Introduction to Electrical Circuits

Final Term
Lecture - 06

Reference Book:

Introductory Circuit Analysis

Robert L. Boylestad, 11th Edition

W10	FC5	Chapter 18	18.2 SUPERPOSITION THEOREM	18.1, 18.2
			18.3 THÉVENIN'S THEOREM	18.8
	FC6	Chapter 18	18.4 NORTON'S THEOREM	18.15
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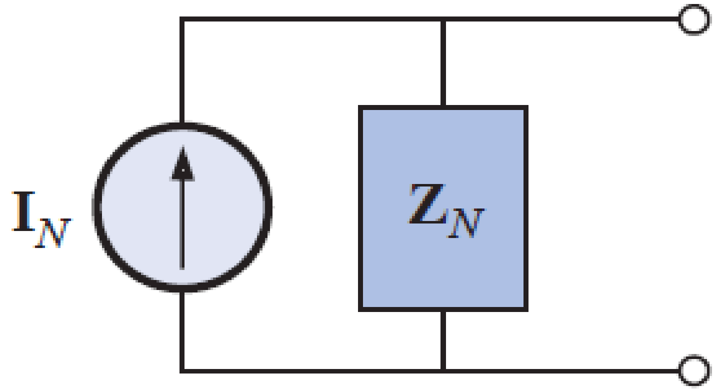
Norton's Theorem

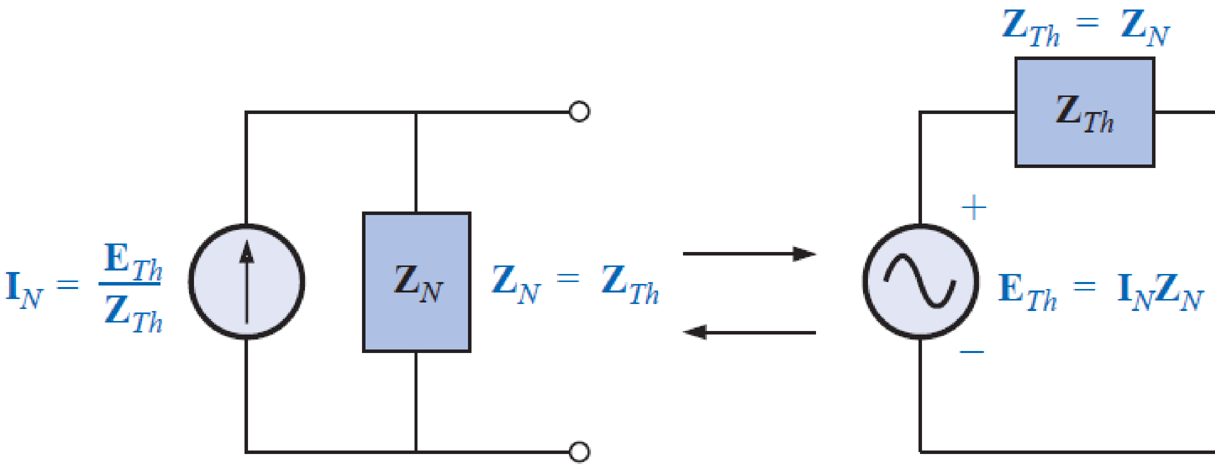
Norton's theorem states the following:

Any two-terminal, linear bilateral network can be replaced by an equivalent circuit consisting of a current source and a parallel impedance, as shown in the following figure.



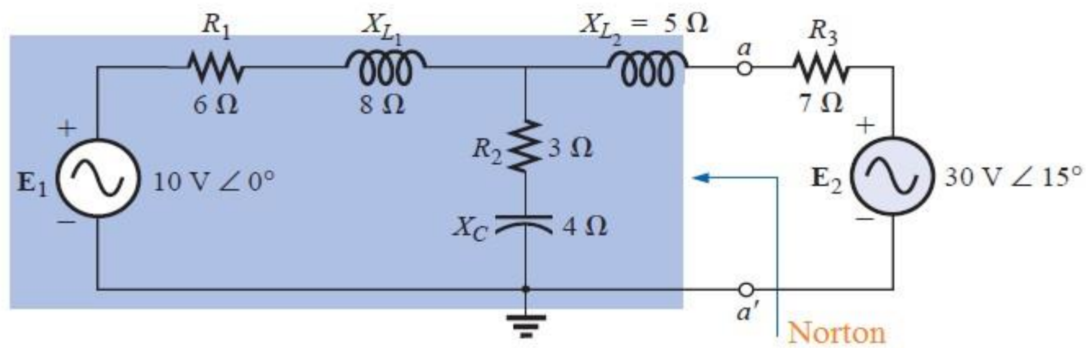
- The Norton and Thévenin equivalent circuits can be found from each other by using the source transformation shown in the following figure.

