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Customer-Based Corporate Valuation for Publicly Traded Noncontractual Firms

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Abstract

There is growing interest in "customer-based corporate valuation"—that is, explicitly tying the value of a firm's customer base to its overall financial valuation using publicly disclosed data. While much progress has been made in building a well-validated customer-based valuation model for contractual (or subscription-based) firms, there has been little progress for noncontractual firms. Noncontractual businesses have more complex transactional patterns because customer churn is not observed, and customer purchase timing and spend amounts are more irregular. Furthermore, publicly disclosed data are aggregated over time and across customers, are often censored, and may vary from firm to firm, making it harder to estimate models for customer acquisition, ordering, and spend per order. The authors develop a general customer-based valuation methodology for noncontractual firms that accounts for these issues. They apply this methodology to publicly disclosed data from e-commerce retailers Overstock.com and Wayfair, provide valuation point estimates and valuation intervals for the firms, and compare the unit economics of newly acquired customers.

Keywords

customer equity, customer lifetime value, unit economics, valuation

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Executives, marketing managers, and financial professionals are increasingly aware that current and future customer relationships are a valuable—if not the most valuable—asset of a firm (Blattberg and Deighton 1996). As a result, customer acquisition, customer retention, and customer lifetime value (CLV) are being discussed and studied more than ever before (Braun, Schweidel, and Stein 2015; Datta, Foubert, and Van Heerde 2015). Accurate estimation of CLV enables external stakeholders (e.g., shareholders, creditors, suppliers, competitors, regulators) to estimate customer equity, or the remaining lifetime value of all existing customers plus the net present value of the CLV of all yet-to-be-acquired customers (Bauer and Hammerschmidt 2005; Kumar and Shah 2009; Rust, Lemon, and Zeithaml 2004). For many firms, customer equity represents the majority of shareholder value of the firm, enabling an explicit link between customer behaviors (i.e., acquisition, retention, and spend) and the overall financial valuation of the firm. Kumar and Shah (2015) provide an excellent summary of the literature on customer equity.

Customer-based corporate valuation (CBCV) is the process of valuing a firm by forecasting current and future customer behavior using customer data in conjunction with traditional financial data. However, the correct valuation framework depends on the nature of the relationship that the firm has with its customers, with a primary distinction being whether a firm is contractual (i.e., subscription-based) or noncontractual. While much progress has been made in performing CBCV for contractual firms, far less progress has been made for noncontractual firms. There are three main challenges to performing CBCV in noncontractual business settings:

1. Whereas customer churn is observable for contractual firms, it is unobservable for noncontractual firms. If customers of an e-commerce retailer decide to end their relationship with the firm, there is no process to inform the firm. Instead, they simply discontinue their purchasing. This complicates the underlying model required to predict future customer activity because noncontractual firms do not when they lose customers and thus cannot publicly disclose customers lost or the total size of their customer base.

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2. Even if the modeler knew that a noncontractual customer was "alive" (i.e., had a nonzero probability of making a future purchase), there are additional complexities associated with repeat purchase and spend behaviors that the modeler must take into account. The number of purchases that customers make, as well as how much they spend on each of those purchases, can be highly variable at noncontractual firms. In contrast, customers of contractual firms have much more predictable ordering and spending patterns.
3. Modeling aggregate customer behavior in a noncontractual setting is doable if the modeler has access to individual-level customer data (Fader, Hardie, and Shang 2010; Schmittlein, Morrison, and Colombo 1987), external stakeholders only have access to highly aggregated, publicly disclosed customer data (e.g., active customers, total orders), which is a collection of repeated cross-sectional summaries (Jerath, Fader, and Hardie 2016). This makes model estimation more challenging and limits the richness of the models that can be considered.

Because of these challenges, many articles in this area have limited their scope to the contractual business setting alone (Bonacchi, Kolev, and Lev 2015; McCarthy, Fader, and Hardie 2017; Schulze, Skiera, and Wiesel 2012). Extant articles that have attempted to perform CBCV in a noncontractual setting have done so with the following workarounds (Gupta, Lehmann, and Stuart 2004; Libai, Muller, and Peres 2009):

1. Assume an observable "retention rate," which may be defined, for instance, as the repeat rate or the proportion of customers who made a purchase last year and made another purchase this year (Farris et al. 2010).
2. Assume that the number of alive customers equals the number of active customers.
3. Proceed with the valuation exercise as if the company operated in a contractual business setting.

While this workaround may seem convenient, it is problematic for several reasons. First, using the repeat rate in a noncontractual setting will typically understate future purchase activity and, thus, will dramatically understate future profits because many customers who have not purchased in one year may still be alive (e.g., 6% of consumer products seller QVC's total sales in 2015 came from customers who had not purchased in over a year; QVC 2015). Second, retention propensities will generally differ across customers, and ignoring this heterogeneity will further undervalue the customer base (Fader and Hardie 2010). Third, different firms define the same metrics differently. For example, at Overstock.com, Wayfair, and Camping World, total "active customers" is defined as the number of customers who placed an order within the preceding 3, 12, and 24 months, respectively. These issues lead to inaccurate projections that are not comparable across firms. Furthermore, they diminish the validity of valuation estimates and create incorrect inferences about the unit economics of customers (e.g., the lifetime value of new customers, customer retention) and their implications for the business (e.g., revenue concentration, reliance on new customer acquisition).

In the pioneering CBCV work of Gupta, Lehmann, and Stuart (2004), both contractual and noncontractual companies were valued with the same model. For noncontractual firms, they used a retention rate proxy as the retention rate in their CLV formula. Of the five companies valued in their empirical analysis, the only two noncontractual businesses, eBay and Amazon, were the two most misvalued.[1] These companies were undervalued by an average of 88% and 83%, respectively.[2] A new model is needed that is specifically suited to noncontractual firms.

Our primary contribution is to develop a new model that can accurately value noncontractual firms. To do so, our underlying model for customer behavior, and the corresponding estimation procedure, are flexible enough to accommodate whatever aggregated, censored customer data the firm publicly discloses. We do not assume that private data are available. Our proposed model reflects important empirical realities associated with noncontractual customer behavior, including latent attrition, repeat purchasing that may vary across customers and over time, and time-varying spend-per-purchase patterns. In addition to providing overall firm valuation insights, our proposed model also allows external stakeholders to look beyond surface-level financial metrics such as revenues to better understand the underlying unit economics of a business. We predict that this deeper examination of the underlying financial health of a firm may be more useful to financial professionals than the valuation estimate itself.

In the next section, we present the model governing customers' acquisition, repeat purchasing, and spend, and explain how this model is used to drive an overall valuation for the firm. We illustrate this methodology by applying it to two publicly traded companies, Overstock.com (Overstock, hereinafter) and Wayfair. After validating the proposed model, we provide overall valuation estimates for both firms and valuation intervals to account for uncertainty in the model forecasts. We then analyze the unit economics of newly acquired customer cohorts and conclude with a discussion of the results.

Model Development

In this section, we specify the individual-level model for the customer, which we use to forecast future customer activity. We show how this model is calibrated on public disclosures

[1] The other three businesses valued—Ameritrade, Capital One, and E-Trade—are all contractual (as defined by Schmittlein, Morrison, and Colombo [1987]) because their churn is observable.

[2] These valuations were performed after the stock market had fallen sharply in the aftermath of the "tech bubble," and prominent Wall Street analysts were publicly questioning Amazon's solvency. This implies that Gupta, Lehmann, and Stuart (2004) undervalued Amazon at a time that Amazon's stock price was depressed.

and embedded within an overall valuation framework for the firm.

Valuation Framework

We adopt the discounted cash flow model as our firm valuation framework (Damodaran 2012; Koller, Goedhart, and Wessels 2010; Schulze, Skiera, and Wiesel 2012). This model is the de facto standard valuation method among financial professionals because of its flexibility and ease of use, which facilitate the adoption of this method by both academics and practitioners. For completeness, we briefly summarize the discounted cash flow valuation procedure next. Shareholder value (SHV) in a particular quarter q is equal to the value of the firm's operating assets (OA) plus the nonoperating assets (NOA), minus the net debt (ND):

$$\mathrm{SHV_q} = \mathrm{OA_q} + \mathrm{NOA_q} - \mathrm{ND_q}. \tag{1}$$

Shareholder value is observed in effectively continuous time for publicly traded firms, whereas operating assets, nonoperating assets, and net debt are observed at the end of each quarter by external stakeholders. The value of a firm's operating assets Q quarters after the beginning of commercial operations ($\mathrm{OA_Q}$) is equal to the sum of all free cash flows (FCFs) the firm will generate thereafter, discounted at the weighted average cost of capital (WACC, assuming quarterly compounding and using a midpoint approximation to account for within-period cash flow timing):

$$\mathrm{OA_Q} = \sum_{q=0}^{\infty} \frac{\mathrm{FCF_{Q+q}}}{(1 + \mathrm{WACC})^{q-1/2}}. \tag{2}$$

Free cash flow is equal to net operating profit after taxes (NOPAT) minus capital expenditures (CAPEX), plus depreciation and amortization (D&A), minus the change in nonfinancial working capital (ΔNFWC) during that quarter:

$$\mathrm{FCF_q} = \mathrm{NOPAT_q} - (\mathrm{CAPEX_q} - \mathrm{D\&A_q}) - \Delta\mathrm{NFWC_q}. \tag{3}$$

Net Operating Profit After Taxes (NOPAT) is equal to total quarterly revenues (QREV) multiplied by the contribution margin ratio (1 − VC), minus fixed operating costs (FC) after taxes (where TR is the tax rate):

$$\mathrm{NOPAT_q} = [\mathrm{QREV_q} \times (1 - \mathrm{VC_q}) - \mathrm{FC_q}] \times (1 - \mathrm{TR}). \tag{4}$$

Financial professionals typically model and forecast revenues and expenses using time-series models. This may be sensible when firm financial disclosures do not include customer data. If customer data are available, however, forecasting can be made more behaviorally appropriate by decomposing revenues (QREV) into total purchases (initial and repeat) and average revenue per order (ARPO). The remaining variables in Equations 1, 2, 3, and 4—NOA, ND, WACC, CAPEX, D&A, ΔNFWC, VC, FC, and TR—are modeled with procedures commonly used by financial professionals, such as obtaining estimates from a third party, or by using a simple time-series extrapolation of historical data.

Our valuation goal, then, is to specify processes for the acquisition of new customers, how many repeat orders these customers place after they have been acquired, and how much they will spend on each of those orders. We estimate the parameters of these models so that the disaggregate behaviors implied by the model are consistent with the quarterly disclosures provided by the firm. We combine the projections from these processes to forecast QREV in Equation 4 into the future, then plug the resulting free cash flow projections into Equation 2 to value the firm's operating assets.

If these processes predict future customer activity well, the valuation forecasts flowing from the model will be more accurate. Financial valuation is a prediction problem, and we assume the modeler is a passive external stakeholder who only has access to publicly disclosed data. As such, our model does not correct for endogeneity and does not include pricing/marketing mix data. While it is possible to incorporate endogenous variables (Schweidel and Knox 2013), this would require additional data that is unavailable to external stakeholders. Furthermore, an endogeneity-corrected valuation model may have lower holdout predictive validity than an uncorrected model because the firm and external stakeholders are unable to observe endogenous variables in the holdout period (Ebbes, Papies, and Van Heerde 2011). Examples of prior literature in which endogeneity-corrected models are proposed that underperform their non-corrected counterparts on holdout data include Besanko, Gupta, and Jain (1998) and Neslin (1990). Endogeneity correction is less helpful in our prediction-focused, limited-data setting. Next, we discuss the customer data that noncontractual firms make available, which motivate the model we specify for customer acquisition, ordering, and spend.

