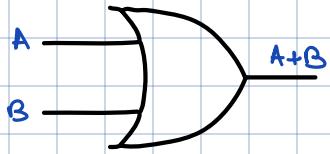


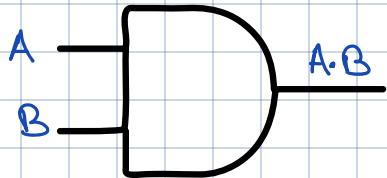
Boolean Addition:

- ↪ Order A, B doesn't matter
- ↪ OR gate



Boolean multiplication:

- ↪ Order of A ∙ B doesn't matter
- ↪ AND gate



Laws of Boolean algebra:

1. Commutative: $A+B=B+A$

2. Associative: $A+(B+C)=(A+B)+C$
 $A(BC)=(AB)C$

3. Distributive: $AB+AC=A(B+C)$

Rules of Boolean algebra:

1. $A+0 = A$
2. $A+1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A+A = A$
6. $A+\bar{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \bar{A} = 0$
9. $\bar{\bar{A}} = A$
10. $A+AB = A$
11. $A+\bar{A}B = A+B$
12. $(A+B)(A+C) = A+BC$

Proofs:

10. $A+AB = A$
 $A(1+B)$
 $A \cdot 1$
 A

12. $(A+B)(A+C) = A+BC$
 $AA+AC+AB+BC$
 $A(1+C+B)+BC$
 $A+BC$

11. $A+\bar{A}B = A+B$

$A+AB + \bar{A}B$
 $AA+AB+\bar{A}B+A\bar{A}$
 $A(A+\bar{A})+B(A+\bar{A})$
 $(A+B)(A+\bar{A})$
 $(A+B) \cdot 1$
 $A+B$

Sum of Products (SOP):

	A	B	C	
m_0	0	0	0	$\bar{A} \bar{B} \bar{C}$
m_1	0	0	1	$\bar{A} \bar{B} C$
m_2	0	1	0	$\bar{A} B \bar{C}$
m_3	0	1	1	$A \bar{B} \bar{C}$
m_4	1	0	0	$\bar{A} B C$
m_5	1	0	1	$A \bar{B} C$
m_6	1	1	0	$A B \bar{C}$
m_7	1	1	1	$A B C$

Conversion to SOP:

$$A + BC$$

$$A(B + \bar{B}) + BC(A + \bar{A})$$

$$AB + A\bar{B} + ABC + \bar{A}BC$$

$$AB(C + \bar{C}) + A\bar{B}(C + \bar{C}) + ABC + \bar{A}BC$$

$$ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$$

$$\begin{matrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

	A	B	C	X
m_0	0	0	0	$\bar{A} \bar{B} \bar{C}$
m_1	0	0	1	$\bar{A} \bar{B} C$
m_2	0	1	0	$\bar{A} B \bar{C}$
m_3	0	1	1	$A \bar{B} \bar{C}$
m_4	1	0	0	$\bar{A} B C$
m_5	1	0	1	$A \bar{B} C$
m_6	1	1	0	$A B \bar{C}$
m_7	1	1	1	$A B C$

Canonical representation:

$$F(A, B, C) = \sum m(3, 4, 5, 6, 7)$$

Product of sums:

	A	B	C	
M ₀	0	0	0	A + B + C
M ₁	0	0	1	A + B + C̄
M ₂	0	1	0	A + B̄ + C
M ₃	0	1	1	A + B̄ + C̄
M ₄	1	0	0	Ā + B + C
M ₅	1	0	1	Ā + B + C̄
M ₆	1	1	0	Ā + B̄ + C
M ₇	1	1	1	Ā + B̄ + C̄

