Performance Comparison of Particle and Extended Kalman Filter Algorithms for GPS C/A Code Tracking and Interference Rejection[1]

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Abstract — **Tracking algorithms for the GPS C/A code with an interference rejection capability based on state estimation technique are considered. A swept tone jamming signal is modeled as an autoregressive (AR) process and estimated jointly with the timing delay. A novel Mean-Value Theorem Particle Filter (MVT-PF) is applied to address the highly nonlinear measurement model and its performance is compared to that of conventional nonlinear estimation algorithms. The simulation results show the MVT-PF outperforms the conventional Extended Kalman Filter and Gaussian Sum Filter.**

I. Summary

The GPS system enables positioning by measuring times of arrivals of direct-sequence signals transmitted from multiple satellites [1]. Typically, the timing phase of the pseudorandom noise sequence is first acquired coarsely by means of a correlation technique and then accurately tracked using a delay-locked loop (DLL) [2]. However, conventional code acquisition and tracking algorithms have little immunity to narrowband jamming.

Previous methods for synchronization of direct-sequence signals in narrowband interference include [3, 4, 5]. In [4] a transform domain rejection method is used to improve acquisition time performance. A median filter is utilized in [5] to mitigate tone jamming in code acquisition. Here, we consider the use of nonlinear state estimation filters to track the GPS C/A code delay in the presence of a swept tone jammer.

Classical approaches to the state estimation problem for a nonlinear stochastic system include the extended Kalman filter (EKF) and the Gaussian sum filter (GSF) [6]. Recently, particle filter techniques have been proposed with promising results [7, 8] and applied to digital communications in [9].

In this paper, the tone jamming signal is modeled by an autoregressive (AR) process and whitened by estimating its AR coefficients jointly with the timing delay. A novel Mean-Value Theorem-Based Particle Filter (MVT-PF) technique [10] is applied to this case and its tracking performance is compared to that of existing EKF and GSF algorithms by simulations. The MVT-PF is structurally close to GSF in that it consists of a bank of EKFs with evolving weights, but it is capable of introducing random jitter to the state estimate based on mean value theorem arguments, leading to potentially better convergence properties. The simulation results here show that the MVT-PF yields an average absolute timing error less than that of the EKF and GSF.

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II. Signal Model

The GPS signal, after downconversion, sampling and lowpass filtering, can be represented as

$$r_{lp}(k) = a(k)PN_{lp}(kT_s + \tau(k)) + j(k) + n(k), \tag{1}$$

where $\tau(k)$ is the C/A code time delay. The signal bandwidth is assumed to be $\frac{2}{T_c}$ and the Nyquist sampling interval T_s equals $\frac{T_c}{4}$. The amplitude sequence $a(k)$ is the product of the binary GPS navigation message and a flat fading amplitude term. The noise samples $n(k)$ are circular white Gaussian with variance σ_n^2. The lowpass jammer $j(k)$ can be modeled as

$$j(k) = \sqrt{J}\exp(j2\pi(\Delta f + \dot{\Delta} f kT_s)kT_s), \tag{2}$$

where J is the tone jammer power, Δf is the frequency offset, and $\dot{\Delta} f$ is the sweep frequency rate. The lowpass filtered PN signal PN_{lp} is given by [11]

$$PN_{lp}(t) = \sum_{m=-\infty}^{\infty}\sum_{k=0}^{1022} c_k \frac{1}{\pi}\left[\mathrm{Si}\left(2\pi\frac{2}{T_c}(t - kT_c - mT_{ca})\right) - \mathrm{Si}\left(2\pi\frac{2}{T_c}(t-(k+1)T_c - mT_{ca})\right)\right], \tag{3}$$

where $c_k \in \{0, 1\}$ is the binary PN sequence determined by the satellite, T_c is the chip duration, and T_{ca} is the period of the C/A code.