Firm Disclosures

At subscription-based firms, the most commonly disclosed customer measures are (1) the number of customers added each quarter and (2) the number of customers at the end of each quarter (McCarthy, Fader, and Hardie 2017).[3] Although noncontractual firms can also observe (and thus disclose) customers added, they cannot disclose the number of customers at the end of each quarter because customer attrition is not observable. Instead, noncontractual firms typically disclose the number of "active" users, or the number of customers who place at least one order over a preceding window of time. In general, noncontractual firms may disclose different metrics summarizing the number of customers, the counts of their activities, and the time frame over which the metrics are defined. While the framework can be adapted to any set of metrics, in this article we focus on the common set of metrics that our two publicly traded empirical examples, Overstock and Wayfair, regularly disclose.

[3] The number of customers lost each quarter can be easily obtained from $\mathrm{QADD_q}$ and $\mathrm{END_q}$.

Both Overstock and Wayfair disclose quarterly customers acquired ($QADD_q$ in quarter q). In addition to $QADD_q$, we consider the number of quarterly and annually active customers (QAU_q and AAU_q, respectively), which are equal to the number of customers who have placed at least one order within the past 3 and 12 months, respectively. Overstock and Wayfair each report only one of these numbers. Finally, we consider the total number of orders that the customers place each quarter (QTO_q).

In addition to customer disclosures, firms also provide financial disclosures. To the best of our knowledge, revenue, the most important financial disclosure, is the only figure that publicly traded firms disclose that identifies how customers spend when they place orders. Because the parameters associated with all processes are estimated jointly, revenue data also statistically identify the acquisition and repeat-purchase processes. This is particularly true for firms that suffer from left-censored customer data. At Wayfair, for example, revenue data alone are available for the first 39 quarters of commercial operations before the firm began disclosing other customer data. We let $QREV_q$ and $AREV_q$ denote the total revenue generated by the firm over the past 3 and 12 months, respectively, at the end of quarter q. We do not model product returns because public companies in general (and Overstock and Wayfair in particular) do not disclose this data.

Although we focus on this set of six common metrics that Overstock and Wayfair disclose—$QADD_q$, QAU_q, AAU_q, QTO_q, $QREV_q$, and $AREV_q$—the estimation method we propose can accommodate other measures that firms may choose to disclose. We use this data to estimate an underlying model for how customers are acquired, make repeat orders, and spend. We specify these three models over the next three subsections.

The Acquisition Process

Our proposed model for the timing of customer adoption consists of three parts: (1) the formation of "pools of prospects" (i.e., those who may be acquired in the future) over time, (2) the conversion of prospects into "intenders" (i.e., those who will eventually be acquired), and (3) the duration of time that elapses from the time a prospect becomes an intender to when the intender is acquired. As in McCarthy, Fader, and Hardie (2017), we drive the creation of prospect pools over time from the population size. At the beginning of the firm's commercial operations, there is an initial prospect pool M(0) which is equal to the population size at the time, POP(0) (e.g., the total number of US households at the time of incorporation). People within this prospect pool may or may not adopt in future weeks w = 1, 2, and so on. The size of the prospect pool in a given week w is equal to population growth during the week:

$$M(w) = POP(w) - POP(w-1), \quad w = 1, 2, \ldots \tag{5}$$

The total number of customers acquired within a particular week w is equal to the number of customers acquired that week across all preceding prospect pools:

$$A(w) = \sum_{i=0}^{w-1} M(i) \times [F_A(w-i|i) - F_A(w-i-1|i)], \tag{6}$$

where $F_A(w - i|i)$ is the probability that an individual from prospect pool i becomes interested and is subsequently acquired by the end of week w.

We model the duration of time until prospects are acquired through a mixture of hazard models. We characterize these processes differently depending on whether the prospect pool was formed before or after the "time of mass awareness," which we denote by w*. Prospect pools formed before and after w* are known as "early prospects" and "late prospects," respectively. The propensity with which prospects become acquired is governed by the following assumptions:

- At the time that their prospect pools are formed, a proportion (π_1) of early prospects are intenders. Intenders' times until acquisition are characterized by a proportional hazards model with a homogeneous Weibull (λ_1) baseline and coefficients associated with the acquisition covariates β_A.[4]
- Early prospects who are not intenders at the time their prospect pool was first formed have zero probability of being acquired before w*.
- In week w*, a proportion π_2 of the early prospects who were not intenders previously will become intenders. The acquisition timing of early prospects who become intenders in week w* is governed by a Weibull distribution with a different λ_2 baseline but the same proportional hazards coefficients β_A for statistical identification.
- Early prospects who are not intenders after week w* (i.e., $[1 - \pi_1] \times [1 - \pi_2]$ of early prospect pools) will never be acquired.
- At the time that their prospect pools are formed, a proportion (π_2) of late prospects are intenders, with times until acquisition governed by a homogeneous Weibull (λ_2) baseline with covariates incorporated through proportional hazards.
- Late prospects who are not intenders at the time their prospect pool was first formed will never be acquired.

Given a prospect's homogeneous baseline propensities to be acquired (λ_1 and λ_2), their corresponding homogeneous acquisition shape parameters (c_1 and c_2), time-varying acquisition covariates ($\mathbf{X}_A(w+1, w') = [\mathbf{x}_A(w+1), \mathbf{x}_A(w+2), \ldots, \mathbf{x}_A(w')]$) and the coefficients associated with those acquisition covariates (β_A), the probability that an individual from prospect pool w is acquired by the end of week w′ is equal to

[4] While we could easily allow for heterogeneity in the Weibull baseline, we do not do so both to maintain model parsimony and because, empirically, heterogeneity has been rejected every time we have applied the model to data allowing for it.

$$F_A[w' - w|w, \mathbf{X}_A(w+1, w'); w^*, \pi_1, \pi_2, \lambda_1, \lambda_2, c_1, c_2, \beta_A] \tag{7}$$

$$= \begin{cases} \pi_1\left(1 - e^{-\lambda_1 B_1(w,w')}\right), & w<w^* \text{ and } w' \leq w^*, \\ \pi_1\left(1 - e^{-\lambda_1 B_1(w,w')}\right) + (1-\pi_1)\pi_2\left(1 - e^{-\lambda_2 B_2(w^*,w')}\right), & w<w^* \text{ and } w'>w^*, \\ \pi_2\left(1 - e^{-\lambda_2 B_2(w,w')}\right) & \text{otherwise,} \end{cases}$$

where

$$B_n(w, w') = \sum_{i=w+1}^{w'} [(i-w)^{c_n} - (i-w-1)^{c_n}] e^{\beta_A^T \mathbf{x}_A(i)}, \quad n \in \{1, 2\}. \tag{8}$$

While the notion of product or service diffusion being driven by two customer segments with distinct start times is not a new one (Van den Bulte and Joshi 2007), the proposed model is the first to integrate this acquisition process with processes for repeat orders and spend per order across multiple cohorts and then calibrate the resulting joint model to empirical data.

There is strong precedent for this approach in extant marketing science literature. The model is akin to Moe and Fader (2002)'s forecasting model for new album sales, which allowed for two classes of prospects: those who place advance orders and those who wait for the "mass market" to emerge. It is consistent with the concept of a firm's sales "takeoff" (Golder and Tellis 1997) being driven by a sharp increase in newly acquired customers, which is preceded by a corresponding sudden expansion in the number of prospects who become interested in the firm. The model can exhibit a dip or "chasm" between the early and late parts of the diffusion curve (Moore 1991).

At both Overstock and Wayfair, we observe clear inflection points in the trajectory with which new customers are acquired that are not well captured by a single Weibull distribution or a mixture of Weibull distributions (Fader, Hardie, and Zeithammer 2003; Schweidel, Fader, and Bradlow 2008). Our proposed two-phase Weibull model allows for slower growth than what a one-phase Weibull model can accommodate near the beginning of commercial operations at Overstock. Conversely, it allows for faster early growth at Wayfair. There is also evidence of a similar inflection point in customer acquisitions at Sirius XM in McCarthy, Fader, and Hardie (2017), but the authors did not pursue this model component.

The Orders Process

Next, we propose a model for repeat-order timing. A customer's relationship with the firm is assumed to have two phases: (s)he is "alive" for some period of time and then churns (i.e., becomes permanently inactive). While alive, the number of orders that (s)he places in week w, o(w), follows a Poisson process with intensity $\lambda_O(w)$. $\lambda_O(w)$ may vary because of unobserved heterogeneity or external factors, such as the state of the macroeconomy or seasonality. We allow for both effects through a log-normal formulation:

$$\lambda_O(w) = \exp\left[b_O + \beta_O^T x_o(w)\right], \tag{9}$$

where the baseline purchase intensity b_O is distributed across the population according to a normal (μ_O, σ_O^2) distribution, $x_O(w)$ are order covariates associated with week w, and β_O represents the coefficients associated with those covariates.

Each week, the customer churns with probability θ. Under these assumptions, the probability that an individual customer acquired in week w places o(w′) orders in week w′ is

$$P_O[o(w')|w; \theta, \lambda_O(w')] = \frac{\exp[-\lambda_O(w')]\lambda_O(w')^{o(w')}}{o(w')!}(1-\theta)^{w'-w}. \tag{10}$$

We let θ vary across the population according to a beta (γ, δ) distribution, independent of the purchase process. While we could allow for customers' ordering behavior to change after the "time of mass awareness," neither of the empirical examples we study showed any evidence of needing to do so.

This process, in combination with the acquisitions process, provides us with quarterly model-based estimates for QAU, AAU, and QTO. Hereinafter, we refer to this repeat purchase model as the beta-geometric/mixed-log-normal model. The fits and forecasts of this model would be, for all intents and purposes, the same as the Pareto/negative binomial distribution (NBD) model (Schmittlein, Morrison, and Colombo 1987), if the Pareto/NBD model were extended to allow for time-varying covariates.

Stochastic repeat purchase models such as this one have proven effective and robust, parsimoniously providing accurate aggregate fits and forecasts for noncontractual customer behavior across many applications (Braun, Schweidel, and Stein 2015; Fader, Hardie, and Shang 2010; Platzer and Reutterer 2016; Schweidel and Knox 2013). Parsimony is particularly important in our setting because of the limited nature of our available data. Our model uses censored data summaries across multiple cohorts, while all the aforementioned studies used granular transaction-level data for individual cohorts.

