

# Introduction to Electrical Circuits

## Mid Term Lecture – 3

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Reference Book:  
**Introductory Circuit Analysis**  
Robert L. Boylestad, 11<sup>th</sup> Edition

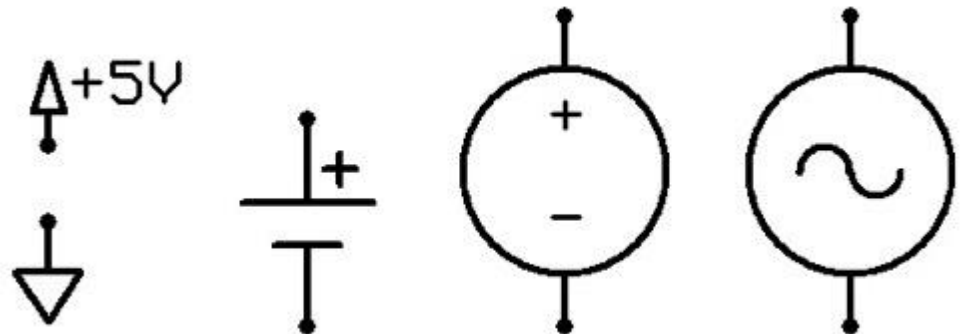
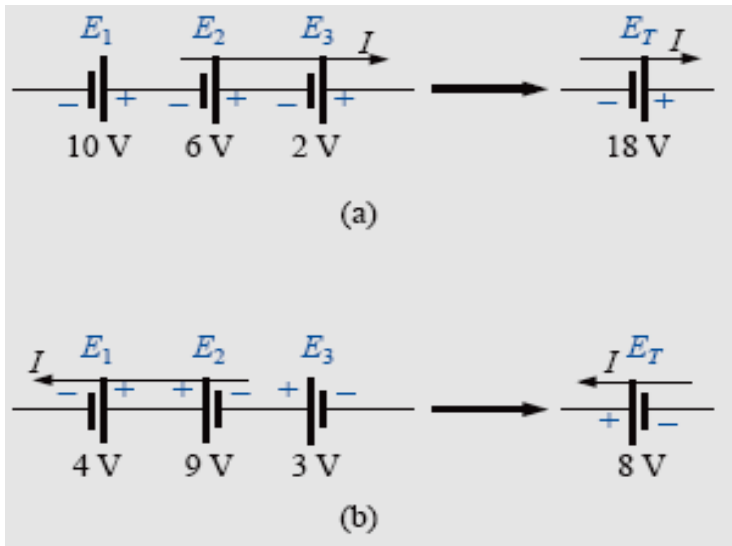




Week No.	Class No.	Chapter No.	Article No., Name and Contents	Example No.	Exercise No.
W2	MC3	Chapter 5	5.7 VOLTAGE DIVISION IN A SERIES CIRCUIT (Voltage Divider Rule only)	5.15	26
		Chapter 6	6.2 PARALLEL RESISTORS	6.1, 6.2	12, 16,
			6.3 PARALLEL CIRCUITS	6.11	17.

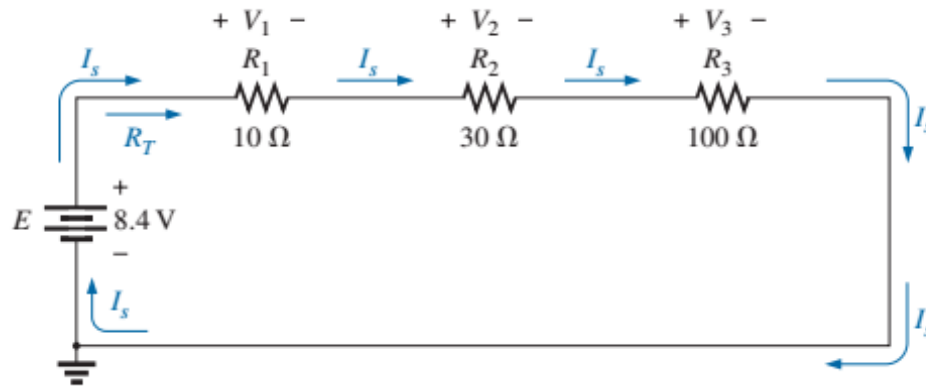


# Voltage Sources in Series



# Voltage Division in a Series Circuit

*The voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.*



**Voltage divider approach:**

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} \times E$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} \times E$$