(Conversion to POS:

$$(A+B)(B+C)$$

$$(A+B+c\bar{C})(B+C+A\bar{A})$$

A BC A BC

$$(A+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C)$$

$$(A+B+C)(A+B+\bar{C})(\bar{A}+B+C)$$

0 0 0 0 0 1 1 0 0

0 0 0

	A	B	C	X
M ₀	0	0	0	A + B + C
M ₁	0	0	1	A + B + C̄
M ₂	0	1	0	A + B̄ + C
M ₃	0	1	1	A + B̄ + C̄
M ₄	1	0	0	Ā + B + C
M ₅	1	0	1	Ā + B + C̄
M ₆	1	1	0	Ā + B̄ + C
M ₇	1	1	1	Ā + B̄ + C̄

\hookrightarrow 1's are SOP

$\bar{A}\bar{B}\bar{C}$
 $\bar{A}BC$
 $A\bar{B}C$
 $AB\bar{C}$
 ABC

SOP

Canonical form:

$$F(A, B, C) = \sum m(0, 1, 4)$$

(Conversion from POS to SOP:

$$F(A, B, C) = \sum m(2, 3, 5, 6, 7)$$

Don't care:

$$F(A, B, C) = \sum m(1, 2, 3) + d(7, 6)$$

	A	B	C	X
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	X
m_7	1	1	1	X

{ Don't care whether it's 1 or 0.

K-maps:

↳ Karnaugh's maps

Rules for Grouping:

1. Maximize the size of groups but groups must be in powers of 2.

$$2^0=1, 2^1=2, 2^2=4, 2^3=8$$

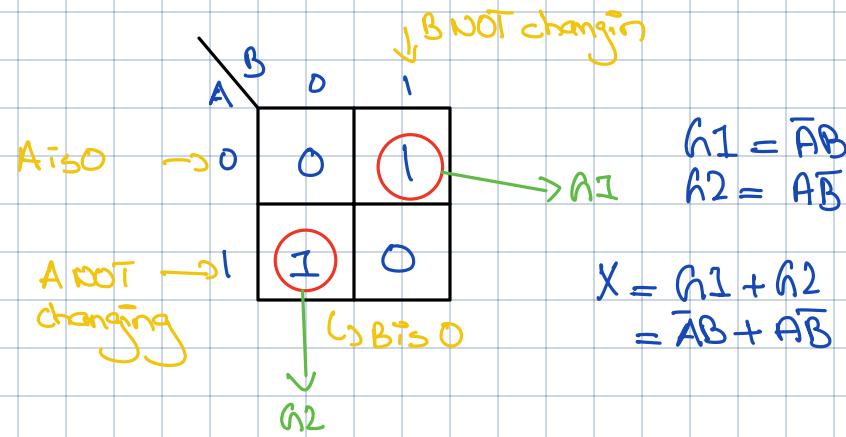
2. Minimize the number of groups

3. Overlapping can be done.

For 2 variables:

	B	0	1
A	0	1 1	1 1
	1	1 1	

A \sqcup B both changing.



	B	0	1
A	0	0	1
	1	1	1

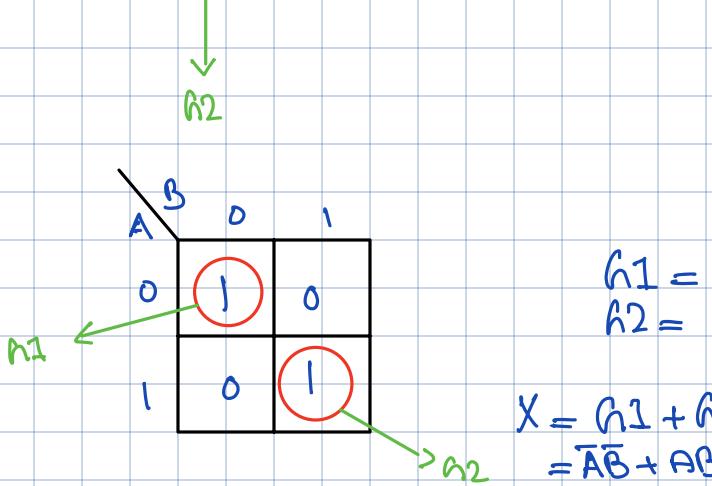
$$\begin{aligned}G_1 &= B \\G_2 &= A \\X &= A + B\end{aligned}$$

