MIDTERM EXAM ANSWER SHEET

NATIONAL UNIVERSITY
OF COMPUTER & EMERGING SCIENCES
LAHORE

FAST

Course Numerical Computing Answer Sheet No. 34875

Student's Name Signature

Rol Section BCS-5L Date 3 NOV, 2023

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QUESTION-02

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$f(x) = e^x - 1 - x - \frac{x^3}{2}$ $[-1, 1]$

Minimum number of iterations for bisection method:

$$\frac{b-a}{2^n} \le \varepsilon$$

$$\frac{b-a}{\varepsilon} \le 2^n$$

$$2^n \geq \frac{b-a}{\varepsilon}$$

Taking log on both sides

$$\log(2^n) \geq \frac{\log(b-a)}{\log(\varepsilon)}$$

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$$n\log(2) \geq \log(b-a) - \log(\varepsilon)$$

we have: $b = 1$, $a = -1$, $\varepsilon = 1\times10^{-5}$

$$n \geq \frac{\log(b-a) - \log(\varepsilon)}{\log(2)}$$

$$n \geq \frac{\log(1+1) - \log(10^{-5})}{\log 2}$$

$n \geq 17.61$

$n \simeq 18$

Thus minimum 18 iterations are required.

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QUESTION-03

Order Of Convergence for Newton Raphson

By Newton Raphson.

$$~~X_n = X_{n-1}~~$$

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)} \quad -①$$

We Know that;

$$\left.\begin{array}{l} X_n = \alpha + \varepsilon_n \\ X_{n+1} = \alpha + \varepsilon_{n+1} \end{array}\right\} -②$$

putting ② in ①

$$\cancel{\alpha} + \varepsilon_{n+1} = \cancel{\alpha} + \varepsilon_n - \frac{f[\alpha + \varepsilon_n]}{f'[\alpha + \varepsilon_n]}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{f(\alpha + \varepsilon_n)}{f'(\alpha + \varepsilon_n)} \quad -③$$

applying taylor series on ③

$$_{1} = \varepsilon_n - \frac{f(\alpha) + \varepsilon_n f'(\alpha) + \frac{\varepsilon_n^2 f''(\alpha)}{2}}{f'(\alpha) + \varepsilon_n f''(\alpha)}$$

$$_{1} = \varepsilon_n - \frac{2f(\alpha) + 2\varepsilon_n f'(\alpha) + \varepsilon_n^2 f''(\alpha)}{2[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

$\because$ Since α is exact root so $f(\alpha) = 0$

$$_{1} = \varepsilon_n - \frac{2\varepsilon_n f'(\alpha) + \varepsilon_n^2 f''(\alpha)}{2[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

$$= \frac{\cancel{2\varepsilon_n f'(\alpha)} + 2\varepsilon_n^2 f''(\alpha) - \cancel{2\varepsilon_n f'(\alpha)} - \varepsilon_n^2 f''(\alpha)}{2[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

Q / Part No.

Rough Work

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2 f''(\alpha)}{2[f'(\alpha) + \varepsilon_n f''(\alpha)]}$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2 f''(\alpha)}{2f'(\alpha)\left[1 + \frac{\varepsilon_n f''(\alpha)}{\cancel{2} f'(\alpha)}\right]}$$

$$\because \left[1 + \frac{\varepsilon_n f''(\alpha)}{f'(\alpha)}\right] \simeq \neq 0 \quad \text{so,}$$

$$\varepsilon_{n+1} = \frac{\varepsilon_n^2 f''(\alpha)}{f'(\alpha) 2}$$

$$\varepsilon_{n+1} = \frac{1}{2} \frac{\varepsilon_n^2 f''(\alpha)}{f'(\alpha)}$$

$$\varepsilon_{n+1} = K \varepsilon_n^2 \quad - (4)$$

$$\therefore K = \frac{f''(\alpha)}{2f'(\alpha)}$$

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By (4)

$$\varepsilon_{n+1} < K \varepsilon_n^2$$

So, order of convergence of newton Raphson is 2.

Q / Part No.

Rough Work

QUESTION-04

$$x = \sqrt[m]{K}$$

$$x^m = K$$

$$x^m - K = 0$$

$$x^m - K = f(x) \quad \text{—(1)}$$

$$f'(x) = m x^{m-1}$$

$$X_n = X - \frac{x^m - K}{m x^{m-1}}$$

Now solving for $\sqrt[3]{25}$ using (1)

$$f(x) = x^3 - 25$$

$$f(2) = -17 \quad , \quad f(3) = 2$$

1) Interval $[2, 3]$

$$X_0 = \frac{2+3}{2} = \boxed{2.5 = X_0}$$

$$X_1 = 2.5 - \frac{(2.5)^3 - 25}{3(2.5)^2}$$

$$\boxed{X_1 = 3}$$

2) $$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

$$X_2 = 3 - \frac{(3)^3 - 25}{3(3)^2}$$

$$\boxed{X_2 = 2.926}$$

3) $$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$$

$$X_3 = 3.07 - \frac{(3.07)^3 - 25}{3(3.07)^2}$$

$$\boxed{X_3 = 2.924}$$

1.5

Rough Work

Q / Part No.

