

Symplectic Geometry and Toric Varieties



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Abstract

In this expository work, I have currently explored the fundamentals of Symplectic Geometry, and also unraveled their local behaviour through symplectic equivalences given by Darboux-Moser-Weinstein Theory. Moreover, I have also talked about the role of closely related Contact Structures and associated Reeb vector fields. Finally, I end it by discussing the natural connections between Symplectic and Complex Geometry, with notions of Dolbeault Cohomology and even pseudo-holomorphic curves being touched upon. It may be noted that as of now, there has not been much exploration into the second part of this thesis: Toric Varieties/Manifolds. This is because I wanted to cover all of the fundamentals of Symplectic Geometry in depth, even if some were relatively unrelated to the study of Toric Manifolds. This will enable me to explore other directions within this exciting field in the future. That said, as far as this thesis is concerned, I will now be moving on to study Hamiltonian actions and moment maps, until eventually breaking upon the grand correspondence between Delzant Polytopes and Toric Varieties.

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Chapter 1

Symplectic Structure

1.1 Symplectic Vector Spaces

1.2 Symplectic Manifolds

1.3 Lagrangian Submanifolds