20

Example on ISD-IFAM

20.1 Problem Definition

This chapter provides an example that illustrates the application of the integrated schedule design problem based on the itinerary-based fleet assignment model (ISD-IFAM). The example presented here extends the example presented in Chapter 17 to consider the case in which the schedule can be adjusted by possibly eliminating some of its optional flights. Figure 20.1 shows the flight schedule of a hypothetical airline that consists of six flights serving between LAX, ORD, and BOS. These six flights are identified as LAX-ORD ($f1$), ORD-BOS ($f2$), LAX-BOS ($f3$), BOS-ORD ($f4$), ORD-LAX ($f5$), and BOS-LAX ($f6$). They represent the set of master flights. Flights $f1$, $f3$, $f5$, and $f6$ are assumed to be mandatory, while flights $f2$ and $f4$ are optional.

As shown in the figure, each of the city-pairs LAX-ORD, ORD-LAX, ORD-BOS, and BOS-ORD is served by nonstop flights. The city-pairs LAX-BOS and BOS-LAX are served by both nonstop and connecting itineraries. For example, the city-pair LAX-BOS is served by the nonstop itinerary $f3$ and the connecting itinerary $f1-f2$. Similarly, the city-pair BOS-LAX is served by the nonstop itinerary $f6$ and the connecting itinerary $f4-f5$. Accordingly, the network includes a total of eight itineraries, which are LAX-ORD, ORD-BOS, LAX-BOS, BOS-ORD, ORD-LAX, BOS-LAX, LAX-ORD-BOS, and BOS-ORD-LAX. The passenger demand and average fare of each itinerary are given in Table 20.1.

Assume that each of these six flights can be assigned to aircraft of fleet types $e1$ or $e2$ with seat capacity of 150 and 250 seats, respectively. The two aircraft types are assumed to have the same speed. Thus, each flight has the same arrival time regardless of its assigned fleet. The operating cost of each of the six flights considering the two fleet types are given in Table 20.2.

Airline Network Planning and Scheduling, First Edition. Ahmed Abdelghany and Khaled Abdelghany.

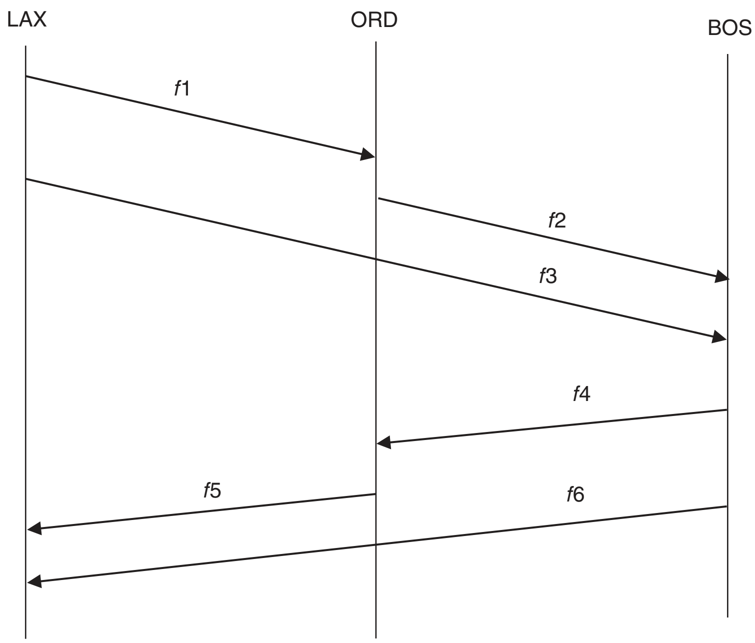


Figure 20.1 An example of a hypothetical airline with six flights for the ISD-IFAM problem.

Table 20.1 The passenger demand and fares on each of the eight itineraries for the ISD-IFAM problem.

Itinerary	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX
Passenger demand	50	50	300	50	50	300	50	50
Fare ($)	200	225	350	225	200	350	300	300

Table 20.2 The operating cost of each flight for each fleet type for the ISD-IFAM problem.

		Fleet	
Flight		***e*1 ($)**	***e*2 ($)**
LAX-ORD	*f*1	30,000	16,600
ORD-BOS	*f*2	15,100	13,500
LAX-BOS	*f*3	15,500	45,000
BOS-ORD	*f*4	57,000	13,500
ORD-LAX	*f*5	42,000	16,600
BOS-LAX	*f*6	15,500	45,000

With six flights and two fleet types, the problem has 12 assignment decision variables, where each of the six flights is assigned to one of the two fleet types. Table 20.3 gives the notation of the decision variables of the problem. For example, $x_{5,1}$ is equal to 1 if flight 5 is assigned to fleet $e1$, and zero otherwise.

To consider the balance constraints of the problem, the interconnection nodes are defined at all three airports. As both fleet types have the same speed, their interconnection nodes are identical as shown in Figure 20.2. As illustrated, there are two interconnection nodes at LAX, two interconnection nodes at ORD, and one interconnection node at BOS.

Table 20.3 The decision variables of fleet assignment for the ISD-IFAM example.

Flight	***e*1**	***e*2**
LAX-ORD	$x_{1,1}$	$x_{1,2}$
ORD-BOS	$x_{2,1}$	$x_{2,2}$
LAX-BOS	$x_{3,1}$	$x_{3,2}$
BOS-ORD	$x_{4,1}$	$x_{4,2}$
ORD-LAX	$x_{5,1}$	$x_{5,2}$
BOS-LAX	$x_{6,1}$	$x_{6,2}$

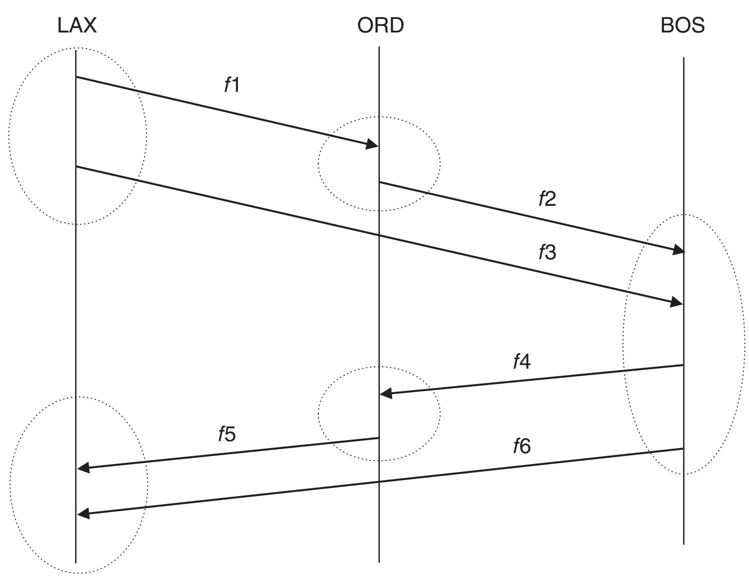


Figure 20.2 The interconnection nodes for the ISD-IFAM example.

Table 20.4 The decision variables for the ground arcs and the overnight arcs for the ISD-IFAM example.

$\text{RON}_{\text{LAX } e1}$	Number of aircraft of type $e1$ that remain overnight at LAX
$\text{RON}_{\text{ORD } e1}$	Number of aircraft of type $e1$ that remain overnight at ORD
$\text{RON}_{\text{BOS } e1}$	Number of aircraft of type $e1$ that remain overnight at BOS
$y_{(1-2)\text{ LAX } e1}$	Number of aircraft of type $e1$ on the ground links and connects interconnection nodes 1 and 2 at LAX
$y_{(1-2)\text{ ORD } e1}$	Number of aircraft of type $e1$ on the ground links and connects interconnection nodes 1 and 2 at ORD
$\text{RON}_{\text{LAX } e2}$	Number of aircraft of type $e2$ that remain overnight at LAX
$\text{RON}_{\text{ORD } e2}$	Number of aircraft of type $e2$ that remain overnight at ORD
$\text{RON}_{\text{BOS } e2}$	Number of aircraft of type $e2$ that remain overnight at BOS
$y_{(1-2)\text{ LAX } e2}$	Number of aircraft of type $e2$ on the ground links and connects interconnection nodes 1 and 2 at LAX
$y_{(1-2)\text{ ORD } e2}$	Number of aircraft of type $e2$ on the ground links and connects interconnection nodes 1 and 2 at ORD

Given the interconnection nodes, the list of decision variables that are related to the ground arcs and the overnight arcs are given in Table 20.4.

20.2 The Constraints of the Problem

The coverage constraints of problem entail that each flight is assigned to one fleet type only. The problem has six coverage constraints as follows.

For the mandatory flights

$$x_{1,1} + x_{1,2} = 1$$

$$x_{3,1} + x_{3,2} = 1$$

$$x_{5,1} + x_{5,2} = 1$$

$$x_{6,1} + x_{6,2} = 1$$

For the optional flights

$$x_{2,1} + x_{2,2} \leq 1$$

$$x_{4,1} + x_{4,2} \leq 1$$

The problem includes 10 balance constraints that are defined as follows:

$$\mathrm{RON}_{\mathrm{LAX}\,e1} = y_{(1-2)\mathrm{LAX}\,e1} + x_{1,1} + x_{3,1}$$

$$y_{(1-2)\mathrm{LAX}\,e1} + x_{5,1} + x_{6,1} = \mathrm{RON}_{\mathrm{LAX}\,e1}$$

$$\mathrm{RON}_{\mathrm{LAX}\,e2} = y_{(1-2)\mathrm{LAX}\,e2} + x_{1,2} + x_{3,2}$$

$$y_{(1-2)\mathrm{LAX}\,e2} + x_{5,2} + x_{6,2} = \mathrm{RON}_{\mathrm{LAX}\,e2}$$

$$\mathrm{RON}_{\mathrm{ORD}\,e1} + x_{1,1} = y_{(1-2)\mathrm{ORD}\,e1} + x_{2,1}$$

$$y_{(1-2)\mathrm{ORD}\,e1} + x_{4,1} = \mathrm{RON}_{\mathrm{ORD}\,e1} + x_{5,1}$$

$$\mathrm{RON}_{\mathrm{ORD}\,e2} + x_{1,2} = y_{(1-2)\mathrm{ORD}\,e2} + x_{2,2}$$

$$y_{(1-2)\mathrm{ORD}\,e2} + x_{4,2} = \mathrm{RON}_{\mathrm{ORD}\,e2} + x_{5,2}$$

$$\mathrm{RON}_{\mathrm{BOS}\,e1} + x_{2,1} + x_{3,1} = \mathrm{RON}_{\mathrm{BOS}\,e1} + x_{4,1} + x_{6,1}$$

$$\mathrm{RON}_{\mathrm{BOS}\,e2} + x_{2,2} + x_{3,2} = \mathrm{RON}_{\mathrm{BOS}\,e2} + x_{4,2} + x_{6,2}$$

For simplicity, it is assumed that there is no limitation on resources, and other constrains such as maintenance and crew constraints are ignored.

20.3 The Objective Function

The problem minimizes an objective function in the form of the sum of the operating cost, the spill cost minus the recapture revenue, and the change in unconstrained revenue due to schedule change ($S - M + \Delta R$). To explain the different components of the ISD-IFAM, consider the Excel sheets given in Figures 20.3 through 20.35. These figures describe the solution of the ISD-IFAM for the hypothetical airline network presented above. The spreadsheets give the formulas used for the calculations as well as the resulting values.

In Figures 20.3 and 20.4, column B (rows 4–9) gives the list of mandatory and optional flights considered in the schedule. Each flight is defined by its origin–destination pair. Columns C and D give the operating cost associated with assigning each of the six flights to fleet types *e*1 and *e*2, respectively. For example, it costs the airline $15,500 and $45,000 to operate flight LAX-BOS using fleet type *e*1 and fleet type *e*2, respectively. Column B (rows 10–12) also gives the variables for the overnight arcs (RON) for the three stations LAX, ORD, and BOS. Finally, the variables of the ground arcs between the interconnection nodes are given in rows 13 and 14.

Columns C and D give the cost associated with an aircraft remaining overnight at each of the three stations for each fleet type, which are assumed to be zero in this example. In addition, the cost associated with each aircraft that remains idle between the interconnection nodes for each fleet type is given.

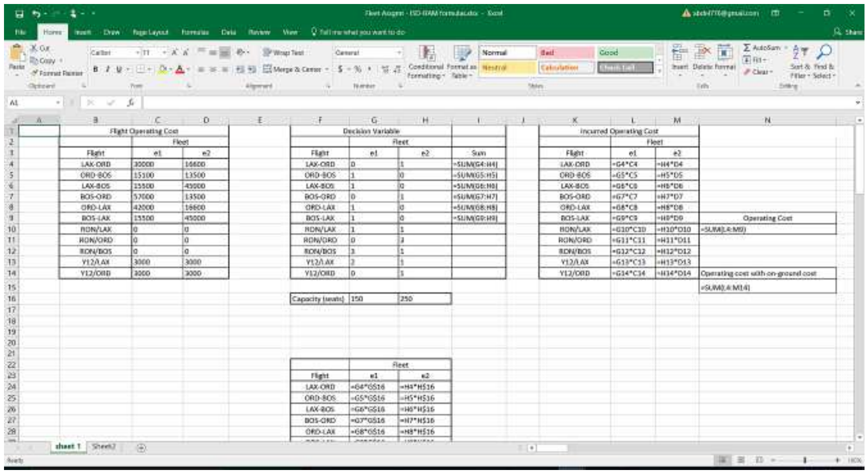


Figure 20.3 The decision variables and formulas for cost calculations for the ISD-IFAM example.

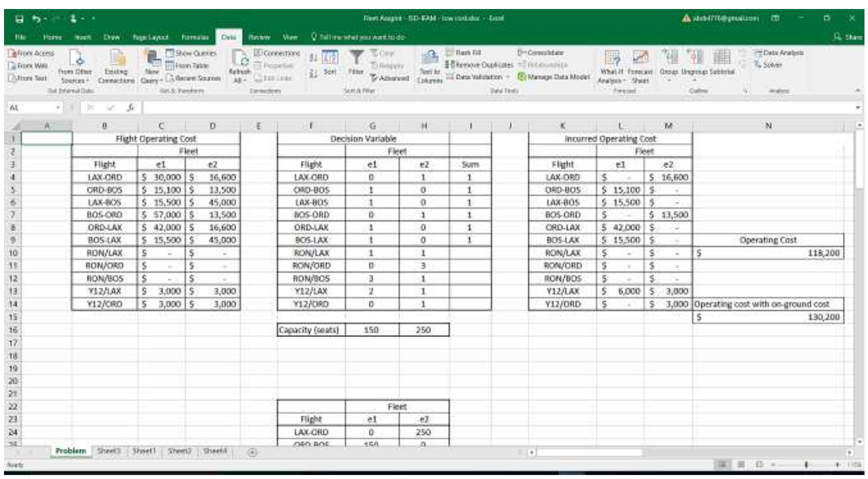


Figure 20.4 The decision variables and initial cost values for the ISD-IFAM example.

It costs the airline \$3000 for each aircraft of both fleet types to remain on the ground between the defined interconnection nodes.

Columns G and H give the initial values of the assignment decision variables. For example, if the value of cell H4 is equal to 1, it implies that the LAX-ORD flight is assigned initially to fleet type *e*2. The value of cell G12 implies that there are three aircraft of type *e*1 to remain overnight at BOS. The value of cell H14 implies that there is one aircraft of fleet type *e*2 to remain on the ground during the time between the two interconnection nodes at ORD.

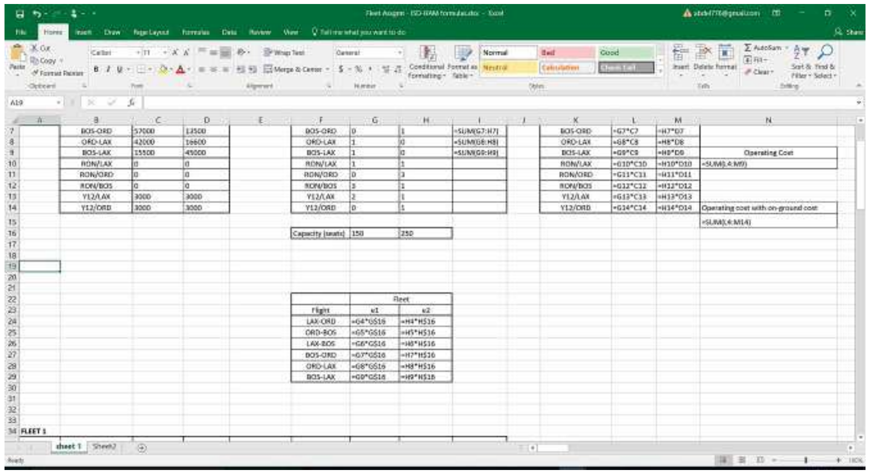


Figure 20.5 The formulas representing the seat capacity of each flight for the ISD-IFAM example.

