8TH EDITION

INTERMEDIATE MICROECONOMICS

A MODERN APPROACH

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Game Theory

Economic agents can interact strategically in a variety of ways, and many of these have been studied by using the apparatus of game theory. Game theory is concerned with the general analysis of strategic interaction.

- Players: who play the game. Normally for simplicity we assume to be two players, one positioned in the left (strategies indicated in rows) and the other in the top (strategies indicated in columns).
- Rules/Strategies: policies/actions players can undertake. Strategies are indicated either in rows or in columns. Strategic interaction can involve many players and many strategies, but we'll limit ourselves to two-person games with a finite number of strategies.
- Payoff: the set of results/outcomes from possible actions/strategies. The first payoff in a cell is the payoff of the player positioned in Left with actions in rows and the later payoff in a cell is the payoff of the player positioned in Top with actions in columns.
- Example:

		Player B	
		Left	Right
Player A	Top	(Top, Left)	(Top, Right)
	Bottom	(Bottom, Left)	(Bottom, Right)

- Dominant Strategy: A dominant strategy for a player is a strategy that is best for that player no matter what other strategies the other player follows. A Nash Equilibrium with each players' dominant strategies is called "Dominant Strategy Nash Equilibrium".

CHAPTER 28

GAME THEORY

The previous chapter on oligopoly theory presented the classical economic theory of strategic interaction among firms. But that is really just the tip of the iceberg. Economic agents can interact strategically in a variety of ways, and many of these have been studied by using the apparatus of **game theory**. Game theory is concerned with the general analysis of strategic interaction. It can be used to study parlor games, political negotiation, and economic behavior. In this chapter we will briefly explore this fascinating subject to give you a flavor of how it works and how it can be used to study economic behavior in oligopolistic markets.

28.1 The Payoff Matrix of a Game

Strategic interaction can involve many players and many strategies, but we'll limit ourselves to two-person games with a finite number of strategies. This will allow us to depict the game easily in a **payoff matrix**. It is simplest to examine this in the context of a specific example.

Suppose that two people are playing a simple game. Person A will write one of two words on a piece of paper, "top" or "bottom." Simultaneously, person B will independently write "left" or "right" on a piece of paper. After they do this, the papers will be examined and they will each get the payoff depicted in Table 28.1. If A says top and B says left, then we examine the top left-hand corner of the matrix. In this matrix the payoff to A is the first entry in the box, 1, and the payoff to B is the second entry, 2. Similarly, if A says bottom and B says right, then A will get a payoff of 1 and B will get a payoff of 0.

Person A has two strategies: he can choose top or he can choose bottom. These strategies could represent economic choices like "raise price" or "lower price." Or they could represent political choices like "declare war" or "don't declare war." The payoff matrix of a game simply depicts the payoffs to each player for each combination of strategies that are chosen.

What will be the outcome of this sort of game? The game depicted in Table 28.1 has a very simple solution. From the viewpoint of person A, it is always better for him to say bottom since his payoffs from that choice (2 or 1) are always greater than their corresponding entries in top (1 or 0). Similarly, it is always better for B to say left since 2 and 1 dominate 1 and 0. Thus we would expect that the equilibrium strategy is for A to play bottom and B to play left.

In this case, we have a **dominant strategy**. There is one optimal choice of strategy for each player no matter what the other player does. Whichever choice B makes, player A will get a higher payoff if he plays bottom, so it makes sense for A to play bottom. And whichever choice A makes, B will get a higher payoff if he plays left. Hence, these choices dominate the alternatives, and we have an equilibrium in dominant strategies.

A payoff matrix of a game. **Table 28.1**

		Player B	
		Left	Right
Player A	Top	1, 2	0, 1
	Bottom	2, 1	1, 0

If there is a dominant strategy for each player in some game, then we would predict that it would be the equilibrium outcome of the game. For a dominant strategy is a strategy that is best no matter what the other player does. In this example, we would expect an equilibrium outcome in which A plays bottom, receiving an equilibrium payoff of 2, and B plays left, receiving an equilibrium payoff of 1.

28.2 Nash Equilibrium

Dominant strategy equilibria are nice when they happen, but they don't happen all that often. For example, the game depicted in Table 28.2 doesn't have a dominant strategy equilibrium. Here when B chooses left the payoffs to A are 2 or 0. When B chooses right, the payoffs to A are 0 or 1. This means that when B chooses left, A would want to choose top; and when B chooses right, A would want to choose bottom. Thus A's optimal choice depends on what he thinks B will do.

Table 28.2

A Nash equilibrium.

		Player B	
		Left	Right
Player A	Top	2, 1	0, 0
	Bottom	0, 0	1, 2

However, perhaps the dominant strategy equilibrium is too demanding. Rather than require that A's choice be optimal for *all* choices of B, we can just require that it be optimal for the *optimal* choices of B. For if B is a well-informed intelligent player, he will only want to choose optimal strategies. (Although, what is optimal for B will depend on A's choice as well!)

We will say that a pair of strategies is a **Nash equilibrium** if A's choice is optimal, given B's choice, *and* B's choice is optimal given A's choice.[1] Remember that neither person knows what the other person will do when he has to make his own choice of strategy. But each person may have

[1] John Nash is an American mathematician who formulated this fundamental concept of game theory in 1951. In 1994 he received the Nobel Prize in economics, along with two other game theory pioneers, John Harsanyi and Reinhard Selten. The 2002 film *A Beautiful Mind* is loosely based on John Nash's life; it won the Academy Award for best movie.

some expectation about what the other person's choice will be. A Nash equilibrium can be interpreted as a pair of expectations about each person's choice such that, when the other person's choice is revealed, neither individual wants to change his behavior.

In the case of Table 28.2, the strategy (top, left) is a Nash equilibrium. To prove this note that if A chooses top, then the best thing for B to do is to choose left, since the payoff to B from choosing left is 1 and from choosing right is 0. And if B chooses left, then the best thing for A to do is to choose top since then A will get a payoff of 2 rather than of 0.

Thus if A chooses top, the optimal choice for B is to choose left; and if B chooses left, then the optimal choice for A is top. So we have a Nash equilibrium: each person is making the optimal choice, *given* the other person's choice.

The Nash equilibrium is a generalization of the Cournot equilibrium described in the last chapter. There the choices were output levels, and each firm chose its output level taking the other firm's choice as being fixed. Each firm was supposed to do the best for itself, assuming that the other firm continued to produce the output level it had chosen—that is, it continued to play the strategy it had chosen. A Cournot equilibrium occurs when each firm is maximizing profits given the other firm's behavior; this is precisely the definition of a Nash equilibrium.

The Nash equilibrium notion has a certain logic. Unfortunately, it also has some problems. First, a game may have more than one Nash equilibrium. In fact, in Table 28.2 the choices (bottom, right) also comprise a Nash equilibrium. You can either verify this by the kind of argument used above, or just note that the structure of the game is symmetric: B's payoffs are the same in one outcome as A's payoffs are in the other, so that our proof that (top, left) is an equilibrium is also a proof that (bottom, right) is an equilibrium.

The second problem with the concept of a Nash equilibrium is that there are games that have no Nash equilibrium of the sort we have been describing at all. Consider, for example, the case depicted in Table 28.3. Here a Nash equilibrium of the sort we have been examining does not exist. If player A plays top, then player B wants to play left. But if player B plays left, then player A wants bottom. Similarly, if player A plays bottom, then player B will play right. But if player B plays right, then player A will play top.

28.3 Mixed Strategies

However, if we enlarge our definition of strategies, we can find a new sort of Nash equilibrium for this game. We have been thinking of each agent as choosing a strategy once and for all. That is, each agent is making one choice and sticking to it. This is called a **pure strategy**.

Table 28.3 **A game with no Nash equilibrium (in pure strategies).**

		Player B: Left	Player B: Right
Player A	Top	0, 0	0, –1
	Bottom	1, 0	–1, 3

Another way to think about it is to allow the agents to *randomize* their strategies—to assign a probability to each choice and to play their choices according to those probabilities. For example, A might choose to play top 50 percent of the time and bottom 50 percent of the time, while B might choose to play left 50 percent of the time and right 50 percent of the time. This kind of strategy is called a **mixed strategy**.

If A and B follow the mixed strategies given above, of playing each of their choices half the time, then they will have a probability of 1/4 of ending up in each of the four cells in the payoff matrix. Thus the average payoff to A will be 0, and the average payoff to B will be 1/2.

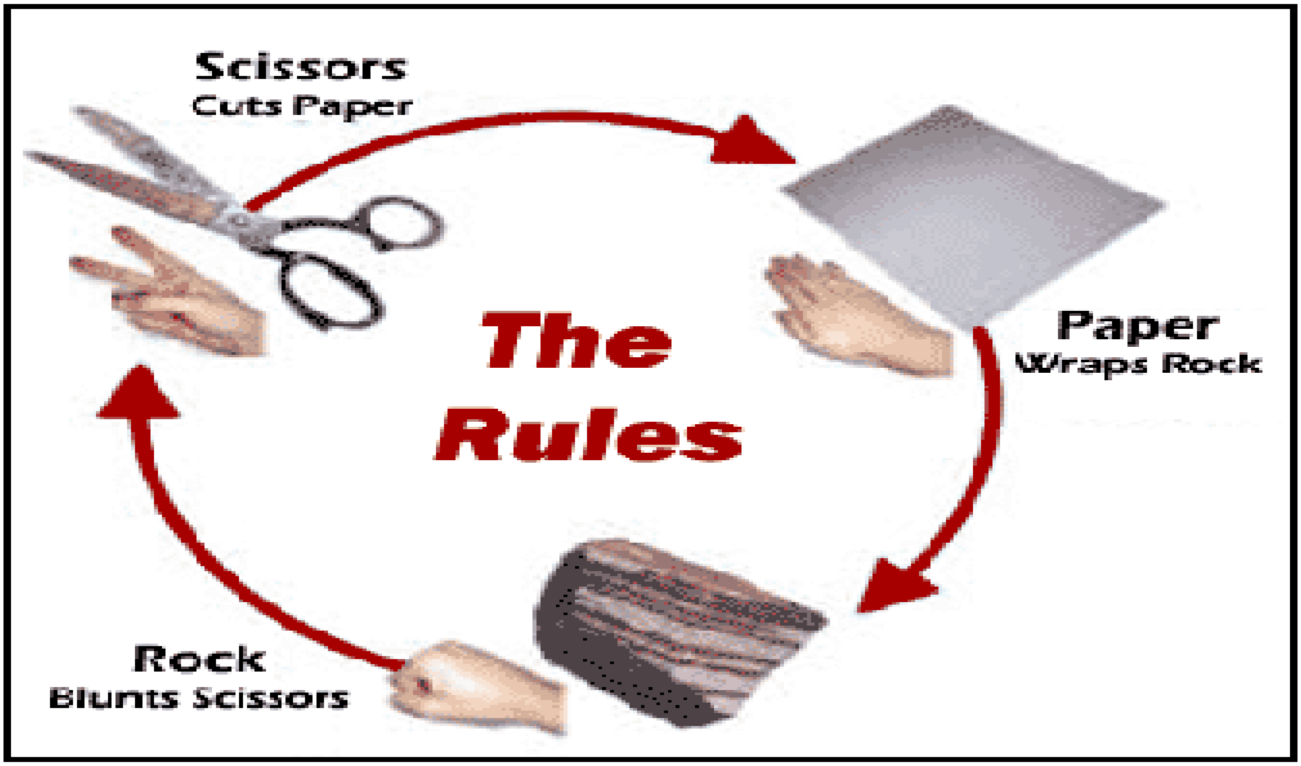
A Nash equilibrium in mixed strategies refers to an equilibrium in which each agent chooses the optimal frequency with which to play his strategies given the frequency choices of the other agent.

