Simultaneous Localization and Mapping in Domestic Environments

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Abstract

This paper describes an accurate and robust algorithm for Simultaneous Localization and Map Building (SLAM). The objective of SLAM is to enable a mobile robot to build an internal representation (Map) of an unexplored environment while simultaneously using that map to navigate. An EKF filter approach is used to process the information acquired by the sonar sensors mounted on the robot. A method for recovering from failures of the SLAM algorithm is presented for increasing the robustness of the general EKF method. Real experiments are presented considering a Nomadic SuperScout mobile robot navigating in a domestic environment.

1 Introduction

The objective of this work is to enable an autonomous robot to navigate in a domestic environment without relying on *a priori* maps and without using artificial landmarks. Navigation in a domestic setting is essential for performing tasks. The error growth rates of dead-reckoning is usually unacceptable and sensing is therefore needed. Different sensory modalities are available for mobile robots, such as ultra-sonic sonar, laser systems, infrared systems and vision. Sonar, even if it is not the most accurate sensor, is the preferred solution due to its limited cost and its limited computational requirement for extracting information from the environment, which is a great advantage if the robot is intended to have an accessible price. Accurate positioning is crucial for the safety of the robot and the performance of the task it performs. There is, therefore, a need for automatic mapping and methods for safe navigation in such settings. The problem of Simultaneous Localization and Map Building is a significant open problem in mobile robotics which is difficult because of the following paradox: to localize itself the robot needs the map of the environment, and, for building a map the robot location must be known precisely. This problem has been studied in the past few years by different researchers. One solution was provided by Smith, Self and Cheesman [1] who developed an EKF approach for building a "stochastic map" of spatial relationships, Moutarlier and Chatila [2] have implemented a framework similar to the one presented by Smith *et al.* using laser range data. Due to the high computational complexity of stochastic maps (in a two-dimensional environment containing n geometric features the complexity of the EKF is $O(n^3)$[3]) real time performance becomes impossible for environments with more than a few hundreds features. Therefore methods for reducing the computational requirements have been proposed. Failures of strategies which ignore the correlations has been demonstrated by Uhlmann *et al.* [4] and Castellanos *et al.* [5]. Leonard and Feder [7] developed a method for splitting the map into multiple globally-referenced sub-maps keeping the complexity bounded, Durrant-Whyte, Dissanayake and Gibbens [8] presented a method for choosing the best features in the environment to best maintain the performance of the SLAM algorithm.

In our work we do not worry about the complexity of the EKF approach to the SLAM problem since the environment in which the robot performs its task is relatively small and the number of feature extracted is small enough to run the algorithm in real time. This paper studies the possibility of using SLAM for automatic mapping in combination with methods for automatic initialization and recovery from failures.

In Section 2 the standard EKF approach to the SLAM problem is described. In Section 3 of this paper we describe the method used for extracting natural landmark features from the environment. In Section 4 a method for detecting and recovering from failures of the algorithm is proposed. In Section 5 some experimental results are presented.

2 Stochastic mapping

The two seminal research effort in feature-based simultaneous localization and mapping were performed by Smith Self and Cheesman [1] and Moutalier and Chatila [2] who respectively published the stochastic mapping algorithm and provided the first implementation with real data. Other implementations of variations of stochastic mapping were presented by Castellanos *et al.* [5] and Chong and Kleeman [9]. Stochastic map is a special way of organizing the states in an Extended Kalman Filter for the purpose of feature relative navigation. The measurements are used to create a map of the environment which, in turn, is used to localize the robot. In our implementation we use

$$\mathbf{x}_{k|k} = \mathbf{X}_k + \eta_k \tag{1}$$

to represent the full system state vector $\mathbf{x} = [\mathbf{x}_r^T\, \mathbf{x}_1^T\, \mathbf{x}_2^T \ldots \mathbf{x}_N^T]^T$, where $\mathbf{x}_r = [x_r\, y_r\, \theta_r]^T$ is the estimate of the robot position and $\mathbf{x}_i = [x_i\, y_i]^T$ is the estimate of the landmark state. The estimate error covariance, $\mathbf{P_{k|k}} = E[\eta_k \eta_k^T]$, of the system state is

$$\mathbf{P}_{k|k} = \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{r1} & \cdots & \mathbf{P}_{rN} \\ \mathbf{P}_{1r} & \mathbf{P}_{11} & \cdots & \mathbf{P}_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{Nr} & \mathbf{P}_{N1} & \cdots & \mathbf{P}_{NN} \end{bmatrix} \tag{2}$$

