

# **CS 401 Artificial Intelligence**

## **MiniMax Search**

## **Lecture 7**

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# The Minimax Procedure

- Two person games are more complicated than simple puzzles because of the existence of a “hostile” and essentially unpredictable opponent.
- The minimax method allows us to tackle the problem of opponents in a relatively simple manner.
- First we need to introduce some terms that relate to game playing:

**Game Tree:** A tree consisting of nodes that represent the possible moves that each player can make in a two-player game.

**Ply:** A single move by either the player or their opponent.

**Move:** A set of plies, whereby each player makes a move.

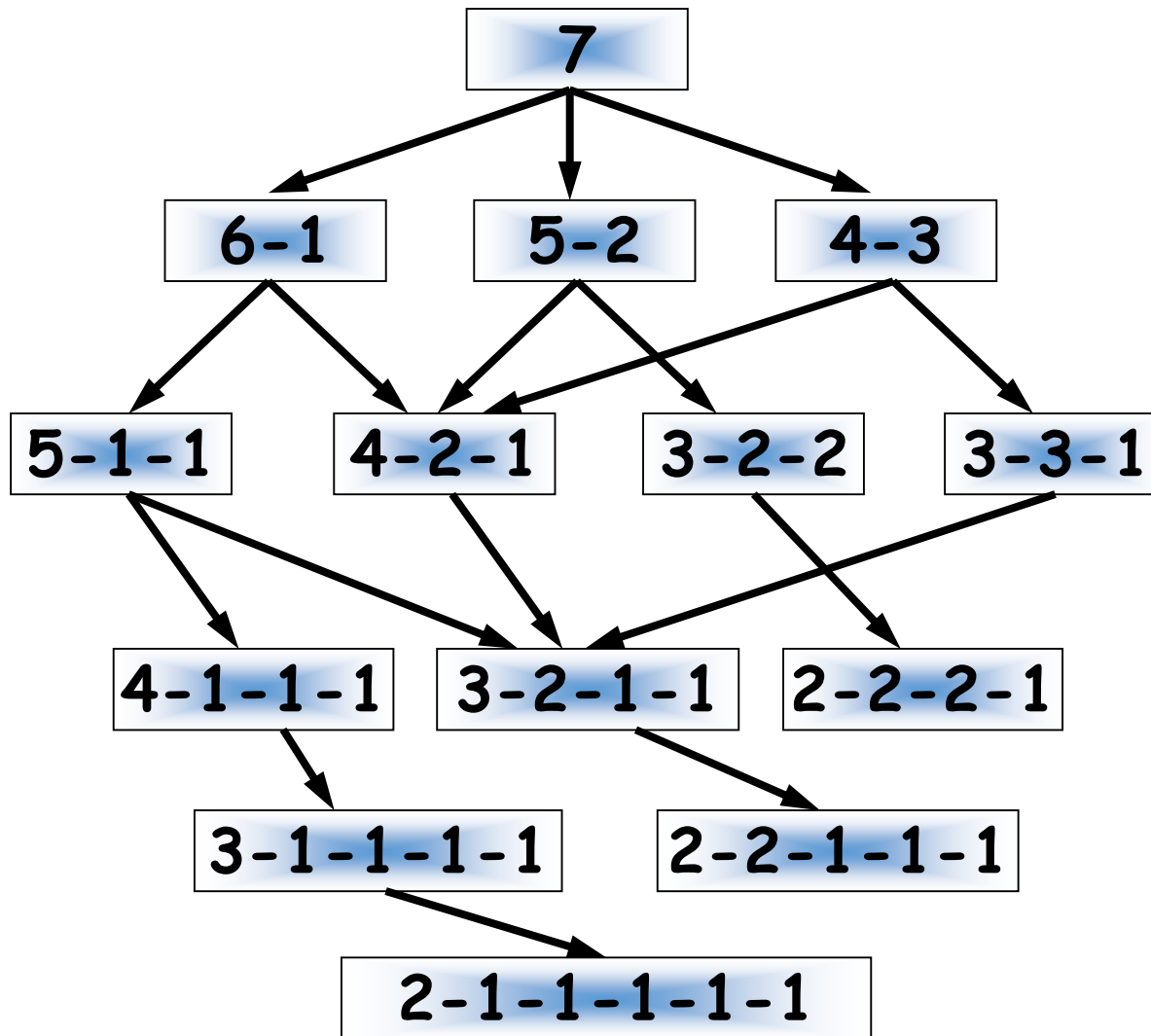
# The Minimax Procedure ...

- The games that we will be considering are two player games, so each move will consist of two plies.
- Let us assume that our opponent is playing to win, a fair assumption, and that they will always take the move that will give them the best advantage.
- Given this, we need to develop a technique that will take into account the opponent's best moves yet still give us the winning strategy.
- This is what Minimax attempts to do, but how?

