MAE 6263 Computational Fluid Dynamics

Project 1

<u>*Instructions:*</u>

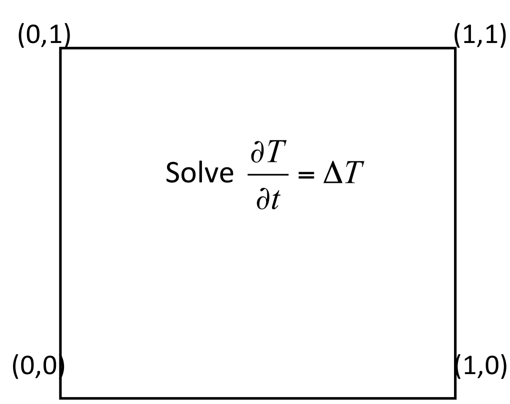
1. *Refer to the 'Academic Professionalism' and 'Academic Dishonesty' sections of the course outline document before you start working on this homework (HW) set.*
2. *The* ***<u>typed</u>*** *HW assignments should be handed over in class 03/29/2018. These need to be <u>submitted as a soft copy along with the source code</u>.*
3. *The source codes can be in any well known languages such as as Matlab, python, C, C++, F77, F90, java etc. Please discuss with me in case you have questions or need special accommodation.*
4. *Please write your full name, CWID and page number on the top right corner of each sheet of paper used in your HW assignment. The page numbers need to be written as 1/x,2/x,3/x….to ensure the integrity of your submitted assignments and neatly stapled.*

<u>Motivation</u>: This project will focus on your ability to understand the basic concepts of finite difference algorithms such as finite difference equation, consistency, stability, modified equation and solution methods. The presentation of the results from this project is intended to train you to develop meaningful reports of your work as you embark on your research career.

<u>Problem</u>: Solve the unsteady heat diffusion equation given below over a square domain of unit length.

$$\frac{\partial T}{\partial t} = \Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \text{ over } \Omega$$

$$At\ t = 0 \left\{ \begin{array}{l} T(x,y) = 0 \ \ for\ \ 0 < x < 1, 0 < y < 1; \\ T(x,y) = x + y \ \ otherwise; \end{array} \right\}$$



Discretization Scheme: Central difference for the spatial derivative and both backward (implicit) and forward (explicit) difference for the temporal derivative.

Assignment:

1. Neatly state the problem statement, boundary and initial conditions as you would in the project report and develop the finite difference equation.
2. Code Development: Develop a computer program to solve this unsteady-diffusion equation using both implicit and explicit strategies in a general purpose programming language such as python, matlab,Fortran, C, C++. The implicit method will require inversion of a coefficient matrix to solve $\overline{\overline{A}}\,\overline{U}^{n+1} = \overline{b}^{n}$. Use a Guass-Seidel or ADI (with Thomas algorithm) method to compute this matrix inversion and a convergence tolerance of 1.0e-4 in the sense of an $L_{1,norm}$.
    a. Neatly document the numerical/computational method used with descriptive text, mathematical equations i.e. give the structure of tensors A,b and how you would represent the explicit and implicit numerical methods and iterative Gauss-Seidel algorithm. The descriptive text should include variable definitions and a simple pseudo-code of the algorithm.
    b. Perform Von Neumann stability analysis for the two FDEs corresponding to implicit and explicit methods and develop the expressions for the amplification factor for the two FDEs as well as the exact solution.
    c. Include/print actual code text as an appendix to the project report
3. Numerical experiments: You will have to use 3 mesh resolutions including 100x100, 50x50 and 20x20. You will have to choose your time-step sizes by selecting a value for Γ. For the explicit method choose Γ =0.15,0.3. For the implicit method, choose Γ as 0.15,0.3 and 1.5. Not all of these simulations will be needed for the analysis. I suggest reading the following section 4 to come up with a finalized list of simulations that you will need to perform.
4. Numerical Analysis:
    a. Plot the amplification factor G for the exact solution, implicit and explicit schemes for the three values of Γ mentioned in step 3. Comment on the values of the amplification factor and its (potential?) impact on the numerical solution.
    b. Consider the result obtained from the implicit numerical method with 100x100 resolution and Γ=0.15 as the exact solution. Compare the solution obtained with the explicit and implicit methods by plotting the iso-contours of the error, i.e. difference, $\left|T_{explicit} - T_{implicit}\right|$ in addition to the plots of isocontours of $T_{explicit}$

and $T_{implicit}$. These should be plotted at t=0.1s. Comment on the error contours and can these be related to the amplification factor plots in step 4a.

c. At t=0.05, 0.1, 0.15 and 0.2s make the following line plots of T vs. x at y=0.5.
    i. *Impact of time-step size*: Plot T-x at y=0.5 for implicit method and resolution 100x100 at t=0.1s for the various values of Γ (one plot). Do the same for the explicit scheme with different values of Γ and include the exact solution as the 3rd curve (one plot). Comment on the plots.
    ii. *Impact of spatial resolution*: Plot T-x at y=0.5 for implicit method and Γ =0.15 at t=0.1s for the various mesh sizes (one plot). Do the same for the explicit scheme (one plot). Comment on the plots.
    iii. *Impact of numerical method*: Plot T-x at y=0.5 for implicit & explicit methods with Γ=0.15 and 100x100 resolution at t=0.1s (one plot), two more plots at t=0.15s and 0.2s. Comment on the plots.

d. Comment on the efficacy, i.e. computational expense relative to stability & accuracy of the explicit and implicit numerical methods for this problem of interest.