The sub-matrices, $\mathbf{P}_{rr}$, $\mathbf{P}_{ri}$ and $\mathbf{P}_{ii}$ are, respectively, the robot to robot, robot to feature and feature to feature covariances. The robot and the map are represented by a single state vector $\mathbf{x}$ with the relative estimate error covariance $\mathbf{P}$ at each time step. Given the system equations

$$\mathbf{x}_{r_{k+1}} = \mathbf{f}(\mathbf{X}_k, \mathbf{u}_k) + \mathbf{q}_k, \tag{3}$$

and

$$\mathbf{x}_{i_{k+1}} = \mathbf{x}_{i_k}. \tag{4}$$

An EKF is employed to estimate the state $\mathbf{x}$ and the covariance $\mathbf{P}$ given the measurement $\mathbf{z}$. The estimation occurs through a prediction step

$$\begin{aligned} \mathbf{x}_{r_{k+1|k}} &= E\left[\mathbf{f}(\mathbf{X}_k, \mathbf{u}_k)\right], & (5) \\ \mathbf{P}_{k+1|k} &= \mathbf{J}_x \mathbf{P}_{k|k} \mathbf{J}_x^T + \mathbf{Q}_k, & (6) \\ & & (7) \end{aligned}$$

where $\mathbf{Q}_k = E[\mathbf{q}_k \mathbf{q}_k^T]$ and $\mathbf{J}_x$ is the Jacobian of $\mathbf{f}$ with respect of $\mathbf{X}$ evaluated at $\mathbf{x}_{k+1|k}$. And an update step which is done when a feature is re-observed, defining $\overline{x}_i = x_{i_{k+1|k}} - x_{r_{k+1|k}}$ and $\overline{y}_i = y_{i_{k+1|k}} - y_{r_{k+1|k}}$ the observation model for the feature i takes the form

$$\begin{aligned} \mathbf{z}_{ik+1} &= \begin{bmatrix} r_{ik+1} \\ \theta_{ik+1} \end{bmatrix} = \begin{bmatrix} \sqrt{\overline{x}_i^2 + \overline{y}_i^2} \\ \arctan \frac{\overline{y}_i}{\overline{x}_i} - \theta_{r_{k+1|k}} \end{bmatrix} + \mathbf{n}_{zi} \\ &= \mathbf{h}_i(\mathbf{x}_{k+1|k}) + \mathbf{n}_{zi}. \end{aligned} \tag{8}$$

The noise process $\mathbf{n}_{zi}$ is assumed to be white Gaussian with covariance $\mathbf{R}_i$. If the N features are observed the observation model becomes

$$\begin{aligned} \mathbf{z}_{k+1} &= \begin{bmatrix} \mathbf{z}_{1k+1} \\ \vdots \\ \mathbf{z}_{Nk+1} \end{bmatrix}, \\ \mathbf{h} &= \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_N \end{bmatrix}, \\ \mathbf{R}_{k+1} &= \begin{bmatrix} \mathbf{R}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{R}_N \end{bmatrix}. \end{aligned} \tag{9}$$

With the Jacobian of $\mathbf{h}$ given by H_x the update step of the EKF becomes

$$\begin{aligned} \mathbf{x}_{k+1|k+1} &= \mathbf{x}_{k+1|k} + \mathbf{K}_{k+1}(\mathbf{z}_{k+1} - \mathbf{h}(\mathbf{x}_{k+1|k})), & (10) \\ \mathbf{P}_{k+1|k+1} &= (\mathbf{I} - \mathbf{K}_{k+1}\mathbf{H}_x)\mathbf{P}_{k+1|k}, & (11) \\ \mathbf{K}_{k+1} &= \mathbf{P}_{k+1|k}\mathbf{H}_x^T(\mathbf{H}_x \mathbf{P}_{k+1|k}\mathbf{H}_x^T + \mathbf{R}_{k+1})^{-1} & (12) \end{aligned}$$

