# Practise Question's From CH 5

Sec - 5.1 Problems 5-5 to 5-13

Sec 5.3 Problems 5-22 to 5-26

Sec 5.4 // 5-33 to 5-38

| | | 1 | 2 | 3 | |
|---|---|---|---|---|---|
| Plant | 1 | 600 $ | 700 $ | 400 $ | 25 |
| | 2 | 320 $ | 300 $ | 350 $ | 40 |
| | 3 | 500 $ | 480 $ | 450 $ | 30 |
| Demand | | 30 | 35 | 25 | 90 \ 95 |

(1)

Additional Information:

During Month of ~~Ran~~ August, there is 20% Increase in demand.

$\Rightarrow$ Increase 20% demand of city 1 $= 30 + \frac{30}{100} \times 20 = 36$

" " " City 2 $= 35 + \frac{35}{100} \times 20 = 42$

" " " city 3 $= 25 + \frac{25}{100} \times 20 = 30$

Purchase electricity from another network at a rate of 1000 $ per million kWh. The network is not linked to city 3.

| | City | 1 | 2 | 3 | |
|---|---|---|---|---|---|
| Plant | 1 | 600 | 700 | 400 | 25. |
| | 2 | 320 | 300 | 350 | 40 |
| | 3 | 500 | 480 | 450 | 30 |
| | 4 | 1000 | 1000 | M | 13 |
| Demand. | | 36 | 42 | 30 | 108 \ 108 |

Assign a very Large cost M to the route from Plant 4 to city 3.

Let $M = 10000$

## Apply Least Cost Method:

Cities

| | 1 ($v_1=400$) | 2 ($v_2=380$) | 3 ($v_3=350$) | |
|---|---|---|---|---|
| $u_1=0$ 1 | 600 X | −380 700 X | −150 400 $\boxed{25}$ | ~~25~~ 0 |
| Plant 2 $u_2=-80$ | 0 320 X | 0 300 $\boxed{40}$ | −80 X 350 | ~~40~~ 0 |
| 3 $u_3=160$ | 500 $\boxed{23}$ | 480 $\boxed{2}$ | 450 $\boxed{5}$ | ~~30~~ ~~25~~ ~~23~~ 0 |
| 4 $u_4=600$ | 1000 $\boxed{13}$ | −20 1000 X | −50 X 10000 | ~~13~~ 6 |
| | ~~36~~ ~~13~~ 6 | ~~42~~ ~~2~~ 0 | ~~30~~ ~~5~~ 0 | 108 / 108 |

Total Cost = $400 \times 25 + 300 \times 40 + 480 \times 2 + 500 \times 23 + 450 \times 5$

$+ 1000 \times 13 = 49710\,\$$

### Check the optimality

Basic variables: $x_{13}=25,\ x_{22}=40,\ x_{31}=23,\ x_{32}=2,\ x_{41}=13$

$u_1 + v_1 = 400$    $u_1=0,\ v_1=400$

$u_2 + v_2 = 300$    $\boxed{u_2=-80}$

$u_3 + v_1 = 560$    $\boxed{u_3=160}$

$u_3 + v_2 = 480$    $\boxed{v_2=380}$

$u_3 + v_3 = 450$    $\boxed{v_3=350}$

$u_4 + v_1 = 1000$    $\boxed{u_4=600}$

Non-Basic ~~coef~~ variable coeff $C^*_{ij} = (u_i + v_j) - c_{ij}$

$C^*_{11} = 0 + 400 - 600 = -200$

$C^*_{12} = 0 + 380 - 700 = -320$

All negative value ~~of~~ shows that our solution is optimal

4 iii)

# Problem 5-24

~~5~~ (7)

pg 241

(3)

| | | | | supply |
|---|---|---|---|---|
| | 5 | 1 | 7 | 10 |
| | 6 | 4 | 6 | 80 |
| | 3 | 2 | 5 | 15 |
| demand | 75 | 20 | 50 | 105 / 145 |

Demand > supply.

