Probability:

- How to determine if high correlation b/w two variables is statistically significant?
- Provides mathematical tools to reason about uncertainty.

Sample space:

- The set of outcomes of an experiment is written as Ω

Exercise

1. $P(A) \times P(B) \rightarrow$ Both A & B work
2. $1 - P(A) \times P(B) \rightarrow$ Atleast one fails
3. $[1-P(A)] \times [1-P(B)] \rightarrow$ None work
4. $1 - [1-P(A)] \times [1-P(B)] \rightarrow$ Some work

Example 2.20

$$P[A \cap B \cup C \cap (D \cup E)]$$

$P(A \cap B) = (0.92)^2 = 0.8464 \rightarrow$ A & B work

$P(D \cup E) = 1 - [(1-0.92)(1-0.92)] = 0.9936$ D or E works

$P(C \cap D \cup E) = 0.92 \times 0.9936 = 0.9141$ C & D or E works

$P[A \cap B \cup C \cap (D \cup E)] = 1 - [(1-0.9141)(1-0.9936)]$ A & B works OR C & D or E works

$= 0.9868$

Exercise

2.1

$$P(\text{both are defective}) = \frac{1}{6C2}$$
$$= 1/15$$

2.2

$P(MB) = 0.4$, $P(HD) = 0.3$, $P(MB \cap HD) = 0.15$

$$P(MB \cup HD) = 0.4 + 0.3 - 0.15$$
$$= 0.55$$

$$P(MB \cap HB) = 1 - P(MB \cup HD)$$
$$= 1 - 0.55$$
$$= 0.45$$

2.3

$P(\text{Email}) = 0.3$

$P(\text{internet}) = 0.4$

$P(\text{Email} \cap \text{internet}) = 0.15$

$$P(\text{Email} \cup \text{internet}) = 0.3 + 0.4 - 0.15$$
$$= 0.55$$

$$(\text{No Virus}) = 1 - 0.55$$
$$= 0.45$$

2.4

$P(C) = 0.7$ $P(F) = 0.6$ $P(C \cap F) = 0.5$

(a) $P(\bar{F}) = 1 - 0.6$
$= 0.4$

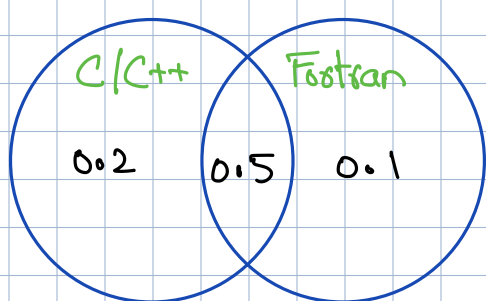
(b) 0.2

(c) 0.2

(d) 0.1

(e) 0.833

(f) 0.714



$$P(C|F) = \frac{P(C \cap F)}{P(F)}$$
$$= \frac{0.5}{0.6}$$

$$P(F|C) = \frac{0.5}{0.7}$$

2.5

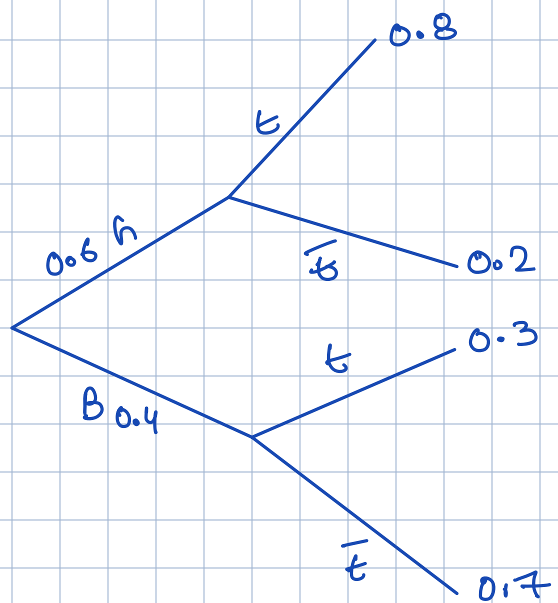
$P(A) = 0.2,\ P(B) = 0.3,\ P(C) = 0.5$

$P(\text{atleast one}) = 1 - [(1-0.2) \times (1-0.3) \times (1-0.5)]$

$= 0.72$

2.6

$P(A) = 0.8,\ P(B) = 0.3,\ P(C) = 0.6$



$P(\text{on time}) = 0.6 \times 0.8 + 0.4 \times 0.3$
$= 0.6$

2.8

$P(A) = 0.01,\ P(B) = 0.02,\ P(C) = 0.02$

$P(\text{launched on time}) = ((1-0.01) \times (1-0.02) \times (1-0.02))$

$= 0.95$

2.9

$P(A) = 0.96,\ P(B) = 0.95,\ P(C) = 0.90$

$P(\text{atleast one fail}) = 1 - 0.96 \times 0.95 \times 0.90$
$= 0.1792$

2.10

$P(A) = 0.4,\ P(B) = 0.5,\ P(C) = 0.2$

$P(\text{Damaged}) = 1 - ((1-0.4)(1-0.5)(1-0.2))$

$= 0.76$

2.11

$P(A) = 0.1,\ P(B) = 0.2,\ P(C) = 0.3,\ P(D) = 0.4,\ P(E) = 0.5$

(a) $P(\text{atleast one}) = 1 - [(1-0.1)\cdots(1-0.5)]$

$= 0.8488$

(b) $P(\text{atleast two}) = P(A) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D}) \times P(\bar{E}) + \cdots + P(\bar{A}) \cdots P(E)$

$= 0.3714$

(c) $P(\text{All five}) = 0.1 \times 0.2 \times 0.3 \times 0.4 \times 0.5$

$= 0.0012$

2.12

$P(A) = P(B) = 0.6$

$P(\text{atleast one}) = 1 - ((1-0.6)^4)$

$= 0.9744$

2.13

$P(A) = P(B) = P(C) = 0.8$

$P(\text{atleast one}) = 1 - ((1-0.8)^3)$

$= 0.992$

2.14

(a) $P(\text{6 different lower case}) = \dfrac{1{,}000{,}000}{26 \times 25 \times 24 \times 23 \times 22 \times 21}$

(b) $P(\text{6 different, some upper case}) = \dfrac{1{,}000{,}000}{52 \times 51 \times 50 \times 49 \times 48 \times 47}$

(c) $P(\text{any 6 letters, upper OR lower case}) = \dfrac{1{,}000{,}000}{52 \times 52 \times 52 \times 52 \times 52 \times 52}$