When a new feature $\mathbf{L}_{new} = [r\ \theta]$ is observed and validated the new feature state $\mathbf{x}_{N+1}$ is incorporated in the system vector state

$$\begin{aligned} \mathbf{x}_{N+1} &= \mathbf{m}(\mathbf{x}_{k+1|k+1}, \mathbf{L}_{new}) \\ &= \begin{bmatrix} x_{r_{k+1|k+1}} + r\cos(\theta_{r_{k+1|k+1}} + \theta) \\ y_{r_{k+1|k+1}} + r\sin(\theta_{r_{k+1|k+1}} + \theta) \end{bmatrix} \end{aligned} \tag{13}$$

$$\mathbf{x}_{k+1|k+1} \leftarrow \begin{bmatrix} \mathbf{x}_{k+1|k+1} \\ \mathbf{x}_{N+1} \end{bmatrix}, \tag{14}$$

$$\mathbf{P}_{N+1N+1} = \mathbf{J}_{\mathbf{x}_r} \mathbf{P}_{rr_{k+1|k+1}} \mathbf{J}_{\mathbf{x}_r}{}^T + \mathbf{J}_{\mathbf{z}} \mathbf{R} \mathbf{J}_{\mathbf{z}}{}^T, \tag{15}$$

$$\mathbf{P}_{rN+1} = \mathbf{P}_{N+1r}^T = \mathbf{P}_{rr_{k+1|k+1}} \mathbf{J}_{\mathbf{x}_r}{}^T. \tag{16}$$

$$\mathbf{P}_{N+1i_{k+1|k+1}} = \mathbf{P}_{iN+1_{k+1|k+1}}^T = \mathbf{J}_{\mathbf{x}_r} \mathbf{P}_{ri_{k+1|k+1}}^T. \tag{17}$$

Where $\mathbf{J}_{\mathbf{x}_r}$ and $\mathbf{J}_{\mathbf{z}}$ are the Jacobians of $\mathbf{m}$ with respect to the robot state $\mathbf{x}_r$ and to $\mathbf{L}_{new}$

A critical aspect for the SLAM algorithm in a real world scenario is the data association. The objective of data association is to assign measurements to the features from which they originate. In this work the measurement to feature association is performed using a gating approach in the innovation space [10], incorporating both measurement uncertainty and robot uncertainty. The innovation matrix $\mathbf{S}_i$ for the feature i is given by:

$$\mathbf{S}_i = \mathbf{H}_{\mathbf{x}_i} \begin{bmatrix} \mathbf{P}_{rr} & \mathbf{P}_{ri} \\ \mathbf{P}_{ir} & \mathbf{P}_{rr} \end{bmatrix} \mathbf{H}_{\mathbf{x}_i}^{\mathbf{T}} + \mathbf{R}_i. \tag{18}$$

Where

$$\mathbf{H}_{\mathbf{x}_i} = \frac{d\mathbf{h}_i([\mathbf{x}_r \mathbf{x}_i])}{d[\mathbf{x}_r \mathbf{x}_i]}. \tag{19}$$

Defining the innovation $\nu_i = \mathbf{z}_i - \mathbf{h}_i(\mathbf{x}_i)$ the validation gate is given by:

$$\nu_i^T \mathbf{S}_i^{-1} \nu_i \leq \gamma. \tag{20}$$

For a system with 2 degrees of freedom, a value of $\gamma = 9.0$ yelds the region of minimum volume that contains the measurement with a probability of 98.9% [10]. Such a validation procedure defines where a measurement is expected to be found. The initiation of a new feature is performed using a nearest neighbor gating technique described in [11].