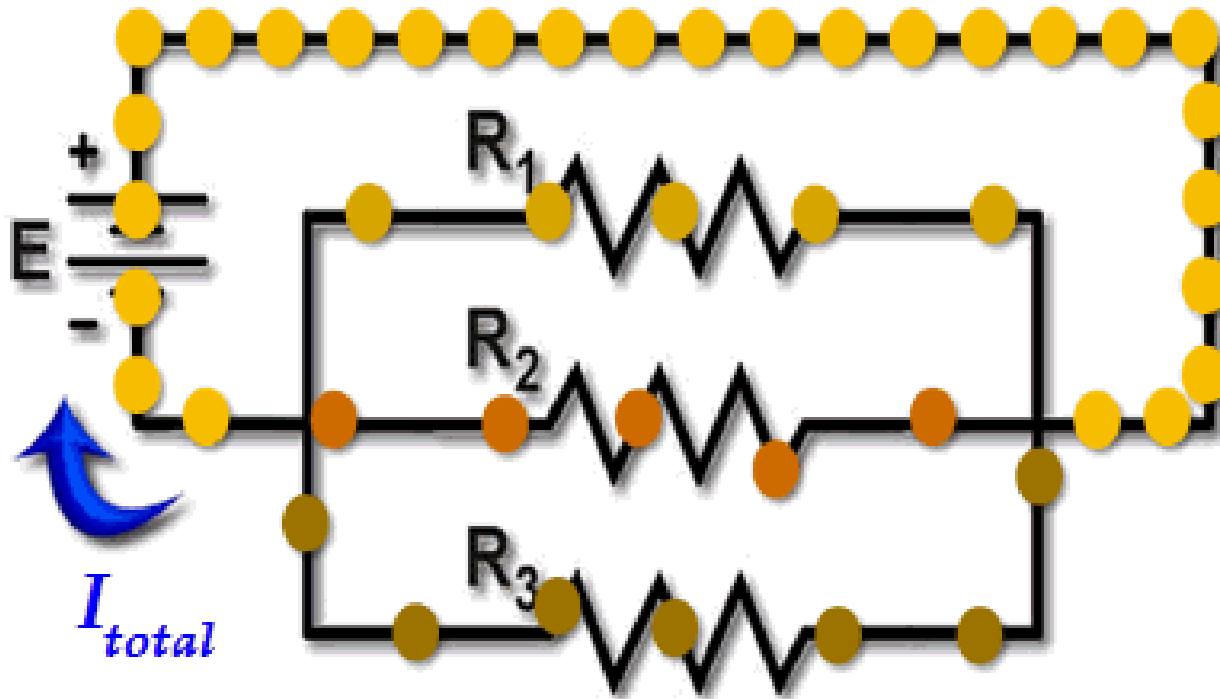
$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} \times E$$

**General formula:**

$$V_x = R_x \frac{E}{R_T} \quad (\text{voltage divider rule})$$



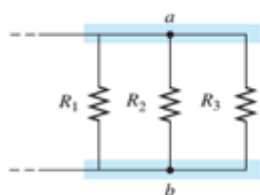
# Parallel Circuits



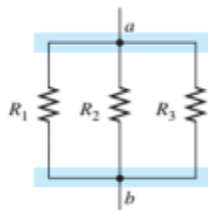
## 6.2 PARALLEL RESISTORS

# CHAPTER 6

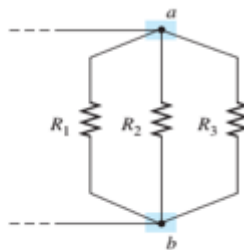
Two elements, branches, or circuits are in parallel if they have two points in common. The total resistance of parallel resistors is always less than the value of the smallest resistor. The total resistance of parallel resistors will always drop as new resistors are added in parallel, irrespective of their value.