Rough Work

4) $X_4 = X_3 - \frac{f(X_3)}{f'(X_3)}$

$X_4 = 2.924 - \frac{(2.924)^3 - 25}{3(2.924)^2}$

$\boxed{X_4 = 2.924}$

⇒ Values have started to repeat so;

the root of $\sqrt[3]{25} = 2.924$.

QUESTION - 05

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 8 \\ 1 & -5 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \\ -4 \end{bmatrix}$$

CROUTS Method. $U_{ii} = 1$

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 8 \\ 1 & -5 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & -1 \\ -1 & 2 & 8 \\ 1 & -5 & 1 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}\begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

- $\boxed{L_{11} = 3}$
- $(L_{11})(U_{12}) = 1$

  $\boxed{U_{12} = 1/3}$
- $(L_{11})(U_{13}) = -1$

  $\boxed{U_{13} = -1/3}$
- $L_{11}(L_{21}) = -1$
- $(L_{21})(U_{12}) + L_{22} = 2$

  $(-1)(1/3) + L_{22} = 2$

  $\boxed{L_{22} = 7/3}$
- $(L_{21})(U_{13}) + (L_{22})(U_{23}) = 8$

  $(-1)(-1/3) + (7/3)(U_{23}) = 8$

  $\boxed{U_{23} = 23/7}$
- $(L_{31})(1) = 1$

  $\boxed{L_{31} = 1}$
- $(L_{31})(U_{12}) + L_{32} = -5$

  $(1)(1/3) + L_{32} = -5$

  $\boxed{L_{32} = \frac{-16}{3}}$
- $(L_{31})(U_{13}) + (L_{32})(U_{23}) + (L_{33})(1) = 1$

  $(1)(-1/3) + (-16/3)(23/7) + L_{33} = 1$

  $\boxed{L_{33} = \frac{132}{7}}$

Q / Part No. Rough Work

We Know that

$$AX = B$$
$$(LU)(X) = B$$
$$\because UX = Y$$
$$\therefore LY = B$$

1) $LY = B$

$$\begin{bmatrix} 3 & 0 & 0 \\ -1 & 7/3 & 0 \\ 1 & -16/3 & 132/7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \\ -4 \end{bmatrix}$$

$$3y_1 = 8$$
$$\boxed{y_1 = 8/3}$$

$$-y_1 + \frac{7}{3} y_2 = -8$$
$$-\frac{8}{3} + \frac{7}{3} y_2 = -8$$
$$\boxed{y_2 = \frac{-16}{7}}$$

$$y_1 - \frac{16}{3} y_2 + \frac{132}{7} y_3 = -4$$
$$\frac{8}{3} - \frac{16}{3}\left[\frac{-16}{7}\right] + \frac{132}{7} y_3 = -4$$

$$\cancel{\boxed{y_3 = \frac{-19}{33}}} \quad \boxed{y_3 = -1}$$

2) $UX = Y$

$$\begin{bmatrix} 1 & 1/3 & -1/3 \\ 0 & 1 & 23/7 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8/3 \\ -16/7 \\ -1 \end{bmatrix}$$

$$\cancel{\boxed{x_3 = \frac{-19}{33}}} \quad \boxed{x_3 = -1}$$

Q / Part No.

$$\cancel{x_2 + \frac{23}{7}x_3 = \frac{-16}{7}}$$

$$\cancel{\boxed{x_2 = -\frac{395}{759}}}$$

$$\cancel{x_1 + \frac{1}{3}x_2 - \frac{1}{3}x_3 = \frac{8}{3}}$$

$$\cancel{\boxed{x_1 = \frac{670}{253}}}$$

$$x_2 + \frac{23}{7}x_3 = \frac{-16}{7}$$

$$\boxed{x_2 = 1}$$

$$x_1 + \frac{1}{3}x_2 - \frac{1}{3}x_3 = \frac{8}{3}$$

$$\boxed{x_1 = 2}$$

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Hence, Values:

$$\boxed{x_1 = 2\ ,\quad x_2 = 1\ ,\quad x_3 = -1}$$

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QUESTION-01

$$\int_0^1 f(x)\,dx$$

Extrapolation Table

SS	$O(h^4)$	$O(h^6)$
h	I_h	
		$I'h$
$h/2$	$I_{h/2}$	

$$I'h = \frac{16B - A}{15}$$

$$I'h = \frac{16(0.6321221415) - 0.632134175}{15}$$

$$\boxed{I'h = 0.6321205643}$$