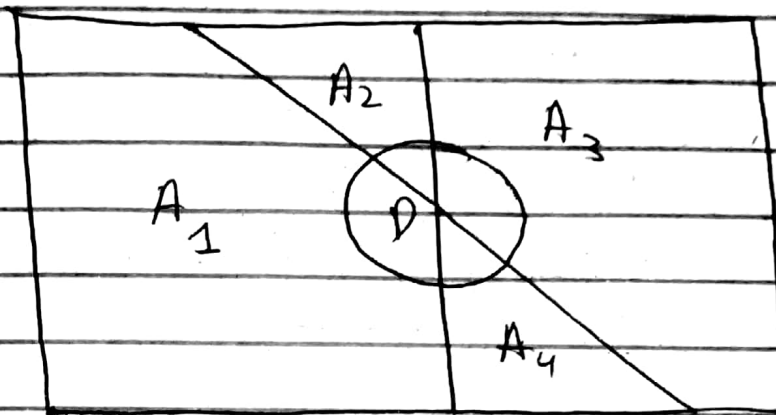


21/09/23

# Baye's Theorum

→ Bayes Thomas



$A_1, A_2, A_3$  and  $A_4$  are mutually independent events and exhaustive because their addition will complete the sample space (union) and their sum of probability will be 1

$$P(D) = P(A_1)P(D|A_1) + P(A_2)P(D|A_2) +$$

★

$$P(D) = P(A_1)P(D|A_1) + P(A_2)P(D|A_2) + P(A_3)P(D|A_3) + P(A_4)P(D|A_4)$$

★

$$P(A_i | D) = \frac{P(A_i)P(D|A_i)}{\sum_{i=1}^n P(A_i)P(D|A_i)}$$

individual

$$\text{Prior probability} = P(A_1) + P(A_2) + P(A_3) = 1$$

$$\text{Posterior probability} = P(A_1|D) = ? \quad P(A_3|D)$$

$$= P(M_2 | D) = \frac{0.75 \times 0.02}{0.75 \times 0.02 + 0.25 \times 0.03}$$

$$= \frac{0.015}{0.015 + 0.0075} =$$

$$= \frac{0.75 \times 0.02}{0.75 \times 0.02 + 0.25 \times 0.03}$$

$$= \frac{0.0015}{0.0015 + 0.0075} = \frac{0.0015}{0.009}$$

$$= 0.1667$$

A paint store produces and sales latex and semi gloss paints. Based on the sales the probability that a customer will purchase latex paint is ~~0.07~~ 0.75 of those who purchase latex paint 60% also purchase rollers - But only 30% semi-gloss paint buyers purchase rollers.

If randomly selected buyer purchases a roller - And a can of paint.  
Find prob that the paint is latex.

$$P(L) = 0.75$$

$$P(S) = 0.25$$

$$P(R|L) = 0.6$$

$$P(R|S) = 0.3$$

$$P(R) = 0.1$$

$$P(L) = 0.75$$

$$P(S) = 0.25$$

$$P(R|L) = 0.6$$

$$P(R|S) = 0.3$$

$$P(L|R) = \frac{P(L) P(R|L)}{P(L) P(R|L) + P(S) P(R|S)}$$

$$= \frac{P(L) P(R|L)}{P(L) P(R|L) + P(S) P(R|S)}$$

$$P(L) \times P(R|L) + P(S) P(R|S)$$

$$\text{Ans} = \frac{0.75 \times 0.6}{0.75 \times 0.6 + 0.25 \times 0.30}$$

$$= 0.85$$

Ex: 2095, 20101, 2099, 2078, 96

Spam Email

Classification



0.3  
0.5      0.3

$$P(S|E) = \frac{P(S) P(E|S)}{P(S) P(E|S) + P(\bar{S}) P(E|\bar{S})}$$

$S$  = spam email       $\bar{S}$  = not spam email  
 $E$  = ~~email~~ word

$$P(S|E_1 \cap E_2) = \frac{P(S) P(E_1|S) P(E_2|S)}{P(S) P(E_1|S) P(E_2|S) + P(\bar{S}) P(E_1|\bar{S}) P(E_2|\bar{S})}$$

Spam ke chances given  $E_1$  and  $E_2$  occur

⇒ If prior probability is not given then we can keep the prior probability as 1 / (total number of events)

e.g. spam and not spam are 2 events  
 So their prior probability is 50%.

(Q) <sup>66</sup> Rolex <sup>99</sup> occurs in  $\frac{250}{2000} = 0.125$

Known to be spam and 5 of 1000 messages known to be not spam.  
 estimate that an incoming message contain word rolex is spam. Also assume that it is equally likely that an incoming message is spam or not spam. If threshold is 0.9 to reject a message as spam. Will we reject such messages.

$$P(W|S) = 0.125$$

$$P(W|\bar{S}) = 0.005$$

$$P(S) = 0.5$$

$$P(\bar{S}) = 0.5$$

$$P(S|W) = ?$$

$$\frac{P(S) P(W|S)}{P(S) P(W|S) + P(\bar{S}) P(W|\bar{S})}$$

$$= \frac{0.5 \times 0.125}{0.5 \times 0.125 + 0.5 \times 0.005}$$

$$= \frac{0.0625}{0.0625 + 0.0025} = \frac{0.0625}{0.065}$$

$$= 0.00961 = \boxed{0.96\%}$$

So the message will be spam