The table in columns K, L, and M gives the incurred operating cost based on the initial values of the decision variables. The total operations cost is given in cell N15, which is the sum of the cost elements given in columns L and M (rows 4–14). For example, given the initial values of the decision variables, the operating cost is $130,200. This cost corresponds to the value C in the objective function of the ISD-IFAM problem $(C+S-M+\Delta R)$. Finally, column I (rows 4–9) gives the sum of the flight assignment decision variables for all fleet types. This value is used to set the coverage constraints of the fleet assignment problem for the mandatory and optional flights.

In Figures 20.5 and 20.6, cells G16 and H16 give the seat capacity of the aircraft of fleet types $e1$ and $e2$, respectively. An aircraft belonging to fleet type $e1$ has a seat capacity of 150 seats, while an aircraft of fleet type $e2$ has a seat capacity of 250 seats. Columns G and H (rows 24–29) give the seat capacity of each flight based on the initial values of the decision variables. The spreadsheet in Figure 20.5 gives the formula of the cells, while the spreadsheet in Figure 20.6 gives the corresponding values. For example, since the value in cell H4 is set to one, it implies that flight LAX-ORD is initially assigned to fleet type $e2$ and has a seat capacity of 250 seats.

Figures 20.7 and 20.8 show the balance constraints (starting at row 35) of the ISD-IFAM problem. The spreadsheet in Figure 20.7 gives the formula of the cells, while the spreadsheet in Figure 20.8 gives the corresponding values. As given above, there are five interconnection nodes for each fleet type, and hence ten balance constraints are defined for the problem. For each constraint, the values of the right-hand side (RHS) and the left-hand side (LHS) are given. At this stage, the balance constraints are not satisfied yet, and the RHS and the LHS of each constraint might not be equal.

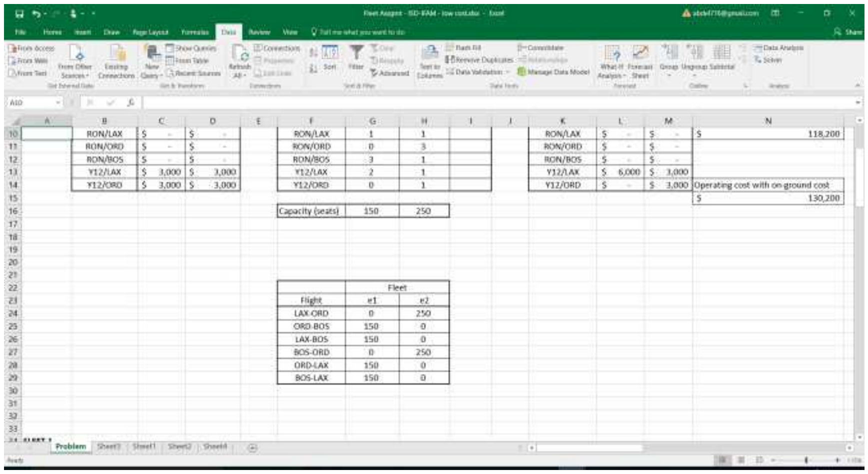


Figure 20.6 The initial values of the seat capacity of each flight for the ISD-IFAM example.

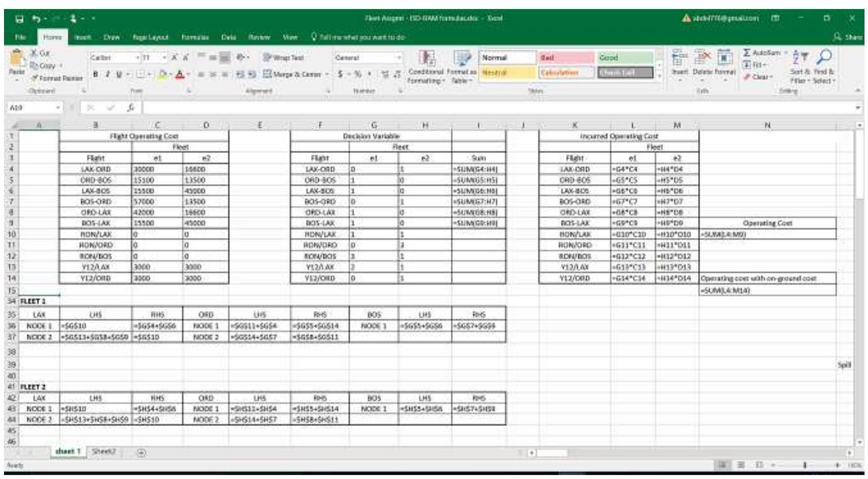


Figure 20.7 The formulas for the balance constraints for the ISD-IFAM example.

In Figures 20.9 and 20.10, the table in columns Q–X (rows 4–9) gives the itinerary–flight incidence matrix. The matrix defines the relationship between the flights and the itineraries in the network. In this matrix, the flights are given in the rows, while the itineraries are given in the columns. A cell in the matrix is equal to 1, if the itinerary represented by the cell includes the flight that corresponds to the same cell, and zero otherwise. For example, the value of cell W5 is equal to 1 because flight ORD-BOS is part of itinerary

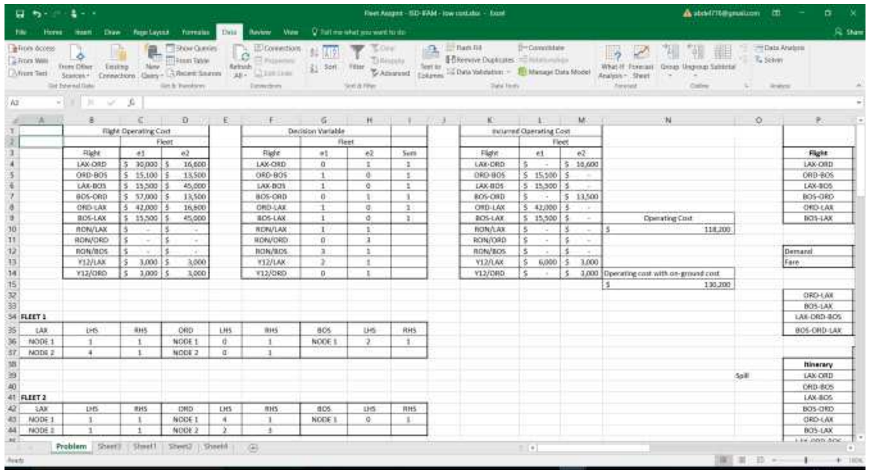


Figure 20.8 The initial values of the balance constraints for the ISD-IFAM example.

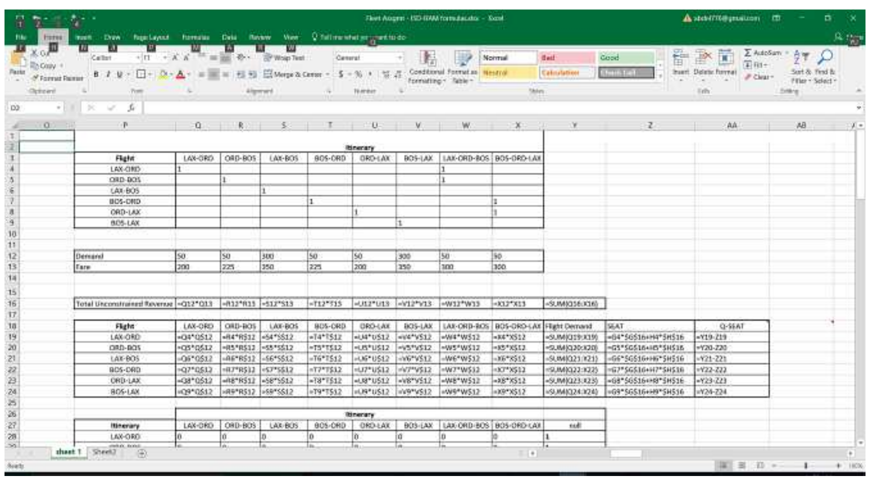
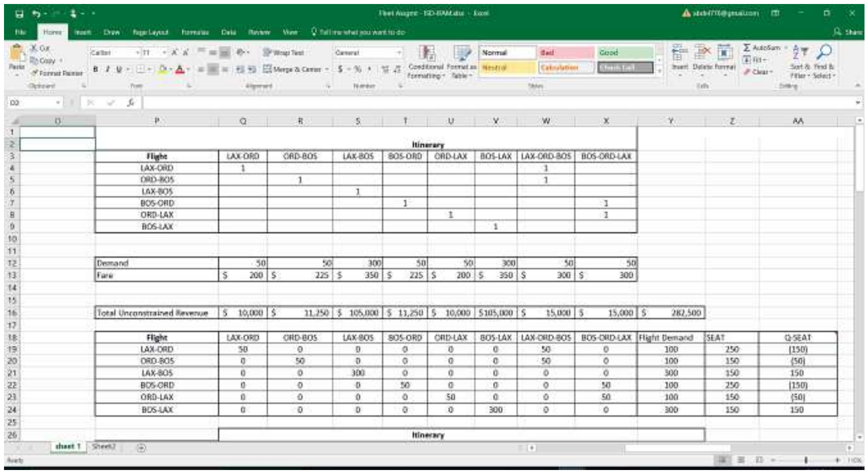


Figure 20.9 The formulas for the unconstrained demand for each flight for the ISD-IFAM example.

LAX-ORD-BOS. Similarly, itinerary BOS-ORD-LAX has two flights BOS-ORD and ORD-LAX. Thus, X7 and X8 are equal to 1.

Rows 12 and 13 (columns Q–X) give the unconstrained passenger demand and the fare for each of the eight itineraries. The unconstrained demand of an itinerary is defined as the number of passengers that choose this itinerary assuming unlimited seat capacity. The unconstrained revenue is calculated in row 16. The total unconstrained revenue is given in cell Y16.



Flight	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX
LAX-ORD	1						1	
ORD-BOS		1					1	
LAX-BOS			1					
BOS-ORD				1				1
ORD-LAX					1			1
BOS-LAX						1		

Demand	50	50	300	50	50	300	50	50	
Fare	$ 200	$ 225	$ 350	$ 225	$ 200	$ 350	$ 300	$ 300	
Total Unconstrained Revenue	$ 10,000	$ 11,250	$ 105,000	$ 11,250	$ 10,000	$105,000	$ 15,000	$ 15,000	$ 282,500

Flight	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX	Flight Demand	SEAT	Q-SEAT
LAX-ORD	50	0	0	0	0	0	50	0	100	250	(150)
ORD-BOS	0	50	0	0	0	0	50	0	100	150	(50)
LAX-BOS	0	0	300	0	0	0	0	0	300	150	150
BOS-ORD	0	0	0	50	0	0	0	50	100	250	(150)
ORD-LAX	0	0	0	0	50	0	0	50	100	150	(50)
BOS-LAX	0	0	0	0	0	300	0	0	300	150	150

Itinerary

Figure 20.10 The values for the unconstrained demand for each flight for the ISD-IFAM example.

Figures 20.9 and 20.10 also give the calculations of the unconstrained demand at the flight level. First, the unconstrained demand of each itinerary is mapped to all flights included in the itinerary. Then, the unconstrained demand of each flight is calculated as the sum of the unconstrained demand of all itineraries that include this flight. For example, flight BOS-ORD is included in two itineraries, which are BOS-ORD and BOS-ORD-LAX. Each of these itineraries has 50 passengers (unconstrained demand). Thus, the unconstrained demand of this flight is equal to 100, which is the sum of the unconstrained demand of these two itineraries. The total unconstrained demand (Q) of each flight is given in column Y (rows 19–24). Column Z (rows 19–24) gives the seat capacity of each flight. The seat capacity of each flight depends on the fleet assignment solution of the flight. The fleet assignment solutions are given in columns G and H (rows 4–9). For example, the initial value of the assignment decision variable of flight LAX-BOS indicates that this flight is assigned to fleet type $e1$ (i.e. cell G6 = 1). Hence, the seat capacity of this flight is 150 seats. Finally, column AA (rows 19–24) gives the difference between the unconstrained demand of the flight and its seat capacity. For example, flight LAX-BOS (row 21) has a total demand of 300 passengers and a seat capacity of 150 seats, resulting in an excess demand of 150 passengers (cell AA21).

Figure 20.11 shows the itinerary–itinerary spill–recapture relationship matrix. This matrix is given in columns Q–Y and rows 28–35. The rows in the matrix represent itineraries that spill demand, while the columns represent itineraries that recapture demand. The cells in the matrix indicate whether there is a spill–recapture relationship between the two itineraries.

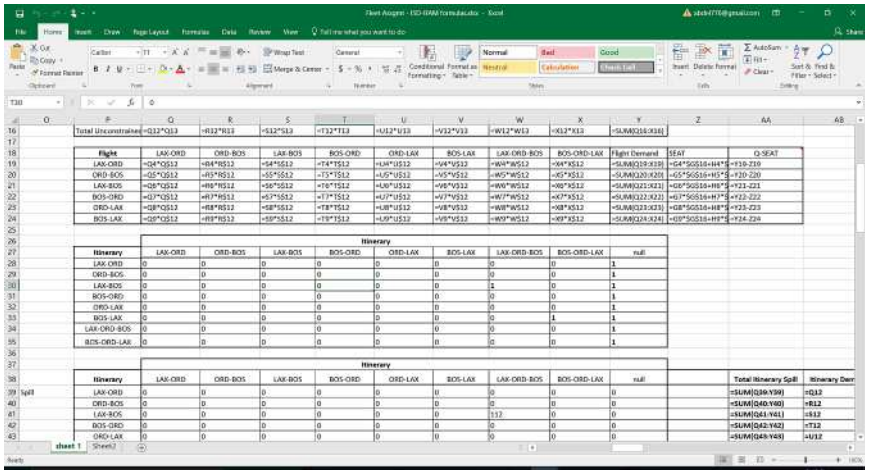


Figure 20.11 The itinerary–itinerary spill–recapture relationship matrix for the ISD-IFAM example.

For example, having cell W30 equal to 1 indicates that itinerary LAX-BOS can spill demand to itinerary LAX-ORD-BOS. Each itinerary can spill demand to a dummy itinerary, which represents the case when the airline cannot recapture the spilled demand on any of its itineraries.

Figure 20.12 shows the spilled demand decision variables. These variables are given in columns Q–Y and rows 39–46. It gives the number of spilled and recaptured passengers among itineraries. For example, it is initially assumed that 107 passengers are to be spilled from itinerary BOS-LAX to BOS-ORD-LAX (cell X44). These numbers are associated with the itinerary–itinerary spill–recapture relationship matrix defined above. In other words, the number of passengers to be spilled and recaptured between any two itineraries is valid, only if the corresponding value in the itinerary–itinerary spill–recapture relationship matrix is equal to 1. At this stage, preliminary numbers are given in the table. Solving the ISD-IFAM problem, the optimal values of the spilled demand are determined in conjunction with the optimal fleet assignment.

Figures 20.13 and 20.14 give the calculations of the total spilled demand of each itinerary. The total itinerary's spilled demand is the sum of the demand spilled from all recommended (substitute) itineraries including the dummy itinerary. Column AA gives the calculations of the total spilled demand of each itinerary (rows 39–46). Columns AB and AC (rows 39–46) give the itinerary demand and the itinerary fares. Column AD (rows 39–46) gives the spilled revenue of each itinerary. The spilled revenue is calculated by multiplying the spilled demand of the itinerary by its fare. Finally, cell AD47 gives the total spilled revenue, which is the sum of spill revenue of all itineraries. For example,

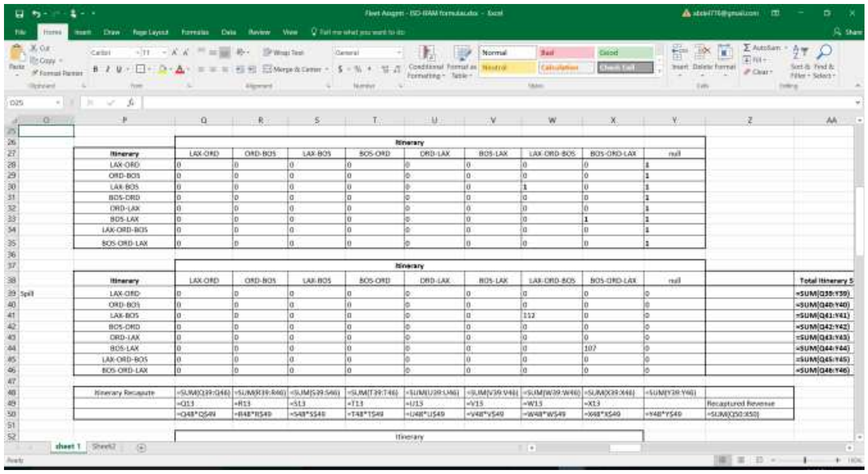


Figure 20.12 The initial values of the spilled demand decision variables for the ISD-IFAM example.

