NUMERICAL COMPUTING NOTES:

QUESTION # 1a:

$$\frac{b-a}{2^N} = 0.0000001$$

$$\frac{3-1}{2^N} = 0.0000001$$

$$\frac{2}{2^N} = 0.0000001$$

$$2^N = \frac{2}{0.0000001}$$

$$2^N = 20000000$$

$$ln2^N = ln20000000$$

$$N * ln2 = ln20000000$$

$$N = \frac{ln20000000}{ln2}$$

$$N = 24.2534967$$

QUESTION # 1b:

$$f(1) = 3sin1 - \frac{1^2}{2} - 1 - ln1 = 1.0244$$

$$f(3) = 3sin3 - \frac{3^2}{2} - 3 - ln3 = -8.1753$$

BISECTION:

$$[P_0, P_1] = [1, 3]$$

$$p_2 = \frac{p_0 + p_1}{2}$$

$$= \frac{1+3}{2}$$

$$= 2$$

$$f(p_2) = 3sin2 - \frac{2^2}{2} - 2 - ln2$$

$$= -1.9653$$

$$\frac{b-a}{\varepsilon} = 2^{n}$$

$$X = \sqrt{7}$$
$$X^2 = 7$$
$$X^2 - 7 = 0$$

$$\ln\left(\frac{1}{\varepsilon}\right) = n \ln 2$$

$(2,3)$

$X -$

$$f(x) = x^2 - 7$$
$$f'(x) = 2x$$

$$P_i = P_{i-1} - \frac{f(P_{i-1})}{f'(P_{i-1})}$$

$$P_i = P_{i-1} - \frac{P_{i-1}^2 - 7}{2P_{i-1}}$$

$$P_1 = P_0 - \frac{P_0^2 - 7}{2P_0}$$

$$P_1 = 2.65,$$
$$P_2 = 2.6457547417$$
$$P_3 = 2.6457513311$$

$P_0 = 2.5$

$a = f(P_0)$
$b = f\,\text{Prime}(P_0)$
$P_1 = P_0 - \frac{a}{b}$

$[P_0, P_1] = [1, 2]$

$$p_2 = \frac{1+2}{2}$$

$$= 1.5$$

$$f(p_2) = 3sin1.5 - \frac{1.5^2}{2} - 1.5 - ln1.5$$

$$= -0.03798$$

FALSE POSITION:

$[P_0, P_1] = [1, 1.5]$

$$p_2 = p_1 - \frac{f(p_1) * (p_1 - p_0)}{f(p_1) - f(p_0)}$$

$$= 1.5 - \frac{-0.03798 * (1.5 - 1)}{-0.03798 - 1.0244}$$

$$= 1.5 - 0.01787496$$

$$= 1.4821$$

$$f(p_2) = 3sin1.4821 - \frac{1.4821^2}{2} - 1.4821 - ln1.4821$$

$$= 0.0143$$

$[P_0, P_1] = [1.4821, 1.5]$

$$p_2 = p_1 - \frac{f(p_1) * (p_1 - p_0)}{f(p_1) - f(p_0)}$$

$$= 1.5 - \frac{-0.03798 * (1.5 - 1.4821)}{-0.03798 - 0.0143}$$

$$= 1.5 - 0.013003$$

$$= 1.486996$$

$$f(p_2) = 3sin1.486996 - \frac{1.486996^2}{2} - 1.486996 - ln1.486996$$

$$= 0.00014$$

QUESTION # 2:

STEP # 1: Find L and U from A = L * U

$$\begin{bmatrix} 4 & 11 & -1 \\ 6 & 3 & 12 \\ 8 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{10} & 1 & 0 \\ l_{20} & l_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{00} & u_{01} & u_{02} \\ 0 & u_{11} & u_{12} \\ 0 & 0 & u_{22} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 11 & -1 \\ 6 & 3 & 12 \\ 8 & -3 & 2 \end{bmatrix} = \begin{bmatrix} u_{00} & u_{01} & u_{02} \\ l_{10}u_{00} & l_{10}u_{01} + u_{11} & l_{10}u_{02} + u_{12} \\ l_{20}u_{00} & l_{20}u_{01} + l_{21}u_{11} & l_{20}u_{02} + l_{21}u_{12} + u_{22} \end{bmatrix}$$

