

Q3.1  $P(\text{first file}) = 0.4, P(\text{second file}) = 0.3$

$$P(X=0) = (1-0.4) \times (1-0.3) \\ = 0.42$$

$$P(X=1) = 0.4 \times (1-0.3) + (1-0.4) \times 0.3 \\ = 0.46$$

$$P(X=2) = 0.4 \times 0.3 \\ = 0.12$$

$$X = \begin{cases} 0.42, & x=0 \\ 0.46, & x=1 \\ 0.12, & x=2 \\ 0, & \text{otherwise} \end{cases}$$

Q3.2

$$E[X] = 0 \times 0.7 + 1 \times 0.2 + 2 \times 0.1 \\ = 0.4$$

$$\text{Expected loss} = 0.4 \times 500 \\ = 200$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 \\ = [0^2 \times 0.7 + 1^2 \times 0.2 + 2^2 \times 0.1] - (0.4)^2 \\ = 0.44$$

$$\text{Variance} = (500)^2 \times 0.44 \\ = 110,000$$

Q3.3

$$P(E \text{ in } 1) = \frac{4C2 \times 1C1}{5C3} \\ = 0.6$$

$$P(E \text{ in } 0) = \frac{4C3 \times 1C0}{5C3} \\ = 0.4$$

$x$	0	1
$P(x)$	0.4	0.6

$$E[X] = 0.6 \quad \text{Var}[X] = 0.24$$

Q3.4

X	1	2	3	4	5	6
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E[X] = 3.5$$

$$\begin{aligned} \text{Var}[X] &= 15.1 - (3.5)^2 \\ &= 2.94 \end{aligned}$$

3.5

$$\begin{aligned} \text{(a)} \quad P(X \geq 2) &= \frac{{}^7C_2 \times {}^5C_2}{{}^{12}C_4} + \frac{{}^7C_1 \times {}^5C_3}{{}^{12}C_4} + \frac{{}^7C_0 \times {}^5C_4}{{}^{12}C_4} \\ &= 0.576 \end{aligned}$$

(c)

X	0	1	2	3	4
P(X)	0.071	0.35	0.423	0.142	0.0101

$$E[X] = 1.66$$

Q3.6

X	0	1
P(X)	0.667	0.333

$$E[X] = 0.333$$

$$P(X=0) = \frac{{}^5C_2 \times {}^1C_0}{{}^6C_2}$$

$$P(X=1) = \frac{{}^5C_1 \times {}^1C_1}{{}^6C_2}$$

Q3.7

x	P(x)
0	0.4
1	0.4
2	0.2

$$\begin{aligned} E[X] &= 0 \times 0.4 + 1 \times 0.4 + 2 \times 0.2 \\ &= 0.8 \end{aligned}$$

$$\begin{aligned} \text{Var}[X] &= 1.2 - (0.8)^2 \\ &= 0.56 \end{aligned}$$

$$Y = X_1 + X_2$$

$$E[Y] = E[X_1] + E[X_2]$$

$$= 0.8 + 0.8$$

$$= 1.6$$

$$\text{Var}[Y] = \text{Var}[X_1] + \text{Var}[X_2]$$

$$= 1.12$$

Q 3.8

$$X = \begin{cases} 0 & 1/4 \\ 1 & 3/4 \times 1/3 \\ 2 & 3/4 \times 2/3 \times 1/2 \\ 3 & 3/4 \times 2/3 \times 1/2 \times 1 \end{cases}$$

$$E[X] = 1.5 \quad \text{Var}[X] = 1.25$$

3.10

$$P(X_{\text{Fri}} > X_{\text{Thu}}) = P(X_{\text{Thu}} = 0) \times P(X_{\text{Fri}} > 0) + P(X_{\text{Thu}} = 1) \times P(X_{\text{Fri}} > 1) \\ + P(X_{\text{Thu}} = 2) \times P(X_{\text{Fri}} > 2)$$