Average Revenue per Order

If suitably rich data were available, we could consider a model for spend per order that explicitly allows for variation in expected spend across customers [e.g., Fader, Hardie, and Lee (2005)'s so-called "gamma-gamma" model] and over time. In practice, we have not found any company, contractual or non-contractual, that discloses the metrics that would enable a modeler to statistically identify heterogeneity in expected spend across customers. The only spend-related metric that firms (including Overstock and Wayfair) disclose are total revenue figures. For this reason, we use a simple time-series regression model to project ARPO, similar in spirit to what was done in McCarthy, Fader, and Hardie (2017). We model weekly ARPO as a function of an intercept, a linear time trend, and weekly time varying covariates $x_S(w)$:

$$\mathrm{ARPO}(w) = \beta_{S,0} + \beta_{S,w} \times w + \beta_S^T x_S(w) + \epsilon(w), \quad \mathbb{E}[\epsilon(w)] = 0. \tag{11}$$

Because we cannot explicitly model heterogeneity in spend amounts across customers, we cannot estimate the distribution of CLV across customers. However, the proposed ARPO model can, in conjunction with the acquisition and repeat-order processes, provide us with model-based estimates of QREV and AREV, allowing us to estimate the overall valuation of the firm without bias. An ARPO model for spend also allows us to estimate the expected unit economics of newly acquired customers. These outputs are likely the most valuable outputs of a customer-based valuation model. In the next section, we describe how to estimate the parameters of the three submodels we have summarized.

Parameter Estimation

After deriving efficient expressions for model-based estimates of the available data (for these derivations, see the Appendix)—$\widehat{\mathrm{QADD}}_q$, $\widehat{\mathrm{QAU}}_q$, $\widehat{\mathrm{AAU}}_q$, $\widehat{\mathrm{QTO}}_q$, $\widehat{\mathrm{QREV}}_q$, and $\widehat{\mathrm{AREV}}_q$—it may seem natural to estimate the parameters of the acquisition, repeat-order, and ARPO processes $(\lambda_1, \lambda_2, c_1, c_2, \pi_1, \pi_2, \beta_A, w^*, \mu_O, \sigma_O^2, \beta_O, \gamma, \delta, \beta_{S,0}, \beta_{S,w}, \beta_S)$ using nonlinear least squares (Srinivasan and Mason 1986), estimating the parameters that minimize the sum-of-squared errors (SSE):

$$\begin{aligned}\mathrm{SSE} = &\sum_{q=1}^{Q_{QADD}} \left(\mathrm{QADD}_{(q)} - \widehat{\mathrm{QADD}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{QAU}} \left(\mathrm{QAU}_{(q)} - \widehat{\mathrm{QAU}}_{(q)}\right)^2 \\ &+ \sum_{q=1}^{Q_{AAU}} \left(\mathrm{AAU}_{(q)} - \widehat{\mathrm{AAU}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{QTO}} \left(\mathrm{QTO}_{(q)} - \widehat{\mathrm{QTO}}_{(q)}\right)^2 \\ &+ \sum_{q=1}^{Q_{QREV}} \left(\mathrm{QREV}_{(q)} - \widehat{\mathrm{QREV}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{AREV}} \left(\mathrm{AREV}_{(q)} - \widehat{\mathrm{AREV}}_{(q)}\right)^2,\end{aligned} \tag{12}$$

where Q_C is the number of times that metric C is disclosed, and $C_{(q)}$ is the qth disclosure of metric C, recognizing that an "empty sum" $\sum_{i=1}^{0} a_i$ is equal to zero.

Although this procedure would provide us with asymptotically unbiased estimates of the model parameters, it would be highly inefficient because the available data operate on very different scales. For example, because the average amount spent on each order at Overstock and Wayfair is over \$180, QREV figures are 200–600 times larger than QAU and QADD figures, causing the procedure to heavily overweight revenue data relative to all other data. Therefore, we prestandardize each measure to have mean zero and unit variance before applying the nonlinear least squares estimation procedure. Data prestandardization is a very common and important preprocessing step in data mining and machine learning (Al Shalabi, Shaaban, and Kasasbeh 2006; Mohamad and Usman 2013).

We estimate the parameters that minimize the following prestandardized sum-of-squared errors (PSSE):

$$\begin{aligned}\mathrm{PSSE} = &\sum_{q=1}^{Q_{QADD}} \left(\widetilde{\mathrm{QADD}}_{(q)} - \widehat{\widetilde{\mathrm{QADD}}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{QAU}} \left(\widetilde{\mathrm{QAU}}_{(q)} - \widehat{\widetilde{\mathrm{QAU}}}_{(q)}\right)^2 \\ &+ \sum_{q=1}^{Q_{AAU}} \left(\widetilde{\mathrm{AAU}}_{(q)} - \widehat{\widetilde{\mathrm{AAU}}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{QTO}} \left(\widetilde{\mathrm{QTO}}_{(q)} - \widehat{\widetilde{\mathrm{QTO}}}_{(q)}\right)^2 \\ &+ \sum_{q=1}^{Q_{QREV}} \left(\widetilde{\mathrm{QREV}}_{(q)} - \widehat{\widetilde{\mathrm{QREV}}}_{(q)}\right)^2 + \sum_{q=1}^{Q_{AREV}} \left(\widetilde{\mathrm{AREV}}_{(q)} - \widehat{\widetilde{\mathrm{AREV}}}_{(q)}\right)^2,\end{aligned} \tag{13}$$

where

$$\tilde{C}_{(q)} = \frac{C_{(q)} - m_C}{s_C} \quad \text{and} \quad \hat{\tilde{C}}_{(q)} = \frac{\hat{C}_{(q)} - m_C}{s_C}$$

and m_C and s_C are plug-in estimates of the mean and standard deviation of metric C:

$$m_C = \sum_{q=1}^{Q_C} C_{(q)}/Q_C \quad \text{and} \quad s_C = \sqrt{\frac{1}{Q_C - 1}\sum_{q=1}^{Q_C}\left(C_{(q)} - m_C\right)^2}.$$

This automatically adjusts for the different scales on which each metric may be measured. The estimation procedure can easily be extended if other metrics were available, as we would simply prestandardize those metrics and add them to the PSSE expression in Equation 13.[5]

Valuation Procedure

As noted in the Valuation Framework section, we value the firm by forecasting QREV into the future, using Equations 3 and 4 to obtain free cash flow projections, then plugging these free cash flow projections into Equation 2 to value the firm's operating assets. Our procedure for forecasting QREV comes as a direct result of the aforementioned estimation procedure—instead of estimating QREV over the Q-quarter calibration

[5] The code is available on request from the authors.

period, we extend it an additional Q* quarters, far enough into the future that the net present value of all future cash flows is effectively zero (we use $Q^* = 200$ in the empirical analyses that follow, corresponding to a 50-year forecasting horizon). After we forecast long-term revenues, the rest of the valuation model is effectively the same as a typical discounted cash flow valuation model.

Our forecasting horizon is long, which makes the forecast uncertain. At the same time, long forecasting horizons are an unavoidable reality of financial valuation. Whenever a financial professional builds a discounted cash flow model, (s)he makes projections over an infinite horizon. This long horizon is usually cloaked through an arbitrary "terminal value" calculation (Courteau, Kao, and Richardson 2001).

While we cannot avoid long-range forecasts, we should account for the fact that they are uncertain. We do so by constructing a valuation distribution that explicitly incorporates the degree of uncertainty in the model forecasts and thus the resulting overall firm valuations. Companies with more uncertain future prospects will have more diffuse valuation distributions, lessening the significance of observed stock price departures from fair value. In turn, this can influence the relative attractiveness of a potential investment (Sharpe 1964).

To create the valuation distribution, we draw a new set of data by bootstrap-resampling the residuals of the original fitted model (Efron and Tibshirani 1994). We estimate the model parameters associated with the bootstrapped data, which we use to obtain projections of the customer metrics and the resulting valuation of the firm. Note too that the magnitude of the valuation uncertainty may not be as large as it may seem, because the time value of money decreases the valuation impact of longer-range projections (e.g., the discount factor associated with a 10% WACC 20 years into the future is less than .15).

Next, we illustrate the entire valuation procedure for two publicly traded companies. Web Appendices A and B detail the steps we take to estimate valuation components other than quarterly revenues.

Empirical Analysis

We apply the proposed customer-based valuation methodology to data from Overstock (Nasdaq: OSTK), an e-commerce retailer selling a wide assortment of products. We validate the model's fit, compare the results with alternative methodologies, estimate the model's valuation, and obtain a valuation distribution that accounts for uncertainty in the fitted model parameters. Next, we apply the methodology to data from Wayfair (NYSE: W), a large Internet-based home goods seller. Wayfair is complementary to Overstock for four reasons:

1. Wayfair is a high-growth business while Overstock is more mature. For example, while Overstock's quarterly revenues grew by 5% from Q1 2016 to Q1 2017, Wayfair grew by 29% over the same period.
2. Wayfair has more limited data than Overstock. Wayfair did not disclose any metrics for the first six quarters of its commercial operations, creating severe left-censoring. Over the next 33 quarters, Wayfair disclosed one metric, AREV, only once per year in the fourth quarter. Wayfair only began disclosing a complete set of metrics in Q1 2013, its 44th quarter of commercial operations.
3. Wayfair discloses different data from Overstock, providing AREV in addition to QREV, and AAU instead of QAU. These may seem like minor distractions, but we would not want to ignore AREV disclosures when our data are so limited, and treating AAU and QAU interchangeably would strongly bias the resulting parameter estimates.
4. While both Overstock and Wayfair are e-commerce retailers, Wayfair has a narrower product assortment, almost entirely focused on home furnishings as opposed to a much broader array of items, including furniture, that Overstock carries. Net revenues per order were \$166 and \$228 at Overstock and Wayfair in Q1 2017, respectively.

We make Overstock and Wayfair's publicly disclosed data available in Web Appendix C.

Overstock

While Overstock began commercial operations in Q1 1999, it began disclosing QADD, QREV, and QTO two years later in Q1 2001 and began disclosing QAU in Q3 2004. Overstock stopped disclosing QADD in Q1 2016 but has continued to disclose all other metrics. We model these data up to and including Q1 2017, so $Q = 73$.

Overstock's revenues come from U.S. customers, so our unit of population is the U.S. labor force. The labor force is a proxy for the number of people who have the means and, perhaps, the need to pay for Overstock products. The Bureau of Labor Statistics' Current Population Survey discloses the U.S. labor force monthly. We include just one time-varying covariate to account for seasonality in the acquisition, repeat-order, and spend processes: a dummy variable equal to one in calendar Q4 and zero otherwise (the coefficients corresponding to this covariate across the three processes are denoted by $\beta_{A,Q4}$, $\beta_{O,Q4}$, and $\beta_{S,Q4}$).

Parameter Estimates and Model Fit

As noted in the "Parameter Estimation" subsection, we estimate the parameters of the acquisition, repeat-order, and ARPO processes using nonlinear least squares from the prestandardized data, finding the set of parameters that jointly minimize PSSE in Equation 13. Table 1, Panel A, contains the parameter estimates along with their standard errors, and the "time of mass awareness" w^* is estimated to be 306, corresponding to

Table 1. Parameter Estimates.

