

## SeriesNumber 28

### Problem 1

Chicken feed can be mixed from four ingredients: I1, I2, I3, and I4. The table below (fictitious) shows the current cost per tonne of each of these ingredients, together with the percentage of recommended daily allowances for protein, fat and fibre that a serving of it fulfils.

	<i>Ingredients</i>			
	<i>I1</i>	<i>I2</i>	<i>I3</i>	<i>I4</i>
Protein (%)	40	40	100	80
Fat(%)	50	70	90	80
Fibre(%)	68	63	60	11
Cost (£)	140	105	155	20

What is the minimal cost of a tonne of feed that satisfies at least 46% of the daily allowance for protein and at least 60% for fibre, while not exceeding 84% of the fat allowance.

### Problem 2

Smart Rug Manufacturers has available 1800 square meters of wool and 800 square meters of nylon for the manufacture of two grades of carpeting: high-grade, which sells for £850 per roll, and low-grade, which sells for £650 per roll.

35 square meters of wool and 15 square meters of nylon are used in a roll of high-grade carpet.

15 square meters of wool and 45 square meters of nylon are used in a roll of low-grade carpet.

25 work-hours are required to manufacture each roll of the high-grade carpet, and 20 work-hours are required for each roll of the low-grade carpet, at an average cost of £7 per work-hour. A maximum of 1000 work-hours are available.

The cost of wool is £4 per square meter and the cost of nylon is £3 per square meter.

What is the maximum income that can be achieved by manufacturing the carpets?

*Hint* :  $\text{Income} = \text{revenue from sale} - (\text{production cost for material} + \text{labor})$

### Problem 3

VirtualFood.com is a new company that allows customers to do grocery shopping over the Internet.

Fresh items, such as fruit, require special handling so they always look fresh and do not spoil.

Frozen items need cool or frozen storage so that they do not warm up. All the other items, including canned goods, paper products, and kitchenware, require minimal special attention and are just stocked on shelves.

The warehouse for storing these items has 85000 square meters of storage space.

The Marketing Department thinks that no more than 55000 square meters should be devoted to fresh goods, 45000 to frozen goods, and 40000 to all other items.

The accounting department estimates that the storage cost is £10 per square meter for fresh items, £19 per square meter for frozen items, and £1.5 per square meter for all other items. Company policy limits the storage cost to £705000.

Frozen goods have the highest profit margin at £50 per square meter, followed by fresh items at £20 per square meter, and £16 per square meter from the other items.

What is the maximal profit (profit from goods minus storage costs) that can be achieved by designing the optimal floorspace allocation?

#### Problem 4

The X Feed Company makes a feed from four ingredients - oats, corn, soybeans, and a vitamin supplement.

The company has 500 kg of oats, 500 kg of corn, 400 kg of soybeans, and 300 kg of vitamin supplement available for the mix. The company has the following recipe for the mix.

At least 40% of the mix must be soybeans

The ratio of corn to oats cannot exceed 4 to 1

At least 20% of the mix must be the vitamin supplement

The mix must be at least 998 kg

A kilogram of oat costs £0.4; a kilogram of corn, £0.9; a kilogram of soybeans, £0.2 and a kilogram of vitamin supplement, £1.7.

What is the minimum possible cost of the mix prepared according to the recipe above?

#### Problem 5

A company is designing a production schedule for manufacturing a summer product, demand on which is highly seasonal.

The demands for the next four quarters are 584, 1000, 1350, and 1150 units respectively.

The company can produce 1167 units per quarter, but inventories must be built up to meet larger demand at a holding cost of £8 per unit per each quarter.

If the company does not meet its demand, then it pays a penalty of £10 in quarter 1, £8 in quarter 2, £12 in quarter 3, and £15 in quarter 4 per unit of unsatisfied demand.

What is the minimal total cost (holding costs plus penalties) that the company can achieve over next four quarters?

#### Problem 6

A manufacturing company has received a contract to deliver a certain amount of units of some product over the next 6 months. The successive demands for the six periods are as follows.

Month	1	2	3	4	5	6
Demand	2900	2400	2300	2800	2800	2300

Production cost per unit of the product varies from month to month depending on labour, material, and utility costs. The company estimates the production cost per unit of the product over the next 6 months to be £8, £7, £7, £9, £6, and £8, respectively.

To take advantage in manufacturing cost, the company may decide to produce more than is needed to satisfy the demand of a given month and stock and stock the excess units for delivery in later months. This will, however incur storage costs at the rate of £0.5 per unit per month assessed on end-of-month inventory.

What is the minimal production cost, if the company designs an optimal production plan?

#### Problem 7

A trust officer for a bank in West Midlands wants to invest in the following bonds:

<i>Bond</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>Yield</i>	5%	8%	7%	6%	11%
<i>Maturity</i>	Short	Long	Short	Short	Long
<i>Risk</i>	High	Low	High	Low	Low
<i>Tax-Free</i>	Yes	No	Yes	Yes	No

She has £110000 available for the investments. To achieve a diversity in her investments she decided to satisfy the following constraints:

- At least £26000 must be placed in short maturity bonds.
- No more than £32000 may be invested in high-risk bonds.
- Total funds invested in low-risk bonds must be less than or equal to total funds placed in long-maturity bonds.

The interest income derived from tax-free bonds must be at least 10% of the total income.

What is the maximal return (in £) that the bank can achieve?

### Problem 8

A firm producing a single product has three plants and four customers.

The three plants will produce 2400, 3800 and 5500 units, respectively, during the next time period.

The firm has made a commitment to sell 3100 units to customer 1, 4650 units to customer 2, and at least 990 units to customer 3. Both customers 3 and 4 also want to buy as many of the remaining units as possible.

The profit associated with shipping a unit from plant  $i$  to customer  $j$  is given in the table below.

From	To Customer (£)			
	1	2	3	4
Plant 1	45	35	35	45
Plant 2	40	30	20	35
Plant 3	50	50	30	30

What is the maximal profit which the company can achieve using an optimal transportation plan?

### Problem 9

Sanders Fishing Supply of Naples, Florida, manufactures a variety of fishing equipment, which it sells throughout the United States.

For the next three months, Sanders estimates demand for a particular product at 110, 155, and 160 units, respectively. Sanders can supply this demand by producing on regular time or overtime.

Because of other commitments and anticipated cost increases in month 3, the production capacities in units and the production costs per unit are as follows:

<i>Production</i>	<i>Capacity</i>	<i>Cost/unit</i>
Month 1 - regular	85	\$45.00
Month 1 - overtime	115	\$55.00
Month 2 - regular	130	\$45.00
Month 2 - overtime	145	\$70.00
Month 3 - regular	130	\$75.00
Month 3 - overtime	25	\$100.00

Inventory may be carried from one month to the next, but the holding cost is \$12 per unit per month.

What is the minimal cost of meeting demands in the next three months?

### **Problem 10**

Ann is considering her investment policy for the next 4 years. She will have £6500 available at the beginning of each of the next four years.

Each year Ann expects to have available both certificates of deposit returning 5.3% after 1 year and ones returning 15.99% after 2 years.

At the beginning of the first year she will also have an opportunity to make a special investment that would return 33.15% after four years. She is, however, allowed to invest no more than £3250 into this option.

Assume that all funds can be reinvested at maturity. What is the maximal total interest (in pounds) that Ann can earn by the beginning of the fifth year?