- The source transformation is applicable for any Thévenin or Norton equivalent circuit determined from a network with any combination of independent or dependent sources.



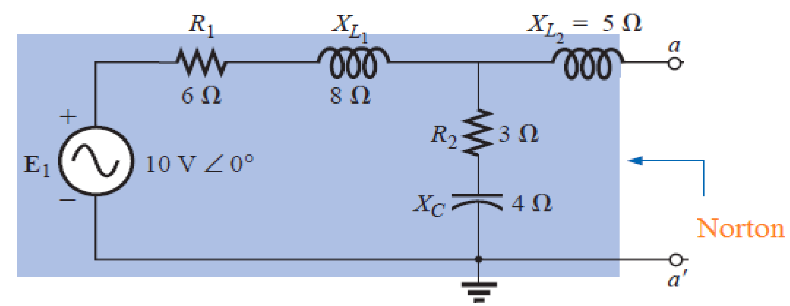
Conversion between the Thévenin and Norton equivalent circuits.

Calculation procedure of Norton's impedance is same as that of calculation of Thevenin's impedance.

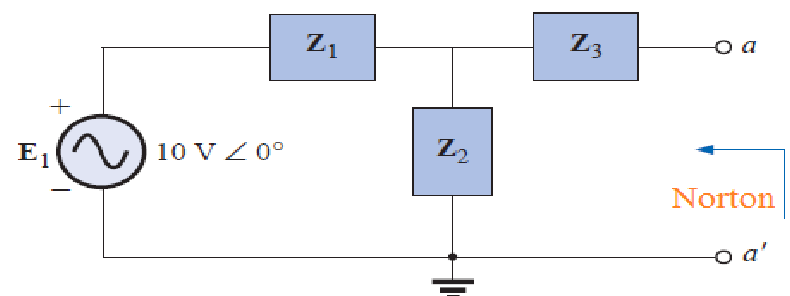


Calculate the Norton's Impedance (Z_N):

Step 1: Remove that portion of the network across which the Norton equivalent circuit is to be found.



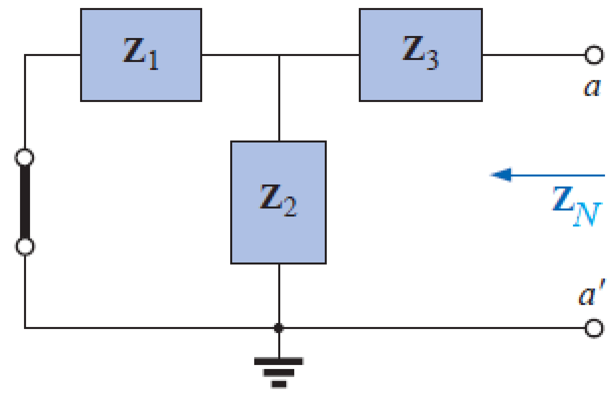
Step 2: Mark the terminals of the remaining two-terminal network.



$Z_1 = R_1 + jX_{L1} = 6 + j8\ \Omega = 10\angle 53.13^\circ\ \Omega$

$Z_2 = R_2 - jX_C = 3 - j4\Omega = 5\angle -53.13^\circ\Omega$

$Z_3 = jX_{L2} = j5\ \Omega = 5\angle 90^\circ\ \Omega$



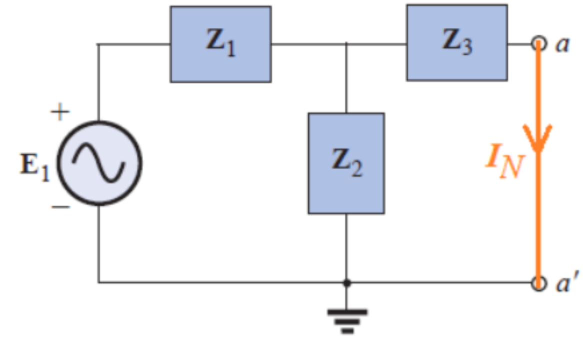
Step 3: **Set all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits). If the internal impedance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.**

Step 4: **Find the impedance between the two marked terminals.**

$$Z_N = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} = j5 + \frac{(6 + j8)(3 - j4)}{6 + j8 + 3 - j4}$$

$$= 4.64 + j2.94\ \Omega = 5.49\angle 32.36^\circ$$

Calculate the Norton's Current (I_N):



Step 1: Remove that portion of the network across which the Norotn's equivalent circuit is to be found.