	Acquisition			Repeat Order			ARPO	
	Est.	SE		Est.	SE		Est.	SE
A: Overstock								
λ_1	.04425	.01826	μ_O	−7.97451	5.09510	$\beta_{S,0}$	55.10163	18.53716
λ_2	.00067	.00173	σ_O^2	.66638	12.87912	$\beta_{S,w}$	1.53010	.32636
c_1	3.17910	.95208	$\beta_{O,Q4}$	1.09160	.22328	$\beta_{S,Q4}$	−22.29522	5.22029
c_2	1.49161	.17929	γ	2.47301	90.68178			
π_1	.04425	.01826	δ	53.34601	83276.03001			
π_2	.99912	.13481						
$\beta_{A,Q4}$	.52584	.09883						
B: Wayfair								
λ_1	.00149	.00115	μ_O	−8.94484	.90564	$\beta_{S,0}$	17.85236	5.12480
λ_2	.00019	.00008	σ_O^2	2.10116	2.96532	$\beta_{S,w}$	3.81151	.14068
c_1	1.45165	.12742	γ	98.15855	19.94462	$\beta_{S,Q4}$	−7.58639	5.64184
c_2	2.29469	.12208	δ	4743.89763	1083.69253			
π_1	.10872	.10034						
π_2	.41178	.14483						
$\beta_{A,Q4}$	.27326	.03303						

Q2 2003. For exposition, we report the results assuming a quarterly unit of time for both Overstock and Wayfair.[6]

Although w* is estimated from the data and not through a subjective assessment of the business, its value is consistent with a major transition in Overstock's business model. Overstock dramatically changed its company strategy from being both a business-to-business and a business-to-consumer company to becoming almost entirely a mass-market business-to-consumer company in 2003. For example, Overstock's business-to-business activity with Safeway alone fell from 16% of sales in 2002 to less than 1% by 2004, and Overstock's year-on-year percentage increases in marketing spend were 50%, 133%, and 101% in 2002, 2003, and 2004, respectively. This pivot in strategy brought with it a large influx of "intenders"; whereas only 4.4% of early prospects were intenders when they first became prospects, nearly 100% of them became intenders after w*. The effect of Q4 seasonality is very strong. We can infer that Overstock acquires significantly more new customers and that existing customers place more orders in November/December, but the amount that is spent on orders during this period is significantly lower than during the rest of the year.

In Figure 1, we plot estimated and actual QADD, QTO, and QREV data over the calibration period in the top, middle, and bottom rows, respectively. In the left-hand column, we show the incremental quarter-by-quarter figures; in the right-hand column, we show the corresponding cumulative figures. The resulting model fit is good. Although there is some variation in the observed data around our model-based estimates, the model does an adequate job of capturing the baseline trends. It may be tempting to add covariates to the model to capture every peak and trough, but doing so is unlikely to help—and may even hurt—model forecasts. The in-sample fit for Overstock's QAU data is as good as the fits shown here. We show this plot in Web Appendix D, though it also appears in the top-right panel of Figure 2 as part of our model comparison analysis in the next subsection.

Strong seasonal variation is evident in these plots. We also observe some evidence of a slowdown in new customer acquisitions, which may hamper Overstock's future sales growth.

Model Comparison

We compare our proposed model's fit with that of three benchmark models: (1) Gupta, Lehmann, and Stuart (2004), (2) Schulze, Skiera, and Wiesel (2012), and (3) Libai, Muller, and Peres (2009). All benchmark models do not incorporate submodels for repeat orders, opting instead to set revenue (or margin) per alive customer equal to the trailing four-quarter average. To obtain implied QTO estimates for these models while remaining consistent with their assumption for revenue per alive customer, we set future orders per alive customer and future revenue per order to be equal to their respective trailing four-quarter averages. Both Libai, Muller, and Peres (2009) and Schulze, Skiera, and Wiesel (2012) model the total number of alive customers over time, so we follow their convention of assuming that the total number of alive customers is equal to the total active customer count (QAU for Overstock).

In Figure 2, we plot the model fits for Gupta, Lehmann, and Stuart's (2004) model, Schulze, Skiera, and Wiesel's (2012) model, Libai, Muller, and Peres's (2009) model, and the proposed model. Gupta, Lehmann, and Stuart's model severely underestimates future acquisitions. This underestimation is primarily due to the inability of the Bass-like technology substitution model to model right-skewed data (unlike, for example,

[6] Model fits and forecasts remain the same when we use a weekly unit of time; however, some parameter estimates change a multiplicative factor. We divide λ_1 and λ_2 by 13^c. We divide μ_p and σ_p by log(13). Finally, we divide $\beta_{S,w}$ by 13. All other parameters remain the same.

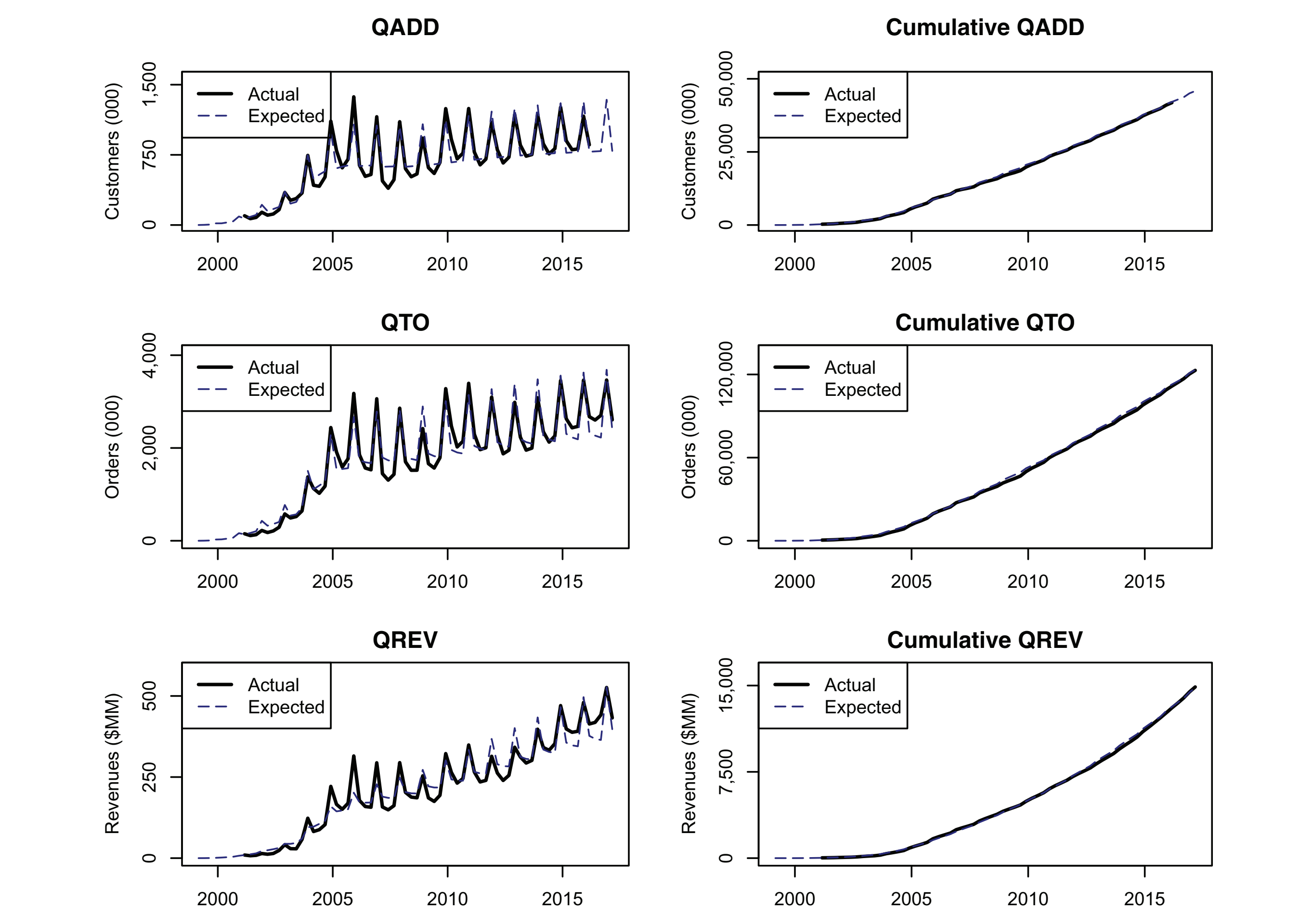

Figure 1. Overstock: quarterly customer additions, total orders, and revenues.

the Weibull distribution). By modeling QAU directly, Schulze, Skiera, and Wiesel's model and Libai, Muller, and Peres's model have better fits in general, particularly to QADD, QAU, and QTO. This result is consistent with the alternative comparisons analysis in McCarthy, Fader, and Hardie (2017). At the same time, Schulze, Skiera, and Wiesel and Libai, Muller, and Peres have difficulty capturing the slow ramp-up of QAU at the beginning of Overstock's commercial operations, which we are able to accommodate through our proposed changepoint-like acquisition process. As a result, they overestimate QAU (and thus, QADD) in the 2000–2005 period and underestimate it in the 2006–2017 period. Although ARPO has been increasing over time, all three methods set spend per order equal to the average of the four most recent quarters, so they overestimate QREV earlier on but are likely to underestimate it in the future because ARPO is likely to continue to increase. Our model explicitly allows for time trend and seasonal effects in ARPO, providing us with a better fit for the QREV data.

The number of estimated parameters for the proposed model, Gupta, Lehmann, and Stuart's (2004) model, Schulze, Skiera, and Wiesel's (2012) model, and Libai, Muller, and Peres's (2009) model are 15, 6, 6, and 8, respectively. While the number of estimated parameters for all models is far less than the number of data points, at 242, the proposed model will naturally have a better in-sample fit because it has more estimated parameters. Next, we turn to a comparison of predictive accuracy in a holdout setting, which does not favor models with more parameters. This is a practically important test as well, because the quality of stock price estimates from a discounted cash flow valuation model is driven by the predictive accuracy of its revenue (and customer acquisition) projections. We assess predictive validity by performing a rolling holdout validation analysis in which we vary the calibration period, predict future customer metrics, and then compare those predictions with the observed data for each model we consider. We consider calibration periods $Q = 34, 35, \ldots, 73$, corresponding to all possible calibration periods for which we have at least three years (12 quarters) of customer data. We train our model on all data up to and including quarter Q, and then predict QADD, QAU, QTO, and QREV over the next two years (i.e., $Q + q^*$ for $q^* = 1, 2, \ldots, 8$). We summarize the predictive accuracy of our model by computing the mean absolute percentage error (MAPE) of our two-year predictions for each metric. After running this analysis for our proposed model, we repeat the