(d) $P(\text{any 6 letter / digits}) = \dfrac{1{,}000{,}000}{(62)^6}$

2.21

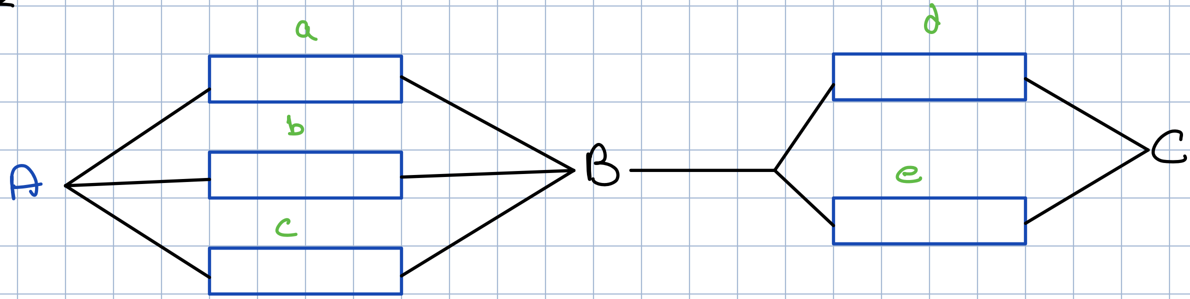
$(C \cup D \cup E) \cap (F \cup G)$

$P(C \cup D) = 1 - (0.3)^3$
$= 0.973$

$P(F \cup G) = 1 - (0.3)^2$
$= 0.91$

$P(C \cup D \cup E) \cap P(F \cup G) = 0.973 \times 0.91$
$= 0.88543$

2.22



(a) $P(a \cup b \cup c) = 1 - (0.2)^3$
$= 0.992$

$P(d \cup e) = 1 - (0.2)^2$
$= 0.96$

$P(a \cup b \cup c) \cap P(d \cup e) = 0.992 \times 0.96$
$= 0.95232$

(b) $P(a \cup b \cup c \cup d) = 1 - (0.2)^4$
$= 0.9984$

$P(e \cup f \cup g) = 1 - (0.2)^3$
$= 0.992$

$P(a \cup b \cup c) \cap P(e \cup f \cup g) = 0.992 \times 0.992$
$= 0.984064$

$P(a a \cup b \cup c \cup d) \cap P(e \cup f) = 0.9984 \times 0.96$
$= 0.958464$

2.23

(a) $P(\text{exactly } 2) = \dfrac{5C2 \times 5C4}{10C6}$

$= 0.23$

(b) $P(X=2 \mid X \geq 2) = \dfrac{P(X=2 \cap X>2)}{P(X>2)}$

$P(X>2) = 0.738$

$= \dfrac{0.23}{0.9761}$

$= 0.2356$

2.26

$P(\text{No defects}) = \dfrac{2C0 \times 4C3}{6C3}$

$= 0.2$

2.27

$P(\text{No defects}) = \dfrac{6C0 \times 12C5}{18C5}$

$= 0.0926$

2.24

$P(X \geq 2) = 1 - P(X<2)$

$P(X<2) = \dfrac{5C0 \times 4C4}{9C4} + \dfrac{5C1 \times 4C3}{9C4}$

$P(X \geq 2) = 0.833$

David forsyth:

Chapter: 02

Exercise

3.2 $3! = 6$

3.4 No. of ways to assign roles (2, 1, 0 provinces) $= 3!$

No. of ways to choose 2 provinces: $3C2 = 3$

Total: $3! \times 3$

$= 18$

3.6

$P(\text{king of hearts}) = \frac{1}{52}$

3.7

$P(\text{ball in slot 2}) = \frac{1}{38}$

3.8

$P(\text{Drink alcohol}) = \frac{1}{2}$ $P(\text{smoke cigs}) = \frac{1}{3}$

(a) $P(\text{Do none}) = 1 - \left(\frac{1}{2} + \frac{1}{3}\right)$

$= \frac{1}{6}$

