

Subject: //

position of helicopter  $\longrightarrow$  how to move control sticks  
 state  $s \longrightarrow$  action  $a$

Option 1:  $\rightarrow$  NOT good

use supervised learning



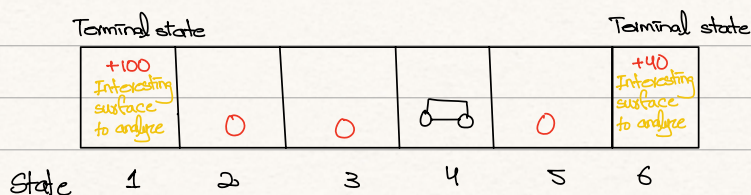
Option 2:

Reinforcement learning  $\rightarrow$  What to do NOT How

Reward function:

flying well  $+1$   
 Crash  $-1000$

Max Rover Example:



state	4	3	2	1	state	4	5	6	
	0	0	0	100		0	0	40	$(s, a, R(s), s')$

$\hookrightarrow$  A new state

state	4	5	4	3	2	1
	0	0	0	0	0	100

The Return of RL:  $\rightarrow$  how to know if a particular reward is better than another.

$$\text{Return} = 0 + 0(0.9) + 0(0.9)^2 + (0.9)^3 100 = 72.9$$

$$= R_1 + \gamma R_2 + \gamma R_3 + \dots \text{until terminal state}$$

Subject:  $\rightarrow$  A little bit less than 1

Discount factor  $\gamma = 0.9$

• This way, getting reward sooner would result in higher return

Policy:

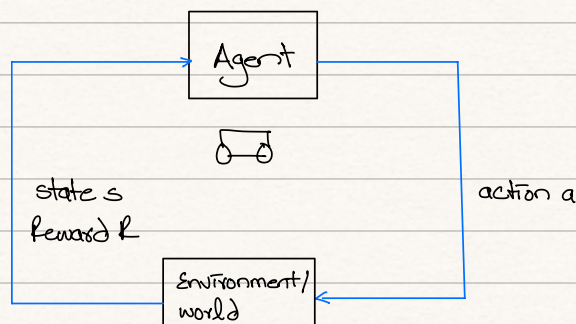
state  $\xrightarrow[\pi]{\text{policy}}$  action  
 $s \quad a$

Find a policy  $\pi$  that tells you what action ( $a = \pi(s)$ ) to take in every state ( $s$ ) so as to maximize the return

Markov Decision Process (MDP):

- states
- actions
- Rewards
- Discount factor  $\gamma$
- Return
- policy  $\pi$

• Future depends on where you are now NOT how you got here



Subject:

State action value function (Q-function)

↳ How good is it?

$Q(s, a)$  = Return if you

- start in state  $s$
- Take action  $a$  (once)
- Then behave optimally after that

$Q(2, \leftarrow)$   $Q(2, \rightarrow)$

100	100	50	12.5	25	6.25	12.5	10	6.25	20	40	40
100	0	0	0	0	0	0	0	0	0	40	40
State	1	2	3	4	5	6					

$\gamma = 0.5$

+100 Interesting surface to analyze	50 ← 0	25 ← 0	12.5 ← 0	20 → 0	+40 Interesting surface to analyze	
State	1	2	3	4	5	6

- The best possible return from state  $s$  is  $\max_a Q(s, a)$
- The best possible action in state  $s$  is the action that gives  $\max_a Q(s, a)$

NOTE:

$Q = Q^*$  = Optimal Q function

Bellman equation:

$s$  = current state

$R(s)$  = Reward of current state

$a$  = current action

$s'$  = state you get after action  $a$

$a'$  = action you take in state  $s'$



Subject:

$$Q(s, a) = R(s) + \gamma \max_{a'} Q(s', a')$$

Reward you get right away  
Return from behaving optimally starting from state  $s'$

$Q(2, \leftarrow)$   $Q(2, \rightarrow)$

100	100	50	12.5	25	6.25	12.5	10	6.25	20	40	40
100	0	0	0	0	0	0	0	0	0	40	40

$$\gamma = 0.5$$

State 1 2 3 4 5 6

Example 1:

$$s = 2$$

$$a = \rightarrow$$

$$s' = 3$$

Example 2:

$$s = 4$$

$$a = \leftarrow$$

$$s' = 3$$

$$Q(2, \rightarrow) = R(2) + 0.5 \max_{a'} Q(3, a')$$

$$= 0 + (0.5) 25 = 12.5$$

$$Q(4, \leftarrow) = R(4) + 0.5 \max_{a'} Q(3, a')$$

$$= 0 + 0.5 (25)$$

$$= 12.5$$

For terminal state,

$$Q(s, a) = R(s)$$

The best possible return from state  $s'$  is  $\max_{a'} Q(s', a')$

Random (stochastic) environment:

$\leftarrow$	$\leftarrow$	$\leftarrow$	$\leftarrow$	$\rightarrow$	
100	0	0	0	0	40

State 1 2 3 4 5 6

4	3	2	1		
0	0	0	100		
4	3	4	3	2	1
0	0	0	0	0	100
4	5	6			
0	0	40			

slips & goes to 4 instead

Subject: / /

$$\text{Expected return} = \text{Return} = \text{Average} (R_1 + \gamma R_2 + \gamma^2 R_3 + \dots) \\ = E[R_1 + \gamma R_2 + \gamma^2 R_3 + \dots]$$

Bellman eq:

$$Q(s, a) = R(s) + \gamma E[\max_{a'} Q(s', a')]$$

Continuous state spaces:

For a truck:

$$s = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

*Annotations:*  
→ How quickly  $x$  is changing (points to  $\dot{x}$ )  
→ " $y$ " (points to  $\dot{y}$ )  
→ " $\theta$ " (points to  $\dot{\theta}$ )

Lunar Lander:

$a$  = Do nothing

left thruster ←

Main thruster ↓

Right thruster →

$$s = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \\ \theta \\ \dot{\theta} \\ l \\ d \end{bmatrix}$$

*Annotations:*  
? left or right (points to  $l$ )  
? leg on ground (0 or 1) (points to  $d$ )

Reward:

Land: 100 - 140

Crash: -100

Soft landing: +100

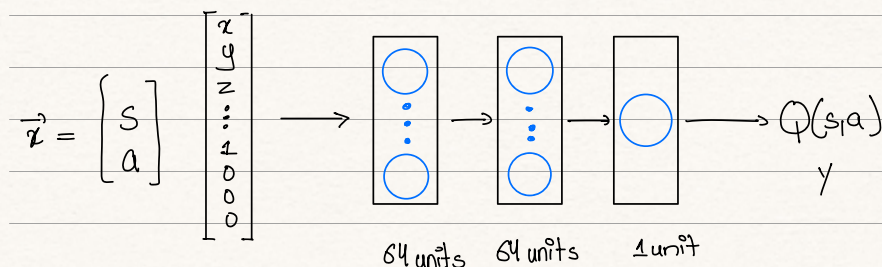
leg grounded: +10

Fire main engine: -0.3

Fire side thrusters: -0.03

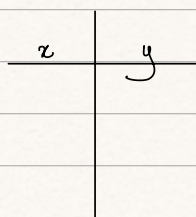
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Learning the state value function:



In state  $s$ , use neural network to compute  $Q(s, \text{nothing})$ ,  $Q(s, \text{left})$ ,  $Q(s, \text{right})$ ,  $Q(s, \text{main})$ . Pick the action  $a$  that maximizes  $Q(s,a)$ .

$$Q(s,a) = \underbrace{R(s)}_z + \underbrace{\gamma \max_{a'} Q(s',a')}_y$$



{ create examples  
from simulation

$$(s,a, R(s), s')$$

$$\begin{aligned} & \underbrace{(s^{(1)}, a^{(1)})}_x, \underbrace{R(s^{(1)}), s'^{(1)}}_y \\ & (s^{(2)}, a^{(2)}), R(s^{(2)}), s'^{(2)} \\ & \vdots \end{aligned}$$

$$(s^{(m)}, a^{(m)}, R(s^{(m)}), s'^{(m)})$$

z	y
$x^{(1)} = (s^{(1)}, a^{(1)})$	$y^{(1)}$
$x^{(2)} = (s^{(2)}, a^{(2)})$	$y^{(2)}$

$$\begin{aligned} y^{(1)} &= R(s^{(1)}) + \gamma \max_{a'} Q(s'^{(1)}, a') \\ y^{(2)} &= R(s^{(2)}) + \gamma \max_{a'} Q(s'^{(2)}, a') \end{aligned}$$

- Initialize neural network randomly as guess of  $Q(s,a)$ .
- Repeat { ?  $\Rightarrow$  greedy policy

Take actions in lunar lander, get  $(s,a, R(s), s')$

Store the 10,000 more recent  $(s,a, R(s), s')$  tuples

↳ replay buffer



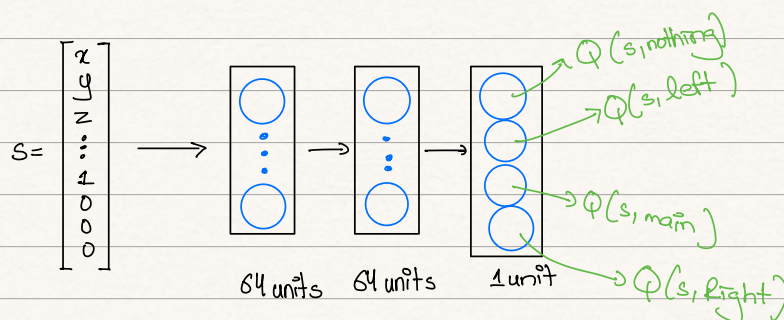
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Train neural network.

