Customer-Based Corporate Valuation for Publicly Traded Non-Contractual Firms

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Abstract

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There is growing interest in “customer-based corporate valuation,” explicitly tying the value of a firm’s customer base to its overall financial valuation using publicly disclosed data. While much progress has been made in building a well-validated customer-based valuation model for contractual (or subscription-based) firms, there has been little progress for non-contractual firms. Non-contractual businesses have more complex transactional patterns because customer churn is not observed, and customer purchase timing and spend amounts are more irregular. Furthermore, publicly disclosed data are aggregated over time and across customers, are often censored, and may vary from firm to firm, making it harder to estimate models for customer acquisition, ordering, and spend per order. The authors develop a general customer-based valuation methodology for non-contractual firms which accounts for these issues. They apply this methodology to publicly disclosed data from e-commerce retailers Overstock.com and Wayfair, providing valuation point estimates and valuation intervals for the firms, and comparing the unit economics of newly acquired customers.

Keywords: customer lifetime value; customer equity; valuation; unit economics

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Executives, marketing managers, and financial professionals are increasingly aware that current and future customer relationships are a valuable – if not the most valuable – asset of a firm (Blattberg and Deighton 1996). As a result, customer acquisition, customer retention, and customer lifetime value (CLV) are being discussed and studied more than ever before (Braun, Schweidel, and Stein 2015, Datta, Foubert, and Van Heerde 2015). Accurate estimation of CLV gives external stakeholders (e.g., shareholders, creditors, suppliers, competitors, and regulators) the ability to estimate customer equity, or the remaining lifetime value of all existing customers plus the net present value of the CLV of all yet-to-be-acquired customers (Bauer and Hammerschmidt 2005, Kumar and Shah 2009, Rust, Lemon, and Zeithaml 2004). For many firms, customer equity represents the majority of shareholder value of the firm, enabling an explicit link between customer behaviors (i.e., acquisition, retention, and spend) and the overall financial valuation of the firm. Kumar and Shah (2015) provides an excellent summary of the literature on customer equity.

Customer-based corporate valuation (CBCV) is the process of valuing a firm by forecasting current and future customer behavior using customer data in conjunction with traditional financial data. However, the correct valuation framework depends on the nature of the relationship that the firm has with its customers, with a primary distinction being whether a firm is contractual (i.e., subscription-based), or non-contractual. While much progress has been made in performing CBCV for contractual firms, far less progress has been made for non-contractual firms. There are three main challenges to performing CBCV in non-contractual business settings:

1. While customer churn is observable for contractual firms, it is *unobservable* for non-contractual firms. If customers of an e-commerce retailer decide to end their relationships with the firm, there is no process to inform the firm. Instead, they simply discontinue their purchasing. This complicates the underlying model required to predict future customer activity because non-contractual firms do not know and thus cannot publicly disclose customers lost or the total size of their customer bases.

2. Even if the modeler knew that a non-contractual customer was "alive" (i.e., had a non-zero probability of making a future purchase), there are additional complexities

associated with repeat purchase and spend behaviors that the modeler must take into account. The number of purchases that customers make, and how much they spend on each of those purchases, can be highly variable at non-contractual firms. In contrast, customers of contractual firms have much more predictable ordering and spending patterns.

3. While modeling aggregate customer behavior in a non-contractual setting is a solved problem if the modeler has access to individual-level customer data (Fader, Hardie, and Shang 2010, Schmittlein, Morrison, and Colombo 1987), external stakeholders only have access to highly aggregated, publicly disclosed customer data (e.g., active customers, or total orders), which is a collection of repeated cross-sectional summaries (Jerath, Fader, and Hardie 2016). This makes model estimation more challenging, and limits the richness of the models that can be considered.

Because of these challenges, many articles in this area have limited their scope to the contractual business setting alone (Bonacchi, Kolev, and Lev 2015, McCarthy, Fader, and Hardie 2017, Schulze, Skiera, and Wiesel 2012). Extant articles that have attempted to perform CBCV in a non-contractual setting have done so with the following workarounds (Gupta, Lehmann, and Stuart 2004, Libai, Muller, and Peres 2009):

1. Assume an observable "retention rate" which may be defined, for instance, as the repeat rate, or proportion of customers who made a purchase last year who made another purchase this year (Farris et al. 2010).

2. Assume the number of alive customers equals the number of *active* customers.

3. Proceed with the valuation exercise as if the company operated in a contractual business setting.

While this workaround may seem convenient, it is problematic for a number of reasons. First, using the repeat rate in a non-contractual setting will typically understate future purchase activity and thus future profits dramatically because many customers who have not purchased in one year may still be alive [for example, 6% of consumer products seller

QVC's total sales in 2015 came from customers who had not purchased in over a year (QVC 2015)]. Second, retention propensities will generally differ across customers, and ignoring this heterogeneity will further undervalue the customer base (Fader and Hardie 2010). Third, different firms define the same metrics differently. For example, at Overstock.com, Wayfair, and Camping World, total "active customers" is defined as the number of customers who placed an order within the preceding three, 12, and 24 months, respectively. These issues lead to inaccurate projections that are not comparable across firms. Furthermore, they diminish the validity of valuation estimates and create incorrect inferences about the unit economics of customers (e.g., the lifetime value of new customers and customer retention) and their implications for the business (e.g., revenue concentration and reliance on new customer acquisition).

In the pioneering CBCV work of Gupta, Lehmann, and Stuart (2004), both contractual and non-contractual companies were valued with the same model. For non-contractual firms, a retention rate proxy was used as the retention rate in their CLV formula. Of the five companies valued in their empirical analysis, the only two non-contractual businesses, eBay and Amazon, were the two most mis-valued.[1] These companies were undervalued by an average of 88% and 83%, respectively.[2] A new model is needed which is specifically suited to non-contractual firms.

Our primary contribution is to develop a new model that can accurately value non-contractual firms. To do so, our underlying model for customer behavior, and the corresponding estimation procedure, are flexible enough to accommodate whatever aggregated, censored customer data the firm publicly discloses. We do not assume that private data are available. Our proposed model reflects important empirical realities associated with non-contractual customer behavior, including latent attrition, repeat purchasing which may vary across customers and over time, and time-varying spend-per-purchase patterns. In addition to providing overall firm valuation insights, our proposed model also allows external stakeholders to go beneath surface-level financial metrics such as revenues to better understand the underlying unit economics of a business. We predict this deeper look into the underlying financial health of a firm may be more useful to financial professionals than the valuation estimate itself.

In the next section, we present the model governing customers' acquisition, repeat purchasing, and spend, and how this model is used to drive an overall valuation for the firm. We illustrate this methodology by applying it to two publicly traded companies, Overstock.com (hereafter, Overstock) and Wayfair. After validating the proposed model, we provide overall valuation estimates for both firms, and valuation intervals to account for uncertainty in the model forecasts. We then analyze the unit economics of newly acquired customer cohorts and conclude with a discussion of the results.

MODEL DEVELOPMENT

In this section, we specify the individual-level model for the customer which we will use to forecast future customer activity, and show how this model is calibrated off of public disclosures and embedded within an overall valuation framework for the firm.

Valuation Framework

We adopt the discounted cash flow model as our firm valuation framework (Damodaran 2012, Koller, Goedhart, and Wessels 2010, Schulze, Skiera, and Wiesel 2012). This model is the de facto standard valuation method among financial professionals due to its flexibility and ease of use, facilitating adoption of these methods by both academics and practitioners. For completeness, we briefly summarize the discounted cash flow valuation procedure below. Shareholder value (SHV) in a particular quarter q is equal to the value of the firm's operating assets (OA) plus the non-operating assets (NOA), minus the net debt (ND):

$$\mathrm{SHV}_q = \mathrm{OA}_q + \mathrm{NOA}_q - \mathrm{ND}_q. \tag{1}$$

Shareholder value is observed in effectively continuous time for publicly traded firms, while operating assets, non-operating assets, and net debt are observed at the end of each quarter by external stakeholders. The value of a firm's operating assets Q quarters after the beginning of commercial operations (OA_Q) is equal to the sum of all free cash flows (FCF's) the firm will generate thereafter, discounted at the weighted average cost of capital (WACC, assuming quarterly compounding and using a midpoint approximation to account for within-period cash flow timing):

$$\mathrm{OA}_Q = \sum_{q=0}^{\infty} \frac{\mathrm{FCF}_{Q+q}}{(1+\mathrm{WACC})^{q-1/2}}, \tag{2}$$

Free cash flow is equal to net operating profit after taxes (NOPAT) minus capital expenditures (CAPEX), plus depreciation and amortization (D&A), minus the change in non-financial working capital (ΔNFWC) during that quarter:

$$\mathrm{FCF}_q = \mathrm{NOPAT}_q - (\mathrm{CAPEX}_q - \mathrm{D\&A}_q) - \Delta\mathrm{NFWC}_q. \tag{3}$$

NOPAT is equal to total quarterly revenues (QREV) multiplied by the contribution margin ratio (1−VC), minus fixed operating costs (FC) after taxes (where TR is the tax rate):

$$\mathrm{NOPAT}_q \quad = \quad [\mathrm{QREV}_q \times (1 - \mathrm{VC}_q) - \mathrm{FC}_q] \times (1 - \mathrm{TR}). \tag{4}$$

Financial professionals typically model and forecast revenues and expenses using time-series models. This may be sensible when firm financial disclosures do not include customer data. If customer data are available, however, forecasting can be made more behaviorally appropriate by decomposing revenues (QREV) into total purchases (initial and repeat) and average revenue per order. The remaining variables in equations 1, 2, 3, and 4 – NOA, ND, WACC, CAPEX, D&A, ΔNFWC, VC, FC, and TR – are modeled with procedures commonly used by financial professionals, such as obtaining estimates from a third party, or by using a simple time-series extrapolation of historical data.

Our valuation goal, then, is to specify processes for the acquisition of new customers, how many repeat orders these customers place after they have been acquired, and how much they will spend on each of those orders. We estimate the parameters of these models so that the disaggregate behaviors implied by the model are consistent with the quarterly disclosures provided by the firm. We combine the projections from these processes to forecast QREV in Equation 4 into the future, then plug the resulting free cash flow projections into Equation 2 to value the firm's operating assets.

If these processes predict future customer activity well, the valuation forecasts flowing from the model will be more accurate. Financial valuation is a prediction problem, and we assume the modeler is a passive external stakeholder who only has access to publicly disclosed data. As such, our model does not correct for endogeneity and does not include pricing/marketing mix data. While it is possible to incorporate endogenous variables (Schweidel and Knox 2013), this would require additional data that is unavailable to external stakeholders. Furthermore, an endogeneity-corrected valuation model may have lower holdout predictive validity than an uncorrected model because the firm and external stakeholders are unable to observe endogenous variables in the holdout period (Ebbes, Papies, and Van Heerde 2011). Examples of prior literature in which endogeneity-corrected models are proposed that underperform their non-corrected counterparts on holdout data include Besanko, Gupta, and Jain (1998) and Neslin (1990). Endogeneity correction is less helpful in our prediction-focused, limited-data setting.

Next, we discuss the customer data that non-contractual firms make available, which will motivate the model we specify thereafter for customer acquisition, ordering, and spend.

Firm Disclosures

At subscription-based firms, the most commonly disclosed customer measures are (1) the number of customers added each quarter and (2) the number of customers at the end of each quarter (McCarthy, Fader, and Hardie 2017).[3] While non-contractual firms can also observe (and thus disclose) customers added, they cannot disclose ending customers because customer attrition is not observable. Instead, non-contractual firms typically disclose the number of "active" users, or the number of customers who place at least one order over a preceding window of time. In general, non-contractual firms may disclose different metrics summarizing the number of customers, the counts of their activities, and the time frame over which the metrics are defined. While the framework can be adapted to any set of metrics, we focus in this article upon the common set of metrics that our two publicly traded empirical examples, Overstock and Wayfair, regularly disclose.

Both Overstock and Wayfair disclose quarterly customers acquired (QADD_q in quarter q). In addition to QADD_q, we consider the number of quarterly and annually active customers (QAU_q and AAU_q), which are equal to the number of customers who have placed at least one order within the past three and 12 months, respectively. Overstock and Wayfair each report one but not the other. Finally, we consider the total number of orders that the customers place each quarter (QTO_q).

In addition to customer disclosures, firms also provide financial disclosures. To the best of our knowledge, revenue, the most important financial disclosure, is the only figure which publicly traded firms disclose that identifies how customers spend when they place orders. Because the parameters associated with all processes are estimated jointly, revenue data also statistically identify the acquisition and repeat purchase processes. This is particularly true for firms that suffer from left censored customer data. At Wayfair, for example, revenue data alone are available for the first 39 quarters of commercial operations before the firm began disclosing other customer data. We let QREV_q and AREV_q denote the total revenue generated by the firm over the past three and 12 months, respectively, at the end of quarter q. We do not model product returns because public companies in general (and Overstock and Wayfair in particular) do not disclose this data.