It can be shown that for the sort of games we are analyzing in this chapter, there will always exist a Nash equilibrium in mixed strategies. Because a Nash equilibrium in mixed strategies always exists, and because the concept has a certain inherent plausibility, it is a very popular equilibrium notion in analyzing game behavior. In the example in Table 28.3 it can be shown that if player A plays top with probability 3/4 and bottom with probability 1/4, and player B plays left with probability 1/2 and right with probability 1/2, this will constitute a Nash equilibrium.

EXAMPLE: Rock Paper Scissors

But enough of this theory. Let's look at an example that really matters: the well-known pastime of "rock paper scissors." In this game, each player simultaneously chooses to display a fist (rock), a palm (paper), or his first two fingers (scissors). The rules: rock breaks scissors, scissors cuts paper, paper wraps rock.

Throughout history, countless hours have been spent in playing this game. There is even a professional society, the RPS Society, that pro-



Rock, Paper, Scissors	Rock	Paper	Scissors
Rock	0, 0	-1, 1	1, -1
Paper	1, -1	0, 0	-1, 1
Scissors	-1, 1	1, -1	0, 0

motes the game. It offer both a Web site and a movie documenting the 2003 championships in Toronto.

Of course, game theorists recognize that the equilibrium strategy in rock paper scissors is to randomly choose one of the three outcomes. But humans are not necessarily so good at choosing totally random outcomes. If you can predict your opponent's choices to some degree, you can have an edge in making your own choices.

According to the somewhat tongue-in-cheek account of Jennifer 8. Lee, psychology is paramount.[2] In her article she writes that "most people have a go-to throw, reflective of their character, when they are caught off guard. Paper, considered a refined, even passive, throw, is apparently favored by literary types and journalists."

What is the go-to throw of economists, I wonder? Perhaps it is scissors, since we like to cut to the essential forces at work in human behavior. Should you play rock against an economist, then? Perhaps, but I wouldn't rely on it . . .

~~28.4 The Prisoner's Dilemma~~

~~Another problem with the Nash equilibrium of a game is that it does not necessarily lead to Pareto efficient outcomes. Consider, for example, the game depicted in Table 28.4. This game is known as the **prisoner's dilemma**. The original discussion of the game considered a situation where two prisoners who were partners in a crime were being questioned in separate rooms. Each prisoner had a choice of confessing to the crime, and thereby implicating the other, or denying that he had participated in the crime. If only one prisoner confessed, then he would go free, and the authorities would throw the book at the other prisoner, requiring him to spend 6 months in prison. If both prisoners denied being involved, then both would be held for 1 month on a technicality, and if both prisoners confessed they would both be held for 3 months. The payoff matrix for this game is given in Table 28.4. The entries in each cell in the matrix represent the utility that each of the agents assigns to the various prison terms, which for simplicity we take to be the negative of the length of their prison terms.~~

~~Put yourself in the position of player A. If player B decides to deny committing the crime, then you are certainly better off confessing, since then you'll get off free. Similarly, if player B confesses, then you'll be better off confessing, since then you get a sentence of 3 months rather than a sentence of 6 months. Thus *whatever* player B does, player A is better off confessing.~~

[2] ~~Yes, "8" really is her middle name. "Rock, Paper, Scissors: High Drama in the Tournament Ring" was published in the *New York Times* on September 5, 2004.~~

Types of Games:

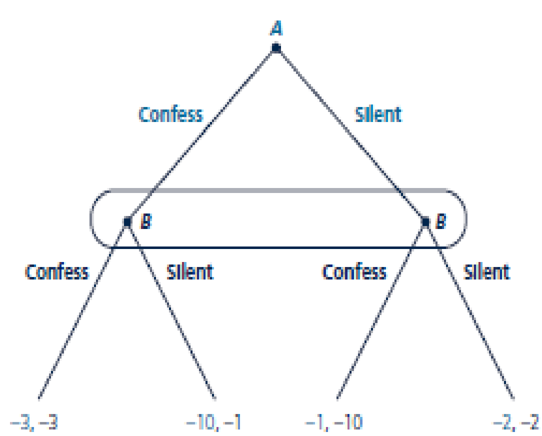
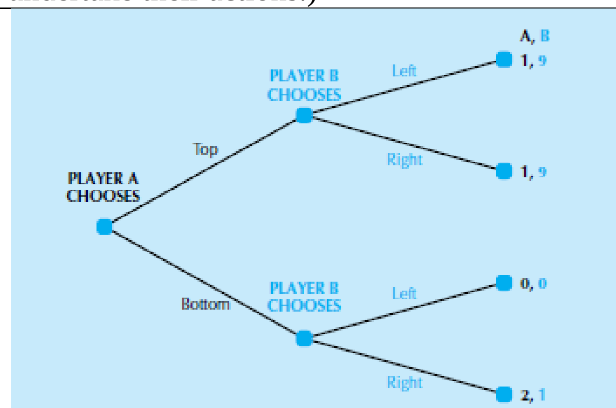
- Based on Strategy: Pure Strategy and Mixed Strategy
- Based on Mutual Assistance: Cooperative and Non-cooperative.
- Based on Repeatition: One shot and Repeated.
- Based on Timing/Information Asymmetry: Simultaneous and Sequential.

Presentation of a Game:

- Short form:

		Player B	
		Left	Right
Player A	Top	1, 9	1, 9
	Bottom	0, 0	2, 1

- Extensive/Extended (Tree) form:

<table>
<tr><td>Simultaneous (When one player’s action is unknown to the other player, no matter whenever the players undertake their actions.)</td><td>Sequential (When one player’s action is known to the other player and the other player acts accordingly, no matter whenever the players undertake their actions.)</td></tr>
<tr><td>
</td><td>

Extensive form of the game. This way of depicting a game indicates the order in which the players move.</td></tr>
</table>

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Table 28.4 **The prisoner's dilemma.**

		Player B	
		Confess	Deny
Player A	Confess	−3, −3	0, −6
	Deny	−6, 0	−1, −1

The same thing goes for player B—he is better off confessing as well. Thus the unique Nash equilibrium for this game is for both players to confess. In fact, both players confessing is not only a Nash equilibrium, it is a dominant strategy equilibrium, since each player has the same optimal choice independent of the other player.

But if they could both just hang tight, they would each be better off! If they both could be sure the other would hold out, and both could agree to hold out themselves, they would each get a payoff of -1, which would make each of them better off. The strategy (deny, deny) is Pareto efficient—there is no other strategy choice that makes both players better off—while the strategy (confess, confess) is Pareto inefficient.

The problem is that there is no way for the two prisoners to coordinate their actions. If each could trust the other, then they could both be made better off.

The prisoner's dilemma applies to a wide range of economic and political phenomena. Consider, for example, the problem of arms control. Interpret the strategy of "confess" as "deploy a new missile" and the strategy of "deny" as "don't deploy." Note that the payoffs are reasonable. If my opponent deploys his missile, I certainly want to deploy, even though the best strategy for both of us is to agree not to deploy. But if there is no way to make a binding agreement, we each end up deploying the missile and are both made worse off.

Another good example is the problem of cheating in a cartel. Now interpret confess as "produce more than your quota of output" and interpret deny as "stick to the original quota." If you think the other firm is going to stick to its quota, it will pay you to produce more than your own quota. And if you think that the other firm will overproduce, then you might as well, too!

The prisoner's dilemma has provoked a lot of controversy as to what is the "correct" way to play the game—or, more precisely, what is a reasonable way to play the game. The answer seems to depend on whether you are playing a one–shot game or whether the game is to be repeated an indefinite number of times.

If the game is going to be played just one time, the strategy of defecting—in this example, confessing—seems to be a reasonable one. After all, whatever the other fellow does, you are better off, and you have no way of influencing the other person's behavior.

28.5 Repeated Games

In the preceding section, the players met only once and played the prisoner's dilemma game a single time. However, the situation is different if the game is to be played repeatedly by the same players. In this case there are new strategic possibilities open to each player. If the other player chooses to defect on one round, then you can choose to defect on the next round. Thus your opponent can be "punished" for "bad" behavior. In a repeated game, each player has the opportunity to establish a reputation for cooperation, and thereby encourage the other player to do the same.

Whether this kind of strategy will be viable depends on whether the game is going to be played a *fixed* number of times or an *indefinite* number of times.

Let us consider the first case, where both players know that the game is going to be played 10 times, say. What will the outcome be? Suppose we consider round 10. This is the last time the game will be played, by assumption. In this case, it seems likely that each player will choose the dominant strategy equilibrium, and defect. After all, playing the game for the last time is just like playing it once, so we should expect the same outcome.

Now consider what will happen on round 9. We have just concluded that each player will defect on round 10. So why cooperate on round 9? If you cooperate, the other player might as well defect now and exploit your good nature. Each player can reason the same way, and thus each will defect.

Now consider round 8. If the other person is going to defect on round 9 ... and so it goes. If the game has a known, fixed number of rounds, then each player will defect on every round. If there is no way to enforce cooperation on the last round, there will be no way to enforce cooperation on the next to the last round, and so on.

Players cooperate because they hope that cooperation will induce further cooperation in the future. But this requires that there will always be the possibility of future play. Since there is no possibility of future play in the last round, no one will cooperate then. But then why should anyone cooperate on the next to the last round? Or the one before that? And so it goes—the cooperative solution "unravels" from the end in a prisoner's dilemma with a known, fixed number of plays.

But if the game is going to be repeated an indefinite number of times, then you *do* have a way of influencing your opponent's behavior: if he refuses to cooperate this time, you can refuse to cooperate next time. As long as both parties care enough about future payoffs, the threat of non-cooperation in the future may be sufficient to convince people to play the Pareto efficient strategy.

This has been demonstrated in a convincing way in a series of experiments run by Robert Axelrod.[3] He asked dozens of experts on game theory to submit their favorite strategies for the prisoner's dilemma and then ran a "tournament" on a computer to pit these strategies against each other. Every strategy was played against every other strategy on the computer, and the computer kept track of the total payoffs.

The winning strategy—the one with the highest overall payoff—turned out to be the simplest strategy. It is called "tit for tat" and goes like this. On the first round, you cooperate—play the "deny" strategy. On every round thereafter, if your opponent cooperated on the previous round, you cooperate. If your opponent defected on the previous round, you defect. In other words, do whatever the other player did in the last round.

The tit-for-tat strategy does very well because it offers an immediate punishment for defection. It is also a forgiving strategy: it punishes the other player only once for each defection. If he falls into line and starts to cooperate, then tit for tat will reward the other player with cooperation. It appears to be a remarkably good mechanism for achieving the efficient outcome in a prisoner's dilemma that will be played an indefinite number of times.

28.6 Enforcing a Cartel

In Chapter 27 we discussed the behavior of duopolists playing a price-setting game. We argued there that if each duopolist could choose his price, then the equilibrium outcome would be the competitive equilibrium. If each firm thought that the other firm would keep its price fixed, then each firm would find it profitable to undercut the other. The only place where this would not be true was if each firm were charging the lowest possible price, which in the case we examined was a price of zero, since the marginal costs were zero. In the terminology of this chapter, each firm charging a zero price is a Nash equilibrium in pricing strategies—what we called a Bertrand equilibrium in Chapter 27.