Step 2: Short the terminals of the remaining two-terminal network.

Step 3: Calculate the current which pass through the shorted circuit.

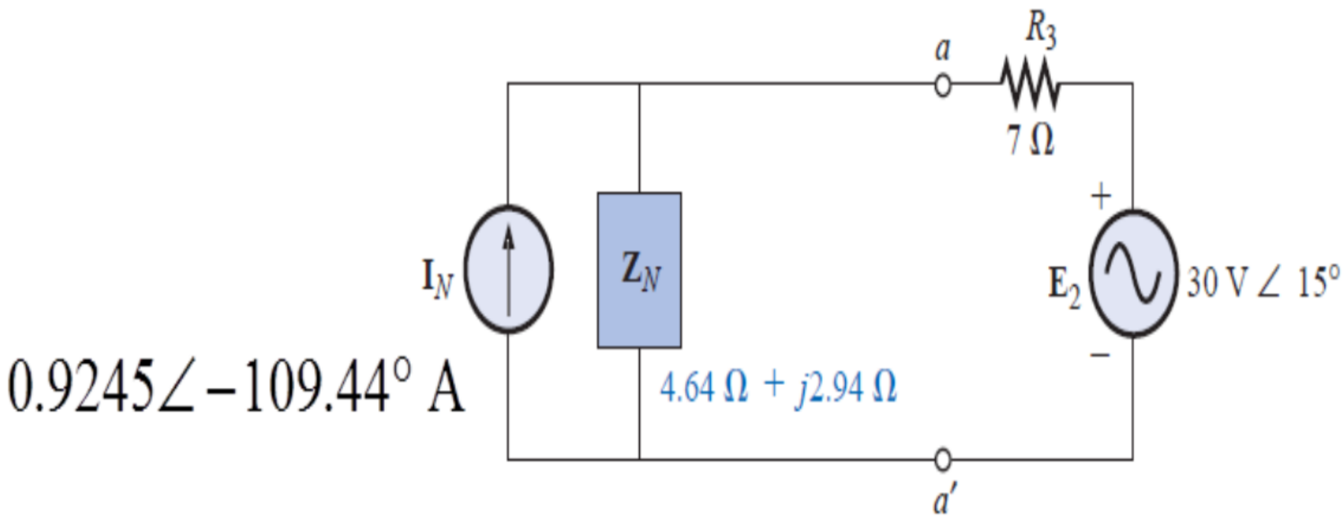
$$\boldsymbol{Z}_{23} = \frac{\boldsymbol{Z}_2 \boldsymbol{Z}_3}{\boldsymbol{Z}_2 + \boldsymbol{Z}_3} = 7.5 + j2.5\ \Omega$$

$$\boldsymbol{V}_{23} = \frac{\boldsymbol{Z}_{23}}{\boldsymbol{Z}_1 + \boldsymbol{Z}_{23}} \boldsymbol{E}_1 = 4.35 - j1.5385\ \text{V}$$

(VDR)

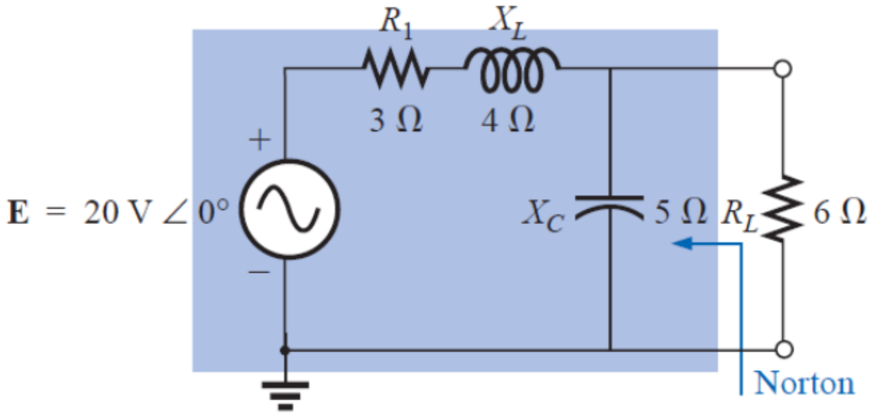
$$\boldsymbol{I}_N = \frac{\boldsymbol{V}_{23}}{\boldsymbol{Z}_3} = -0.3077 - j0.8718 = 0.9245\angle -109.44^\circ\ \text{A}$$

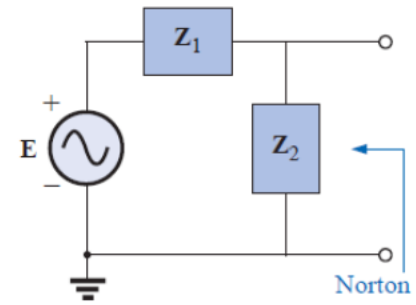
Finally draw the Norton's equivalent circuit by connecting the removed part.



