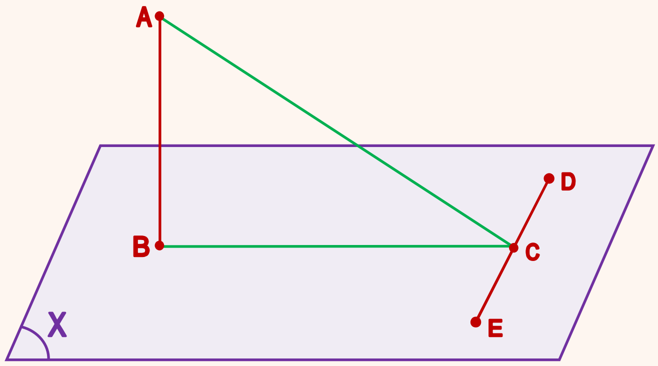
$\overleftrightarrow{DE} \perp (F)$ *(Conclusion of theorem 9)*

$\overleftrightarrow{DE} \perp (Z)$ *(A line that perpendicular on one of two parallel planes is also perpendicular to the other)*

(Q.E.D)

Theorem of Three Perpendiculars

If AB is perpendicular to a plane (X) and if from B, the foot of the perpendicular, a straight line BC is drawn perpendicular to any straight line ED in the plane, then AC is also perpendicular to ED



$\overline{AB} \perp (X)$ on B , $\overline{ED} \subset (X)$

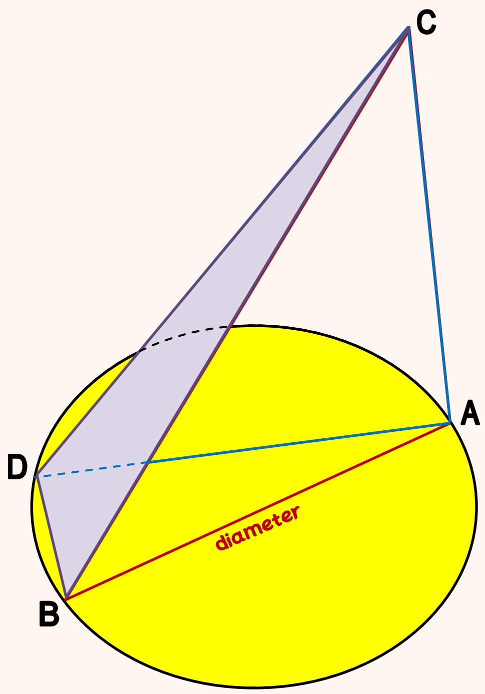
$\overline{BC} \perp \overline{ED} \quad \rightarrow \quad \overline{AC} \perp \overline{ED}$

$\overline{AB} \perp (X)$ on B , $\overline{ED} \subset (X)$

$\overline{AC} \perp \overline{ED} \quad \rightarrow \quad \overline{BC} \perp \overline{ED}$

Q6) A circle with diameter $\overline{AB}$, $\overline{AC}$ perpendicular on its plane , D is point on the circle ; prove that (CDA) is perpendicular on (CDB)

Given :



$\overline{AB}$ is diameter of circle

$\overline{AC}$ is perpendicular to the circle

D is a point belongs to the circle

Required :

$(CDA) \perp (CDB)$

Proof :

$\because$ $\overline{AB}$ is diameter of circle (Given)

$\therefore m \sphericalangle ADB = 90^o$ (The inscribed angle opposite a semicircle is a right angle)

$\because \overline{AC} \perp (ADB)$ (Given)

$\overline{AD} \perp \overline{DB}$ (By proof)

$\therefore \overline{CD} \perp \overline{DB}$ (Theorem of Three Perpendiculars)