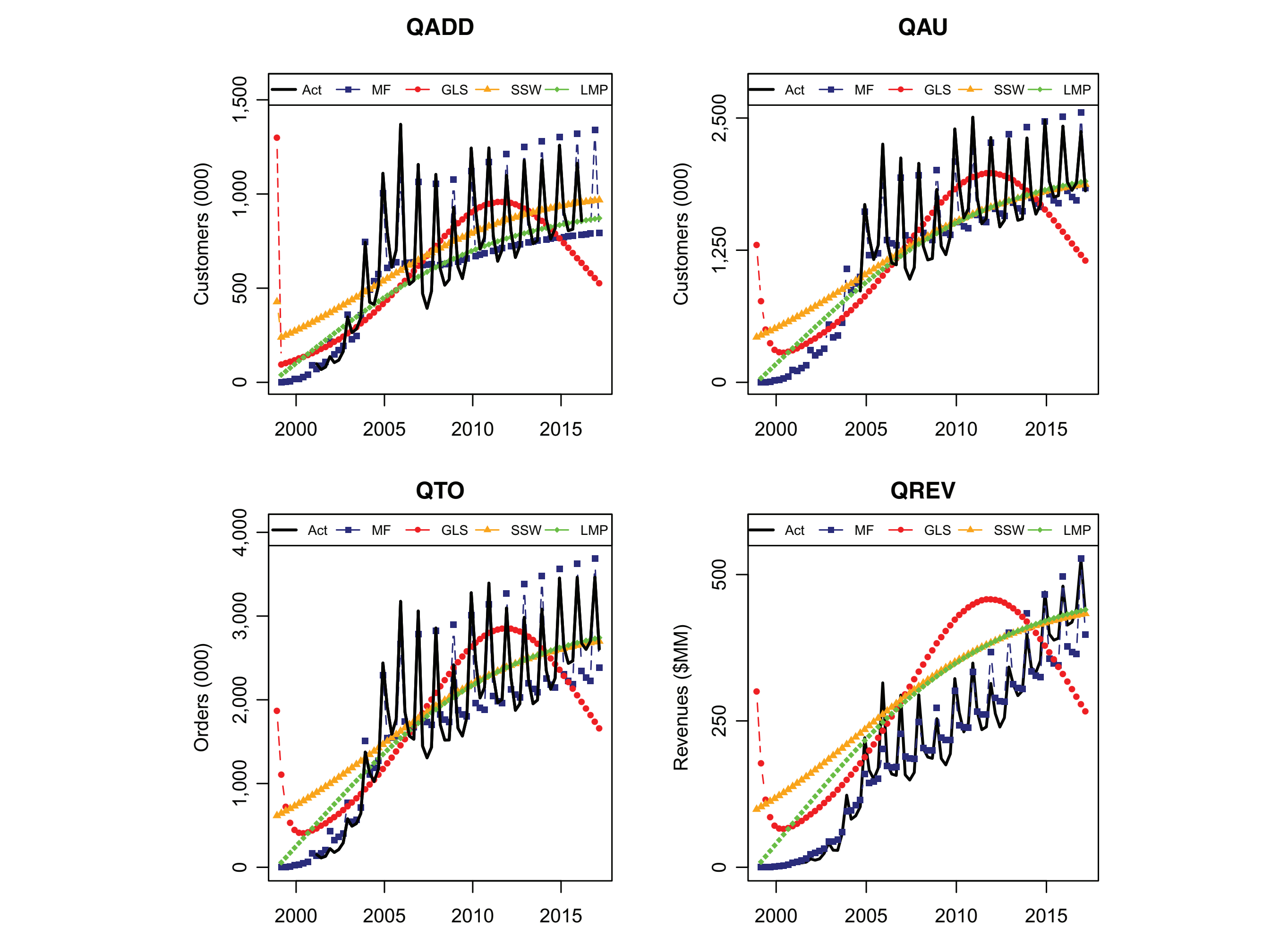

Figure 2. Overstock: model fit comparison against benchmark models and actual. Act = actual; MF = proposed model; GLS = Gupta, Lehmann, and Stuart (2004); SSW = Schulze, Skiera, and Wiesel (2012); LMP = Libai, Muller, and Peres (2009).

exercise for the three benchmark models, Gupta, Lehmann, and Stuart; Schulze, Skiera, and Wiesel; and Libai, Muller, and Peres. We present the results in Table 2, Panel A.

The MAPE figures in Table 2 are consistent with the in-sample goodness-of-fit results. While the model fits for Schulze, Skiera, and Wiesel (2012) and Libai, Muller, and Peres (2009) are generally comparable to one another, Libai, Muller, and Peres is more accurate across all metrics. Gupta, Lehmann, and Stuart's (2004) model has the lowest predictive accuracy of all alternative models by a wide margin. Our proposed model has the highest predictive accuracy, with a MAPE that is, on average, approximately 80, 30, and 18% smaller than Gupta, Lehmann, and Stuart, Schulze, Skiera, and Wiesel, and Libai, Muller, and Peres, respectively.

Table 2. MAPE of Rolling Two-Year Predictions of Customer Metrics.

Metric	GLS	SSW	LMP	Proposed
A: Overstock				
QADD	67.6	18.8	18.0	11.8
QAU	64.2	16.5	16.2	10.5
QTO	61.9	17.0	14.5	15.3
QREV	56.3	23.2	16.2	15.0
B: Wayfair				
QADD	17.8	21.0	15.0	4.9
AAU	19.9	3.0	2.7	2.1
QTO	20.5	8.8	8.8	5.8
QREV	18.9	5.9	5.6	4.9

Valuation

Having established the in-sample and out-of-sample performance of our proposed model, we turn next to estimating Overstock's overall valuation. We begin by projecting QREV far into the future. We do so by first forecasting the size of the U.S. labor force. The annual average growth rate of the U.S. labor force over the period from Q1 1999 to Q1 2017 was 0.8%, so we assume that the labor force will continue growing at this rate into the future.

We drive a detailed discounted cash flow valuation model using the forecasted QREV figures, which we use to estimate

Table 3. Valuation Summaries (End of Q1 2017).

Overstock	
Value of operating assets	$354.1 million
Nonoperating assets − Net debt	$72.7 million
Shareholder value	$426.8 million
Shares outstanding	25.3 million
Implied stock price	$16.88
Actual stock price	$15.50
Over(under)estimation	8.9%
Wayfair	
Value of operating assets	$825.4 million
Nonoperating assets − Net debt	$55.5 million
Shareholder value	$880.8 million
Shares outstanding	86.0 million
Implied stock price	$10.24
Actual stock price	$64.16
Over(under)estimation	(84.0%)

the other variables in Equations 1, 2, 3, and 4. Web Appendix B contains all the assumptions driving non-QREV figures. We summarize the model-based estimates of the value of the operating assets, nonoperating assets, net debt, and shareholder value (Equation 1), as well as the actual stock price at market close the day that Overstock released Q1 2017 results, in Table 3, Panel A.

We estimate a valuation for Overstock of $16.88 per share, approximately 9% above Overstock's Q1 2017 stock price of $15.50. The valuation estimates corresponding to Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), and Libai, Muller, and Peres (2009) are $0, $8.11, and $10.28 per share, respectively. We then implement the procedure to construct Overstock's valuation distribution. In Figure 3, we plot the resulting sampled QREV forecasts (Panel A) and valuation distribution (Panel B) after repeating this procedure 2,500 times.

There is a fair and growing amount of uncertainty in our projections. For example, these sampled realizations of the data suggest that peak quarterly acquisitions may have already occurred, or may occur 18 years from now. Similarly, while we expect quarterly revenues five years into the future to be $483 million, our 95% interval for revenues suggests that it could be as low as $389 million or as high as $524 million. The resulting 95% valuation distribution implies that Overstock's fair value lies between $7.2 and $26.9 per share. The observed stock price of $15.50 falls on the 38th percentile of the valuation distribution, only 12% away from the median and well within the 95% valuation interval. We conclude that the observed stock price is not practically or statistically significantly different from our fair-value estimate.

Wayfair

Next, we perform a customer-based valuation analysis for Wayfair, a large Internet-based home goods retailer. As with Overstock, all Wayfair's data were disclosed in its Securities and Exchange Commission filings and investor presentations. We use the same model specification for Wayfair that we used for Overstock, except that we remove the seasonal covariate for Q4 from the repeat-order process. Overstock and Wayfair also have different population sizes. While Wayfair primarily sells in the United States, it began selling in Canada and the United Kingdom in 2008, and in Germany in 2009. Therefore, the population size at Wayfair is equal to the U.S. labor force prior to 2008, the sum total of the U.S., Canadian, and U.K. labor forces in 2008, and the sum total of the U.S., Canadian, U.K., and German labor forces in and after 2009. Wayfair has not entered a new market in eight years and has not indicated that it intends to in coming years.

We estimate the parameters of the acquisition, repeat-order, and ARPO processes using the PSSE equation (Equation 13). In Table 1, Panel B, we provide the parameter estimates, where w^* is estimated to be 731, corresponding to Q4 2012. As with Overstock, while w^* is estimated from the data, its value also makes intuitive sense given the evolution of the business. From 2002 to 2011, the company had been bootstrapped by its founders and operated as a collection of hundreds of niche websites. In 2012, the company made the strategic decision to close and permanently redirect over 240 of these niche websites into Wayfair.com, making it a one-stop shop. The data would suggest that many prospects became intenders shortly after this change—whereas only 11% of early prospects were intenders at the time their prospect pool first formed, 41% of those remaining became intenders at the time of mass awareness.

Figure 4 contains the in-sample fits for QADD, QTO, and QREV in the top, middle, and bottom rows, respectively.[7] The left-hand column corresponds to the incremental quarter-by-quarter figures and the right-hand column corresponds to cumulative figures. We overplot black dots in the cumulative revenues plot (see the bottom-right panel) because Wayfair disclosed AREV figures once per year for ten years before it began regularly disclosing QREV. The corresponding plot of expected quarter-by-quarter QREV (i.e., the dashed line in the bottom-left panel) belies a fundamentally different customer acquisition trajectory before (vs. after) the time of mass awareness and cannot be adequately modeled with a single Weibull distribution. The model provides adequate fits for all metrics, including AAU (included in Web Appendix D).

To provide insight into the predictive validity of the proposed model, we repeat the two-year rolling holdout validation analysis that we had performed for Overstock. As with Overstock, we consider all possible calibration periods for which we have at least three years (12 quarters) of data on which to calibrate our model, so that $Q = 55, 56, \ldots, 60$. We then compare the MAPE of our proposed model's two-year predictions to the corresponding predictions for Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), Libai, Muller, and Peres (2009). Table 2, Panel B, summarizes the results.

[7] Technically, we provide in-sample fits for quarterly direct revenues. Wayfair also generates a small and declining amount of sales from third-party websites. These other sales represent 1.8% of sales in Q1 2017, which is 54% less than the same proportion in the previous year's period (see Web Appendix A).

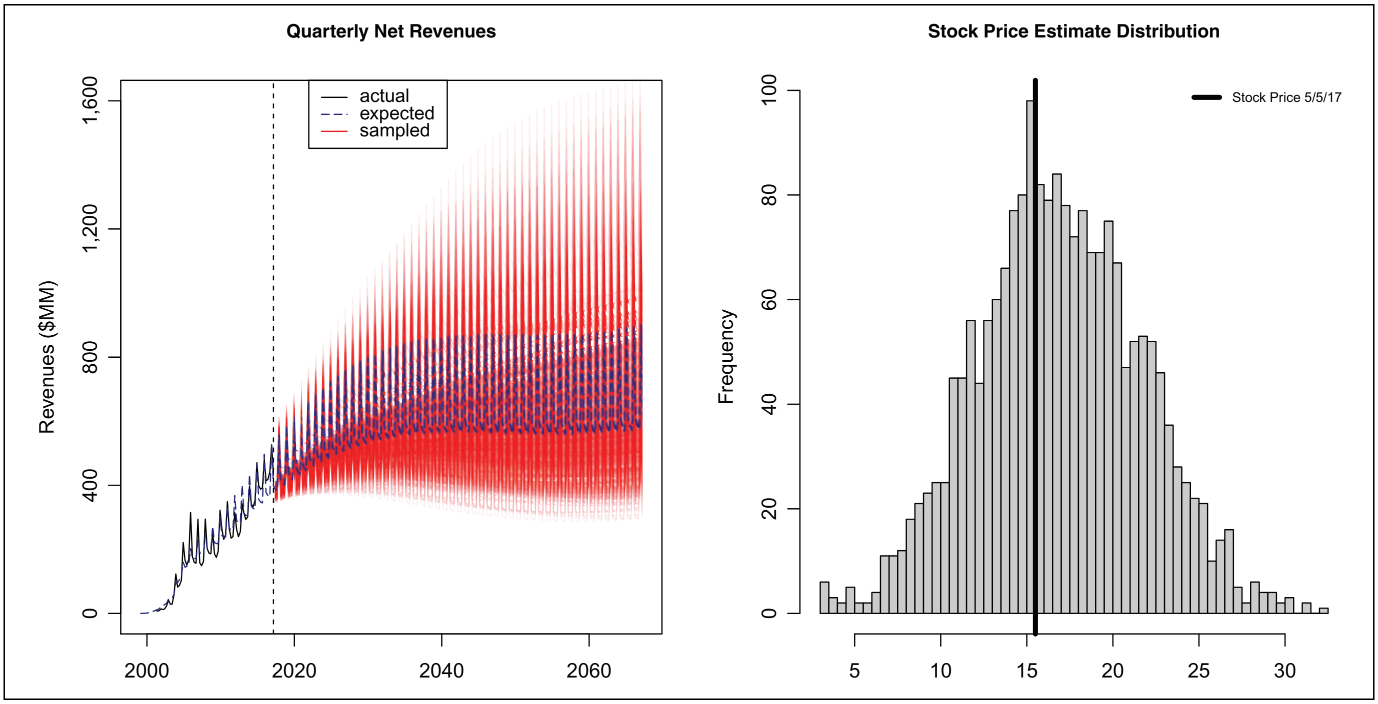

Figure 3. Overstock: QREV forecasts and stock price distribution.

