A Gentle Introduction to
Dynamic Programming and the Viterbi Algorithm

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Multistage decision problems | Problem statement
Multistage decision algorithm
Underlying principle

Quiz: Bridge traversal I

Problem: Four persons want to traverse a suspension bridge at night.

▶ The bridge can carry no more than two persons at a time.

▶ The four persons take 5, 10, 20 and 25min respectively to traverse.

▶ Each party must carry a torch while traversing the bridge.

▶ The torch lasts no longer than 60 minutes.

Can you help these people?

How did you arrive at your plan?

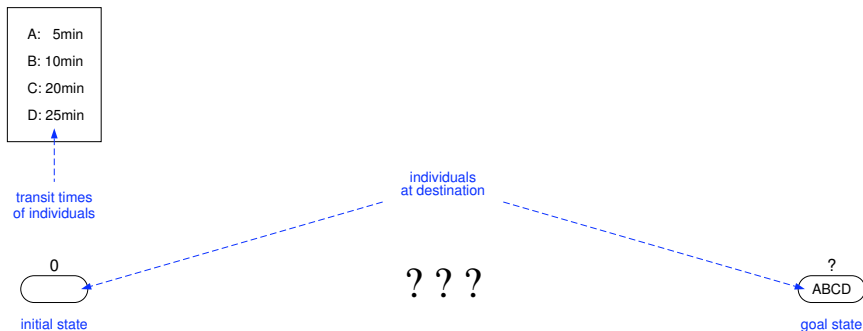Can you formulate a general policy (i.e. an algorithm)?

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Quiz: Bridge traversal II

Figure: The problem stated graphically.

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Quiz: Bridge traversal III

A: 5min
B: 10min
C: 20min
D: 25min

individuals
at destination
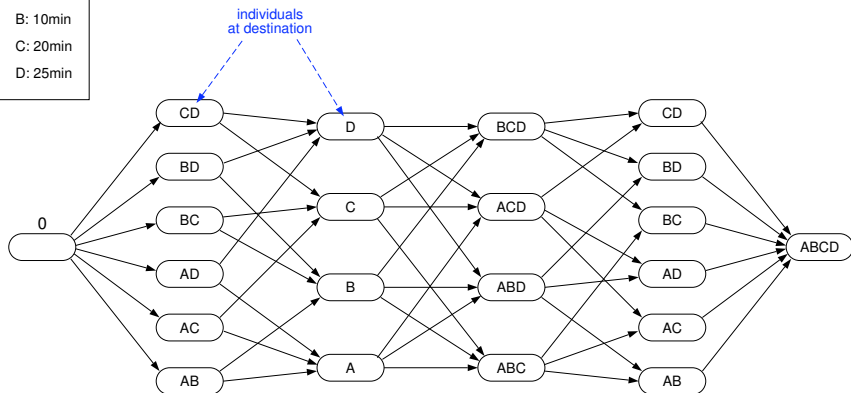
Figure: The solution space drawn as a trellis graph.

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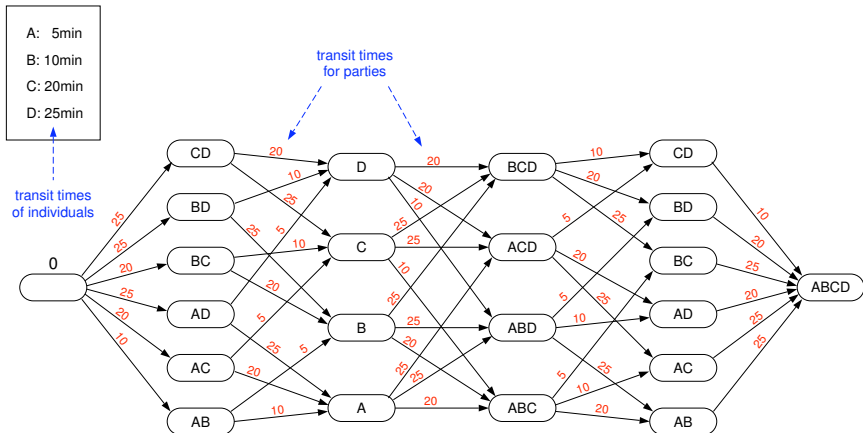
Quiz: Bridge traversal IV