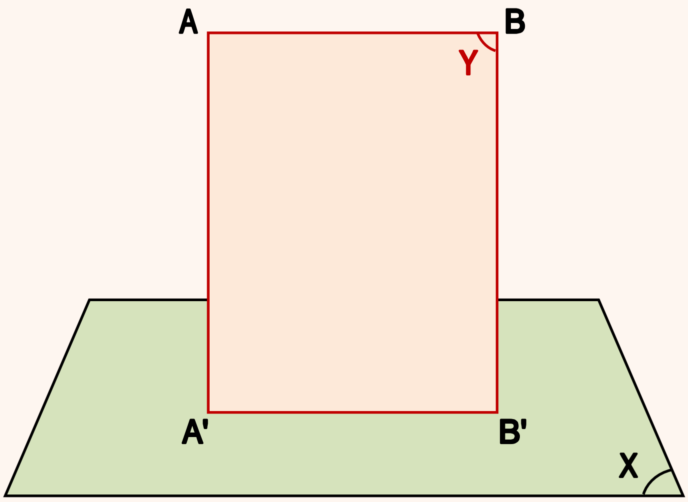
$\therefore \overline{DB} \perp (CDA)$ (A perpendicular line on two intersect lines at intersect point is perpendicular on their plane)

$\therefore (CDA) \perp (CDB)$ (Theorem 8)

(Q.E.D)

Exercises (6 -2)

Q1) *Prove that the length of line segment parallel to known plane equals to length of its projection on known plane and parallel to it*



Given :

$\overline{A'B'}$ *is projection of* $\overline{AB}$ *on* (X)

$\overline{AB} \,//\, (X)$

Required :

$\overline{AB} \,//\, \overline{A'B'}$, $AB = A'B'$

Proof :

$\because$ $\overline{AA'}$ *and* $\overline{BB'}$ *are perpendicular on* (X) *(definition of projection)*

$\therefore$ $\overline{AA'} \,//\, \overline{BB'}$ *(Two line perpendicular to same plane are parallel)*

plot the plane (Y) *by the two parallel lines* $\overleftrightarrow{AA'}$, $\overleftrightarrow{BB'}$

(for each two parallel lines, there is one unique plane containing them)

$\because$ $\overline{AB} \,//\, (X)$ *(Given)*

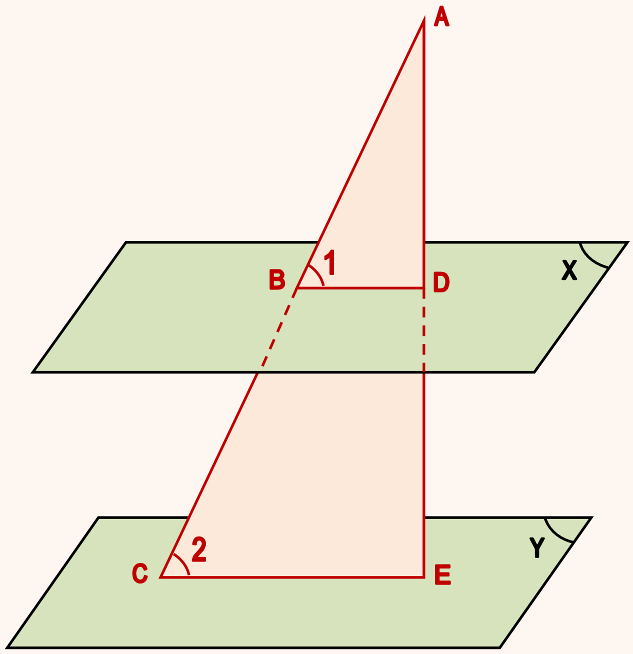
$\therefore$ $\overline{AB} \,//\, \overline{A'B'}$ *(If a line parallel to a plane then it is parallel to all lines resulting from intersect of this plane and planes in that line)*

$\therefore$ $ABB'A'$ *is a parallelogram* *(opposite sides are parallel)*

$\therefore AB = A'B'$ *(Opposite sides are equal in parallelogram)*

(Q.E.D)

Q2) ***Prove that; the line that intersect two parallel planes, its inclination on one of them equal to its inclination on the other***



Given :

$(X) \parallel (Y)$

$\overleftrightarrow{AC}$ intersect (X) in B and (Y) in C

$\sphericalangle 1$ incline angle of $\overleftrightarrow{AB}$ on (X)

$\sphericalangle 2$ incline angle of $\overleftrightarrow{AC}$ on (Y)

Required :

$m\sphericalangle 1 = m\sphericalangle 2$

Proof :

Draw $\overleftrightarrow{AD} \perp (X)$ (There exists a unique line perpendicular on given plane through the given point)

$\therefore \overleftrightarrow{AD} \perp (Y)$ in E (If line is perpendicular to one of two parallel planes then it is perpendicular to the other plane also)