While we focus upon this set of six common metrics which Overstock and Wayfair disclose – QADD_q, QAU_q, AAU_q, QTO_q, QREV_q, and AREV_q – the estimation method we will propose can accommodate other measures that firms may choose to disclose.

We use this data to estimate an underlying model for how customers are acquired, make repeat orders, and spend. We specify these three models over the next three sections.

The Acquisition Process

Our proposed model for the timing of customer adoption consists of three parts: (1) the formation of "pools of prospects" (i.e., those who *may* be acquired in the future) over time, (2) the conversion of prospects into "intenders" (i.e., those who *will* eventually be acquired), and (3) the duration of time that elapses from the time a prospect becomes an intender to when the intender is acquired. As in McCarthy, Fader, and Hardie (2017), we drive the creation of prospect pools over time off of the population size. At the beginning of the firm's commercial operations, there is an initial prospect pool $\mathrm{M}(0)$ which is equal to the population size at the time, $\mathrm{POP}(0)$ (e.g., the total number of US households at the time of incorporation). Individuals within this prospect pool may or may not adopt in future weeks $w = 1, 2$, and so on. The size of the prospect pool in a given week w is equal to population growth during the week:

$$\mathrm{M}(w) = \mathrm{POP}(w) - \mathrm{POP}(w-1), \quad w = 1, 2, \ldots \tag{5}$$

The total number of customers acquired within a particular week w is equal to the number of customers acquired that week across all preceding prospect pools:

$$\mathrm{A}(w) = \sum_{i=0}^{w-1} \mathrm{M}(i) \times [F_A(w-i|i) - F_A(w-i-1|i)], \tag{6}$$

where $F_A(w-i|i)$ is the probability that an individual from prospect pool i becomes interested and is subsequently acquired by the end of week w.

We model the duration of time until prospects are acquired through a mixture of hazard models. We characterize these processes differently depending upon whether the prospect pool was formed before or after the "time of mass awareness," which we denote by $w^\star$. Prospect pools formed before and after $w^\star$ are known as "early prospects" and "late prospects," respectively. The propensity with which prospects become acquired is governed by the following assumptions:

- At the time that their prospect pools are formed, a proportion, π_1, of early prospects are intenders. Intenders' times until acquisition are characterized by a proportional hazards model with a homogeneous Weibull(λ_1) baseline and coefficients associated with the acquisition covariates $\boldsymbol{\beta}_A$.[4]

- Early prospects who are not intenders at the time their prospect pool was first formed have zero probability of being acquired before $w^\star$.

- In week $w^\star$, a proportion π_2 of the early prospects who were not intenders previously will become intenders. The acquisition timing of early prospects who become intenders in week $w^\star$ is governed by a Weibull distribution with a different λ_2 baseline but the same proportional hazards coefficients $\boldsymbol{\beta}_A$ for statistical identification.

- Early prospects who are not intenders after week $w^\star$ [i.e., $(1-\pi_1) \times (1-\pi_2)$ of early prospect pools] will never be acquired.
- At the time that their prospect pools are formed, a proportion, π_2, of late prospects are intenders, with times until acquisition governed by a homogeneous Weibull(λ_2) baseline with covariates incorporated through proportional hazards.
- Late prospects who are not intenders at the time their prospect pool was first formed will never be acquired.

Given a prospect's homogeneous baseline propensities to be acquired (λ_1 and λ_2), their corresponding homogeneous acquisition shape parameters (c_1 and c_2), time-varying acquisition covariates ($\mathbf{X}_A(w+1, w') = [\mathbf{x}_A(w+1), \mathbf{x}_A(w+2), \ldots, \mathbf{x}_A(w')]$), and the coefficients associated with those acquisition covariates ($\boldsymbol{\beta}_A$), the probability that an individual from prospect pool w is acquired by the end of week w' is equal to

$$\mathrm{F}_A[w'-w|w, \mathbf{X}_A(w+1,w'); w^\star, \pi_1, \pi_2, \lambda_1, \lambda_2, c_1, c_2, \boldsymbol{\beta}_A] \tag{7}$$
$$= \begin{cases} \pi_1 \left(1 - e^{-\lambda_1 B_1(w,w')}\right), & w < w^\star \text{ and } w' \leq w^\star, \\ \pi_1 \left(1 - e^{-\lambda_1 B_1(w,w')}\right) + (1-\pi_1)\pi_2 \left(1 - e^{-\lambda_2 B_2(w^\star,w')}\right), & w < w^\star \text{ and } w' > w^\star, \\ \pi_2 \left(1 - e^{-\lambda_2 B_2(w,w')}\right) & \text{otherwise,} \end{cases}$$

where

$$B_n(w,w') = \sum_{i=w+1}^{w'} \left[(i-w)^{c_n} - (i-w-1)^{c_n}\right] e^{\boldsymbol{\beta}_A^T \mathbf{x}_A(i)}, \quad n \in \{1,2\}. \tag{8}$$

While the notion of product or service diffusion being driven by two customer segments with distinct start times is not a new one (Van den Bulte and Joshi 2007), the proposed model is the first to integrate this acquisition process with processes for repeat orders and spend per order across multiple cohorts and then calibrate the resulting joint model to empirical data.

There is strong precedent for this approach in extant marketing science literature. The model is akin to Moe and Fader (2002)'s forecasting model for new album sales, which allowed for two classes of prospects: those who place advance orders and those who wait for the "mass market" to emerge. It is consistent with the concept of a firm's sales "takeoff" (Golder and Tellis 1997) being driven by a sharp increase in newly acquired customers, which is preceded by a corresponding sudden expansion in the number of prospects who become interested in the firm. The model can exhibit a dip or "chasm" between the early and late parts of the diffusion curve (Moore 1991).

At both Overstock and Wayfair, we observe clear inflection points in the trajectory with which new customers are acquired that are not well captured by a single Weibull distribution or a mixture of Weibull distributions (Fader et al. 2003), Schweidel, Fader, and Bradlow 2008). Our proposed two-phase Weibull model allows for slower growth than a one-phase Weibull model can accommodate near the beginning of commercial operations at Overstock. Conversely, it allows for faster early growth at Wayfair. There is also evidence of a similar inflection point in customer acquisitions at Sirius XM in McCarthy, Fader, and Hardie (2017), but the authors did not pursue this model component.

The Orders Process

Next, we propose a model for repeat order timing. A customer's relationship with the firm is assumed to have two phases: he/she is "alive" for some period of time, then churns (i.e., becomes permanently inactive). While alive, the number of orders that he/she places in week w, $o(w)$, follows a Poisson process with intensity $\lambda_O(w)$. $\lambda_O(w)$ may vary due to unobserved heterogeneity, or due to external factors such as the state of the macroeconomy or seasonality. We allow for both effects through a log-normal formulation:

$$\lambda_O(w) = \exp[b_O + \boldsymbol{\beta}_O^T \mathbf{x}_o(w)], \tag{9}$$

where the baseline purchase intensity b_O is distributed across the population according to a normal(μ_O, σ_O^2) distribution, $\mathbf{x}_O(w)$ are order covariates associated with week w, and $\boldsymbol{\beta}_O$ represents the coefficients associated with those covariates.

Each week, the customer churns with probability θ. Under these assumptions, the probability that an individual customer acquired in week w places $o(w')$ orders in week w' is

$$\mathrm{P}_O\left[o(w')|w;\theta,\lambda_O(w')\right] = \frac{\exp\left[-\lambda_O(w')\right]\lambda_O(w')^{o(w')}}{o(w')!}(1-\theta)^{w'-w}. \tag{10}$$

We let θ vary across the population according to a beta(γ, δ) distribution, independent of the purchase process. While we could allow for customers' ordering behavior to change after the "time of mass awareness," neither of the empirical examples we will study showed any evidence of needing to do so.

This process, in combination with the acquisitions process, provide us with quarterly model-based estimates for QAU, AAU, and QTO. Going forward, we will refer to this repeat purchase model as the Beta-Geometric/Mixed-Lognormal model. The fits and forecasts of this model would be for all intents and purposes the same as the Pareto-NBD model (Schmittlein, Morrison, and Colombo 1987), if the Pareto-NBD model were extended to allow for time-varying covariates.

Stochastic repeat purchase models such as this one have proven effective and robust, parsimoniously providing accurate aggregate fits and forecasts for non-contractual customer behavior across many applications (Braun, Schweidel, and Stein 2015, Fader et al. 2010, Platzer and Reutterer 2016, Schweidel and Knox 2013). Parsimony is particularly important in our setting because of the limited nature of our available data. Our model uses censored data summaries across multiple cohorts, while all of the aforementioned papers used granular transaction-level data for individual cohorts.

Average Revenue per Order

If suitably rich data were available, we could consider a model for spend per order that explicitly allows for variation in expected spend across customers (e.g., the so-called "gamma-

gamma" model of Fader, Hardie, and Lee 2005), and over time. In practice, we have not found any company, contractual or non-contractual, that discloses the metrics that would enable a modeler to statistically identify heterogeneity in expected spend across customers. The only spend-related metric that firms (including Overstock and Wayfair) disclose are total revenue figures. For this reason, we use a simple time-series regression model to project average revenue per order (ARPO), similar in spirit to what was done in McCarthy, Fader, and Hardie (2017). We model weekly ARPO as a function of an intercept, a linear time trend, and weekly time varying covariates $\mathbf{x}_S(w)$:

$$\text{ARPO}(w) = \beta_{S,0} + \beta_{S,w} \times w + \boldsymbol{\beta}_S^T \mathbf{x}_S(w) + \epsilon(w), \quad \mathbb{E}\left[\epsilon(w)\right] = 0. \tag{11}$$

Because we cannot explicitly model heterogeneity in spend amounts across customers, we cannot estimate the distribution of CLV across customers. However, the proposed ARPO model can, in conjunction with the acquisition and repeat order processes, provide us with model-based estimates of QREV and AREV, allowing us to estimate the overall valuation of the firm without bias. An ARPO model for spend also allows us to estimate the expected unit economics of newly acquired customers. These outputs are likely the most valuable outputs of a customer-based valuation model.

In the next section, we describe how to estimate the parameters of the three sub-models summarized above.

Parameter Estimation

After deriving efficient expressions for model-based estimates of the available data (see Appendix A for these derivations) – $\widehat{\text{QADD}}_q$, $\widehat{\text{QAU}}_q$, $\widehat{\text{AAU}}_q$, $\widehat{\text{QTO}}_q$, $\widehat{\text{QREV}}_q$, and $\widehat{\text{AREV}}_q$ – it may seem natural to estimate the parameters of the acquisition, repeat order and average revenue per order processes $(\lambda_1, \lambda_2, c_1, c_2, \pi_1, \pi_2, \boldsymbol{\beta}_A, w^\star, \mu_O, \sigma_O^2, \boldsymbol{\beta}_O, \gamma, \delta, \beta_{S,0}, \beta_{S,w}, \boldsymbol{\beta}_S)$ using nonlinear least squares (Srinivasan and Mason 1986), estimating the parameters that minimize the sum-of-squared errors (SSE):

$$\begin{aligned}\mathrm{SSE} = &\sum_{q=1}^{Q_{\mathrm{QADD}}} (\mathrm{QADD}_{(q)} - \widehat{\mathrm{QADD}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{QAU}}} (\mathrm{QAU}_{(q)} - \widehat{\mathrm{QAU}}_{(q)})^2 \\ &+ \sum_{q=1}^{Q_{\mathrm{AAU}}} (\mathrm{AAU}_{(q)} - \widehat{\mathrm{AAU}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{QTO}}} (\mathrm{QTO}_{(q)} - \widehat{\mathrm{QTO}}_{(q)})^2 \\ &+ \sum_{q=1}^{Q_{\mathrm{QREV}}} (\mathrm{QREV}_{(q)} - \widehat{\mathrm{QREV}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{AREV}}} (\mathrm{AREV}_{(q)} - \widehat{\mathrm{AREV}}_{(q)})^2,\end{aligned} \tag{12}$$

where Q_{C} is the number of times that metric C is disclosed, and $\mathrm{C}_{(q)}$ is the q^{th} disclosure of metric C, recognizing that an "empty sum" $\sum_{i=1}^{0} a_i$ is equal to zero.

While this procedure would provide us with asymptotically unbiased estimates of the model parameters, it would be highly inefficient because the available data operate on very different scales. For example, because the average amount spent on each order at Overstock and Wayfair is over \$180, QREV figures are 200 to 600 times larger than QAU and QADD figures, causing the procedure to heavily overweight revenue data relative to all other data. Therefore, we pre-standardize each measure to have mean zero and unit variance before applying the nonlinear least squares estimation procedure. Data pre-standardization is a very common and important pre-processing step in data mining and machine learning (Al Shalabi, Shaaban, and Kasasbeh 2006, Mohamad and Usman 2013).

