Date:

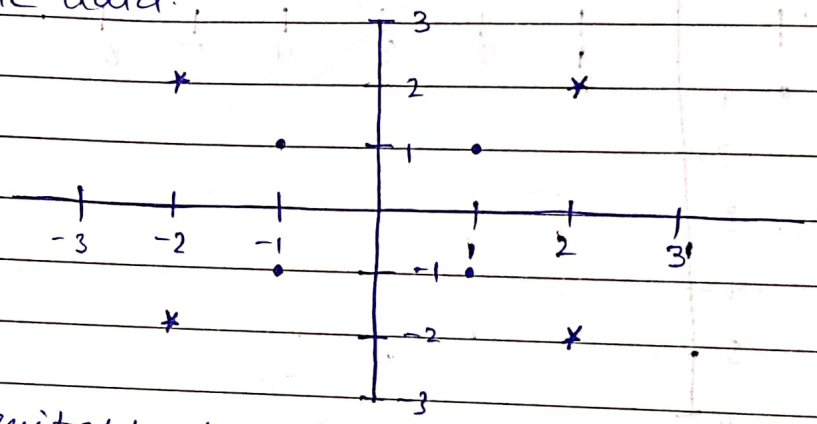
* radical bases function RBF

Q2: Nonlinear SVM

$$\left\{\begin{pmatrix}2\\2\end{pmatrix},\begin{pmatrix}2\\-2\end{pmatrix},\begin{pmatrix}-2\\-2\end{pmatrix},\begin{pmatrix}-2\\2\end{pmatrix}\right\}$$

$$\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\end{pmatrix},\begin{pmatrix}-1\\1\end{pmatrix}\right\}$$

① plot the data:



labels.
• -1
* 1

② use a suitable kernel to transform data

→ the data is non-linearly seperable so, we will use radial bases function i.e RBF Kernel: $e^{-\gamma(a-b)^2}$

$$\Theta\begin{pmatrix}x_1,x_2\end{pmatrix}=\begin{cases}\begin{pmatrix}4-x_2+|x_1-x_2|\\4-x_1+|x_1-x_2|\end{pmatrix} & \text{if } \sqrt{x_1^2+x_2^2}>2\\ (x_1,x_2) & \text{otherwise.}\end{cases}$$

+ve eg $\left\{\begin{pmatrix}2\\2\end{pmatrix},\begin{pmatrix}6\\2\end{pmatrix},\begin{pmatrix}6\\6\end{pmatrix},\begin{pmatrix}2\\6\end{pmatrix}\right\}$

-ve eg $\left\{\begin{pmatrix}1\\1\end{pmatrix},\begin{pmatrix}1\\-1\end{pmatrix},\begin{pmatrix}-1\\-1\end{pmatrix},\begin{pmatrix}-1\\1\end{pmatrix}\right\}$

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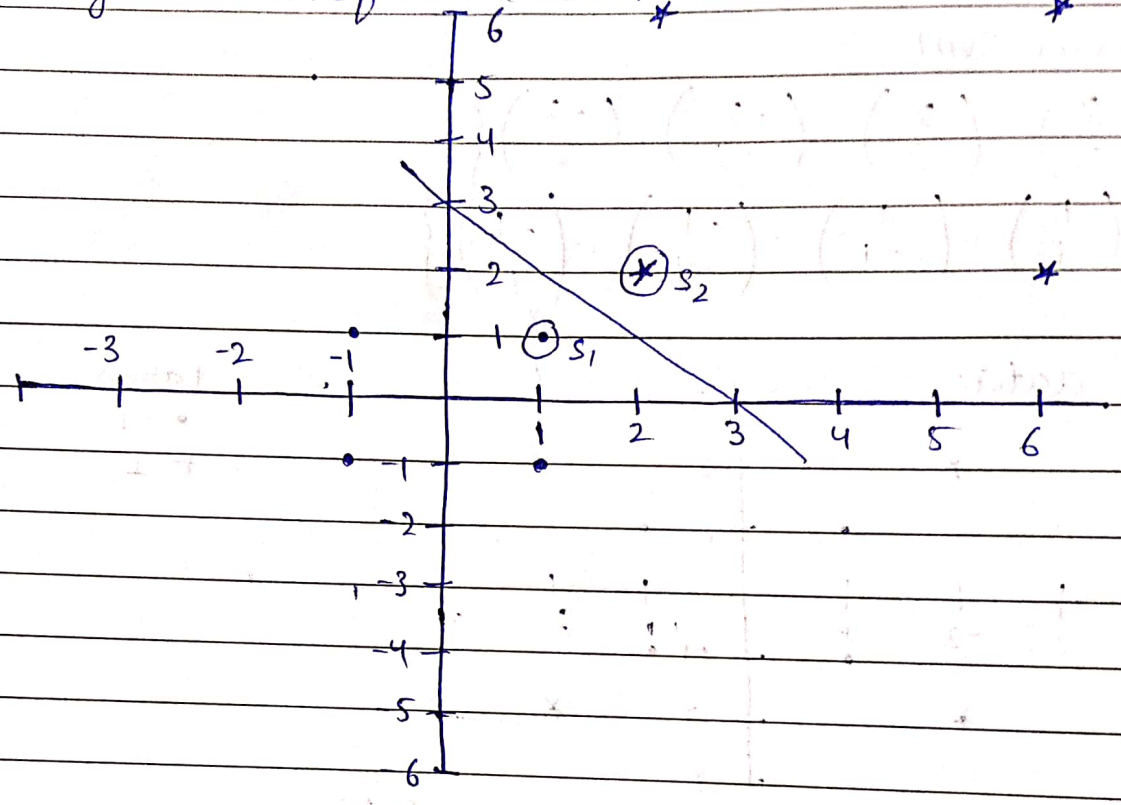
→ plotting the transformed data:



③ determine SVs.

$$S_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$S_1' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \quad S_2' = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

using kernel

$$a_1 S_1' S_1' + a_2 S_1' S_2' = -1$$
$$a_1 S_2' S_1' + a_2 S_2' S_2' = 1$$

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$$a_1\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\2\\1\end{pmatrix} + a_2\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix} = -1$$

$$3a_1 + 5a_2 = -1$$

$$a_1\begin{pmatrix}2\\2\\1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} + a_2\begin{pmatrix}2\\2\\1\end{pmatrix}\begin{pmatrix}2\\2\\1\end{pmatrix} = 1$$

$$5a_1 + 9a_2 = 1$$

$5\times$ $\quad 3a_1 + 5a_2 = -1$

$3\times$ $\quad 5a_1 + 9a_2 = 1$

$$15a_1 + 25a_2 = -5$$

$$-15a_1 \pm 27a_2 = \pm 3$$

$$25a_2 - 27a_2 = -5 - 3$$

$$-2a_2 = -8$$

$$\boxed{a_2 = 4}$$ plug in.

$$3a_1 + 5(4) = -1$$

$$3a_1 = -1 - 20$$

$$\boxed{a_1 = -\frac{19}{3}} \approx -6.34 \approx -7$$

(b) calculate slope of decision boundary

$$\tilde{w} = \sum a_i s_i$$

$$\tilde{w} = -6.34\begin{pmatrix}1\\1\\1\end{pmatrix} + 4\begin{pmatrix}2\\2\\1\end{pmatrix} = \begin{pmatrix}1\\1\\-3\end{pmatrix}$$

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$$y = wx + b$$

$$w = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b = -3.$$

(5) plot plane. (on figure)