$\therefore \overline{DB}$ is a projection of $\overline{AB}$ on (X)
$\therefore \overline{EC}$ is a projection of $\overline{AC}$ on (Y)
(Definition of projection)

$m\sphericalangle 1 = m\sphericalangle 2$ (Corresponding angles)

(Q.E.D)

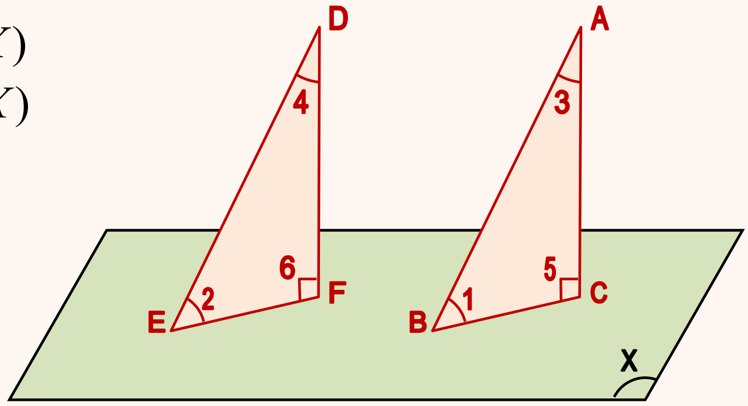
Q3) Prove that ; the parallel lines which intersects a plane have the same inclination on the plane

Given :

$\overline{AB} \parallel \overline{DE}$

$\sphericalangle 1$ incline angle of $\overline{AB}$ on (X)

$\sphericalangle 2$ incline angle of $\overline{DE}$ on (X)



Required :

$m\sphericalangle 1 = m\sphericalangle 2$

Proof :

$\left.\begin{array}{l}\overline{BC} \text{ is a projection of } \overline{AB} \text{ on } (X) \\ \overline{EF} \text{ is a projection of } \overline{DE} \text{ on } (X)\end{array}\right\}$ (Definition of projection)

$\because\ \overline{AC} \perp (X)\ ,\ \overline{DF} \perp (X)$ (By proof)

$\therefore\ \overline{AC} \perp \overline{BC}\ ,\ \overline{DF} \perp \overline{EF}$ (A line that perpendicular on a plane is perpendicular on all lines contained in that plane and passing through it)

$\therefore\ m\sphericalangle 5 = m\sphericalangle 6$ (Right angles)

$\overline{AB} \parallel \overline{DE}$ (Given)

$\overline{AC} \parallel \overline{DF}$ (If two lines perpendicular to the same plane then they are parallel to each other)

$\therefore\ m\sphericalangle 3 = m\sphericalangle 4$ (If two sides of an angle parallel to two sides of another angle then their measurement are equal)

$\therefore\ m\sphericalangle 1 = m\sphericalangle 2$ (The sum of interior angles of a triangle is 180°)

(Q.E.D)

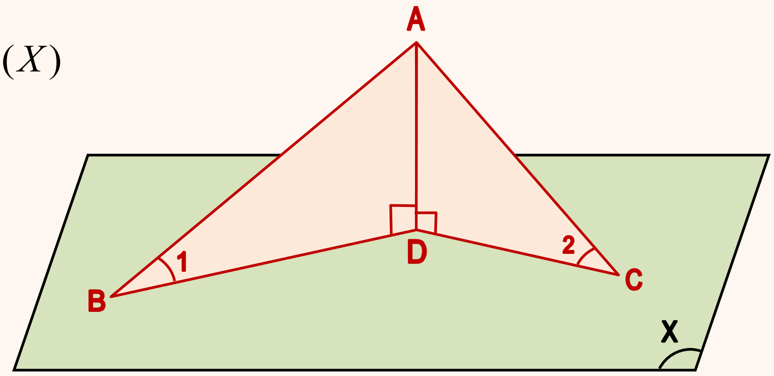
Q4) **Prove that ; if two inclines, different in length are drawn from a point not belonging to a given plane, the longest has smaller angle of inclination than angle of inclination of the other**

Given :

$\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ **two inclined lines on** (X) **,** $AB > AC$

$\sphericalangle$ **1 incline angle of** $\overleftrightarrow{AB}$ **on** (X)

