CSCE 221 Cover Page

Homework Assignment #3

Due June 26 at 23:59 pm to eCampus

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more on Aggie Honor System Office website: `http://aggiehonor.tamu.edu/`

Type of sources				
People				
Web pages (provide URL)				
Printed material				
Other Sources				

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.
On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.

Your Name Abdullah Ahmad   Date 6/27/2020

Homework 3 (100 points)

due June 26 at 11:59 pm to eCampus.

Write clearly and give full explanations to solutions for all the problems. Show all steps of your work.

Reading assignment:

- Balanced Binary Search Trees
- Skip Lists
- Hash Tables
- Heap and Priority Queue
- Graphs

Problems.

1. (10 points) For the following statements about red-black trees, provide a justification for each true statement and a counterexample for each false one.

   (a) A subtree of a red-black tree is itself a red-black tree.
   False, if a subtree has a red root. The root must be a black node for a red-black tree by the definition of a red-black tree

   (b) The sibling of an external node is either external or red.
   True, if an external node has a black internal sibling then the 2 siblings don't have the same black depth, and in a red-black tree all leaves must have the same black depth.

   (c) There is a unique 2-4 tree associated with a given red-black tree.
   True, there is unique 2-4 tree assosciated with a red-black tree. Every node with red children is represented as a 4 node and each node that has one red child is represented as a 3 node, and a node with no red children is a 2 node.

   (d) There is a unique red-black tree associated with a given 2-4 tree.
   False, since there is two possible representations of a 3-node in a red black tree.

2. (10 points) Modify this skip list after performing the following series of operations: `erase(38)`, `insert(48,x)`, `insert(24,y)`, `erase(42)`. Provided the recorded coin flips for `x` and `y`.

$-\infty$	—-	—–	—–	—–	—–	—–	$+\infty$
$-\infty$	—	—–	17	—–	—–	—–	$+\infty$
$-\infty$	—-	—–	17	—–	—–	—	$+\infty$
$-\infty$	—-	—–	17	—–	—–	—	$+\infty$
$-\infty$	—-	12	17	—–	48	—	$+\infty$
$-\infty$	24	12	17	20	48	—	$+\infty$

For X 2 flips occured until head (1 head and then 1 tails) and for Y 1 flip occured (1 tails,).

3. (10 points) Draw the 17-entry hash table that results from using the has function: $h(k) = ((3k + 5) \mod 11)$, to hash the keys: 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5, assuming collisions are handled by double hashing using the secondary hash function: $h_s(k) = (7 - (k \mod 7))$

0	13
1	94
2	
3	
4	39
5	44
6	
7	
8	12
9	16
10	20
11	88
12	
13	23
14	11
15	5
16	

4. (10 points) An airport is developing a computer simulation of air-traffic control that handles events such as landings and takeoffs. Each event has a *time-stamp* that denotes the time when the event occurs. The simulation program needs to efficiently perform the following two fundamental operations:

- Insert an event with a given time-stamp (that is, add a future event)
  - Extract the event with a smallest time-stamp (that is, determine the next event to process)

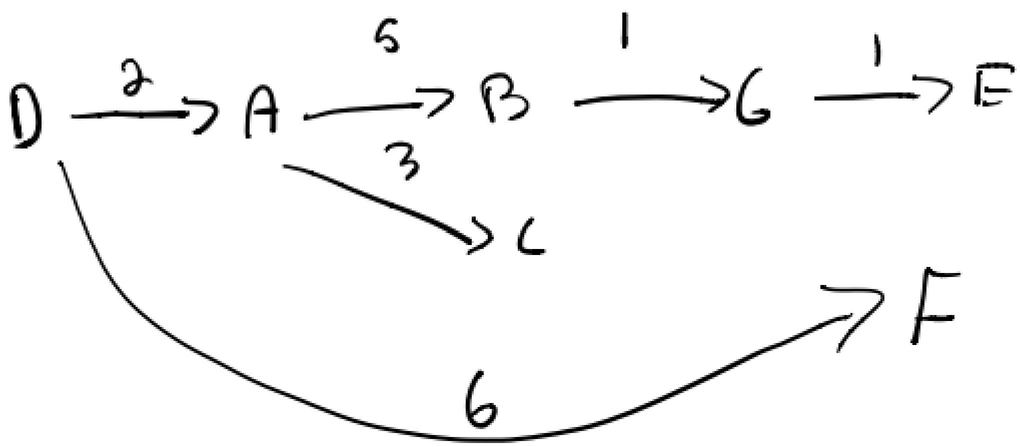
  Which data structure should be used for the above operations? Why? Provide big-O asymptotic complexity for each operation.

  A binary heap data structure can be used, in a minimum priority queue. Because it will have a O(logn) complexity which is fast and it supports buth operations easily.

5. (15 points) Find the shortest path from D to all other vertices for the graph below.

