Permutation

IQ

→ arrangement of all or a part of a set of objects

→ Number of different arrangements of n objec is $n!$ (n-factorial)

⇒ $$^nP_r = \frac{n!}{(n-r)!}$$

n = total objects r = objects to select/arrange
When we have to arrange a part of a group

⇒ To arrange objects around a circle

$$(n-1)!$$

(making 1 object as pivot)

⇒ $$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

When similarity occurs between a no. of objects in a group.

Combination

⇒ Permutation but without regard of order
e.g selecting a TA without any criteria.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

e.g 10 As, 5 S
How many ways to select 3a and 2S?

$$\binom{10}{3} = \frac{10!}{3!(10-3)!} = 120$$

$$\binom{5}{2} = \frac{5!}{2!3!} = 10$$

Example: STA $\boxed{TIS}$ TICS

→ Possible arrangements?

S = 3 T = 3 C = 1
I = 2 A = 1

$$\frac{10!}{3!3!2!} = 50,400.$$

2021

2027, 2023, 2,37, 2.46, 2.44, 2.43, 2.39, 2.35
2032, 2035, 2039, 2040, 2041, 242, 43, 44, 45

IQ Permutation :-

$\Rightarrow$ $n!$ → distinct objects

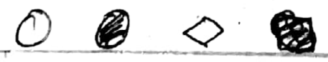
$\Rightarrow$ $(n-1)!$ → circular arrangement

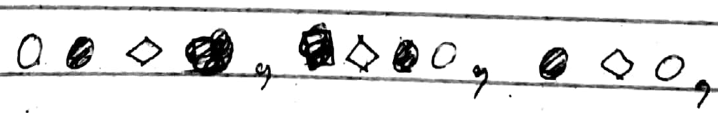
$\Rightarrow$ $^nP_r = \dfrac{n!}{(n-r)!}$ → selected part

$= \dfrac{n!}{n_1!\, n_2! \cdots n_k!}$ → similar objects excluded

$\Rightarrow$ $^nC_r = \dfrac{n!}{(n-r)! \times r!}$ → selected part w/o any order
order is ignored

$=$

e.g 

$n! =$ 

14 September 2023 Thursday

$\boxed{Q}$

In how many ways can 7 grads be assigned ~~2 1 3~~ to 1 triple and 2 double hotel rooms.

$$\frac{\cancel{7C}}{\cancel{2!2!}} = \frac{n!}{r!(n-r)!} = \frac{7!}{2!(n-2)!}$$

$$\boxed{\frac{7!}{3! \times 2! \times 2!} = 210 \text{ combinations}}$$

$\boxed{Q}$ STATISTICS

$$\frac{10!}{3!2!2!} = 50,400$$

$\boxed{Q}$ 4 married couples have bought 8 seats in same row

|| || || || $8!$

$\frac{4!}{}$

a) with no restrictions

b) each couple is to sit together

Q.2.

a) $8! = 40320$

b) || || || ||

$$\frac{\cancel{8}!}{(\cancel{8-02})!} \quad \cancel{40320}$$

$$4! \times 2! \times 4 = 192$$

MF MF M F MF

c) $\boxed{\text{MF FM MF FM}}$

$\boxed{\text{FM MF FM MF}}$

$$2 \times 4! = 48$$

$\boxed{Q}$ 4 boys 5 girls sit in a row if boys and girsl alternate

$$\frac{9!}{(9-2)!} = 72$$

Probability of An Event

IQ

i ★ Classical Approach
ii ★ Axiomatic approach
iii ★ Relative frequency

i) $\frac{n}{N} = P(A)$

ii) $P(S) = 1$, $P(\text{Impossible event}) = 0$

iii) $\lim_{n \to \infty} \frac{n}{N}$ $\quad P(A \cup B) = P(A) + P(B)$

n = number of favorable outcomes
N = number of total outcomes

(i) Classical approach is used when one occurrance is likely to occur as the other one. For example, out of 10 balls, 5 are red, 5 are blue. In first attempt $\frac{1}{2}$ is probability to pic a red ball

$$P(A) = (n/N)$$

(ii) Relative frequency is practially used very less and it is just modification on (i) to calculate experiments many times.

$$\lim_{n \to \infty} \frac{n}{N}$$

Axiomatic Approach when occurances are not equally likely to occur.

$P(S) = 1$.

$P(\text{impossible event}) = 0 = \boxed{P(A \cup B) = P(A) + P(B)}$

$\boxed{2024}$, $\boxed{2025}$, $\boxed{2027}$, $\boxed{2028}$

Q (Q 2024)

Students: freshman, sophomore, junior, senior
: male, female

f, s, j, se
m, fe

~~6x6 = 36~~ Total no. of possible classifications = 8

(Q 2025) 6 styles 5 colors
$6 \times 5 = 30$ pairs of shoes.

