National University of Computer and Emerging Sciences, Lahore Campus

Course:	Design & Analysis of Algorithms	Course Code:	CS2009
Program:	BS (Computer Science)	Semester:	Spring 2023
Duration:	20 Minutes	Total Marks:	15
Paper Date:	21-Feb-2023	Weight	2.5
Section:	J	Page(s):	2
Exam:	Quiz 1	Reg. No.	

Instruction/Notes:

11/15

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Question 1: [5 marks]

For the following functions f(n) and g(n), indicate whether f(n) = O(g(n)), f(n) = Ω(g(n)), or both, i.e. f(n) = θ(g(n)). Justify your answer.

f(n)= $n^2 \log n$

g(n) = $2^{1/2} n^2 \log(2^{1/2} n)$

Assuming we are taking log base of 2

$O(n) \Rightarrow f(n) \leq c \cdot g(n)$

$n^2 \log n \leq c \cdot 2^{1/2} n^2 \log(2^{1/2} n)$

$\log n \leq c \cdot 2^{1/2} \log(2^{1/2} n)$

$\frac{\log n}{\sqrt{2} \log(\sqrt{2} n)} \leq c$

assume if $c = 3$

$\frac{\log n}{\sqrt{2} \log_2(\sqrt{2} n)} \leq 3 \Rightarrow$ if $n_0 = 10$

$\frac{\log_2 10}{\sqrt{2} \log_2(\sqrt{2} \times 10)} = 1.202$

$0.615 \leq 3$ [illegible]

$\Omega(n) \Rightarrow c \cdot g(n) \leq f(n)$

$c \cdot 2^{1/2} n^2 \log(2^{1/2} n) \leq n^2 \log n$

$c \cdot 2^{1/2} \log(\sqrt{2} n) \leq \log n$

$c \leq \frac{\log n}{\sqrt{2} \log(\sqrt{2} n)}$

Assume $c = 0.5$

$0.5 \leq \frac{\log n}{\sqrt{2} \log(\sqrt{2} n)}$

if $n_0 = 5$

$0.5 \leq 0.581$

Therefore $O(n)$ & $\Omega(n)$ justified.

Question 2: [5 Marks]

Find big-theta of the function $f(n) = n/18 - 19n^{1/2} + 20$, give the constants c_1, c_2, n_0

$O(n) = n/18 - 19n^{1/2} + 20$

$f(n) \leq c \cdot g(n)$

$n/18 - 19n^{1/2} + 20 \leq c \cdot n$

Assume $c = 10$

if $n_0 = 15$

$15/18 - 19(15)^{1/2} + 20 \leq 10 \cdot 10(15)$

$-52.8 \leq 150$

if $\Omega(n) \Rightarrow c \cdot g(n) \leq f(n)$

$c \cdot n \leq n/18 - 19n^{1/2} +$

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Question 3: [5 marks]

Write down the Running Time equation (T(n)) of the following algorithm and analyze its time complexity. Show complete steps. If you make any assumptions, state them clearly.

```
int algo(input, n)
{
If (n<=0)
        {return 0}
X = algo(A, n/2)
Y = algo(A, n/4)
Z = A[(n/2) + (n/4) +1]
return (X+Y+Z)
}
```

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + cn$$