$\sphericalangle$ **2 incline angle of** $\overleftrightarrow{AC}$ **on** (X)



Required :

m $\sphericalangle$ **1** $<$ **m** $\sphericalangle$ **2**

Proof :

Draw $\overline{AD} \perp (X)$ **(There exists a unique line perpendicular on given plane through the given point)**

$\overline{BD}$ *is a projection of* $\overline{AB}$ *on* (X)
$\overline{CD}$ *is a projection of* $\overline{AC}$ *on* (X) } **(Definition of projection)**

$\because\ AB > AC$ **(Given)**

$\frac{1}{AB} < \frac{1}{AC}$ **(Properties of inequality)**

$\frac{AD}{AB} < \frac{AD}{AC}$

$\therefore\ \sin \sphericalangle 1 < \sin \sphericalangle 2$

$\therefore$ **m** $\sphericalangle$ **1** $<$ **m** $\sphericalangle$ **2**

(Q.E.D)

Q5) ***Prove that ; if two inclines are drawn from some point to a plane, the one with smaller inclination is the longest***

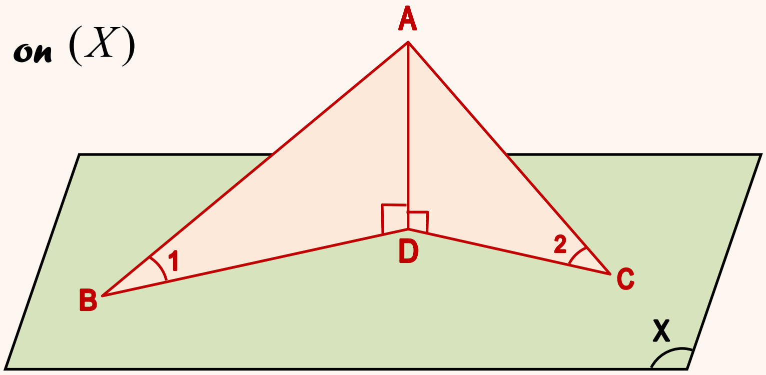
Given :

$\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ **two inclined lines on** (X)

$\sphericalangle$ **1 is the angle of incline** $\overleftrightarrow{AB}$ **on** (X)

$\sphericalangle$ **2 is the angle of incline** $\overleftrightarrow{AC}$ **on** (X)

$m\sphericalangle 1 < m\sphericalangle 2$



Required : $AB > AC$

Proof :

Draw $\overleftrightarrow{AD} \perp (X)$ ***(There exists a unique line perpendicular on given plane through given point)***

$\left.\begin{array}{l} \overline{BD} \text{ is a projection of } \overline{AB} \text{ on } (X) \\ \overline{CD} \text{ is a projection of } \overline{AC} \text{ on } (X) \end{array}\right\}$ ***(definition of projection)***

$\overleftrightarrow{AD} \perp \overline{BD}, \overline{CD}$ ***(A line that perpendicular on a plane is perpendicular on all lines contained in that plane and passing through it)***

$\because m\sphericalangle 1 < m\sphericalangle 2$ ***(Given)***

$\therefore \sin\sphericalangle 1 < \sin\sphericalangle 2$

$\frac{AD}{AB} < \frac{AD}{AC} \rightarrow \frac{1}{AB} < \frac{1}{AC} \rightarrow AB > AC$ ***(Properties of inequality)***

(Q.E.D)

Q6) ***Prove that angle of inclination between line and its projection on a plane is smaller than the angle bounded by the line itself and any other line drawn from position within that plane***