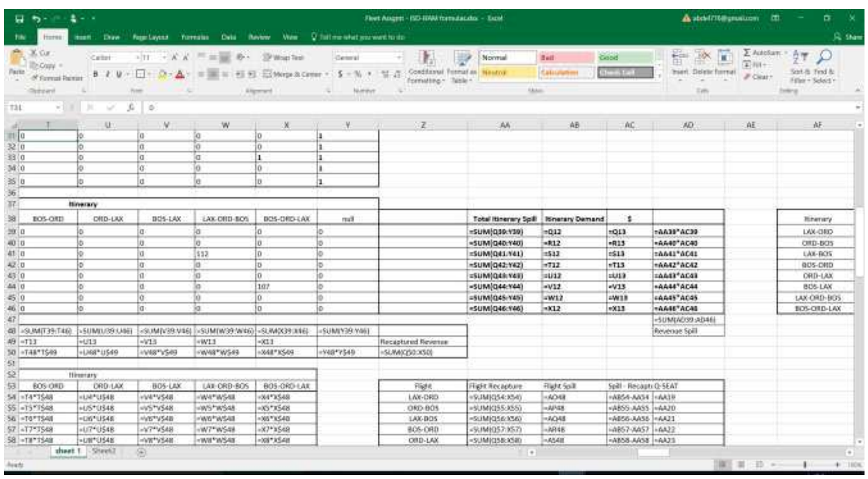


Figure 20.13 The formulas for the calculations of the total demand spill for each itinerary for the ISD-IFAM example.

given the initial values of the decision assignment and the spill variables, the spill revenue is $76,650. The spilled revenue is the value of the variable S in the objective function $(C + S - M + \Delta R)$.

Figure 20.15 gives the itinerary–flight incidence matrix after transposing. As mentioned above, the incidence matrix defines the relation between the

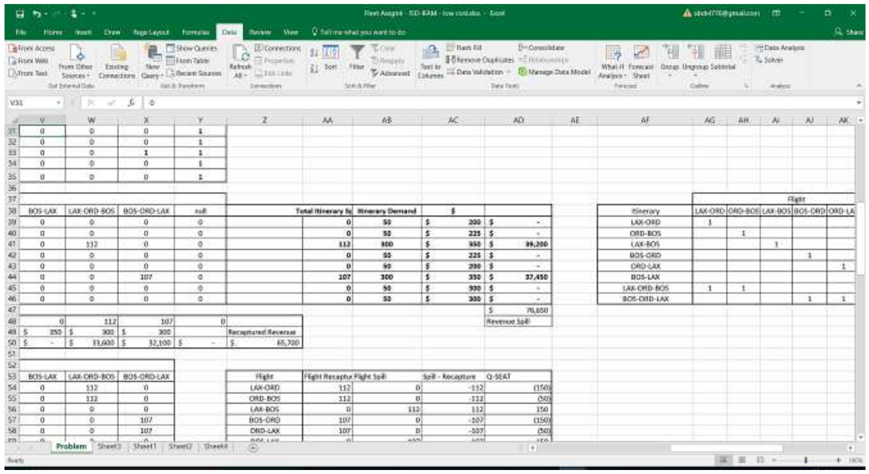


Figure 20.14 The initial values of the total demand spill for each itinerary for the ISD-IFAM example.

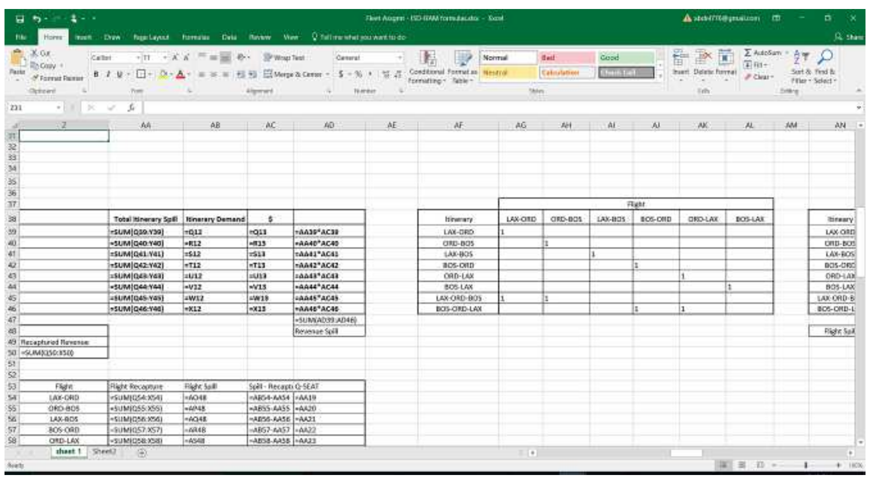


Figure 20.15 The transposed form of the itinerary–flight incidence matrix for the ISD-IFAM example.

flights and the itineraries in the network. The transposed matrix is given in columns AG–AL (rows 39–46), where the columns define the flights and the rows define the itineraries.

In Figures 20.16 and 20.17, the table given in columns AO–AT (rows 39–46) shows the calculations of the spilled demand at the flight level. The logic here is that in case a demand is spilled of an itinerary, it is by definition spilled from

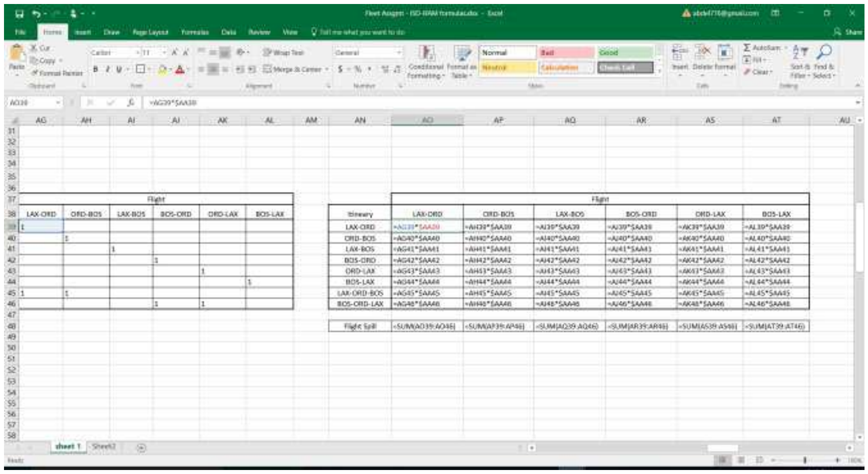


Figure 20.16 The formulas for the calculations of the demand spill at the flight level for the ISD-IFAM example.

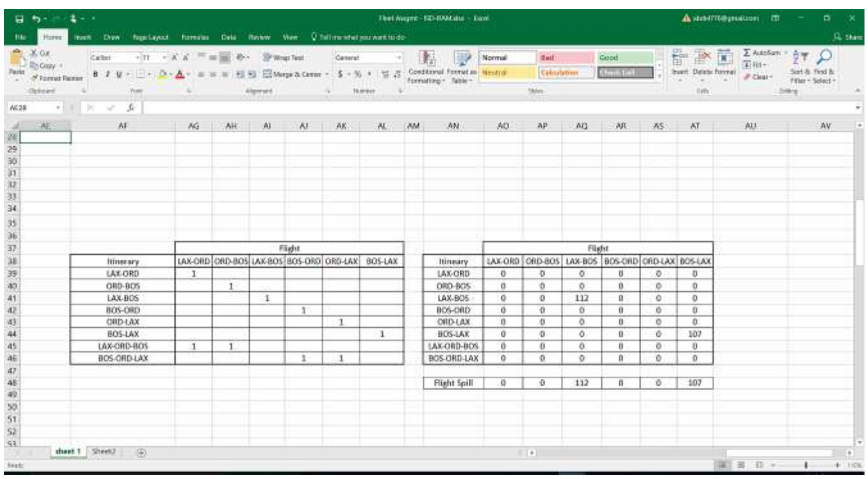


Figure 20.17 The initial values of the demand spill at the flight level for the ISD-IFAM example.

all flights of that itinerary. A flight could be part of more than one itinerary. Hence, a flight's spilled demand is calculated as the sum of the spilled demand of all itineraries that include the flight. The spilled demand of the flights is given in row 48 (columns AO–AT).

Figures 20.18 and 20.19 give the itineraries' recaptured demand. As described earlier, in case demand is spilled from an itinerary, the airline suggests another

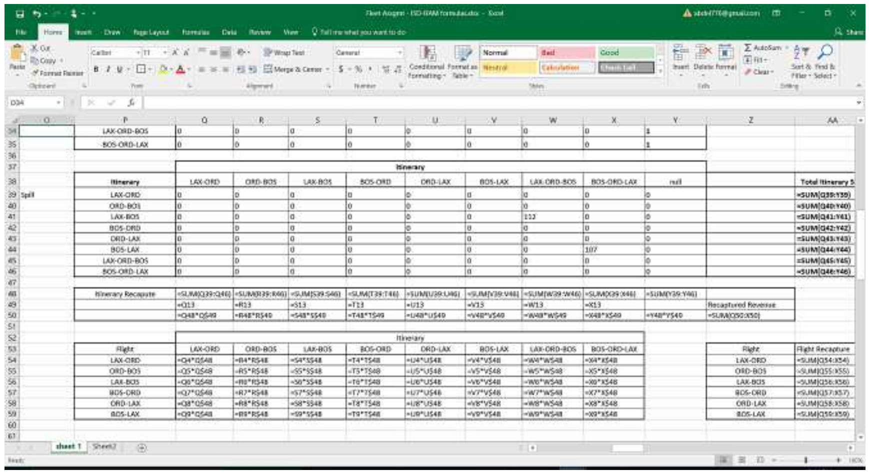


Figure 20.18 The formulas for the calculations of the recaptured demand and recaptured revenue for the ISD-IFAM example.

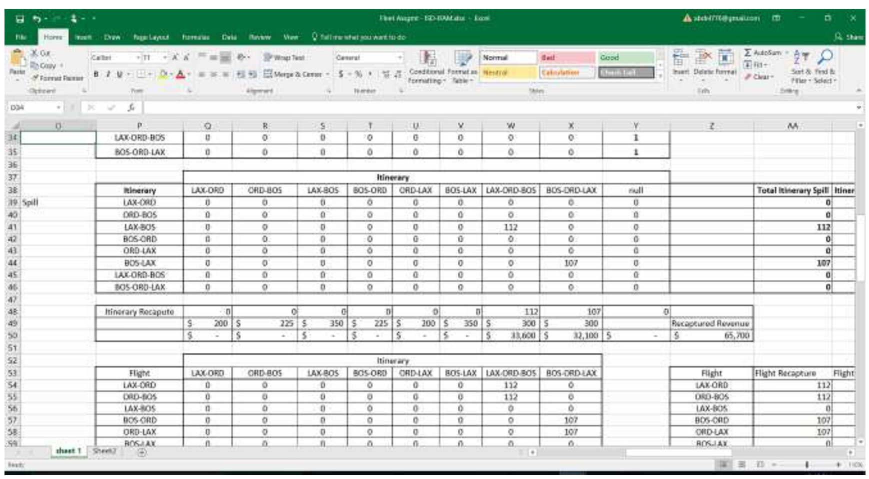


Figure 20.19 The initial values of the recaptured demand and recaptured revenue for the ISD-IFAM example.

itinerary (itineraries) to recapture this demand. An itinerary can recapture demand spilled from more than one itinerary. Thus, the demand recaptured on an itinerary is the sum of the demand spilled from other itineraries to this itinerary. Row 48 (columns Q–Y) gives the demand recaptured on each itinerary including the dummy itinerary.

At this stage, the values of the recaptured demand of each itinerary depend on the initial values of the decision variables (i.e. the fleet assignment variables and the demand spill variables). These initial values are not necessarily optimal or satisfying the problem's constraints. Accordingly, the recaptured demand of each itinerary is not necessarily satisfying the seat capacity constraints of the itineraries. Row 49 (columns Q–Y) gives the itinerary fare. Finally, the recaptured revenue of each itinerary is estimated by multiplying the recaptured demand of the itinerary by its fare. The total recaptured revenue of the airline is given in cell Z50, which is the sum of the recaptured revenue of all itineraries except for that of the dummy itinerary. For example, given the initial values of the fleet assignment and spill decision variables, the recaptured revenue is \$65,700. The recaptured revenue corresponds to the value M in the objective function ($C + S - M + \Delta R$).

Figures 20.18 and 20.19 also show the calculations of the recaptured demand at the flight level in the table given in columns Q–X (rows 54–59). The logic here is that if a demand is recaptured on an itinerary, it is recaptured on all flights that belong to the itinerary. A flight could be part of more than one itinerary. Thus, the demand recaptured on a flight is calculated as the sum of the demand recaptured on all itineraries that include the flight. The recaptured demand of each of the six flights is given in column AA (rows 54–59).

Figures 20.20 and 20.21 give the main statistics for the flights. The table in columns AA–AD (rows 54–59) gives the recaptured demand (column AA) and spilled demand (column AB) for each flight. The difference between

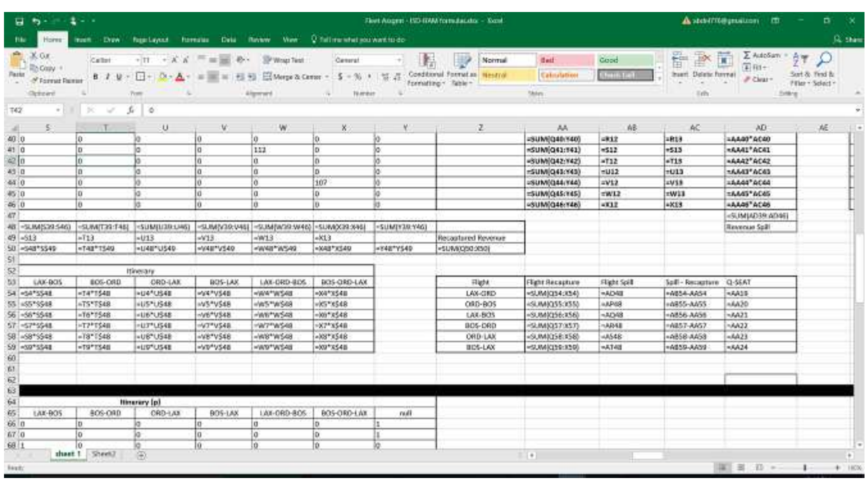


Figure 20.20 The calculations of the flight characteristics and the objective function of the ISD-IFAM example.

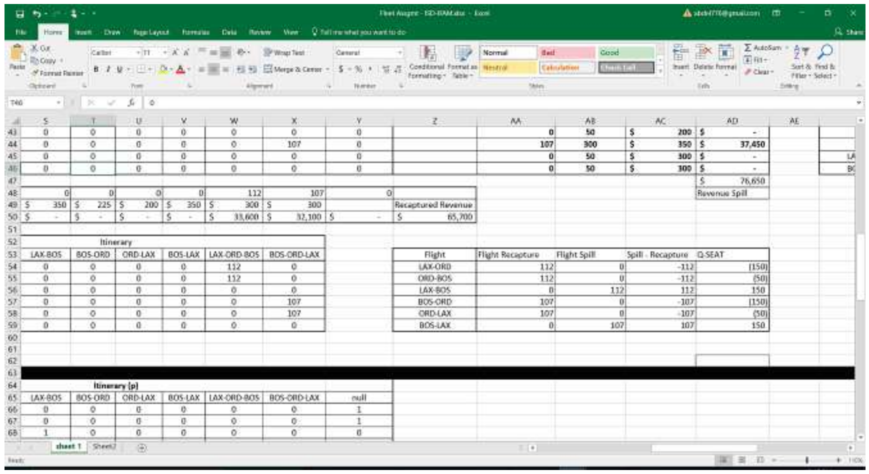
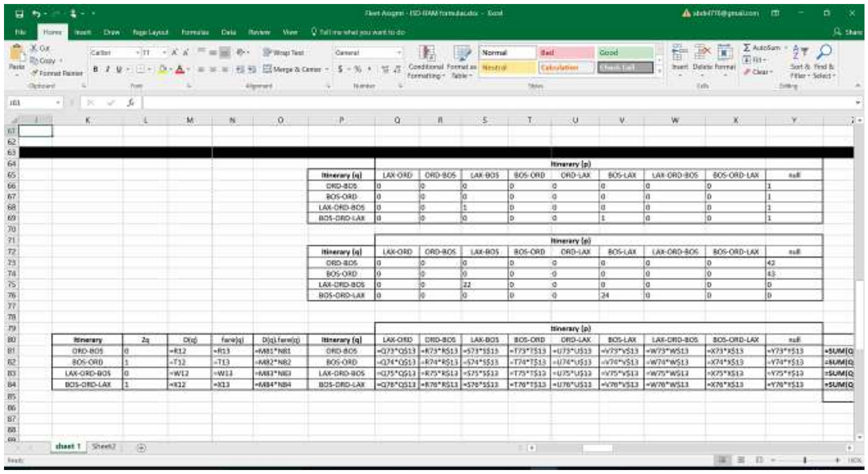


Figure 20.21 The initial values of the flight characteristics and the objective function of the ISD-IFAM example.

recaptured and spilled demand for each flight is given in column AC. Finally, the difference between the unconstrained demand and the seat capacity for each flight is given in column AD.

