Dual

Primal	Dual	
objective	Objective	Constraint type
max	min	$\geq$
min	max	$\leq$

Primal : Max $z = 5x_1 + 12x_2 + 4x_3$

subject to

$$x_1 + 2x_2 + x_3 \leq 10 \qquad y_1$$

$$2x_1 - x_2 + 3x_3 = 8 \qquad y_2$$

$$x_1, x_2, x_3 \geq 0$$

Dual : Min $w = 10y_1 + 8y_2$

subject to

$$y_1 + 2y_2 \geq 5$$

$$2y_1 - y_2 \geq 12$$

$$y_1 + 3y_2 \geq 4$$

⇒ If dual is minimize tw
Agr y_i ki equation $\leq$ hui tw
wo $y \geq 0$, agr $\geq$ hui ty $y \leq 0$
and agr $=$ hui tw unrestricted.

⇒ If dual is max tw
constraint $\geq$ → variable ≥ 0
constaint $\leq$ → variable ≤ 0
constraint $=$ → variable unrestr

Determining Dual Values.

Method 1:

(optimal value of dual variable y_i) = (optimal primal z-coeff of starting basic variable x_i + original objective coeffecient of x_i)

Method 2:

(optimal values of dual variables) = (Row vector of original obj coeff of optimal primal basic variables) $\times$ (optimal primal inverse)

Primal Dual Objective values.

$$\begin{pmatrix}\text{objective value}\\ \text{of maximization}\\ \text{problem}\end{pmatrix} \leq \begin{pmatrix}\text{objective value}\\ \text{of minimization}\\ \text{problem}\end{pmatrix}$$

if $z < w$ → not optimal.

Simplex Tableau computations

Formula 1: constraint column computation

$$\begin{pmatrix}\text{constraint}\\ \text{col in}\\ \text{iteration } i\end{pmatrix} = \begin{pmatrix}\text{inverse}\\ \text{in itera-}\\ \text{tion } i\end{pmatrix} \times \begin{pmatrix}\text{original}\\ \text{contraint}\\ \text{column}\end{pmatrix}$$

Formula 2: Objective z-row computation

$$\begin{pmatrix}\text{Primal } z\text{-}\\ \text{coeff of}\\ \text{variable } x_j\end{pmatrix} = \begin{pmatrix}\text{left hand}\\ \text{side of } j\text{th}\\ \text{dual constraint}\end{pmatrix} - \begin{pmatrix}\text{right hand}\\ \text{side of } j\text{th}\\ \text{dual constraint}\end{pmatrix}$$

to check feasibility => inverse matrix × sol col

to check optimality => original z-coeff × inverse matrix.

ADDITIONAL SIMPLEX METHODS

Dual — infeasible
Primal — non-optimal
Generalized — infeasible + non optimal

=> It is infeasible if solution column has any -ve value.

Dual Simplex Requirements.
=> objective function must be optimal
=> constraints must be of type ($\leq$)
↳ agr koi constraint ($\leq$) nhi ho ga tw us eq ki dono sides of -1 se multiply kra k sign change krain ge.

⇒ If LP includes (=) constraint then replace it by two inequalities

e.g. $x_1 + x_2 = 1$

$\Downarrow$

1) $x_1 + x_2 \leq 1$

2) $x_1 + x_2 \geq 1 \Rightarrow -x_1 - x_2 \leq -1$

For optimality ⇒ z-row values in table should be -ve in min

⇒ z-row values in table should be +ve in max

⇒ most -ve from sol column.

⇒ least +ve from ratio $\left(\frac{c_j}{\alpha_j}, \alpha_j < 0 \right)$

* If α_j is ≥ 0 for all non basic variables then problem has no feasible solution.

continue untill we have all +ve values in the sol column.

CHAPTER # 05

transportation model

- Balanced
- unbalanced
  - demand > supply
    - * add dummy row
  - demand < supply
    - * add dummy column.

feasibility $m+n-1$ = no. of allocated cells.

North-Corner Problem

$\boxed{5}$ 10	$\boxed{10}$ 2	20	11	~~15~~ ~~10~~ 0
12	$\boxed{5}$ 7	$\boxed{15}$ 9	$\boxed{5}$ 20	~~25~~ ~~20~~ ~~5~~ 0
4	14	16	$\boxed{10}$ 18	~~10~~ 0
~~5~~ 0	~~15~~ ~~5~~ 0	~~15~~ 0	~~15~~ ~~10~~ 0	50 / 50

feasibility = $n+m-1 = 4+3-1 = 6$

Thus, it is feasible.