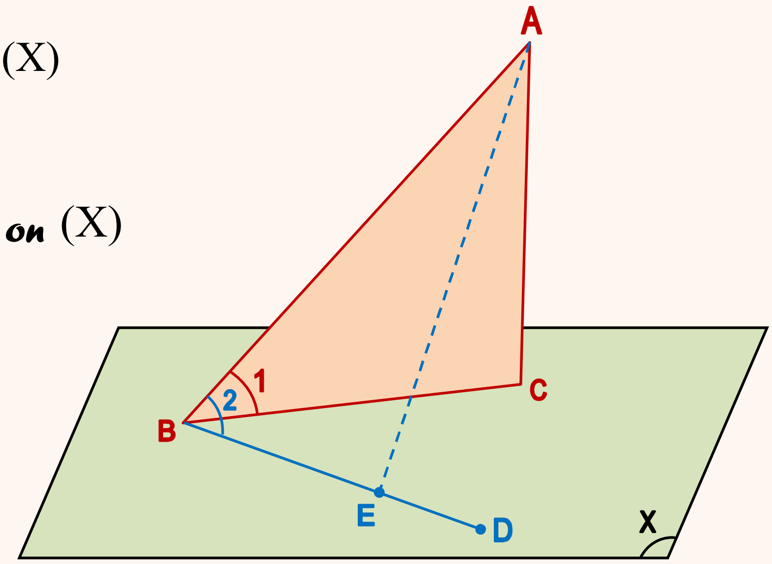
Given :

$\overline{BC}$ is a projection of $\overline{AB}$ on (X)

$\overleftrightarrow{BD} \subset (X)$

$\measuredangle 1$ *is the angle of incline* $\overleftrightarrow{AB}$ *on* (X)

$\measuredangle 2 = \measuredangle ABD$



Required :

$m\measuredangle 1 < m\measuredangle 2$

Proof :

Let $E \in \overleftrightarrow{BD}$ *such that* $BC = BE$

Draw $\overline{AE}$

$\because \overline{AC} \perp (X)$ *(Definition of projection)*

$\therefore AC < AE$ *(The perpendicular line is the shortest distance between the point and plane)*

$BC = BE$ *(By proof)*

$AB = AB$ *(Common)*

$\therefore m\measuredangle 1 < m\measuredangle 2$ *(If two sides of triangle is equal to two sides of another triangle and the third side is different , then the smallest one corresponds the smallest angle)*

(Q.E.D)

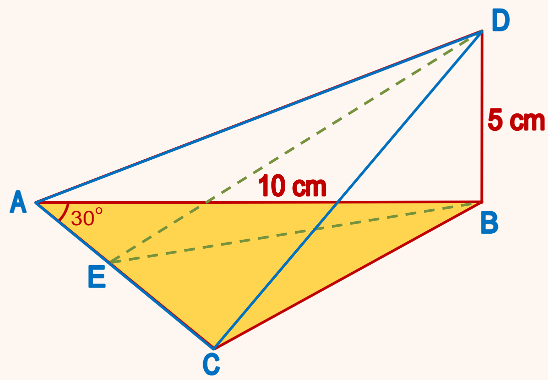
Ex (1): In ΔABC : $\overline{BD} \perp (ABC)$, m$\sphericalangle A$ $=30°$, $AB = 10cm$, $BD = 5cm$

Find measurement of dihedral angle $(D) - \overline{AC} - (B)$

Given : $\overline{BD} \perp (ABC)$, m$\sphericalangle A$ $=30°$, $AB = 10cm$, $BD = 5cm$

Required : Find measurement of dihedral angle $(D) - \overline{AC} - (B)$

Proof :



In plane (ABC) : draw $\overline{BE} \perp \overline{AC}$ at E

(In a plane , there is only one line perpendicular on another line at given point)

$\because \overline{BD} \perp (ABC)$ (Given)

$\therefore \overline{DE} \perp \overline{AC}$ (Three perpendicular line theorem)

$\sphericalangle DEB$ Plane angle of dihedral angle $\overline{AC}$ (By definition of plane angle of dihedral angle)

$\overline{DB} \perp \overline{BE}$ (A line that is perpendicular on a plane is perpendicular on all lines contained in that plane and passing through it)

ΔDBE right angle at B

In ΔBEA right angle at E

$$\sin 30^o = \frac{BE}{BA} \Rightarrow \frac{1}{2} = \frac{BE}{10} \Rightarrow BE = 5\,cm$$

In ΔDBE which is right angle at B

$$\tan(BED) = \frac{BD}{BE} = \frac{5}{5} = 1$$

m$\sphericalangle BED = 45^o$

m$\sphericalangle (D) - \overleftrightarrow{AC} - (B) = 45^o$ (Measure of a dihedral angle is equal to measure of its plane angle and vice versa)

(Q.E.D)

Ex (2): Let ABC a triangle such that :