The payoff matrix for the duopoly game in pricing strategies has the same structure as the prisoner's dilemma. If each firm charges a high price, then they both get large profits. This is the situation where they are both cooperating to maintain the monopoly outcome. But if one firm is charging

[3] Robert Axelrod is a political scientist from the University of Michigan. For an extended discussion, see his book *The Evolution of Cooperation* (New York: Basic Books, 1984).

a high price, then it will pay the other firm to cut its price a little, capture the other fellow's market, and thereby get even higher profits. But if both firms cut their prices, they both end up making lower profits. Whatever price the other fellow is charging, it will always pay you to shave your price a little bit. The Nash equilibrium occurs when each fellow is charging the lowest possible price.

However, if the game is repeated an indefinite number of times, there may be other possible outcomes. Suppose that you decide to play tit for tat. If the other fellow cuts his price this week, you will cut yours next week. If each player knows that the other player is playing tit for tat, then each player would be fearful of cutting his price and starting a price war. The threat implicit in tit for tat may allow the firms to maintain high prices.

Real-life cartels sometimes appear to employ tit-for-tat strategies. For example, the Joint Executive Committee was a famous cartel that set the price of railroad freight in the United States in the late 1800s. The formation of this cartel preceded antitrust regulation in the United States, and at the time was perfectly legal.[4]

The cartel determined what market share each railroad could have of the freight shipped. Each firm set its rates individually, and the JEC kept track of how much freight each firm shipped. However, there were several occasions during 1881, 1884, and 1885 where some members of the cartel thought that other member firms were cutting rates so as to increase their market share, despite their agreement. During these periods, there were often price wars. When one firm tried to cheat, all firms would cut their prices so as to "punish" the defectors. This kind of tit-for-tat strategy was apparently able to support the cartel arrangement for some time.

EXAMPLE: Tit for Tat in Airline Pricing

Airline pricing provides an interesting example of tit-for-tat behavior. Airlines often offer special promotional fares of one sort or another; many observers of the airline industry claim that these promotions can be used to signal competitors to refrain from cutting prices on key routes.

A senior director of marketing for a major U.S. airline described a case in which Northwest lowered fares on night flights from Minneapolis to various West Coast cities in an effort to fill empty seats. Continental Airlines interpreted this as an attempt to gain market share at its expense and responded by cutting *all* its Minneapolis fares to Northwest's night-fare

[4] For a detailed analysis, see Robert Porter, "A Study of Cartel Stability: the Joint Executive Committee, 1880–1886," *The Bell Journal of Economics*, 14, 2 (Autumn 1983), 301–25.

level. However, the Continental fare cuts were set to expire one or two days after they were introduced.

Northwest interpreted this as a signal from Continental that it was not serious about competing in this market, but simply wanted Northwest to retract its night-fare cuts. But Northwest decided to send a message of its own to Continental: it instituted a set of cheap fares to the West Coast for its flights departing from Houston, Continental's home base! Northwest thereby signaled that it felt its cuts were justified, while Continental's response was inappropriate.

All these fare cuts had very short expiration dates; this feature seems to indicate that they were meant more as messages to the competition than as bids for larger market share. As the analyst explained, fares that an airline doesn't want to offer "should almost always have an expiration date on them in the hopes that the competition will eventually wake up and match."

The implicit rules of competition in duopoly airline markets seem to be the following: if the other firm keeps its prices high, I will maintain my high prices; but if the other firm cuts its prices, I will play tit for tat and cut my prices in response. In other words, both firms "live by the Golden Rule": do unto others as you would have them do unto you. This threat of retaliation then serves to keep all prices high.[5]

28.7 Sequential Games

Up until now we have been thinking about games in which both players act simultaneously. But in many situations one player gets to move first, and the other player responds. An example of this is the Stackelberg model described in Chapter 27, where one player is a leader and the other player is a follower.

Let's describe a game like this. In the first round, player A gets to choose top or bottom. Player B gets to observe the first player's choice and then chooses left or right. The payoffs are illustrated in a game matrix in Table 28.5.

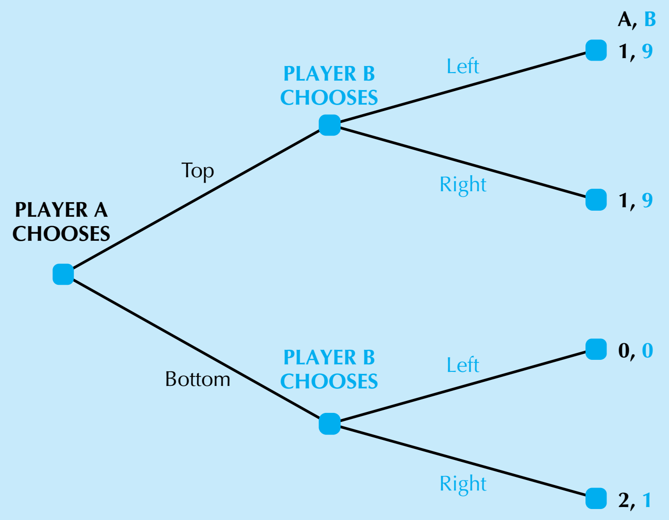
Note that when the game is presented in this form it has two Nash equilibria: (top, left) and (bottom, right). However, we'll show below that one of these equilibria isn't really reasonable. The payoff matrix hides the fact that one player gets to know what the other player has chosen before he makes his choice. In this case it is more useful to consider a diagram that illustrates the asymmetric nature of the game.

Figure 28.1 is a picture of the game in **extensive form**—a way to represent the game that shows the time pattern of the choices. First, player A

[5] Facts taken from A. Nomani, "Fare Warning: How Airlines Trade Price Plans," *Wall Street Journal,* October 9, 1990, B1.

The payoff matrix of a sequential game. **Table 28.5**

		Player B	
		Left	Right
Player A	Top	1, 9	1, 9
	Bottom	0, 0	2, 1



Extensive form of the game. This way of depicting a game indicates the order in which the players move. **Figure 28.1**

has to choose top or bottom, and then player B has to choose left or right. But when B makes his choice, he will know what A has done.

The way to analyze this game is to go to the end and work backward. Suppose that player A has already made his choice and we are sitting in one branch of the game tree. If player A has chosen top, then it doesn't matter what player B does, and the payoff is (1,9). If player A has chosen bottom, then the sensible thing for player B to do is to choose right, and the payoff is (2,1).

Now think about player A's initial choice. If he chooses top, the outcome will be (1,9) and thus he will get a payoff of 1. But if he chooses bottom, he gets a payoff of 2. So the sensible thing for him to do is to choose bottom. Thus the equilibrium choices in the game will be (bottom, right), so that the payoff to player A will be 2 and to player B will be 1.

The strategies (top, left) are not a reasonable equilibrium in this sequential game. That is, they are not an equilibrium given the order in which the players actually get to make their choices. It is true that if player A chooses top, player B could choose left—but it would be silly for player A to ever choose top!

From player B's point of view this is rather unfortunate, since he ends up with a payoff of 1 rather than 9! What might he do about it?

Well, he can *threaten* to play left if player A plays bottom. If player A thought that player B would actually carry out this threat, he would be well advised to play top. For top gives him 1, while bottom—if player B carries out his threat—will only give him 0.

But is this threat credible? After all, once player A makes his choice, that's it. Player B can get either 0 or 1, and he might as well get 1. Unless player B can somehow convince player A that he will really carry out his threat—even when it hurts him to do so—he will just have to settle for the lower payoff.

Player B's problem is that once player A has made his choice, player A expects player B to do the rational thing. Player B would be better off if he could *commit* himself to play left if player A plays bottom.

One way for B to make such a commitment is to allow someone else to make his choices. For example, B might hire a lawyer and instruct him to play left if A plays bottom. If A is aware of these instructions, the situation is radically different from his point of view. If he knows about B's instructions to his lawyer, then he knows that if he plays bottom he will end up with a payoff of 0. So the sensible thing for him to do is to play top. In this case B has done better for himself by *limiting* his choices.

28.8 A Game of Entry Deterrence

In our examination of oligopoly we took the number of firms in the industry as fixed. But in many situations, entry is possible. Of course, it is in the interest of the firms in the industry to try to prevent such entry. Since they are already in the industry, they get to move first and thus have an advantage in choosing ways to keep their opponents out.

Suppose, for example, that we consider a monopolist who is facing a threat of entry by another firm. The entrant decides whether or not to come into the market, and then the incumbent decides whether or not to cut its price in response. If the entrant decides to stay out, it gets a payoff of 1 and the incumbent gets a payoff of 9.

If the entrant decides to come in, then its payoff depends on whether the incumbent fights—by competing vigorously—or not. If the incumbent fights, then we suppose that both players end up with 0. On the other hand, if the incumbent decides not to fight, we suppose that the entrant gets 2 and the incumbent gets 1.

Note that this is exactly the structure of the sequential game we studied earlier, and thus it has a structure identical to that depicted in Figure 28.1. The incumbent is player B, while the potential entrant is player A. The top strategy is to stay out, and the bottom strategy is to enter. The left strategy is to fight and the right strategy is not to fight. As we've seen in this game, the equilibrium outcome is for the potential entrant to enter and the incumbent *not* to fight.

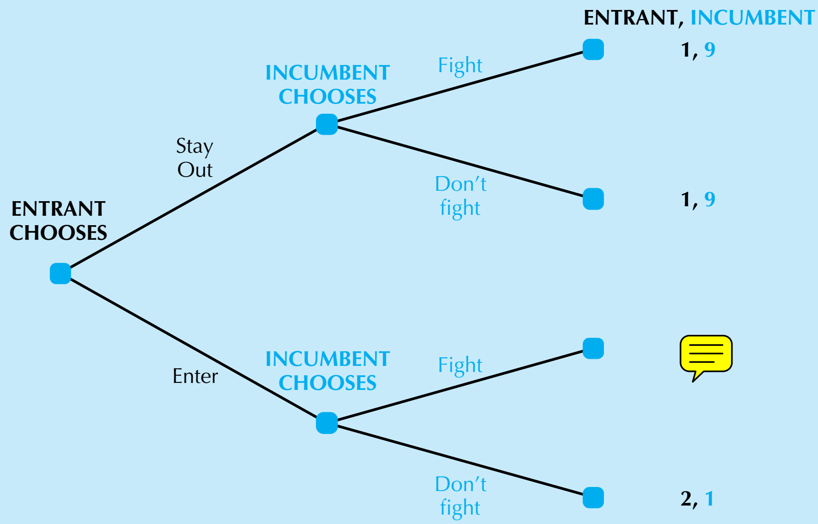


Figure 28.1

The new entry game. This figure depicts the entry game with the changed payoffs.

The incumbent's problem is that he cannot precommit himself to fighting if the other firm enters. If the other firm enters, the damage is done and the rational thing for the incumbent to do is to live and let live. Insofar as the potential entrant recognizes this, he will correctly view any threats to fight as empty.

But suppose that the incumbent can purchase some extra production capacity that will allow him to produce more output at his current marginal cost. Of course, if he remains a monopolist, he won't want to actually use this capacity since he is already producing the profit-maximizing monopoly output.

But, if the other firm enters, the incumbent will now be able to produce so much output that he may well be able to compete much more successfully against the new entrant. By investing in the extra capacity, he will lower his costs of fighting if the other firm tries to enter. Let us assume that if he purchases the extra capacity and if he chooses to fight, he will make a profit of 2. This changes the game tree to the form depicted in .

Now, because of the increased capacity, the threat of fighting is credible. If the potential entrant comes into the market, the incumbent will get a payoff of 2 if he fights and 1 if he doesn't; thus the incumbent will rationally choose to fight. The entrant will therefore get a payoff of 0 if he enters, and if he stays out he will get a payoff of 1. The sensible thing for the potential entrant to do is to stay out.

