Random Variables:

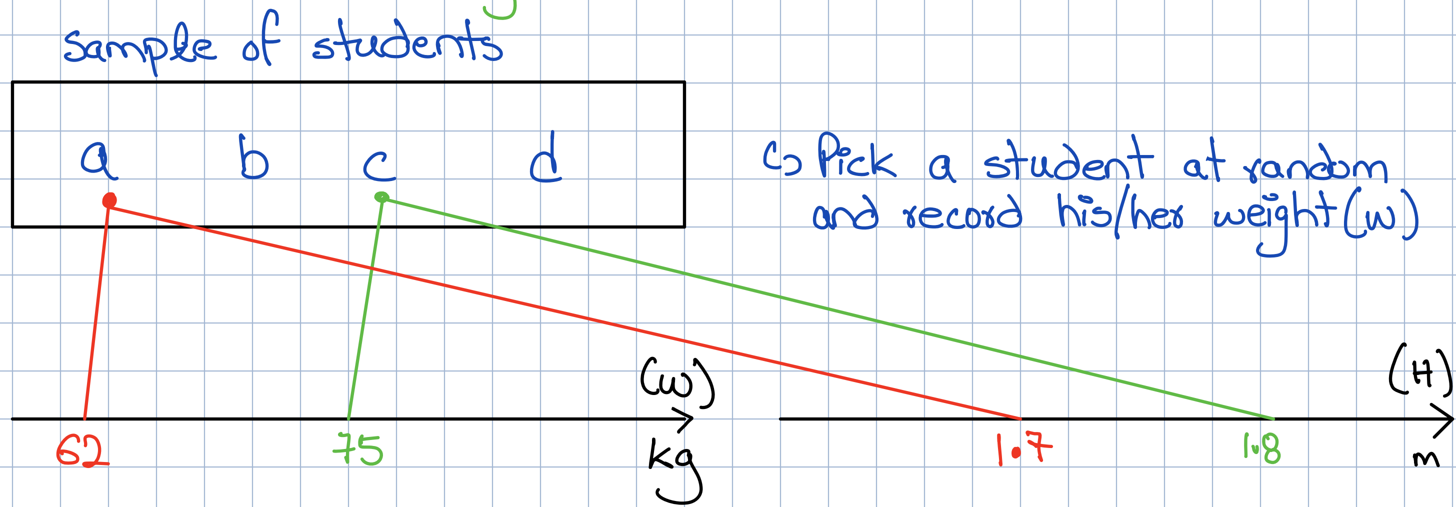
- Discrete
- Continuous

↳ Basic probability tells you the chance of each outcome, but random variables help you calculate the average payout.

↳ Random Variable map to some numerical value once the outcome of an event is determined, that you can measure

$X = f(\omega)$ $\qquad$ $X : \Omega \rightarrow \mathbb{R}$

↳ Domain of X is sample space.
↳ Range can be $\mathbb{R}$, $(0, \infty)$, $(0,1)$ etc

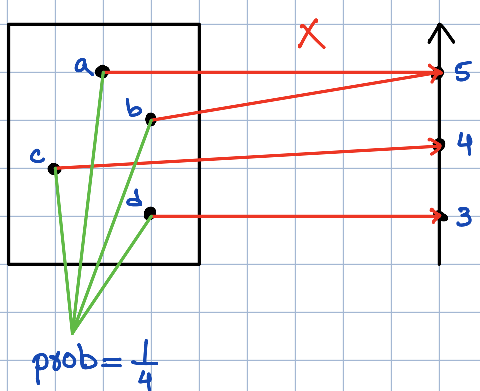


↳ Pick a student at random and record his/her weight (w)

↳ Random Variable is a function that maps to some numerical value.
↳ Probabilistic experiment can have several associated RVs W & H.
↳ We can create a third RV like $BMI = W/H^2$.

Probability Mass function:

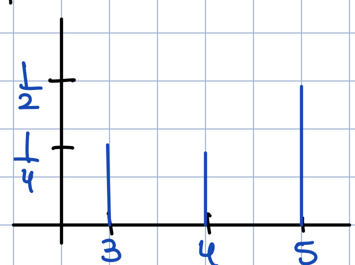
↳ Some outcomes of an event are more OR less likely than others.



↳ $X = x$ is an event.

