MAE 6263 Computational Fluid Dynamics

Project 2

Instructions:

1. *Refer to the 'Academic Professionalism' and 'Academic Dishonesty' sections of the course outline document before you start working on this homework (HW) set.*
2. *The **typed** project report should be handed over by the end of 04/15/2018. These need to be submitted as a soft copy along with the source code, input files and plotting script.*
3. *The source codes need to be in one of the lower level languages like C, C++, F77, F90, java or Matlab/python (without leveraging inbuilt functionality not available in F90/C/C++) etc. Please discuss with me in case you have questions or need special accommodation.*
4. *Please write your full name, CWID and page number on the top right corner of each sheet of paper used in your HW assignment. The page numbers need to be written as 1/x,2/x,3/x….to ensure the integrity of your submitted assignments and neatly stapled.*

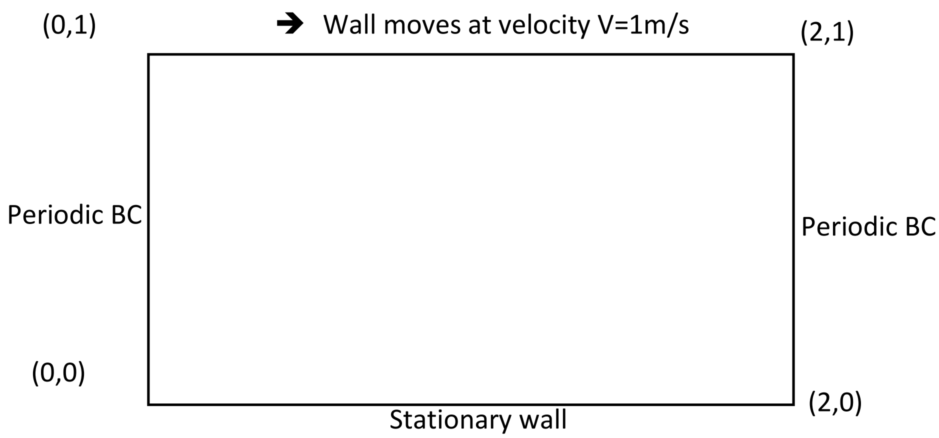
Motivation: This project will focus on your ability to apply the basic concepts of finite difference algorithms to the solution of a model equation of the Navier-Stokes – the Burger's equation. This equation is the momentum equation without the pressure gradient and will train you to develop simulation models using finite difference method for flows with boundary layer type phenomena. The presentation of the results from this project is intended to train you to develop professional reports of your work as you embark on your research career. So, please take this seriously.

Problem: Solve the 2-D non-linear Burgers equation given below over the domain as shown.

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}.\nabla\vec{u} = \upsilon\Delta\vec{u} \ over \ \Omega$$

$$At \ t=0 \left\{ \vec{u}\left(x,y\right)=0 \ \ for \ 0<x<2, 0<y<1; \right\}$$

$$BC: \begin{Bmatrix} \vec{u}\left(x,y\right)=1 \ \ for \ y=1; \\ \vec{u}\left(x,y\right)=0 \ \ for \ y=0; \\ \vec{u}\left(x,y\right)=\vec{u}\left(x-2,y\right) \ \ for \ x=2; \\ \vec{u}\left(x,y\right)=\vec{u}\left(x+2,y\right) \ \ for \ x=0; \end{Bmatrix}$$



<u>Discretization Scheme</u>: Central difference for the viscous term and both (i) central difference and (ii) first order upwind for the advective terms. Use (a) forward Euler and (b) backward Euler (implicit) for the temporal derivative.

<u>Assignment</u>:

1. Neatly state the problem, boundary and initial conditions as you would in the project report and develop the finite difference equation.
2. <u>Code Development</u>: Develop a computer program (i.e. build on project 1) to solve this unsteady-convection-diffusion equation using both implicit and explicit strategies in a general purpose programming language such as Fortran, C, C++. The implicit treatment will require inversion of a coefficient matrix to solve $\overline{\overline{A}}\,\overline{U}^{n+1} = \overline{b}^{n}$. Use the Guass-Seidel method developed in project 1 to compute this matrix inversion and a convergence tolerance of 1.0e-4 for the iteration in the sense of an $L_{1,norm}$.
    a. Neatly document the numerical/computational method used with descriptive text, mathematical equations i.e. give the structure of tensors A,b and how you would represent the explicit and implicit numerical methods and iterative Gauss-Seidel algorithm. The descriptive text should include variable definitions and a simple pseudo-code of the algorithm (again build on project 1 material as much as possible).
    b. Perform Von Neumann stability analysis for the two FDEs corresponding to implicit and explicit methods (for both diffusion and advection terms) and develop the expressions for the amplification factor for the two FDEs. Derive their stability constraints and any other constraint for ensuring a physically correct solution.
    c. Include/print actual code text as an appendix to the project report along with submission of the actual source code.
3. <u>Designing Numerical experiments</u>: In this problem the key to designing a simulation is to (i) identify the global Reynolds number as $\mathrm{Re} = VL_y / \upsilon$ (ii) Decide on the local Peclet

number, $Pe_{\Delta x} = V\Delta x / \upsilon$ to get $\Delta x = \Delta y$ and (iii) Decide on a value for $\gamma = \upsilon \Delta t / \Delta x^2$ to get Δt

4. <u>Numerical Simulations and Analysis</u>: We will consider the following choices of simulation parameters. Two values of $\mathrm{Re} = VL_y / \upsilon$ as 40 and 200. Two values of $Pe_{\Delta x} = V\Delta x / \upsilon$ as 1 and 5. Two values of $\gamma = \upsilon \Delta t / \Delta x^2$ as 0.1 and 1.0. Note: You do not need to perform all these runs. Choose ones that you need for analysis below. According to my count you will need to perform 8(?) different runs in total.
    a. <u>Choice of implicit vs Explicit (2 explicit and 2 implicit runs)</u>: Use Re=40 (FOU for advection and CD for diffusion) and perform explicit and implicit numerical simulations upto steady state for $\gamma = 0.1$ and $Pe_{\Delta x} = 1.0$. Is there any advantage for choosing the explicit vs Implicit for this case? Discuss by comparing computational expense and also plot Vx vs y the implicit and explicit method. What happens if you choose $Pe_{\Delta x} = 5.0$? How would your answer change? Show proof by carrying out those simulations. Also, plot the profile of the x-component of velocity with height, Vx vs y for each of these runs.
    b. <u>Choice of advection scheme (2 runs for each advection scheme)</u>: Use Re=40 (FOU & CD for advection and CD for diffusion) and perform ***explicit*** numerical simulations upto steady state in a time-dependent fashion for $\gamma = 0.1$ and $Pe_{\Delta x} = 1.0 \textit{ and } 5.0$. Does the choice of local Peclet number impact the solution for this problem? Discuss this by showing the plots of Vx vs Y at 4- different instances in time between t=0 to $t=t_{steady}$. Choose your time snapshots wisely to make your point.
    c. <u>Impact of Reynolds number (1 run for each Re):</u> Choose FOU for advection, CD for diffusion term and $Pe_{\Delta x} = 5.0$ and $\gamma = 1.0$. Perform two simulations using ***implicit*** method for $\mathrm{Re} = 40 \textit{ and } 200$ up to steady state. Compare their steady state solutions using plots of Vx vs y and comment on the boundary layer thickness in both these runs.