Design Case Study

- A logic circuit has two inputs X & Y each is a 2-bit unsigned number. It has an output number Z such that $Z = X^2 + Y^2$.
- What is the minimum number of bits required for the output number Z?
- Construct the truth table of the circuit.
- Derive the Boolean expressions of the two least significant output bits (Z_0, Z_1) using basic gates.

$Z = X^2 + Y^2$

Q1.

X	Y
00	00
01	01
10	10
11 → 3	11 → 3

$Z = X^2 + Y^2$

$3^2 + 3^2 = 9$ → 5 bits

Q2.

X_1	X_0	Y_1	Y_0	X_0X_1	Y_0Y_1	Z_4 Z_3 Z_2 Z_1 Z_0 2^4 2^3 2^2 2^1 2^0	
0	0	0	0	0	0	0 0 0 0 0	1
0	0	0	1	0	0	0 0 0 0 1	2
0	0	1	0	0	0	0 0 1 0 0	3
0	0	1	1	0	1	0 1 0 0 1	4
0	1	0	0	0	0	0 0 0 0 1	5
0	1	0	1	0	0	0 0 0 1 0	6
0	1	1	0	0	0	0 0 1 0 1	7
0	1	1	1	0	1	0 1 0 1 0	8
1	0	0	0	0	0	0 0 1 0 0	9
1	0	0	1	0	0	0 0 1 0 1	10
1	0	1	0	0	0	0 1 0 0 0	11
1	0	1	1	0	1	0 1 1 0 1	12
1	1	0	0	1	0	0 1 0 0 1	13
1	1	0	1	1	0	0 1 0 1 0	14
1	1	1	0	1	0	0 1 1 0 1	15
1	1	1	1	1	1	1 0 0 1 0	16

4+9

9+4

9+9 = 18

Z_0

X_1X_0 \ Y_1Y_0	00	01	11	10
00	0	1	1	0
01	1	0	0	1
11	1	0	0	1
10	0	1	1	0

$Q1 = \bar{X}_0 Y_0$

$Q2 = X_0 \bar{Y}_0$

$X = \bar{X}_0 Y_0 + X_0 \bar{Y}_0$

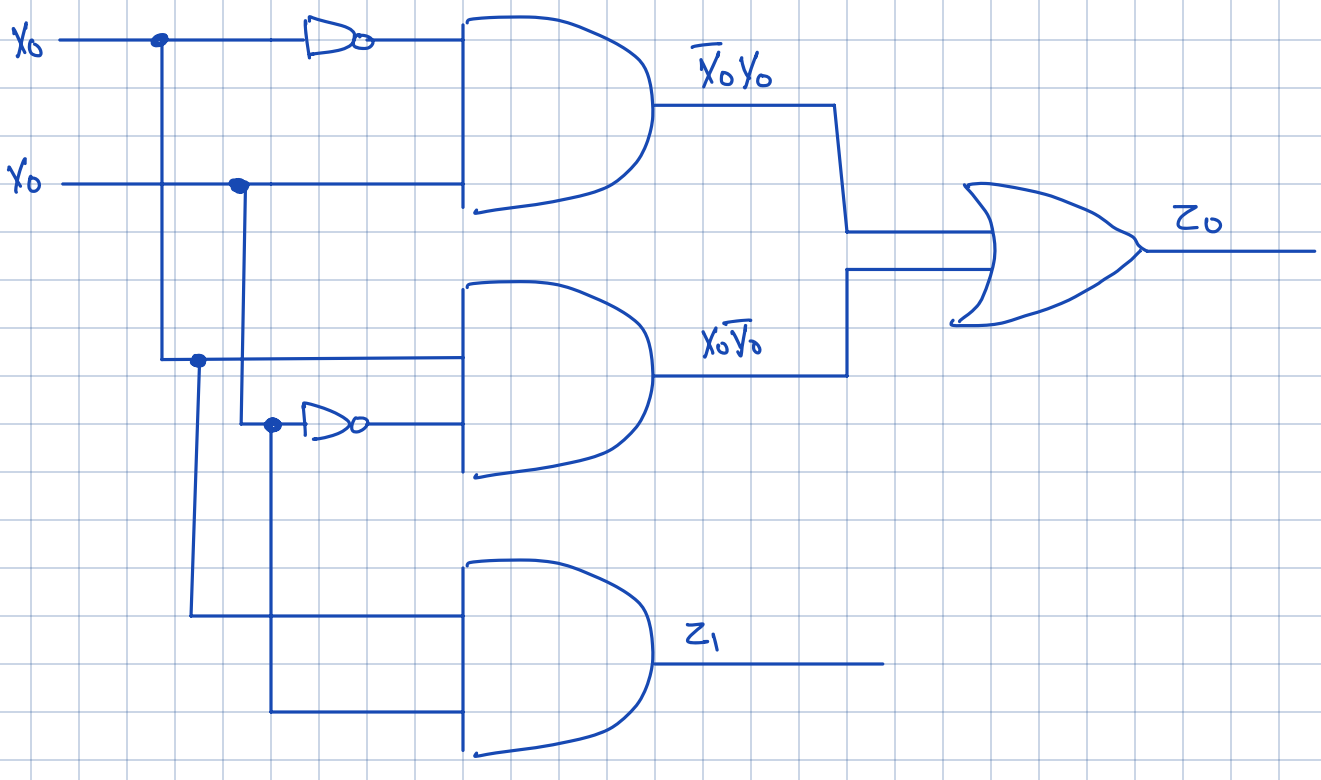
Z_1

X_1X_0 \ Y_1Y_0	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

$Q1 = Y_0 X_0$

$X = X_0 Y_0$

Q 3.



↳ Check out slides