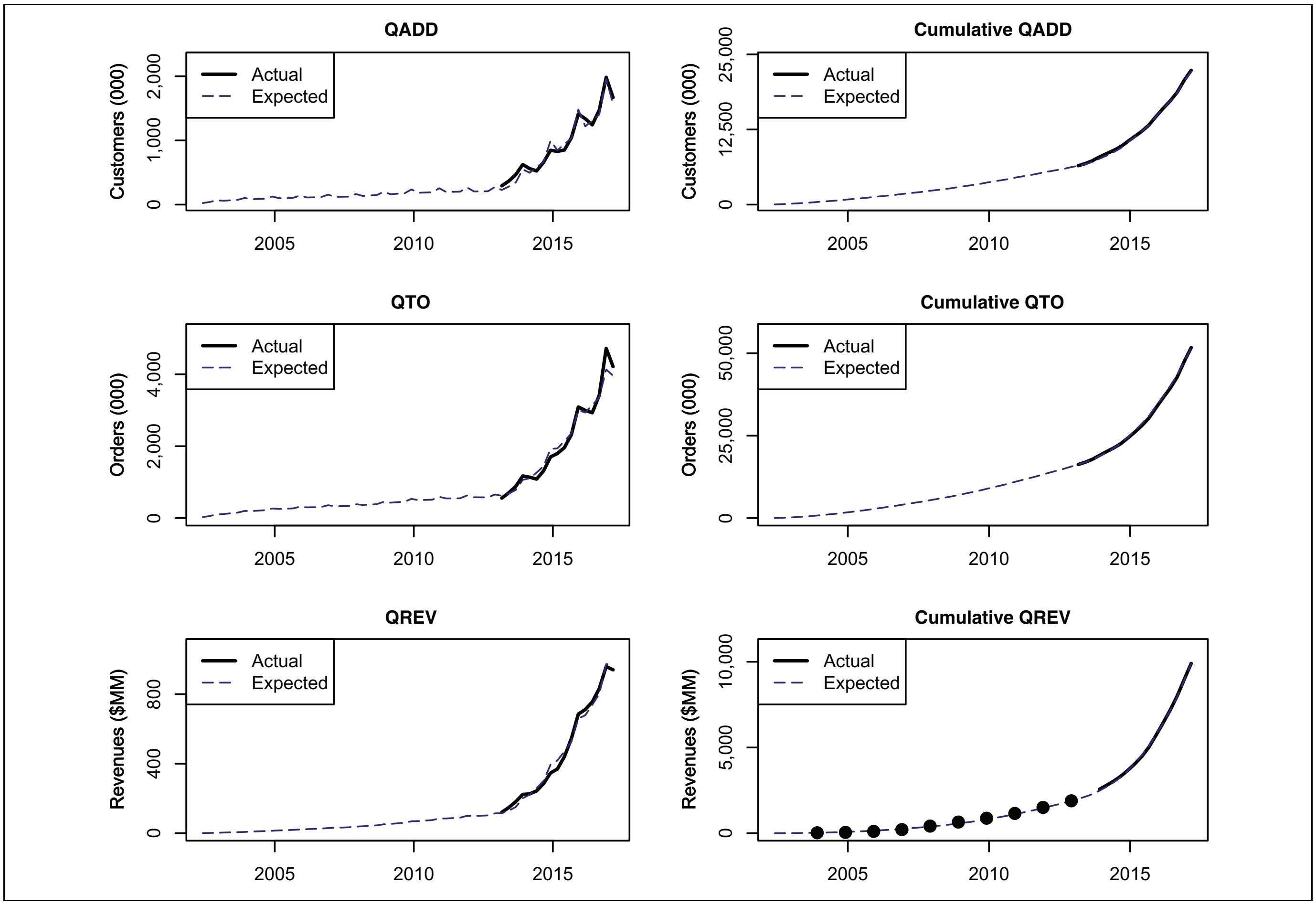

Figure 4. Wayfair: quarterly customer additions, total orders, and revenues.

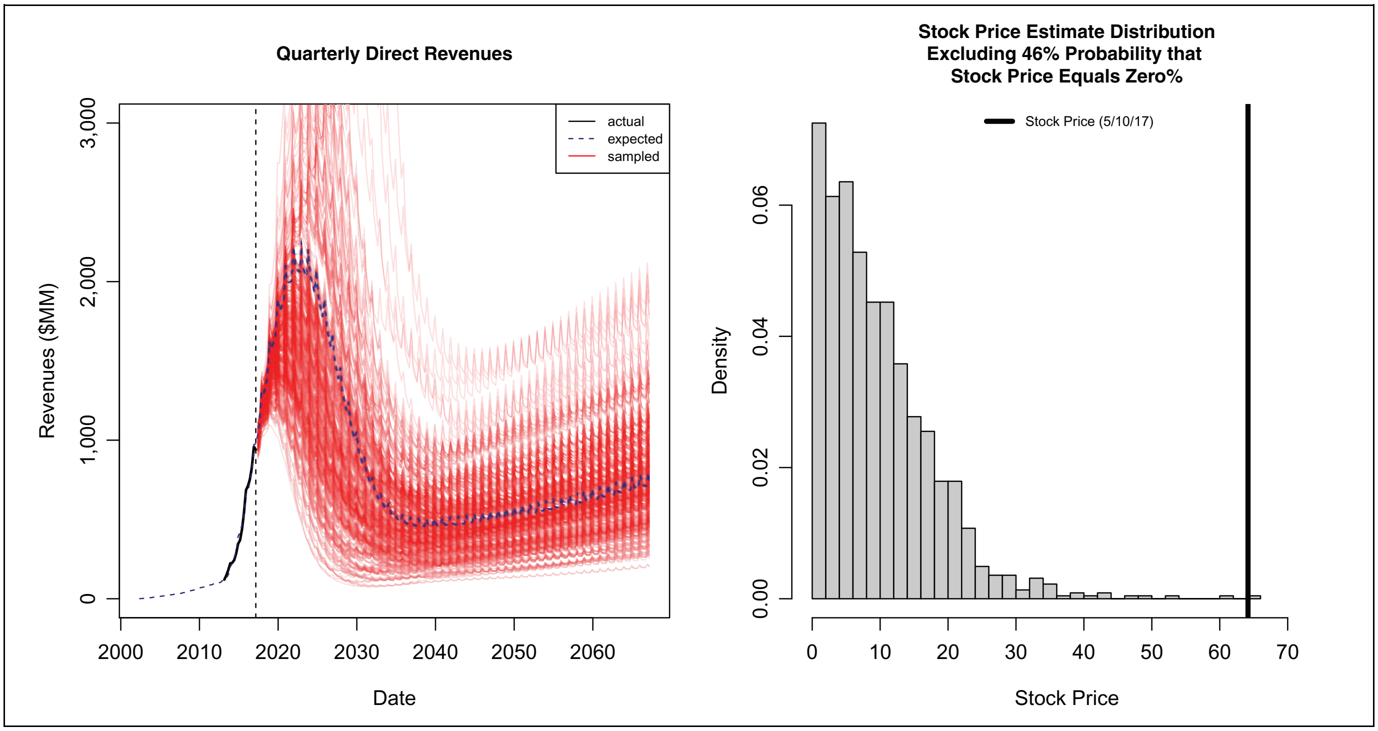

Figure 5. Wayfair: QREV forecasts and stock price distribution.

While the rank ordering of the average accuracy of the methods is the same for Wayfair as it was for Overstock, the relative outperformance of the proposed method is more significant. The average holdout MAPE of our proposed method is approximately 80%, 40%, and 35% smaller than that of Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), Libai, Muller, and Peres (2009), respectively.

While there is an intuitive appeal to the notion that our model would yield better predictions for more mature companies than for companies earlier in their life cycle, the results from the rolling validation analysis do not suggest that this is true. Holdout prediction accuracy is dramatically higher at Wayfair than at Overstock. Moreover, when we segment Overstock model predictions into three groups on the basis of the length of the calibration period, prediction accuracy is best for the mid-length calibration periods. That being said, our results are based on only two companies. A proper meta-analysis would be required to evaluate this theory more conclusively.

We project revenues over the next 50 years to drive our model for Wayfair's overall valuation. We assume the long-term growth rate of the labor forces in Canada, the United Kingdom, and Germany are equal to their historical averages of 1.2, .9, and .4%, respectively. For a detailed account of the assumptions underlying this valuation, see Web Appendix A. Table 3, Panel B, summarizes the resulting valuation. We estimate a fair valuation for Wayfair of $10.24 per share, substantially below its then-current stock price of $64.16. This overvaluation is very robust to departures from our base case assumptions for key nonrevenue projections, such as Wayfair's WACC and future gross margin percentage.

We then obtain a valuation distribution for Wayfair using the same bootstrapping procedure that we used for Overstock. As with Overstock, we generate 2,500 bootstrapped realizations of revenues. Figure 5, Panel A, shows the corresponding bootstrapped revenue forecasts. While our baseline expectation is that quarterly revenues will peak at $2.3 billion in Q4 2022, more than double its current level, there are bootstrapped realizations of peak QREV exceeding $12 billion. Even in this most optimistic revenue scenario, however, the resulting fair valuation for Wayfair's stock is $57.03, which is still 11.1% below Wayfair's observed stock price. In addition, these results are robust to assumptions regarding Wayfair's future margins, which we assume will rise to the midpoint of long-term margin goals that Wayfair management has provided on recent conference calls within five years. They are also robust to assumptions regarding Wayfair's discount rate, which we assume is equal to Bloomberg's estimated Q1 2017 WACC figure.

