# Probability and Statstics

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# Textbooks

- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

- **Elementary Statistics: Picturing the World,** $6^{th}$ Edition, Ron Larson and Betsy Farber

- **Elementary Statistics**, $13^{th}$ Edition, Mario F. Triola

# Reference books

- **Probability Demystified**, Allan G. Bluman
- **Schaum's Outline of Probability and Statistics**
- **MATLAB Primer**, Seventh Edition
- **MATLAB Demystified** by McMahon, David

# References

Readings for these lecture notes:

- **Schaum's Outline of Probability, Second Edition (Schaum's Outlines)**

  by Seymour Lipschutz, Marc Lipson

- **Probability & Statistics for Engineers & Scientists**, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

These notes contain material from the above resources.

# Finite Stochastic Processes And Tree Diagrams

- A (finite) sequence of experiments in which each experiment has a finite number of outcomes with given probabilities is called a **(finite)** hypothetical/random **stochastic process**.

- A convenient way of describing such a process and computing the probability of any event is by a **tree diagram.**

**Example:**

We are given three boxes as follows:

**Box 1** has **10** light bulbs of which **4** ~~axe~~ are defective.

**Box 2** has **6** light bulbs of which **1** is defective.

**Box 3** has **8** light bulbs of which **3** are defective.

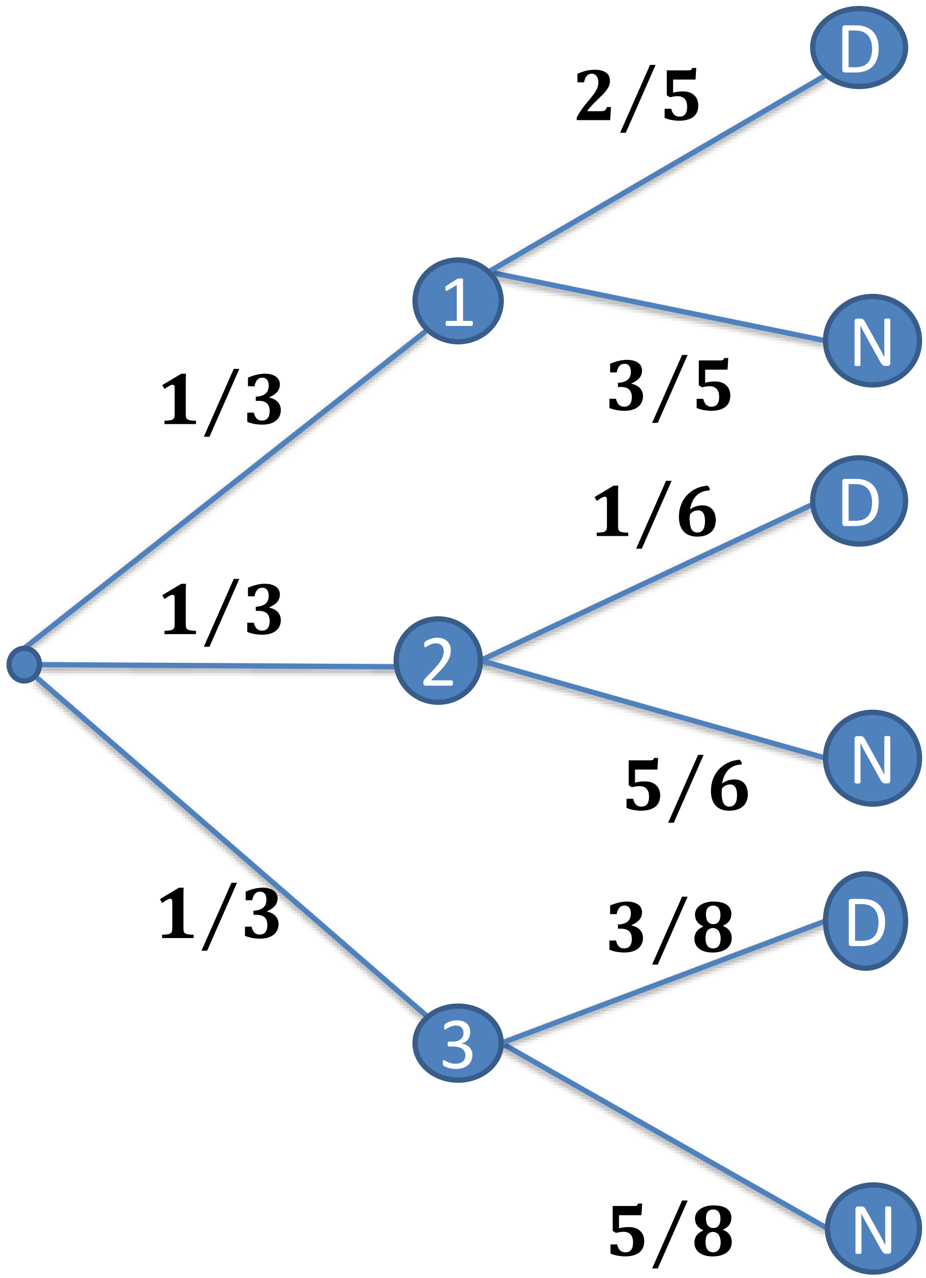
We select a box at random and then draw a bulb at random. What is the probability **p** that the **bulb** is **defective**?