As mentioned above, the set of optional flights includes two flights, which are ORD-BOS and BOS-ORD. The decision to include these flights as part of the schedule is obtained based on the solution of this problem. The set of itineraries P^o that include these two optional flights includes four itineraries, which are ORD-BOS, BOS-ORD, LAX-ORD-BOS, and BOS-ORD-LAX. As explained above, if any of these itineraries is removed from the network, the unconstrained demand of the city-pair and the unconstrained demand of the alternative itineraries of the city-pair are expected to change. As mentioned above, the amount of change in the unconstrained demand of each itinerary depends on its attractiveness compared with other competing itineraries. The estimation of the share of each itinerary can be estimated using a methodology similar to the methodology presented in Chapter 8.

Figures 20.22 and 20.23 give the relationship between the itineraries in the set P^o, indexed by q, and alternative itineraries in the network. As mentioned above, the set P^o includes four itineraries that are included in the airline's schedule, only if the two optional flights ORD-BOS and BOS-ORD are scheduled. These itineraries are shown in rows 65–69. The table in columns Q–Y (rows 65–69) gives the incidence matrix for the alternative itineraries for each of the optional itineraries. In case an optional itinerary is removed from the schedule, the demand of this optional itinerary is shifted to alternative



Itinerary (q)	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX	null
ORD-BOS	0	0	0	0	0	0	0	0	1
BOS-ORD	0	0	0	0	0	0	0	0	1
LAX-ORD-BOS	0	0	1	0	0	0	0	0	1
BOS-ORD-LAX	0	0	0	0	0	1	0	0	1

Itinerary (q)	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX	null
ORD-BOS	0	0	0	0	0	0	0	0	42
BOS-ORD	0	0	0	0	0	0	0	0	43
LAX-ORD-BOS	0	0	22	0	0	0	0	0	0
BOS-ORD-LAX	0	0	0	0	0	24	0	0	0

Itinerary	Zq	D(q)	fare(q)	D(q).fare(q)	Itinerary (q)	LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	LAX-ORD-BOS	BOS-ORD-LAX	null	
ORD-BOS	0	=R12	=R13	=M81*N81	ORD-BOS	=Q73*Q$13	=R73*R$13	=S73*S$13	=T73*T$13	=U73*U$13	=V73*V$13	=W73*W$13	=X73*X$13	=Y73*Y$13	=SUM(Q
BOS-ORD	1	=T12	=T13	=M82*N82	BOS-ORD	=Q74*Q$13	=R74*R$13	=S74*S$13	=T74*T$13	=U74*U$13	=V74*V$13	=W74*W$13	=X74*X$13	=Y74*Y$13	=SUM(Q
LAX-ORD-BOS	0	=W12	=W13	=M83*N83	LAX-ORD-BOS	=Q75*Q$13	=R75*R$13	=S75*S$13	=T75*T$13	=U75*U$13	=V75*V$13	=W75*W$13	=X75*X$13	=Y75*Y$13	=SUM(Q
BOS-ORD-LAX	1	=X12	=X13	=M84*N84	BOS-ORD-LAX	=Q76*Q$13	=R76*R$13	=S76*S$13	=T76*T$13	=U76*U$13	=V76*V$13	=W76*W$13	=X76*X$13	=Y76*Y$13	=SUM(Q

Figure 20.22 The formulas that give revenue change due to the suggested deletion of the optional itineraries for the ISD-IFAM example (the left-hand side).

itineraries. The alternative itineraries include a dummy itinerary, which represents the case when demand of the optional itinerary is spilled to competitors or other transportation modes. The value of a cell in this matrix is equal to 1 if the itinerary in the column is an alternative to the optional itinerary given in the row. For example, the optional itinerary LAX-ORD-BOS has the itinerary LAX-BOS and the dummy itinerary as alternative itineraries. The optional itinerary ORD-BOS has no alternative itineraries except the dummy itinerary. The table in column Q–Y (rows 73–76) gives the shifted demand from each optional itinerary to its alternative itineraries, in case the optional itinerary is removed from the schedule. The values in the cells of this table give the change in the demand of the alternative itinerary in case the optional itinerary is removed. For example, in case itinerary LAX-ORD-BOS is removed from the schedule, the demand of the alternative itinerary LAX-BOS is expected to increase by 22 passengers. The values in the cells of this table correspond to the values ΔD_q^p defined above, where q and p refer to the optional and alternative itineraries, respectively.

The table in columns K–AA (rows 81–84) gives the revenue change due to the deletion of the optional itineraries. The left side of this table is shown in Figures 20.22 and 20.23. The right side is shown in Figures 20.24 and 20.25. Column K (rows 81–84) gives the list of optional itineraries, as defined above. Column L (rows 81–84) gives the values of the decision variable Z_q for each optional itinerary q. As defined above, the decision variable z_q is equal to 1, if itinerary q is included in the proposed schedule, and zero otherwise. For example, the initial values suggest that the optional itineraries ORD-BOS and LAX-ORD-BOS are to be removed from the schedule. The values given at

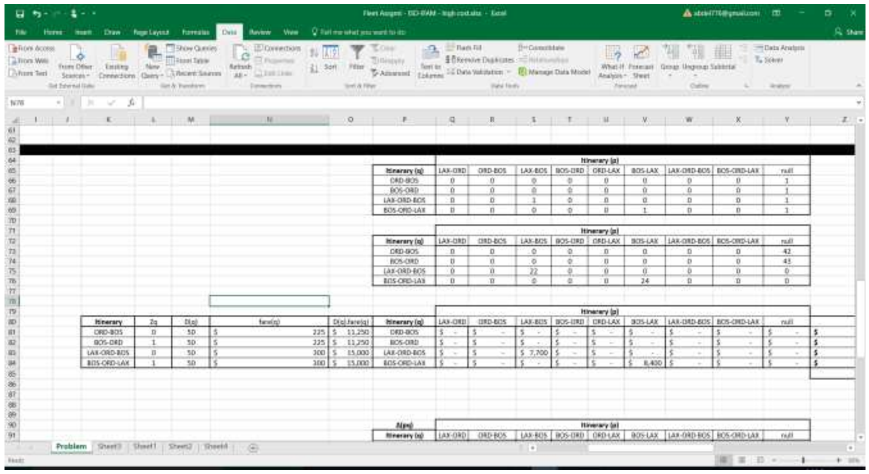


Figure 20.23 The revenue change due to the suggested deletion of the optional itineraries for the ISD-IFAM example (the left-hand side).

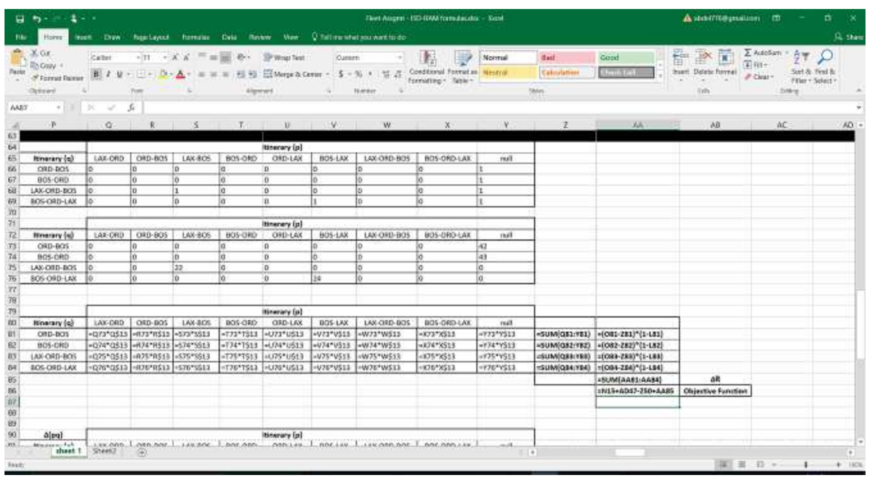


Figure 20.24 The formulas that give revenue change due to the suggested deletion of the optional itineraries for the ISD-IFAM example (the right-hand side).

this stage are initial values, which are not necessarily feasible nor optimum. The columns M and N (rows 81–84) give the demand and fare for each of the optional itineraries. These values are as defined in rows 12 and 13 (columns Q–X), respectively. The revenue of each of the optional itineraries is given in column O. This revenue is lost in case the optional itinerary is removed from the network. The revenues recaptured on alternative itineraries, if the

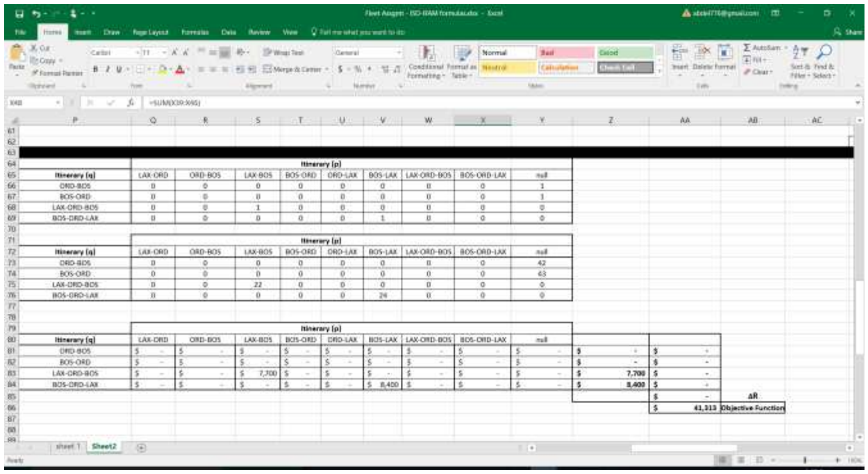
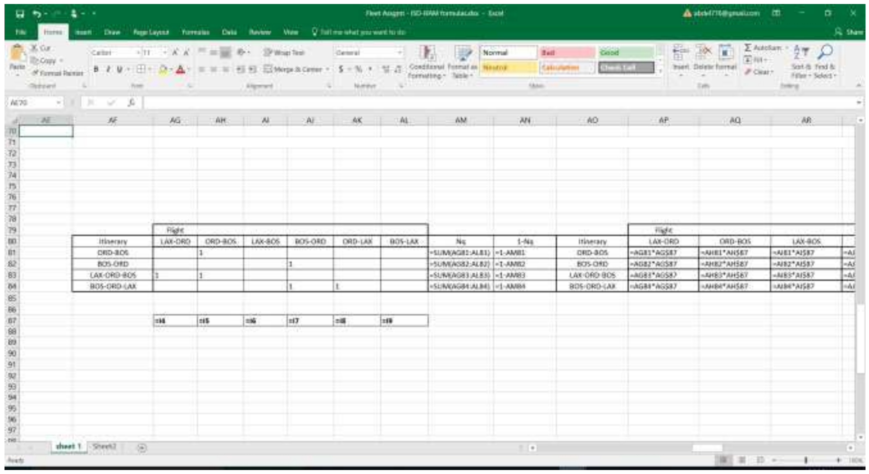


Figure 20.25 The revenue change due to the suggested deletion of the optional itineraries for the ISD-IFAM example (the right-hand side).

optional itineraries are removed, are given in column Z. The difference between the revenue loss and the recaptured revenue due to the removal of each of the optional itineraries is given in column AA (rows 81–84). Cell AA85 gives the term ΔR, which is the change in unconstrained revenue due to the elimination of the optional itineraries. Finally, cell AB86 gives the value of the objective function of the problem, which is defined as $C+S-M+\Delta R$. As shown in Figure 20.25, the initial value of the objective function is $41,313.

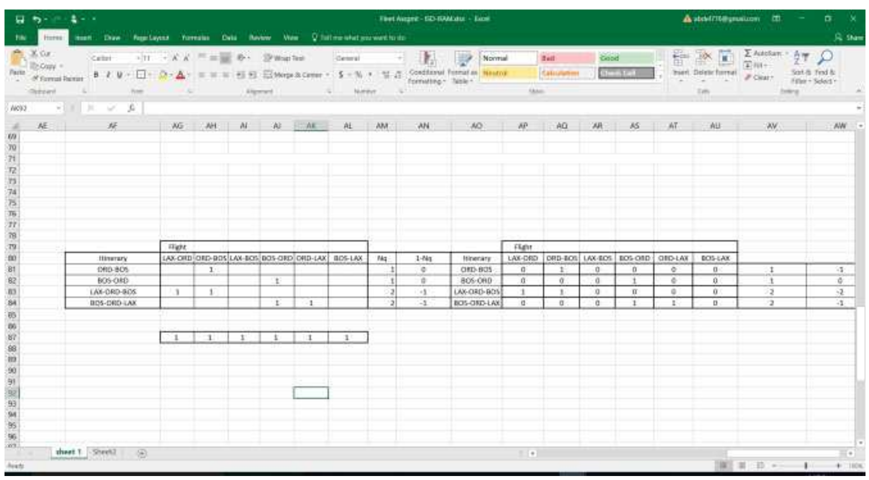
Figures 20.26 through 20.29 give additional calculations at the flight level. For example, the table in columns AG–AL (rows 81–84) gives the itinerary–flight incidence matrix for the optional itineraries. Columns AM and AN give the number of flights N_q and the value $1-N_q$ for each optional itinerary q. Row 87 (columns AG–AL) gives the RHS of the coverage constraints of each flight. This value is equal to 1, if the flight is included in the schedule, and zero otherwise. Columns AP–AU (rows 81–84) redefine the itinerary–flight incidence matrix only if the flight is considered in the solution (i.e. when the RHS of the coverage constraint is not equal to 0). Columns AV and AW in Figures 20.28 and 20.29 give the value of the terms $\left(\sum_{f\in L(q)}\sum_{e\in E}x_{fe}\right)$ and $\left(Z_q-\sum_{f\in L(q)}\sum_{e\in E}x_{fe}\right)$ for each optional flight, respectively.

In Figures 20.30 and 20.31, row 96 (columns Q–Y) gives the change in the demand for each of the alternative itineraries resulting from removing the optional itineraries. Row 97 (columns Q–Y) gives the total demand for each alternative itinerary. This total demand is calculated as the unconstrained demand of the itinerary, which is given in row 12 (columns Q–Y) plus the



Itinerary	Flight: LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	Nq	1-Nq	Itinerary	Flight: LAX-ORD	ORD-BOS	LAX-BOS
ORD-BOS		1					=SUM(AG81:AL81)	=1-AM81	ORD-BOS	=AG81*AG$87	=AH81*AH$87	=AI81*AI$87
BOS-ORD				1			=SUM(AG82:AL82)	=1-AM82	BOS-ORD	=AG82*AG$87	=AH82*AH$87	=AI82*AI$87
LAX-ORD-BOS	1	1					=SUM(AG83:AL83)	=1-AM83	LAX-ORD-BOS	=AG83*AG$87	=AH83*AH$87	=AI83*AI$87
BOS-ORD-LAX				1	1		=SUM(AG84:AL84)	=1-AM84	BOS-ORD-LAX	=AG84*AG$87	=AH84*AH$87	=AI84*AI$87
	=I4	=I5	=I6	=I7	=I8	=I9						

Figure 20.26 The formulas that show additional calculations for the optional flights for the ISD-IFAM example (part 1).



Itinerary	Flight: LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX	Nq	1-Nq	Itinerary	Flight: LAX-ORD	ORD-BOS	LAX-BOS	BOS-ORD	ORD-LAX	BOS-LAX		
ORD-BOS		1					1	0	ORD-BOS	0	1	0	0	0	0	1	-1
BOS-ORD				1			1	0	BOS-ORD	0	0	0	1	0	0	1	0
LAX-ORD-BOS	1	1					2	-1	LAX-ORD-BOS	1	1	0	0	0	0	2	-2
BOS-ORD-LAX				1	1		2	-1	BOS-ORD-LAX	0	0	0	1	1	0	2	-1
	1	1	1	1	1	1											

Figure 20.27 Additional calculations for the optional flights for the ISD-IFAM example (part 1).

additional demand calculated in row 96 (columns Q–Y). The table in rows 103–107 (columns Q–Y) shows the calculations of the additional demand at the flight level, which is shifted to the alternative itineraries, if the optional itineraries are eliminated. For example, given the initial values of the decision variable Z_q, there are 22 additional passengers on the flight LAX-BOS (cell S104).