$\overline{AF} \perp (ABC)$

$\overline{BD} \perp \overline{CF}$

$\overline{BE} \perp \overline{CA}$

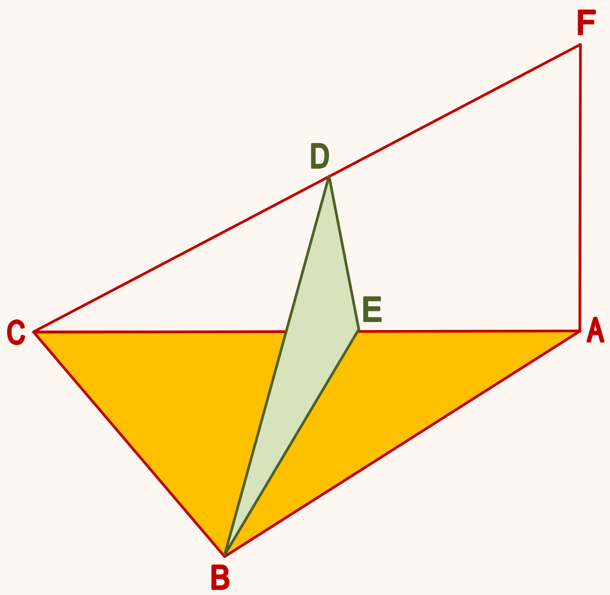
Proof that $\overline{BE} \perp (CAF)$, $\overline{ED} \perp \overline{CF}$

Given :

$\overline{AF} \perp (ABC)$

$\overline{BD} \perp \overline{CF}$

$\overline{BE} \perp \overline{CA}$



Required :

$\overline{BE} \perp (CAF)$

$\overline{ED} \perp \overline{CF}$

Proof :

$\because \overline{AF} \perp (ABC)$ **(Given)**

$\therefore (CAF) \perp (ABC)$ **(Theorem 8)**

$\because \overline{BE} \perp \overline{CA}$ **(Given)**

$\therefore \overline{BE} \perp (CAF)$ **(Theorem 7)**

$\because \overline{BD} \perp \overline{CF}$ **(Given)**

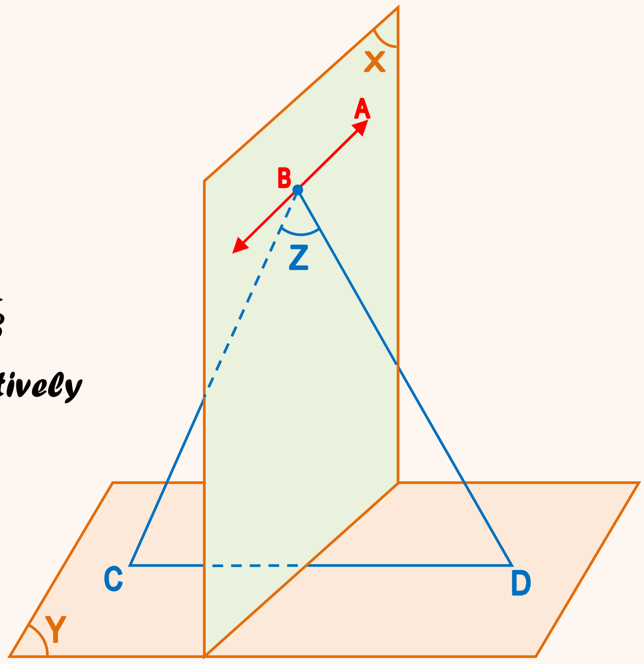
$\therefore \overline{ED} \perp \overline{CF}$ **(Three perpendicular line theorem)**

(Q.E.D)

Ex (3): $(Y),(X)$ **two orthogonal planes :** $\overline{AB} \subset (X)$
$\overleftrightarrow{CB}, \overleftrightarrow{BD}$ **are perpendicular on** $\overleftrightarrow{AB}$
and intersect (Y) **at** C, D **respectively**
prove that : $\overleftrightarrow{CD} \perp (X)$

Given :

$(X) \perp (Y)$
$\overline{AB} \subset (X)$
$\overleftrightarrow{CB}, \overleftrightarrow{BD}$ **are perpendicular on** $\overleftrightarrow{AB}$
and intersect (Y) **at** C, D **respectively**



Required :

$\overleftrightarrow{CD} \perp (X)$

Proof :