# The Minimax Procedure – Example

- Let us first consider the game of nim, a game whose state space may be exhaustively searched.
- To play nim, a number of matches are placed on a table between the two opponents.
- At each move, the player must divide a pile of matches into two nonempty piles with different numbers of matches in each pile.
- Thus, 6 matches may be divided into piles of 5 and 1, 4 and 2, and.
- The first player who can no longer make a move loses the game.
- For a reasonable number of matches, the state space can be exhaustively searched.

# The Minimax Procedure – Example



State Space for the game of nim.

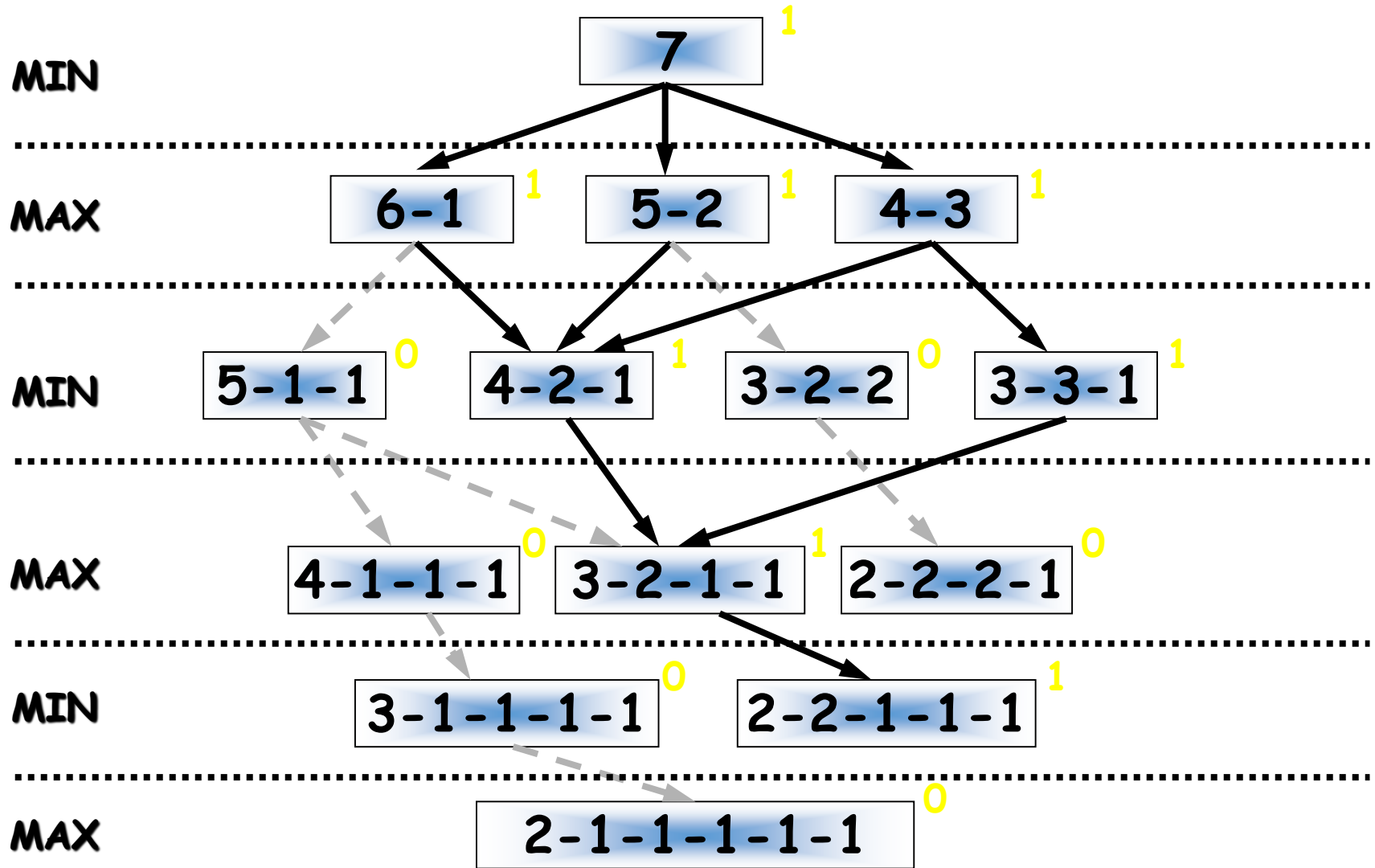
# The Minimax Procedure – Example

- The opponents in a typical game are referred to as MIN and MAX.
- The significance of these names is straight forward:
  - MAX represents the player trying to win, or MAXimize her advantage.
  - MIN is the opponent who attempts to MINimize MAX's score.
- We assume that MIN uses the same information and always attempts to move to a state that is worst for MAX.
- In implementing minimax, we label each level in the search space according to whose move is at that point in the game, MIN or MAX.

# The Minimax Procedure – Example

- Lets us assume that MIN is allowed to move first.
- Each leaf node in the game tree is given a value of 1 or 0, depending on whether it is a win for MAX or for MIN.
- Minimax propagates these values up the tree through successive parent nodes according to the rule:
  - If the parent state is a MAX node, give it the maximum value among its children.
  - If the parent is a MIN node, give it the minimum value of its children.
- The value that is thus assigned to each state indicates the value of the best state that the player can hope to achieve (assuming the opponent plays as predicted by the minimax algorithm).