The signal-to-noise ratio (SNR) in dB is defined by

$$\mathrm{SNR} = 10\log_{10}\frac{|a|^2}{\sigma_n^2/2}. \tag{4}$$

The jammer-to-signal power ratio, J/S is given by

$$J/S = 10\log_{10}\frac{J}{|a|^2}. \tag{5}$$

The jammer plus noise process is assumed to be Gaussian and autoregressive of order N_α [12]. That is,

$$j(k) + n(k) \approx -\sum_{m=1}^{N_\alpha}\alpha_m(j(k-n) + n(k-m)) + \epsilon(k), \tag{6}$$

where $\epsilon(k)$ is a circular white Gaussian process with variance σ_ϵ^2. Using the model (1), an additive white Gaussian noise model for the received signal is obtained as

$$\begin{aligned} r_{lp}(k) \approx & -\sum_{m=1}^{N_\alpha} \alpha_m r_{lp}(k-m) \\ & + \sum_{m=0}^{N_\alpha} \alpha_m a(k) PN_{lp}((k-m)T_s + \tau(k)) + \epsilon(k). \end{aligned} \tag{7}$$

III. Summary of EKF and GSF Algorithms for C/A Code Tracking

For mathematical simplicity, a scalar measurement sequence $r(n) \triangleq r_{lp}(n)$ is considered. $r(n)$ can also be represented by

$$r(n) = h(\mathbf{x}(n)) + \epsilon(n), \tag{8}$$

where $\mathbf{x}(n)$ is a complex state vector defined as the concatenation of the signal amplitude, delay, and jammer AR coefficients.

$$\mathbf{x}(n) = [\tau(n),\ a(n),\ \alpha_1(n),\ \alpha_2(n),\ \cdots,\ \alpha_{N_\alpha}(n)]^T, \tag{9}$$

and the measurement function is determined from the AR model (7).

$$\begin{aligned} h(\mathbf{x}(n)) = & -\sum_{m=1}^{N_\alpha} \alpha_m r_{lp}(n-m) \\ & + \sum_{m=0}^{N_\alpha} \alpha_m a(n) PN_{lp}((n-m)T_s + \tau(n)) \end{aligned} \tag{10}$$

Note that $r(n)$ is dependent on $r(n-m)$ for $m = 1, 2, \cdots, N_\alpha$. However, this measurement model is still consistent with the Kalman filter derivation because only $r(n)$ *conditioned on* $\mathbf{r}^{n-1} \triangleq \{r(n-1), r(n-2), \cdots, r(1)\}$ is involved.

By linearizing the measurement function as

$$h(\mathbf{x}(n)) \approx h(\hat{\mathbf{x}}(n|n-1)) + \mathbf{h}(\hat{\mathbf{x}}(n|n-1))^T(\mathbf{x}(n) - \hat{\mathbf{x}}(n|n-1)), \tag{11}$$

with the Jacobian $\mathbf{h}(\mathbf{x})^T$ defined by the row vector

$$\mathbf{h}(\mathbf{x})^T = \frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}, \tag{12}$$

the complex-valued EKF equations are given by [12]

$$\begin{aligned} & \mathbf{P}(n|n) = \mathbf{P}(n|n-1) \\ & - \mathbf{P}(n|n-1)\mathbf{h}(n)^* \Sigma(n|n-1)^{-1} \mathbf{h}(n)^T \mathbf{P}(n|n-1) \\ & \hat{\mathbf{x}}(n|n) = \hat{\mathbf{x}}(n|n-1) \\ & + \mathbf{P}(n|n-1)\mathbf{h}(n)^* \Sigma(n|n-1)^{-1}[r(n) - h(\hat{\mathbf{x}}(n|n-1))] \\ & \Sigma(n|n-1) = \mathbf{h}(n)^T \mathbf{P}(n|n-1)\mathbf{h}(n)^* + \sigma_\epsilon^2, \end{aligned} \tag{13}$$

where the shorthand $\mathbf{h}(n)$ was used for $\mathbf{h}(\hat{\mathbf{x}}(n|n-1))$. The one-step prediction follows the standard Kalman filter equations

$$\begin{aligned} \hat{\mathbf{x}}(n+1|n) &= \mathbf{F}\hat{\mathbf{x}}(n|n) \\ \mathbf{P}(n+1|n) &= \mathbf{F}\mathbf{P}(n|n)\mathbf{F}^T + \mathbf{Q}, \end{aligned} \tag{14}$$

where the process model parameters are chosen empirically and are given by diagonal matrices

$$\begin{aligned} \mathbf{F} &= \mathrm{diag}\{f_\tau\ f_a\ f_\alpha\ \cdots\ f_\alpha\} \\ \mathbf{Q} &= \mathrm{diag}\{\sigma_\tau^2\ \sigma_a^2\ \sigma_\alpha^2\ \cdots \sigma_\alpha^2\}. \end{aligned} \tag{15}$$