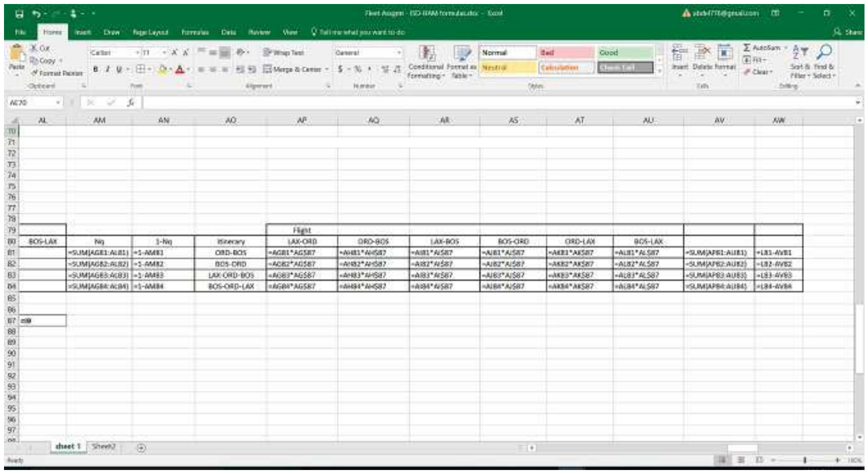


Figure 20.28 The formulas that show additional calculations for the optional flights for the ISD-IFAM example (part 2).

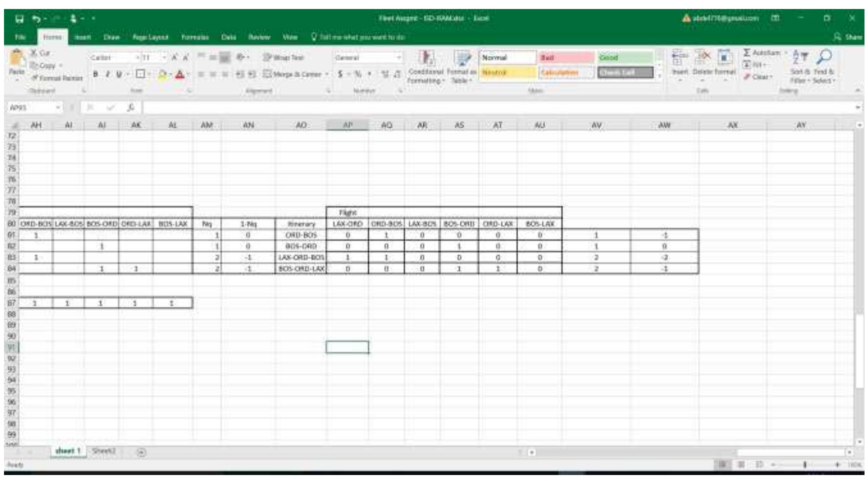


Figure 20.29 Additional calculations for the optional flights for the ISD-IFAM example (part 2).

In Figures 20.32 and 20.33, column AF (rows 102–107) gives the total additional demand shifted to each flight. Columns AB (rows 102–107) and AC (rows 102–107) in this table give the recaptured and spilled demand at the flight level. The difference between the recaptured and spilled demand at the flight level is given in column AE (rows 102–107). Finally, column AG (rows 102–107) gives the sum of columns AE and AF.

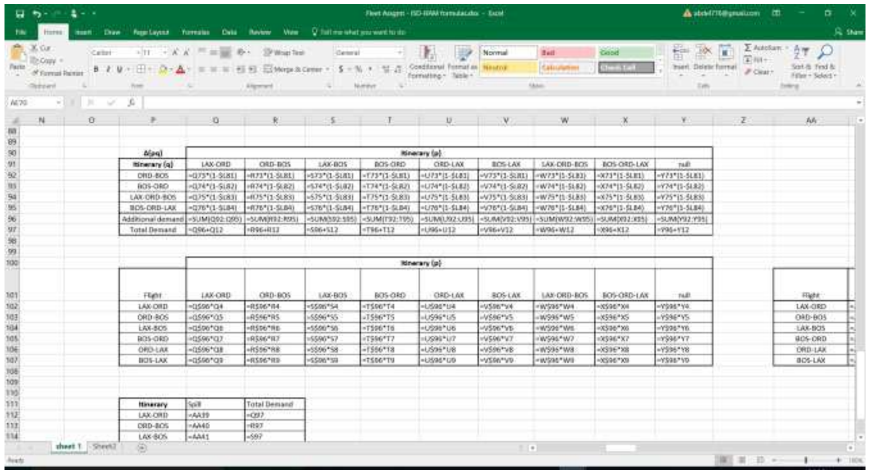


Figure 20.30 The formulas for the calculations of the demand change for each of the alternative itineraries, because of removing the optional itineraries for the ISD-IFAM example.

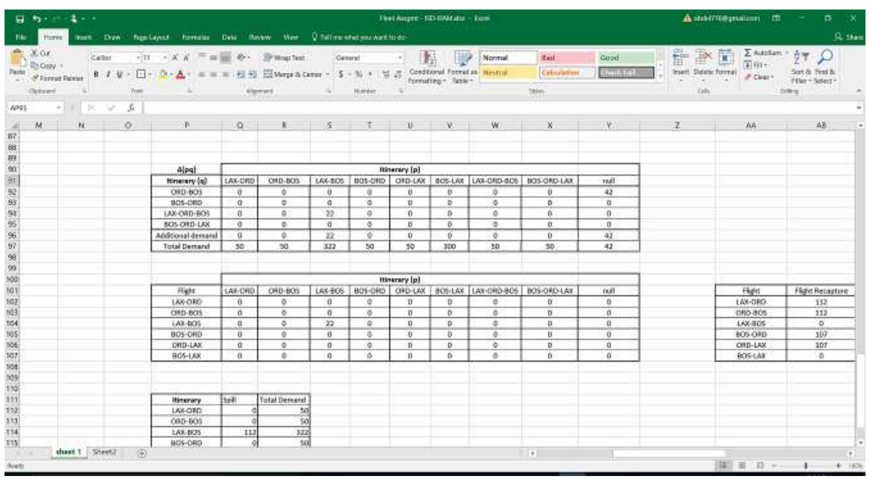


Figure 20.31 The demand change for each of the alternative itineraries, because of removing the optional itineraries for the ISD-IFAM example.

In Figures 20.34 and 20.35, the total demand spilled of each itinerary and the total demand of each itinerary are given in columns Q (rows 112–119) and R (rows 112–119), respectively. These two variables are used in the demand constraints, as illustrated in the next section.

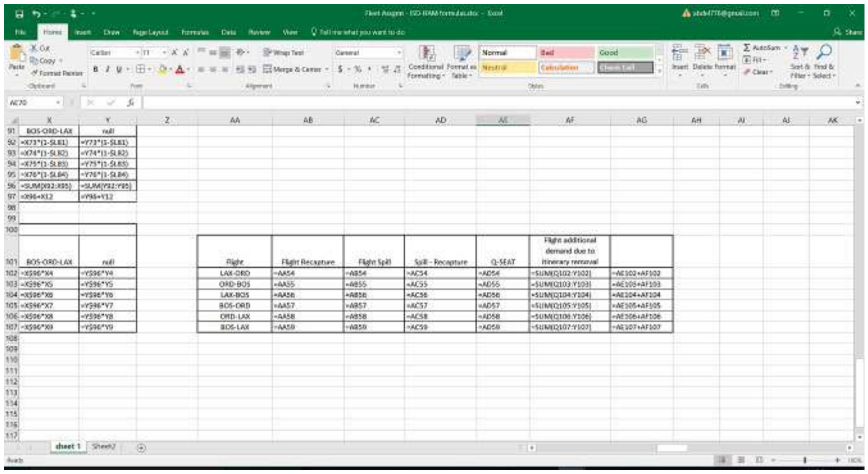


Figure 20.32 Formulas for calculating main flight characteristics for ISD-IFAM example.

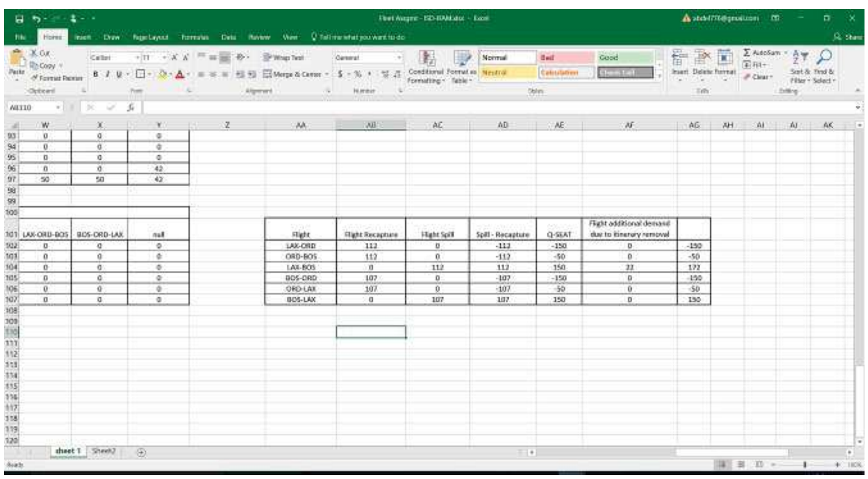


Figure 20.33 Main flight characteristics for ISD-IFAM example.

20.4 Problem Solving

The Excel Solver is used to solve the problem as shown in Figures 20.36 through 20.38. A minimization objective function is defined in cell AA86. The decision variables of the problem are also defined. The decision variables include the flight-fleet assignment variables, which are given in columns G and H (rows 4–9). They also include the number of aircraft to stay on ground between the

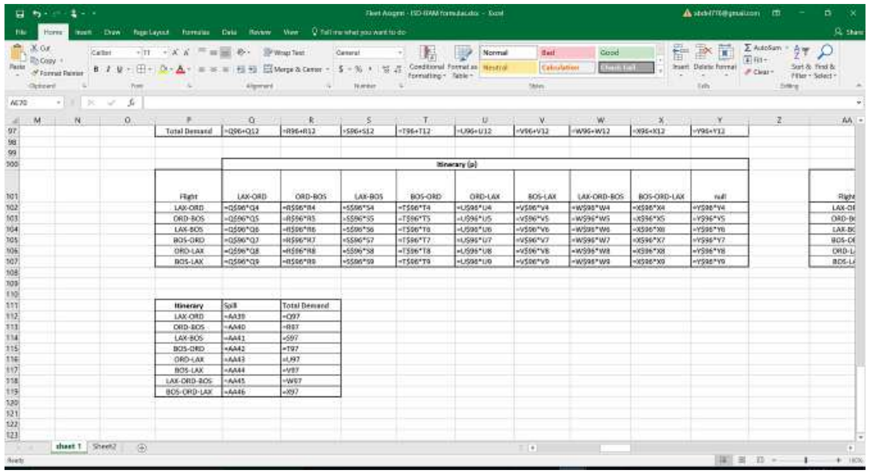


Figure 20.34 The formulas for the total demand spilled for each itinerary and the total demand of each itinerary for the ISD-IFAM example.

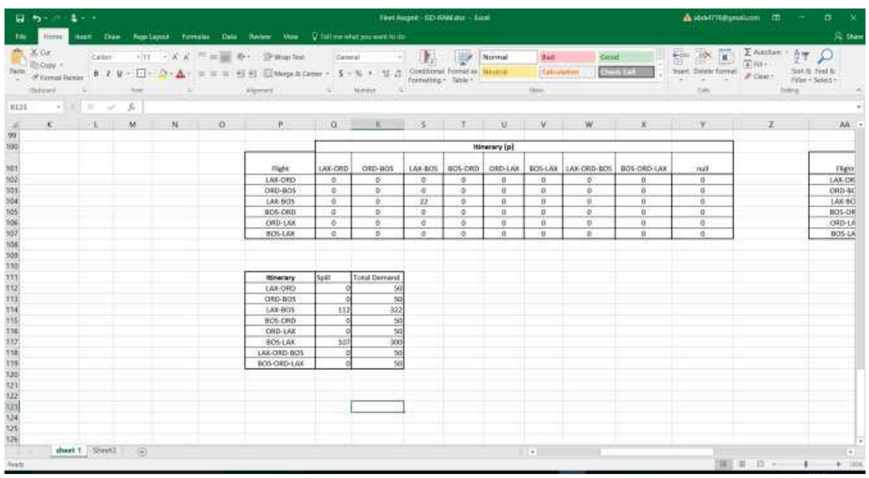


Figure 20.35 The total demand spilled for each itinerary and the total demand of each itinerary for the ISD-IFAM example.

interconnection nodes and overnight at the different stations for each fleet type, which are given in columns G and H (rows 10–14). The amount of demand spilled among itineraries is also included in the decision variables, which are cell W41, cell X44, and cells Y39–Y46. Finally, the decision variable Z_q for each optional itinerary q is given in cells L81–L84.

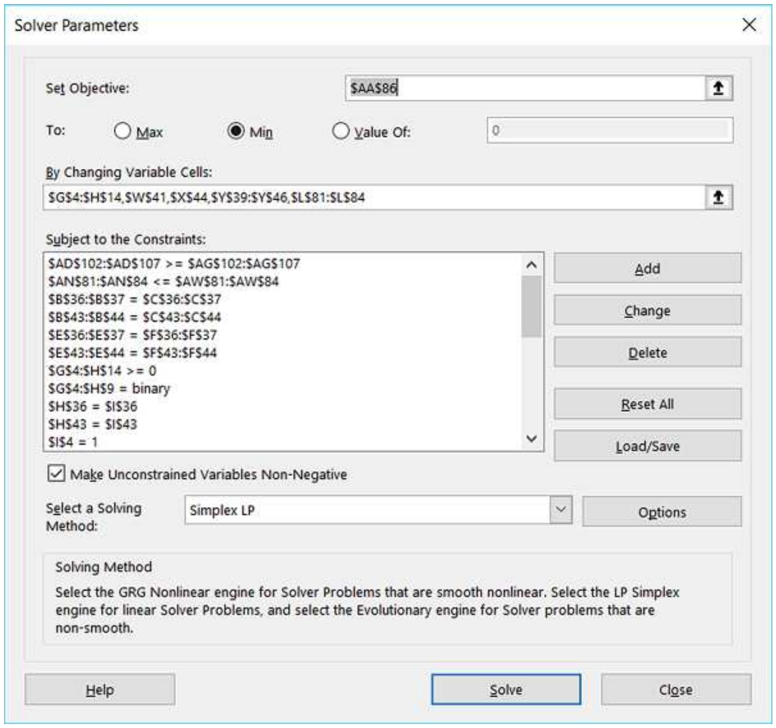


Figure 20.36 The Solver Parameters window for the ISD-IFAM example (part 1).

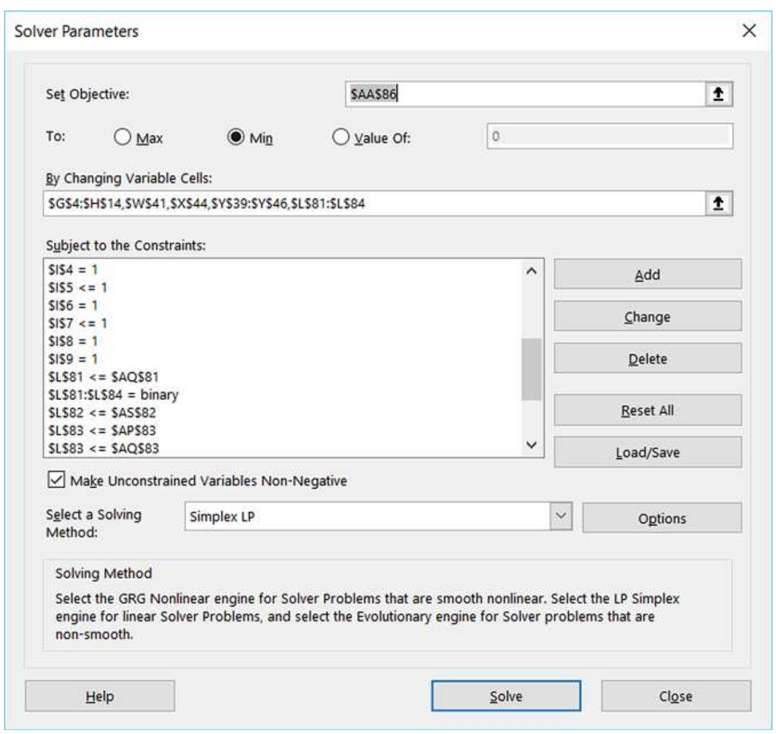


Figure 20.37 The Solver Parameters window for the ISD-IFAM example (part 2).