$x = 5$, $\{\omega : X(\omega) = 5\} \rightarrow \{a, b\}$

PMF:



↳ $P_X(x) >= 0$
↳ $P_X(x_1) + \cdots + P_X(x_n) = 1$

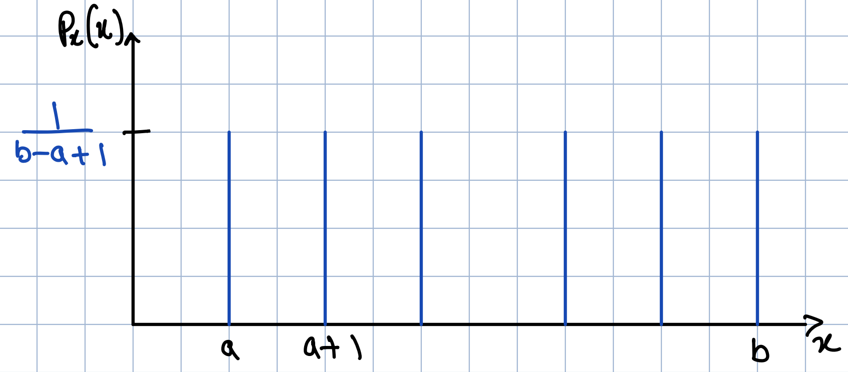
↳ $P_X(x) = P(X = x) \rightarrow P(\{\omega \in \Omega \text{ st } X(\omega) = x\})$

Bernoulli Random Variable with parametre [0,1]

$$X = \begin{cases} 1, & p \\ 0, & 1-p \end{cases}$$

↳ Models a single trail that results in success/failure.

Discrete Uniform Random Variable:



↳ $a <= b$, ints
↳ All equally likely
↳ $\Omega = \{a, a+1, \dots, b\}$

Bionomial Random Variable:

↳ Represents number of successes in a fixed number of independent trials with binary outcomes.

↳ For example,
- ↳ Flip a coin.
  - ↳ Success would be landing head
  - ↳ Failure would be landing tails
- ↳ If we repeat this n times & count the success, then this count is a bionomial Random Variable

Geometric Random Variable

↳ A type of Random Variable that counts no. of trials needed to get the first success.

↳ For example,
- ↳ A factory produces a working bulb with $p = 0.9$.
- ↳ X: no. of bulbs until first working bulb is found.

$$P(X=k) = p(1-p)^{k-1} \quad \text{for } k = 1, 2, 3, \dots$$

Expectation:

↳ Average gain.

$\mathcal{E}[X] = \sum_x P_X(x)$

Expectation as a population average:

↳ Consider n students
↳ Weight of ith student: x_i
↳ Experiment: Pick a student at random, all equally likely
↳ Random Variable X: Weight of selected student

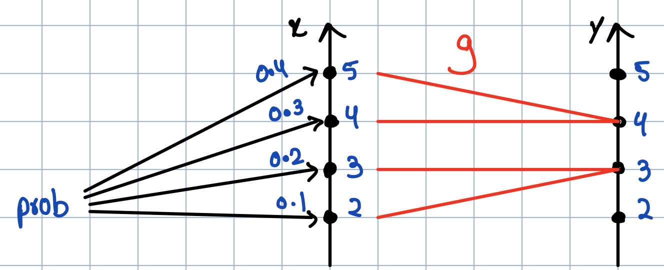
$$P_X(x_i) = \frac{1}{n}$$

$$\mathcal{E}[X] = \sum_i x_i \cdot \frac{1}{n}$$
$$= \frac{1}{n} \sum_i x_i$$
$$= \text{mean}\{X\}$$

The Expected value rule for calculating $\mathcal{E}[g(X)]$

↳ Let X be a RV & let $Y = g(X)$

$$\mathcal{E}[Y] = \sum P_Y(y)$$
$$\mathcal{E}[g(x)] = \sum g(x) P_X(x)$$



Variance of Random Variable:

$$\sigma^2 = \text{Var}(X)$$
$$= \mathcal{E}[X - \mathcal{E}[X]]^2$$
$$= \sum_x (x-\mu)^2 P(x)$$
$$= \mathcal{E}[X^2] - (\mathcal{E}[X])^2$$

$\sigma = std(X) = \sqrt{Var(X)}$

PMFs of Random Variables:

↳ If there are RVs X & Y, then (X, Y) is a random vector and it's distribution is called joint distribution.

$$P_{X,Y}(x, y) = P(X = x, Y = y) \quad \text{for} \quad -\infty < x, y > \infty$$

↳ Individual distributions of X & Y are called marginal distribution.
↳ It can be obtained by summing over the other variable.

Covariance & Correlation:

$$Cov(X, Y) = \mathcal{E}\{(X - \mathcal{E}[X])(Y - \mathcal{E}(Y])\}$$
$$= \mathcal{E}[XY] - \mathcal{E}[X]\mathcal{E}[Y]$$

$Cov(X, Y) > 0$: X & Y +evly related

$Cov(X, Y) < 0$: X & Y -evly related

$Cov(X, Y) = 0$: X & Y NOT related

↳ This has unit according to unit of X & Y & $Cov(X, Y)$ is unbounded. So, we can't tell if X & Y are strongly & weakly related.

↳ So, we need correlation ρ

$$\rho = \frac{Cov(X, Y)}{std(X)\, std(Y)}$$