Example

Determine the Norton equivalent circuit for the network external to the 6 Ω resistor of following figure.

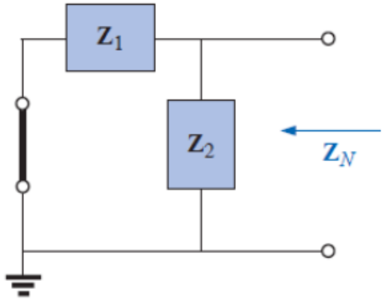




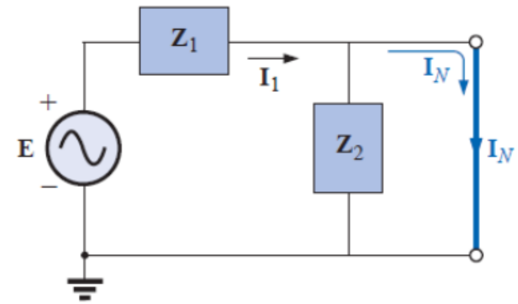
$$Z_1 = 3 + j4\ \Omega = 5\angle 53.13°\ \Omega$$

$$Z_2 = -j5 = 5\angle -90°\Omega$$

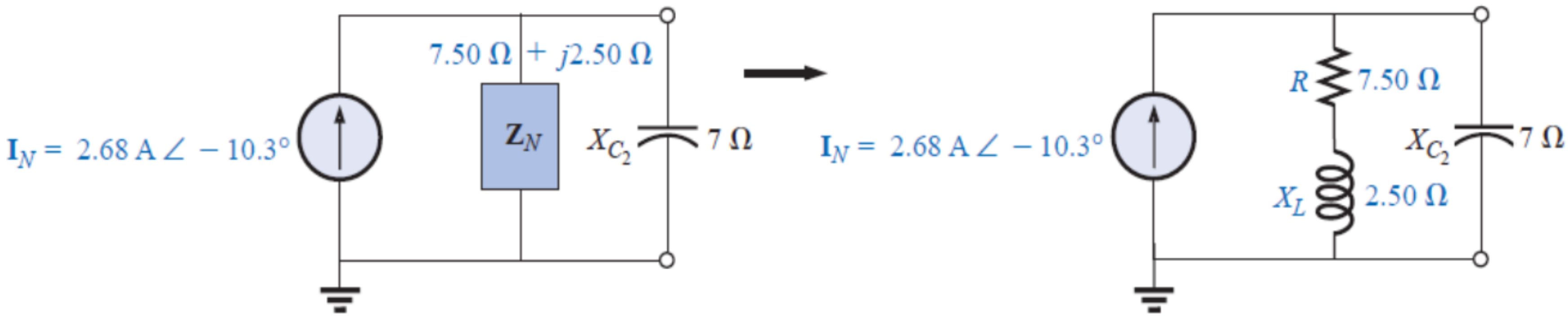
$$Z_N = \frac{Z_1 Z_2}{Z_1 + Z_2} = 7.5 - j2.5 = 7.91\angle -18.44°\ \Omega$$