We estimate the parameters that minimize the following pre-standardized sum-of-squared errors (PSSE):

$$\begin{aligned}\mathrm{PSSE} = &\sum_{q=1}^{Q_{\mathrm{QADD}}} (\widetilde{\mathrm{QADD}}_{(q)} - \widehat{\widetilde{\mathrm{QADD}}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{QAU}}} (\widetilde{\mathrm{QAU}}_{(q)} - \widehat{\widetilde{\mathrm{QAU}}}_{(q)})^2 \\ &+ \sum_{q=1}^{Q_{\mathrm{AAU}}} (\widetilde{\mathrm{AAU}}_{(q)} - \widehat{\widetilde{\mathrm{AAU}}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{QTO}}} (\widetilde{\mathrm{QTO}}_{(q)} - \widehat{\widetilde{\mathrm{QTO}}}_{(q)})^2 \\ &+ \sum_{q=1}^{Q_{\mathrm{QREV}}} (\widetilde{\mathrm{QREV}}_{(q)} - \widehat{\widetilde{\mathrm{QREV}}}_{(q)})^2 + \sum_{q=1}^{Q_{\mathrm{AREV}}} (\widetilde{\mathrm{AREV}}_{(q)} - \widehat{\widetilde{\mathrm{AREV}}}_{(q)})^2,\end{aligned} \tag{13}$$

where

$$\widetilde{\mathrm{C}}_{(q)} = \frac{\mathrm{C}_{(q)} - m_{\mathrm{C}}}{s_{\mathrm{C}}} \quad \text{and} \quad \widehat{\widetilde{\mathrm{C}}}_{(q)} = \frac{\widehat{\mathrm{C}}_{(q)} - m_{\mathrm{C}}}{s_{\mathrm{C}}}$$

and m_{C} and s_{C} are plug-in estimates of the mean and standard deviation of metric C:

$$m_{\mathrm{C}} = \sum_{q=1}^{Q_{\mathrm{C}}} \mathrm{C}_{(q)}/Q_{\mathrm{C}} \quad \text{and} \quad s_{\mathrm{C}} = \sqrt{\frac{1}{Q_{\mathrm{C}}-1}\sum_{q=1}^{Q_{\mathrm{C}}}(\mathrm{C}_{(q)} - m_{\mathrm{C}})^2}.$$

This automatically adjusts for the different scales upon which each metric may be measured. The estimation procedure can easily be extended if other metrics were available, as we would simply pre-standardize those metrics and add them to the PSSE expression in Equation 13.[5]

Valuation Procedure

As noted in the Valuation Framework section, we value the firm by forecasting QREV into the future, using Equations 3 and 4 to obtain free cash flow projections, then plugging these free cash flow projections into Equation 2 to value the firm's operating assets. Our procedure for forecasting $QREV$ comes as a direct result of the aforementioned estimation procedure – instead of estimating $QREV$ over the Q-quarter calibration period, we extend it an additional $Q^\star$ quarters, far enough into the future that the net present value of all future cash flows is effectively zero (we use $Q^\star = 200$ in the empirical analyses below, corresponding to a 50-year forecasting horizon). After we have forecasted long term revenues, the rest of the valuation model is effectively the same as a typical discounted cash flow valuation model.

Our forecasting horizon is long, which makes the forecast uncertain. At the same time, long forecasting horizons are an unavoidable reality of financial valuation. Whenever a financial professional builds a discounted cash flow model, he/she makes projections over an infinite horizon. This long horizon is usually cloaked through an arbitrary "terminal value" calculation (Courteau, Kao, and Richardson 2001).

While we cannot avoid long-range forecasts, we should account for the fact that they are uncertain. We do so by constructing a valuation distribution which explicitly incorporates the degree of uncertainty in the model forecasts and thus the resulting overall firm valuations. Companies with more uncertain future prospects will have more diffuse valuation distributions, lessening the significance of observed stock price departures from fair value. In turn, this can influence the relative attractiveness of a potential investment (Sharpe 1964).

To create the valuation distribution, we draw a new set of data by bootstrap resampling the residuals of the original fitted model (Efron and Tibshirani 1994). We estimate the model parameters associated with the bootstrapped data, which we use to obtain projections of the customer metrics and the resulting valuation of the firm. Note too that the magnitude of the valuation uncertainty may not be as large as it may seem, because the time value of money decreases the valuation impact of longer-range projections (for example, the discount factor associated with a 10% WACC 20 years into the future is less than 0.15).

We illustrate the entire valuation procedure for two publicly traded companies next. All of the steps that we take to estimate valuation components other than quarterly revenues are listed out in detail in Web Appenices A and B.

EMPIRICAL ANALYSIS

We apply the proposed customer-based valuation methodology to data from Overstock (Nasdaq: OSTK), an e-commerce retailer selling a wide assortment of products. We validate the model's fit, compare the results to alternative methodologies, estimate its valuation, then obtain a valuation distribution which accounts for uncertainty in the fitted model parameters. Next, we apply the methodology to data from Wayfair (NYSE: W), a large internet-based home goods seller. Wayfair is complementary to Overstock for four reasons:

1. Wayfair is a high-growth business while Overstock is more mature. For example, while Overstock's quarterly revenues grew by 5% from Q1 2016 to Q1 2017, Wayfair grew by 29% over the same period.

2. Wayfair has more limited data than Overstock. Wayfair did not disclose any metrics for the first six quarters of its commercial operations, creating severe left censoring. Over the next 33 quarters, Wayfair disclosed one metric, AREV, only once per year in the fourth quarter. Wayfair only began disclosing a complete set of metrics in Q1 2013, its 44th quarter of commercial operations.

3. Wayfair discloses different data from Overstock, providing AREV in addition to QREV, and AAU instead of QAU. These may seem like minor distractions, but we would not

want to ignore AREV disclosures when our data is so limited, and treating AAU and QAU interchangeably would strongly bias the resulting parameter estimates.

4. While both Overstock and Wayfair are e-commerce retailers, Wayfair has a narrower product assortment, almost entirely focused on home furnishings as opposed to a much broader array of items, including furniture, that Overstock carries. Net revenues per order were \$166 and \$228 at Overstock and Wayfair in Q1 2017, respectively.

We make Overstock and Wayfair's publicly disclosed data available in Web Appendix C.

Overstock.com

While Overstock began commercial operations in Q1 1999, it began disclosing QADD, QREV, and QTO two years later in Q1 2001, and began disclosing QAU in Q3 2004. Overstock stopped disclosing QADD in Q1 2016, but has continued to disclose all other metrics. We model these data up to and including Q1 2017, so $Q = 73$.

Overstock's revenues come from US customers, so our unit of population is the US labor force. The labor force is a proxy for the number of individuals who have the means and perhaps, the need to pay for Overstock products. The Bureau of Labor Statistics' Current Population Survey discloses the US labor force monthly. We include just one time-varying covariate to account for seasonality in the acquisition, repeat order and spend processes: a dummy variable which is equal to one in calendar Q4 and zero otherwise (the coefficients corresponding to this covariate across the three processes are denoted by $\beta_{A,Q4}$, $\beta_{O,Q4}$, and $\beta_{S,Q4}$ below).

Parameter Estimates and Model Fit

As noted in the Parameter Estimation section, we estimate the parameters of the acquisition, repeat order, and ARPO processes using nonlinear least squares off of the pre-standardized data, finding the set of parameters that jointly minimize PSSE in Equation 13. The top half of Table 1 contains the parameter estimates along with their standard errors, and the "time of mass awareness" $w^\star$ is estimated to be 306, corresponding to Q2 2003. For exposition, we report the results assuming a quarterly unit of time for both Overstock and Wayfair.[6]

While $w^\star$ is estimated from the data and not through a subjective assessment of the business, its value is consistent with a major transition in Overstock's business model. 2003 was the first year that Overstock dramatically changed its company strategy from being both a business-to-business and a business-to-consumer company to becoming almost entirely a mass-market business-to-consumer company. For example, Overstock's business-to-business activity with Safeway alone fell from 16% of sales in 2002 to less than 1% by 2004, and Overstock's year on year percentage increase in marketing spend were 50%, 133%, and 101% in 2002, 2003, and 2004, respectively. This pivot in strategy brought with it a large influx of "intenders" – whereas only 4.4% of early prospects were intenders when they first became prospects, nearly 100% of them became intenders after $w^\star$. The effect of Q4 seasonality is very strong. We can see that Overstock acquires significantly more new customers and that existing customers place more orders during the US holiday season, but the amount that is spent on orders during the holiday season is significantly lower.

Table 1: Parameter Estimates

Overstock.com

	Acquisition			Repeat Order			ARPO	
	Est	*SE*		*Est*	*SE*		*Est*	*SE*
λ_1	.04425	.01826	μ_O	-7.97451	5.09510	$\beta_{S,0}$	55.10163	18.53716
λ_2	.00067	.00173	σ_O^2	.66638	12.87912	$\beta_{S,w}$	1.53010	.32636
c_1	3.17910	.95208	$\beta_{O,Q4}$	1.09160	.22328	$\beta_{S,Q4}$	-22.29522	5.22029
c_2	1.49161	.17929	γ	2.47301	90.68178			
π_1	.04425	.01826	δ	53.34601	83276.03001			
π_2	.99912	.13481						
$\beta_{A,Q4}$	.52584	.09883						

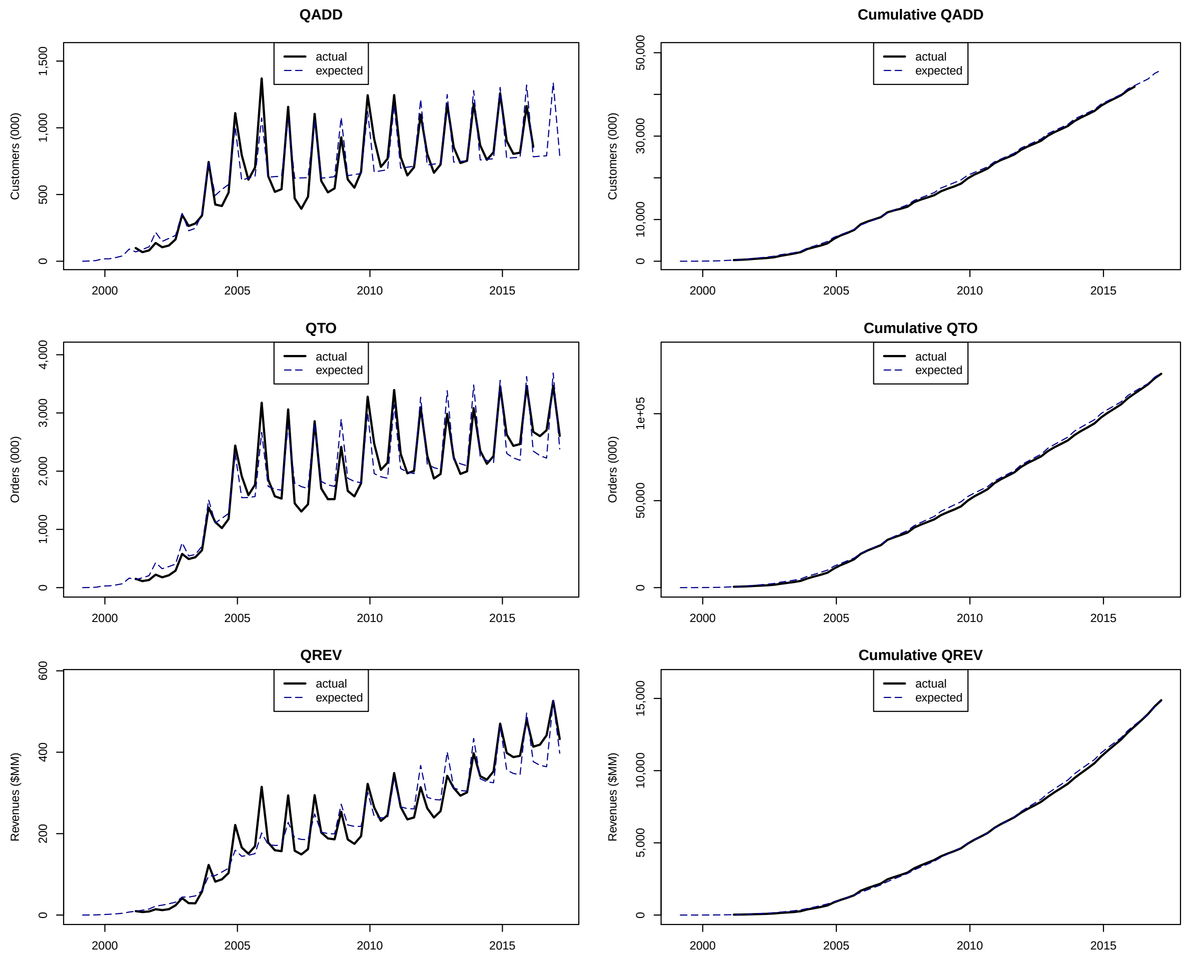
Wayfair

	Acquisition			Repeat Order			ARPO	
	Est	*SE*		*Est*	*SE*		*Est*	*SE*
λ_1	.00149	.00115	μ_O	-8.94484	.90564	$\beta_{S,0}$	17.85236	5.12480
λ_2	.00019	.00008	σ_O^2	2.10116	2.96532	$\beta_{S,w}$	3.81151	.14068
c_1	1.45165	.12742	γ	98.15855	19.94462	$\beta_{S,Q4}$	-7.58639	5.64184
c_2	2.29469	.12208	δ	4743.89763	1083.69253			
π_1	.10872	.10034						
π_2	.41178	.14483						
$\beta_{A,Q4}$	.27326	.03303						

In Figure 1 we plot estimated and actual QADD, QTO and QREV data over the calibration period in the top, middle, and bottom rows, respectively. In the leftmost column, we show the incremental quarter-by-quarter figures while in the rightmost column, we show the corresponding cumulative figures. The resulting model fit is good. While there is some variation in the observed data around our model-based estimates, the model does an adequate job of capturing the baseline trends. It may be tempting to add additional covariates to the model to capture every peak and trough, but doing so is unlikely to help, and may even hurt, model forecasts. The in-sample fit for Overstock's QAU data is as good as the fits shown here. We include it in Web Appendix D for space reasons, although it can also be seen in the top-right panel of Figure 2 as part of our model comparison analysis in the next section.