(b) $P(\text{None}) = \frac{1}{3}$

$P[A \cup S] = 1 - \frac{1}{3}$

$= \frac{2}{3}$

$P(A \cap S) = P(A) + P(S) - P(A \cup S)$

$= \frac{1}{2} + \frac{1}{3} - \frac{2}{3}$

$= \frac{1}{6}$

3.13

↳ 4 balls into 2 bucket

↳ 1 red, one green, 2 white

?

(a)

$P(\text{Colored balls in one bucket}) = \frac{2C1 \times 2C0}{2}$

(b) $4P2 = 12$

(c) $P(\text{colored ball in each bucket}) = \frac{4 \times 2!}{4P2}$

$= \frac{2}{3}$

3.14

$P(HTH) = \frac{1}{8}$

H	H	H
H	H	T
H	T	H ✓
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

3.15

(a) $P(\text{king}) = \frac{4}{52}$

(b) $P(\text{Heart}) = \frac{13}{52}$

(c) $P(\text{Heart or diamond}) = \frac{26}{52}$

3.18

(a) $P(\text{king}) = \frac{3}{51}$

(b) $P(\text{Heart}) = \frac{12}{51}$

3.19

(a) $P(\text{same suit}) = \frac{13C4 \times 4}{52C4}$

(b) $P(\text{All four red}) = \frac{26C4}{52C4}$

(c) $P(\text{All four diff}) = \frac{13^4}{52C4}$ → For each card, I have 13 options

3.22 $P(\text{No ace}) = \frac{(52-4)C7}{52C7}$

3.24

(a) $P(\text{red king}) = \frac{1}{(52-13)C1}$

Cheat sheet:

1. n^r:
   - ↳ Independent/with replacement
   - ↳ For each r, you have n choices

2. $n!$:
   - ↳ Ways to arrange n

3. nCr:
   - ↳ Ways to choose r from n
   - ↳ Order doesn't matter

4. nPr:
   - ↳ Ways to choose r from n
   - ↳ Order matters

(b) $P(\text{spade}) = \dfrac{13}{(52-13)C1}$

3.26

$P(5\text{ reds}) = \dfrac{26C5 \times 26C5}{52C10}$

Deck: 40

P1 = 10 LC

P2 = 20 LC

3.27

(a) $P(\text{Each has 4 Land cards}) = \dfrac{10C4 \times 30C3}{40C7} \times \dfrac{20C4 \times 20C3}{40C7}$

(b) $P(\text{P1 2L \& P2 3L}) = \dfrac{10C2 \times 30C5}{40C7} \times \dfrac{20C3 \times 20C4}{40C7}$