The Gaussian Sum Filter (GSF) is based on the argument that the probability density can be approximated as closely as desired by a weighted sum of Gaussian densities [6]. More precisely, assume that the *pdf* of the state vector, given the cumulative measurement history $\mathbf{r}^{n-1}$ has the form

$$p(\mathbf{x}(n)|\mathbf{r}^{n-1}) = \sum_{i=1}^{N_G} g_i(n-1)\mathcal{N}(\mathbf{x}(n); \mathbf{m}_i(n|n-1), \mathbf{B}_i(n|n-1)). \tag{16}$$

Then it can be shown that the posterior density $p(\mathbf{x}(n)|\mathbf{r}^n)$ is

$$p(\mathbf{x}(n)|\mathbf{r}^n) = \sum_{i=1}^{N_G} g_i(n)\mathcal{N}(\mathbf{x}(n); \mathbf{m}_i(n|n), \mathbf{B}_i(n|n)), \tag{17}$$

where the weights $g_i(n)$ are calculated by

$$g_i(n) = \frac{1}{c} g_i(n-1)\mathcal{N}(r(n); h(\mathbf{m}_i(n|n-1)), \Omega_i(n|n-1)) \tag{18}$$

with a normalization constant c. $\mathbf{m}_i(n|n)$, $\mathbf{B}_i(n|n)$ and $\Omega_i(n|n-1)$ are obtained by Eq. (13) with $\hat{\mathbf{x}}$, $\mathbf{P}$ and $\Sigma(n|n-1)$ replaced by $\mathbf{m}_i$, $\mathbf{B}_i$ and $\Omega_i(n|n-1)$, respectively, for $i = 1, \cdots, N_G$. Similarly, the time-update for $\mathbf{m}_i$ and $\mathbf{B}_i$ follows from Eq. (14).

Now the GSF state estimate and its covariance are easily seen to be

$$\begin{aligned} \hat{\mathbf{x}}^G(n|n) &= \sum_{i=1}^{N_G} g_i(n)\mathbf{m}_i(n|n) \qquad (19) \\ \mathbf{P}^G(n|n) &= \sum_{i=1}^{N_G} g_i(n) \Big\{ \mathbf{B}_i(n|n) + [\hat{\mathbf{x}}^G(n|n) - \mathbf{m}_i(n|n)] \\ & \quad \cdot [\hat{\mathbf{x}}^G(n|n) - \mathbf{m}_i(n|n)]^H \Big\}. \end{aligned}$$

IV. The MVT-PF Algorithm for C/A Code Tracking

The derivation of the MVT-PF in the presence of the AR-modeled jammer is analogous to [10]. The exact linearization of the measurement function is obtained by means of the linearization process $\mathbf{c}_i(n)$ based on the mean-value theorem as

$$\begin{aligned} r(n) = & h(\hat{\mathbf{x}}_i(n|n-1)) \\ & + \mathbf{h}(\hat{\mathbf{x}}_i(n|n-1) + \mathbf{c}_i(n))^T(\mathbf{x}(n) - \hat{\mathbf{x}}_i(n|n-1)), \end{aligned} \tag{20}$$

where $\mathbf{c}_i(n) = (1-\alpha)(\mathbf{x}(n) - \hat{\mathbf{x}}_i(n|n-1))$ for some $\alpha \in [0,1]$, $\hat{\mathbf{x}}_i(n|n-1) \triangleq E\{\mathbf{x}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}\}$ and $\mathbf{c}_i^n \triangleq \{\mathbf{c}_i(n), \cdots, \mathbf{c}_i(0)\}$.

The importance-sampling (IS) estimate is defined by

$$\hat{\mathbf{x}}(n|n) = \sum_{i=1}^{N_P} \hat{\mathbf{x}}_i(n|n) w_i(n). \tag{21}$$

Here, the conditional state estimate $\hat{\mathbf{x}}_i(n|n) \triangleq E\{\mathbf{x}(n)|\mathbf{r}^n, \mathbf{c}_i^n\}$ and the weights $w_i(n)$ are defined by

$$w_i(n) = \frac{1}{c'} \frac{p(\mathbf{c}_i^n|\mathbf{r}^n)}{\pi(\mathbf{c}_i^n|\mathbf{r}^n)}, \tag{22}$$

where c' is a normalization constant. The simulation density $\pi(\mathbf{c}_i^n|\mathbf{r}^n)$ is restricted to the following form to obtain a recursive particle filter [8].