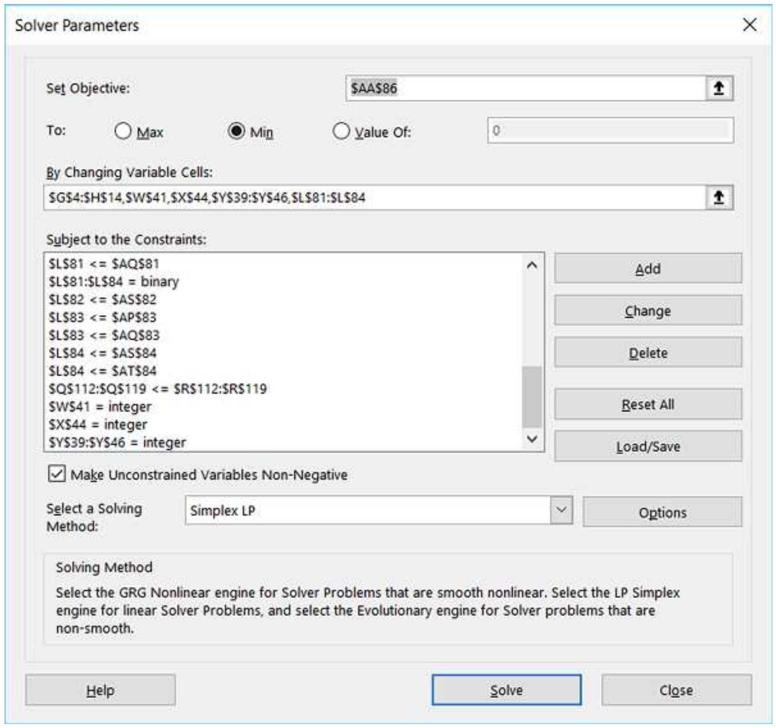


Figure 20.38 The Solver Parameters window for the ISD-IFAM example (part 3).

Next, the different constraints of the problem are defined. These constraints include the coverage constraints for the mandatory flights and the optional flights, the balance constraints, the flight capacity constraints, the itinerary demand constraints, and the constraints that link the optional flights to their corresponding itineraries. This final set of constraints ensure that if any optional flight is removed from the schedule, all itineraries that include this flight are removed from the schedule, and vice versa. Considering such setting for the problem, it is solved as indicated in Figure 20.39.

20.5 Solution Interpretation

The optimal solution of the problem is given in Figures 20.40 through 20.50. As shown in Figure 20.40, the solution recommends that the two optional flights are included in the schedule. The two flights LAX-BOS and BOS-LAX are assigned to fleet type e_1, while the remaining four flights are assigned to fleet type e_2.

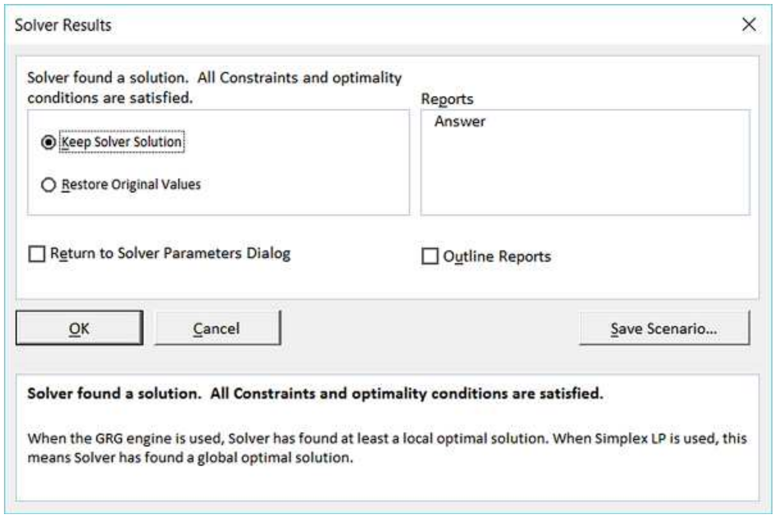


Figure 20.39 The Solver Results window for the ISD-IFAM example.

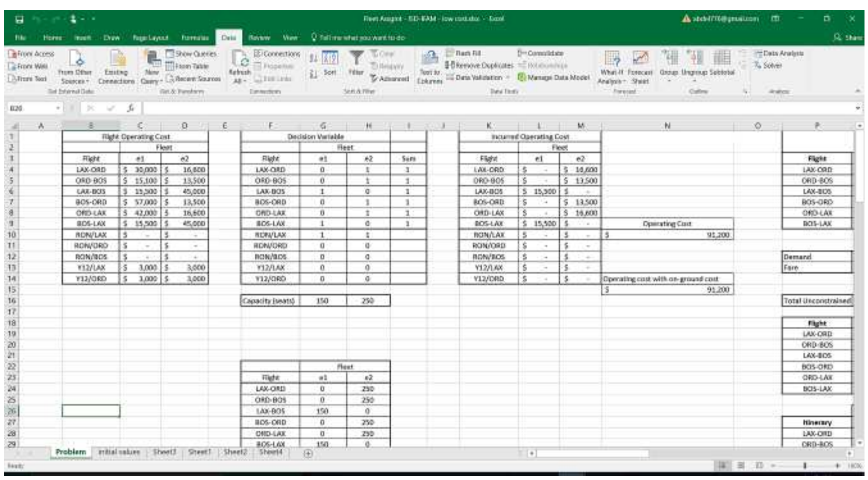


Figure 20.40 The values of the decision variables at the optimal solution for the ISD-IFAM example.

The solution also requires that one aircraft of each fleet type is to remain overnight at LAX. Figure 20.41 shows that all balance constrains are satisfied.

Figure 20.42 shows the results of the spill variables. As shown in the figure, the model suggests that 150 passengers are spilled from itinerary LAS-BOS to itinerary LAX-ORD-BOS. Another 150 passengers are spilled from itinerary

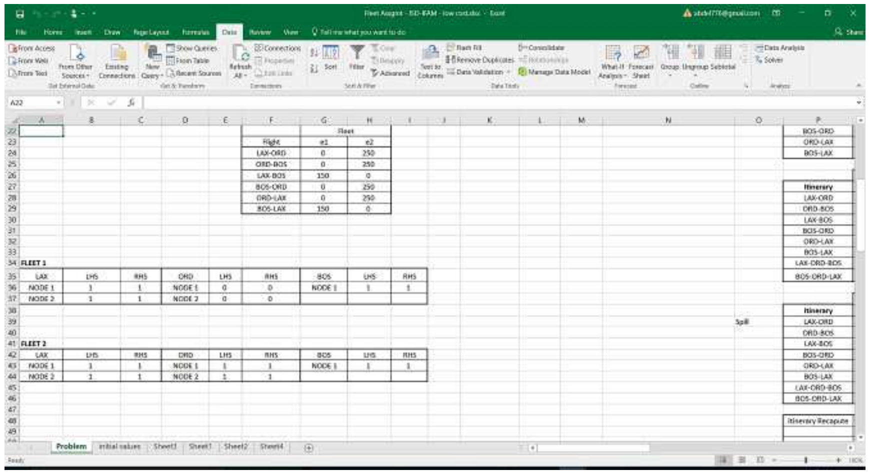


Figure 20.41 The values of the RHS and LHS for the balance constraints at the optimal solution for the ISD-IFAM example.

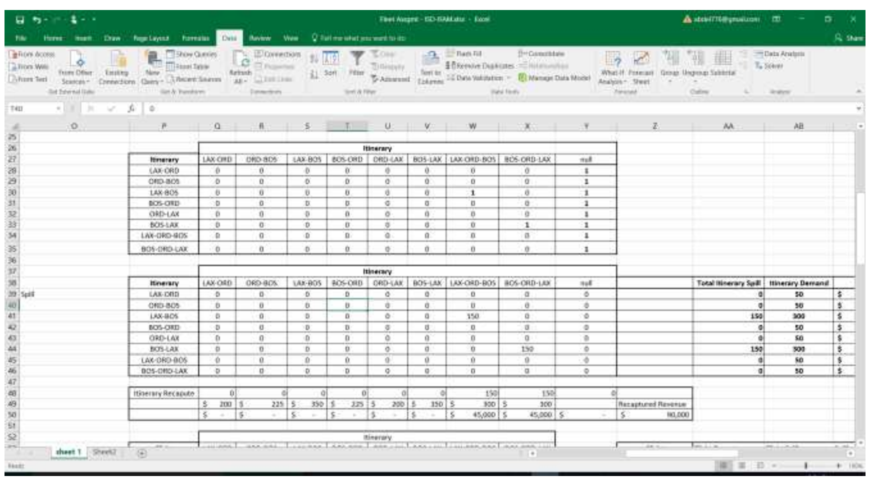


Figure 20.42 The values of the spill variables at the optimal solution for the ISD-IFAM example.

BOS-LAX to itinerary BOS-ORD-LAX. As shown in Figure 20.43, the recaptured revenue is \$90,000, and the spilled revenue is \$105,000. The recaptured and spilled demand at the flight level is given in Figure 20.44.

Figure 20.45 shows the values of the decision variable Z_q for each optional itinerary. As shown in column L (rows 81–84), $Z_q = 1\ \forall q$. In other words, all optional itineraries are included in the schedule. The value of the objective

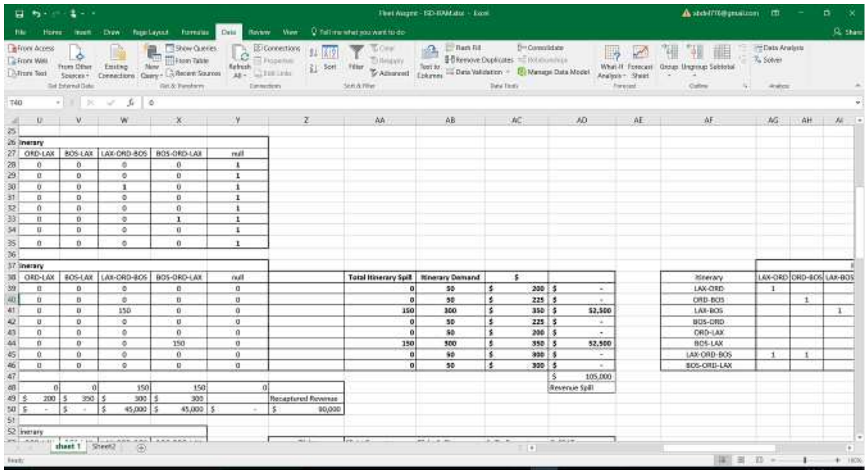


Figure 20.43 The value of the total revenue spill and the total revenue recaptured at the optimal solution for the ISD-IFAM example.

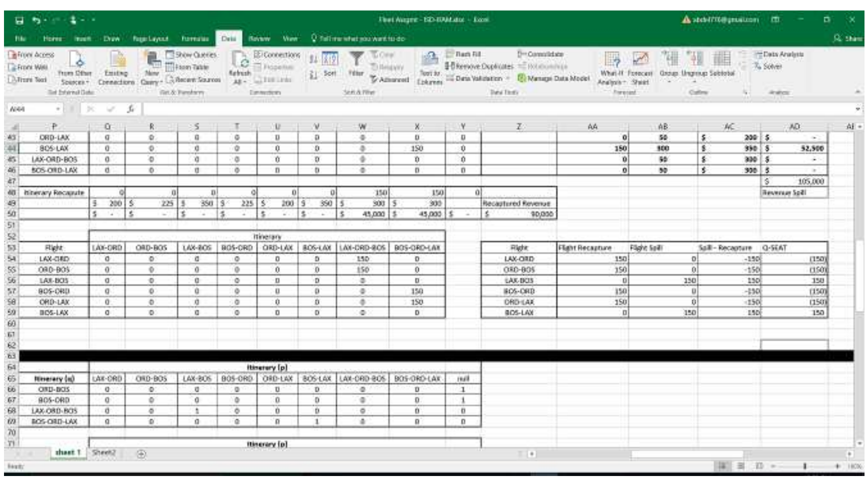


Figure 20.44 The value of demand spill and recapture at the flight level at the optimal solution for the ISD-IFAM example.

function is given in cell AA86, as shown in Figure 20.46, which is equal to \$106,200. Figures 20.47 through 20.50 give the different values of the remaining elements of the problem. As shown in these figures, all constraints are satisfied at the optimal values.

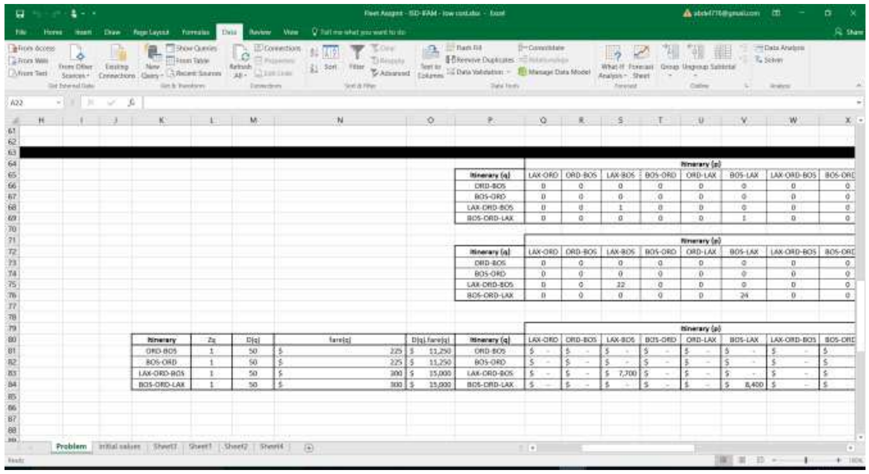


Figure 20.45 The values of the decision variable Z_q for each optional itinerary at the optimal solution for the ISD-IFAM example.

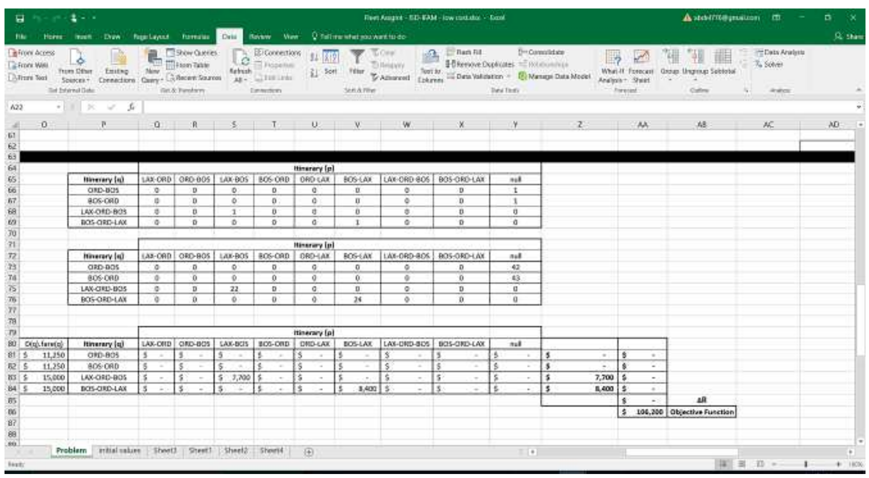


Figure 20.46 The value of the objective function at optimal solution for the ISD-IFAM example.

20.6 Changing the Operations Cost

This example is solved again after significantly increasing the operations cost of the two optional flights ORD-BOS and BOS-ORD for the two fleet types considered in the problem. The new cost values are given in Table 20.5.

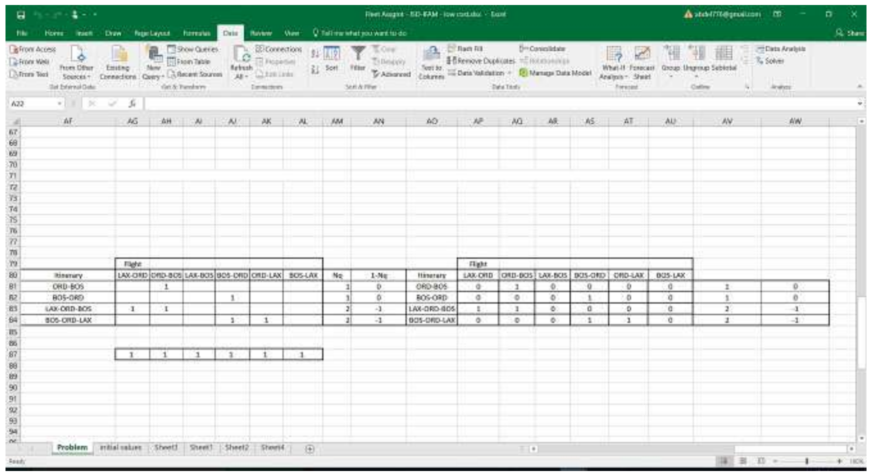


Figure 20.47 The values of the problem elements at optimal solution for the ISD-IFAM example (part 1).