- Create training set of 10,000 examples using  
 $x = (s, a)$        $y = R(s) + \gamma \max_{a'} Q(s, a')$
- Train  $Q_{\text{new}}$  such that  $Q_{\text{new}}(s, a) \approx y$
- set  $Q = Q_{\text{new}}$

}

Improved NN.



Epsilon greedy policy: *How pick a while reasoning*

Option 1:

pick  $a$ , that maxs  $Q(s, a)$

Option 2:

with probability 0.95, pick  $a$  that maxs  $Q(s, a)$  "greedy, exploitation"  
with probability 0.05, pick  $a$  randomly "exploration"

• This is better because if  $Q(s, \text{main})$  set to low in step 1 then, it will never learn to fire main thruster.



$\epsilon$  greedy policy ( $\epsilon = 0.05$ )

NOTE:  
start  $\epsilon$  high  
Decrease gradually

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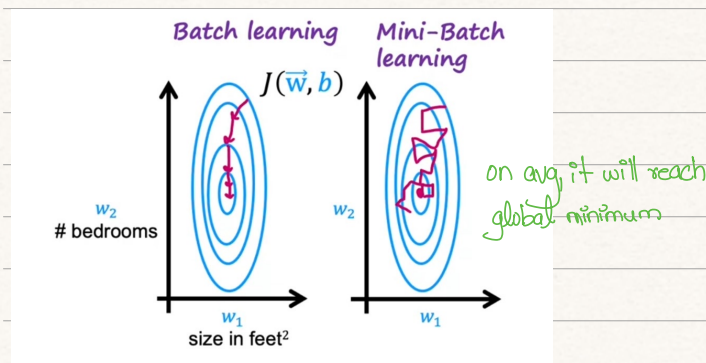
Mini-batches & soft updates:

If  $m$  is very large, we use a mini batch.

$x$	$y$
2104	400
1416	232
$\vdots$	$\vdots$
3210	870

$\left. \begin{array}{l} \text{2104} \\ \text{1416} \end{array} \right\} \text{mini batch 1}$   
 $\left. \begin{array}{l} \vdots \\ \text{3210} \end{array} \right\} \text{mini batch 2}$

• Every iteration look at a portion of dataset



Soft updates:

Set  $Q = Q_{\text{new}}$

$\begin{array}{cc} \uparrow \uparrow & \uparrow \uparrow \\ w & b \end{array} \quad \begin{array}{cc} \uparrow \uparrow & \uparrow \uparrow \\ w_{\text{new}} & b_{\text{new}} \end{array}$

$$\begin{aligned} W &= 0.01 W_{\text{new}} + 0.99 W \\ B &= 0.01 W_{\text{new}} + 0.99 B \end{aligned} \quad \left\{ \begin{array}{l} 99\% \text{ old } W \\ 1\% \text{ new } B \end{array} \right.$$



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**Algorithm 1: Deep Q-Learning with Experience Replay**

```
1 Initialize memory buffer  $D$  with capacity  $N$ 
2 Initialize  $Q$ -Network with random weights  $w$ 
3 Initialize target  $\hat{Q}$ -Network with weights  $w^- = w$ 
4 for episode  $i = 1$  to  $M$  do
5   Receive initial observation state  $S_1$ 
6   for  $t = 1$  to  $T$  do
7     Observe state  $S_t$  and choose action  $A_t$  using an  $\epsilon$ -greedy policy
8     Take action  $A_t$  in the environment, receive reward  $R_t$  and next state  $S_{t+1}$ 
9     Store experience tuple  $(S_t, A_t, R_t, S_{t+1})$  in memory buffer  $D$ 
10    Every  $C$  steps perform a learning update:
11    Sample random mini-batch of experience tuples  $(S_j, A_j, R_j, S_{j+1})$  from  $D$ 
12    Set  $y_j = R_j$  if episode terminates at step  $j + 1$ , otherwise set  $y_j = R_j + \gamma \max_{a'} \hat{Q}(S_{j+1}, a')$ 
13    Perform a gradient descent step on  $(y_j - Q(S_j, A_j; w))^2$  with respect to the  $Q$ -Network weights  $w$ 
14    Update the weights of the  $\hat{Q}$ -Network using a soft update
15  end
16 end
```