Homework # 1 – Interpolation

Due date: **Sep 13, Thursday,** class time.

Please bring hard copy of your solution(s) in a report form (could be typed or hand written or mixed, and stamped if possible). Always provide its caption when you present a computer generated figure. You should also discuss your observations (i.e., one paragraph stating the strengths and weaknesses of the methods, summarizing your observations etc). I am not collecting code scripts (i.e., feel free to use any language you like, some examples are given online (e.g., www.cfdlab.org or https://numerics.stanford.edu/ta/index.html).

1. [20pt] Derive an expression for the derivative of a Lagrange polynomial of order n at a point x between the data points (Exercise 2 from Chapter 1 in your textbook). Please see wiki snap below for the final solution.

Derivatives [edit]

The dth derivatives of the lagrange polynomial can be written as

$$L^{(d)}(x) := \sum_{j=0}^{k} y_j \ell_j^{(d)}(x).$$

For the first derivative, the coefficients are given by

$$\ell_j^{(1)}(x) := \sum_{i=0, i\neq j}^{k} \left[\frac{1}{x_j - x_i} \prod_{m=0, m\neq(i,j)}^{k} \frac{x - x_m}{x_j - x_m} \right]$$

and for the second derivative

$$\ell_j^{(2)}(x) := \sum_{i=0, i\neq j}^{k} \frac{1}{x_j - x_i} \left[\sum_{m=0, m\neq(i,j)}^{k} \left(\frac{1}{x_j - x_m} \prod_{l=0, l\neq(i,j,m)}^{k} \frac{x - x_l}{x_j - x_l} \right) \right].$$

Through recursion, one can compute formulas for higher derivatives.

Figure 1: Lagrange polynomial derivatives.

2. [50pt] Write a computer program for Lagrange interpolation of the function

$$f(x) = \cos(10x)\sin(x) \tag{1}$$

in the interval $-1 \leq x \leq 1$. Use equally distributed grid of 9, 17, 33, 65 grid points (e.g., 9 grid points: n=8, you can construct an 8th order Lagrange polynomial by using 9 points). For each data set (i.e., n=8, 16, 32, 64):

   i. Present graphs of $f(x)$ and its Lagrange polynomial approximation $p(x)$ in the same figure. Here $p(x)$ will be obtained from $n+1$ given discrete data points and refers to an approximation to $f(x)$ once you construct a Lagrange interpolation using $n+1$ data points. You should obtain data points directly from given $f(x)$ (i.e., using equidistant intervals for given n). Plot 4 different figures for each set with different n.

   ii. Evaluate the derivative of your Lagrange Polynomial approximation $p'(x)$ and compare your results to the exact derivative of the function $f'(x)$ and plot your results.

3. [30pt] Write a computer program for the cubic spline interpolation for the same problem described in 2.

   i. Present graphs of $f(x)$ and its cubic spline polynomial approximation $p(x)$. Plot 4 different figures for each n. You can use natural spline boundary condition (i.e., $f''(x_0) = f''(x_n) = 0$).