NOTE:

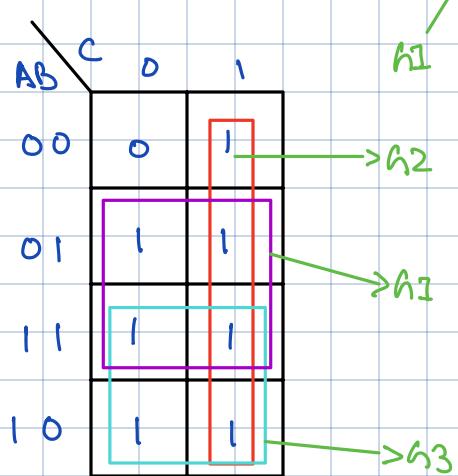
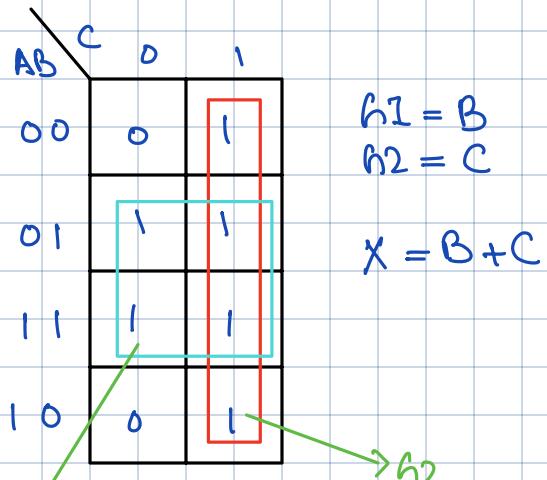
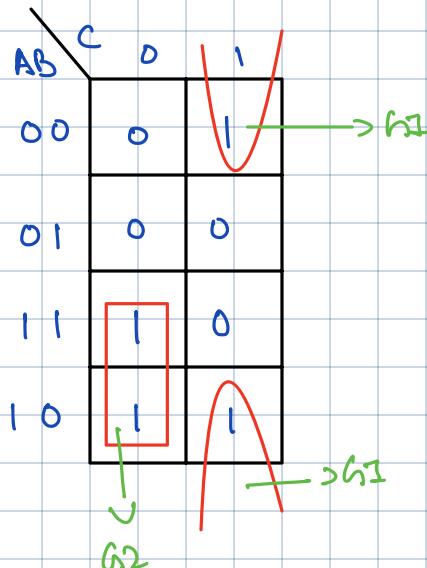
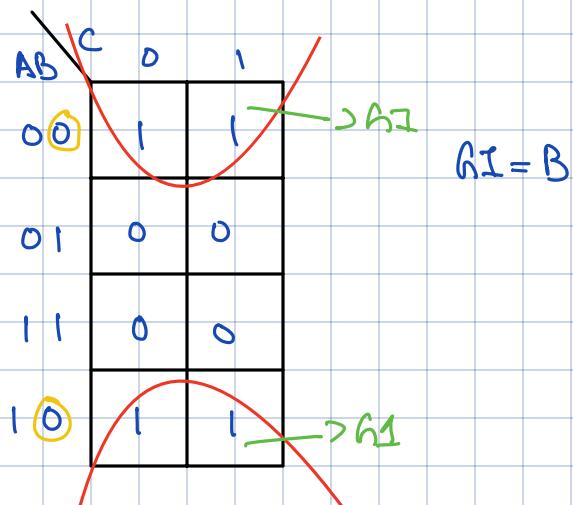
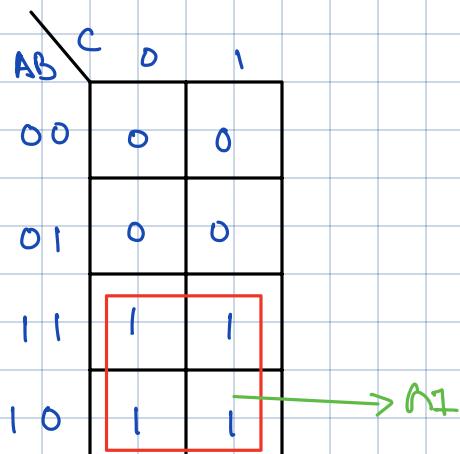
$$\hookrightarrow 0 = \bar{A}$$

$$\hookrightarrow 1 = A$$

↳ The one which is changing is omitted.