Strong seasonal variation is evident in these plots. We also see some evidence of a slowdown in new customer acquisitions, which may hamper Overstock's future sales growth.

Figure 1: Overstock: Quarterly Customer Additions, Total Orders, and Revenues

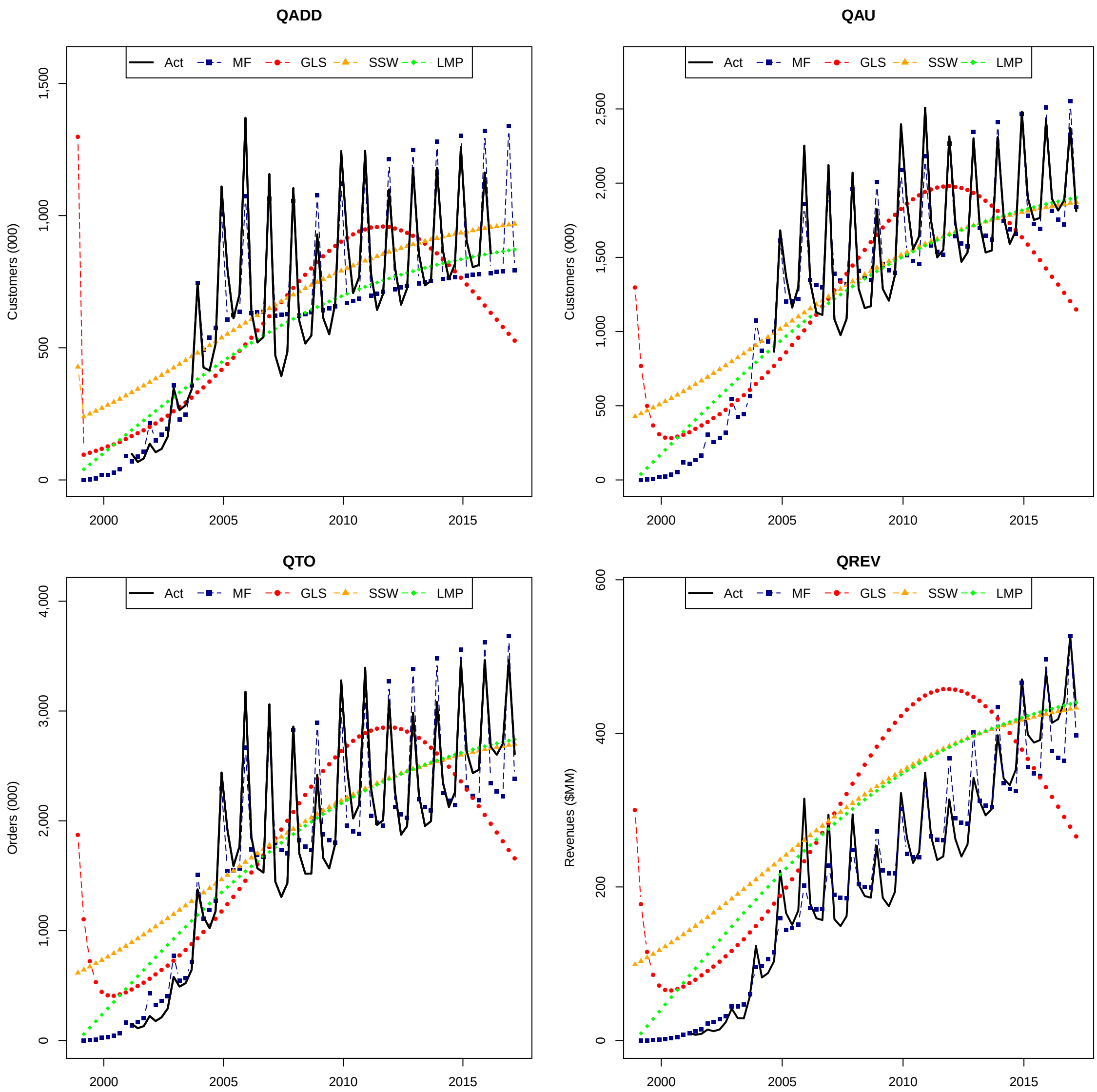


Model Comparison

We compare our proposed model's fit to that of three benchmark models: (1) Gupta, Lehmann, and Stuart (2004), (2) Schulze, Skiera, and Wiesel (2012), and (3) Libai, Muller, and Peres (2009). All benchmark models do not incorporate sub-models for repeat orders, opting instead to set revenue (or margin) per alive customer equal to the trailing four quarter average. To back out implied QTO estimates for these models while remaining consistent with their assumption for revenue per alive customer, we set future orders per alive customer and future revenue per order to be equal to their respective trailing four quarter averages. Both Libai, Muller, and Peres (2009) and Schulze, Skiera, and Wiesel (2012) model the total number of alive customers over time, so we follow their convention of assuming that the total number of alive customers is equal to the total active customer count (QAU for Overstock).

In Figure 2, we plot the model fits for Gupta, Lehmann, and Stuart (2004) (referred to in the figure as GLS), Schulze, Skiera, and Wiesel (2012) (SSW), Libai, Muller, and Peres (2009) (LMP), and the proposed model (MF). Gupta, Lehmann, and Stuart (2004) severely underestimates future acquisitions. This underestimation is primarily due to the inability of the Bass-like technology substitution model to model right-skewed data (unlike, for example, the Weibull distribution). By modeling QAU directly, Schulze, Skiera, and Wiesel (2012) and Libai, Muller, and Peres (2009) have better fits in general, particularly to QADD, QAU, and QTO. This result is consistent with the alternative comparisons analysis in McCarthy, Fader, and Hardie (2017). At the same time, Schulze, Skiera, and Wiesel (2012) and Libai, Muller, and Peres (2009) have difficulty capturing the slow ramp up of QAU at the beginning of Overstock's commercial operations, which we are able to accommodate through our proposed changepoint-like acquisition process. As a result, they overestimate QAU (and thus QADD) in the 2000-2005 period, then underestimate it in the 2006-2017 period. ARPO has been increasing over time while all three methods set spend per order equal to the average of the four most recent quarters, so they overestimate QREV earlier on but are likely to underestimate it in the future, because ARPO is likely to continue increasing as we move into the future. Our model explicitly allows for time trend and seasonal effects in ARPO, providing us with a better fit for the QREV data.

Figure 2: Overstock.com: Model Fit Comparison Against Benchmark Models



The number of estimated parameters for the proposed model, Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), and Libai, Muller, and Peres (2009) are 15, 6, 6, and 8, respectively. While the number of estimated parameters for all models is far less than the number of data points, at 242, the proposed model will naturally have a better in-sample fit because it has more estimated parameters. We turn next to a comparison of predictive accuracy in a holdout setting, which does not favor models with more parameters. This is a practically important test as well, because the quality of stock price estimates from a discounted cash flow valuation model is driven by the predictive accuracy of its revenue (and customer acquisition) projections. We assess predictive validity by performing a rolling holdout validation analysis in which we vary the calibration period, predict future customer metrics and then compare those predictions to the observed data for each model we consider. We consider calibration periods Q $= 34,\ 35,\ \ldots,\ 73$, corresponding to all possible calibration periods for which we have at least three years (12 quarters) of customer data. We train our model upon all data up to and including quarter Q, then predict QADD, QAU, QTO, and QREV over the next two years (i.e., Q+q^* for q^*=1, 2, . . ., 8). We summarize the predictive accuracy of our model by computing the mean absolute percentage error (MAPE) of our two-year predictions for each metric. After running this analysis for our proposed model, we repeat the exercise for the three benchmark models, Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), and Libai, Muller, and Peres (2009). We present the results in the top half of Table 2.

Table 2: MAPE of Rolling Two-Year Predictions of Customer Metrics

Company	Metric	GLS	SSW	LMP	Proposed
Overstock	QADD	67.6	18.8	18.0	11.8
	QAU	64.2	16.5	16.2	10.5
	QTO	61.9	17.0	14.5	15.3
	QREV	56.3	23.2	16.2	15.0
Wayfair	QADD	17.8	21.0	15.0	4.9
	AAU	19.9	3.0	2.7	2.1
	QTO	20.5	8.8	8.8	5.8
	QREV	18.9	5.9	5.6	4.9

The MAPE figures in Table 2 are consistent with the in-sample goodness of fit results. While the model fits for Schulze, Skiera, and Wiesel (2012) and Libai, Muller, and Peres (2009) are generally comparable to one another, Libai, Muller, and Peres (2009) is more accurate across all metrics. Gupta, Lehmann, and Stuart (2004) has the lowest predictive accuracy of all alternative models by a wide margin. Our proposed model has the highest predictive accuracy, with a MAPE that is, on average, approximately 80%, 30%, and 18% smaller than Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012) and Libai, Muller, and Peres (2009), respectively.

Valuation

Having established the in-sample and out-of-sample performance of our proposed model, we turn next to estimating Overstock's overall valuation. We begin by projecting QREV far into the future. We do by first forecasting the size of the US labor force. The annual average growth rate of the US labor force over the period from Q1 1999 to Q1 2017 was 0.8%, so we assume that the labor force will continue growing at this rate into the future.

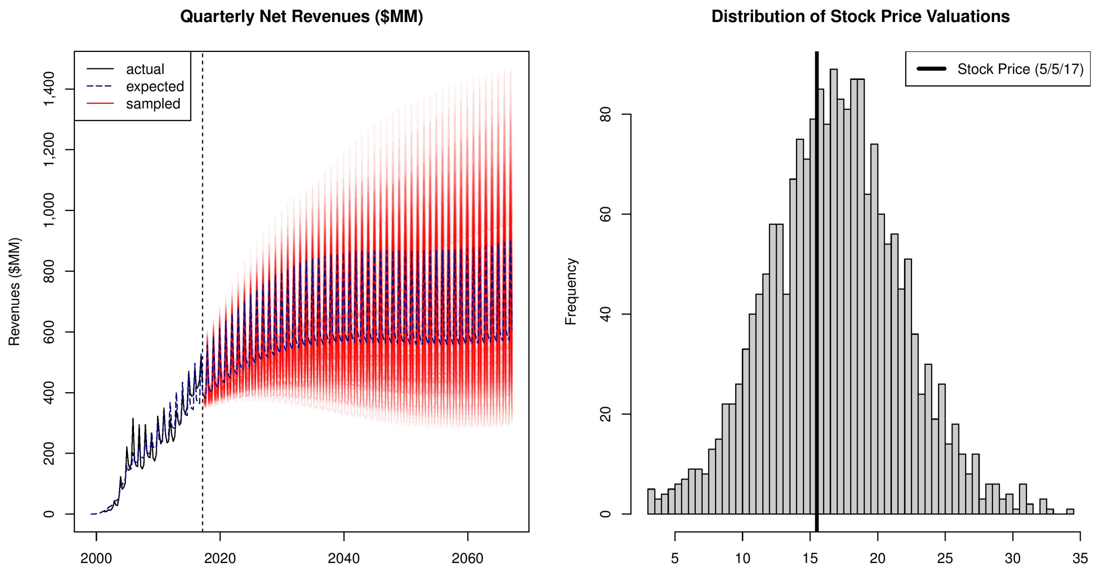
We drive a detailed discounted cash flow valuation model off of the forecasted QREV figures, which we use to estimate the other variables in Equations 1, 2, 3, and 4. Web Appendix B contains all of the assumptions driving non-QREV figures. We summarize the model-based estimates of the value of the operating assets, non-operating assets, net debt, and shareholder value (Equation 1), as well as the actual stock price at market close the day that Overstock released Q1 2017 results, in the top half of Table 3.