3.37

(a) $P(RK \mid \text{king is removed}) = \dfrac{P(RK \cap \text{king removed})}{P(\text{king removed})}$

$= \dfrac{\frac{2}{52}}{\frac{4}{52}}$

(b) $P(\text{Draw a red king} \mid \text{Red king is removed}) = \dfrac{P(\text{Draw a RK} \cap \text{RK is removed})}{P(\text{RK is removed})}$

$= \dfrac{\frac{1}{51}}{\frac{1}{26}}$

$= \dfrac{26}{51}$

(c) $P(\text{Draw a red king} \mid \text{Remove black ace}) = \dfrac{2}{51}$

3.39

(a) (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

$$\frac{15}{30} = \frac{1}{2}$$

(b) $\frac{9}{15}$

3.40

$$P(\text{Draw red cards} \mid \text{Removed black cards}) = \frac{\frac{3}{39}}{\frac{13}{52}}$$

$$= \frac{4}{13}$$

3.41

(a) $$P(\text{All 2and} \mid \text{Choose one 2and}) = \frac{\frac{10C7}{40C7}}{\frac{10C1 \times 30C6}{40C7}}$$

(b) $$P(\text{Only 1 2and} \mid \text{Choose one 2and}) = \frac{10C1 \times 30C6}{40C7}$$

Micheal Baron Practice:

1. $P(A) \times P(B) \to$ Both A & B work

2. $1 - P(A) \times P(B) \to$ Atleast one fails

3. $[1-P(A)] \times [1-P(B)] \to$ None work

4. $1 - [1-P(A)] \times [1-P(B)] \to$ Some work

After 10 years:

2.2 $P(MB) = 0.4 \quad P(HD) = 0.3, \quad P(MB \cap HD) = 0.15$

$P(MB \cup HD) = 0.4 + 0.3 - 0.15 = 0.55$

$P(\overline{MB \cap HD}) = 1 - 0.55$
$= 0.45$

2.3

$P(A) = 0.3 \qquad P(B) = 0.4 \qquad P(A \cap B) = 0.15$

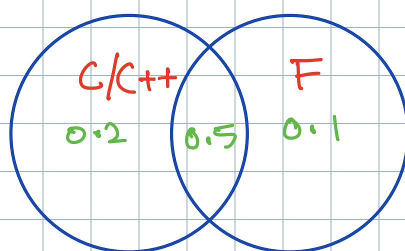
$P(A \cup B) = 0.3 + 0.4 - 0.15$
$= 0.55$