3 Variables:



AB	C	0	1
00	0	1	
01	1	0	
11	0	1	
10	1	0	

$$f_1 = \bar{A}\bar{B}C$$

$$X = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C}$$

$$f_2 = \bar{A}B\bar{C}$$

$$f_3 = ABC$$

$$f_4 = A\bar{B}\bar{C}$$

Assignment No 1:

$$Q_1. F = \overline{ABC} + \overline{AC} + \overline{C}$$

$$\bar{A}(1+C) + \bar{B}(1+C) + \bar{C}$$

$$\bar{A} + \bar{B} + \bar{C}$$

$$\bar{A}C(\bar{B} + \bar{B}) + \bar{B}C(A + \bar{A})$$

$$\bar{A} + \bar{B} + \bar{C} + (\bar{A} + \bar{B})C$$

$$\bar{A} + \bar{B} + \bar{C} + \bar{A}C + \bar{B}C$$

$$\bar{A}(\bar{B} + \bar{B}) + \bar{B}(A + \bar{A}) + \bar{C}(A + \bar{A})$$

$$\bar{A}BC + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}\bar{B}C$$

x x x x

$$\bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B} + A\bar{C} + \bar{A}\bar{C}$$

$$\bar{A}B + A\bar{B} + \bar{A}\bar{B} + A\bar{C} + \bar{A}\bar{C}$$

$$= \bar{A}B(C + \bar{C}) + A\bar{B}(C + \bar{C}) +$$

$$\bar{A}\bar{B}(C + \bar{C}) + A\bar{C}(B + \bar{B})$$

$$+ \bar{A}\bar{C}(B + \bar{B})$$

	A	B	C	X
m_0	0	0	0	1 $A + B + C$
m_1	0	0	1	1 $A + B + \bar{C}$
m_2	0	1	0	1 $A + \bar{B} + C$
m_3	0	1	1	1 $A + \bar{B} + \bar{C}$
m_4	1	0	0	1 $\bar{A} + B + C$
m_5	1	0	1	1 $\bar{A} + B + \bar{C}$
m_6	1	1	0	1 $\bar{A} + \bar{B} + C$
m_7	1	1	1	0 $\bar{A} + \bar{B} + \bar{C}$

$$= \bar{A}BC + \bar{A}\bar{B}\bar{C} + A\bar{B}C + A\bar{B}\bar{C}$$

$$+ \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$$

x

$$+ \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$POS = (\bar{A} + \bar{B} + \bar{C})$$

$$= \bar{A}BC + \bar{A}\bar{B}\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C$$

$$+ \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$000 \quad 110$$

$$Q_2. AB + A(B+C) + B(B+C)$$

$$AB + AB + AC + BB + BC$$

$$AB + AC + B + BC$$

$$AB + AC + B$$

$$B(1+A) + AC$$

$$B+AC$$

$$AB(C+\bar{C}) + AC(B+\bar{B}) + B(A+\bar{A}) + BC(A+\bar{A})$$

$$ABC + AB\bar{C} + ABC + \cancel{A}\bar{B}C + AB + \cancel{A}B + \cancel{A}BC + \cancel{A}\bar{B}C$$

$$ABC + AB\bar{C} + A\bar{B}C + AB + \cancel{A}B + \cancel{A}\bar{B}C$$

$$AB(C+\bar{C}) + \cancel{A}\bar{B}(C+\bar{C})$$

$$\cancel{ABC} + AB\bar{C} + \cancel{A}\bar{B}C + \cancel{A}\bar{B}\bar{C}$$

$$ABC + AB\bar{C} + A\bar{B}C + \cancel{A}\bar{B}C + \cancel{A}\bar{B}\bar{C}$$

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & & 1 & & 1 \\ \hline 1 & 1 & 0 & 1 & 1 \end{array}$$