**Solution:**

Here we perform a sequence of two experiments:

(i) select one of the three boxes;

(ii) select a bulb which is either defective ***(D)*** or nondefective ***(N).***


2/5
D
1
N
3/5
1/3
1/6
D
1/3
2
N
5/6
1/3
3/8
D
3
5/8
N

Theorem of total probability or rule of elimination

Thus by the multiplication theorem,

Prob of defective1 given that box1 is selected

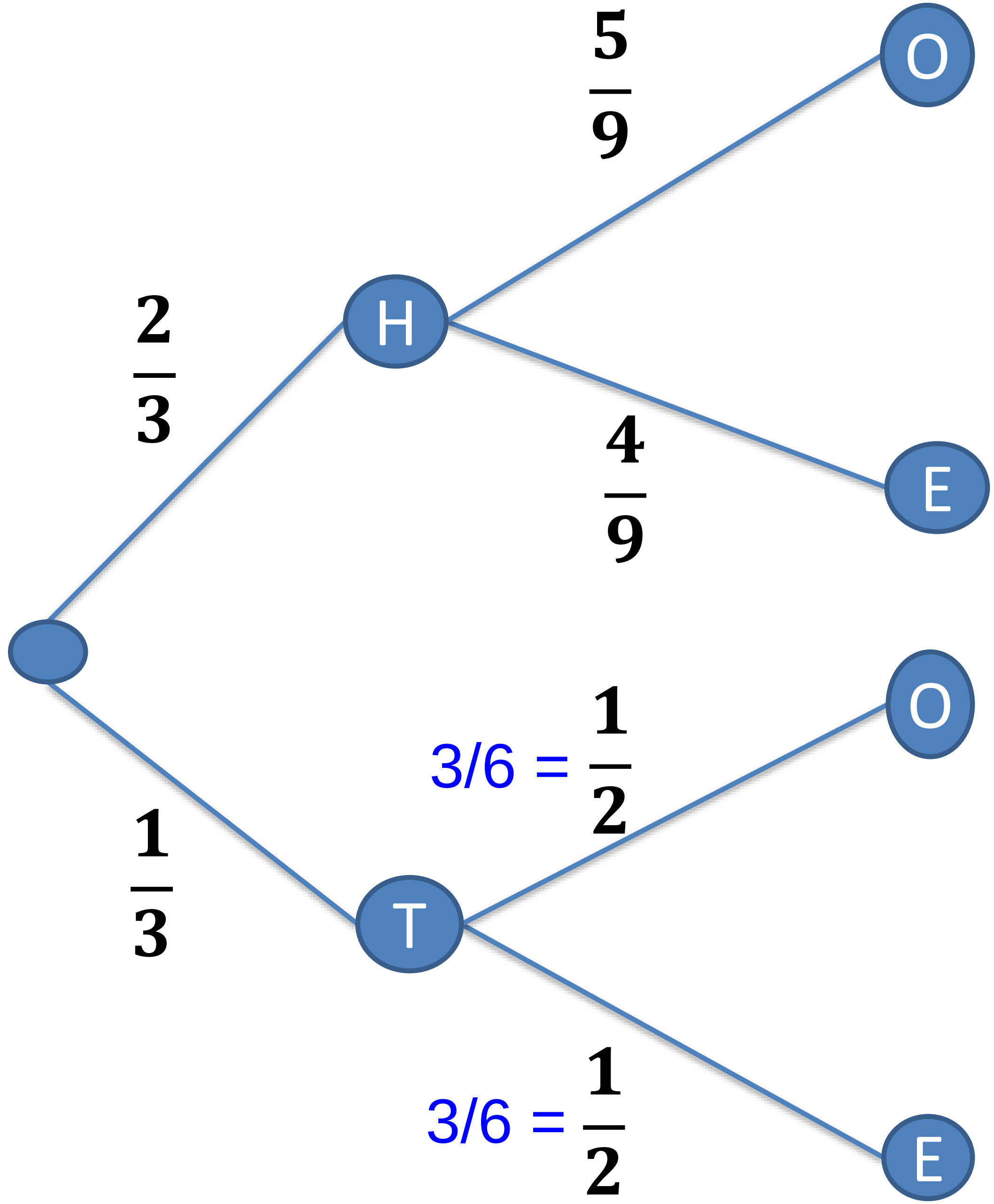
$$\mathbf{p} = \left(\frac{1}{3}\right)\left(\frac{2}{5}\right)+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)+\left(\frac{1}{3}\right)\left(\frac{5}{8}\right) = \frac{\mathbf{113}}{\mathbf{360}}$$

P(box1 | defect1) = P(box1 and defect1) / P(defec1)

=> P(box1 and defect1) = P(defect1) x P(box1 | defect1)

**Example :** A coin, weighted so that **P(H) =** $\mathbf{\frac{2}{3}}$ and **P(T) =** $\mathbf{\frac{1}{3}}$, is tossed. If **heads** appears, then a number is selected at random from the numbers **1 through 9**; if **tails** appears, then a number is selected at random from the numbers **1 through 6.**

Find the probability p that an **even number** is selected.


$\frac{5}{9}$
O
$\frac{2}{3}$
H
$\frac{4}{9}$
E
O
$3/6 = \frac{1}{2}$
$\frac{1}{3}$
T
$3/6 = \frac{1}{2}$
E

**Probability of even using 1 through 9:**

$$P(E) = \frac{4}{9}$$

**Probability of even using 1 through 6:**

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

$$\mathbf{p} = (\frac{2}{3})(\frac{4}{9})+(\frac{1}{3})(\frac{1}{2}) = \mathbf{\frac{25}{54}}$$