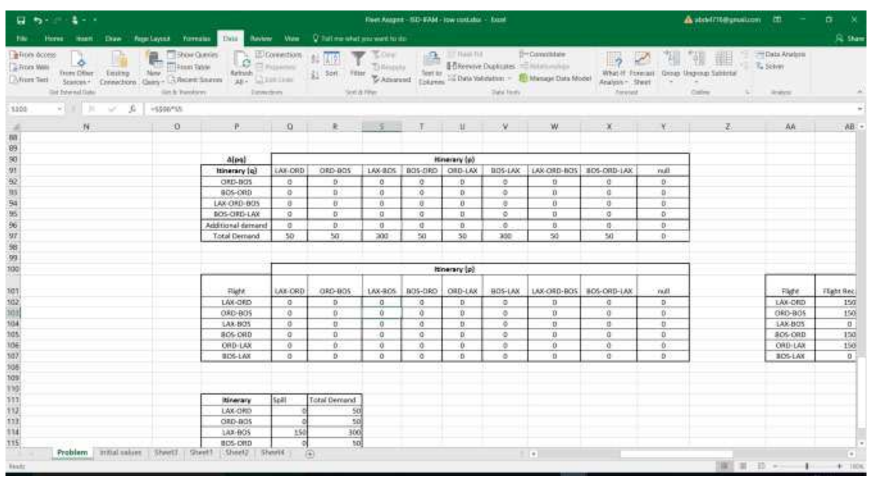


Figure 20.48 The values of the problem elements at optimal solution for the ISD-IFAM example (part 2).

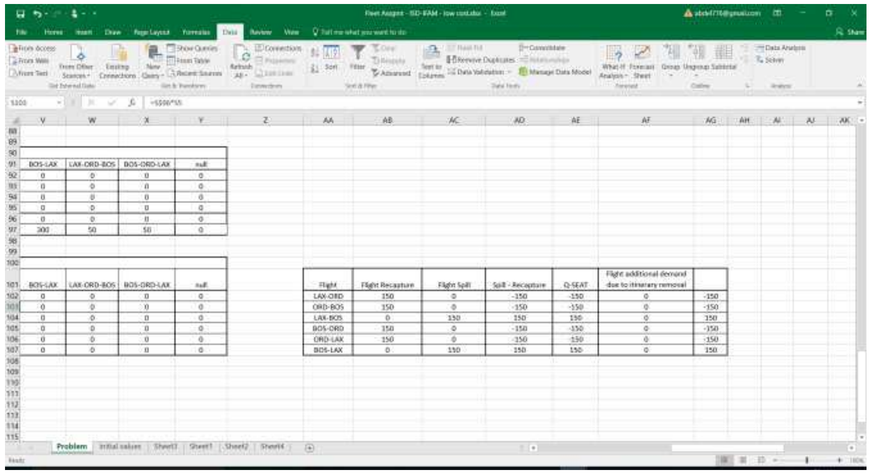


Figure 20.49 The values of the problem elements at optimal solution for the ISD-IFAM example (part 3).

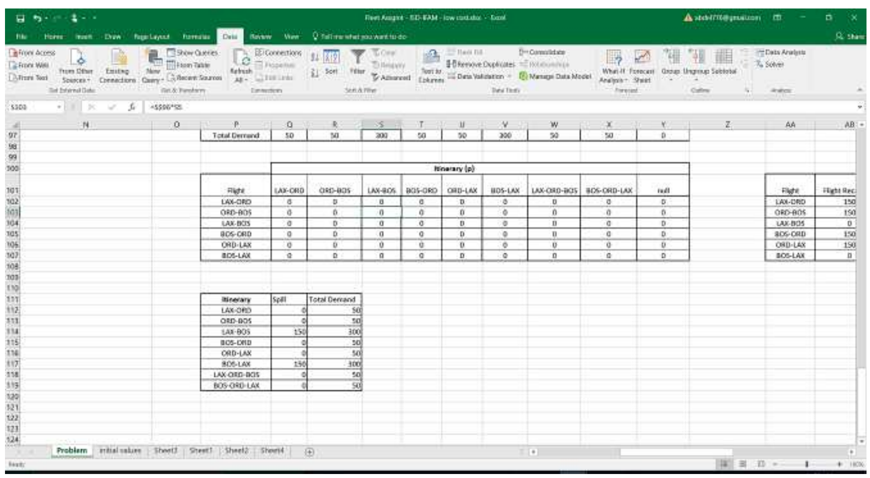


Figure 20.50 The values of the problem elements at optimal solution for the ISD-IFAM example (part 4).

Table 20.5 The new operating cost of each flight for each fleet type for the ISD-IFAM problem.

Flight		Fleet	
		e1 ($)	**e2 ($)**
LAX-ORD	$f1$	30,000	16,600
ORD-BOS	$f2$	95,000	95,000
LAX-BOS	$f3$	15,500	45,000
BOS-ORD	$f4$	95,000	95,000
ORD-LAX	$f5$	42,000	16,600
BOS-LAX	$f6$	15,500	45,000

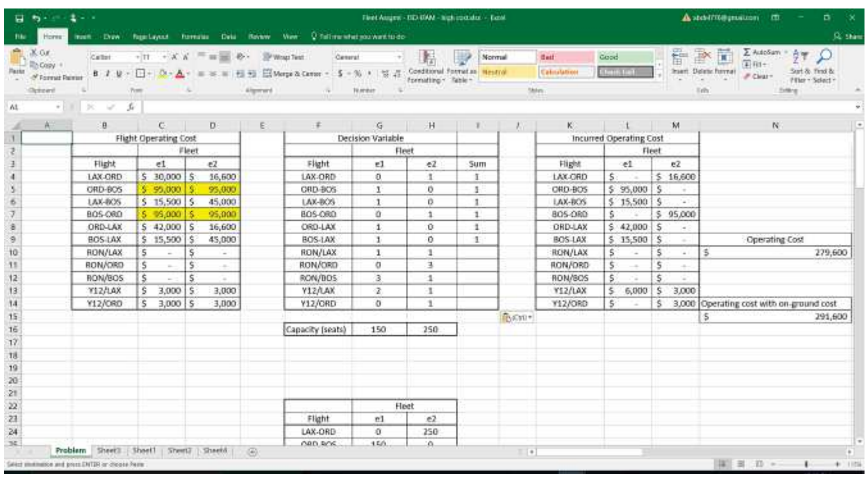


Figure 20.51 The decision variables and formulas for cost calculations for the modified ISD-IFAM example.

As given in the table, the costs of the two flights ORD-BOS and BOS-ORD are increased to \$95,000, when assigned to fleet type e_1 or fleet type e_2.

Similar to the previous example, Figures 20.51 through 20.54 show several snapshots for the modified ISD-IFAM problem. Except the assignment costs of the two flights ORD-BOS and BOS-ORD, all initial values are similar to the ones given in the previous example. The problem is again solved using the Excel Solver as shown in Figures 20.55 and 20.56.

Figures 20.57 through 20.68 give the values of the different elements of the problem at optimality. As shown in Figure 20.57, the model suggests that the

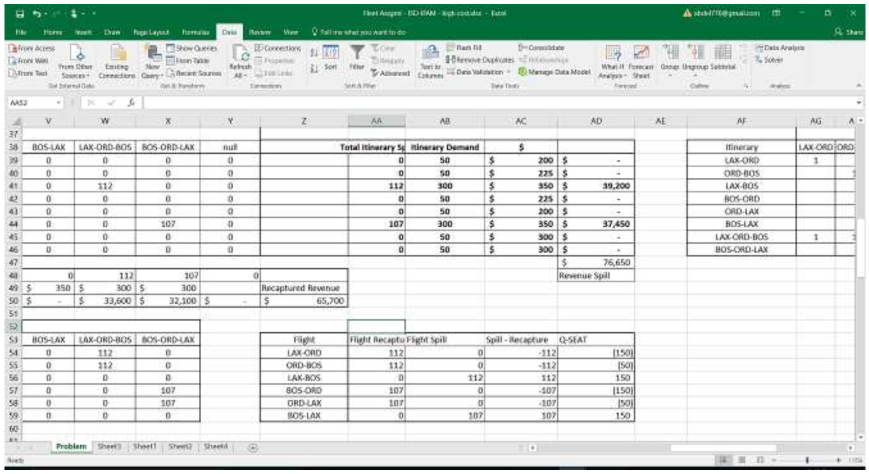


Figure 20.52 The revenue spill and the revenue recaptured for the modified ISD-IFAM example.

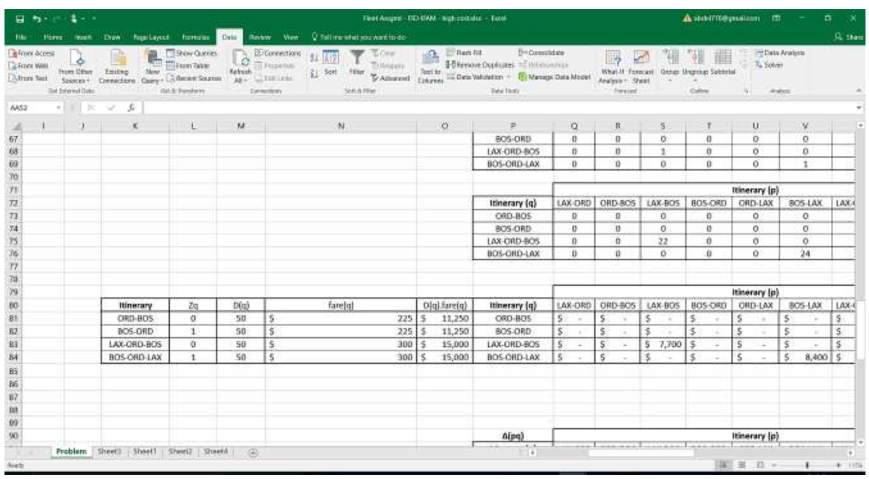


Figure 20.53 The revenue change due to the suggested deletion of the optional itineraries for the modified ISD-IFAM example.

flights ORD-BOS and BOS-ORD are to be removed from the schedule as they are expensive to operate. All four mandatory flights are assigned to fleet type e_2. The solution requires two aircraft of type e_2 to remain overnight at LAX and one aircraft of type e_2 to remain on the ground between the two interconnection nodes at ORD. Figure 20.58 shows that all balance constraints are satisfied at the optimal solution. Figure 20.59 shows the spilled demand among

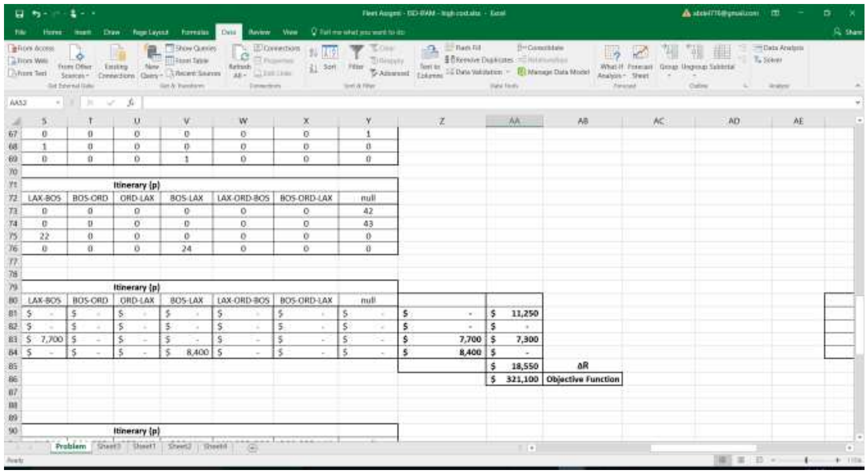


Figure 20.54 The initial value of the objective function for the modified ISD-IFAM example.

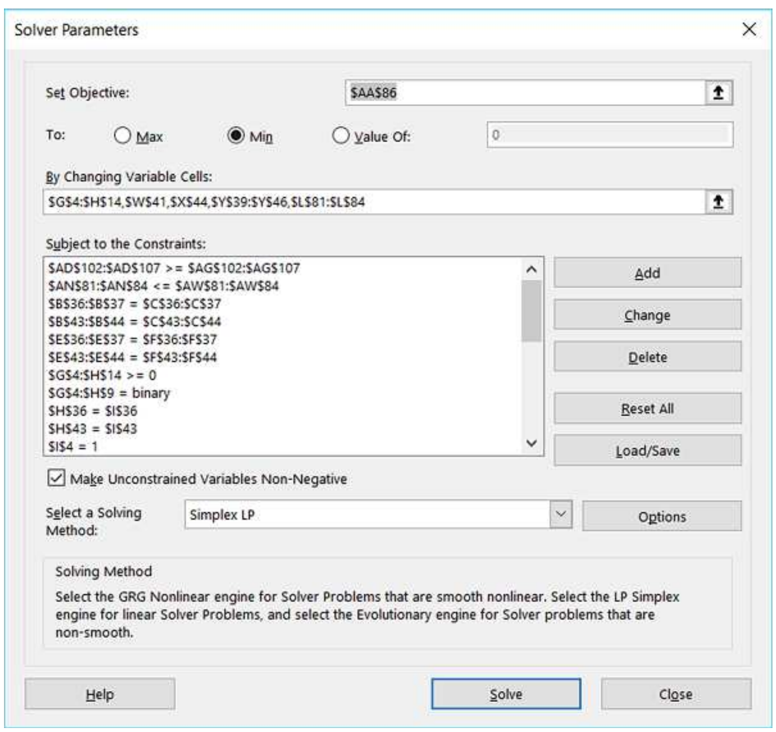


Figure 20.55 The Solver Parameters window for the modified ISD-IFAM example.

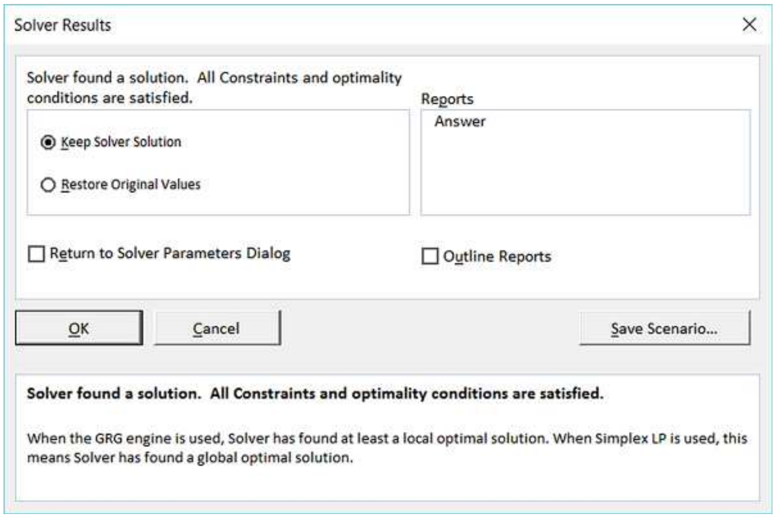


Figure 20.56 The Solver Results window for the modified ISD-IFAM example.

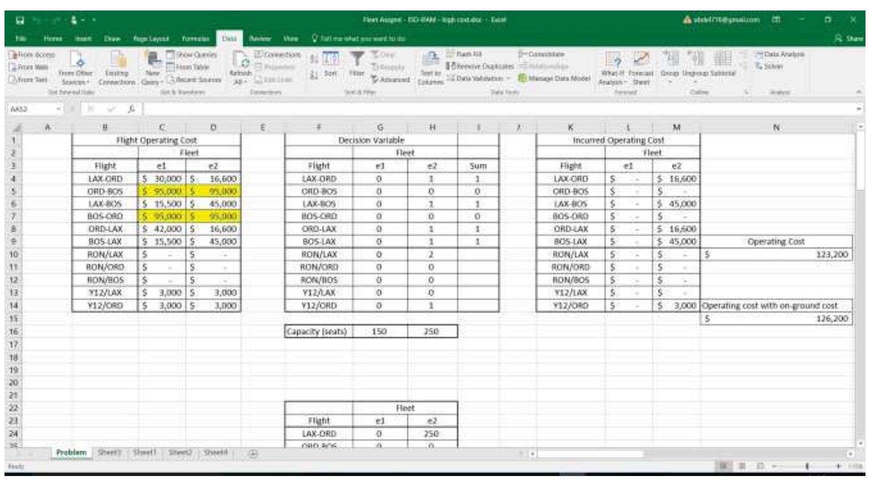


Figure 20.57 The values of the decision variables at the optimal solution for the modified ISD-IFAM example.