But this means that the incumbent will remain a monopolist and never have to use his extra capacity! Despite this, it is worthwhile for the monopolist to invest in the extra capacity in order to make credible the *threat* of fighting if a new firm tries to enter the market. By investing in "excess" capacity, the monopolist has signaled to the potential entrant that he will be able to successfully defend his market.

Summary

1. A game can be described by indicating the payoffs to each of the players for each configuration of strategic choices they make.

2. A dominant strategy equilibrium is a set of choices for which each player's choices are optimal *regardless* of what the other players choose.

3. A Nash equilibrium is a set of choices for which each player's choice is optimal, given the choices of the other players.

4. The prisoner's dilemma is a particular game in which the Pareto efficient outcome is strategically dominated by an inefficient outcome.

5. If a prisoner's dilemma is repeated an indefinite number of times, then it is possible that the Pareto efficient outcome may result from rational play.

6. In a sequential game, the time pattern of choices is important. In these games, it can often be advantageous to find a way to precommit to a particular line of play.

REVIEW QUESTIONS

1. Consider the tit-for-tat strategy in the repeated prisoner's dilemma. Suppose that one player makes a mistake and defects when he meant to cooperate. If both players continue to play tit for tat after that, what happens?

2. Are dominant strategy equilibria always Nash equilibria? Are Nash equilibria always dominant strategy equilibria?

3. Suppose your opponent is *not* playing her Nash equilibrium strategy. Should you play your Nash equilibrium strategy?

4. We know that the single-shot prisoner's dilemma game results in a dominant Nash equilibrium strategy that is Pareto inefficient. Suppose we allow the two prisoners to retaliate after their respective prison terms. Formally, what aspect of the game would this affect? Could a Pareto efficient outcome result?

5. What is the dominant Nash equilibrium strategy for the repeated prisoner's dilemma game when both players know that the game will end after one million repetitions? If you were going to run an experiment with human players for such a scenario, would you predict that players would use this strategy?

6. Suppose that player B rather than player A gets to move first in the sequential game described in this chapter. Draw the extensive form of the new game. What is the equilibrium for this game? Does player B prefer to move first or second?

CHAPTER 29

GAME APPLICATIONS

In the last chapter we described a number of important concepts in game theory and illustrated them using a few examples. In this chapter we examine four important issues in game theory—cooperation, competition, coexistence, and commitment—and see how they work in various strategic interactions.

In order to do this, we first develop an important analytic tool, **best response curves**, which can be used to solve for equilibria in games.

29.1 Best Response Curves

Consider a two-person game, and put yourself in the position of one of the players. For any choice the other player can make, your **best response** is the choice that maximizes your payoff. If there are several choices that maximize your payoff, then your best response will be the set of all such choices.

For example, consider the game depicted in Table 29.1, which we used to illustrate the concept of a Nash equilibrium. If the column player chooses left, row's best response is to choose top; if column chooses right, then

A simple game

Table 29.1

		Column: Left	Column: Right
Row	Top	2, 1	0, 0
	Bottom	0, 0	1, 2

row's best response is to choose bottom. Similarly, the best responses for column are to play left in response to top and to play right in response to bottom.

We can write this out in a little table:

Column's choice:	Left	Right
Row's best response:	Top	Bottom
Row's choice:	Top	Bottom
Column's best response:	Left	Right

Notice that if column thinks that row will play top, then column will want to play left, *and* if row thinks that column will play left, row will want to play top. So the pair of choices (top, left) are mutually consistent in the sense that each player is making an optimal response to the other player's choice.

Consider a general two-person game in which row has choices $r_1, \ldots, r_R$ and column has choices $c_1, \ldots, c_C$. For each choice r that row makes, let $b_c(r)$ be a best response for column, and for each choice c that column makes, let $b_r(c)$ be a best response for row. Then a **Nash equilibrium** is a pair of strategies (r^*, c^*) such that

$$\begin{aligned} c^* &= b_c(r^*) \\ r^* &= b_r(c^*). \end{aligned}$$

The concept of Nash equilibrium formalizes the idea of "mutual consistency." If row expects column to play left, then row will choose to play top, and if column expects row to play top, column will want to play left. So it is the *beliefs* and the *actions* of the players that are mutually consistent in a Nash equilibrium.

Note that in some cases one of the players may be indifferent among several best responses. This is why we only require that c^* be *one* of column's best responses, and r^* be *one* of row's best responses. If there is a unique best response for each choice then the best response *curves* can be represented as best response *functions.*

This way of looking at the concept of a Nash equilibrium makes it clear that it is simply a generalization of the Cournot equilibrium described in Chapter 27. In the Cournot case, the choice variable is the amount of output produced, which is a continuous variable. The Cournot equilibrium has the property that each firm is choosing its profit-maximizing output, given the choice of the other firm.

The Bertrand equilibrium, also described in Chapter 27, is a Nash equilibrium in pricing strategies. Each firm chooses the price that maximizes its profit, given the choice that it thinks the other firm will make.

These examples show how the best response curve generalizes the earlier models, and allows for a relatively simple way to solve for Nash equilibrium. These properties make best response curves a very helpful tool to solve for an equilibrium of a game.

29.2 Mixed Strategies

Let us use best response functions to analyze the game shown in Table 29.2.

Table 29.2

Solving for Nash equilibrium.

		Ms. Column	
		Left	Right
Mr. Row	Top	2, 1	0, 0
	Bottom	0, 0	1, 2

We are interested in looking for mixed strategy equilibria as well as pure strategy equilibria, so we let r be the probability that row plays top, and $(1-r)$ the probability that he plays bottom. Similarly, let c be the probability that column plays left, and $(1-c)$ the probability that she plays right. The pure strategies occur when r and c equal 0 or 1.

Let us calculate row's expected payoff if he chooses probability r of playing top and column chooses probability c of playing left. Look at the following array

Combination	Probability	Payoff to Row
Top, Left	rc	2
Bottom, Left	$(1-r)c$	0
Top, Right	$r(1-c)$	0
Bottom, Right	$(1-r)(1-c)$	1

The expected payoff to the players are:

$$\Pi_r = 2rc + (1-r)(1-c) = 2rc + 1 - r - c + rc$$
$$= 3r + 1 - r - c = 1 - c + r(3c-1)$$

Π_r is max if $r = 1$ against $c > 1/3$

$r = 0$ " $c < 1/3$

$0 < r < 1$ " $c = 1/3$

$$\Pi_c = rc + 2(1-r)(1-c) = rc + 2 - 2c - 2r + 2rc$$
$$= 3rc + 2 - 2c - 2r = 2(1-r) + c(3r-2)$$

Π_c is max if $c = 1$ against $r > 2/3$

$c = 0$ " $r < 2/3$

$0 < c < 1$ " $r = 2/3$

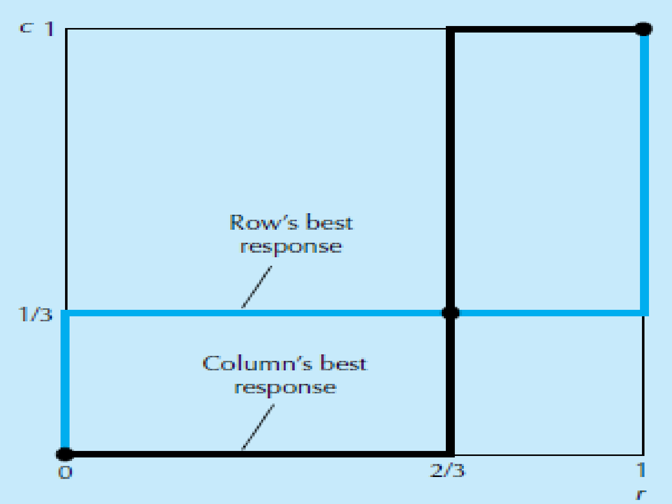


Figure 29.1 **Best response curves.** The two curves depict the best response of row and column to each other's choices. The intersections of the curves are Nash equilibria. In this case there are three equilibria, two with pure strategies and one with mixed strategies.

These curves are depicted in Figure 29.1. It is easy to see that they cross in three places: $(0, 0)$, $(2/3, 1/3)$, and $(1, 1)$, which correspond to the three Nash equilibria of this game. Two of these strategies are pure strategies, and one is a mixed strategy.

Combination	Probability	Payoff to Row
Top, Left	rc	2
Bottom, Left	$(1-r)c$	0
Top, Right	$r(1-c)$	0
Bottom, Right	$(1-r)(1-c)$	1

To calculate the expected payoff to row, we weight row's payoffs in the third column by the probability that they occur, given in the second column, and add these up. The answer is

$$\text{Row's payoff} = 2rc + (1-r)(1-c),$$

which we can multiply out to be

$$\text{Row's payoff} = 2rc + 1 - r - c + rc.$$

Now suppose that row contemplates increasing r by Δr. How will his payoff change?

$$\begin{aligned}\Delta\text{payoff to row} &= 2c\,\Delta r - \Delta r + c\,\Delta r \\ &= (3c-1)\Delta r.\end{aligned}$$

This expression will be positive when $3c > 1$ and negative when $3c < 1$. Hence, row will want to increase r whenever $c > 1/3$, decrease r when $c < 1/3$, and be happy with any value of $0 \leq r \leq 1$ when $c = 1/3$.

Similarly, the payoff to column is given by

$$\text{Column's payoff} = cr + 2(1-c)(1-r).$$

Column's payoff will change when c changes by Δc according to

$$\begin{aligned}\Delta\text{ payoff to column} &= r\,\Delta c + 2r\,\Delta c - 2\Delta c \\ &= (3r-2)\Delta c.\end{aligned}$$

Hence column will want to increase c whenever $r > 2/3$, decrease c when $r < 2/3$, and be happy with any value of $0 \leq c \leq 1$ when $r = 2/3$.

We can use this information to plot the best response curves. Start with row. If column choses $c = 0$, row will want to make r as small as possible, so $r = 0$ is the best response to $c = 0$. This choice will continue to be the best response up until $c = 1/3$, at which point *any* value of r between 0 and 1 is a best response. For all $c > 1/3$, the best response row can make is $r = 1$.

These curves are depicted in Figure 29.1. It is easy to see that they cross in three places: $(0, 0)$, $(2/3, 1/3)$, and $(1, 1)$, which correspond to the three Nash equilibria of this game. Two of these strategies are pure strategies, and one is a mixed strategy.

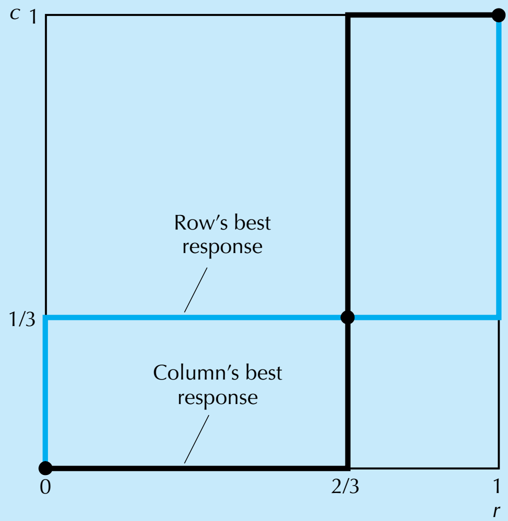


Figure 29.1 **Best response curves**. The two curves depict the best response of row and column to each other's choices. The intersections of the curves are Nash equilibria. In this case there are three equilibria, two with pure strategies and one with mixed strategies.