Table 3: Valuation Summaries (End of Q1 2017)

Overstock.com	
Value of Operating Assets	\$354.1MM
Non-Operating Assets - Net Debt	\$72.7MM
Shareholder Value	\$426.8MM
Shares Outstanding	25.3MM
Implied Stock Price	\$16.88
Actual Stock Price	\$15.50
Over(under)-estimation	8.9%
Wayfair	
Value of Operating Assets	\$825.4MM
Non-Operating Assets - Net Debt	\$55.5MM
Shareholder Value	\$880.8MM
Shares Outstanding	86.0MM
Implied Stock Price	\$10.24
Actual Stock Price	\$64.16
Over(under)-estimation	(84.0%)

We estimate a valuation for Overstock of \$16.88 per share, approximately 9% above Overstock's Q1 2017 stock price of \$15.50. The valuation estimates corresponding to Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), and Libai, Muller, and Peres (2009) are \$0, \$8.11, and \$10.28 per share, respectively. We then implement the procedure to construct Overstock's valuation distribution. In Figure 3, we plot the resulting sampled QREV forecasts (left panel) and valuation distribution (right panel) after repeating this procedure 2,500 times.

Figure 3: Overstock.com: QREV Forecasts (left) and Stock Price Distribution (right)



There is a fair and growing amount of uncertainty in our projections. For example, these sampled realizations of the data suggest that peak quarterly acquisitions may have already occurred, or may occur 18 years from now. Similarly, while we expect quarterly revenues five years into the future to be $483MM, our 95% interval for revenues suggests that it could be as low as $389MM or as high as $524MM. The resulting 95% valuation distribution implies that Overstock's fair value lies between $7.2 and $26.9 per share. The observed stock price of $15.50 falls upon the 38th percentile of the valuation distribution, only 12% away from the median and well within the 95% valuation interval. We conclude that the observed stock price is not practically or statistically significantly different from our fair-value estimate.

Wayfair

Next, we perform a customer-based valuation analysis for Wayfair, a large internet-based home goods retailer. As with Overstock, all of Wayfair's data were disclosed in its SEC filings and investor presentations. We use the same model specification for Wayfair that we used for Overstock, except that we remove the seasonal covariate for Q4 from the repeat order process. Overstock and Wayfair also have different population sizes. While Wayfair primarily sells into the US, it began selling into Canada and the UK in 2008, and into Germany in 2009. Therefore, the population size at Wayfair is equal to the US labor force prior to 2008, the sum total of the US, Canadian, and UK labor forces in 2008, and the sum total of the US, Canadian, UK, and German labor forces in and after 2009. Wayfair has not entered a new market in eight years and has not indicated that it intends to in coming years.

We estimate the parameters of the acquisition, repeat order, and ARPO processes using the PSSE equation (Equation 13). In the bottom half of Table 1, we provide the parameter estimates, where $w^\star$ is estimated to be 731, corresponding to Q4 2012. As with Overstock, while $w^\star$ estimated from the data, its value also makes intuitive sense given the evolution of the business. From 2002 to 2011, the company had been bootstrapped by its founders and operated as a collection of hundreds of niche websites. 2012 was the first year that the company made the strategic decision to close and permanently redirect over 240 of these niche websites into `wayfair.com`, making it a one-stop shop. The data would suggest that many prospects became intenders shortly after this change – while only 11% of early prospects were intenders at the time their prospect pool first formed, 41% of those remaining became intenders at the time of mass awareness.

Figure 4 contains the in-sample fits for QADD, QTO and QREV in the top, middle, and bottom rows, respectively.[7] As before, the leftmost column corresponds to the incremental quarter-by-quarter figures while the rightmost column corresponds to cumulative figures. We overplot black dots in the cumulative revenues plot (see the bottom-right panel) because Wayfair disclosed AREV figures once per year for 10 years before it began regularly disclosing QREV. The corresponding plot of expected quarter-by-quarter QREV (i.e., the blue dotted line in the bottom-left panel) belies a fundamentally different customer acquisition trajectory before the time of mass awareness versus after it, and cannot be adequately modeled with a single Weibull distribution. The model provides adequate fits for all metrics, including AAU, included in Web Appendix D for space reasons.

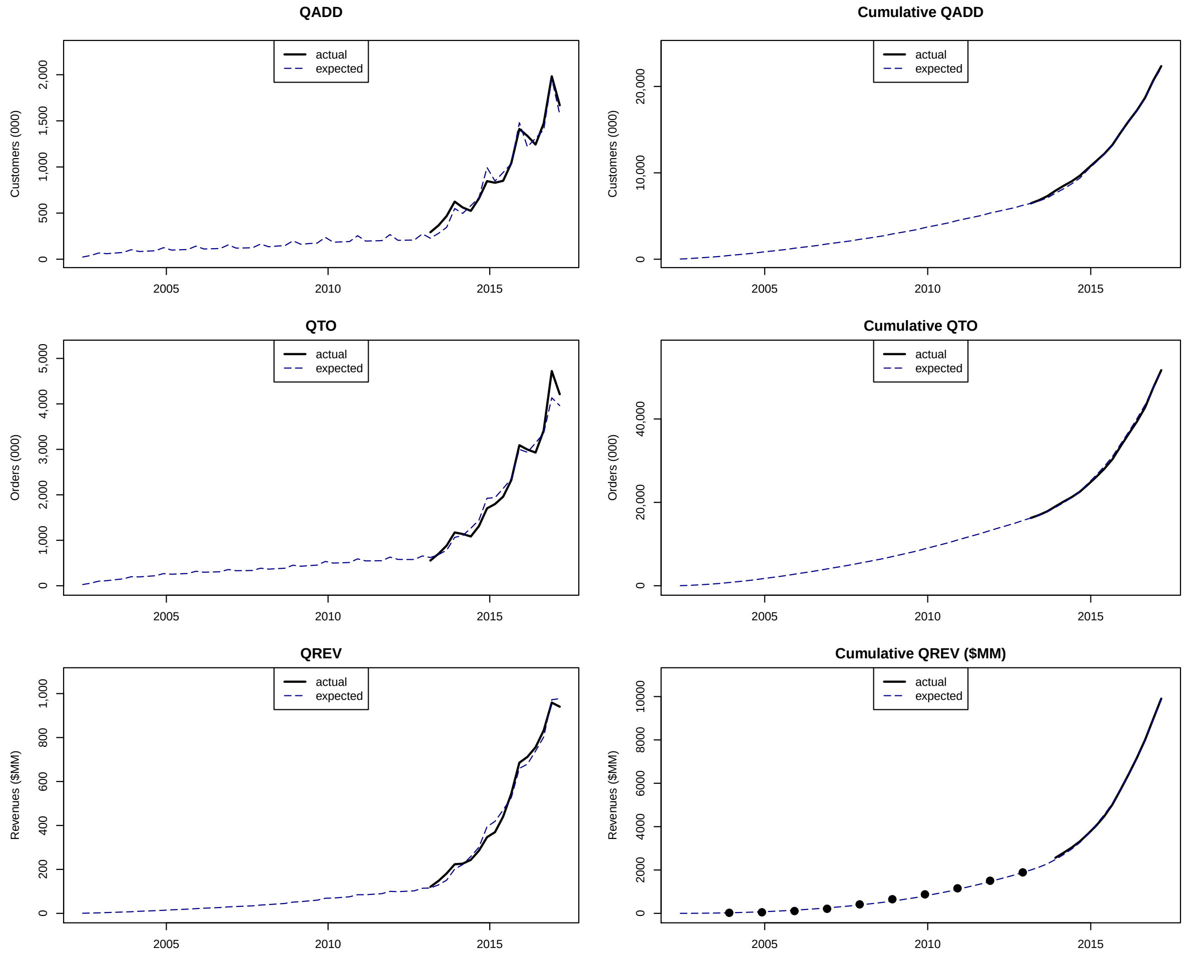
To provide insight into the predictive validity of the proposed model, we repeat the two-year rolling holdout validation analysis that we had performed for Overstock. As with Overstock, we consider all possible calibration periods for which we have at least three years (12 quarters) of data upon which to calibrate our model, so that Q =55, 56, . . ., 60. We then compare the MAPE of our proposed model's two-year predictions to the corresponding predictions for Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), Libai, Muller, and Peres (2009). The bottom half of Table 2 summarizes the results. While the rank ordering of the average accuracy of the methods is the same for Wayfair as it was for Overstock, the relative outperformance of the proposed method is more significant. The average holdout MAPE of our proposed method is approximately 80%, 40%, and 35% smaller than that of Gupta, Lehmann, and Stuart (2004), Schulze, Skiera, and Wiesel (2012), Libai, Muller, and Peres (2009), respectively.

While there is an intuitive appeal to the notion that our model would predict better for more mature companies than for companies earlier on in their lifecycle, the results from the rolling validation analysis do not suggest that this is true. Holdout prediction accuracy is dramatically higher at Wayfair than at Overstock. Moreover, when we segment Overstock model predictions into three groups based upon the length of the calibration period, prediction accuracy is best for the mid-length calibration periods. That being said, our results are based on only two companies. A proper meta-analysis would be required to evaluate this theory more conclusively.

We project revenues over the next 50 years to drive our model for Wayfair's overall valuation. We assume the long-term growth rate of the labor forces in Canada, the UK, and Germany are equal to their historical averages of 1.2%, 0.9%, and 0.4%, respectively. For a detailed account of the assumptions underlying this valuation, see Web Appendix A. The bottom half of Table 3 summarizes the resulting valuation. We estimate a fair valuation for Wayfair of $10.24 per share, substantially below its then-current stock price of $64.16. This overvaluation is very robust to departures from our base case assumptions for key non-revenue projections, such as Wayfair's WACC and future gross margin percentage.

We then obtain a valuation distribution for Wayfair using the same bootstrapping procedure that we used for Overstock. As with Overstock, we generate 2,500 bootstrapped

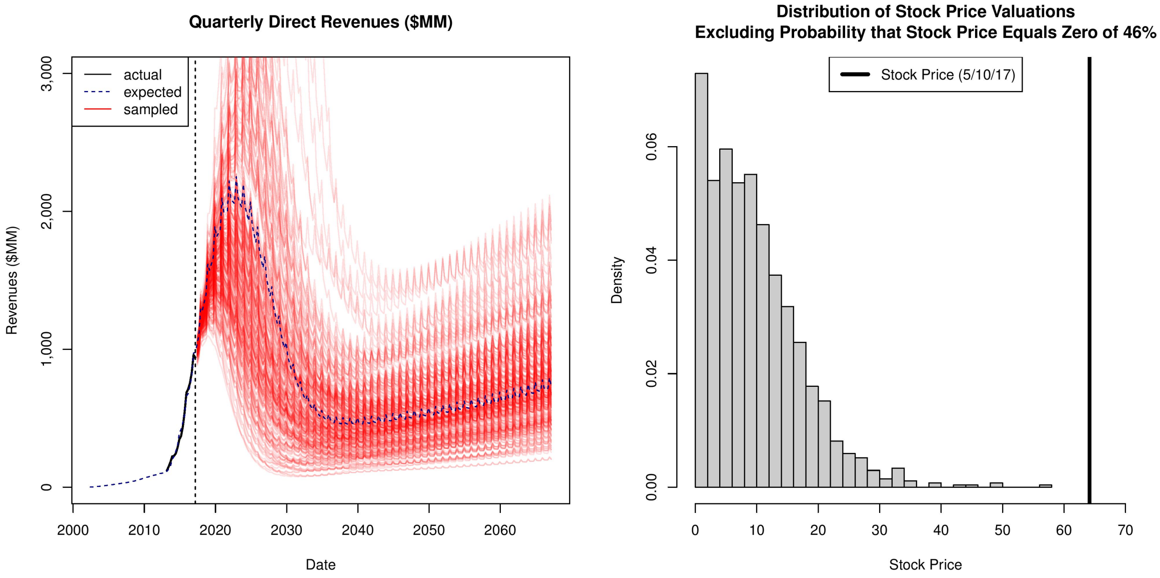
Figure 4: Wayfair: Quarterly Customer Additions, Total Orders, and Revenues



realizations of revenues. The leftmost panel of Figure 5 shows tThe corresponding bootstrapped revenue forecasts. While our baseline expectation is that quarterly revenues will peak at $2.3B in Q4 2022, more than double its current level, there are bootstrapped realizations of peak QREV exceeding $12B. Even in this most optimistic revenue scenario, however, the resulting fair valuation for Wayfair's stock is $57.03, which is still 11.1% below Wayfair's observed stock price. Additionally, these results are robust to assumptions regarding Wayfair's future margins, which we assume will rise to the midpoint of long-term margin goals that Wayfair management has provided on recent conference calls within five years. They are also robust to assumptions regarding Wayfair's discount rate, which we assume is equal to Bloomberg's estimated Q1 2017 WACC figure.