$$\pi(\mathbf{c}_i^n|\mathbf{r}^n) = \pi(\mathbf{c}_i(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1})\pi(\mathbf{c}_i^{n-1}|\mathbf{r}^{n-1}) \tag{23}$$

We consider the choice of simulation density $\pi(\mathbf{c}_i(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1}) = p(\mathbf{c}_i(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1})$ to minimize the variance of the weights w.r.t. the distribution of $\mathbf{c}(n)$ [8]. It is shown in [10] that a practical sampling density can be obtained under suitable approximations as

$$\pi(\mathbf{c}_i(n)|\mathbf{c}_i^{n-1}, \mathbf{r}^n) = \mathcal{N}(\mathbf{c}_i(n); 0, (1-\alpha)^2 \mathbf{P}_i(n|n-1)), \quad (24)$$

where $\mathbf{P}_i(n|n-1) \triangleq E\{[\mathbf{x}(n) - \hat{\mathbf{x}}_i(n|n-1)][\mathbf{x}(n) - \hat{\mathbf{x}}_i(n|n-1)]^H|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}\}$. The weight update is also shown to be

$$w_i(n) = \frac{1}{c''} w_i(n-1)\mathcal{N}(r(n); h(\hat{\mathbf{x}}_i(n|n-1)), \Sigma_i(n|n-1)), \quad (25)$$

where $\Sigma_i(n|n-1) \triangleq E\{|r(n) - h(\hat{\mathbf{x}}_i(n|n-1))|^2|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}\}$ and c'' is a normalization constant. $\mathbf{P}_i$, $\hat{\mathbf{x}}_i$ and Σ_i are obtained by Eq. (13) with $\hat{\mathbf{x}}$, $\mathbf{P}$, $\Sigma(n|n-1)$ and $\mathbf{h}(n)$ replaced by $\hat{\mathbf{x}}_i$, $\hat{\mathbf{P}}_i$, $\Sigma_i(n|n-1)$ and $\mathbf{h}(\hat{\mathbf{x}}_i(n|n-1) + \mathbf{c}_i(n))$, respectively, for $i = 1, \cdots, N_P$. Similarly, the time-update for $\hat{\mathbf{x}}_i$ and $\mathbf{P}_i$ follows from Eq. (14).

V. Simulation Results

The tracking performance of the proposed MVT-PF algorithm is compared with that of the EKF and GSF algorithm by simulation. We use an SNR of -12dB, $J/S = 30$dB, a sweep rate of $\dot{\Delta} f = 2 \times 10^9$ Hz/sec, and a flat fading channel with a doppler spread of 5kHz. The order of the AR model for the jammer is $N_\alpha = 3$ and $N_G = N_P = 5$ are used. The true timing delay $\tau(n)$ is fixed at -1. For the sampling of the particles, $\alpha = 0$ is used to maximize the randomization of $\mathbf{c}_i(n)$. However, noting that $h(\mathbf{x})$ has a nonlinear dependence only on $\tau(n)$, we reset the value of $\mathbf{c}_i(n)$ to be zero except for the first component corresponding to $\tau(n)$, effectively suppressing the randomization for $a(n)$ and $\alpha_1(n), \cdots, \alpha_{N_\alpha}(n)$.

Fig. 1 and Fig. 2 show the averaged absolute timing delay estimates obtained from the EKF, GSF and MVT-PF algorithms over 64 realizations, without and with the jammer present, respectively. The graph clearly demonstrates that the performance of the newly proposed MVT-PF algorithm is superior to that of the conventional algorithms. Also, it can be seen that the GSF gives exactly the same performance as the EKF after about 12,000 chips. This is because the weights for some EKFs in the GSF effectively vanish at that point, while the estimates of the rest of the EKFs simply merge to a single value.

To see how the statistical advantage of the MVT-PF is obtained, an example sample path of the MVT-PF algorithm is depicted in Fig. 3. The circles depict the estimates conditioned on each particle history while the solid line represents the unconditional estimate of the timing delay. The dashed line corresponds to the true timing delay. At the time instant of about $n = 880$, it is observed that the divergent delay estimate is brought back to the neighborhood of the correct value as the weight for the particle near the true value starts to dominate. Moreover, it is seen that the Sampling/Importance Resampling (SIR) [8] procedure soon removes the divergent particles.