29.3 Games of Coordination

Armed with the tools of the last section we can examine our first class of games, **coordination games**. These are games where the payoffs to the players are highest when they can coordinate their strategies. The problem, in practice, is to develop mechanisms that enable this coordination.

Battle of the Sexes

The classic example of a coordination game is the so-called battle of the sexes. In this game, a boy and a girl want to meet at a movie but haven't had a chance to arrange which one. Alas, they forgot their cell phones, so they have no way to coordinate their meeting and have to guess which movie the other will want to attend.

The boy wants to see the latest action flick, while the girl would rather go to an art film, but they would both rather go to the same movie than not meet up at all. Payoffs consistent with these preferences are shown in

The battle of the sexes.

Table 29.3

		Girl	
		Action	Art
Boy	Action	2, 1	0, 0
	Art	0, 0	1, 2

Table 29.3. Note the defining feature of coordination games: the payoffs are higher when the players coordinate their actions than when they don't.

What are the Nash equilibria of this game? Luckily, this is just the game we used in the last section to illustrate best response curves. We saw there that there are three equilibria: both choose action, both choose art, or each chooses his or her preferred choice with probability 2/3.

Since all of these are possible equilibria, it is hard to say what will happen from this description alone. Generally, we would look to considerations outside the formal description of the game to resolve the problem. For example, suppose that the art film was a closer destination for one of the two players. Then both players might reasonably suppose that would be the equilibrium choice.

When players have good reasons to believe that one of the equilibria is more "natural" than the others, it is called a **focal point** of the game.

Prisoner's Dilemma

The prisoner's dilemma, which we discussed extensively in the last chapter, is also a coordination game. Recall the story: two prisoners can either confess, thereby implicating the other, or deny committing a crime. The payoffs are shown in Table 29.4.

The striking feature of the prisoner's dilemma is that confessing is a dominant strategy, even though coordination (both choose deny) is far superior in terms of the total payoff. Coordination would allow the prisoners to choose the best payoff, but the problem is that there is no easy way to make it happen in a single-shot game.

One way out of the prisoner's dilemma is to enlarge the game by adding new choices. We saw in the last chapter that an indefinitely repeated prisoner's dilemma game could achieve the cooperative outcome via strategies like tit for tat, in which players rewarded cooperation and punished lack of cooperation through their future actions. The extra strategic consideration

Table 29.4

The prisoner's dilemma.

		Player B: Confess	Player B: Deny
Player A	Confess	–3, –3	0, –6
	Deny	–6, 0	–1, –1

here is that refusing to cooperate today may result in extended punishment later on.

Another way to "solve" the prisoner's dilemma is to add the possibility of contracting. For example, both players could sign a contract saying that they will stick with the cooperative strategy. If either of them reneges on the contract, he or she will have to pay a fine or be punished in some way. Contracts are very helpful in achieving all sorts of outcomes, but they rely on the existence of a legal system that will enforce such contracts. This makes sense for business negotiations but is not an appropriate assumption in other contexts, such as military games or international negotiations.

Assurance Games

Consider the U.S.-U.S.S.R. arms race of the 1950s in which each country could build nuclear missiles or refrain from building them. The payoffs to these strategies might look like those shown in Table 29.5. The best outcome for both parties is to refrain from building the missiles, giving a payoff of (4, 4). But if one refrains while the other builds, the payoff will be 3 to the builder and 1 to the refrainer. The payoff if they both build missile sites is (2, 2).

It is not hard to see that there are two pure strategy Nash equilibria, (refrain, refrain) and (build, build). However, (refrain, refrain) is better for both parties. The trouble is, neither party knows which choice the other will make. Before committing to refrain, each party wants some *assurance* that the other will refrain.

One way to achieve this assurance is for one of the players to move first, by opening itself to inspection, say. Note that this can be unilateral, at least as long as one believes the payoffs in the game. If one player announces that it is refraining from deploying nuclear missiles and gives the other player sufficient evidence of its choice, it can rest assured that the other player will also refrain.

An arms race.

Table 29.5

		U.S.S.R. Refrain	U.S.S.R. Build
U.S.	Refrain	4, 4	1, 3
	Build	3, 1	2, 2

Chicken

Our last coordination game is based on an automobile game popularized in the movies. Two teenagers start at opposite ends of the street and drive in a straight line toward each other. The first to swerve loses face; if neither swerves, they both crash into each other. Some possible payoffs are shown in Table 29.6.

There are two pure strategy Nash equilibria, (row swerves, column doesn't) and (column swerves, row doesn't). Column prefers the first equilibrium and row the second, but each equilibrium is better than a crash. Note the difference between this and the assurance game; there, both players were better off doing the same thing (building or refraining) than doing different things. Here, both players are worse off doing the same thing (driving straight or swerving) than if they did different things.

Each player knows that if he can commit himself to driving straight, the other will chicken out. But of course, each player also knows that it would be crazy to crash into each other. So how can one of the players enforce his preferred equilibrium?

One important strategy is commitment. Suppose that row ostentatiously fastened a steering wheel lock on his car before starting out. Column, recognizing that row now has no choice but to go straight, would choose to swerve. Of course if both players put on a lock, the outcome would be disastrous!

How to Coordinate

If you are a player in a coordination game, you may want to get the other player to cooperate at an equilibrium that you both like (the assurance game), cooperate an an equilibrium one of you likes (battle of the sexes), play something other than the equilibrium strategy (the prisoner's dilemma), or make a choice leading to your preferred outcome (chicken).

In the assurance game, the battle of the sexes, and chicken, this can be accomplished by one player's moving first, and committing herself to a particular choice. The other player can then observe the choice and respond accordingly. In the prisoner's dilemma, this strategy doesn't work: if one player chooses not to confess, it is in the other's interest to do so. Instead of sequential moves, repetition and contracting are major ways to "solve" the prisoner's dilemma.

Table 29.6

Chicken.

		Column: Swerve	Column: Straight
Row	Swerve	0, 0	–1, 1
	Straight	1, –1	–2, –2

29.4 Games of Competition

The opposite pole from cooperation is competition. This is the famous case of **zero-sum games**, so called because the payoff to one player is equal to the losses of the other.

Most sports are effectively zero-sum games: a point awarded to one team is equivalent to a point subtracted from the other team. Competition is fierce in such games because the players' interests are diametrically opposed.

Let us illustrate a zero-sum game by looking at soccer, known as football in most of the world. Row is kicking a penalty shot and column is defending. Row can kick to the left or kick to the right; column can favor one side and defend to the left or defend to the right in order to deflect the kick.

We will express the payoffs to these strategies in terms of expected points. Obviously row will be more successful if column jumps the wrong way. On the other hand, the game may not be perfectly symmetric since row may be better at kicking in one direction than another and column may be better at defending one direction or the other.

Let us assume that row will score 80 percent of the time if he kicks to the left and column jumps to the right but only 50 percent of the time if column jumps to the left. If row kicks to the right, we will assume that he succeeds 90 percent of the time if column jumps to the left but 20 percent

Table 29.7

Penalty point in soccer.

		Column: Defend left	Column: Defend right
Row	Kick left	50, –50	80, –80
	Kick right	90, –90	20, –20

of the time if column jumps to the right. These payoffs are illustrated in Table 29.7.

Note that the payoffs in each entry sum to zero, indicating that the players have diametrically opposed goals. Row wants to maximize his expected payoff, and column wants to maximize her expected payoff—which means she wants to minimize row's payoff.

Obviously, if column knows which way row will kick she will have a tremendous advantage. Row, recognizing this, will therefore try to keep column guessing. In particular, he will kick sometimes to his strong side and sometimes to his weak side. That is, he will pursue a **mixed strategy**.

If row kicks left with probability p, he will get an expected payoff of $50p + 90(1 - p)$ when column jumps left and $80p + 20(1 - p)$ when column jumps right. Row wants to make this expected payoff as big as possible, and column wants to make it as small as possible.

For example, suppose that row chooses to kick left half the time. If column jumps left, row will have an expected payoff of $50 \times 1/2 + 90 \times 1/2 = 70$, and if column jumps right, row will have an expected payoff of $80 \times 1/2 + 20 \times 1/2 = 50$.

Column, of course, can carry through this same reasoning. If column believes that row will kick to the left half the time, then column will want to jump to the right, since this is the choice that minimizes row's expected payoff (thereby maximizing column's expected payoff).

Figure 29.2 shows row's expected payoffs for different choices of p. This simply involves graphing the two functions $50p+90(1-p)$ and $80p+20(1-p)$. Since these two expressions are linear functions of p, the graphs are straight lines.

Row recognizes that column will always try to minimize his expected payoff. Thus, for any p, the best payoff he can hope for is the *minimum* of the payoffs given by the two strategies. We've illustrated this by the colored line in Figure 29.2.

Where does the maximum of these minimum payoffs occur? Obviously, it occurs at the peak of the colored line, or, equivalently, where the two

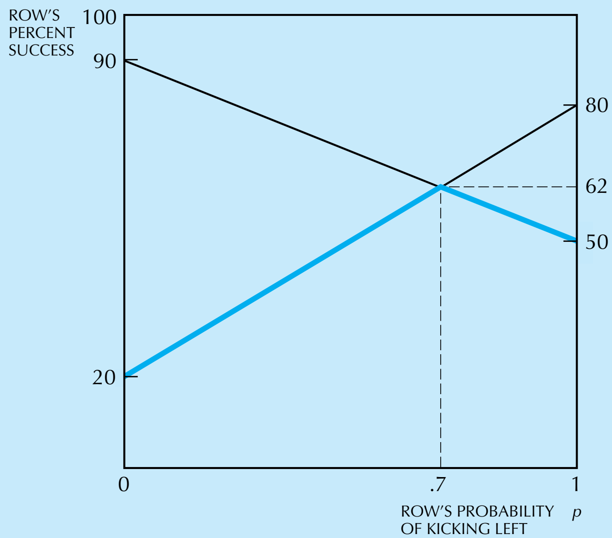


Figure 29.2 **Row's strategy**. The two curves show row's expected payoff as a function of p, the probability that he kicks to the left. Whatever p he chooses, column will try to minimize row's payoff.

lines intersect. We can calculate this value algebraically by solving

$$50p + 90(1 - p) = 80p + 20(1 - p)$$

for p. You should verify that the solution is $p = .7$.

Hence, if row kicks to the left 70 percent of the time and column responds optimally, row will have an expected payoff of $50 \times .7 + 90 \times .3 = 62$.

What about column? We can perform a similar analysis for her choices. Suppose column decides to jump to the left with probability q and jump to the right with probability $(1 - q)$. Then row's expected payoff will be $50q + 80(1 - q)$ if column jumps to the left and $90q + 20(1 - q)$ if column jumps to the right. For each q, column will want to *minimize* row's payoff. But column recognizes that row wants to *maximize* this same payoff.

Hence, if column chooses to jump to the left with probability 1/2, she recognizes that row will get an expected payoff of $50 \times 1/2 + 80 \times 1/2 = 65$ if row kicks left and $90 \times 1/2 + 20 \times 1/2 = 55$ if row kicks right. In this case row will, of course, choose to kick left.

We can plot the two payoffs in Figure 29.3, which is analogous to the previous diagram. From column's viewpoint, it is the maximum of the two lines that is relevant, since this reflects row's optimal choice for each choice of q. Hence, the diagram depicts these lines in color. Just as before we can find the best q for column—the point where row's maximum payoff is minimized. This occurs where

$$50q + 80(1 - q) = 90q + 20(1 - q),$$

which implies $q = .6$.