Penalty cost per unit for on unsatisfied demand are 2$, 5$, 3$. for destination 1, 2, 3 respectively

## Least cost Method

| | $v_1 = 2$ | $v_2 = 1$ | 2 | |
|---|---|---|---|---|
| $u_1 = 0$ | $0+2-5=-3$ 5 x | 1 (10) | $0+2-7=-5$ 7 x | ~~10~~ 0 |
| $u_2 = 4$ | 6 (30) | $\boxed{1}$ 4 X | 6 (50) | ~~80~~ 30 |
| $u_3 = 1$ | 3 (5) | 2 (10) | -2 5 X | ~~15~~ ~~5~~ 0 |
| $u_4 = 0$ | 2 (40) 0 $0+2-2=0$ | -4 5 X | -1 3 X | ~~40~~ 0 |
| | ~~75~~ ~~35~~ ~~30~~ 0 | ~~20~~ ~~10~~ 0 | ~~50~~ 0 | 145 / 145 |

Total cost = 10 × 1 + 30 × 6 + 50 × 6 + 5 × 3 + 10 × 2
+ 40 × 2
= 605$

Now check optimality Apply UV method



$u_1 + v_2 = 1$ $\quad u_1 = 0,\ v_2 = 1$

$u_2 + v_1 = 6$ $\quad \boxed{u_2 = 4}$

$u_2 + v_3 = 6$ $\quad \boxed{v_3 = 2}$

$u_3 + v_1 = 3$ $\quad \boxed{v_1 = 2}$

$u_3 + v_2 = 2$ $\quad \boxed{u_3 = 1}$

$u_4 + v_1 = 2$ $\quad \boxed{u_4 = 0}$

$C_{ij}^* = u_i + v_j - C_{ij}$

Because $C_{22}^* = 1 > 0$, So soln is not optimal.

Assign $\theta$ to that variable and Balance the Eqn

| | | |
|---|---|---|
| 5 | 10 1 | 7 |
| 6 30−θ | θ 4 | 50 6 |
| 3 5+θ | 10−θ 2 → Become non Basic | 5 |
| 2 40 | 5 | 3 |

Put $\theta = 10$, such that $10 - \theta \geq 0$.

| | $v_1 = 3$ | $v_2 = 1$ | $v_3 = 3$ |
|---|---|---|---|
| $u_1 = 0$ | 5 −2 | 10 1 | −4 7 |
| $u_2 = 3$ | 6 20 | 10 4 | 50 6 |
| $u_3 = 0$ | 3 15 | 2 −1 | −2 5 |
| $u_4 = -1$ | 2 40 | 5 −5 | −1 3 |

$u_1 + v_2 = 1$ $\quad u_1 = 0,\ v_2 = 1$

$u_2 + v_1 = 6$ $\quad \boxed{v_1 = 3}$

$u_2 + v_2 = 4$ $\quad \boxed{u_2 = 3}$

$u_3 + v_1 = 3$ $\quad \boxed{u_3 = 0}$

$u_4 + v_1 = 2$ $\quad \boxed{u_4 = -1}$

$u_{\cancel{2}} + v_3 = 6$ $\quad \boxed{v_3 = 3}$

All non-Basic variable have neg coefficient so, this is optimal tableau

$$\text{Total cost} = 1 \times 10 + 6 \times 20 + 4 \times 10 + 3 \times 15 + 6 \times 50 + 2 \times 40$$
$$= 595\ \$$$

Destination 1 will be 40 units short.

roblem 3–26 Pg 241 ~~OK 2~~ ~~5~~ 6

Initially

| | | Destination 2 | | |
|---|---|---|---|---|
| Source | 1 | 2 | 3 | Supply |
| 1 | 1$ | 2$ | 1$ | 20 |
| 2 | 3 | 4 | 5 | 40 |
| 3 | 2 | 3 | 3 | 30 |
| Demand | 30 | 20 | 20 | 70 \ 90 |

Supply > Demand : Add dummy colum.

Additional Information:

The source 2 must be shipped out (means, we cannot shipped any materiel from source 2 to dummy Destination). Also if a source is not shipped out to any destination, it charge 5$, 4$, 3$ per unit ~~source~~ for source 1, 2, 3 respectively

Apply Vogel's Method (Read Pg #    6K-3    6    5

7

| $u_i \backslash v_j$ | 1 | 2 | 1 | 2 | Supply | Row penalty | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | $1^{0}$ (0) | $2^{X}$ (0+2-2=0) | $1^{20}$ (0) | $X\,5$ (-3) | ~~20~~ 0 | 1-1=0 | 0 | 1 | — | — |
| 2 | $3^{20}$ | $4^{20}$ | $5\,X$ (-2) | $X\,M = 100$ (-96) | ~~40~~ ~~20~~ 0 | 4-3=1 | 1 | 1 | 1 | 1 |
| 1 | $2^{10}$ | $3\,X$ (0) | $3\,X$ (-1) | $3^{20}$ | ~~30~~ 10 | 3-2=1 | 1 | 1 | 1 | — |
| | ~~30~~ ~~30~~ ~~20~~ 0 | ~~20~~ 0 | ~~20~~ 0 | ~~20~~ 0 | 90 / 90 | | | | | |

Column penalty.