Fig. 4 shows the frequency response of the effective time-varying filter defined by the α_ℓ coefficients for interference rejection. Note that the position of the tone is periodically reset to the left band of the C/A signal bandwidth. Fig. 5 shows the case where a frequency-hopping tone jammer with a hop rate of 120 hops/sec is present. It is seen the "notch" of the filter tracks the jamming signals successfully.

VI. Conclusion

The performance of the particle and extended Kalman filter algorithms for GPS C/A code tracking in the presence of tone jamming was considered and compared. An appropriate state-space model for the tracking problem was formulated using an AR model for the tone interferer. The computer simulation showed that the newly proposed MVT-PF algorithm not only successfully rejects the swept tone jammer but also gives better performance than the conventional EKF and GSF algorithms in terms of the mean absolute timing error.

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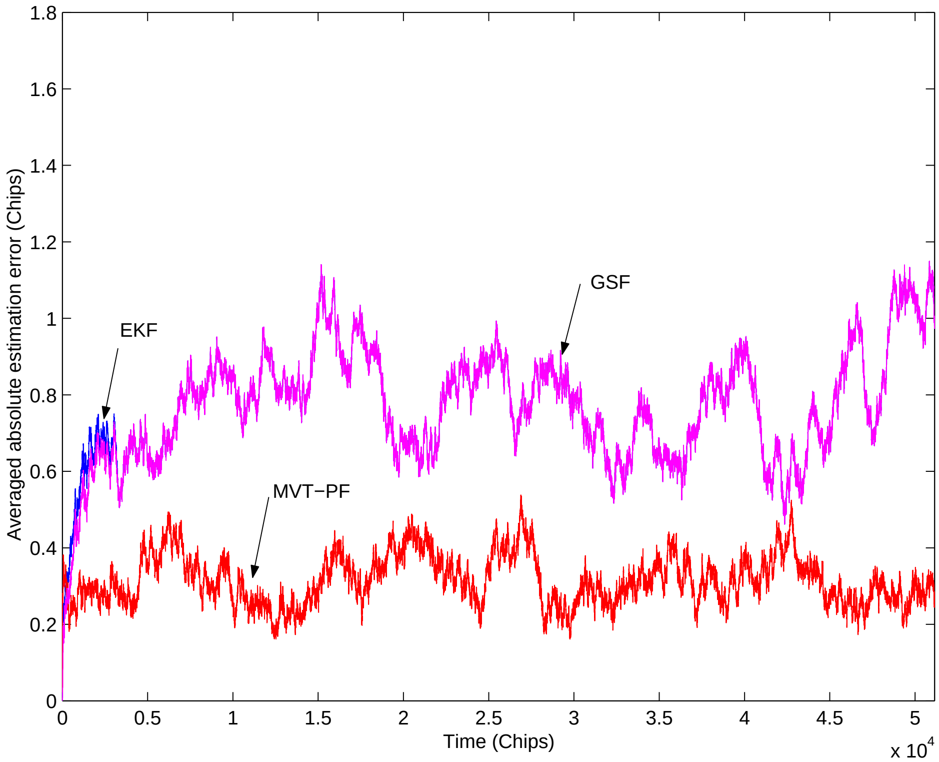