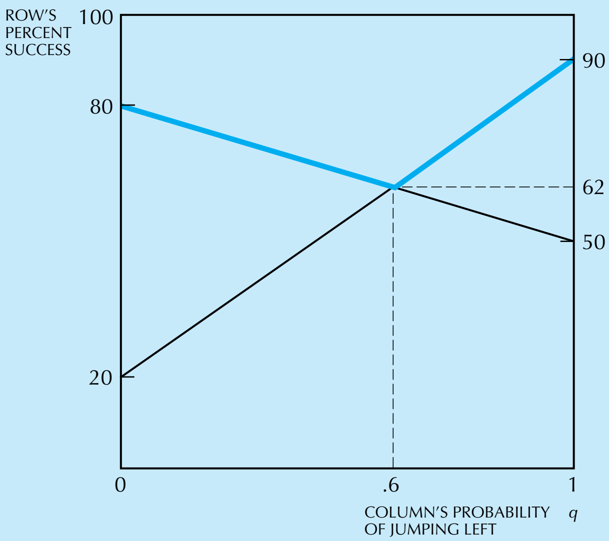


Figure 29.3

Column's strategy. The two lines show row's expected payoff as a function of q, the probability that column jumps to the left. Whatever q column chooses, row will try to maximize his own payoff.

We have now calculated the equilibrium strategies for each of the two players. Row should kick to the left with probability .7, and column should jump to the left with probability .6. These values were chosen so that row's payoffs and column's payoffs will be the same, whatever the other player does, since we found the values by equating the payoffs from the two strategies the opposing player could choose.

So when row chooses .7, column is indifferent between jumping left and jumping right, or, for that matter jumping left with any probability q. In particular, column is perfectly happy jumping left with probability .6.

Similarly, if column jumps left with probability .6, then row is indifferent between kicking left and kicking right, or any mixture of the two. In particular, he is happy to kick left with probability .7. Hence these choices are a Nash equilibrium: each player is optimizing, given the choices of the other.

In equilibrium row scores 62 percent of the time and fails to score 38 percent of the time. This is the best he can do, if the other player responds optimally.

What if column responds nonoptimally? Can row do better? To answer this question, we can use the best response curves introduced at the beginning of this chapter. We have already seen that when p is less than .7, column will want to jump left, and when p is greater than .7, column will want to jump right. Similarly when q is less than .6, row will want to kick left, and when q is greater than .6, row will want to kick right.

Figure 29.4 depicts these best response curves. Note that they intersect at the point where $p = .7$ and $q = .6$. The nice thing about the best response curves is that they tell each player what to do for every choice the other player makes, optimal or not. The only choice that is an optimal response to an optimal choice is where the two curves cross—the Nash equilibrium.

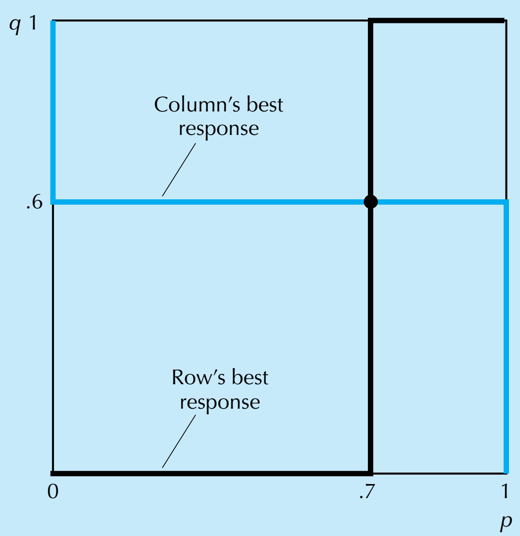


Figure 29.4 **Best response curves.** These are the best response curves for row and column, as a function of p, the probability that row kicks to the left, and q, the probability that column jumps to the left.

29.5 Games of Coexistence

We have interpreted mixed strategies as randomization by the players. In the penalty kick game, if row's strategy is to play left with probability .7 and right with probability .3, then we think that row will "mix it up" and play left 70 percent of the time and right 30 percent of the time.

But there is another interpretation. Suppose that kickers and goalies are matched up at random and that 70 percent of the kickers always kick left and 30 percent always kick right. Then, from the goalie's point of view, it is just like facing a single player who randomizes with those probabilities.

This isn't all that compelling as a story for soccer games, but it is a reasonable story for animal behavior. The idea is that various kinds of behavior are genetically programmed and that evolution selects the mixtures of the population that are stable with respect to evolutionary forces. In recent years, biologists have come to regard game theory as an indispensible tool to study animal behavior.

The most famous game of animal interaction is the **hawk-dove game**. This doesn't refer to a game between hawks and doves (which would have a pretty predictable outcome) but rather to a game involving a single species that exhibits two kinds of behavior.

Think of a wild dog. When two wild dogs come across a piece of food, they have to decide whether to fight or to share. Fighting is the hawkish strategy: one will win and one will lose. Sharing is a dovish strategy: it works well when the other player is also dovish, but if the other player is hawkish, the offer to share is rejected and the dovish player will get nothing.

A possible set of payoffs is given in Table 29.8.

Table 29.8 **Hawk-dove game.**

		Column	
		Hawk	Dove
Row	Hawk	–2, –2	4, 0
	Dove	0, 4	2, 2

If both wild dogs play dove, they end up with (2, 2). If one plays hawk and the other plays dove, the hawkish player wins everything. But if both play hawk, each dog will be seriously injured.

It obviously can't be an equilibrium if everyone plays hawk, since if some dog played dove, it would end up with 0 rather than -2. And if all dogs played dove, it would pay someone to deviate and play hawk. So there will have to be some mixture of hawk types and dove types in equilibrium. What sort of mixture should we expect?

Suppose that the fraction playing hawk is p. Then a hawk will meet another hawk with probability p and meet a dove with probability $1-p$. The expected payoff to the hawk type will be

$$H = -2p + 4(1-p).$$

The expected payoff to the dove type will be

$$D = 2(1-p).$$

Suppose that the type that has the higher payoff reproduces more rapidly, passing its tendency to play hawk or dove on to its offspring. So if $H > D$, we would see the fraction of hawk types in the population increase, and if $H < D$, we would expect to see the number of dove types increase.

The only way the population can be in equilibrium is if the payoffs to each type are the same. This requires

$$H = -2p + 4(1-p) = 2(1-p) = D,$$

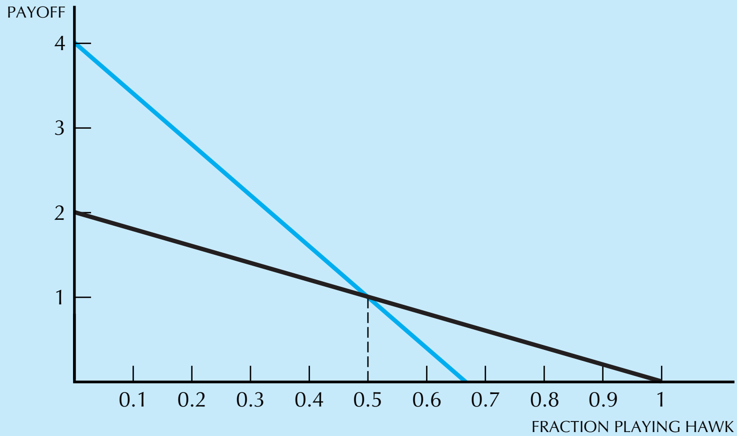
which solves for $p = 1/2$.

We have found that a 50-50 mixture of doves and hawks is an equilibrium. Is it stable, in some sense? We plot the payoffs to hawk and dove as a function of p, the fraction of the population playing hawk in Figure 29.5. Note that when $p > 1/2$, the payoff to playing hawk is less than that of playing dove, so we would expect to see the doves reproduce more rapidly, moving us back to the equilibrium 50-50 ratio. Similarly, when $p < 1/2$, the payoff to hawk is greater than the payoff to dove, leading the hawks to reproduce more rapidly.

This argument shows that not only is $p = 1/2$ an equilibrium but it is also stable under evolutionary forces. Considerations of this sort lead to a concept known as an **evolutionarily stable strategy** or an **ESS**.[1] Remarkably, an ESS turns out to be a Nash equilibrium, even though it was derived from quite different considerations.

The Nash equilibrium concept was designed to deal with calculating, rational individuals, each of whom is trying to devise a strategy appropriate for the best strategy the other player might choose. The ESS was designed to model animal behavior under evolutionary forces, where strategies that had greater fitness payoffs would reproduce more rapidly. But the ESS equilibria are also Nash equilibria, giving another argument for why this particular concept in game theory is so compelling.

[1] See John Maynard Smith, *Evolution and the Theory of Games*, (Cambridge University Press, 1982).



Payoffs in the hawk-dove game. The payoff to hawk is depicted in color; the payoff to dove is in black. When $p > 1/2$, the payoff to hawk is less than dove and vice versa, showing that the equilibrium is stable.

Figure 29.5

29.6 Games of Commitment

The previous examples involving games of cooperation and competition have been concerned with games with **simultaneous moves**. Each player had to make his or her choice without knowing what the other player was choosing (or had chosen). Indeed, games of coordination or competition can be quite trivial if one player knows the other's choices.

In this section we turn our attention to games with **sequential moves**. An important strategic issue that arises in such games is **commitment**. To see how this works, look back at the game of chicken described earlier in this chapter. We saw there that if one player could force himself to choose straight, the other player would optimally choose to swerve. In the assurance game, the outcome would be better for both players if one of them moved first.

Note that this committed choice must be both irreversible and observable by the other player. Irreversibility is part of what it means to be committed, while observability is crucial if the other player is going to be persuaded to change his or her behavior.

The Frog and the Scorpion

We begin with the fable of the frog and the scorpion. They were standing on the bank of the river, trying to figure out a way across. "I know," said the scorpion "I will climb on your back and you can swim across the river." The frog said, "But what if you sting me with your stinger?" The scorpion said, "Why would I do that? Then we would both die."

The frog found this convincing, so the scorpion climbed on his back and they started across the river. Halfway across, at the deepest point, the scorpion stung the frog. Writhing in pain, the frog cried out, "Why did you do that? Now we are both doomed!" "Alas," said the scorpion, as he sank into the river, "it is my nature."

Let's look at the frog and the scorpion from the viewpoint of game theory. Figure 29.6 depicts a sequential game with payoffs consistent with the story. Start at the bottom of the game tree. If the frog refuses the scorpion, both get nothing. Looking up one line, we see that if the frog carries the scorpion, he receives utility 5, for doing a good deed, and the scorpion receives a payoff of 3, for getting across the river. In line where the frog is stung, he receives a payoff of -10, and the scorpion gets a payoff of 5, representing the satisfaction from fulfilling his natural instincts.

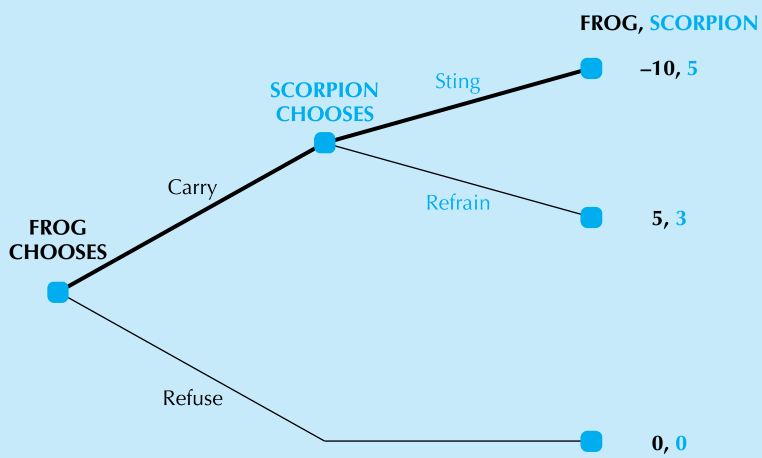


Figure 29.6 **The frog and the scorpion**. If the frog chooses to carry the scorpion, the scorpion will choose to sting him and both will die.

