Propositional Logic

Course Code: CSC 1204 Course Title: Discrete Mathematics

Dept. of Computer Science
Faculty of Science and Technology

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Lecture Outline

1.1 Propositional Logic

- Logic
- Propositional Logic
- Propositions
- Propositional Variables
- Compound Propositions
- Logical Operators
- Truth Value & Truth Table
- Truth Tables of Compound Propositions (next class)
- Conditional Statements (next class)
- Logic and Bit Operations (next class)

Objectives and Outcomes

- **Objectives**: To understand the importance of logic in mathematical reasoning, to understand proposition and propositional logic, symbol and usage of different types of logical operators.
- **Outcomes**: Students are expected to be able to apply logical operators and analyze logical propositions via truth tables, be able to construct a truth table for a given compound proposition.

Key Terms

- **Logic:** Logic is the discipline that deals with the methods of reasoning.
  - Logic is the basis of all mathematical reasoning
  - The rules of logic specify the meaning of mathematical statements

- **Propositional Logic**: The area of logic that deals with ***propositions*** is called the propositional logic.

Key Terms

- **Proposition:** A ***proposition*** is a declarative statement that's either **TRUE** or **FALSE**, but not both.

- Statements that are not propositions *include*
  - questions
  - Commands

Key Terms

- **Propositional variable:** A variable that represents a proposition. The conventional letters used for propositional variables are *p, q, r, s, t,..*
- **Compound proposition:** A proposition constructed by combining two or more propositions using *logical operators* (**AKA** : *logical connectives*)
- **Logical Operators**: Operators used to combine propositions
- **Truth Value**: **The truth value** of a proposition is **true**, denoted by **T**, if it is a true statement and **false**, denoted by **F**, if it is a false statement. **Truth Value ==>** Either **True** or **False**
- **Truth Table:** A table displaying the truth values of propositions.

Proposition: Examples

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Trenton is the capital of New Jersey.
  c) Toronto is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

Logical Operators

- Logical Operators ==> **unary**, **binary**
- **Unary**:
  - Negation
- **Binary**
  - Conjunction
  - Disjunction
  - Exclusive OR
  - Conditional/Implication
  - Biconditional

Logical Operators: Symbols & Usage

Operator	Symbol	Usage
Negation	$\neg$	NOT
Conjunction	$\wedge$	AND
Disjunction	$\vee$	OR
Exclusive or	$\oplus$	XOR
Conditional	$\rightarrow$	if, then
Biconditional	$\leftrightarrow$	iff

Propositional Logic : Negation

- Let *p* be a proposition. The *negation of p*, denoted by $\neg p$ (or $\bar{p}$), is the statement "It is not the case that *p*."
- The proposition $\neg p$ *is read "not p"*
- The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Truth table for Negation of a Proposition

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Propositional Logic : **Negation**

- Negation just turns *a* ***false*** proposition to ***true*** and the opposite for a true proposition.

- Example1: P: I am going to town

  $\neg$ P: I am not going to town; or,

  It is not the case that I am going to town

- Example2: *p :* “23 = 15 +7”

  p happens to be false, so $\neg$ *p* is true.

Conjunction

- Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q."
- The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.
- Conjunction corresponds to English "**AND**".

- Example: Liana is curious AND clever.

Truth Table for Conjunction

TABLE 2 **The Truth Table for the Conjunction of Two Propositions.**

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction: Example

- **Example:** p : 'I am going to town'

  q : 'It is going to rain'

  $p \land q$: 'I am going to town and it is going to rain.'

- **Note:** **Both p and q** must be true to $p \land q$ be **true**

Disjunction

- Let p and q be propositions.
- The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q."
- The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.
- Disjunction is true when at least one of the components is true.
- Disjunction corresponds to English "**OR**".
- Example: Abdullah is brave OR intelligent.

Truth Table for Disjunction

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Examples of Conjunction & Disjunction

Let,

$p : 5 < 9$

$q : 9 < 7.$

Construct the propositions $p \wedge q$ and $p \vee q$.

Solution:

- The conjunction of the propositions p and q is the proposition

  $p \wedge q$: 5 < 9 and 9 < 7

- The disjunction of the propositions p and q is the proposition

  $p \vee q$: 5 < 9 or 9 < 7

Question: What are the truth values of $\boldsymbol{p \wedge q}$ *and* $\boldsymbol{p \vee q}$?

Exclusive Or

- Let *p* and *q* be propositions.
- The *exclusive or* of *p* and *q*, denoted by $p \oplus q$, is the proposition that is **true** when exactly one of *p* and *q* is true and is **false** otherwise.
- "Exclusive Or" - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get both soup and salad. This is the meaning of Exclusive Or (xor).

Truth Table of Exclusive Or

TABLE 4 The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Books

- *Discrete Mathematics and its applications with combinatorics and graph theory (7th edition)* by Kenneth H. Rosen [Indian Adaptation by KAMALA KRITHIVASAN], published by McGraw-Hill

References

1. Discrete Mathematics, *Richard Johnsonbaugh*, Pearson education, Inc.
2. Discrete Mathematical Structures, *Bernard Kolman*, *Robert C. Busby*, *Sharon Ross,* Prentice-Hall, Inc.
3. *SCHAUM'S outlines Discrete Mathematics(2nd edition)*, by *Seymour Lipschutz*, *Marc Lipson*