These results are consistent with the polarized market sentiment surrounding Wayfair.

Figure 5: Wayfair: QREV Forecasts (left) and Stock Price Distribution (right)



One the one hand, many market participants are very optimistic about the company's future prospects. For example, Patrick McKeever, managing director at MKM Partners, believes the company is undervalued, stating that Wayfair's most important key performance indicators are all "moving up and to the right" (McKeever 2018, January 24). John Blackledge, senior internet analyst at Cowen, also believes that Wayfair is undervalued and is "well positioned to reap the benefits from major investments the company has made" (Blackledge et al. 2018, January 11). On the other hand, a number of hedge funds and other investment professionals have publicly expressed skepticism about Wayfair's valuation. For example, Citron Research's Andrew Left said "Wayfair is a throwback to 1999, a business where there's never EBIDTA, just cumulative losses" (Left 2017). Marketing experts and media outlets such as the Wall Street Journal have conveyed similar sentiments (Winkler 2018, February 22). Scott Galloway, in his well-followed predictions for 2018, foresees "online furniture Wayfair shares crash" due to "weak customer loyalty" (Galloway 2017). At the time of this analysis, Wayfair's stock is the fifth most shorted in the Internet Retail sector (Dusaniwsky 2017), with over 40% of its tradeable shares sold short (i.e., held by investors who will earn a profit if the stock falls below the price they shorted the stock at). The availability of shares to short tightened to the point that investors must pay a 6% fee to borrow the shares needed to establish their short position. These data points suggest considerable disagreement among market participants about the future direction of Wayfair's stock price.

Our valuation model also suggests considerable disagreement in future valuations "across worlds." For example, the standard deviation of the percentage difference between the bootstrapped valuations and our expected one is 72% for Wayfair, versus 28% for Overstock. However, all future stock price realizations lie below Wayfair's observed stock price.

Unit Economics Analysis

In addition to providing point estimates and intervals for firm valuation, our model can also go beneath surface-level metrics such as revenues to better understand the underlying unit economics of a business. One quantity of managerial interest is the expected cash flows associated with a newly acquired customer – both the initial spend that the firm incurs to acquire the customer, and the stream of future marginal cash flows associated with the customer thereafter. Accounting for the time value of money properly, summing these quantities provides us with the expected CLV of newly acquired customers. Successful businesses are able to acquire many high CLV customers. While businesses that acquire low (or negative) CLV customers may nevertheless report strong revenue growth if they grow customer acquisition expenses quickly enough, they will be very reliant upon a continued high rate of new customer acquisition and will have greater difficulty growing themselves into profitability.

In this section, we compare the unit economics of Overstock to Wayfair. To do so, we must estimate (1) the average customer acquisition cost per newly acquired customer (CAC), and (2) the expected stream of marginal profits per customer after acquisition:

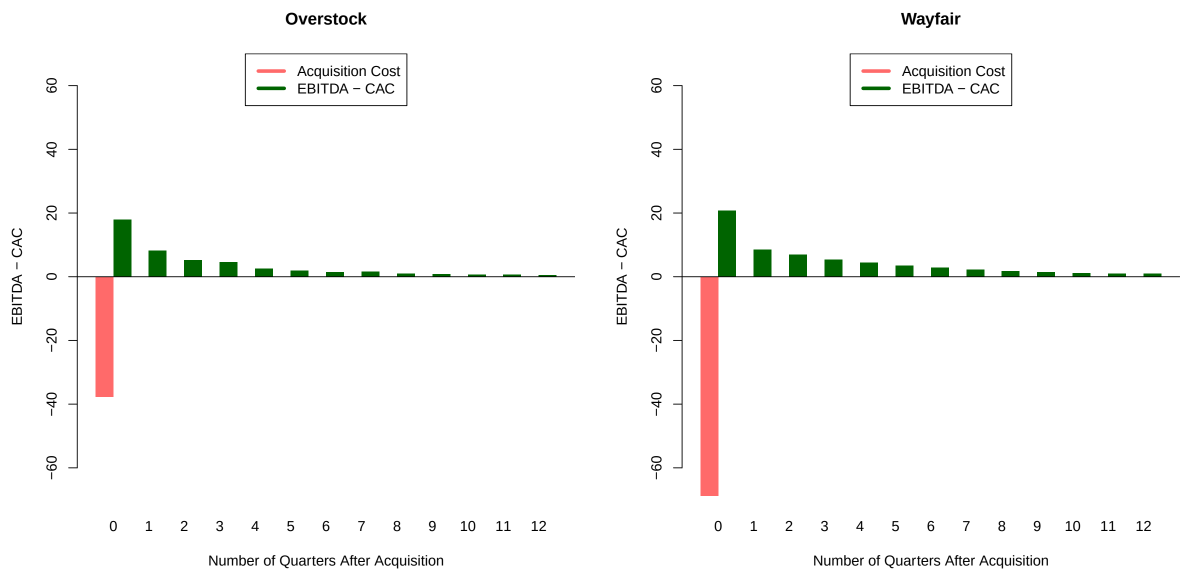
- We assume that CAC is equal to the trailing 12 month ratio of total advertising expense to the expected total number of customers acquired, which is consistent with statements made by both Overstock (Overstock 2017) and Wayfair (Wayfair 2017), as well as prior academic literature (Gupta et al. 2004). This also ensures that our accounting is consistent across both companies, because Overstock and Wayfair both disclose advertising expense each quarter in their financial statements.

- As in the expected residual lifetime value calculation of McCarthy, Fader, and Hardie

(2017), we operationalize marginal profit after the customer has been acquired to be equal to earnings before interest, taxes, depreciation, amortization, and acquisition expenses, which we call "EBITDA−CAC." Our model provides these cash flows by taking the expected QREV each quarter associated with one customer acquired at the end of Q1 2017, then multiplying these future expected QREV figures by the firm's overall ratio of expected EBITDA−CAC to total revenues.

In Figure 6, we present the resulting comparison of expected cash flows over the next three years at the two firms. We assume that CAC is incurred at the time the customer is acquired (i.e., zero quarters after acquisition) while the customer's initial purchase occurs immediately after the customer is acquired (i.e., in the first quarter after acquisition).

Figure 6: Expected Cash Flows of a Customer Acquired in Q1 2017



Wayfair customers generate more profits than Overstock customers after acquisition – the net present value of future profits after acquisition are \$59 and \$47 per customer at Wayfair and Overstock, respectively. However, Wayfair spends far more than Overstock to acquire new customers. Wayfair's CAC is \$69, nearly double Overstock's \$38. While one could argue that some of Wayfair's advertising expenses are earmarked for customer retention, the proportion is likely to be small (Thomas 2001), and because Overstock is a relatively more mature business, its corresponding proportion is likely larger than Wayfair's.

If Wayfair were able to reduce its CAC to Overstock's level, we estimate that Wayfair's expected valuation would more than double, all else being equal. Of course, all else is not equal – Overstock is pursuing a more selective customer acquisition strategy, avoiding the much higher acquisition costs that Wayfair has been willing to incur. We estimate that Overstock earns approximately \$9 per acquired customer, while Wayfair incurs a loss of approximately \$10 per customer in Q1 2017. While we anticipate that the unit economics of Wayfair's newly acquired customers will improve in the future as their variable contribution margin is expected to expand, challenging unit economics are a reality for the business, and are an important driver behind their relatively modest valuation.

DISCUSSION

The main contribution of this article is to propose a methodology with which customer disclosures are used to estimate a latent-variable model for customer acquisition, repeat purchase, and spend at publicly traded non-contractual firms. We use this latent-variable model to obtain a point estimate and distribution for the overall valuation of the firm, and study the unit economics of newly acquired customers.

The valuation distribution is particularly useful when we are analyzing young firms with less historical data – for these firms, the valuation distribution will widen to account for parameter uncertainty. It would be natural to further assume that the extent of our uncertainty in what future revenues (and thus free cash flows) will be may influence the firm's discount rate. This would be true if the revenue uncertainty were non-diversifiable (i.e., the uncertainty could not be reduced to zero if it were added to a diversified investment portfolio). A promising area of future work would be to specify a customer-based corporate valuation model which accounts for risk in a way that is consistent with the capital asset pricing model.

The methodology could also be applied, in theory, to private companies. Companies pursuing merger and acquisition opportunities, private equity firms, and venture capital firms may not be able to get access to the full transaction log of potential privately held investments until the later stages of the due diligence process, if at all. It may be easier for these firms to get access to a small collection of quarterly data summaries quickly.

The methodology has uses that extend beyond corporate valuation. For example, it may be useful for expert testimony in litigation cases where firms would like to provide enough information to confirm or deny specific points raised within the case, but no more than that. Furthermore, it may often be the case that external stakeholders are primarily interested in sales forecasts [e.g., for forecasts of overall economic activity, or for valuation via a multiple of future sales (Liu, Nissim, and Thomas 2002)], which the proposed methodology provides.

One of the limitations of this work is that we have assumed that the disclosure decision is not strategic. It could be that there is a forward-looking component to firms' decision to disclose (Mintz et al. 2016), but the large-scale meta-analysis of firm disclosures needed to carry out such an analysis is beyond the scope of this work. Another limitation is that we are not making any normative statements about the information content of the customer metrics themselves (e.g., which metrics provide stakeholders with the most accurate predictions of future revenues). We focus upon a practical, general methodology for performing firm valuation in a non-contractual setting with whatever data the firm happens to provide, regardless of its quality, and leave metric optimization to future work.

The scope of the proposed method is limited to passive investors valuing a business on a going-concern basis. While this may be the most common setting in which public company valuation is performed, there are other settings. For example, there are stakeholders who may be interested in using CBCV to both measure and manage the value of a firm over time. For example, so-called "activist investors" may make an investment in a firm and then actively seek to increase the value of their investment, typically by changing high-level resource allocations (George and Lorsch 2014). The valuation of the firm after these allocations have been made will then be a very important consideration for whether the activist will make the investment in the first place. In this setting, it is more important to understand the causal relationship between marketing actions/strategy and overall firm valuation, so the modeler may desire an endogeneity-corrected CBCV model, raising interesting trade-offs and methodological challenges.

This work highlights an important under-appreciated use for customer disclosures. These metrics need not be ends in their own right as standalone key performance indicators. They can be leveraged to better understand the true underlying propensity of customers to acquire services, make purchases, and spend, and how these propensities vary across customers. The proposed methodology turns backward-looking customer metrics into important forward-looking measures, which should decrease investor uncertainty regarding future cash flows and thus increase the value of the firm (Bayer, Tuli, and Skiera 2017).

As investors realize these many uses for customer metrics, the demand for their disclosure will continue to grow. This would not be the first time – one of the most commonly disclosed and tracked retail metrics, same store sales (SSS), became popular after a Wall Street analyst used it to uncover the true underlying financial condition of a fast-growing retailer in the 1970's (Blumenthal 2008). Customer metrics like the ones discussed here allow investors to track the quality of existing customers much the same way that SSS allows investors to track the quality of existing stores. With physical stores ceding share to internet-based retail, the need for such metrics is more important than ever.

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Notes

[1]The other three businesses valued – Ameritrade, Capital One, and E*Trade – are all contractual (as defined by Schmittlein, Morrison, and Colombo 1987) because their churn is observable.

[2]These valuations were performed after the stock market had fallen sharply in the aftermath of the "tech bubble," and prominent Wall Street analysts were publicly questioning Amazon's solvency. This implies that Gupta, Lehmann, and Stuart (2004) undervalued Amazon at a time that Amazon's stock price was depressed.

[3]The number of customers lost each quarter can be easily backed out from QADD_q and END_q.

[4]While we could easily allow for heterogeneity in the Weibull baseline, we do not to maintain model parsimony and because empirically, heterogeneity has been rejected every time we have applied the model to data allowing for it.

[5]The code is available upon request from the authors.

[6]Model fits and forecasts remain the same using a weekly unit of time, however some parameter estimates change a multiplicative factor. We divide λ_1 and λ_2 by 13^c. We divide μ_p and σ_p by $\log(13)$. Finally, we divide $\beta_{S,w}$ by 13. All other parameters remain the same.

[7]Technically, we provide in-sample fits for quarterly direct revenues. Wayfair also generates a small and declining amount of sales from third party websites. These other sales represent 1.8% of sales in Q1 2017, which is 54% less than the same proportion in the year-ago period. See Web Appendix A.