| | | | |
|---|---|---|---|
| 2-1=1 | 3-2=1 | 3-1=2 | 5-3=2 X |
| 1 | 1 | 2 X | — |
| 1 X | 1 | — | — |
| 1 X | 1 | — | — |
| — | — | — | — |

Total cost $= 0\times1 + 20\times1 + 20\times3 + 20\times4 + 10\times2 + 20\times3$

$= 240$ \$

Now check the optimality:

$u_1 + v_1 = 1$    $u_1 = 0$, $\boxed{v_1 = 1}$

$u_1 + v_3 = 1$    $\boxed{v_3 = 1}$

$u_2 + v_1 = 3$    $\boxed{u_2 = 2}$

$u_2 + v_2 = 4$    $\boxed{v_2 = 2}$

$u_3 + v_1 = 2$    $\boxed{u_3 = 1}$

$u_3 + v_4 = 3$    $\boxed{v_4 = 2}$

The solution is optimal because all non Basic have neg or zero coefficient.

Source 2 consume or shift to Destination 1 and 2 with units ~~[illegible]~~ 20\$ and 20 respectively.

(8)

Problem 5-11 Pg 238

Q No 11

Retailer

| | | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|---|
| orchard | 1 | 1\$ | 2\$ | 3\$ | 2\$ | 150 |
| | 2 | 2 | 4 | 1 | 2 | 200 |
| | 3 | 1 | 3 | 5 | 3 | 250 |
| | | 150 | 150 | 400 | 100 | 800 total demand |

150 + 200 + 250 → 600

orchard one & two they could supply more crates daily

so

Final

Retailer

| | | 1 | 2 | 3 | 4 | Dummy | | |
|---|---|---|---|---|---|---|---|---|
| orchard | 1 | 1 | 2 | 3 | 2 | 0\$ | 150+200 | 350 |
| | 2 | 2 | 4 | 1 | 2 | 0\$ | 200+200 | 400 |
| | 3 | 1 | 3 | 5 | 3 | M\$ | 250 | 250 |
| | | 150 | 150 | 400 | 100 | 200 | 1000 | 1000 |

Solve By Vogel Method, Let M = 106

| | | | | | | Row Penalty | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $1^{\times}$ | $2$ (150) | $3^{\times}$ | $2$ (~~100~~) | $0$ (200) | ~~350~~ ~~200~~ 100 | 1 | 1 | 2 | 2 | – | – |
| $2^{\times}$ | $4^{\times}$ | $1$ (400) | $2^{\times}$ | $0^{\times}$ | ~~400~~ 0 | 1 | 2 | 2 | 2 | 2 | 2 |
| $1$ (150) | $3^{\times}$ | $5^{\times}$ | $3$ (100) | ~~M~~ 100 | ~~250~~ 100 | 2 | 2 | 0 | 0 | 93 | – |
| ~~150~~ 0 | ~~150~~ 0 | ~~400~~ 0 | ~~100~~ 0 | 200 | 1000 / 1000 | | | | | | |

Column Penalty

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 2 | 0 | 0 |
| 0 | 1 | – | 0 | 0 |
| – | 1 | – | 0 | 0 |
| – | 1 | – | 1 | 0 |
| – | 1 | – | 1 | 0 |
| – | ~~1~~ | – | – | 0 |

# Assignment Model

Supply = Demand = 1

↓

Then development of Hungarian method.

9

Jobs

Workers

$$
\begin{array}{c|cccc|c}
 & 1 & 2 & 3 \cdots & m & \\
\hline
1 & c_{11} & c_{12} & \cdots & c_{1m} & 1 \\
2 & c_{21} & c_{22} & \cdots & c_{2m} & 1 \\
3 & \vdots & & \vdots & \vdots & \\
\vdots & \vdots & & & \vdots & \\
N & c_{n1} & c_{n2} & \cdots & c_{nm} & 1 \\
\hline
 & 1 & 1 & & 1 &
\end{array}
$$

## Q 5-33

The goal is to minimum cost assignment to jobs.

→ Deals with matching workers to jobs.

→ Special case of transportation model

| | | | |
|---|---|---|---|
| 50 | 50 | M=100 | $\boxed{20}$ |
| 70 | 40 | $\boxed{20}$ | 30 |
| 90 | $\boxed{30}$ | 50 | M=100 |
| 70 | $\boxed{20}$ | 60 | 70 |

i) Row Reduction

workers - sources

jobs - destination

→ Thus supply (demand) at each source (destination) exactly equals 1.