	A	B	C	X
--	---	---	---	---

m_0	0	0	0	0	$A+B+C$
m_1	0	0	1	0	$A+B+\bar{C}$
m_2	0	1	0	1	$A+\bar{B}+C$
m_3	0	1	1	1	$A+\bar{B}+\bar{C}$
m_4	1	0	0	0	$\bar{A}+B+C$
m_5	1	0	1	1	$\bar{A}+B+\bar{C}$
m_6	1	1	0	1	$\bar{A}+\bar{B}+C$
m_7	1	1	1	1	$\bar{A}+\bar{B}+\bar{C}$

$$POS = (A+B+C)(A+B+\bar{C})(\bar{A}+B+C)$$

$$POS = (\bar{A}+\bar{B}+\bar{C})$$

$$F = \left[\overline{\{A + \bar{B}\}} \overline{\{AB\}} \right] + \left[\overline{\{A+B\}} \overline{\{\bar{A}\bar{B}\}} \right]$$

$$(A + \bar{B}) + (\bar{A}B) + (A+B)(A + \bar{B})$$

$$A + \bar{B} + \bar{A} + \bar{B} \quad \overline{(A+B)} + \overline{(A+\bar{B})}$$

$$1 + \bar{B} \quad + \quad \overline{\bar{A}\bar{B}} + \overline{A \cdot B}$$

$$\begin{array}{c} \bar{A}(\bar{B}+B) \\ \bar{A} \end{array}$$

$$F = 1 + \bar{A}$$

$$SOP: \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

POS: 1

Practice from book:

Q1.

$$\begin{array}{lll} (a) \overline{ABC} + (\overline{D+E}) & (b) \overline{(A+B)C} & (c) \overline{A+B+C} + \overline{DE} \\ \bar{A} + \bar{B} + \bar{C} + \bar{D} \cdot \bar{E} & (\overline{A+B}) + \bar{C} & (\bar{A}\bar{B}\bar{C}) + \bar{D} + \bar{E} \\ \bar{A} + \bar{B} + \bar{C} + D\bar{E} & (\bar{A}\bar{B}) + \bar{C} & (\bar{A}\bar{B}\bar{C}) + D + \bar{E} \end{array}$$

Example 4.9

$$AB + A(B+C) + B(B+C)$$

$$AB + AB + AC + BB + BC$$

$$AB + AC + B + BC$$

$$AC + AB + B + BC$$

$$AC + B(A + 1 + B)$$

$$B + AC$$

Example 4.11

$$\bar{A}BC + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$\bar{A}BC + \bar{B}\bar{C}(A+\bar{A}) + AC(\bar{B}+B)$$

$$\bar{A}BC + \bar{B}\bar{C} + AC$$

$$\bar{B}\bar{C} + C(A+\bar{A}B)$$

$$\bar{B}\bar{C} + C(A+B)$$

Example 4.10

$$[A\bar{B}(C+BD) + \bar{A}\bar{B}]C$$

$$[A\bar{B}C + A\bar{B}BC + \bar{A}\bar{B}]C$$

$$[A\bar{B}C + \bar{A}\bar{B}]C$$

$$A\bar{B}C + \bar{A}\bar{B}C$$

$$\bar{B}C(A+\bar{A})$$

$$\bar{B}C$$

Example 4.12

$$\overline{AB+AC+\bar{A}\bar{B}\bar{C}}$$

$$(\bar{A}B)(\bar{A}C) + \bar{A}\bar{B}\bar{C}$$

$$(\bar{A}+\bar{B})(\bar{A}+\bar{C}) + \bar{A}\bar{B}\bar{C}$$

$$\bar{A}\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$\bar{A} + \bar{A}\bar{C} + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$B\bar{C} + AC + BC$$