It is best to start with the final move of the game: the scorpion's choice of sting or refrain. Stinging has a higher payoff to the scorpion because "it is his nature" to sting. Hence the frog should rationally choose to refuse to carry the scorpion. Unfortunately, the frog didn't understand the scorpion's payoffs; apparently, he thought that the scorpion's payoffs looked something like those in Figure 29.7. Alas, this mistake was fatal for the frog.

A smart frog would figure out some way to make the scorpion commit to not stinging. He could, for example, tie his tail. Or he could hire a hit frog, who would retaliate against the scorpion's family. Whatever the strategy, the critical thing for the frog to do is to change the payoffs to the scorpion by making stinging more costly or refraining more rewarding.

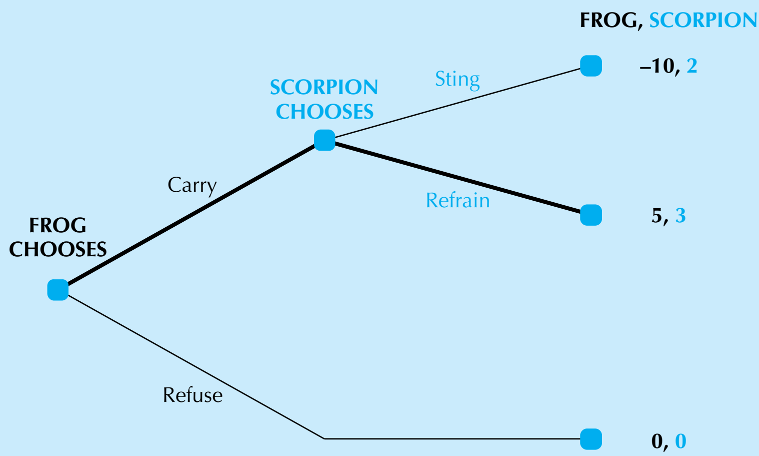


Figure 29.7 **The frog and the scorpion**. With these payoffs, if the frog chooses to carry the scorpion, the scorpion will not choose to sting him, and both will make it across the river safely.

The Kindly Kidnapper

Kidnapping for ransom is a big business in some parts of the world. In Columbia, it is estimated that there are over 2,000 kidnappings for ransom per year. In the former Soviet Union, kidnappings rose from 5 in 1992 to 105 in 1999. Many of the victims are Western businesspeople.

Some countries, such as Italy, have laws against paying ransom. The reasoning is that if the victim's family or employers can commit themselves not to pay ransom, then the kidnappers will have no motive to abduct the victim in the first place.

The problem is, of course, once a kidnapping has taken place, a victim's family will prefer to pay the kidnappers, even if it is illegal to do so. Hence penalties for paying ransom may not be effective as a commitment device.

Suppose some kidnappers abduct a hostage and then discover that they can't get paid. Should they release the hostage? The hostage, of course, promises not to reveal the identity of the kidnappers. But will he keep this promise? Once he is released, he has no incentive to do so—and every incentive to try to punish the kidnappers. Even if the kidnappers want to let the hostage go, they can't do so for fear of being identified.

Figure 29.8 depicts some possible payoffs. The kidnapper would feel bad about killing the hostage, receiving a payoff of -3. Of course, the hostage would feel even worse, receiving a payoff of -10. If the hostage is released, and refrains from identifying the kidnapper, the hostage gets a payoff of 3 and the kidnapper gets a payoff of 5. But if the hostage does identify the kidnapper, he gets a payoff of 5, leaving the kidnapper with a payoff of -5.

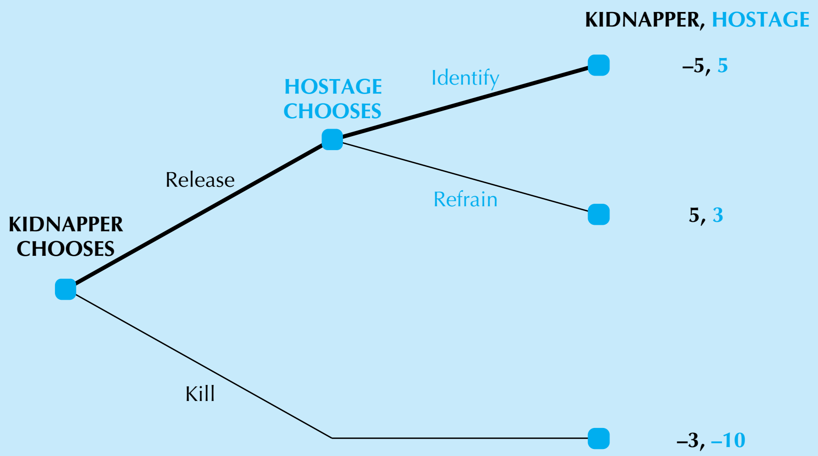


Figure 29.8 **Kidnap game.** The kidnapper would like to release the hostage, but if he does, the hostage will identify him.

Now it is the hostage who has the commitment problem: how can he convince the kidnappers that he won't renege on his promise and reveal their identity?

The hostage needs to figure out a way to change the payoffs of the game. In particular, he needs to find a way to impose a cost on himself if he identifies the kidnappers.

Thomas Schelling, an economist at the University of Maryland who has worked extensively on strategic analysis in dynamic games, suggests that the hostage might have the kidnappers photograph him in some embarrassing act and leave them with the photos. This effectively changes the payoffs from his subsequently revealing the identity of the kidnappers, since they then have the option of revealing the embarrassing photograph.

This sort of strategy is known as an "exchange of hostages." In the Middle Ages, when two kings wanted to ensure a contract wouldn't be broken, they would exchange hostages such as family members. If either king broke the agreement, the hostages would be sacrificed. Neither wanted to sacrifice their family members, so each king would have an incentive to respect the terms of their contract.

In the case of the kidnapping, the embarrassing photo would impose costs on the hostage if it were released, thereby ensuring that he will stick to his agreement not to reveal the identity of the kidnappers.

When Strength Is Weakness

Our next example comes from the world of animal psychology. It turns out that pigs quickly establish dominance-subordinateness relations, in which the dominant pig tends to boss the subordinate pig around.

Some psychologists put two pigs, one dominant, one subordinate, in a long pen.[2] At one end of the pen was a lever that would release a portion of food to a trough located at the other end of the pen. The question of interest was this: which pig would push the lever and which would eat the food?

Somewhat surprisingly the outcome of the experiment was that the dominant pig pressed the lever, while the subordinate pig waited for the food. The subordinate pig then ate most of the food, while the dominant pig rushed as fast as it could to the trough end of the pen, ending up with only a few scraps. Table 29.9 depicts a game that illustrates the problem.

Pigs pressing levers. **Table 29.9**

		Dominant Pig	
		Don't press lever	Press lever
Subordinate Pig	Don't press lever	0, 0	4, 1
	Press lever	0, 5	2, 3

[2] The original reference is Baldwin and Meese, "Social Behavior in Pigs Studied by Means of Operant Conditioning," (*Animal Behavior,* (1979)). I draw on the description of John Maynard Smith, *Evolution and the Theory of Games* (Cambridge University Press, 1982).

The subordinate pig compares a payoff of (0, 4) to (0, 2) and concludes, sensibly enough, that pressing the lever is dominated by not pressing it. Given that the subordinate pig doesn't press the lever, the dominant pig has no choice but to do so.

If the dominant pig could refrain from eating all the food and reward the subordinate pig for pressing the lever, it could achieve a better outcome. The problem is that pigs have no contracts, and the dominant pig can't help being a hog!

As in the case of the kindly kidnapper, the dominant pig has a commitment problem. If he could only commit to not eating all the food, he would end up much better off.

Savings and Social Security

Commitment problems aren't limited to the animal world. They also show up in economic policy.

Saving for retirement is an interesting and timely example. Everyone gives lip service to the fact that saving is a good idea. Unfortunately, few people actually do it. Part of the reason for the reluctance to save is that individuals recognize that society won't let them starve, so there is a good chance they will be bailed out later on.

To formulate this in a game between the generations, let's consider two strategies for the older generation: save or squander. The younger generation likewise has two strategies: support their elders or save for their own retirement. A possible game matrix is shown in Table 29.10.

Table 29.10 **Intergenerational conflict over savings.**

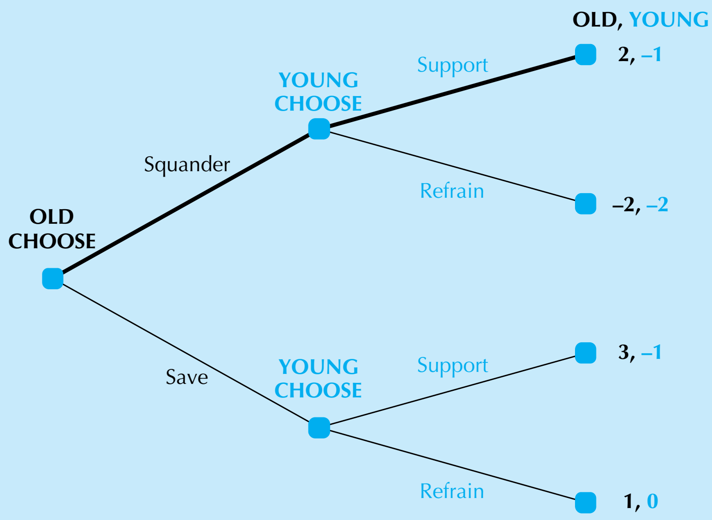
		Younger Generation	
		Support	Refrain
Older Generation	Save	3, –1	1, 0
	Squander	2, –1	–2, –2

If the older generation saves and the younger generation also supports them, the old folks end up with a utility level of 3 and the young folks end up with -1. If the older generation squanders and the younger generation supports them, the elders end up with a utility of 2 and the young folks end up with -1.

If the younger generation refrains from providing support to their elders and the older generation saves, the old folks get 1 and the young folks get 0. Finally, if the old folks squander and the young folks neglect them, each ends up with utility of -2, the old folks from starving and the young folks from having to watch.

It is not hard to see that there are two Nash equilibria in this game. If the old folks choose to save, then the young folks will choose optimally to neglect them. But if the old folks choose to squander, then it is optimal for the younger generation to support them. And of course, given that the younger generation will support their elders, it is optimal for their elders to squander!

However, this analysis ignores the time structure of the game: one of the (few) advantages of being old is that you get to move first. If we draw out the game tree, the payoffs become those in Figure 29.9.



The savings game in extended form. Knowing that the younger generation will support them, the older generation chooses to squander. The subgame perfect equilibrium is (support, squander).

Figure 29.9

If the oldsters save, the youngsters will choose to neglect them, so the oldsters end up with a payoff of 1. If the oldsters squander, they know that the youngsters won't be able to bear watching them starve, so the oldsters end up with a payoff of 3. Hence the sensible thing for the oldsters to do is to squander, knowing they will be bailed out later on.

Of course, most developed countries now have a program like the U.S. Social Security program that forces each generation to save for retirement.