# The Minimax Procedure – Example



Exhaustive minimax for the game of nim.



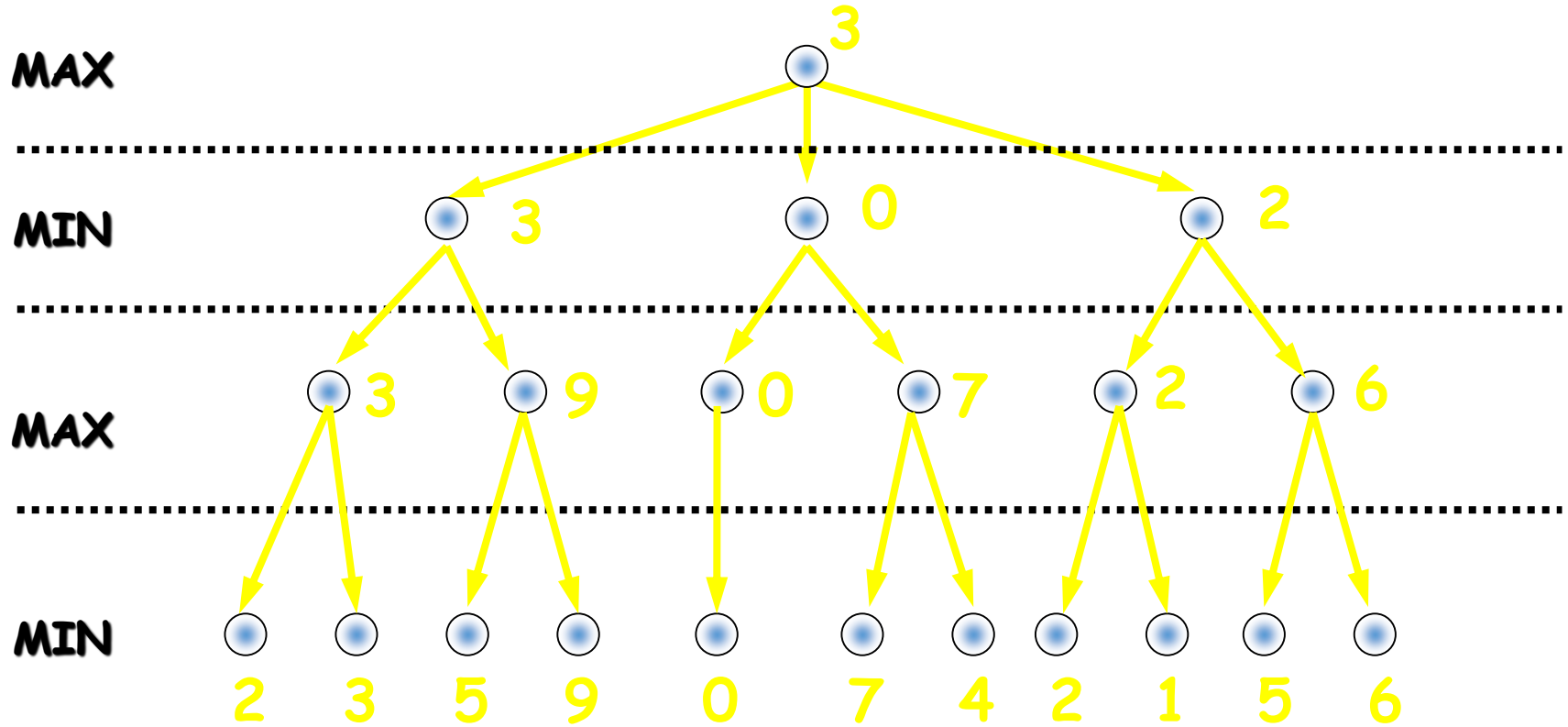
# Minimax to Fixed Ply Depth

- In applying minimax to more complicated games, it is seldom possible to expand the state space graph out to the leaf nodes.
- Instead, the state space is searched to a predefined number of levels, as determined by the available resources of time and memory.
- This strategy is called **n-move look-ahead**, where  $n$  is the number of levels explored.
- As the leaves of this sub-graph are not final states of the game, it is not possible to give them values that reflect a win or a loss.
- Instead, each node is given a value according to some heuristic evaluation function.

# Minimax to Fixed Ply Depth

- After assigning an evaluation to each state on the selected ply, the program propagates a value up to each parent state.
- If the parent is on a MIN level, the minimum value of the children is backed up.
- If the parent is a MAX node, minimax assigns it the maximum value of its children.
- Maximizing for MAX parents and minimizing for MIN, the values go back up the game tree to the children of the current state.
- These values are then used by the current state to select among its children.
- Figure on next slide shows minimax on a hypothetical state space with a four-ply look-ahead.

# Minimax to a Hypothetical State space



Leaf values show heuristic values;  
Internal states show backed-up values