

Decimal numbers:

↳ Base - 10

↳ 10^3 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3}

Example,

↳ 9240 can be written as:

$$9 \times 10^3 + 2 \times 10^2 + 4 \times 10^1 + 0 \times 10^0$$

$$9000 + 200 + 40 + 0$$

↳ 480.52

$$4 \times 10^2 + 8 \times 10^1 + 0 \times 10^0 + 5 \times 10^{-1} + 2 \times 10^{-2}$$

Binary Numbers

↳ Base - 2

↳ Binary to Decimal:

1) 1 0 0 1 0

$$1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$16 + 0 + 0 + 2 + 0$$

$$18$$

2) 1 0 1 0 . 1 0

$$1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2}$$


$$8 + 0 + 2 + 0 + 0.5$$

$$10.5$$

↳ Decimal to Binary:

1)

2	512	
2	256	0
2	128	0
2	64	0
2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	0



1 0 0 0 0 0 0 0 0 0

2) 512.25

$$0.25 \times 2 = 0.5 \quad 0$$

$$0.5 \times 2 = 1.0 \quad 1$$

$$0.0 \times 2 = 0$$

1000000000.01

MSB
1 0 0 1
LSB

↳ Furthest 1 is Most significant bit
↳ Last bit is Least

↳ Binary addition:

$$\begin{array}{r} 00111 \\ + 10101 \\ \hline 11100 \end{array}$$

1 + 1 = 2 = 10
1 + 1 + 0 = 2 = 10
1 + 1 + 1 = 3 = 11

1M power 5 drops 1 time

3 bottle 5 drops 3 time remaining

2 tablets 3 times

↳ Binary subtraction:

$$\begin{array}{r} 01011 \\ - 00111 \\ \hline 01110 \end{array}$$

1 - 1 = 0
10 = 2 - 1 = 1

OR

$$\begin{array}{r} 00111 \\ + 11000 \\ \hline 11001 \end{array}$$

Discard

$$\begin{array}{r} 10101 \\ + 11001 \\ \hline 01110 \end{array}$$

Binary multiplication:

$$\begin{array}{r} 11101 \\ \times 1001 \\ \hline 11101 \\ 00000 \\ 00000 \\ 11101 \\ \hline 100000101 \end{array}$$

Binary division:

$$\begin{array}{r} 101 \overline{) 101110} \\ - 101 \\ \hline 0011 \\ - 0011 \\ \hline 000 \\ - 000 \\ \hline 101 \end{array}$$

101 → 5

$$\begin{array}{r} 101 \overline{) 1010} \\ - 101 \\ \hline 001 \\ 01 \\ 00 \\ 10 \end{array}$$

$$\begin{array}{r} 10 \\ 2 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 11001 \\ \overline{) 11011} \\ - 11 \\ \hline 000 \\ - 000 \\ \hline 001 \\ - 001 \\ \hline 000 \\ - 000 \\ \hline 11 \end{array}$$

001

10

00 → 0

00

OR

↳ Sub Divisor from Dividend till you get a number less than Divisor

$$10 \div 3$$

$$\begin{array}{r} 1010 \\ - 0011 \\ \hline \end{array} \quad \begin{array}{r} 0011 \\ 1100 \\ + \quad 1 \\ \hline 1101 \end{array}$$

OR

+

$$\begin{array}{r} 1010 \\ + 1101 \\ \hline 0111 \end{array} \quad \begin{array}{l} 1 \\ 7 < 3 \end{array}$$

$$\begin{array}{r} 0111 \\ + 1101 \\ \hline 0100 \end{array} \quad \begin{array}{l} 1 \\ 4 < 3 \end{array}$$

$$\begin{array}{r} 0100 \\ + 1101 \\ \hline 0001 \end{array} \quad \begin{array}{l} 1 \\ 1 < 3 \text{ YES} \end{array}$$

R ←

$$1 + 1 + 1 = 3 = (11)_2 \quad \hookrightarrow Q$$

Signed Binary Numbers

↳ Range: $(-2^{k-1}, 2^{k-1} - 1)$

↳ 8 bits

00111010
 ↓ sign ↓ magnitude

Example,

11000110

$$- 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0$$

$$-128 + 64 + 0 + 0 + 0 + 4 + 2 + 0$$

-58

Q. Given +15 (00001111) find -15?

$$\begin{array}{r}
 00001111 \\
 11110000 \\
 + \quad \quad \quad 1 \\
 \hline
 11110001
 \end{array}$$

Arithmetic operation:

↳ Add:

$$\begin{array}{r}
 1111 \\
 00001111 \quad 15 \\
 00001111 \quad 15 \\
 + 00011110
 \end{array}$$

$$\begin{array}{r}
 111 \\
 00001110 \quad 14 \\
 11101111 \quad -17 \\
 + 11111101
 \end{array}$$

$$\begin{array}{r}
 1111 \\
 11111111 \\
 11111000 \\
 + 11110111
 \end{array}$$

↳ Sub:

$$\begin{array}{r}
 00011110 \quad +30 \\
 -00001111 \quad -(+15) \\
 111 \quad \hookrightarrow 2's \text{ comp} \\
 00011110 \\
 + 11110001 \\
 + 00001111
 \end{array}$$

$$\begin{array}{r}
 00001110 \quad +14 \\
 -11101111 \quad -(-17) \\
 111 \quad \hookrightarrow 2's \text{ comp} \\
 00001110 \quad +14 \\
 00010001 \quad +17 \\
 00011111
 \end{array}$$