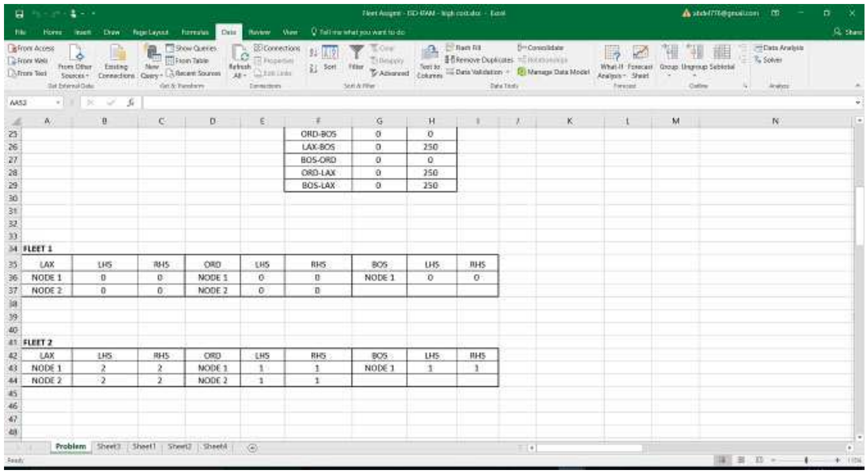


Figure 20.58 The values of the RHS and LHS for the balance constraints at the optimal solution for the modified ISD-IFAM example.

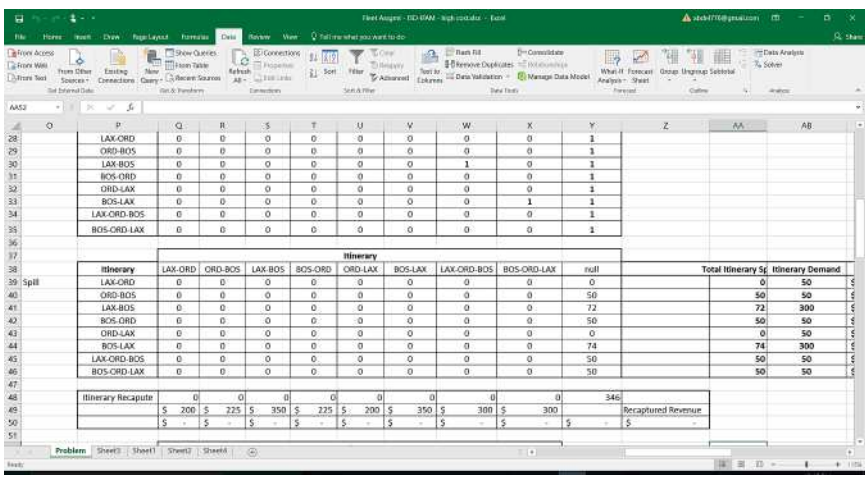


Figure 20.59 The demand spilled between itineraries at the optimal solution for the modified ISD-IFAM example.

itineraries. As shown in the figure, all demand is spilled to the dummy itinerary. The total recaptured revenue is $0, since all demand is spilled to the dummy itinerary. The total spilled revenue is $103,600. The calculation of the spilled revenue is given in Figure 20.60.

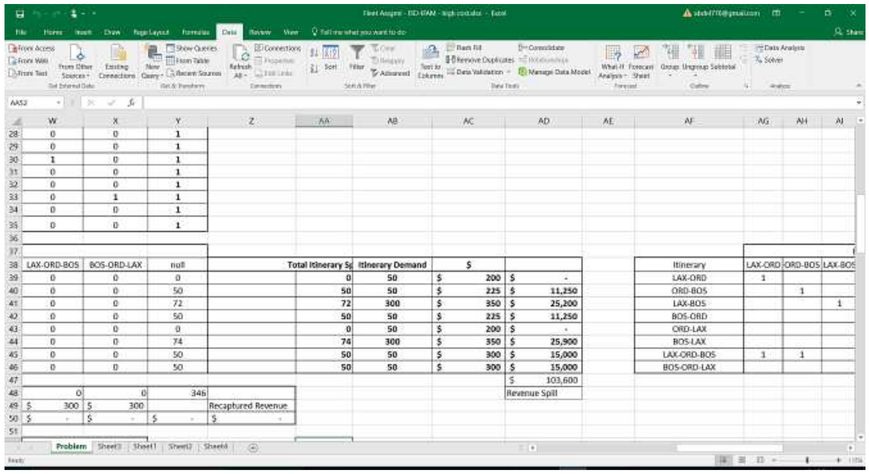


Figure 20.60 The spilled and recaptured revenue at the optimal solution for the modified ISD-IFAM example.

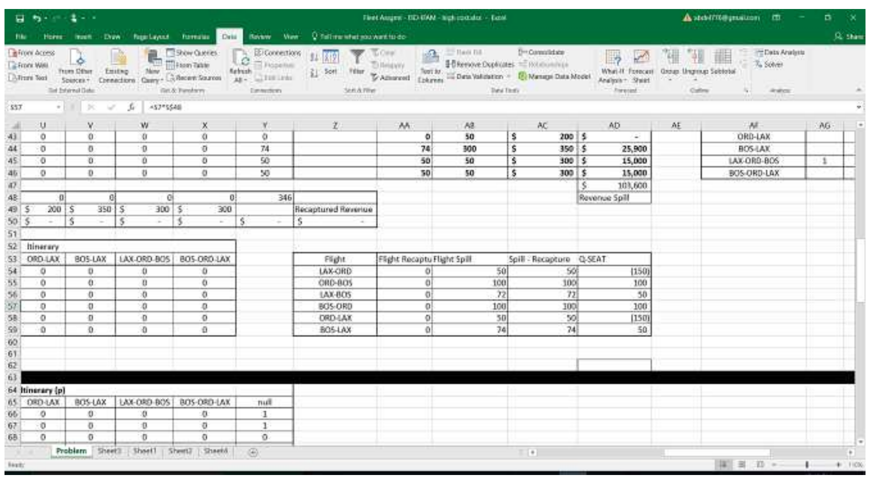


Figure 20.61 The demand spill and recapture at the flight level at the optimal solution for the modified ISD-IFAM example.

Figure 20.61 shows the recaptured and spilled demand at the flight level. Figure 20.62 shows the values of the decision variable Z_q for each optional itinerary. As shown in column L (rows 81–84), $Z_q = 0$, $\forall q$. In other words, all optional itineraries are removed from the schedule. The value of the objective

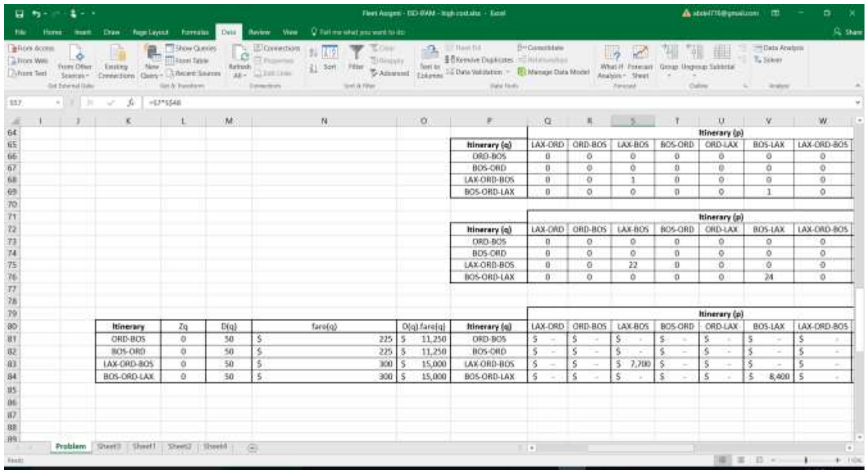


Figure 20.62 The values of the decision variable Z_q for each optional itinerary at the optimal solution for the modified ISD-IFAM example.

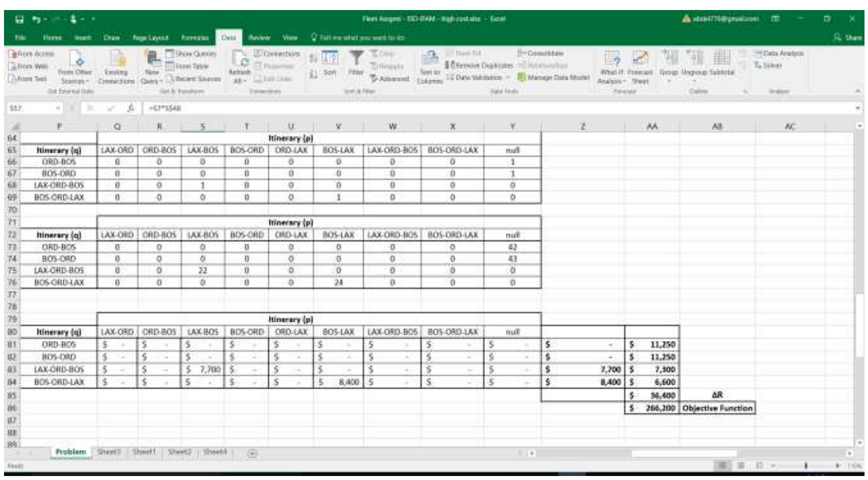


Figure 20.63 The value of the objective function at the optimal solution for the modified ISD-IFAM example.

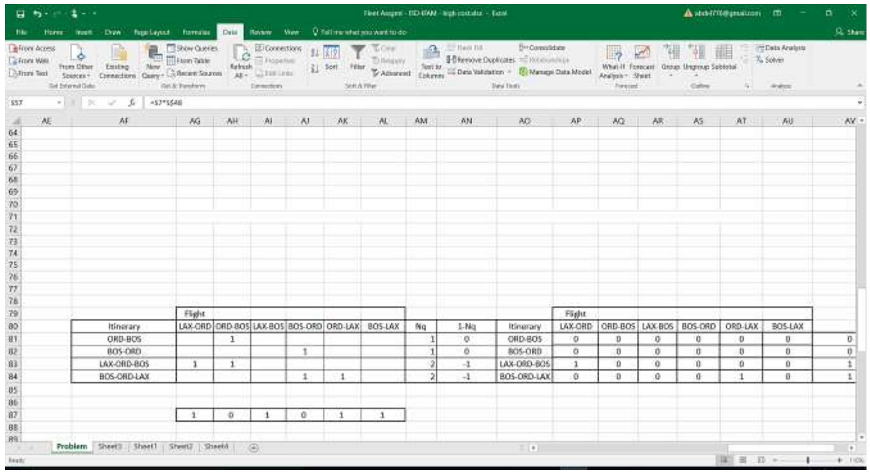


Figure 20.64 The values of the problem elements at optimal solution for the modified ISD-IFAM example (part 1).

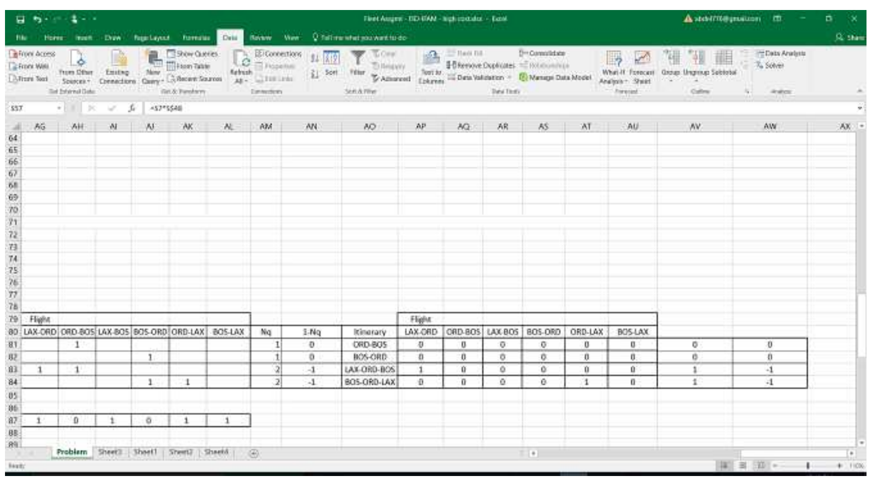


Figure 20.65 The values of the problem elements at optimal solution for the modified ISD-IFAM example (part 2).

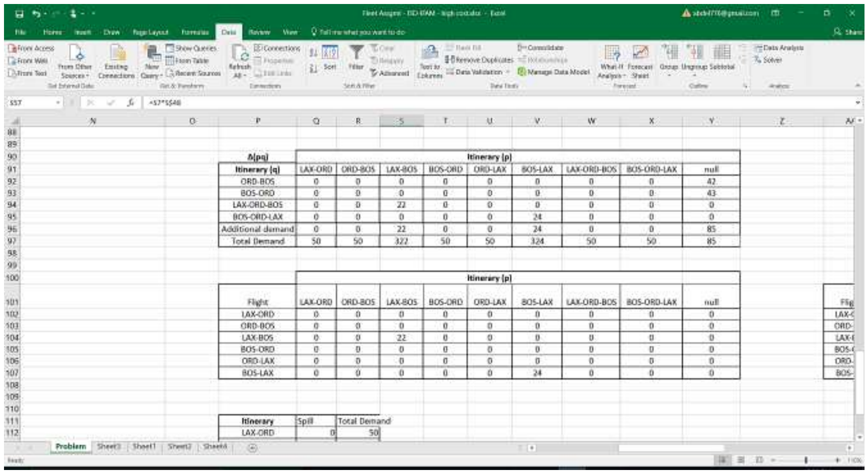


Figure 20.66 The values of the problem elements at optimal solution for the modified ISD-IFAM example (part 3).

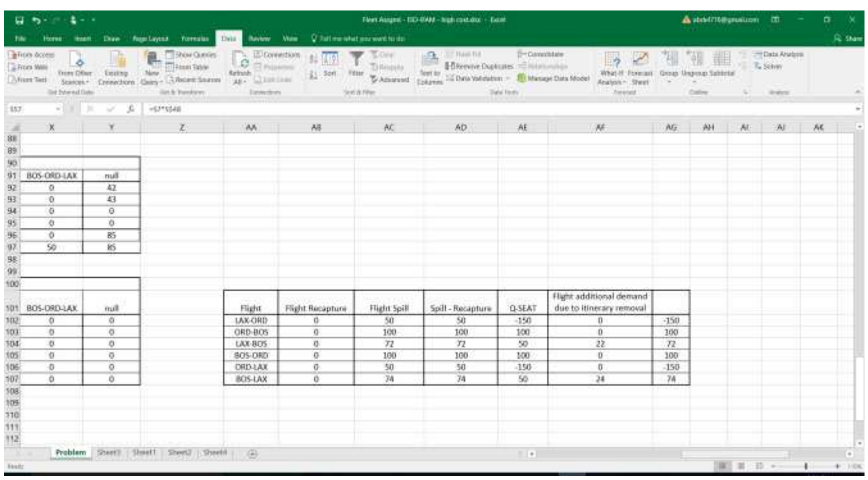
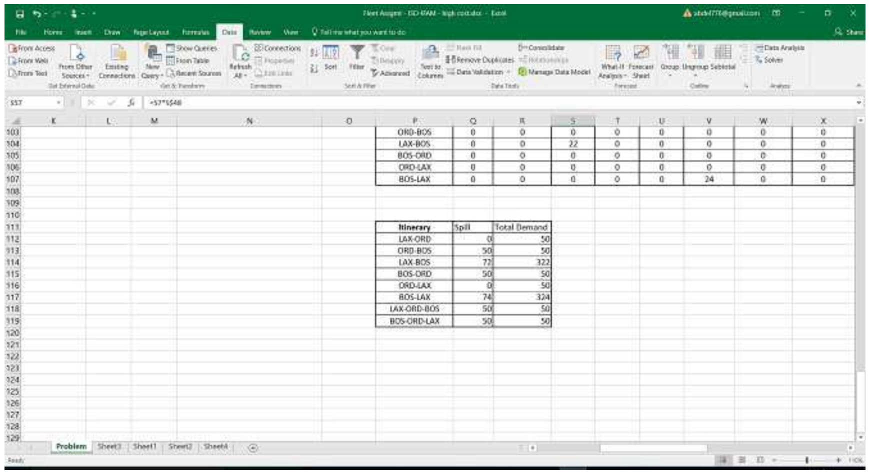


Figure 20.67 The values of the problem elements at optimal solution for the modified ISD-IFAM example (part 4).



P	Q	R	S	T	U	V	W	X
ORD-BOS	0	0	0	0	0	0	0	0
LAX-BOS	0	0	22	0	0	0	0	0
BOS-ORD	0	0	0	0	0	0	0	0
ORD-LAX	0	0	0	0	0	0	0	0
BOS-LAX	0	0	0	0	0	24	0	0

Itinerary	Spill	Total Demand
LAX-ORD	0	50
ORD-BOS	50	50
LAX-BOS	72	322
BOS-ORD	50	50
ORD-LAX	0	50
BOS-LAX	74	324
LAX-ORD-BOS	50	50
BOS-ORD-LAX	50	50

Figure 20.68 The values of the problem elements at optimal solution for the modified ISD-IFAM example (part 5).

function is given in cell AA86, as shown Figure 20.63, which is equal to $266,200. Finally, Figures 20.64 through 20.68 give the different values of the problem elements at optimality. As shown in these figures, all constraints are satisfied at the optimal solution.