cost = $5 \times 10 + 10 \times 2 + 5 \times 7 + 15 \times 9$
$+ 5 \times 20 + 10 \times 18$
$= 520$

least cost method

10	2 $\boxed{15}$	20	11	$\cancel{15}$ 0
12	7	$\boxed{15}$ 9	$\boxed{10}$ 20	$\cancel{25}$ $\cancel{15}$ $\cancel{10}$ 0
4 $\boxed{5}$	14	16	$\boxed{5}$ 18	$\cancel{10}$ $\cancel{5}$ 0
$\cancel{5}$ 0	$\cancel{15}$ 0	$\cancel{15}$ 0	$\cancel{15}$ $\cancel{10}$ 0	50 / 50

cost = $15 \times 2 + 15 \times 9 + 10 \times 20 +$
$5 \times 4 + 5 \times 18 = 475$

* table 5.10 pg 217 (why are we allocating 0 at 11)

VAM

16	[15] 2	20	11	~~15~~ 0	8	9	
12	7	[15] 9	[10] 20	~~25~~ ~~10~~	2	2	11
[5] 4	14	16	[5] 18	~~10~~ ~~5~~ 0	10	2	2
~~5~~ 0	~~15~~ 0	~~15~~ 0	~~15~~ ~~10~~ 0	50 / 50			
6	5	7	7				
		7	2				

$$\text{cost} = 15 \times 2 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = 475$$

Method of Multipliers.

First solve by any method.

Starting iteration

	V_1		V_2		V_3		V_4		
u_1	$\boxed{5}$	10	$\boxed{10}$	2	$\boxed{-16}$	20	$\boxed{4}$	11	15
v_2	$\boxed{3}$	12	$\boxed{5}$	7	$\boxed{15}$	9	$\boxed{5}$	20	25
u_3	$\boxed{9}$	4	$\boxed{-9}$	14	$\boxed{-9}$	16	$\boxed{10}$	18	10
	5		15		15		15		50 / 50

for basic variables

making equations. $\boxed{u_1 = 0}$

① $u_1 + v_1 = 10$ $u_1 = 0$

$v_1 = 10$

② $u_1 + V_2 = 2$ $u_1 = 0$

$V_2 = 2$

③ $u_2 + v_2 = 7$ $v_2 = 2$

$u_2 = 5$

(4) $U_2 + V_3 = 9$ $U_2 = 5$

$V_3 = 4$

(5) $U_2 + V_4 = 20$ $U_2 = 5$

$V_4 = 15$

(6) $U_3 + V_4 = 18$ $V_4 = 15$

$U_3 = 3$

Now, we have

$U_1 = 0$ $U_2 = 5$ $U_3 = 3$

$V_1 = 10$ $V_2 = 2$ $V_3 = 4$ $V_4 = 15$

to evaluate non-basic variables

$U_i + V_j - C_{ij}$

$x_{13} = U_1 + V_3 - C_{13}$

$x_{13} = 0 + 4 - 20 = -16$

$x_{14} = U_1 + V_4 - C_{14}$

$= 0 + 15 - 11$

$x_{14} = 4$

$$x_{21} = U_2 + V_1 - C_{21}$$
$$= 5 + 10 - 12 = 3$$

$$x_{31} = U_3 + V_1 - C_{31}$$
$$= 3 + 10 - 4 = 9 \rightarrow \text{entering variable (most +ve)}$$

$$x_{32} = U_3 + V_2 - C_{32}$$
$$= 3 + 2 - 14 = -9$$

$$x_{33} = U_3 + V_3 - C_{33}$$
$$= 3 + 4 - 16 = -9$$

iteration 1:

$\boxed{5}$ $5-\theta$ 10	$\boxed{10}$ $10+\theta$ 2	$\boxed{-16}$ 20	$\boxed{4}$ 11	15
$\boxed{3}$ 12	$\boxed{5}$ $5-\theta$ 7	$\boxed{15}$ 9	$\boxed{5}$ $5+\theta$ 20	25
$\boxed{9}$ θ 4	$\boxed{-9}$ 14	$\boxed{-9}$ 16	$\boxed{10}$ $10-\theta$ 18	10
5	15	15	15	

$\theta = 5$

entering variable : most +ve non-basic variable

leaving variable : basic variable with smallest value in corner of loop where sign is -ve.

<u>iteration 2</u> :

$\boxed{0}$ -9 10	$\boxed{15}$ $15-\theta$ 2	$\boxed{-16}$ 20	θ $\boxed{4}$ 11
$\boxed{3}$ -6 12	$\boxed{0}$ $0+\theta$ 7	$\boxed{15}$ 9	$\boxed{10}$ $10-\theta$ 20
$\boxed{5}$ 4	$\boxed{-9}$ 14	$\boxed{-9}$ 16	$\boxed{5}$ 18

