

A Gentle Introduction to Dynamic Programming and the Viterbi Algorithm

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Quiz: Bridge traversal I

Problem: Four persons want to traverse a suspension bridge at night.

- ▶ The bridge can carry no more than two persons at a time.
- ▶ The four persons take 5, 10, 20 and 25min respectively to traverse.
- ▶ Each party must carry a torch while traversing the bridge.
- ▶ The torch lasts no longer than 60 minutes.

Can you help these people?

How did you arrive at your plan?

Can you formulate a general policy (i.e. an algorithm)?

Quiz: Bridge traversal II

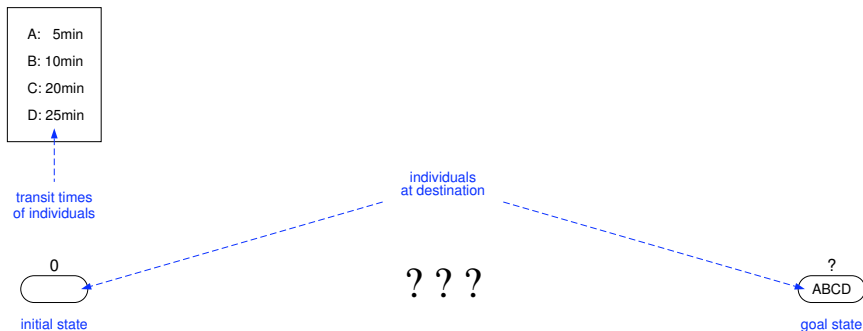


Figure: The problem stated graphically.

Quiz: Bridge traversal III

A: 5min

B: 10min

C: 20min

D: 25min

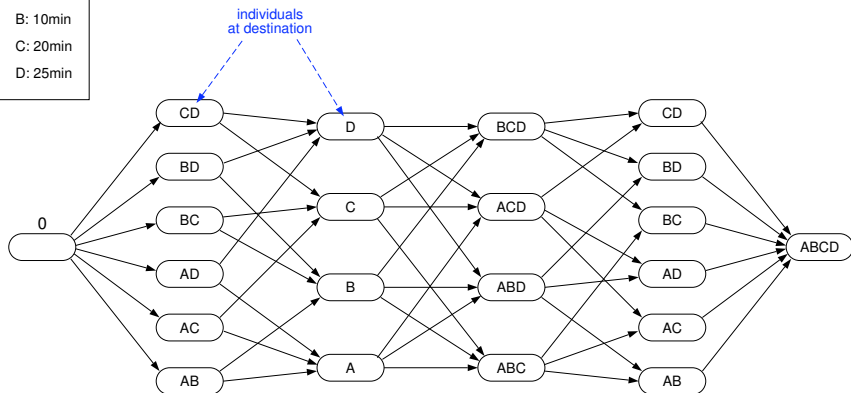


Figure: The solution space drawn as a trellis graph.

Quiz: Bridge traversal IV

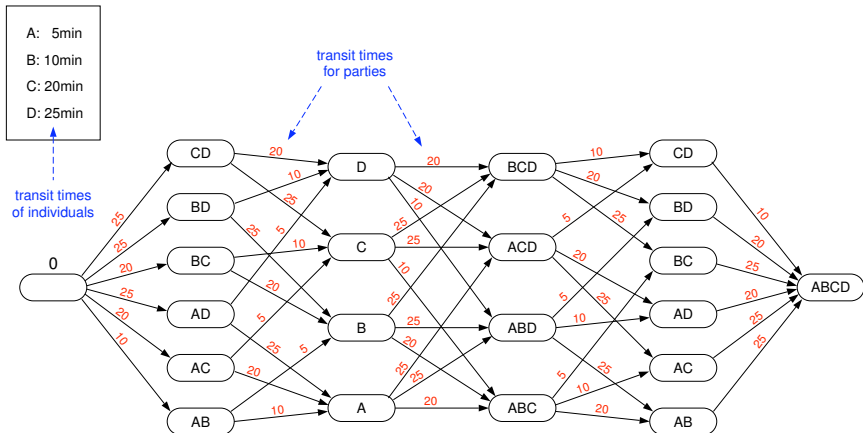


Figure: ... with branch metrics assigned to all state transitions (edges).