Fig. 1: Mean absolute timing error under no jamming

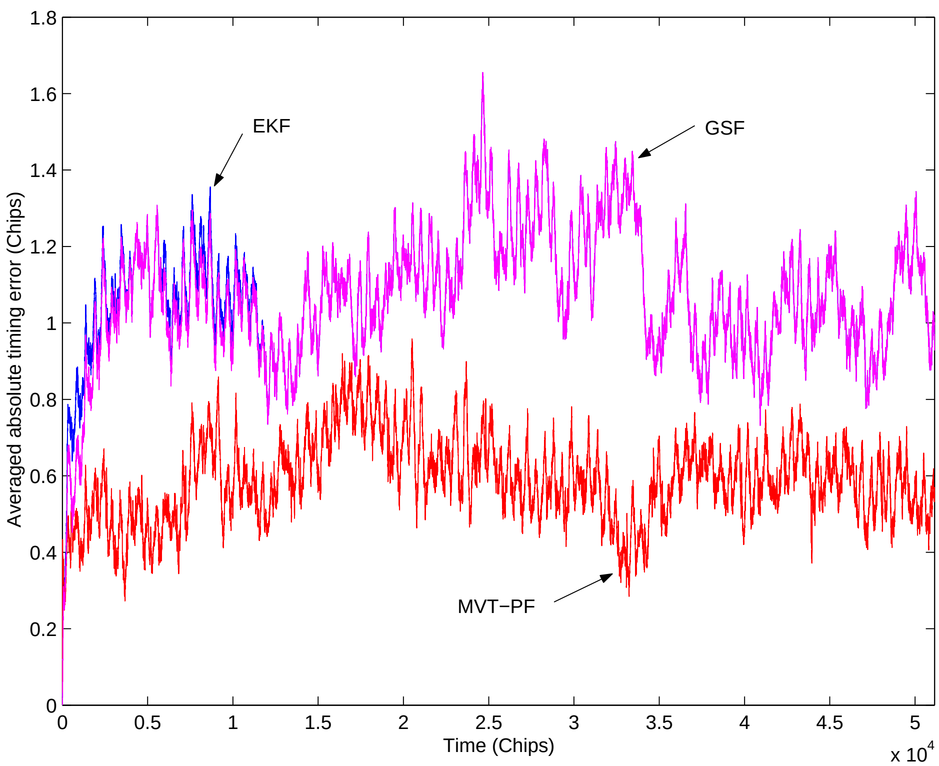

Fig. 2: Mean absolute timing error under swept tone jamming

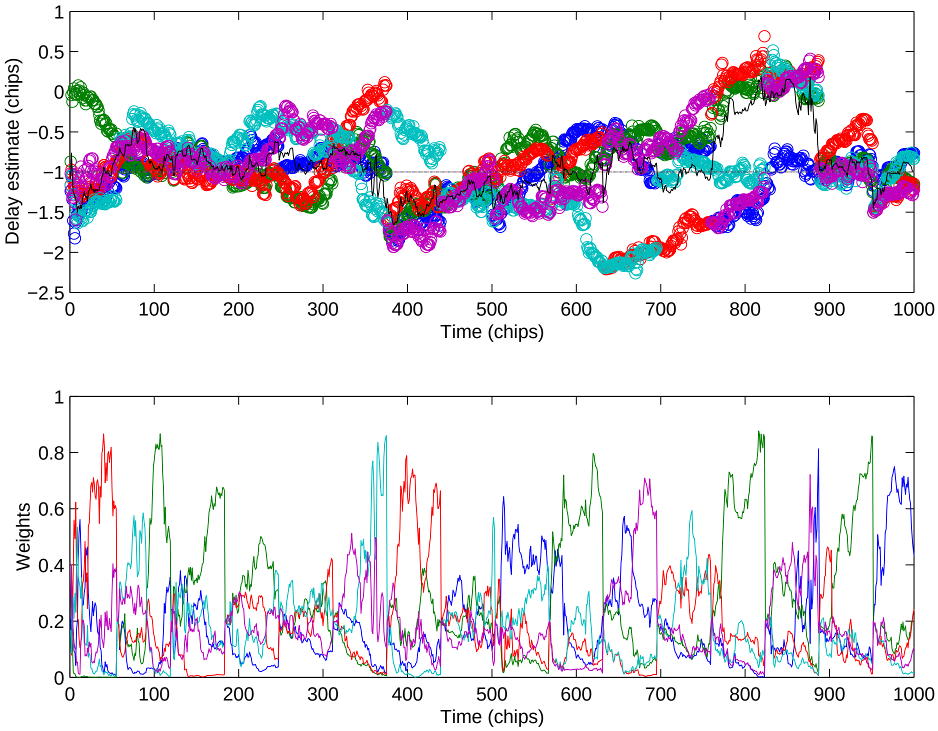

Fig. 3: Conditional and unconditional delay estimate and corresponding weights when $N_P = 5$