Let (Z) **a plane of intersect lines** $\overleftrightarrow{CB}, \overleftrightarrow{BD}$ **(For two intersect lines, there is one unique plane containing them)**

$\because \overleftrightarrow{AB} \perp \overleftrightarrow{BC}, \overleftrightarrow{BD}$ **(Given)**

$\therefore \overleftrightarrow{AB} \perp (Z)$ **(A perpendicular line on two intersect lines at intersect point is perpendicular on their plane)**

$\because \overleftrightarrow{AB} \subset (X)$ **(Given)**

$\therefore (X) \perp (Z)$ **(Theorem 8)**

$\because (X) \perp (Y)$ **(Given)**
And $(Z) \cap (Y) = \overleftrightarrow{CD}$ **(Because it is contained in both)**
$\therefore \overleftrightarrow{CD} \perp (X)$ **(Conclusion of theorem 9)**

(Q.E.D)

Ex (5): In ΔABC: $\overline{BC} \subset (X)$, the measurement of dihedral angle between triangle plane (ABC) and plane (X) = 60°, if $AB = AC = 13cm, BC = 10cm$

Find projection of triangle (ABC) on (X) then find project area ΔABC on (X)

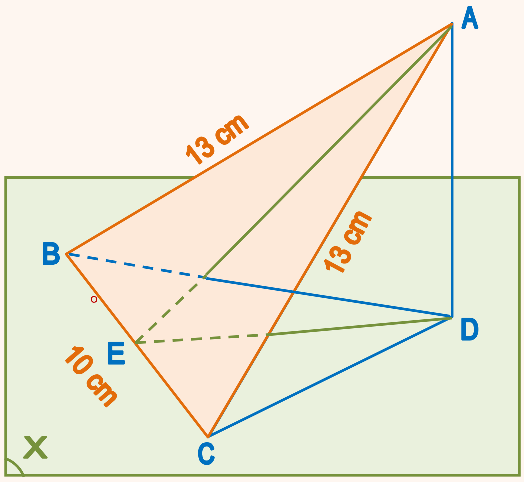
Given :

ΔABC: $\overline{BC} \subset (X)$, $(ABC) - \overline{BC} - (X) = 60^o$, $AB = AC = 13cm, BC = 10cm$

Required :

Find projection of triangle (ABC) on (X) then find project area ΔABC on (X)

Proof :



Draw $\overline{AD} \perp (X)$ at D (There exists a unique line perpendicular on given plane through the given point)

$\left.\begin{array}{l}\overline{CD} \text{ is project of } \overline{AC} \\ \overline{BD} \text{ is project of } \overline{AB}\end{array}\right\}$ (Definition of projection)

$\overline{BC}$ is project of itself on (X)

$\therefore \Delta BCD$ is project of ΔABC on (X)

In (ABC) we draw $\overline{BC} \perp \overline{AE}$ in E

(In a plane , there exists a unique line perpendicular on given line through the given point)

$\because AC = AB$ (Given)

$\therefore EC = BE = 5\,cm$ (The perpendicular line from vertex of isosceles triangle at the base, divides the base into half)

$\therefore \overline{ED} \perp \overline{BC}$ (postulate of three columns theorem)

$\therefore \sphericalangle DEA$ Plane angle of dihedral $\overline{BC}$ (By definition of plane angle of dihedral angle)

However, measurement of dihedral angle $\overline{BC} = 60^o$ (Given)

In $\Delta \mathrm{AEB}$ right angle in E:

$$(\mathrm{AB})^2 = (\mathrm{AE})^2 + (\mathrm{BE})^2 \Rightarrow 169 = (\mathrm{AE})^2 + 25 \Rightarrow (\mathrm{AE})^2 = 144 \Rightarrow \mathrm{AE} = 12\,\mathrm{cm}$$

In $\Delta \mathrm{AED}$ right angle in D :

$$\cos 60^o = \frac{\mathrm{ED}}{\mathrm{AE}} \quad \Rightarrow \quad \frac{1}{2} = \frac{\mathrm{ED}}{12} \quad \Rightarrow \quad \mathrm{ED} = 6\,\mathrm{cm}$$

Area of $\Delta\mathrm{BCD} = \frac{1}{2} \times 10 \times 6 = 30\ cm^2$

(Q.E.D)