Solutions

1. Which option is equivalent to $\log_{16} 2$?

A. $\dfrac{\log_2 16}{\log_{16} 2}$

B. $\dfrac{\ln 16}{\ln 2}$

C. $\dfrac{4 \log 2}{\log 16}$

D. $\dfrac{\log_7 2}{\log_7 16}$

Solution.

$$\log_{16} 2 = \frac{\ln 2}{\ln 16} = \frac{\ln 2}{\ln(2^4)} = \frac{\ln 2}{4 \ln 2} = \frac{1}{4}.$$

Change of base with any common base k gives $\log_{16} 2 = \dfrac{\log_k 2}{\log_k 16}$. Only choice D has that exact structure.

$$\boxed{\textbf{D}}$$

2. Mara's 6-year-old car is currently valued at \$18500. If the car depreciated at a continuous rate of 9% per year, which equation correctly represents the value C of the car when it was <u>new</u>?

A. $C = 18500(1 - 0.09)^6$

B. $C = 18500\left(1 + \dfrac{0.09}{12}\right)^{72}$

C. $C = 18500\, e^{0.54}$

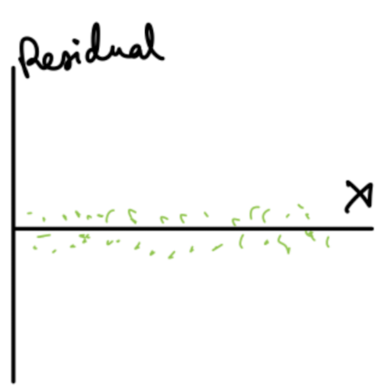
D. $C = 18500\, e^{-0.54}$

Solution. Continuous decay formula:

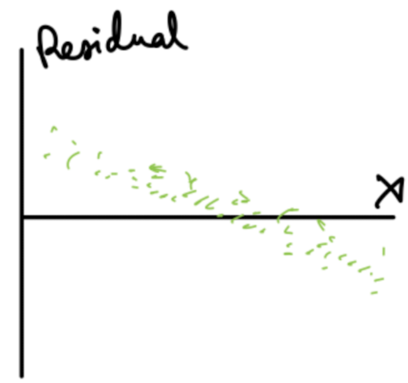
$$P_{\text{now}} = Ce^{-rt} \quad \Longrightarrow \quad C = P_{\text{now}} e^{rt} = 18500\, e^{0.09 \cdot 6} = 18500\, e^{0.54}.$$

$$\boxed{\textbf{C}}$$

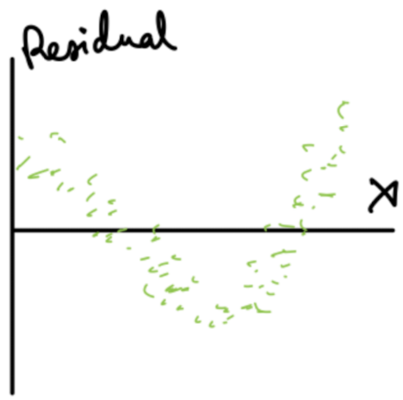
3. Linear regressions were constructed for four data sets, and the residual plots are shown below. Which residual plot indicates that an exponential function would be a better model for the relationship between the two variables?



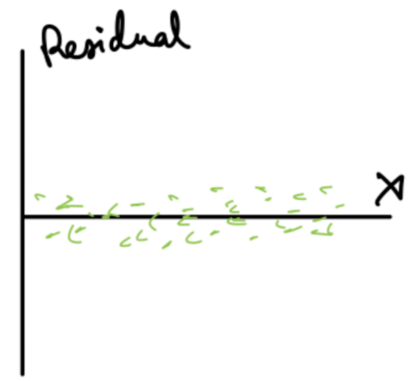
A.



B.



C.



D.

Solution. Residual Plot C displays a clear U-shape: positive residuals at low and high x, negative in the middle. Such systematic curvature means a straight line is missing multiplicative structure—typical of underlying exponential data. Plots A, B, D show no such pattern.

$$\boxed{\textbf{C}}$$

4. Solve $\frac{1}{2}\left(5^{3x-2}\right) = 40$.

A. $x = \log_5\left(\frac{8}{5}\right) + \frac{2}{3}$

B. $x = \dfrac{\log_5 80 + 2}{3}$

C. $x = \dfrac{2 - \log_5 80}{3}$

D. $x = \dfrac{\log_2 80 - 2}{3}$

Solution.

$$\frac{1}{2}\,5^{3x-2} = 40 \implies 5^{3x-2} = 80 \implies 3x - 2 = \log_5 80 \implies x = \frac{\log_5 80 + 2}{3}.$$

$$\boxed{\textbf{B}}$$

5. Elena deposits \$15 000 into a savings account that compounds monthly at an annual rate of 3%.

Which function matches this scenario and provides the correct amount of time Elena must wait for her deposit to reach at least \$25 000?

A. $A(t) = 15,000\left(1+\frac{0.03}{12}\right)^{12t}$ — predicts **19** years

B. $A(t) = 15,000\left(1+\frac{0.03}{12}\right)^{12t}$ — predicts **18** years

C. $A(t) = 15,000\left(1+\frac{3}{12}\right)^{12t}$ — predicts **19** years

D. $A(t) = 15,000\left(1+\frac{3}{12}\right)^{12t}$ — predicts **18** years

Solution. Proper monthly-compounding model:

$$A(t) = 15\,000\left(1+\frac{0.03}{12}\right)^{12t}.$$

Set $A = 25\,000$:

$$\left(1+\tfrac{0.03}{12}\right)^{12t} = \frac{25\,000}{15\,000} = 1.666\overline{6} \implies t = \frac{\ln(5/3)}{12\ln(1+0.0025)} \approx 18.8 \text{ yr.}$$

Rounded up, Elena must wait **19 years**. Choice A alone uses the correct formula and states 19 years.

$\boxed{\textbf{A}}$