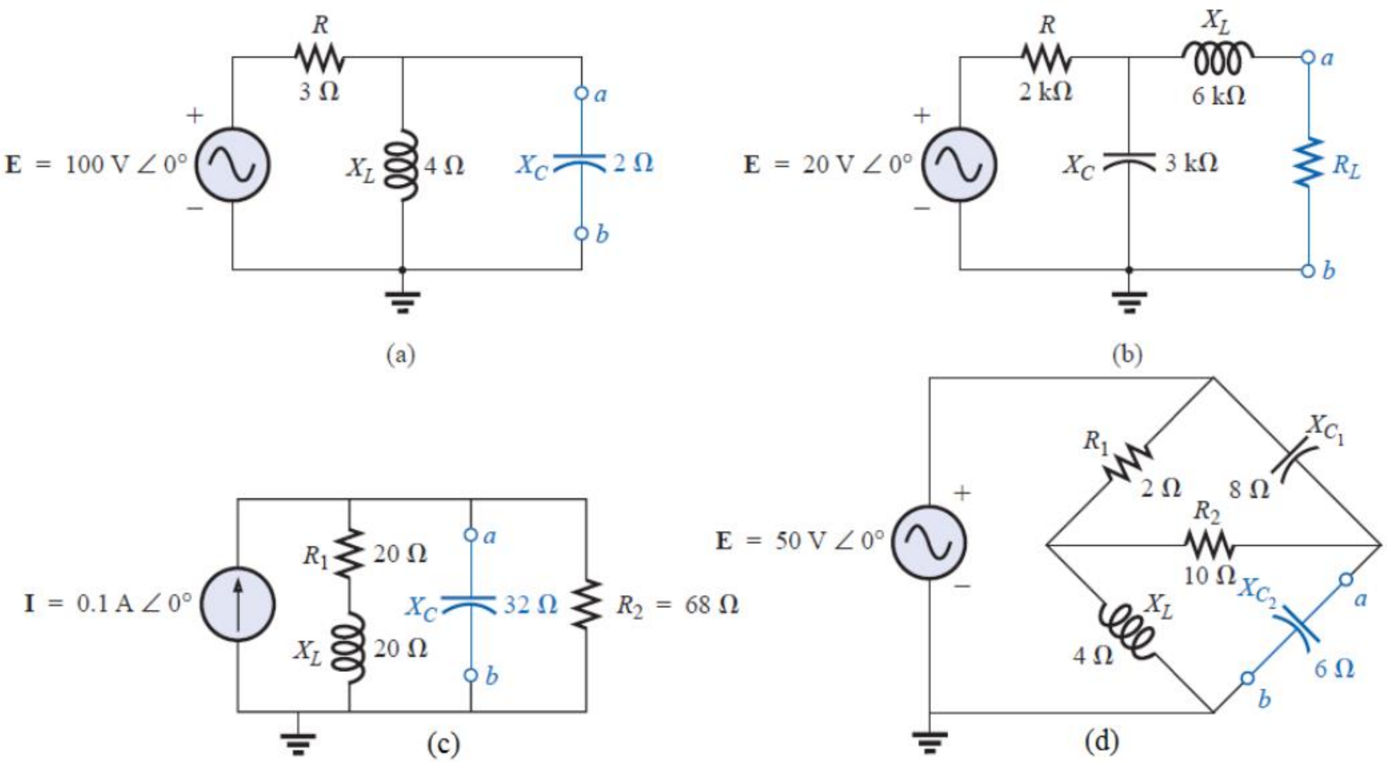
$$I_N = I_1 = \frac{E}{Z_1} = \frac{20\angle 0°}{5\angle 53.13°} = 4\angle -53.13° \text{ A}$$



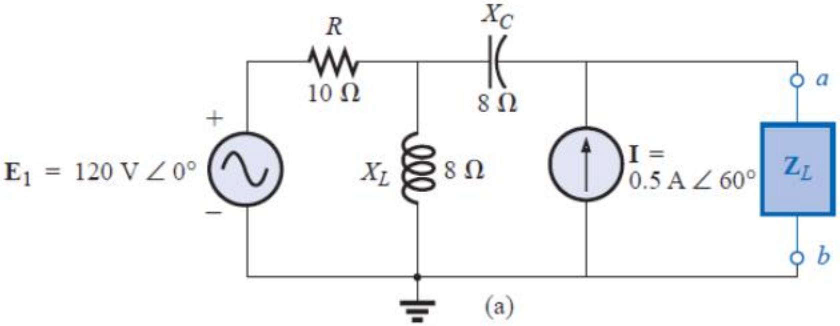
Norton equivalent circuit shown in the following figure.



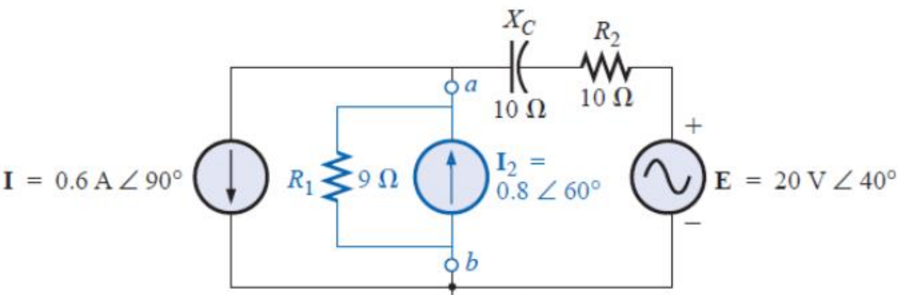
Problem 1: Find the Norton equivalent circuit for the portions of the networks of following figures external to the elements between points a and b.