↳ Adding same numbers:

$$\begin{array}{r}
 11111111 \\
 01111111 \quad 127 \\
 01111111 \quad 127 \\
 + 01111110
 \end{array}$$

↳ Add one bit

$$\begin{array}{r}
 1 \\
 10000001 \quad -127 \\
 10000001 \quad -127 \\
 + 10000001
 \end{array}$$

↳ Don't discard

Hexadecimal:

↳ Binary to hexa:

$$\begin{array}{cccc}
 \underline{1001} & \underline{0110} & \underline{0000} & \underline{1110} \\
 9 & 6 & 0 & E
 \end{array}$$

↳ Decimal to Hexa:

16	100
	6 — 4

$$(100)_{10} = (64)_{16}$$

16	1600
16	100 — 0
	6 — 4

$$(1600)_{10} = (640)_{16}$$

$$1600 \cdot 7$$

$$0.7 \times 16 = 11.2 \quad 11$$

$$20.2 \times 16 = 323.2 \quad 3$$

$$(1600 \cdot 7)_{10} = (640 \cdot 833 \dots)_{16}$$

Hexa to binary

$$(F40)_{16} = (111101000000)_{16}$$

\swarrow \downarrow \searrow
 1111 0100 0000

Octadecimal:

↳ Same as above but now groups in 3

Binary coded decimal:

↳ To display on seven segmented display

↳ Add 110 to no > 9 to separate them

	0	1	0	1	0	→ 10
+			1	1	0	→ 6
	0	0	1	0	0	0
	1			0		

	0	0	1	0	1	0	1	0
					1	1	0	
	0	0	1	1	0	0	0	0
	3				0			

$$(15)_{10} = 0 \oplus 1 \oplus 1 \oplus 1 \oplus 1$$

↓

0 1 0 0 0

$$(16)_{10} = 1 \oplus 0 \oplus 0 \oplus 0 \oplus 0$$

↓

1 1 0 0 0

one bit transition from 15 to 16

Gray Code:

$$(15)_{10} = (01111)_2$$

$$(16)_{10} = (10000)_2$$

Too many bit changes.
Causes errors

Practices

Binary to Decimal:

1 0 0 0 1 0 . 0 1

$2^5 2^4 2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2}$

$$2^5(1) + 0 + 0 + 0 + 2 + 0 + 0.25$$

34.25

Decimal to binary:

315.25

2	315	
2	157	1
2	78	1
2	39	0
2	19	1
2	9	1
2	4	1
2	2	0
	1	0

1 0 0 1 1 1 0 1 1 . 0 1

$$0.25 \times 2 = 0.5$$

carry = 0

$$0.5 \times 2 = 1.0$$

carry = 1

$$0.0 \times 2 = 0$$

Binary addition:

$$215 + 11$$

$$\begin{array}{r} 11010111 \\ + \quad \quad 1011 \\ \hline 11100010 \end{array}$$

Binary sub:

$$215 - 11$$

$$\begin{array}{r} 11010111 \\ - \quad \quad 1011 \\ \hline 11001100 \end{array}$$

Binary multiplication:

$$20 \times 2 = 40$$

$$\begin{array}{r} 10100 \\ \times \quad 10 \\ \hline 00000 \\ 10100 \\ \hline 101000 \end{array}$$

Binary division:

$$\begin{array}{r} 1010 \rightarrow 10 \\ 10 \angle 10 \quad | \quad 10100 \rightarrow 20 \\ - 10 \downarrow \\ \hline 001 \\ 00 \downarrow \\ \hline 10 \\ 10 \\ \hline 00 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 - 0 \end{array}$$

$$\begin{array}{r} 1111 \\ + \quad \quad 1 \\ \hline 1000 \end{array}$$

Binary to Hexa-decimal

$\underbrace{1001}_9 \underbrace{0110}_6 \underbrace{0000}_0 \underbrace{1110}_E$

Hexa to decimal.

$1 \quad A \quad 2 \quad F$
 $16^3 \quad 16^2 \quad 16^1 \quad 16^0$

$$2(16^3) + 2(16^2) + 2(16^1) + 1(15) = 6703$$

Decimal to hexa:

$16 \overline{) 215}$
 $\underline{13} \quad 7$
 $16 \overline{) 215}$
 $\underline{20} \quad 15$
 $\underline{16} \quad 7$

Hexa to octa

$F \quad 7 \quad D$
 $\swarrow \quad | \quad \searrow$
 $1111 \quad 0111 \quad 1101$

$\underbrace{1111}_7 \underbrace{1011}_5 \underbrace{1111}_7 \underbrace{1011}_5$

$2 \quad C$
 $16^1 \quad 16^0$
 $32 + 12$

$\begin{array}{r} 1010 \\ + 110 \\ \hline 100010000 \\ \hline 1 \quad 0 \end{array}$