$P(\overline{A \cap B}) = 1 - 0.55$
$= 0.45$

2.4

(a) $P(\overline{F}) = 0.2$, (b) $P(\overline{C \cap F}) = 0.2$

(c) $P(C \cap \overline{F}) = 0.2$ (d) $P(\overline{C} \cap F) = 0.1$



(e) $P(C|F) = \dfrac{P(C \cap F)}{P(F)}$

$= \dfrac{0.5}{0.6}$

$= 0.83$

(f) $P(F|C) = \dfrac{P(C \cap F)}{P(C)}$

$= \dfrac{0.5}{0.7}$

$= 0.71$

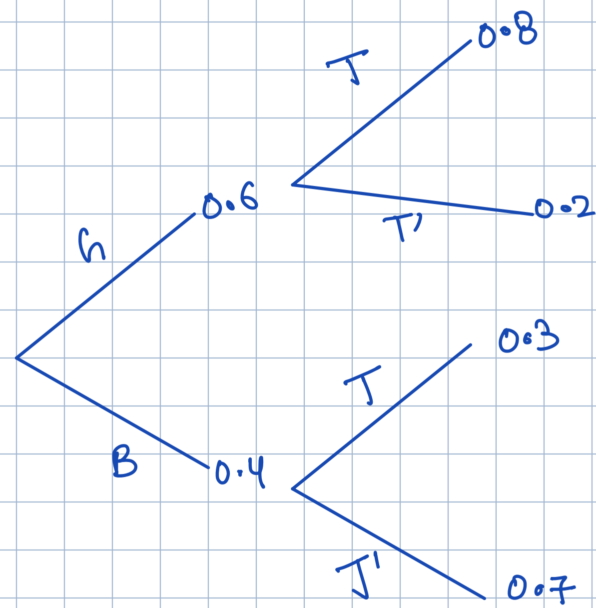
2.25

$P(T_1)=0.2, \; P(T_2)=0.3, \; P(T_3)=0.5$

$P(T_1 \cup T_2 \cup T_3) = 1-[(1-0.2)\times(1-0.3)\times(1-0.5)]$

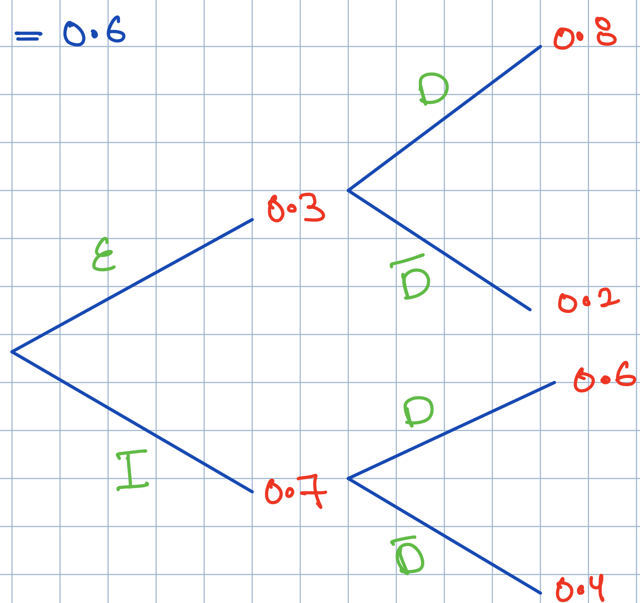
$= 0.72$

2.6



$P(T) = 0.8\times0.6 + 0.4\times0.3$
$= 0.6$

2.7



$P(D) = 0.3\times0.8 + 0.7\times0.6$
$= 0.66$

2.8

$P(F_1)=0.01, \; P(F_2)=0.02, \; P(F_3)=0.02$

$P(W) = (1-0.01)\times(1-0.02)\times(1-0.02)$
$=0.95$

2.9

$P(M1)=0.96$, $P(M2)=0.95$, $P(M3)=0.9$

$P(\text{atleast one fail}) = 1-(0.96 \times 0.95 \times 0.9)$
$= 0.1792$

2.10

$P(A)=0.4$, $P(B)=0.5$, $P(C)=0.2$

$P(\text{Damaged}) = 1-[(1-0.4)\times(1-0.5)\times(1-0.2)]$

$= 0.76$

2.11

$P(D1)=0.1$, $P(D2)=0.2$, $P(D3)=0.3$, $P(D4)=0.4$

$P(D5)=0.5$

(c) $P(\text{All test}) = 0.1 \times 0.2 \times 0.3 \times 0.4 \times 0.5$
$= 0.0012$

(a) $P(\text{atleast } 1) = 1-[(1-0.1)\dots(1-0.5)]$
$= 0.8488$

(b) $P(\text{atleast } 2) =$?

2.14

(a) $P(\text{breaks}) = \dfrac{1000000}{26\times25\times24\times23\times22\times21}$

$= 0.00603$

(b) $P(\text{breaks}) = \dfrac{1000000}{52\times51\times50\times49\times48\times47}$
$= 0.000068$

(c) $P(\text{breaks}) = \dfrac{1000000}{52\times52\times52\times52\times52\times52}$
$= 0.0000505$

(d) $P(\text{breaks}) = \dfrac{1000000}{62^6}$
$= 0.000017$

2.15

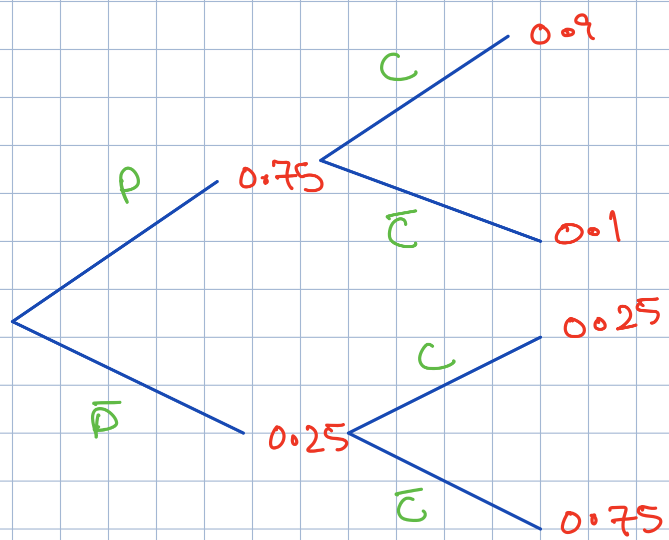
$P(\varepsilon 1) = 0.2 \; , \; P(\varepsilon 2) = 0.3$