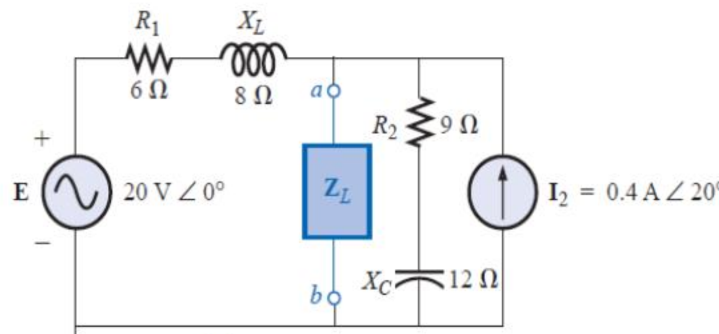
Problem 2: Find the Norton equivalent circuit for the portions of the networks of following figures external to the elements between points a and b.



(a)



(b)

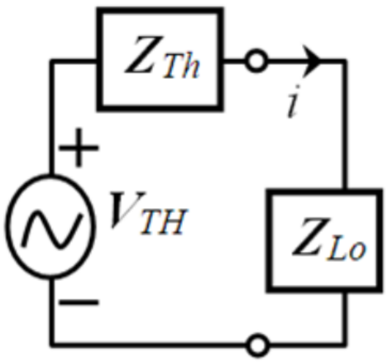


(c)

Maximum Power Transfer Theorem

The maximum power transfer theorem states:

Maximum power will be delivered to a load when the load impedance is the complex conjugate of the Thévenin's impedance across its terminals.



$$\mathbf{Z}_{Th} = R_{Th} + jX_{Th} = Z_{Th}\angle\theta_{Th}$$

$$\mathbf{Z}_{Lo} = R_{Lo} + jX_{Lo} = Z_{Lo}\angle\theta_{Lo}$$

The power transfer to the load is maximum if: $R_{Lo} = R_{Th}$ $X_{Lo} = -X_{Th}$

$$\boxed{\mathbf{Z}_{Lo} = R_{Lo} + jX_{Lo} = R_{Th} - jX_{Th} = \mathbf{Z}_{Th}^{*}}$$

The maximum power is given by:

$$\boxed{P_{L\max} = I^2 R_{Lo} = \frac{V_{Th}^2 R_{Lo}}{(2R_{Lo})^2} = \frac{V_{Th}^2}{4R_{Lo}} = \frac{V_{Th}^2}{4R_{Th}}}$$

Example

Find the load impedance required to be connected across the terminals A-B for the maximum power transfer, in the network shown in the Fig. 2.36. Also find the maximum power delivered to the load.

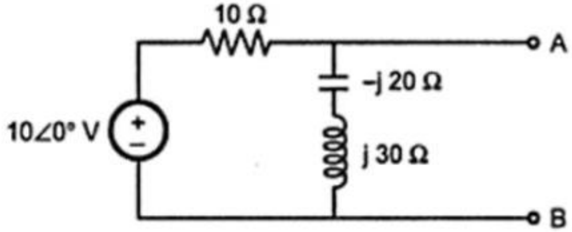


Fig. 2.63

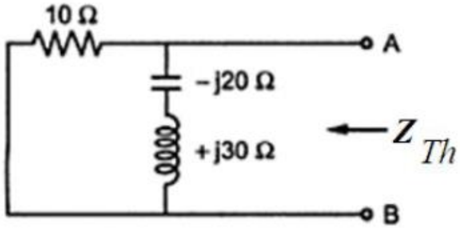


Fig. 2.63 (a)

$$Z_{Th} = \frac{10\times j(30-20)}{10+j(30-20)} = \frac{10\times j10}{10+j10} = \frac{100\angle 90^\circ}{14.14\angle 45^\circ} = 7.07\angle 45^\circ = 5+j5\ \Omega$$

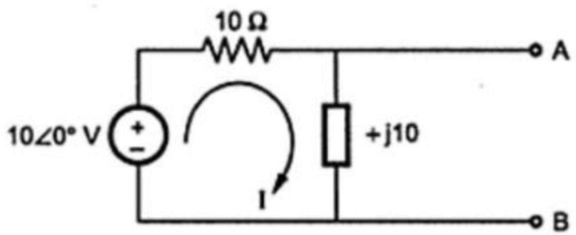


Fig. 2.63 (b)

$$V_{Th} = \frac{10\angle 0^\circ \times j10}{10+j10} = \frac{100\angle 90^\circ}{14.14\angle 45^\circ} = 7.07\angle 45^\circ$$