→ The cost of transporting worker i to job j is $c_{ij}$.

i) Row Reduction

| | | | |
|---|---|---|---|
| $\boxed{30}$ | 30 | 80 | $\boxed{0}$ |
| 50 | 20 | $\boxed{0}$ | 10 |
| 60 | $\boxed{0}$ | 20 | 70 |
| 50 | 0 | 40 | 50 |

Column Reduction

10

2

| Worker \ Job | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $\boxed{0}$ | 30 +10 | 80 +10 | ~~0~~ |
| 2 | 20 | 20 | $\boxed{0}$ | * 10 → minimum uncovered |
| 3 | 30 | $\boxed{0}$ | 20 | 70 −10 |
| 4 | 20 | ~~0~~ | 40 | 50 −10 |

Job 4 is not assigned to anyone so this is not optimal soln.

- Search single zero for row wise or column wise then select it.
- If ~~Searching~~ selected zero is found in row wise searching, cross another zero in associated column and vice versa
- If there is no zero row wise and column wise for then select any zero Arbitrary and cross all the zero in the associated row and column.

Not optimal.

Improvement:

Step 1. Draw minimum possible vertical and Horizontal line to cover all zero [ ~~No of line~~

Step Two

## Step Two

- Select the minimum uncovered (which is not include in these lines) element and subtract it from all uncovered element.

- Add it (minimum uncovered element) to the element lie on the intersection of the line.

11

| | | | |
|---|---|---|---|
| $\boxed{0}$ | 40 | 90 | ×0 |
| 10 | 20 | $\boxed{0}$ | ×0 |
| 20 | $\boxed{0}$ | 20 | 60 |
| 10 | 0× | 40 | 40 |

Job 4 is not Assigned
not optimal.

Jobs

| Workers | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $\boxed{0}$× | 50 | 90 | ③ $\boxed{0}$ |
| 2 | 10 | 30 | ② $\boxed{0}$ | ×0 |
| 3 | 10 | ① $\boxed{0}$ | 10 | 50 |
| 4 | $\boxed{0}$ ④ | 0× | 30 | 30 |

All Jobs are Assigned

Assignments are

1 ⟶ 4, 2 ⟶ 3, 3 ⟶ 2, 4 ⟶ 1.

Total cost = 20 + 20 + 30 + 70 = 140 B

Problem 5-34

~~ERR~~

12

QNo3) if worker 5 are apply for Job then we add a dummy Job for him.

so final Assignment Model is.

| | Jobs 1 | 2 | 3 | 4 | Dummy job |
|---|---|---|---|---|---|
| workers 1 | $\boxed{50}$ | 50 | 100 | $\boxed{20}$ | 0$ |
| 2 | 70 | 40 | $\boxed{20}$ | 30 | 0$ |
| 3 | 90 | 30 | 50 | 100 | 0$ |
| 4 | 70 | $\boxed{20}$ | 60 | 70 | 0$ |
| 5 | 70 | 50 | 30 | 90 | 0$ |

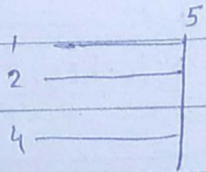
Row Reduction

we get same matric Because every row have least element is zero.

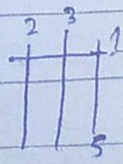
Column Reduction

| | | | | |
|---|---|---|---|---|
| $\boxed{0}$ | 30 | 80 | 0 | 0 |
| 20 | 20 | $\boxed{0}$ | 10 | 0 |
| 40 | 10* | 30 | 80 | $\boxed{0}$ |
| 20 | $\boxed{0}$ | 40 | 50 | 0 |
| 20 | 30 | 10 | 70 | 0 |

Job 4 is not assigned to any one so it's not optimal.




| | | | | |
|---|---|---|---|---|
| $\boxed{0}$ | 30 | 8 | 0 | 10 |
| 20 | 20 | $\boxed{0}$ | 10 | 10 |
| 30 | 0 | 20 | 70 | $\boxed{0}$ |
| 20 | $\boxed{0}$ | 40 | 50 | 10 |
| 10* | 20 | 0 | 60 | 0 |




Job '4' not assigned. so it's not optimal.

555



Dumy Job

| | | | | |
|---|---|---|---|---|
| ~~0~~ | 40 | 18 | $\boxed{0}$ | 20 |
| 10 | 20 | $\boxed{0}$ | ~~0~~ | 10 |
| 20 | ~~0~~ | 20 | 60 | $\boxed{0}$ |
| 10 | $\boxed{0}$ | 40 | 40 | 10 |
| $\boxed{0}$ | 20 | ~~0~~ | 50 | ~~0~~ |

So every Job is assigened to workeos so it's optimal.

~~Job~~ "Dummy Job is assigned to workeo 3"

So workeo 5 must replaced by workeo 3.

optimal cost = 70 + 20 + 20 + 20 + 0

= 130.