3 Landmark Detection

For extraction of landmarks from the environment we use a method proposed by Wijk and Christensen [12] called *Triangulation Based Fusion (TBF)*. This sensor fusion scheme is a computationally efficient voting algorithm for grouping together sonar readings which have hit a mutual vertical edge in the environment. The algorithm uses a basic triangulation technique: consider two sonar readings taken from different positions during the robot motions (see figure 1). The readings are assumed to originate from a vertical edge at position $T = (x_T, y_T)$. If only one reading is considered , the physics of the sonar limit the object position to be somewhere along the corresponding beam arc. When using the information from both the sonar readings the location of the object can be extracted by computing the intersection point $T = (x_T, y_T)$ between the two arcs. The equations used for determining the intersection point are:

$$(x_T - x_{s_i})^2 + (y_T - y_{s_i})^2 = r_i^2 \tag{21}$$

$$\arctan \frac{y_T - y_{s_i}}{x_T - x_{s_i}} \in \left[\gamma_i - \frac{\delta}{2}, \gamma_i + \frac{\delta}{2}\right] \tag{22}$$

Where (x_s, y_s) denotes the sensor position, r the range reading, γ the sensor heading angle and δ the opening angle of the center sonar lobe. Using such an approach with a set of readings stored in a temporal buffer it is possible to estimate the position of invariant features in the environment $(\hat{x}_T, \hat{y}_T)$. Considering, furthermore, the model of the Polaroid 6500 sonar sensor, the set of readings associated to a mutual feature are post-processed for extracting the covariance matrix P_T of the measurement. See [12] for details. An additional parameter n_t is given by the *TBF* algorithm which specifies the number of sensor readings contributing to the measurement. This parameters is very important, the higher is the n_t value the higher is the probability that the object measured is a good landmark. In this work we consider just measurements with $n_t \geq 5$ and $\sqrt{\rho(P_T)} < 50$ mm.

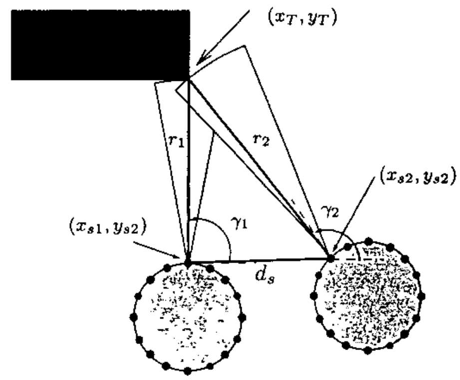

Figure 1: Basic triangulation principle.

4 Recovering from failures

There are three basic modes of failure of the stochastic mapping approach: divergence due to data association errors, map slippage and unexpected perturbation of the robot. The first failure mode occurs when a measurement is associated to an incorrect feature, that might happen when features are very close to each other and the uncertainty in vehicle position is big. The system state vector is, then, updated with erroneous data and the error will drift outside the bound defined by the estimate covariance. The second mode occurs when the robot's position is close to the error bounds and due to the linearization of the non-linear transformations all the features are remapped in new locations which are slightly shifted from the original map. The third mode occurs when the robot is effected by a strong perturbation, which drives the actual error outside the error bound. A simple method for detecting a failure mode is implemented, if the robot does not get any measurements from a location where a landmark is expected, a warn-

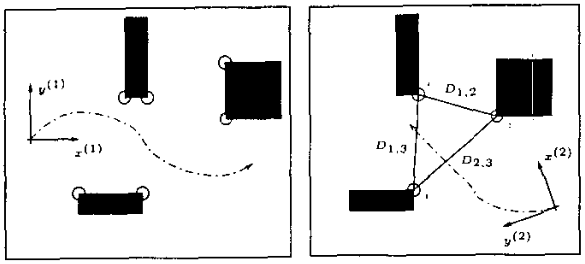

Figure 2: Matching of measurements with landmarks in the reference map.