For maximum power transfer: $\boldsymbol{Z}_{Lo} = \boldsymbol{Z}_{Th}^{*} = 7.07\angle -45° = 5 - j5\ \Omega$

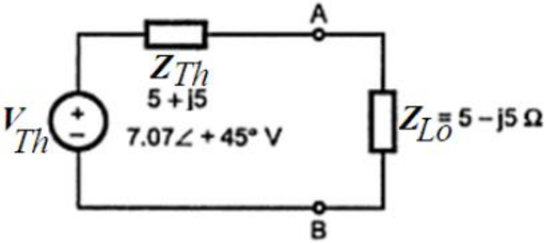


Fig. 2.63 (c)

The maximum power is given by: $P_{L\max} = \dfrac{V_{Th}^2}{4R_{Lo}} = \dfrac{7.07^2}{4\times 5} = 25\ \mathrm{W}$

Example

Find the load impedance in Fig. 18.81 for maximum power to the load, and find the maximum power.

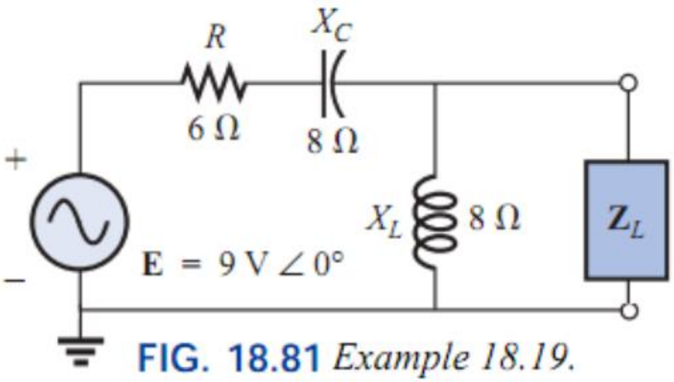


FIG. 18.81 *Example 18.19.*

$$Z_1 = 6 - j8\ \Omega = 10\Omega\angle -53.13^\circ$$

$$Z_2 = j8\ \Omega = 8\Omega\angle 90^\circ$$

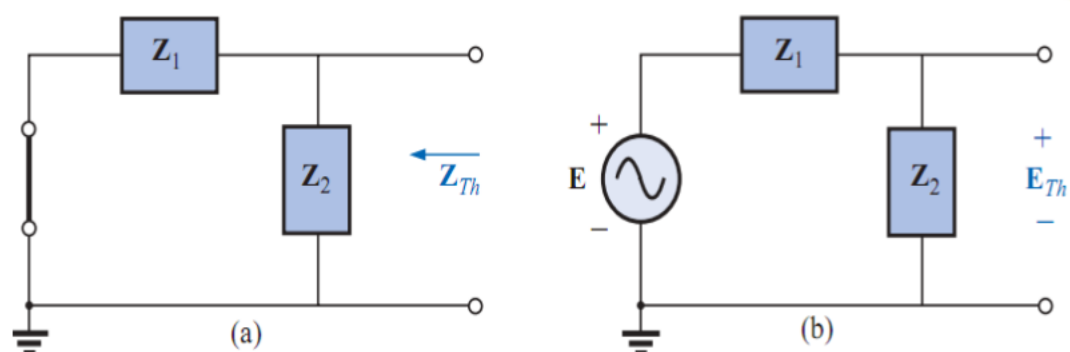


FIG. 18.82

Determining (a) $\mathbf{Z}_{Th}$ and (b) $\mathbf{E}_{Th}$ for the network external to the load in Fig. 18.81.

