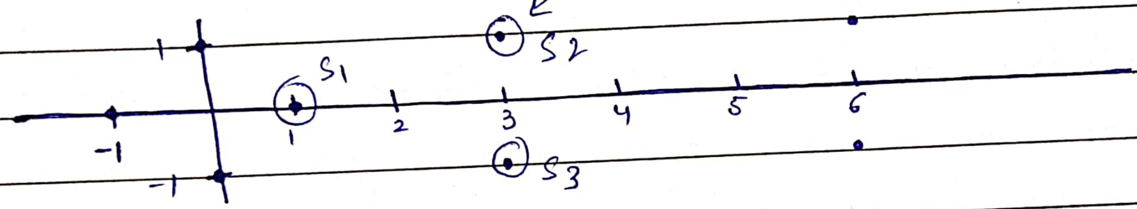
Date:

Q1

data : $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \end{pmatrix} \right\} = -1$ labeled

$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\} = +1$

① plot data: ② find SVs



③ use a suitable kernel.

→ As this data is linearly seperable we will use polynomial kernel:

$$(a * b + r)^d$$

d ← degree of polynomial

a, b: data points

r: coefficient of variable

because we have 3 points (SVs) we will have three combinations from where we draw coefficients of variables.

→

$$\alpha_1 S_1 S_1 + \alpha_2 S_1 S_2 + \alpha_3 S_1 S_3 = -1$$

$$\alpha_1 S_2 S_1 + \alpha_2 S_2 S_2 + \alpha_3 S_2 S_3 = +1$$

$$\alpha_1 S_3 S_1 + \alpha_3 S_3 S_2 + \alpha_3 S_3 S_3 = +1.$$

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Let bias = 1

$$S_1' = \begin{pmatrix}1\\0\\1\end{pmatrix}, \quad S_2' = \begin{pmatrix}3\\1\\1\end{pmatrix}, \quad S_3' = \begin{pmatrix}3\\-1\\1\end{pmatrix}$$

$$\alpha_1 S_1' S_1' + \alpha_2 S_1' S_2' + \alpha_3 S_1' S_3' = 1$$

$$\alpha_1 S_2' S_1' + \alpha_2 S_2' S_2' + \alpha_3 S_2' S_3' = -1$$

$$\alpha_1 S_3' S_1' + \alpha_2 S_3' S_2' + \alpha_3 S_3' S_3' = -1$$

$$\alpha_1 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}3\\-1\\1\end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}3\\1\\1\end{pmatrix}\begin{pmatrix}3\\-1\\1\end{pmatrix} = +1$$

$$\alpha_1 \begin{pmatrix}3\\-1\\1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}3\\-1\\1\end{pmatrix}\begin{pmatrix}3\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}3\\-1\\1\end{pmatrix}\begin{pmatrix}3\\-1\\1\end{pmatrix} = +1$$

$$\alpha_1 (2) + \alpha_2 (4) + \alpha_3 (4) = -1$$

$$\alpha_1 (4) + \alpha_2 (11) + \alpha_3 (9) = +1$$

$$\alpha_1 (4) + \alpha_2 (9) + \alpha_3 (11) = +1$$

By Crammers rule:

$$\begin{bmatrix}2 & 4 & 4\\4 & 11 & 9\\4 & 9 & 11\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix} = \begin{bmatrix}-1\\+1\\+1\end{bmatrix}$$

$$A \times X = B$$

$$\alpha_1 = \frac{|A_x|}{|A|}, \quad \alpha_2 = \frac{|A_y|}{|A|}, \quad \alpha_3 = \frac{|A_z|}{|A|}$$

$$|A_x| = \begin{vmatrix}-1 & 4 & 4\\+1 & 11 & 9\\+1 & 9 & 11\end{vmatrix} = 1(121-81) - 4(-11+9) + 4(-9+11)$$

$$= 40 + 8 + 8 = 7 \; \sqrt{-56}.$$

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$$|A| = \begin{vmatrix} 2 & 4 & 4 \\ 4 & 11 & 9 \\ 4 & 9 & 11 \end{vmatrix} \Rightarrow 2(121-81) - 4(44-36) + 4(36-44)$$
$$\Rightarrow 80 - 32 - 32$$
$$\Rightarrow 80 - 64 \Rightarrow 16.$$

$$|A_y| = \begin{vmatrix} 2 & -1 & 4 \\ 4 & +1 & 9 \\ 4 & +1 & 11 \end{vmatrix} \Rightarrow 2(-11+9) - 1(44-36) + 4(-4+4)$$
$$\Rightarrow -4 - 8 \Rightarrow +12.$$

$$|A_z| = \begin{vmatrix} 2 & 4 & -1 \\ 4 & 11 & +1 \\ 4 & 9 & +1 \end{vmatrix} \Rightarrow 2(-11+9) - 4(-4+4) + 1(36-44)$$
$$\Rightarrow -4 - 8$$
$$\Rightarrow +12.$$

Scanned with CamScanner

$\alpha_1 = \frac{-56}{16}$; $\alpha_2 = \frac{12}{16}$; $\alpha_3 = \frac{12}{16}$

$\alpha_1 = -3.5$ $\alpha_2 = 0.75$, $\alpha_3 = 0.75$

④ find plane (decision boundary)

$\tilde{w}$ = sum of all SVs with their coefficients.

$$\tilde{w} = (\alpha_1 S_1' + \alpha_2 S_2' + \alpha_3 S_3')$$

$$= -3.5\begin{pmatrix}1\\0\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\1\\1\end{pmatrix} + 0.75\begin{pmatrix}3\\-1\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\\-2\end{pmatrix} \quad \begin{pmatrix}1\\0\end{pmatrix} = \tilde{w} \text{ (plane)}.$$

$-2 = b$ (offset)

⑤ plot the line.