Figure: ... with branch metrics assigned to all state transitions (edges).
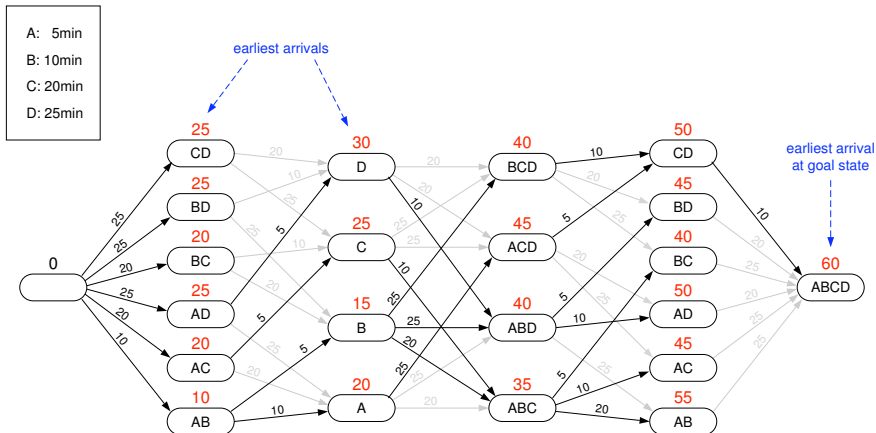
Quiz: Bridge traversal V

Figure: ... with minimum path lengths updated for all states (nodes).

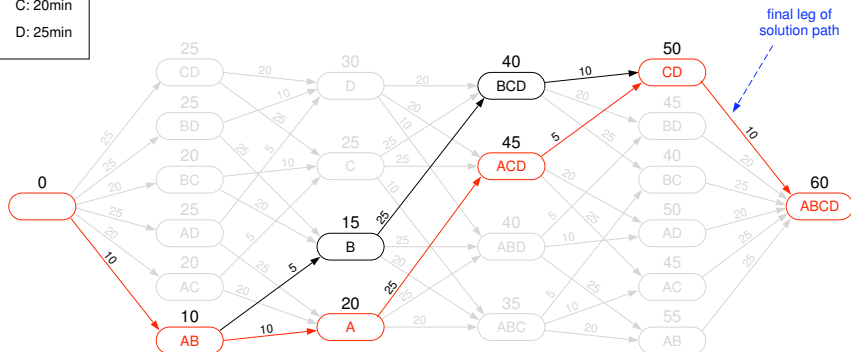# Quiz: Bridge traversal VI

A: 5min
B: 10min
C: 20min
D: 25min

final leg of
solution path

Figure: The surviving trace yields the solution (shortest path through trellis).

General policy

The general policy is known as "Dynamic Programming" and goes:

1. Assign each branch a cost metric.

2. Update all path metrics by discarding those edges
   found to be suboptimal (add-compare-select = ACS).

3. Starting from the final state, trace back the surviving path.
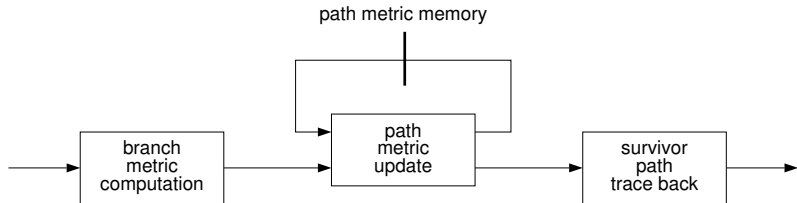
Note:

▶ "Programming" here is a synonym for "finding an optimal plan".

▶ Step 2. is to be carried out recursively stage by stage.

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Richard Bellman's principle of optimality

○ Not all multi-stage decision problems can be solved
with Dynamic Programming.

○ Those that can satisfy Bellman's principle of optimality:

  ▶ The globally optimum solution includes no suboptimal local decision.

  or in Bellman's original words from 1957:

  ▶ "An optimal policy has the property that, regardless of the decisions
  taken to enter a particular state, the remaining decisions made for
  leaving that stage must constitute an optimal policy."

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"Life can only be understood backwards, but must be lived forwards." (Sören Kierkegaard)

Applications

In numerous multistage decision problems of optimum fit such as:

▶ Error correction coding (Viterbi algorithm).

▶ Automatic labelling of speech segments (dynamic time warping).

▶ Video coding.

▶ Digital watermark detection.

▶ Cell library binding (as part of logic optimization).

▶ Flight trajectory planning.

▶ Genome sequencing (Smith-Waterman and Needleman-Wunsch algorithms).

▶ Knapsack problems.

▶ Stereo vision.

▶ ...