5s
4c
$5 \times 4 = 20$

Q (2026) 7 Rules → 11 years ++ (male)
→ 7 years ++ (female)

1) NO smoking
2) reg. exercise
3) no alcohol
4) 7-8 hrs sleep
5) weight
6) eat breakfast
7) no eating b/w meals.

How many ways to adopt any 5.

a) currently violating all 5 -

$\binom{7}{5} = 21$

$$\frac{n!}{r!(n-r)!} = \frac{7!}{5! \times (2)!} = \frac{\cancel{(6)!}\, 6 \times 7}{\cancel{5!}\; 21}$$

$= (6 \times 7) \div 2 = \boxed{21}$

19/9/2023

(2.5) Additive Rules

If A and B are two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

→ Rule for not mutually exclusive (disjoint)

$$P(A \cup B) = P(A) + P(B)$$

→ Rule for mutually exclusive (disjoint) A and B

52

54

59

56

61

58

21/09/23

Independant Events

$P(A|B) = P(A)$ (A is independant of B)

A = ace
B = spades

$P(A) = \frac{4}{52}$, $P(B) = \frac{13}{52}$

$P(B|A) = \frac{13}{52}$ (A has been replaced)

(MULTIPLICATIVE LAW)

$P(A \cap B) = P(A)\, P(B)$ → independant

$P((A) \cap (B)) = P(A)\, P(B/A)$ → dependant

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap A_2 \cap A_3 \cdots A_k)$

$= P(A_1)\, P(A_2)\, P(A_3) \cdots P(A_k)$

Dependant Case:-

$P(A \cap A_1 \cap A_3) = P(A_1)\, P(A_2/A_1)$
$P(A_3/A_1 \cap A_2)$
$P(A_k/A_1 \cap A_2 \cdots \cap A_k)$

	Car	Light Track
US	87.4	193.1
Non US	228.5	148.0

2009, January = 657,000
= 37% less than that of January 2008

3 companies sold 280,500 vehicles
which is 48% less than 2008 Jan.

Toyota / Honda / Nissan = Non - US
Pickup, minivan, SUV, crossover = Light truck

a) 657000 → Jan 2009 } Industry
1042857 → Jan 2008

$x - (0.37 \times x) = 657000$

$0.63x = 657000$

$x =$

US GM, Ford, Chrysther

280500 → Janya 2008
539,423 → January 2009

Non Us Toyota / Honda / Nissan

Jan 2009 Type of Vehicle

	Car	Truck	All
US	87.4	193.1	280.500
NON U.S	228.5	148.0	376.5
Total	315.9	341.1	657

Jan 2008

	Car	Truck	All
			539423
US			~~1042857~~
NON U-S			503434
Total			1042857

Row total, Column total, Probability.

Joint Probability table

	CAR	Truck	ALL
US	0.1330	0.2939	0.42694
NON US	0.3420	0.22526	0.5730
Total	0.4808	0.51917	1

b) marginal probabilities ? Probabilities on the margin of joint prob - table

⇒ Cars = 0.4808

⇒ Truck = 0.51917

⇒ Total US = 0.42694

⇒ Total Non US = 0.5730

c) (i) ~~0.42694 x 0.4808 = 0.20527~~

(ii) ~~0.42694 x 0.51917 = 0.22165~~

d) ~~0.5730 x 0.4808 = 0.2754~~

~~0.5730 x 0.51917 = 0.29748~~

a) $\frac{P(\text{car} \cap \text{US})}{P(\text{US})}$

$= \frac{0.1330}{0.42694}$

b) $\frac{P(\text{truck} \cap \text{US})}{P(\text{US})}$

$= \frac{0.2939}{0.42694} =$

succession = one by one

Q) suppose that we have a box of 20 fuses of which 5 are unusable If 2 fuses are selected at random and removed in succession with no replacement -

Find Prob that both fuses are defectiv

$\frac{5}{20} = \frac{1}{4}$ = Probability of defective

$\frac{1}{4} \times \frac{4}{19} = 0.05263$

$$\frac{\binom{5}{2}\binom{15}{0}}{\binom{20}{2}} = \frac{{}^{15}C_2 \times {}^{15}C_0}{{}^{20}C_2}$$

Q) Ambulame + Fire Truck

⇒ multiplicative rule

→ we need ambulance and Fire truck in emergency - What prob is there that both will be available

IQ (Q2.40) 3 cards drawn in succession with NO Replacement from ordinary deck of playing cards. Find the prob that event

$(A_1 \cap A_2 \cap A_3)$ occurs where

A_1 = red ace $4/52$
A_2 = second card is ten or a jack
A_3 = third card is greater than 3 but less than 7.

Total = 52
Suit 1 = 13 } red → diamond
Suit 2 = 13 → heart
Suit 3 = 13 } black → spade
Suit 4 = 13 → club

Every suit has ace, 2 ---- 10, King, Queen and Jack.

ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, King, Queen, Jack

$$A_1 = \frac{26}{52} \times \frac{1}{13} = 0.03846$$

$A_2 =$