These results are consistent with the polarized market sentiment surrounding Wayfair. On the one hand, many market participants are very optimistic about the company's future prospects. For example, Patrick McKeever, managing director at MKM Partners, believes the company is undervalued, stating that Wayfair's most important key performance indicators are all "moving up and to the right" (McKeever 2018). John Blackledge, senior Internet analyst at Cowen, also believes that Wayfair is undervalued and is "well positioned to reap the benefits from major investments the company has made" (Blackledge et al. 2018). On the other hand, several hedge funds and other investment professionals have publicly expressed skepticism about Wayfair's valuation. For example, Citron Research's

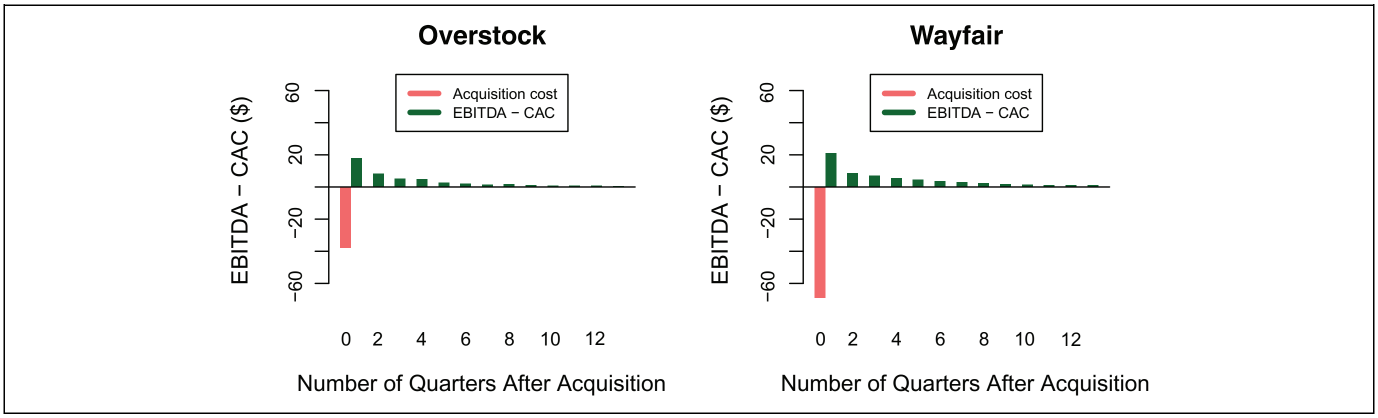

Figure 6. Expected cash flows of a customer acquired in Q1 2017.

Andrew Left said, "Wayfair is a throwback to 1999, a business where there's never EBIDTA [earnings before interest, taxes, depreciation, and amortization], just cumulative losses" (Left 2017). Marketing experts and media outlets such as the Wall Street Journal have conveyed similar sentiments (Winkler 2018). In his predictions for 2018, Scott Galloway foresees "online furniture Wayfair shares crash" due to "weak customer loyalty" (Galloway 2017). At the time of this analysis, Wayfair's stock is the fifth most shorted in the Internet Retail sector (Dusaniwsky 2017), with over 40% of its tradeable shares sold short (i.e., held by investors who will earn a profit if the stock falls below the price at which they shorted the stock). The availability of shares to short tightened to the point that investors must pay a 6% fee to borrow the shares needed to establish their short position. These data points suggest considerable disagreement among market participants about the future direction of Wayfair's stock price.

Our valuation model also suggests considerable disagreement in future valuations "across worlds." For example, the standard deviation of the percentage difference between the bootstrapped valuations and our expected one is 72% for Wayfair versus 28% for Overstock. However, all future stock price realizations lie below Wayfair's observed stock price.

Unit Economics Analysis

In addition to providing point estimates and intervals for firm valuation, our model can also extend beyond surface-level metrics such as revenues to better understand the underlying unit economics of a business. One quantity of managerial interest is the expected cash flows associated with a newly acquired customer—both the initial spend that the firm incurs to acquire the customer and the stream of future marginal cash flows associated with the customer thereafter. Accounting for the time value of money properly, summing these quantities provides us with the expected CLV of newly acquired customers. Successful businesses are able to acquire many high-CLV customers. While businesses that acquire low- (or negative) CLV customers may nevertheless report strong revenue growth if they grow customer acquisition expenses quickly enough, they will be very reliant on a continued high rate of new customer acquisition and will have greater difficulty growing themselves into profitability.

In this section, we compare the unit economics of Overstock with those of Wayfair. To do so, we must estimate (1) the average customer acquisition cost per newly acquired customer (CAC) and (2) the expected stream of marginal profits per customer after acquisition:

- We assume that CAC is equal to the trailing 12-month ratio of total advertising expense to the expected total number of customers acquired, which is consistent with statements made by both Overstock (2017) and Wayfair (2017), as well as prior academic literature (Gupta, Lehmann, and Stuart 2004). This also ensures that our accounting is consistent across both companies, because Overstock and Wayfair both disclose advertising expense each quarter in their financial statements.
- As in the expected residual lifetime value calculation of McCarthy, Fader, and Hardie (2017), we operationalize marginal profit after the customer has been acquired to be equal to earnings before interest, taxes, depreciation, amortization, and acquisition expenses, which we call "EBITDA − CAC." Our model provides these cash flows by taking the expected QREV each quarter associated with one customer acquired at the end of Q1 2017 and then multiplying these future expected QREV figures by the firm's overall ratio of expected EBITDA − CAC to total revenues.

In Figure 6, we present the resulting comparison of expected cash flows over the next three years at the two firms. We assume that CAC is incurred at the time the customer is acquired (i.e., zero quarters after acquisition) and the customer's initial purchase occurs immediately after the customer is acquired (i.e., in the first quarter after acquisition).

Wayfair customers generate more profits than Overstock customers after acquisition—the net present values of future profits after acquisition are \$59 and \$47 per customer at

Wayfair and Overstock, respectively. However, Wayfair spends far more than Overstock to acquire new customers. Wayfair's CAC is $69, nearly double Overstock's $38. While one could argue that some of Wayfair's advertising expenses are earmarked for customer retention, the proportion is likely to be small (Thomas 2001), and because Overstock is a relatively more mature business, its corresponding proportion is likely larger than Wayfair's.

If Wayfair could reduce its CAC to Overstock's level, we estimate that its expected valuation would more than double, all else being equal. Of course, all else is not equal—Overstock is pursuing a more selective customer acquisition strategy, avoiding the much higher acquisition costs that Wayfair has been willing to incur. We estimate that Overstock earns approximately $9 per acquired customer, whereas Wayfair incurred a loss of approximately $10 per customer in Q1 2017. While we anticipate that the unit economics of Wayfair's newly acquired customers will improve in the future as its variable contribution margin is expected to expand, challenging unit economics are a reality for the business and are an important driver behind its relatively modest valuation.

Discussion

The main contribution of this article is to propose a methodology with which customer disclosures are used to estimate a latent-variable model for customer acquisition, repeat purchase, and spend at publicly traded noncontractual firms. We use this latent-variable model to obtain a point estimate and distribution for the overall valuation of the firm and study the unit economics of newly acquired customers.

The valuation distribution is particularly useful when analyzing young firms with less historical data—for these firms, the valuation distribution will widen to account for parameter uncertainty. It would be natural to assume further that the extent of our uncertainty in future revenues (and thus free cash flows) might influence the firm's discount rate. This would be true if the revenue uncertainty were nondiversifiable (i.e., the uncertainty could not be reduced to zero if it were added to a diversified investment portfolio). A promising area of future work would be to specify a CBCV model that accounts for risk in a way that is consistent with the capital asset pricing model.

The methodology could also be applied, in theory, to private companies. Companies pursuing merger and acquisition opportunities, private equity firms, and venture capital firms may not be able to access the full transaction log of potential privately held investments until the later stages of the due diligence process, if at all. It might be easier for these firms to access a small collection of quarterly data summaries quickly.

The methodology has uses that extend beyond corporate valuation. For example, it could be useful for expert testimony in litigation cases in which firms would like to provide enough information to confirm or deny specific points raised within the case, but no more than that. Furthermore, it may often be the case that external stakeholders are primarily interested in sales forecasts (e.g., for forecasts of overall economic activity, for valuation through multiple future sales; Liu, Nissim, and Thomas 2002), which the proposed methodology provides.

One of the limitations of this work is that we have assumed that the disclosure decision is not strategic. It could be that there is a forward-looking component to firms' decision to disclose (Mintz et al. 2016), but the large-scale meta-analysis of firm disclosures necessary to carry out such an analysis is beyond the scope of this work. Another limitation is that we do not make any normative statements about the information content of the customer metrics themselves (e.g., which metrics provide stakeholders with the most accurate predictions of future revenues). We focus on a practical, general methodology for performing firm valuation in a noncontractual setting with whatever data the firm happens to provide, regardless of its quality, and leave metric optimization to future work.

The scope of the proposed method is limited to passive investors valuing a business on a going-concern basis. Although this may be the most common setting in which public company valuation is performed, there are other settings. For example, there are stakeholders who may be interested in using CBCV to both measure and manage the value of a firm over time. For example, so-called "activist investors" may invest in a firm and then actively try to increase the value of their investment, typically by changing high-level resource allocations (George and Lorsch 2014). The valuation of the firm after these allocations have been made will then be a very important consideration for whether the activist will make the investment in the first place. In this setting, it is more important to understand the causal relationship between marketing actions/strategy and overall firm valuation, so the modeler might want an endogeneity-corrected CBCV model, raising interesting trade-offs and methodological challenges.

This work highlights an important underappreciated use for customer disclosures. These metrics need not be ends in their own right as standalone key performance indicators; they can be leveraged to better understand (1) customers' true underlying propensity to acquire services, make purchases, and spend as well as (2) how these propensities vary across customers. The proposed methodology turns backward-looking customer metrics into important forward-looking measures, which should decrease investor uncertainty regarding future cash flows and thus increase the value of the firm (Bayer, Tuli, and Skiera 2017).

As investors realize these many uses for customer metrics, the demand for their disclosure will continue to grow. This would not be the first time such a thing has happened: one of the most commonly disclosed and tracked retail metrics, same store sales, became popular after a Wall Street analyst used it to uncover the true underlying financial condition of a fast-growing retailer in the 1970s (Blumenthal 2008). Customer metrics like the ones discussed here allow investors to track the quality of existing customers in much the same way that same store sales allows investors to track the quality of existing stores. With physical stores ceding share to Internet-based retailers, the need for such metrics is more important than ever.

Appendix: Derivations for Estimates of Public Disclosures

In this section, we derive efficient expressions for model-based estimates of the available data: $\widehat{\mathrm{QADD}}_q$, $\widehat{\mathrm{QAU}}_q$, $\widehat{\mathrm{AAU}}_q$, $\widehat{\mathrm{QTO}}_q$, $\widehat{\mathrm{QREV}}_q$, and $\widehat{\mathrm{AREV}}_q$. All expressions except for $\widehat{\mathrm{QAU}}_q$ and $\widehat{\mathrm{AAU}}_q$ are available in closed form and can thus be computed with no stochastic error. As we show in this Appendix, although our expressions for $\widehat{\mathrm{QAU}}_q$ and $\widehat{\mathrm{AAU}}_q$ require simulation, they are derived in a way that incurs negligible stochastic error.