Quiz: Bridge traversal V

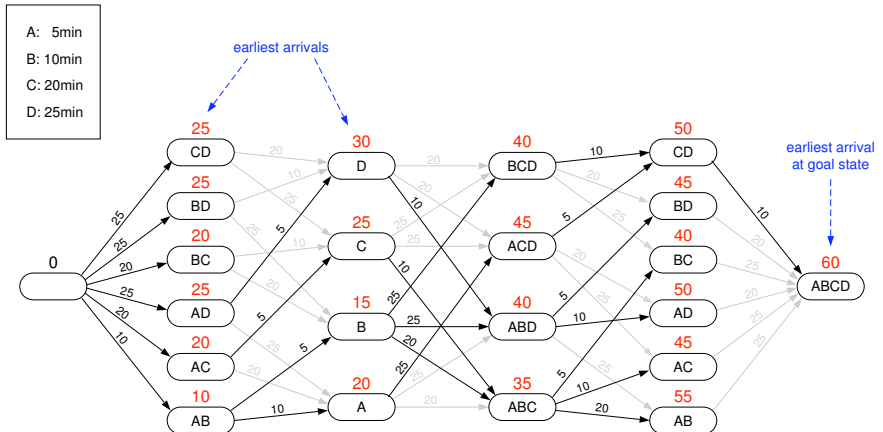


Figure: ... with minimum path lengths updated for all states (nodes).

Quiz: Bridge traversal VI

A: 5min
B: 10min
C: 20min
D: 25min

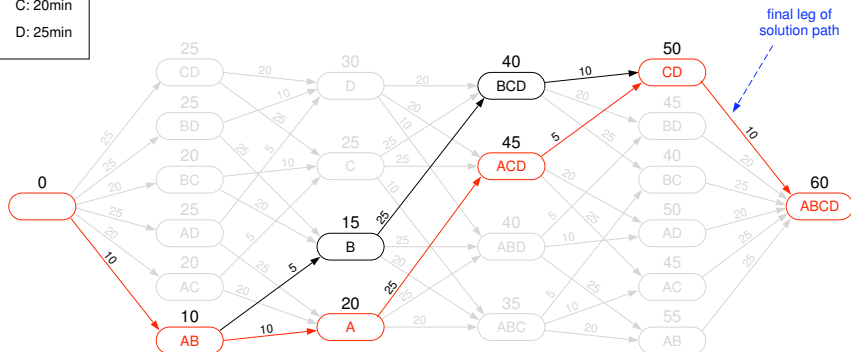
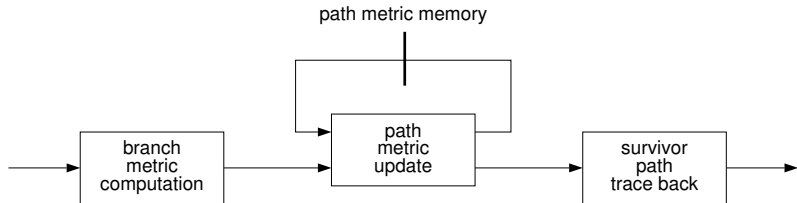


Figure: The surviving trace yields the solution (shortest path through trellis).

General policy

The general policy is known as “Dynamic Programming” and goes:

1. Assign each branch a cost metric.
2. Update all path metrics by discarding those edges found to be suboptimal (add-compare-select = ACS).
3. Starting from the final state, trace back the surviving path.



Note:

- ▶ “Programming” here is a synonym for “finding an optimal plan”.
- ▶ Step 2. is to be carried out recursively stage by stage.

Richard Bellman's principle of optimality

- Not all multi-stage decision problems can be solved with Dynamic Programming.
 - Those that can satisfy Bellman's principle of optimality:
 - ▶ The globally optimum solution includes no suboptimal local decision.
- or in Bellman's original words from 1957:
- ▶ "An optimal policy has the property that, regardless of the decisions taken to enter a particular state, the remaining decisions made for leaving that stage must constitute an optimal policy."

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"Life can only be understood backwards, but must be lived forwards."
(Søren Kierkegaard)

Applications

In numerous multistage decision problems of optimum fit such as:

- ▶ Error correction coding (Viterbi algorithm).
- ▶ Automatic labelling of speech segments (dynamic time warping).
- ▶ Video coding.
- ▶ Digital watermark detection.
- ▶ Cell library binding (as part of logic optimization).
- ▶ Flight trajectory planning.
- ▶ Genome sequencing (Smith-Waterman and Needleman-Wunsch algorithms).
- ▶ Knapsack problems.
- ▶ Stereo vision.
- ▶ ...