1. Illustrate the minimum priority queue at each iteration Dijkstra's algorithm.
2. Draw the Shortest Path Tree.
3. What is the running time of the Dijkstra's algorithm under the assumption that the graph is implemented based on an adjacency list and the minimum priority queue is implemented based on a binary heap?

iter	P[d]	P[a]	P[b]	P[c]	P[g]	P[e]	P[f]
0	0	∞	∞	∞	∞	∞	∞
1		2(D)	∞	∞	∞	∞	6(D)
2			7(A)	5(A)	∞	∞	∞
3				9(B)	8(B)	10(B)	∞
4						9(G)	11(E)
5							10(E)

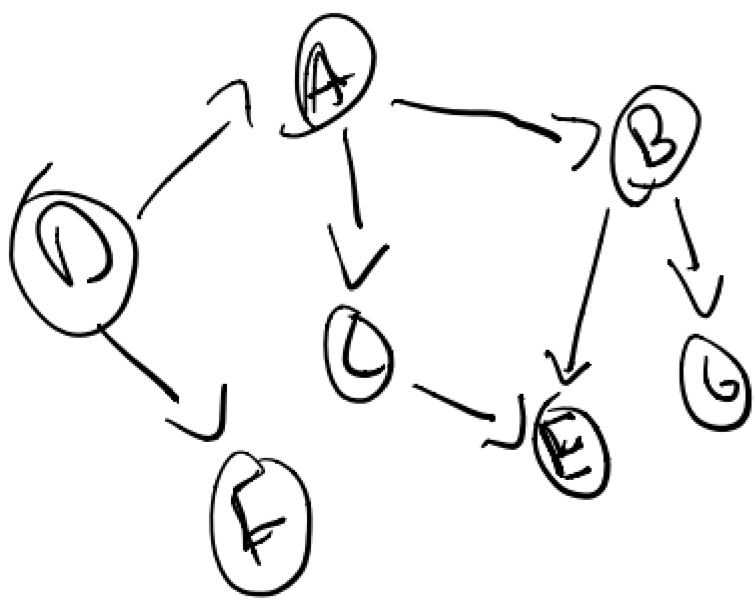


The running time of the Dijkstra's algorithm is O(E LogV) where E is the edges and V is the vertices of the graph.

6. (15 points) Find the shortest unweighted path from D to all other vertices for the graph below. You can measure the distance from D by number of edges.

1. Which graph algorithm can solve the problem?
   Dijkstra's algorthim adapted with breadth first search
2. Draw the Shortest Path Tree.

Source Node	Destination	Path	Cost
D	A	D->A	2
	B	D->A->B	2
	C	D->A->C	2
	E	D->A->B->E or D->A->C->E	3
	G	D->A->B->G	3
	F	D->F	1



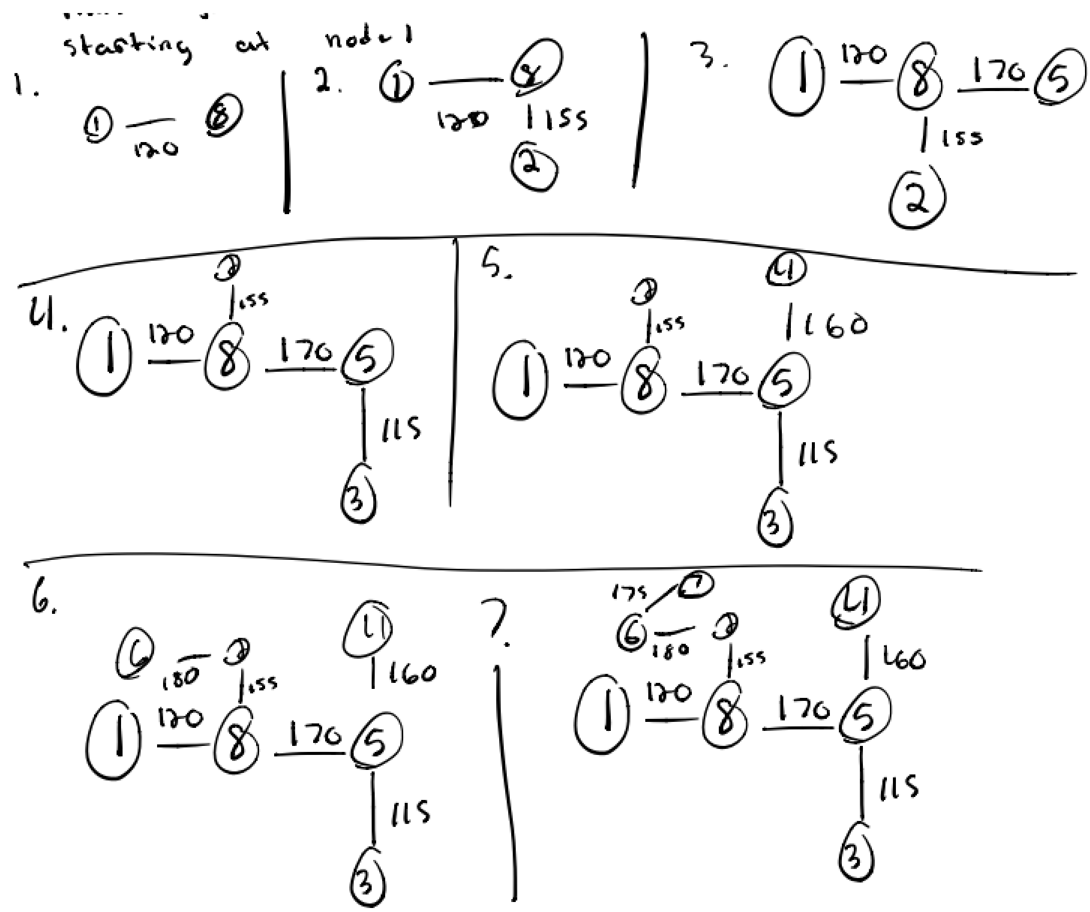
7. (10 points) Apply the Dijkstra's algorithm to find the shortest path from the vertex A to all the vertices in the graph below. Does the algorithm return a correct output? Justify your answer using the Dijkstra's Theorem.