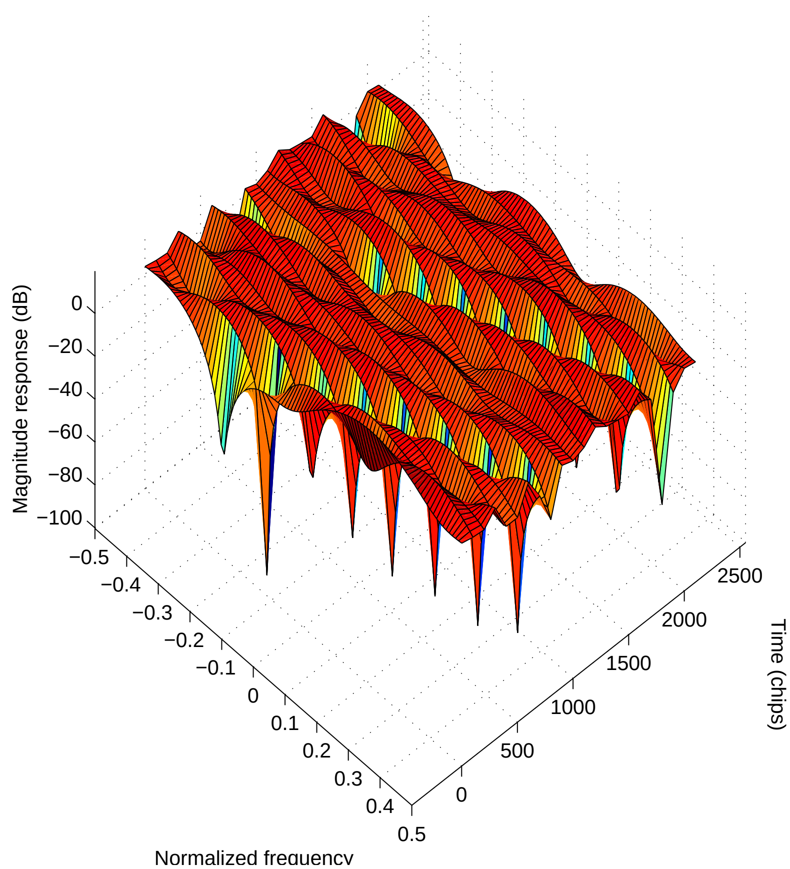

Fig. 4: Frequency response of effective whitening filter for swept tone jammer, $\dot{\Delta} f = 2 \times 10^9$ Hz/sec

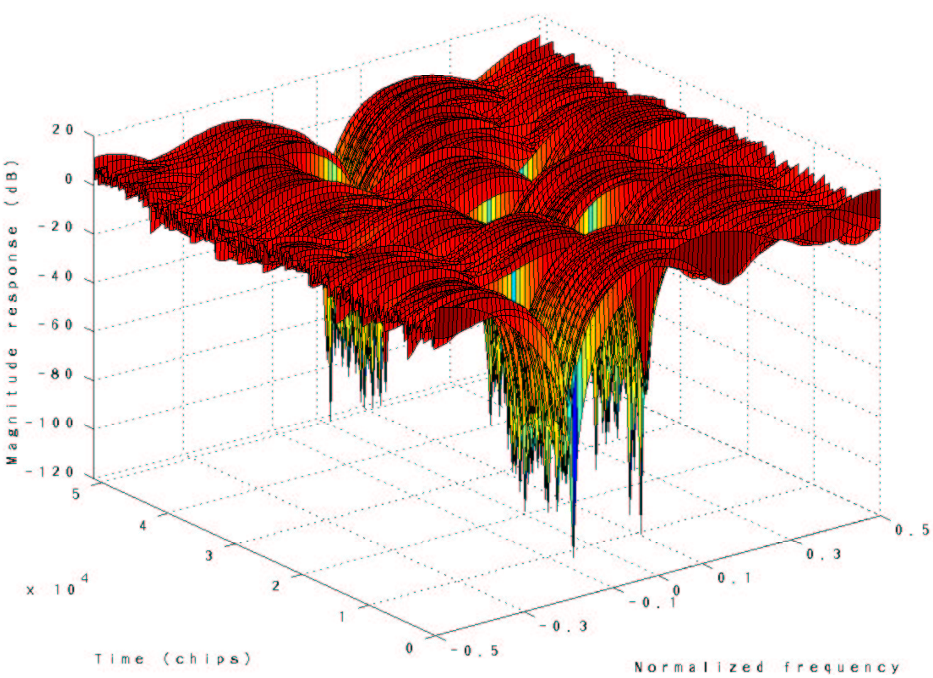

Fig. 5: Frequency response of effective whitening filter for frequency hopping tone jammer with hop rate of 120 hops/sec