(a)



(b)



(c)

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

Since  $G = 1/R$ ,

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N \quad (\text{siemens, S})$$

In general, however, when the total resistance is desired, the following format is applied:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}} \quad (6.3)$$

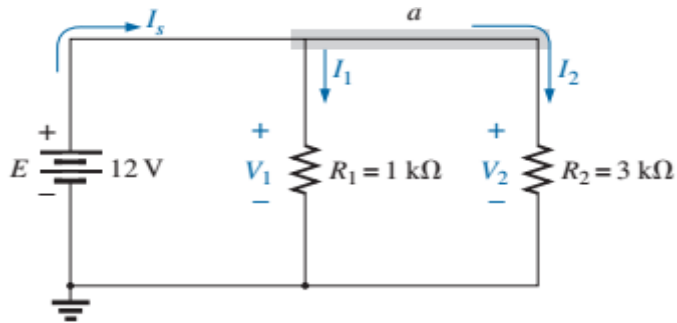
Special Case: Two Parallel Resistors

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$



## 6.3 PARALLEL CIRCUITS

(Equations associated to analyze series network)



1. The voltage is always the same across parallel elements.

$$V_1 = V_2 = E$$

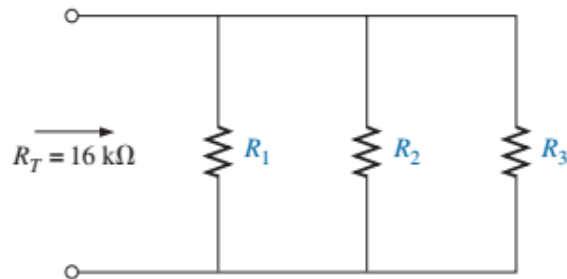
2. Total Resistance:  $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$

3. Total Current:  $I_s = \frac{E}{R_T}$

4. Branch Currents: Ohm's Law approach:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$





**FIG. 6.16**

*Parallel network for Example 6.11.*

**EXAMPLE 6.11** Determine the values of  $R_1$ ,  $R_2$  and  $R_3$  in Fig. 6.16 if  $R_2 = 2R_1$ ,  $R_3 = 2R_2$ , and the total resistance is  $16 \text{ k}\Omega$ .

**Solution:** Eq. (6.1):

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However,  $R_2 = 2R_1$  and  $R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that 
$$\frac{1}{16 \text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and 
$$\frac{1}{16 \text{ k}\Omega} = \frac{1}{R_1} + \frac{1}{2} \left( \frac{1}{R_1} \right) + \frac{1}{4} \left( \frac{1}{R_1} \right)$$

or 
$$\frac{1}{16 \text{ k}\Omega} = 1.75 \left( \frac{1}{R_1} \right)$$

resulting in  $R_1 = 1.75(16 \text{ k}\Omega) = \mathbf{28 \text{ k}\Omega}$

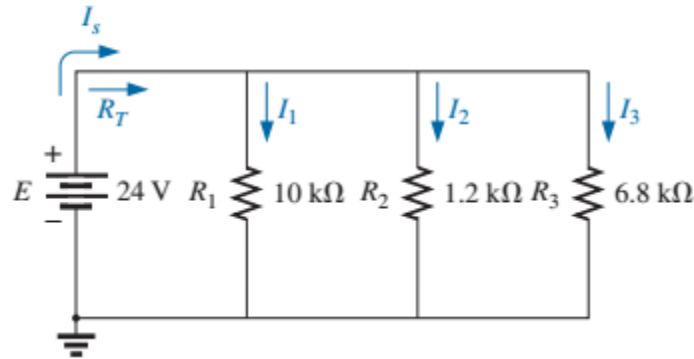
so that  $R_2 = 2R_1 = 2(28 \text{ k}\Omega) = \mathbf{56 \text{ k}\Omega}$

and  $R_3 = 2R_2 = 2(56 \text{ k}\Omega) = \mathbf{112 \text{ k}\Omega}$





12. Repeat the analysis of Problem 10 for the network in Fig. 6.83, constructed of standard value resistors.



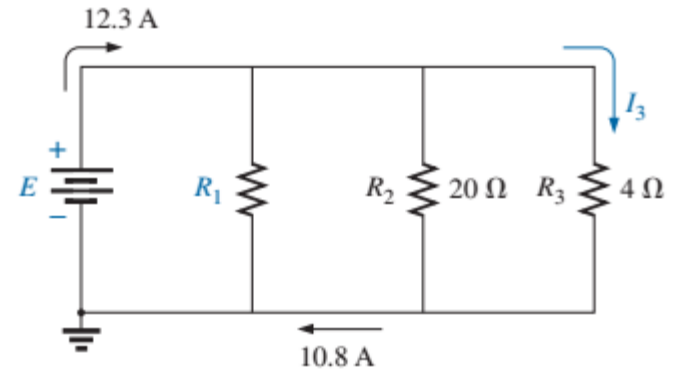
**Solution:**

- a. 
$$R_T = \frac{1}{\frac{1}{10 \text{ k}\Omega} + \frac{1}{1.2 \text{ k}\Omega} + \frac{1}{6.8 \text{ k}\Omega}} = \frac{1}{100 \times 10^{-6} \text{ S} + 833.333 \times 10^{-6} \text{ S} + 147.059 \times 10^{-6} \text{ S}}$$
  