In this case the aglorithm does work, but the Dijkstra's algorithm may fail whenever a graph as a negative weight edge. The reason it may fail with a negative edge is because it can fall into a loop where it goes around the negative cycle infinitely, but that is not the case here.

8. (20 points) There are eight small island in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other one via one or more bridges. The cost of bridge construction is proportional to its length. The distance between pairs of islands are given in the following table.

1. Illustrate the Prim's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

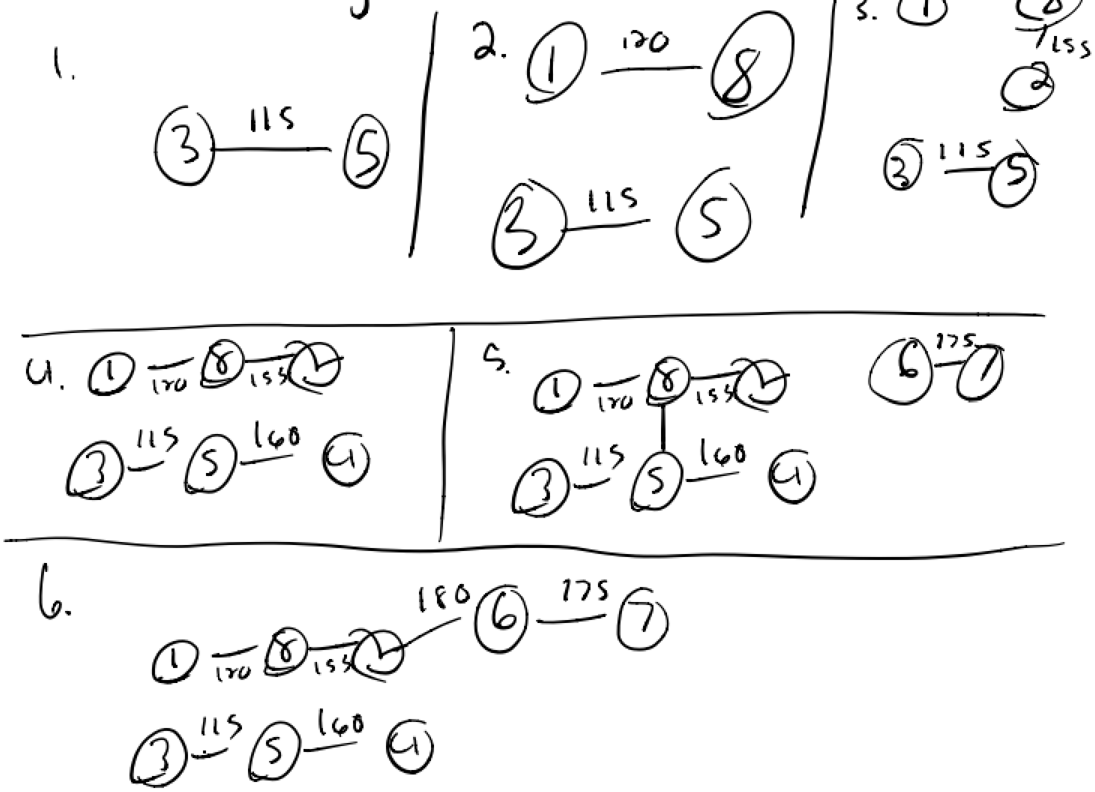


The length of the bridges is given by the edges.

(a) Illustrate the Kruskal's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

Kruskal's algorithm



The length of the bridges is given by the edges.