$\underline{\widehat{\mathrm{QADD}}_q}$:

QADD_q is the sum of weekly customer acquisitions across all weeks during quarter q:

$$\widehat{\mathrm{QADD}}_q = \sum_{w=13q-12}^{13q} \hat{A}(w),$$

As shown in Equation 6, A(w) is a function of M(i) and $F_A(w - i|i)$. The former is equal to the change in population size, which is known. The latter is a function of the model parameters, which we have conditioned on. Multiplying these two terms together and marginalizing across all weeks within quarter q gives us the desired expression.

$\underline{\widehat{\mathrm{QTO}}_q}$:

Let the vector of expected weekly customer additions over the W-week calibration period be defined as

$$\hat{\mathbf{A}} \equiv \left[\hat{A}(1), ..., \hat{A}(W)\right].$$

Let RL be the residual lifetime of a customer (i.e., the duration of time in weeks from the time the customer is born to the time the customer churns). The unconditional probability mass function for RL is

$$\hat{P}(RL = t|\gamma, \delta) = B(\gamma + 1, \delta + t - 1)/B(\gamma, \delta), \quad t = 1, 2, \ldots \tag{A1}$$

We create a W × W lower triangular "expected alive customers" matrix, $\hat{\mathbf{C}}$, whose rows represent time (in weeks), whose columns represent weekly acquisition cohorts, and whose (i, j)th entry represents the number of customers acquired in week j who are still active in week i. The (i, j)th entry of $\hat{\mathbf{C}}$ is equal to

$$\hat{C}_{i,j} = \begin{cases} \hat{A}(j) \times \hat{P}(RL > i - j|\gamma, \delta), & i \geq j \\ 0 & \text{otherwise} \end{cases}$$

where $\hat{A}(j)$ is defined as in Equation 6 and $\hat{P}(RL = t|\gamma, \delta)$ is defined as in Equation A1.

This implies that the number of customers who are expected to be alive in a particular week w—that is, $\widehat{\mathrm{NA}}(w)$—is the sum of the corresponding row of the $\hat{\mathbf{C}}$ matrix:

$$\widehat{\mathrm{NA}}(w) = \sum_{j=1}^{W} \hat{C}_{w,j}.$$

The expected number of orders in a given week w are equal to the expected number of alive customers in week w multiplied by the expected number of repeat orders given that a customer is alive in that week, plus all the initial orders placed by customers expected to be acquired during that week. Letting O(w) denote the number of orders placed in week w,

$$\hat{O}(w) = \widehat{\mathrm{NA}}(w) \times \exp\left[\mu_O + \sigma_O^2/2 + \beta_O^T x_O(w)\right] + \hat{A}(w). \tag{A2}$$

QTO_q is the sum of weekly total orders across all weeks during quarter q:

$$\widehat{\mathrm{QTO}}_q = \sum_{w=13q-12}^{13q} \hat{O}(w).$$

$\underline{\widehat{\mathrm{QREV}}_q}$:

$\widehat{\mathrm{QREV}}_q$ is equal to the sum of the expected revenue generated each week during the quarter, which is itself equal to the product of the expected number of orders placed during the week ($\hat{O}(w)$), multiplied by the expected average revenue per order during the week, $\widehat{\mathrm{ARPO}}(w)$:

$$\widehat{\mathrm{QREV}}_q = \sum_{w=13q-12}^{13q} \hat{O}(w) \times \widehat{\mathrm{ARPO}}(w), \tag{A3}$$

where $\hat{O}(w)$ is defined as in Equation A2 and $\widehat{\mathrm{ARPO}}(w)$ follows easily from Equation 11:

$$\widehat{\mathrm{ARPO}}(w) = \beta_{S,0} + \beta_{S,w} \times w + \beta_S^T x_S(w). \tag{A4}$$

$\underline{\widehat{\mathrm{AREV}}_q}$:

$\widehat{\mathrm{AREV}}_q$ is the sum of $\widehat{\mathrm{QREV}}$ over the preceding four quarters, adjusted to account for the beginning of the time series, where $a \vee b = \max(a, b)$:

$$\widehat{\mathrm{AREV}}_q = \sum_{q'=1\vee(q-3)}^{q} \widehat{\mathrm{QREV}}_{q'}. \tag{A5}$$

$\widehat{\mathrm{QAU}}_q$

The expected number of quarterly active users in quarter q is equal to the number of customers who were acquired during quarter q, plus the expected number of customers born before quarter q who will place at least one repeat order within quarter q. If there were no heterogeneity in the time-invariant baseline propensity with which customers place orders, b_O, then letting QA_q be an indicator variable equal to one if a randomly chosen customer was active in quarter q and zero otherwise and letting $\mathbf{X}_O$ be the time-varying repeat-order covariates ($\mathbf{X}_O = [x_O(1), \ldots, x_O(W)]$),

$$\hat{E}(QAU_q|i, b_O, \mathbf{X}_O, \gamma, \delta) = \sum_{i=13q-12}^{13q} \hat{A}(i) + \sum_{i=1}^{13q-13} \hat{A}(i) \times P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta). \tag{A6}$$

$P(AQ_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta)$ is obtained by conditioning on the number of weeks the customer is alive during the quarter, which we denote by n_q:

$$P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) = \sum_{w=13q-12}^{13q} P(RL = w - i|\gamma, \delta) \times P(QA_q = 1|n_q = w - 13q + 13, b_O, \beta_O, \mathbf{X}_q), \tag{A7}$$

recognizing that a customer born in week i who lives for $w - i$ weeks is alive until week w, which for $w \in \{13q - 12, ..., 13q\}$ implies $w - 13q + 13$ alive weeks during quarter q. Using the "memorylessness" property of the Poisson process and accounting for the fact that time-varying covariates are constant within each quarter,

$$\begin{aligned} &P(QA_q = 1|n_q, b_O, \beta_O, \mathbf{X}_q) \\ &= 1 - \exp\{-\exp[b_O + \beta_O^T x_O(13q)]n_q\}. \end{aligned} \tag{A8}$$

Plugging Equation A8 into Equation A7 in conjunction with Equation A1 gives us a closed-form conditional expression for $\widehat{QAU}_q$ using Equation A6. To remove the conditioning on b_O, we must integrate Equation A8 against the mixing distribution for b_O:

$$\begin{aligned} \widehat{QAU}_q &= \hat{E}(QAU_q|i, \mu_O, \sigma_O^2, \mathbf{X}_O, \gamma, \delta) \\ &= \sum_{i=13q-12}^{13q} \hat{A}(i) + \sum_{i=1}^{13q-13} \hat{A}(i) \\ &\times \int_{-\infty}^{\infty} P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O|\mu_O, \sigma_O^2) db_O. \end{aligned} \tag{A9}$$

Equation A9 does not have a closed-form analytical expression. Instead, we use $\underline{K} = 50{,}000$ one-dimensional Halton draws (Datta, Foubert, and Van Heerde 2015; Train 2000) to approximate the integral:

$$\begin{aligned} &\int_{-\infty}^{\infty} P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O|\mu_O, \sigma_O^2) db_O \\ &\approx \frac{\sum_{k=1}^{K} P\left(QA_q = 1||i, b_O^{(k)}, \mathbf{X}_O, \gamma, \delta\right)}{K}, \end{aligned} \tag{A10}$$

where $b_O^{(k)}$ represents the kth element of a standard Normal Halton sequence. Evaluating Equation A10 using its conditional counterpart (Equation A7) and plugging this into Equation A9 gives us the desired expression.

$\underline{\widehat{AAU}_q}$:

The expression for $\widehat{AAU}_q$ follows the same logic as the expression for $\widehat{QAU}_q$. The expected number of annual active users in quarter q is equal to the number of customers who were acquired during the previous four quarters $\{q - 3, \ldots, q\}$, plus the expected number of customers born before quarter $q - 3$ who will place at least one repeat order within quarters $\{q - 3, \ldots, q\}$. If there were no heterogeneity in the time-invariant baseline propensity with which customers place orders, b_O, then letting AA_q be an indicator variable equal to one if a randomly chosen customer was active in any of quarters $\{q - 3, \ldots, q\}$ and zero otherwise,

$$\hat{E}(AAU_q|i, b_O, \mathbf{X}_O, \gamma, \delta) = \sum_{i=1\vee(13q-51)}^{13q} \hat{A}(i) + \sum_{i=1}^{0\vee(13q-52)} \hat{A}(i) \times P(AA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta). \tag{A11}$$

$P(AA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta)$ is obtained by conditioning on the number of weeks the customer is alive for during each of the preceding four quarters, $[n_{q-3}, n_{q-2}, n_{q-1}, n_q]$:

$$P(AA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) = \sum_{w=1\vee(13q-51)}^{13q} P(RL = w - i|\gamma, \delta) \times P(AA_q = 1|n_{q-3}, n_{q-2}, n_{q-1}, n_q, b_O, \beta_O, \mathbf{X}_q), \tag{A12}$$

where for each summand, letting n_a be the weeks that the customer is alive in quarters $q - 3$ through q, and recalling that $a \wedge b = \min(a, b)$,

$$\begin{aligned} n_a &= w - 13q + 52, \\ n_{q-3} &= n_a \wedge 13, \\ n_{q-2} &= [(n_a - 13) \vee 0] \wedge 13, \\ n_{q-1} &= [(n_a - 26) \vee 0] \wedge 13, \quad \text{and} \\ n_q &= (n_a - 39) \vee 0. \end{aligned} \tag{A13}$$

Using the memorylessness property of the Poisson process and accounting for the fact that time-varying covariates are constant within each quarter,

$$\begin{aligned}&P(AA_q = 1|n_{q-3}, n_{q-2}, n_{q-1}, n_q, b_O, \beta_O, \mathbf{X}_q) \\ &\quad = 1 - \exp\left\{-\exp\left[b_O n_a + \beta^T \sum_{q'=q-3}^{q} x_O(13q')n_{q'}\right]\right\}.\end{aligned} \tag{A14}$$

Plugging Equation A14 into Equation A12 in conjunction with Equation A1 gives us a closed-form conditional expression for $\widehat{AAU}_q$ using Equation A11. To remove the conditioning on b_O, we must integrate Equation A14 against the mixing distribution for b_O:

$$\begin{aligned}\widehat{AAU}_q &= \hat{E}(AAU_q|i, \mu_O, \sigma_O^2, \mathbf{X}_O, \gamma, \delta) = \sum_{i=13q-51}^{13q} \hat{A}(i) \\ &\quad + \sum_{i=1}^{0\vee(13q-52)} \hat{A}(i) \times \int_{-\infty}^{\infty} P(AA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) \\ &\quad \times f\left(b_O|\mu_O, \sigma_O^2\right) db_O.\end{aligned} \tag{A15}$$

Equation A15 does not have a closed-form analytical expression. As with the derivation for $\widehat{QAU}_q$, we use $K = 50{,}000$ one-dimensional Halton draws to approximate the integral:

$$\begin{aligned}&P(AA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) f\left(b_O|\mu_O, \sigma_O^2\right) db_O \\ &\quad \approx \frac{\sum_{k=1}^{K} P\left(AA_q = 1|i, b_O^{(k)}, \mathbf{X}_O, \gamma, \delta\right)}{K},\end{aligned} \tag{A16}$$

where $b_O^{(k)}$ represents the kth element of a standard Normal Halton sequence. Evaluating Equation A16 using its conditional counterpart (Equation A12) and plugging this into Equation A15 gives us the desired expression.

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