$$Y = \max(T_1, T_2)$$

Q 3.11

$X/Y$	1	2	3	4	5	6	$X$
1	$1/36$	$2/36$	$2/36$	$2/36$	$2/36$	$2/36$	
2	$2/36$	$1/36$	$2/36$	$2/36$	$2/36$	$2/36$	
3	$2/36$	$2/36$	$1/36$	$2/36$	$2/36$	$2/36$	
4	$2/36$	$2/36$	$2/36$	$1/36$	$2/36$	$2/36$	
5	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$	$2/36$	
6	$2/36$	$2/36$	$2/36$	$2/36$	$2/36$	$1/36$	
$Y$							

Q 3.12

$X+Y$	$P(X+Y)$	$X-Y$	$P(X-Y)$
0	0.5	-1	0.2
1	0.4	0	0.6
2	0.1	1	0.2

$X+Y \backslash X-Y$	-1	0	1	$X+Y$
0	0	0.5	0	0.5
1	0.2	0	0.2	0.4
2	0	0.1	0	0.1
$X-Y$	0.2	0.6	0.2	

① 3.13

$X \backslash Y$	0	1	2	(a) $Z = X+Y$	$P(Z)$
0	0.2	0	0.3	0	0
1	0	0.1	0	1	0.2
2	0.3	0	0.1	2	0.7
				3	0
				4	0.1

3.17

$X$	$P(X)$	$Y$	$P(Y)$
1	0.5	1	0.5
-1	0.5	-1	0.5

$$E[X] = 0$$

$$\text{Var}[X] = 1$$

$$E[Y] = 0$$

$$\text{Var}[Y] = 1$$

(a)  $A = 100X$

$$E[A] = 100 E[X] = 0$$

$$\text{Var}[A] = 100^2 \text{Var}[X] = 10000$$

$$\text{std}[A] = 10$$

(b)  $C = 50X + 10Y$

$$E[C] = 0$$

$$\text{Var}[C] = 2600$$

$$\text{std}[C] = 50.9$$

(c)  $D = 40X + 12Y$

$$E[D] = 0$$

$$\text{Var}[D] = 1744$$

$$\text{std}[D] = 41.76$$

3.18

If probability distribution is in percentage

$$\text{new RV} = \text{numbers of shares} \times \text{price per share} \times \frac{X}{100}$$

## Chapter 09

Q9.8

(a) 40.77, 43.22

$$\begin{aligned} P(1.23 \leq Z \leq 5.15) &= P(Z \leq 5.15) - P(Z \leq 1.23) \\ &= 1 - 0.89 \\ &= 0.11 \end{aligned}$$

Q9.9

(a)  $\bar{x} = 50$ ,  $s = 20$ 

$$t_{0.05, 2} = 4.303$$

$$[0.31, 99.6]$$

(b)  $H_0: \mu = 80$ 

0.1

$$H_1: \mu \neq 80$$

$$= \frac{50 - 80}{\frac{20}{\sqrt{3}}}$$

$$\alpha = 0.1$$

$$= -2.59$$

$$t_{0.1} = 2.92$$

 $\therefore$ (c)  $\chi^2_{0.95, 2} = 0.103$ 

$$\chi^2_{0.025, 2} = 7.37$$

$$\frac{(3-1)20^2}{7.37} \leq \sigma^2 \leq \frac{(3-1)20^2}{0.0506}$$

$$108 \leq \sigma^2 \leq 15810.2$$

$$10.3 \leq \sigma \leq 125.7$$

Q9.12

$$(a) \bar{x} = 0.62, s = 0.2, N = 52, \alpha = 0.05$$

$$Z_{0.025} = 1.96$$

$$[0.56, 0.67]$$

(b)

$$z = \frac{0.62 - 0.6}{\frac{0.2}{\sqrt{52}}}$$

$$= 0.7211$$

$$P(Z \geq 0.7211) = 1 - P(Z \leq 0.72)$$

$$= 1 - 0.76$$

$$= 0.23$$

Q9.14