$$\boldsymbol{Z}_{Th} = \frac{\boldsymbol{Z}_1\boldsymbol{Z}_2}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{(6 - j8)(j8)}{6 - j8 + j8}$$

$$= \frac{80\ \Omega\angle 36.87^\circ}{6\angle 0^\circ} = 13.33\angle 36.87^\circ$$

$$= 10.66 + j8\ \Omega$$

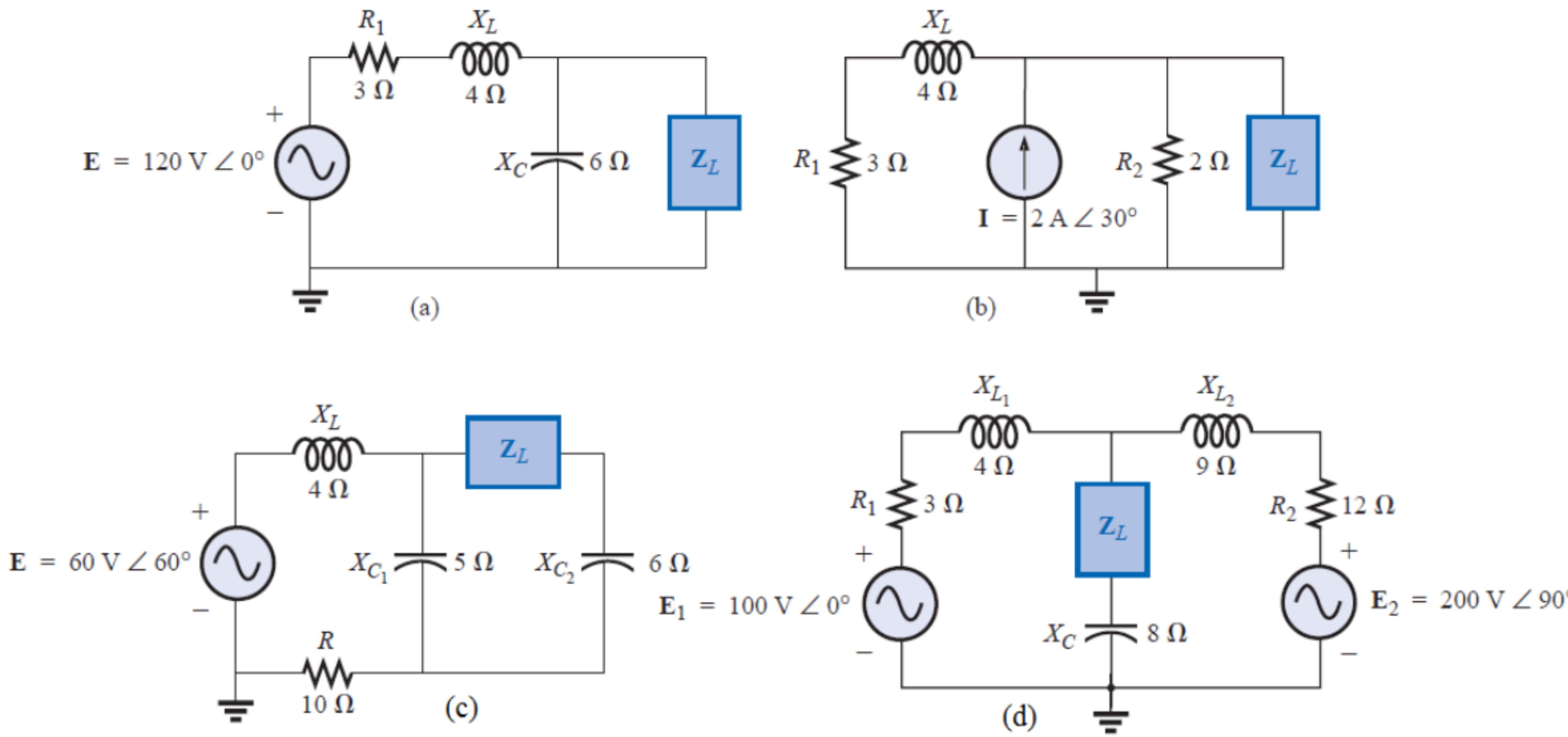
$$\boldsymbol{E}_{Th} = \frac{\boldsymbol{Z}_2\boldsymbol{E}}{\boldsymbol{Z}_1 + \boldsymbol{Z}_2} = \frac{(8\Omega\angle 90^\circ)(9\text{V}\angle 0^\circ)}{j8 + 6 - j8} = \frac{72\,\text{V}\angle 90^\circ}{6\angle 0^\circ} = 12\angle 90^\circ$$

For maximum power transfer: $\boldsymbol{Z}_{Lo} = \boldsymbol{Z}_{Th}^{*} = 13.33\angle -36.87^\circ = 10.66 - j8\ \Omega$

The maximum power is given by:

$$P_{L\max} = \frac{E_{Th}^2}{4R_{Lo}} = \frac{12^2}{4\times 10.66} = 3.38\ \text{W}$$

Problem 1: Find the load impedance ZL for the networks of following figures for maximum power to the load and find the maximum power to the load.



Thank You