APPENDIX A: DERIVATIONS FOR ESTIMATES OF PUBLIC DISCLOSURES

In this section, we derive efficient expressions for model-based estimates of the available data – $\widehat{\text{QADD}}_q$, $\widehat{\text{QAU}}_q$, $\widehat{\text{AAU}}_q$, $\widehat{\text{QTO}}_q$, $\widehat{\text{QREV}}_q$, and $\widehat{\text{AREV}}_q$. All expressions except for $\widehat{\text{QAU}}_q$ and $\widehat{\text{AAU}}_q$ are available in closed-form and can thus be computed with no stochastic error. As we will see below, while our expressions for $\widehat{\text{QAU}}_q$ and $\widehat{\text{AAU}}_q$ require simulation, they are derived in a way that incurs negligible stochastic error.

$\underline{\widehat{\text{QADD}}_q}$:

QADD_q is the sum of weekly customer acquisitions across all weeks during quarter q:

$$\widehat{\text{QADD}}_q = \sum_{w=13q-12}^{13q} \widehat{A}(w),$$

As shown in Equation 6, A(w) is a function of $M(i)$ and $F_A(w-i|i)$. The former is equal to the change in population size, which is known. The latter is a function of the model parameters, which we have conditioned upon. Multiplying these two terms together and marginalizing across all weeks within quarter q gives us the desired expression.

$\underline{\widehat{\text{QTO}}_q}$:

Let the vector of expected weekly customer additions over the W-week calibration period be defined as

$$\widehat{\mathbf{A}} \equiv [\widehat{A}(1), \cdots, \widehat{A}(W)].$$

Let RL be the residual lifetime of a customer (i.e., the duration of time in weeks from the time the customer is born to the time the customer churns). The unconditional probability mass function for RL is

$$\widehat{P}(RL = t|\gamma, \delta) = B(\gamma + 1, \delta + t - 1)/B(\gamma, \delta), \quad t = 1, 2, ... \tag{A1}$$

We create a WxW lower triangular "expected alive customers" matrix, $\widehat{\mathbf{C}}$, whose rows represent time (in weeks), whose columns represent weekly acquisition cohorts, and whose $(i, j)^{\text{th}}$ entry represents the number of customers acquired in week j who are still active in week i. The $(i, j)^{\text{th}}$ entry of $\widehat{\mathbf{C}}$ is equal to

$$\widehat{C}_{i,j} = \begin{cases} \widehat{A}(j) \times \widehat{P}(RL > i - j|\gamma, \delta), & i \geq j \\ 0 & \text{otherwise} \end{cases}$$

where $\widehat{A}(j)$ is defined in Equation 6 and $\widehat{P}(RL = t|\gamma, \delta)$ is defined in Equation A1.

This implies that the number of customers who are expected to be alive in a particular week w, call it $\widehat{NA}(w)$, is the sum of the corresponding row of the $\widehat{\mathbf{C}}$ matrix:

$$\widehat{NA}(w) = \sum_{j=1}^{W} \widehat{C}_{w,j}.$$

The expected number of orders in a given week w are equal to the expected number of alive customers in week w multiplied by the expected number of repeat orders given a customer is alive in that week, plus all the initial orders placed by customers expected to be acquired during that week. Letting $O(w)$ denote the number of orders placed in week w,

$$\widehat{O}(w) = \widehat{NA}(w) \times \exp[\mu_O + \sigma_O^2/2 + \boldsymbol{\beta}_O^T \mathbf{x}_O(w)] + \widehat{A}(w). \tag{A2}$$

$\widehat{\text{QTO}}_q$ is the sum of weekly total orders across all weeks during quarter q:

$$\widehat{\text{QTO}}_q = \sum_{w=13q-12}^{13q} \widehat{O}(w).$$

<u>$\widehat{\text{QREV}}_q$:</u>

$\widehat{\text{QREV}}_q$ is equal to the sum of the expected revenue generated each week during the quarter, which is itself equal to the product of the expected number of orders placed during the week ($\widehat{O}(w)$), multiplied by the expected average revenue per order during the week, $\widehat{\text{ARPO}}(w)$:

$$\widehat{\text{QREV}}_q = \sum_{w=13q-12}^{13q} \widehat{O}(w) \times \widehat{\text{ARPO}}(w), \tag{A3}$$

where $\widehat{O}(w)$ is defined in Equation A2 and $\widehat{\text{ARPO}}(w)$ follows easily from Equation 11:

$$\widehat{\text{ARPO}}(w) = \beta_{S,0} + \beta_{S,w} \times w + \boldsymbol{\beta}_S^T \mathbf{x}_S(w). \tag{A4}$$

<u>$\widehat{\text{AREV}}_q$:</u>

$\widehat{\text{AREV}}_q$ is the sum of $\widehat{\text{QREV}}$ over the preceding four quarters, adjusted to account for the beginning of the time series, where $a \vee b = \max(a, b)$:

$$\widehat{\text{AREV}}_q = \sum_{q'=1\vee(q-3)}^{q} \widehat{\text{QREV}}_{q'}. \tag{A5}$$

<u>$\widehat{\text{QAU}}_q$:</u>

The expected number of quarterly active users in quarter q is equal to the number of customers who were acquired during quarter q, plus the expected number of customers born before quarter q who will place at least one repeat order within quarter q. If there were no heterogeneity in the time-invariant baseline propensity with which customers place orders, b_O, then letting QA_q be an indicator variable equal to one if a randomly chosen customer was active in quarter q and zero otherwise, and $\mathbf{X}_O$ be the time-varying repeat order covariates

$(\mathbf{X}_O = [\mathbf{x}_O(1), \ldots, \mathbf{x}_O(W)])$,

$$\widehat{E}(\text{QAU}_q|i, b_O, \mathbf{X}_O, \gamma, \delta) = \sum_{i=13q-12}^{13q} \widehat{A}(i) + \sum_{i=1}^{13q-13} \widehat{A}(i) \times P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) \quad \text{(A6)}$$

$P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta)$ is obtained by conditioning upon the number of weeks the customer is alive for during the quarter, which we denote by n_q:

$$\begin{aligned} &P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) = &\text{(A7)} \\ &\quad \sum_{w=13q-12}^{13q} P(RL = w - i|\gamma, \delta) P(QA_q = 1|n_q = w - 13q + 13, b_O, \boldsymbol{\beta}_O, \mathbf{X}_q), \end{aligned}$$

recognizing that a customer born in week i who lives for $w - i$ weeks is alive until week w, which for $w \in \{13q - 12, \ldots, 13q\}$ implies $w - 13q + 13$ alive weeks during quarter q. Using the memorylessness property of the Poisson process and accounting for the fact that time-varying covariates are constant within each quarter,

$$P(QA_q = 1|n_q, b_O, \boldsymbol{\beta}_O, \mathbf{X}_q) = 1 - \exp\{-\exp[b_O + \boldsymbol{\beta}_O^T \mathbf{x}_O(13q)] n_q\}. \quad \text{(A8)}$$

Plugging Equation A8 into Equation A7 in conjunction with Equation A1 gives us a closed-form *conditional* expression for $\widehat{\text{QAU}}_q$ using Equation A6. To remove the conditioning upon b_O, we must integrate Equation A8 against the mixing distribution for b_O:

$$\begin{aligned} \widehat{\text{QAU}}_q &= \widehat{E}(\text{QAU}_q|i, \mu_O, \sigma_O^2, \mathbf{X}_O, \gamma, \delta) &\text{(A9)} \\ &= \sum_{i=13q-12}^{13q} \widehat{A}(i) + \sum_{i=1}^{13q-13} \widehat{A}(i) \times \int_{-\infty}^{\infty} P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O|\mu_O, \sigma_O^2) db_O \end{aligned}$$

Equation A9 does not have a closed-form analytical expression. Instead, we use $K = 50,000$ one-dimensional Halton draws (Datta, Foubert, and Van Heerde 2015, Train 2000) to approximate the integral:

$$\int_{-\infty}^{\infty} P(QA_q = 1|i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O|\mu_O, \sigma_O^2) db_O \approx \frac{\sum_{k=1}^{K} P(QA_q = 1|i, b_O^{(k)}, \mathbf{X}_O, \gamma, \delta)}{K}, \quad \text{(A10)}$$

where $b_O^{(k)}$ represents the k^{th} element of a standard Normal Halton sequence. Evaluating Equation A10 using its conditional counterpart (Equation A7), and plugging this into Equation A9 gives us the desired expression.

$\widehat{\text{AAU}}_q$:

The expression for $\widehat{\text{AAU}}_q$ follows the same logic as the expression for $\widehat{\text{QAU}}_q$. The expected number of annual active users in quarter q is equal to the number of customers who were acquired during the previous four quarters $\{q-3,\ldots,q\}$, plus the expected number of customers born before quarter $q-3$ who will place at least one repeat order within quarters $\{q-3,\ldots,q\}$. If there were no heterogeneity in the time-invariant baseline propensity with which customers place orders, b_O, then letting AA_q be an indicator variable equal to one if a randomly chosen customer was active in any of quarters $\{q-3,\ldots,q\}$ and zero otherwise,

$$\widehat{E}(\text{AAU}_q|i,b_O,\mathbf{X}_O,\gamma,\delta)=\sum_{i=1\vee(13q-51)}^{13q}\widehat{A}(i)+\sum_{i=1}^{0\vee(13q-52)}\widehat{A}(i)\times P(AA_q=1|i,b_O,\mathbf{X}_O,\gamma,\delta). \tag{A11}$$

$P(AA_q=1|i,b_O,\mathbf{X}_O,\gamma,\delta)$ is obtained by conditioning upon the number of weeks the customer is alive for during each of the preceding four quarters, $[n_{q-3},n_{q-2},n_{q-1},n_q]$:

$$\begin{aligned}&P(AA_q=1|i,b_O,\mathbf{X}_O,\gamma,\delta)= \qquad\qquad \text{(A12)}\\&\qquad\sum_{w=1\vee(13q-51)}^{13q}P(RL=w-i|\gamma,\delta)P(AA_q=1|n_{q-3},n_{q-2},n_{q-1},n_q,b_O,\boldsymbol{\beta}_O,\mathbf{X}_q),\end{aligned}$$

where for each summand, letting n_a be the weeks that the customer is alive for in quarters $q-3$ through q, and recalling that $a\wedge b=\min(a,b)$,

$$\begin{aligned}n_a&=w-13q+52, \qquad\qquad \text{(A13)}\\n_{q-3}&=n_a\wedge 13,\\n_{q-2}&=((n_a-13)\vee 0)\wedge 13,\\n_{q-1}&=((n_a-26)\vee 0)\wedge 13,\quad\text{and}\\n_q&=(n_a-39)\vee 0.\end{aligned}$$

Using the memorylessness property of the Poisson process and accounting for the fact that time-varying covariates are constant within each quarter,

$$
\begin{aligned}
&P(AA_q = 1 | n_{q-3}, n_{q-2}, n_{q-1}, n_q, b_O, \boldsymbol{\beta}_O, \mathbf{X}_q) = \\
&\qquad\qquad 1 - \exp\left\{ - \exp\left[b_O n_a + \boldsymbol{\beta}^T \sum_{q'=q-3}^{q} \mathbf{x}_O(13q') n_{q'} \right] \right\}.
\end{aligned} \tag{A14}
$$

Plugging Equation A14 into Equation A12 in conjunction with Equation A1 gives us a closed-form *conditional* expression for $\widehat{\text{AAU}}_q$ using Equation A11. To remove the conditioning upon b_O, we must integrate Equation A14 against the mixing distribution for b_O:

$$
\begin{aligned}
\widehat{\text{AAU}}_q &= \widehat{E}(\text{AAU}_q | i, \mu_O, \sigma_O^2, \mathbf{X}_O, \gamma, \delta) \\
&= \sum_{i=13q-51}^{13q} \widehat{A}(i) + \sum_{i=1}^{0 \vee (13q-52)} \widehat{A}(i) \times \int_{-\infty}^{\infty} P(AA_q = 1 | i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O | \mu_O, \sigma_O^2) db_O
\end{aligned} \tag{A15}
$$

Equation A15 does not have a closed-form analytical expression. As with the derivation for $\widehat{\text{QAU}}_q$, we use $K = 50,000$ one-dimensional Halton draws to approximate the integral:

$$
P(AA_q = 1 | i, b_O, \mathbf{X}_O, \gamma, \delta) f(b_O | \mu_O, \sigma_O^2) db_O \approx \frac{\sum_{k=1}^{K} P(AA_q = 1 | i, b_O^{(k)}, \mathbf{X}_O, \gamma, \delta)}{K}, \tag{A16}
$$

where $b_O^{(k)}$ represents the k^{th} element of a standard Normal Halton sequence. Evaluating Equation A16 using its conditional counterpart (Equation A12), and plugging this into Equation A15 gives us the desired expression.