ing flag is set and a counter is incremented, whenever a measurement is matched with a feature the counter is reset to zero. If the counter reaches a threshold value M the robot considers itself lost, at this point if the number of landmarks acquired is greater than a threshold value ψ (in this implementation ψ is set to 5) the initialization procedure is performed, otherwise the robot starts to build a new stochastic map from the beginning. The initialization procedure is divided into a localization step and a restoration step. The first step consists in an absolute localization by matching recently collected landmarks against a reference map of feature [13]. The robots moves performing measurements on a new coordinate system $(x^{(2)}, y^{(2)})$, with the origin chosen in the robot pose when the failure is detected. Consider now the situation where the mobile robot has got a reference map $\chi^{(1)}_{ref} = \{x^{(1)}_{ref,1}, \ldots, x^{(1)}_{ref,N}\}$ and a set of K landmarks recently collected $\chi^{(2)}_{land} = \{x^{(2)}_{land,1}, \ldots, x^{(2)}_{land,K}\}$. The reference map , given in the initial system coordinate $(x^{(1)}, y^{(1)})$, contains the estimate of the position of the features in the environment given by the stochastic map algorithm before the failure occurs. An image of the vector state and of the covariance matrix $(\mathbf{x}_{ref}, P_{ref})$ is saved whenever the system performs a matching between measurement and features in the state vector. Once the robot detects a failure the map used as reference map for the localization process is the one stored immediately after a matching with a consistent feature is performed. The two sets are represented in two different coordinate systems (see fig.2) $(x^{(1)}, y^{(1)})$ and $(x^{(2)}, y^{(2)})$ the relation between these coordinate systems will be a linear transformation τ involving a rotation and a translation. Given $\chi^{(1)}_{ref}$ and $\chi^{(2)}_{land}$, τ is obtained by solving a graph matching problem. Details of this implementation can be found in [13]. This step is followed by an additional confirmation procedure which using an EKF checks whether the position estimate is consistent or not. If the result of this check is positive the localization step returns an estimate of the robot state $\hat{\mathbf{x}}_L$ with the relative error covariance $\mathbf{P}_L$. otherwise this step is repeated.

At this point it is not possible to simply replace $\mathbf{x}_{r_{ref}}$ and $P_{rr_{ref}}$ with $\hat{\mathbf{x}}_L$ and $\mathbf{P}_L$ because the robot-to-feature correlation has not maintained during the localization step and moreover the resulting covariance matrix can no longer be guaranteed to be positive definite [7]. To overcome this problem a strategy similar to the one used in [7] is implemented. The restoration step consists of two sub-steps: de-correlation and updating. In the de-correlation step the robot state estimate of the reference map is randomized and its covariance is highly inflated:

$$\mathbf{x}^{ref}[-] \leftarrow \begin{bmatrix} \phi^{ref} \\ \mathbf{x}^{ref}_f \end{bmatrix}, \tag{23}$$

$$\mathbf{P}^{ref}[-] \leftarrow \begin{bmatrix} \mathbf{P}^{ref}_{rr} + \mathbf{\Phi}^{ref} & \mathbf{P}^{ref}_{rf} \\ \mathbf{P}^{ref}_{fr} & 2\mathbf{P}^{ref}_{ff} \end{bmatrix}, \tag{24}$$

where ϕ^{ref} is a random value uniformly distributed over the reference map and $\mathbf{\Phi}^{ref}$ represents a covariance larger than the size of the ref. map. The updating step consists of an EKF update using $\hat{\mathbf{x}}_L$ as a measurement with covariance $\mathbf{P}_L$:

$$\begin{aligned} \mathbf{K} &= \mathbf{P}^{ref}[-]\mathbf{H}^T(\mathbf{H}\mathbf{P}^{ref}[-])\mathbf{H}^T + \mathbf{P}_L)^{-1}, && (25) \\ \mathbf{x}[+] &= \mathbf{x}^{ref}[-] + \mathbf{K}(\hat{\mathbf{x}}_L - \mathbf{H}\mathbf{x}^{ref}[-]), && (26) \\ \mathbf{P}[+] &= (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^{ref}[-](\mathbf{I} - \mathbf{K}\mathbf{H})^T + \mathbf{K}\mathbf{P}_L\mathbf{K}^T && (27) \end{aligned}$$

where $\mathbf{H}$ is the $3 \times (3 + 2N)$ matrix $[\mathbf{I}\ \mathbf{0}]$. Through these steps the robot can restore the stochastic map algorithm.