$$\bar{A}(1 + \bar{C} + \bar{B} + \bar{B}\bar{C}) + \bar{B}\bar{C}$$

$$\bar{A} + \bar{B}\bar{C}$$

Section 4.5

$$(a) A + AB + A\bar{B}C$$

$$A(1 + B + \bar{B}C)$$

$$A$$

$$(b) (\bar{A} + B)C + ABC$$

$$\bar{A}C + BC + ABC$$

$$\bar{A}C + BC(1 + A)$$

$$\bar{A}C + BC$$

$$C(\bar{A} + B)$$

$$(c) A\bar{B}C(BD + CDE) + A\bar{C}$$

$$A\bar{B}BCD + A\bar{B}CDE + A\bar{C}$$

$$A\bar{B}CDE + A\bar{C}$$

$$A(\bar{B}CDE + \bar{C})$$

Example 4.14

$$(a) AB + B(CD + EF)$$

$$AB + BCD + BEF$$

$$(b) (A + B)(B + C + D)$$

$$AB + AC + AD + B + BC + BD$$

$$(c) \overline{(A+B)} + C$$

$$\overline{(A+B)} \cdot \overline{C}$$

$$AC + BC$$

Example 4.15

$$A\bar{B}C + \bar{A}\bar{B} + A\bar{B}\bar{C}D$$

$$A\bar{B}C(D + \bar{D}) + \bar{A}\bar{B}(C + \bar{C}) + A\bar{B}\bar{C}D$$

$$A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}D$$

$$A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}C(D + \bar{D}) + \bar{A}\bar{B}\bar{C}(D + \bar{D}) + A\bar{B}\bar{C}D$$

$$A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D$$

Example 4.16

$$ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$$

$$\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

$$1$$

$$1$$

$$1$$

Example 4.17

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

$$(A + \overline{B} + C + D\bar{D})(\overline{B} + C + \overline{D} + A\bar{A})(A + \overline{B} + \overline{C} + \overline{D})$$

$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + \overline{D})$$

$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + \overline{D})$$

Example 4.18

$$(A + B + C + D)(A + \overline{B} + \overline{D} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$$

0	0	0	0	0	1	1	0	1	1	1	1
					0			0			0

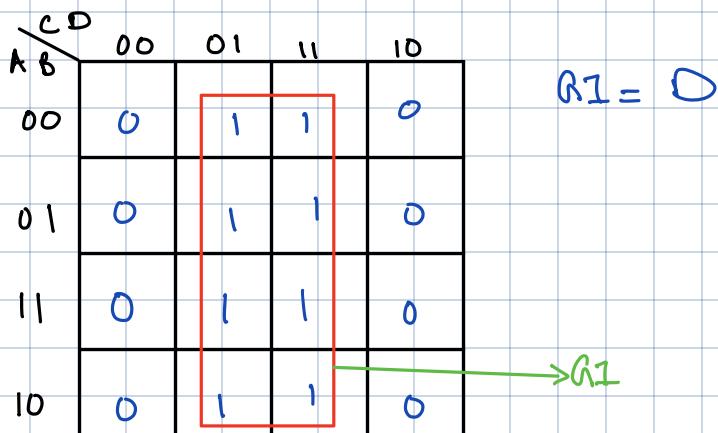
Example 4.19

$$\overline{ABC} + \overline{AB}\overline{C} + \overline{ABC} + A\overline{BC} + ABC$$

000	010	011	101	111	
1	1	1	1	1	
A	B	C	X		
0	0	0	1		$A + B + C$
0	0	1	0		$A + B + \overline{C}$
0	1	0	1		$A + \overline{B} + C$
0	1	1	1		$A + \overline{B} + \overline{C}$
1	0	0	0		$\overline{A} + B + C$
1	0	1	1		$\overline{A} + B + \overline{C}$
1	1	0	0		$\overline{A} + \overline{B} + C$
1	1	1	1		$\overline{A} + \overline{B} + \overline{C}$

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

4-variable k-maps:



		CD	00	01	11	10
		AB	00	01	11	10
00	00		1	0	0	1
			0	0	0	0
11	11		0	0	0	0
			1	0	0	1
10	10		1	0	0	1
			0	0	0	1

		CD	00	01	11	10
		AB	00	01	11	10
00	01		1	1	1	1
			1	1	1	1
11	11		1	1	1	1
			1	1	1	1
10	10		1	1	1	1
			1	1	1	1

		CD	00	01	11	10
		AB	00	01	11	10
00	00		1	1	1	1
			0	0	0	0
11	11		0	0	0	0
			1	1	1	1
10	10		1	1	1	1
			1	1	1	1

		CD	00	01	11	10
		AB	00	01	11	10
00	01		0	0	0	1
			0	0	0	0
11	11		0	0	0	0
			1	0	0	0
10	10		1	0	0	0
			1	1	1	1

		CD	00	01	11	10
		AB	00	01	11	10
00	01		0	1	1	1
			1	1	1	1
11	11		0	1	1	1
			1	0	1	1
10	10		0	0	0	0
			0	0	0	0

		CD	00	01	11	10
		AB	00	01	11	10
00	01		1	0	0	0
			0	0	1	1
11	11		0	0	1	1
			0	0	1	1
10	10		0	0	1	1
			1	0	1	1

		CD	00	01	11	10
		AB	00	01	11	10
00	01		1	0	0	1
			0	0	0	0
11	11		0	0	0	0
			1	1	0	1
10	10		1	0	0	1
			0	1	0	1

		CD	00	01	11	10
		AB	00	01	11	10
00	01		1	0	0	0
			0	1	0	0
11	11		1	0	0	0
			0	1	0	0
10	10		0	1	0	0
			0	0	1	0

	CD	00	01	11	10
AB	00	1	0	1	0
01	0	1	0	1	
11	1	0	1	0	
10	0	1	0	1	

	CD	00	01	11	10
AB	00	1	2	4	3
01	5	6	8	7	
11	13	14	16	15	
10	9	10	12	11	

↳ Output Arrangement

Presence of Don't care (X) in k-maps:

- Include them in group to maximize the size of the group
- Exclude them if all 1's are in groups.

	CD	00	01	11	10
AB	00	X	X	1	X
01		X	1		
11			1		
10	1		1	1	

	CD	00	01	11	10
AB	00	X	K		
01		1	1		X
11					
10				1	X

	CD	00	01	11	10
AB	00	X	X		
01		1		X	
11		X		1	
10				X	

	CD	00	01	11	10
AB	00	1	1		
01		1	1	1	X
11		1	1	1	
10	1	1	1		

	CD	00	01	11	10
AB	00	1	1		
01		1	1	1	X
11		1	1	1	X
10	1	1	1		

Example 4-23

$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

| | | |

0 0 0	$\bar{A}\bar{B}\bar{C}$	0
0 0 1	$\bar{A}\bar{B}C$	1
0 1 0	$\bar{A}B\bar{C}$	1
0 1 1	$\bar{A}B{C}$	0
1 0 0	$A\bar{B}\bar{C}$	0
1 0 1	$A\bar{B}C$	0
1 1 0	$AB\bar{C}$	1
1 1 1	ABC	1

AB	C	0	1
0 0	0	0	1
0 1	0	1	0
1 1	1	1	1
1 0	0	0	0

Example 4-30

$$A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

| | | | |

AB	C	0	1
0 0	1	1	
0 1	0	1	
1 1	0	0	
1 0	1	1	

0 0 0	$\bar{A}\bar{B}\bar{C}$	1
0 0 1	$\bar{A}\bar{B}C$	1
0 1 0	$\bar{A}B\bar{C}$	0
0 1 1	$\bar{A}B{C}$	1
1 0 0	$A\bar{B}\bar{C}$	1
1 0 1	$A\bar{B}C$	1
1 1 0	$AB\bar{C}$	0
1 1 1	ABC	0

$$G1 = B$$

$$G2 = \bar{A}C$$

$$X = B + \bar{A}C$$