$$= \frac{1}{1.080 \times 10^{-3} \text{ S}} = 925.93 \text{ }\Omega$$
- b.  $V_{R_1} = V_{R_2} = V_{R_3} = 24 \text{ V}$
- c.  $I_s = \frac{E}{R_T} = \frac{24 \text{ V}}{925.93 \text{ }\Omega} = 25.92 \text{ mA}$   
 $I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{24 \text{ V}}{10 \text{ k}\Omega} = 2.4 \text{ mA}, I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{24 \text{ V}}{1.2 \text{ k}\Omega} = 20 \text{ mA},$   
 $I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{24 \text{ V}}{6.8 \text{ k}\Omega} = 3.53 \text{ mA}$
- d.  $I_T = 25.92 \text{ mA} = 2.4 \text{ mA} + 20 \text{ mA} + 3.53 \text{ mA} = 25.93 \text{ mA}$  (checks)

# Exercise Problems

16. Given the information provided in Fig. 6.86, find the unknown quantities:  $E$ ,  $R_1$ , and  $I_3$ .



**Solution:**

$$I_3 = \frac{(20 \text{ }\Omega)(10.8 \text{ A})}{20 \text{ }\Omega + 4 \text{ }\Omega} = 9 \text{ A}$$

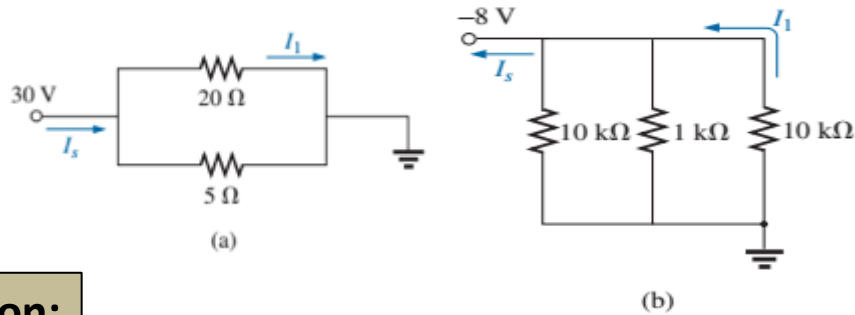
$$E = V_{R_3} = I_3 R_3 = (9 \text{ A})(4 \text{ }\Omega) = 36 \text{ V}$$

$$I_{R_1} = 12.3 \text{ A} - 10.8 \text{ A} = 1.5 \text{ A}$$

$$R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{36 \text{ V}}{1.5 \text{ A}} = 24 \text{ }\Omega$$



17. Determine the currents  $I_1$  and  $I_s$  for the networks in Fig. 6.87.



**Solution:**

a.  $R_T = 20\ \Omega \parallel 5\ \Omega = 4\ \Omega$

$$I_s = \frac{E}{R_T} = \frac{30\ \text{V}}{4\ \Omega} = 7.5\ \text{A}$$

CDR:  $I_1 = \frac{5\ \Omega I_s}{5\ \Omega + 20\ \Omega} = \frac{1}{5}(7.5\ \text{A}) = 1.5\ \text{A}$

b.  $10\ \text{k}\Omega \parallel 10\ \text{k}\Omega = 5\ \text{k}\Omega$

$$R_T = 1\ \text{k}\Omega \parallel 5\ \text{k}\Omega = 0.833\ \text{k}\Omega$$

$$I_s = \frac{E}{R_T} = \frac{8\ \text{V}}{0.833\ \text{k}\Omega} = 9.6\ \text{mA}$$

$$R'_T = 10\ \text{k}\Omega \parallel 1\ \text{k}\Omega = 0.9091\ \text{k}\Omega$$

$$I_1 = \frac{R'_T I_s}{R'_T + 10\ \text{k}\Omega} = \frac{(0.9091\ \text{k}\Omega)(9.6\ \text{mA})}{0.9091\ \text{k}\Omega + 10\ \text{k}\Omega} = \frac{8.727\ \text{mA}}{10.9091} = 0.8\ \text{mA}$$