$P(\varepsilon 1 \cap \varepsilon 2) = 0.2 \times 0.3 = 0.06$

$P(\varepsilon 1 \cup \varepsilon 2) = 0.2 + 0.3 - 0.06$
$= 0.44$

$P(\varepsilon 1 \cap \varepsilon 2 \mid \varepsilon 1 \cup \varepsilon 2) = \dfrac{0.06}{0.44}$
$= 0.136$

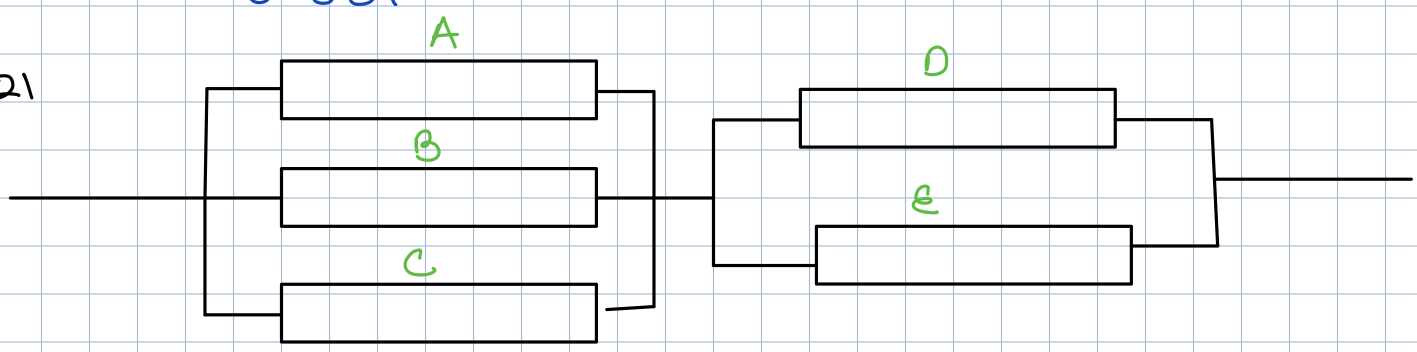
2.18



$P(\bar{P} \mid C) = \dfrac{0.25 \times 0.25}{0.25 \times 0.25 + 0.75 \times 0.9}$

$= 0.084$

2.21



$P[(A \cup B \cup C) \cap (D \cup E)] = 0.88543$

$P(A \cup B \cup C) = 1 - (0.3)^3$
$= 0.973$

$P(D \cup E) = 1 - (0.3)^2$
$= 0.91$

2.26

$$P(\text{None}) = \frac{4C3}{6C3}$$

$= 0.2$

2.27

Defects $= 6$

Fine $= 18 - 6$

$= 12$

$$P(\text{None}) = \frac{12C5}{18C5}$$

$= 0.092$

2.28

$p = \frac{1}{4}$, $q = \frac{3}{4}$, $n = 6$

$$P(\text{passes}, X > 3) = 1 - \sum_{0}^{2} \left[xCn \times (0.25)^x \times (1 - 0.25)^{n-x} \right]$$

$= 0.169$

2.29

Total db $= 9$

$$P(\text{Found}) = 1 - \left[\frac{\underbrace{4C4}_{\text{NOT found}} \times \underbrace{5C0}_{\text{found}}}{9C4} + \frac{\underbrace{4C1}_{\text{NOT found}} \times \underbrace{5C3}_{\text{found}}}{9C4} \right]$$

$= 0.833$

2.30

2
•

(a) $2^3 = 8$

(b) $P(B \cap B) = (0.5)^2 = P(B \cap G) = P(G \cap B)$

(c) $P(\text{Met a boy} \mid BB) = \frac{2}{4}$

$$P(\text{Met a boy} \mid GB) = \frac{1}{4}$$

$$P(\text{Met a boy} \mid BG) = \frac{1}{4}$$

(d) $P(BB \mid \text{met a boy}) = \frac{2}{4}$

$$= \frac{1}{2}$$