Hold Up

Consider the following strategic interaction. You hire a contractor to build a warehouse. After the plans are approved and the construction is almost done, you realize that the color is bad, so you ask the contractor to change the paint, which involves a trivial expense. The contractor comes back and says: "That change order will be $1500, please."

You recognize that it will cost you at least that much to delay completion until you can find a painter, and you really do want the new color, so, muttering under your breath, you pay the cost. Congratulations, you have been held up!

Of course, contractors are not the only party at fault in this sort of game. The clients can "hold up" their payment as well, causing lots of grief for the contractor.

The game tree for the hold-up problem is depicted in Figure 29.10. We suppose that the value the owner places on having the new paint is $1500 and that the actual cost of painting is $200. Starting at the top of leaves of the tree, if the contractor charges $1500, it will realize a profit of $1300, and the client gets a net utility of zero.

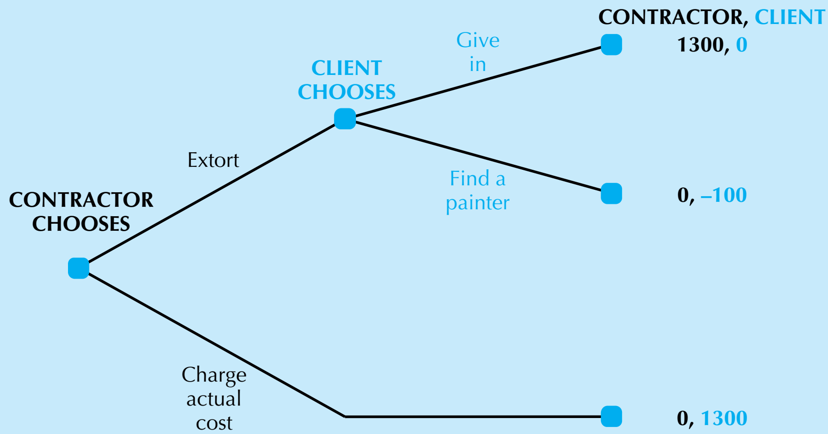
If the client looks for another painter, it will cost him $200 to pay the painter and, say, $1400 in lost time. He gets the color he wants which is worth $1500, buy has to pay $1600 in direct costs and delay costs, leaving him with a net loss of $100.

If the contractor charges the client the actual cost of $200, he breaks even and the client gets a $1500 value for $200, leaving him with a net payoff of $1300.

As can be seen, the optimal choice for the contractor is to extort the payment, and the optimal choice for the customer is to give in. But a sensible client will recognize that change orders will occur in any project. Because of this, the client will be reluctant to hire contractors with a reputation for extortion which is, of course, bad for the contractor.

How do firms solve the hold-up problem? The basic answer is contracts. Normally, contractors negotiate a contract specifying what kinds of change orders are appropriate and how their costs will be determined. Sometimes there are even arbitration or other dispute resolution procedures built into the contracts. A lot of time, energy, and money goes into writing contracts just to make certain that hold up won't occur.

But contracts aren't the only solution. Another way to solve the problem is through commitment. For example, the contractor might post a bond guaranteeing timely completion of the project. Again, there will generally be some objectively specified terms about what constitutes completion.



The hold-up problem. The contractor charges a high price for the change since the client has no alternative.

Figure 29.10

Another important factor is reputation. Obviously, a contractor who persistently tries to extort his customers will get a bad reputation. He won't be hired again by this customer, and he certainly won't get good recommendations. This reputation effect can be examined in a repeated game context in which hold up today will cost the contractor in the future.

29.7 Bargaining

The classical bargaining problem is divide the dollar. Two players have a dollar that they want to divide between them. How do they do it?

The problem, as stated, has no answer since there is too little information to construct a reasonable model. The challenge in modeling bargaining is to find some other dimensions on which the players can negotiate.

One solution, the **Nash bargaining model**, takes an axiomatic approach by specifying certain properties that a reasonable bargaining solution should have and then proving that there is only one outcome that satisfies these axioms.

The outcome ends up depending on how risk averse the players are and what will happen if no bargain is made. Unfortunately, a full treatment of this model is beyond the scope of this book.

An alternative approach, the **Rubinstein bargaining model**, looks at a sequence of choices and then solves for the subgame perfect equilibrium. Luckily the basic insight of this model is easy to illustrate in simple cases.

Two players, Alice and Bob, have \$1 to divide between them. They agree to spend at most three days negotiating over the division. The first day, Alice will make an offer, Bob either accepts or comes back with a counteroffer the next day, and on the third day Alice gets to make one final offer. If they cannot reach an agreement in three days, both players get zero.

We assume Alice and Bob differ in their degree of impatience: Alice discounts payoffs in the future at a rate of α per day, and Bob discounts payoffs at a rate of β per day. Finally, we assume that if a player is indifferent between two offers, he will accept the one that is most preferred by his opponent. This idea is that the opponent could offer some arbitrarily small amount that would make the player strictly prefer one choice and that this assumption allows us to approximate such an "arbitrarily small amount" by zero. It turns out that there is a unique subgame perfect equilibrium of this bargaining game.

We start our analysis at the end of the game, right before the last day. At this point Alice can make a take-it-or-leave-it offer to Bob. Clearly, the optimal thing for Alice to do at this point is to offer Bob the smallest possible amount that he would accept, which, by assumption, is zero. So if the game actually lasts three days, Alice would get \$1 and Bob would get zero (i.e., an arbitrarily small amount).

Now go back to the previous move, when Bob gets to propose a division. At this point Bob should realize that Alice can guarantee herself \$1 on the next move by simply rejecting his offer. A dollar next period is worth α to Alice this period, so any offer less than α would be sure to be rejected. Bob certainly prefers $1-\alpha$ now to zero next period, so he should rationally offer α to Alice, which Alice will then accept. So if the game ends on the second move, Alice gets α and Bob gets $1-\alpha$.

Now move to the first day. At this point Alice gets to make the offer and he realizes that Bob can get $1-\alpha$ if he simply waits until the second day. Hence Alice must offer a payoff that has at least this present value to Bob in order to avoid delay. Thus she offers $\beta(1-\alpha)$ to Bob. Bob finds this (just) acceptable and the game ends. The final outcome is that the game ends on the first move with Alice receiving $1-\beta(1-\alpha)$ and Bob receiving $\beta(1-\alpha)$.

The first panel in Figure 29.11 illustrates this process for the case where $\alpha=\beta<1$. The outermost diagonal line shows the possible payoff patterns on the first day, namely, all payoffs of the form $x_A+x_B=1$. The next diagonal line moving toward the origin shows the present value of the payoffs if the game ends in the second period: $x_A+x_B=\alpha$. The diagonal line closest to the origin shows the present value of the payoffs if the game ends in the third period; the equation for this line is $x_A+x_B=\alpha^2$. The right-angled path depicts the minimum acceptable divisions each period, leading up to the final subgame perfect equilibrium. The second panel in Figure 29.11 shows how the same process might look with more stages in the negotiation.

It is natural to let the horizon go to infinity and ask what happens in the

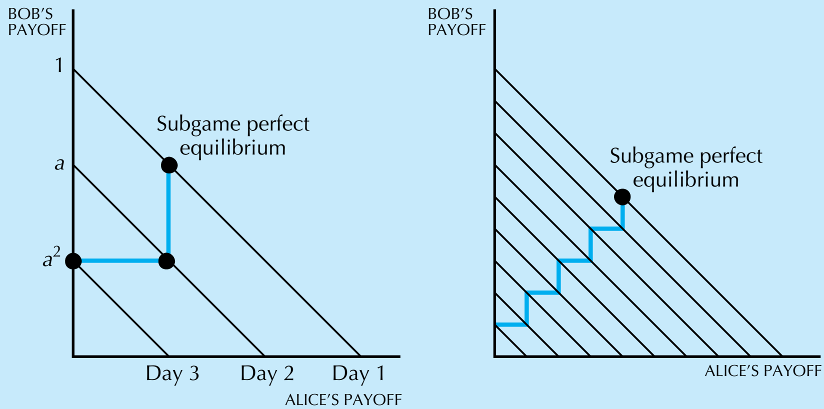


Figure 29.11

A bargaining game. The heavy line connects together the equilibrium outcomes in the subgames. The point on the line that is furthest out is the subgame perfect equilibrium.

infinite game. It turns out that the subgame perfect equilibrium division is

$$\text{Payoff to Alice} = \frac{1-\beta}{1-\alpha\beta}$$

$$\text{Payoff to Bob} = \frac{\beta(1-\alpha)}{1-\alpha\beta}.$$

Note that if $\alpha = 1$ and $\beta < 1$, then Alice receives the entire payoff.

The Ultimatum Game

The Rubinstein bargaining model is so elegant that economists rushed to test it in the laboratory. They found, alas, that elegance does not imply accuracy. Naive subjects (i.e., noneconomics majors) aren't very good at looking ahead more than one or two steps, if that.

In addition, there are other factors that cause problems. To see this, let us examine a one-step version of the bargaining model described above. Alice and Bob still have \$1 to divide between them. Alice proposes a division, and, if Bob agrees, the game ends. The question is, what should Alice say?

According to the theory, she should propose something like 99 cents for Alice, 1 cent for Bob. Bob, figuring that 1 cent is better than nothing, accepts, and Alice goes home happy that she studied economics.

Unfortunately, it doesn't work out like that. A more likely outcome is that Bob, disgusted by the paltry 1 cent, says "No way," and Alice ends up with nothing. Alice, recognizing this possibility, will tend to sweeten the offer. In actual experiments, the average offer for U.S. undergraduates is about 45 cents, and this offer tends to be accepted most of the time.

The offering players are behaving rationally, in the sense that the 45 cent offer is pretty close to maximizing the expected payoff, given the observed frequency of rejection. It is the receiving players who behave differently than the theory predicts, since they reject small offers, even though this makes them worse off.

There are many proposed explanations for this. One view is that too small an offer violates **social norms** of behavior. Indeed, economists have found quite significant cross-cultural differences in behavior in ultimatum games. Another, not inconsistent view, is that receivers get some utility payoff from hurting the offerers, in retaliation for the small offer. After all, if all you are losing is a penny, the satisfaction of striking back at the other player is pretty attractive by comparison. We will the ultimatum game in more detail in the next chapter.

Summary

1. A player's best response function gives the optimal choice for him as a function of the choices the other player(s) might make.

2. A Nash equilibrium in a two-person game is a pair of strategies, one for each player, each of which is a best response to the other.

3. A mixed strategy Nash equilibrium involves randomizing among several strategies.

4. Common games of coordination are the battle of the sexes, where both players want to do the same thing rather than different things; the prisoner's dilemma, where the dominant strategy ends up hurting both players; the assurance game, where both players want to cooperate as long as they think the other will cooperate; and chicken, where players want to avoid doing the same thing.

5. A two-person zero-sum game is one where the payoffs to one player are the negative of the payoffs to the other.

6. Evolutionary games are concerned with outcomes that are stable under population reproduction.

7. In sequential games, players move in turn. Each player therefore has to reason about what the other will do in response to his or her choices.

8. In many sequential games, commitment is an important issue. Finding ways to force commitment to play particular strategies can be important.

REVIEW QUESTIONS

1. In a two-person Nash equilibrium, each player is making a best response to what? In a dominant strategy equilibrium, each player is making a best response to what?

2. Look at the best responses for row and column in the section on mixed strategies. Do these give rise to best response functions?

3. If both players make the same choice in a coordination game, all will be well.

4. The text claims that row scores 62 percent of the time in equilibrium. Where does this number come from?

5. A contractor says that he intends to "low-ball the bid and make up for it on change orders." What does he mean?