Advanced Statistics (DS2003)
BDS-4A, 4B, 4C
Spring 2024

Assignment-1

Instructions:

- This assignment comprises the following problems
- You need to submit your assignment in the hard form complete over A4 or assignment papers ONLY
- For sections B and C: Assignment-1 is due on Wednesday (21-Feb-2024) during the class
- For section A: Assignment-1 is due on Thursday (22-Feb-2024) during the class

1. In the manufacturing of electroluminescent lamps, several different layers of ink are deposited onto a plastic substrate. The thickness of these layers is critical if specifications regarding the final color and intensity of light of the lamp are to be met. Let X and Y denote the thickness of two different layers of ink. It is known that X is normally distributed with a mean of 0.1 millimeter and a standard deviation of 0.00031 millimeter and Y is also normally distributed with a mean of 0.23 millimeter and a standard deviation of 0.00017 millimeter. Assume that these variables are independent.
    a. If a particular lamp is made up of these two inks only, what is the probability that the total ink thickness is less than 0.2337 millimeter?
    b. A lamp with a total ink thickness exceeding 0.2405 millimeters lacks the uniformity of color demanded by the customer. Find the probability that a randomly selected lamp fails to meet customer specifications.
2. The width of a casing for a door is normally distributed with a mean of 24 inches and a standard deviation of 0.125 inch. The width of a door is normally distributed with a mean of 23.875 inches and a standard deviation of 0.0625 inch. Assume independence.
    a. Determine the mean and standard deviation of the difference between the width of the casing and the width of the door.
    b. What is the probability that the width of the casing minus the width of the door exceeds 0.25 inch?
    c. What is the probability that the door does not fit in the casing?

3. A factory produces X_i gadgets on the i$^{\text{th}}$ day. where the $X_i's$ are independent and identically distributed random variables, with mean 5 and variance 9.
   a. Find an approximation to the probability that the total number of gadgets produced in 100 days is less than 440.
   b. Find (approximately) the largest value of N such that $\Pr(X_1 + X_2 + \cdots + X_N \geq 200 + 5N) \leq 0.05$.
   c. Let N be the first day on which the total number of gadgets produced exceeds 1000. Calculate an approximation to the probability that $N \geq 220$
4. A synthetic fiber used in manufacturing carpet has tensile strength that is normally distributed with mean 75.5 psi and standard deviation 3.5 psi. Find the probability that a random sample of N=6 fiber specimens will have sample mean tensile strength that exceeds 75.75 psi.
5. The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean 8.2 minutes and standard deviation 1.5 minutes. Suppose that a random sample of N = 49 customers is observed. Find the probability that the average time waiting in line for these customers is
   a. Less than 10 minutes
   b. Between 5 and 10 minutes
   c. Less than 6 minutes
6. The heights of 1000 students are approximately normally distributed with a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimeter. Determine
   a. the mean and standard deviation of the sampling distribution of $\bar{X}$;
   b. the number of sample means that fall between 172.5 and 175.8 centimeters inclusive;
   c. the number of sample means falling below 172.0 centimeters.
7. In a chemical process, the amount of a certain type of impurity in the output is difficult to control and is thus a random variable. Speculation is that the population mean amount of the impurity is 0.20 gram per gram of output. It is known that the standard deviation is 0.1 gram per gram. An experiment is conducted to gain more insight regarding the speculation that μ = 0.2. The process is run on a lab scale 50 times and the sample average $\bar{X}$ turns out to be 0.23 gram per gram. Comment on the speculation that the mean amount of impurity is 0.20 gram per gram. Make use of the Central Limit Theorem in your work.