$\rightarrow U_1 + V_2 = 2$ $\qquad U_1 = 0$

$V_2 = 2$

$\rightarrow U_2 + V_2 = 7$ $\quad V_2 = 2$

$U_2 = 5$

$\rightarrow U_2 + V_3 = 9$ $\quad U_2 = 5$

$V_3 = 4$

$\rightarrow U_2 + V_4 = 20$ $\quad U_2 = 5$

$V_4 = 15$

$\rightarrow U_3 + V_1 = 4$ $\quad U_3 = 3$

$V_1 = 1$

$\rightarrow U_3 + V_4 = 18$ $\quad V_4 = 15$

$U_3 = 3$

$U_{11} = U_1 + V_1 - C_{11} = 0+1-10 = -9$

$U_{13} = U_1 + V_3 - C_{13} = 0+4-20 = -16$

$U_{14} = U_1 + V_4 - C_{14} = 0+15-11 = 4$

$U_{21} = U_2 + V_1 - C_{21} = 5+1-12 = -6$

$U_{32} = U_3 + V_2 - C_{32} = 3+2-14 = -9$

$U_{33} = U_3 + V_3 - C_{33} = 3+4-16 = -9$

$\boxed{-9}$ 10	$\boxed{5}$ 2	$\boxed{-16}$ 20	$\boxed{10}$ 11
$\boxed{-6}$ 12	$\boxed{10}$ 7	$\boxed{15}$ 9	$\boxed{0}$ 20
$\boxed{5}$ 4	$\boxed{-9}$ 14	$\boxed{-9}$ 16	$\boxed{5}$ 18

→ $U_1 + V_2 = 2$ $\quad$ $U_1 = 0$, $V_2 = 2$

→ $U_1 + V_4 = 11$ $\quad$ $U_1 = 0$, $V_4 = 11$

→ $U_2 + V_2 = 7$ $\quad$ $V_2 = 2$, $U_2 = 5$

→ $U_2 + V_3 = 9$ $\quad$ $U_2 = 5$, $V_3 = 4$

→ $U_3 + V_4 = 4$ $\quad$ $U_3 = 7$, $V_1 = -3$

→ $U_3 + V_4 = 18$ $\quad$ $V_4 = 11$, $U_3 = 7$

$U_{11} = U_1 + V_1 - C_{11} = 0-3-10 = -13$

$U_{13} = U_1 + V_3 - C_{13} = 0+4-20 = -16$

$U_{21} = U_2 + V_1 - C_{21} = 5-3-12 = -10$

$U_{24} = U_2 + V_4 - C_{24} = 5+11-20 = -4$

$U_{32} = U_3 + V_2 - C_{32} = 7+2-14 = -5$

$U_{33} = U_3 + V_3 - C_{33} = 7+4-16 = -5$

<u>iteration 3:</u>

$\boxed{-13}$ 10	$\boxed{5}$ 2	$\boxed{-16}$ 20	$\boxed{10}$ 11
$\boxed{-10}$ 12	$\boxed{10}$ 7	$\boxed{15}$ 9	$\boxed{-4}$ 20
$\boxed{5}$ 4	$\boxed{-5}$ 14	$\boxed{-5}$ 16	$\boxed{5}$ 18

→ stop the iterations when all non-basic variables are -ve.

final cost $= 5\times2 + 10\times11 + 10\times7$
$+ 15\times9 + 5\times4 + 5\times18$
$= 435$

THE ASSIGNMENT MODEL

$$\begin{bmatrix} 15 & 10 & 9 \\ 9 & 15 & 10 \\ 10 & 12 & 8 \end{bmatrix}$$

* select min from each row and subtract

$$\begin{bmatrix} 6 & 1 & 0 \\ 0 & 6 & 1 \\ 2 & 4 & 0 \end{bmatrix}$$

* Now select min from each col and subtract

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 5 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

* take original values at thes positions and add to get cost

total cost $= 9 + 10 + 8$

$= 27$

$$\begin{bmatrix} 1 & 4 & 6 & 3 \\ 9 & 7 & 10 & 9 \\ 4 & 5 & 11 & 7 \\ 8 & 7 & 8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 3 & 2 \\ 0 & 1 & 7 & 3 \\ 3 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & 0 & 2 \\ 0 & 1 & 4 & 3 \\ 3 & 2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 5 & 2 \\ 2 & 0 & \cancel{0} & 2 \\ \cancel{0} & 1 & 4 & 3 \\ 3 & 2 & 0 & \cancel{0} \end{bmatrix}$$

⇒ cuz sb ko allocate nhi ho skt tw agla kam krain ge

* select smallest uncovered entry, subtract from all uncovered entries and add it to every entry at intersection point of lines.

$$\begin{bmatrix} 0 & 2 & 4 & 1 \\ 3 & \cancel{0} & 0 & 2 \\ \cancel{0} & 0 & 3 & 2 \\ 4 & 2 & \cancel{0} & 0 \end{bmatrix}$$

$$\text{cost} = 1 + 5 + 10 + 5$$
$$= 21$$