5 Empirical Evaluation

In this section an experimental result is presented. The test environment is a regular living room set up in our laboratories at the Centre for Autonomous Systems. The room is the size of about 5×9 meters.

The experiment shows a situation where the robot performs a recovery from a failure. The robot starts at the position (3600, 4800), see figure 3-a to explore the unknown environment, during the run we deliberately made an angle perturbation to the robot. As shown in the figure 4 the x-position error grows going outside the boundaries. After detecting the failure the robot starts the initialization procedure. At the time iteration 920 (see the correction in figure 3-a the vehicle performs a localization step, it matches landmarks number 2,12 and 9 (see fig. 3-b) on the reference map, with a set of latest measurements stored into a temporary map. In this case the estimation of the robot position and the corresponding covariance supplied from the localization step are:

$$\hat{\mathbf{x}}_L = \begin{bmatrix} 3723 \\ 3677 \\ 4.57 \end{bmatrix}, \mathbf{P}_L = \begin{bmatrix} 55237 & 0 & 0 \\ 0 & 38988 & 0 \\ 0 & 0 & .305 \end{bmatrix}.$$

These data are entered into the equations 25-27 and the restoration step is completed, then the robot continues with the standard stochastic algorithm. Figure 3-b shows the estimate of the feature position with the corresponding 2σ uncertainty. The resulting feature map results satisfactory with all the features mapped inside the 2σ bounds. Note that features 5,10,15,16,17 in figure 3-b correspond to actual objects not included in the CAD model of the living room such as mobile robots and various equipment.

6 Conclusions and future works

This paper describes an implementation of a SLAM algorithm on a Nomadic SuperScout mobile robot operating in a real domestic environment. A method for recovering from the failures of the EKF approach, consisting of a localization step and a restoration step, is also presented. This method allows to restore the stochastic map algorithm after a failure is detected. The performance of this algorithm are shown to be robust in a medium size room using point features. The use of line feature will be investigate in future works for extending the application to corridors and larger environments. More advanced methods for detecting failures are going to be investigated in order to improve the robustness of the algorithm.

7 Acknowledgment

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(a) Robot trajectory

(b) Feature position estimate

Figure 3: Trajectory of the robot in the living room and landmarks position estimate of the second experimental test.

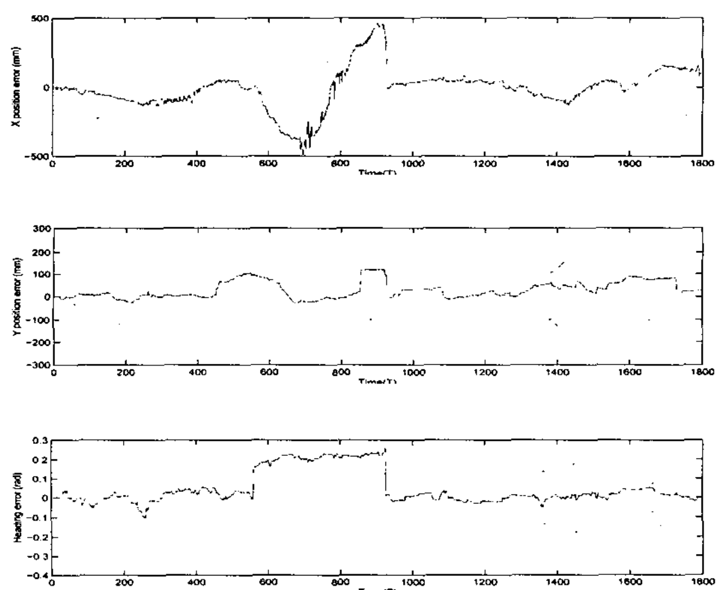

Figure 4: Error and the 2σ bounds of the x-position, y-position and heading.