WEB APPENDIX A: WAYFAIR VALUATION ASSUMPTIONS

Wayfair's Valuation section describes our process for arriving at long-term QREV projections. In this section, we provide all of the assumptions we made to obtain the remaining variables in Equations 1, 2, 3, and 4 – NOA, ND, WACC, CAPEX, D&A, ΔNFWC, VC, FC, and TR – to arrive at a stock price estimate:

- Projecting Other Revenues: Wayfair generates a small and declining amount of revenue through third party websites (e.g., Other Revenues represented 1.8% of sales in Q1 2017, and this percentage declined 54% year-on-year). We assume that Other Revenue declines at the same rate that it has historically until it falls to zero. A simple time trend regression for Other Revenues has a R^2 of 70.2% and implies that Other Revenues will decline by $2.3MM per quarter.

- Projecting FC and VC: We assume that all costs are variable in nature, so that our forecasts are consistent with the forecasts that Wayfair management has made in the "Long-Term Target Model" section of Wayfair's Q1 2017 investor presentation.

- Projecting margins: We assume that all margins – gross margin, customer service plus merchant fees, advertising, merchandising, marketing, sales, operations, technology, general and administrative – improve from their levels in Q1 2017 to the midpoint of Wayfair's long-term targets in the "Long-Term Target Model" section of their Q1 2017 investor presentation. We assume they linearly improve until they achieve these midpoints in five years, or Q1 2022. This forecast is consistent with the proposed QREV forecast, because 2022 is also year that QREV is expected to peak. We assume they maintain these margins after Q1 2022.

- Projecting CAPEX: We assume that capital expenditures as a percentage of sales falls linearly to 1.9% of sales by Q4 2017, the level of D&A. After Q4 2017, CAPEX remains at 1.9% of sales. Wayfair has made substantial capital investments over the past nine quarters, with trailing 12 month CAPEX varying between 7.3% and 10.8% of sales, so this decline in spending will be a dramatic (and beneficial) departure from

Wayfair's past. The company has indicated that the large capital projects they had been spending on, most notably the CastleGate fulfillment network, are largely complete. Therefore, this forecast assumes that the company will undertake no additional large capital projects into the forseeable future.

- Projecting ΔNFWC: We assume that ΔNFWC as a percentage of trailing twelve month sales is equal to its historical two-year average level of -6.7%. This level is stable over time – the corresponding trailing one-year average level is virtually unchanged at -6.8%. As the company grows, this is an important source of positive cash flow.

- Projecting D&A: We assume that D&A as a percentage of sales remains at its Q1 2017 level of 1.9%.

- Estimating NOA: Wayfair has no non-operating assets.

- Estimating ND: We assume that working capital cash represents 5% of trailing twelve month sales. This implies \$97MM in excess cash as of Q1 2017. Total debt in Q1 2017 is \$42MM, so ND is equal to $-\$55$MM.

- Estimating WACC: We assume Wayfair's WACC is equal to 12.9%, which is equal Bloomberg's Q1 2017 estimate. The results are robust to changes in WACC. For example, if we had assumed a WACC of 10%, the expected stock price increases from \$10.24 per share to \$12.92 per share, which remains 80% below the observed stock price.

- Estimating TR: We assume a tax rate of 40%.

WEB APPENDIX B: OVERSTOCK.COM VALUATION ASSUMPTIONS

In this section, we provide all assumptions used to obtain the valuation estimate for Overstock:

- Projecting FC and VC: For consistency with Wayfair, we assume that all operating costs are variable in nature.

- Projecting gross margin: We assume that Overstock's gross margin remains unchanged from its Q1 2017 level of 20.1% of sales.

- Projecting technology plus general and administrative expenses: We assume that technology plus general and administrative expenses improves to its Q4 2016 level of 9.5% of sales. We assume that this figure improves linearly until it reaches this level in Q4 2023. We assume that it remains constant after Q4 2023.

- Projecting sales and marketing expense: We assume that sales and marketing expense per newly acquired customer is equal to its estimated Q1 2017 level of \$47.5. To account for the transition of marketing spend from acquisition to retention as the business matures, and for consistency with Wayfair, we assume that sales and marketing expenses falls to no lower than 7% of sales.

- Projecting CAPEX: We assume 2017 CAPEX of \$25MM, which is the guidance that Overstock's CEO provided on their Q1 2017 quarterly earnings conference call. After 2017, we assume that CAPEX grows at the historical rate of CPI inflation over Overstock's commercial operations, or 0.6%.

- Projecting ΔNFWC: As with Wayfair, we assume that Overstock's ΔNFWC is equal to its trailing two-year average of -3.1% of trailing one-year sales.

- Projecting D&A: We assume that D&A is equal to its trailing one-year average of 1.7% of sales.

- Estimating NOA: Overstock has three non-operating assets – blockchain assets, deferred tax assets, and precious metals investments. The total value of NOA is estimated to be \$84MM in Q1 2017. The assumptions we made to arrive at the valuation estimates for these three assets are as follows: (1) Because we do not know the market value of the blockchain assets, we assume their value is equal to their carrying cost (i.e., the amount that Overstock spent to acquire the assets, less impairment charges) of \$7.7MM. An increase in the market value of any of the blockchain assets would result in these assets being worth more than our estimate. (2) The value of deferred tax assets is disclosed on Overstock's balance sheet. In Q1 2017, these assets are worth \$66.4MM. (3) The value of Overstock's precious metals are also disclosed on Overstock's balance sheet. In Q1 2017, these assets are worth \$9.9MM.

- Estimating ND: As at Wayfair, we assume that working capital cash represents 5% of trailing twelve month sales. This implies \$46MM in excess cash as of Q1 2017. Total debt in Q1 2017 is \$57MM, so ND is equal to $-\$11$MM.

- Estimating WACC: We assume Overstock's WACC is equal to 10.1%, which is equal Bloomberg's Q1 2017 estimate.

- Estimating TR: As with Wayfair, we assume a tax rate of 40%.

WEB APPENDIX C: COMPANY DISCLOSURES

Table A: Overstock.com's Publicly Disclosed Historical Customer Data
Customer Data (000) and Revenue Figures ($000)

Period	QREV	QADD	QTO	QAU	Period	QREV	QADD	QTO	QAU
Q1 1999					Q2 2008	188202	516	1519	1158
Q2 1999					Q3 2008	186007	546	1520	1171
Q3 1999					Q4 2008	253841	929	2418	1824
Q4 1999					Q1 2009	185729	613	1662	1287
Q1 2000					Q2 2009	174898	551	1567	1208
Q2 2000					Q3 2009	193783	669	1791	1381
Q3 2000					Q4 2009	322359	1244	3279	2397
Q4 2000					Q1 2010	264330	915	2473	1872
Q1 2001	9578	99	151		Q2 2010	231253	707	2022	1554
Q2 2001	7407	68	112		Q3 2010	245420	768	2146	1642
Q3 2001	8744	81	131		Q4 2010	348870	1245	3394	2508
Q4 2001	14274	137	223		Q1 2011	265470	780	2297	1743
Q1 2002	12067	105	177		Q2 2011	234992	643	1963	1500
Q2 2002	14380	118	212		Q3 2011	239738	704	2005	1551
Q3 2002	23808	164	291		Q4 2011	314077	1099	3097	2315
Q4 2002	41529	348	579		Q1 2012	262367	804	2268	1736
Q1 2003	29164	264	491		Q2 2012	239536	663	1874	1470
Q2 2003	28833	283	522		Q3 2012	255352	725	1952	1533
Q3 2003	57788	342	643		Q4 2012	342034	1179	2985	2302
Q4 2003	123160	744	1376		Q1 2013	311994	854	2235	1733
Q1 2004	82078	425	1126		Q2 2013	293204	736	1953	1533
Q2 2004	87792	414	1023		Q3 2013	301426	753	1997	1546
Q3 2004	103444	514	1178	864	Q4 2013	397593	1181	3083	2311
Q4 2004	221321	1110	2440	1682	Q1 2014	341207	868	2352	1774
Q1 2005	165881	796	1915	1370	Q2 2014	332545	760	2126	1591
Q2 2005	150638	611	1588	1162	Q3 2014	352991	817	2259	1679
Q3 2005	169323	703	1768	1299	Q4 2014	470360	1259	3454	2480
Q4 2005	315018	1370	3176	2253	Q1 2015	398344	901	2630	1901
Q1 2006	178044	638	1856	1331	Q2 2015	388013	805	2435	1752
Q2 2006	159192	520	1568	1128	Q3 2015	391211	813	2465	1766
Q3 2006	156885	540	1529	1112	Q4 2015	480270	1163	3463	2423
Q4 2006	294029	1157	3061	2122	Q1 2016	413677	857	2675	1894
Q1 2007	157930	471	1447	1083	Q2 2016	418540		2602	1816
Q2 2007	148967	393	1307	976	Q3 2016	441564		2703	1886
Q3 2007	161930	484	1432	1086	Q4 2016	526182		3465	2372
Q4 2007	294516	1104	2858	2071	Q1 2017	432435		2606	1811
Q1 2008	202814	602	1703	1289					

Note: Most of the data presented here were obtained from Overstock's SEC filings. The remainder of the data were mined from figures within documents available on Overstock's Investor Relations page, using a pixel-counting software program.

Table B: Wayfair's Publicly Disclosed Historical Customer Data
Customer Data (000) and Revenue Figures ($000)

Period	QREV	AREV	QADD	QTO	AAU	Period	QREV	AREV	QADD	QTO	AAU
Q2 2002						Q4 2009		224000			
Q3 2002						Q1 2010					
Q4 2002						Q2 2010					
Q1 2003						Q3 2010					
Q2 2003						Q4 2010		279000			
Q3 2003						Q1 2011					
Q4 2003		7700				Q2 2011					
Q1 2004						Q3 2011					
Q2 2004						Q4 2011		349300			
Q3 2004						Q1 2012					1290
Q4 2004		24000				Q2 2012					1280
Q1 2005						Q3 2012					1260
Q2 2005						Q4 2012		386000			1300
Q3 2005						Q1 2013	120617		292	555	1370
Q4 2005		57000				Q2 2013	147748		366	703	1510
Q1 2006						Q3 2013	181693		468	884	1770
Q2 2006						Q4 2013	223388		624	1172	2090
Q3 2006						Q1 2014	226000		561	1138	2410
Q4 2006		105000				Q2 2014	243534		524	1084	2640
Q1 2007						Q3 2014	285502		659	1313	2860
Q2 2007						Q4 2014	346650		846	1702	3220
Q3 2007						Q1 2015	369395		829	1797	3600
Q4 2007		203000				Q2 2015	440297		850	1959	4040
Q1 2008						Q3 2015	544971		1042	2324	4590
Q2 2008						Q4 2015	685575		1412	3091	5360
Q3 2008						Q1 2016	711846		1337	2996	6070
Q4 2008		237000				Q2 2016	755657		1243	2930	6670
Q1 2009						Q3 2016	832398		1473	3416	7360
Q2 2009						Q4 2016	959008		1983	4722	8250
Q3 2009						Q1 2017	940352		1669	4213	8850

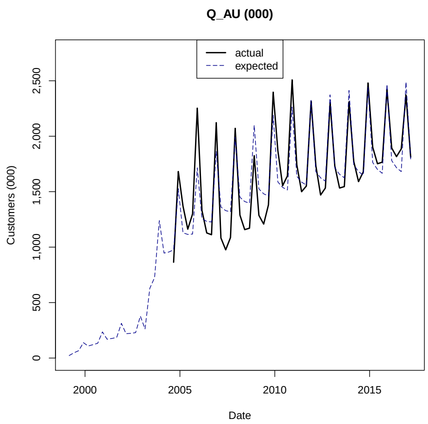
Note: As with Overstock, most of the data presented here were obtained from Wayfair's SEC filings, while the remainder of the data were mined from figures within documents available on Wayfair's Investor Relations page, using a pixel-counting software program.

WEB APPENDIX D: ADDITIONAL MODEL FITS

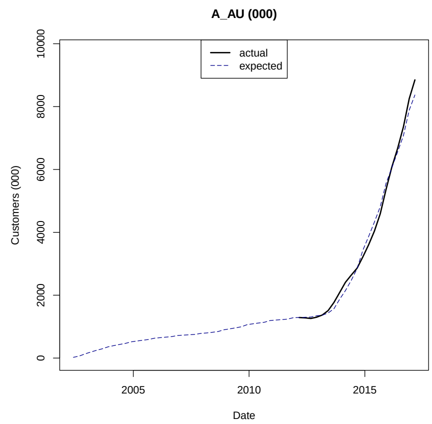
In Figures Aa and Ab, we provide actual and expected QAU and AAU for Overstock and Wayfair, respectively. In Figure Ac, we provide the in-sample fits for QADD, QAU, QTO, and AREV for Gupta, Lehmann, and Stuart 2004 (GLS), Schulze, Skiera, and Wiesel 2012 (SSW), Libai, Muller, and Peres 2009 (LMP), and the proposed model (MF).

Figure A: Additional Model Fits for Overstock.com and Wayfair

(a) Overstock: QAU



(b) Wayfair: AAU



(c) Wayfair: Model Fit Comparison Against Benchmark Models