0
$$u_{00} = 4$$

0a
$$u_{01} = 11$$

0b
$$u_{02} = -1$$

0a
$$l_{10}u_{00} = 6$$

1
$$l_{10}u_{01} + u_{11} = 3$$

1a
$$l_{10}u_{02} + u_{12} = 12$$

0b
$$l_{20}u_{00} = 8$$

1a
$$l_{20}u_{01} + l_{21}u_{11} = -3$$

3
$$l_{20}u_{02} + l_{21}u_{12} + u_{22} = 2$$

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0 $u_{00} = 4$	0a $u_{01} = 11$	0b $u_{02} = -1$
0a $l_{10} = \dfrac{6}{u_{00}}$ $= \dfrac{6}{4}$ $= 1.5$	1 $u_{11} = 3 - l_{10}u_{01}$ $= 3 - 1.5 * 11$ $= -\dfrac{27}{2}$ $= -13.5$	1a $u_{12} = 12 - l_{10}u_{02}$ $= 12 - 1.5 * (-1)$ $= \dfrac{27}{2}$ $= 13.5$
0b $l_{20} = \dfrac{8}{u_{00}}$ $= \dfrac{8}{4}$ $= 2$	1a $l_{21} = \dfrac{-3 - l_{20}u_{01}}{u_{11}}$ $= \dfrac{-3 - 2 * 11}{-13.5}$ $= \dfrac{50}{27}$ $= 1.8518518519$	3 $u_{22} = 2 - l_{20}u_{02} - l_{21}u_{12}$ $= 2 - 2 * (-1) - \dfrac{50}{27} * \dfrac{27}{2}$ $= -21$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 2 & \dfrac{50}{27} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 4 & 11 & -1 \\ 0 & -13.5 & 13.5 \\ 0 & 0 & -21 \end{bmatrix}$$

STEP # 2: Find Y from L * Y = b

$$\begin{bmatrix} 1 & 0 & 0 \\ 1.5 & 1 & 0 \\ 2 & \dfrac{50}{27} & 1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 33 \\ 35 \\ 20 \end{bmatrix}$$

$$y_0 \qquad\qquad = 33$$

$$1.5y_0 + y_1 \qquad = 35$$

$$2y_0 + \frac{50}{27}y_1 + y_2 = 20$$

$y_0 = 33$

$y_1 = 35 - 1.5y_0$

$\quad = 35 - 1.5 * 33$

$\quad = -14.5$

$y_2 = 20 - 2 * 33 - \dfrac{50}{27} * (-14.5)$

$\quad = -19.1481481$

STEP #3: Find X from U * X = Y

$$\begin{bmatrix} 4 & 11 & -1 \\ 0 & -13.5 & 13.5 \\ 0 & 0 & -21 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 33 \\ -14.5 \\ -19.1481481 \end{bmatrix}$$

$$4x_0 + 11x_1 - x_2 = 33$$

$$-13.5x_1 + 13.5x_2 = -14.5$$

$$-21x_2 = -19.1481481$$

$$x_2 = \frac{-19.1481481}{-21}$$

$$= 0.911816576$$

$$x_1 = \frac{-14.5 - 13.5x_2}{-13.5}$$

$$= \frac{-14.5 - 13.5 * 0.911816576}{-13.5}$$

$$= 1.98589065$$

$$x_0 = \frac{33 - 11x_1 + x_2}{4}$$

$$= \frac{33 - 11 * 1.98589065 + 0.911816576}{4}$$

$$= 3.01675485$$

QUESTION # 3:

$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$
$$6x + 3y + 12z = 35$$

$$x = \frac{20 + 3y - 2z}{8}$$
$$y = \frac{33 - 4x + z}{11}$$
$$z = \frac{35 - 6x - 3y}{12}$$

JACOBI:

$$x = \frac{20 + 3 * 0 - 2 * 1}{8} = 2.25$$
$$y = \frac{33 - 4 * 0 + 1}{11} = 3.09090909$$
$$z = \frac{35 - 6 * 0 - 3 * 0}{12} = 2.91666667$$

NUMERICAL COMPUTING NOTES:

GAUSS-SEIDEL:

$$x = \frac{20 + 3 * 3.09090909 - 2 * 2.91666667}{8} = 2.92992424$$

$$y = \frac{33 - 4 * 2.92992424 + 2.91666667}{11} = 2.19972452$$

$$z = \frac{35 - 6 * 2.92992424 - 3 * 2.19972452}{12} = 0.901773417$$