Quiz 1 Solution.

Q1

$f(n) = n^{1/2} \log n^{1/2}$

$g(n) = 30000\, n^{1/2} \log n^{1/4}$

$\boxed{f(n) = \Theta(g(n))}$

$\Rightarrow f(n) = \frac{1}{2}\sqrt{n} \log n \qquad \Rightarrow \qquad g(n) = \frac{30000}{4}\sqrt{n} \log n$

$f(n)$ and $g(n)$ are both of same growth rate ($\sqrt{n} \log n$). So, so particular c_1, c_2 and n_0, $g(n)$ can be upper as well as lower bound of $f(n)$.

$f(n) = \Omega(g(n))$

$f(n) = O(g(n))$

So, $f(n) = \Theta(g(n))$

Q2

$f(n) = \frac{\sqrt{n}}{18} - 19 \log n + 20$

$k_1 g(n) \le f(n) \le k_2 g(n)$

$\boxed{g(n) = \sqrt{n}}$

$k_1 \sqrt{n} \le \frac{\sqrt{n}}{18} - 19 \log n + 20 \le k_2 \sqrt{n}$

for Upper bound:

let $k_2 = 50$, then

$f(n) \le 50\sqrt{n}$ holds for all values of n, so

$n_0 = 1$.

for lower bound:

let $k_1 = \frac{1}{1000}$ $\qquad \frac{\sqrt{n}}{1000} \le \frac{\sqrt{n}}{18} - 19 \log n + 20$

Now, we can't simply find n_0, as for some range of n, the factor $19 \log n$ would be large and make the R.H.S -ve. And since, we cant solve this eq by calculator to find roots, we'll find n_0 by hit and trial.

for $n_0 = 5000000$ $\quad k_1 g(n) \le f(n)$

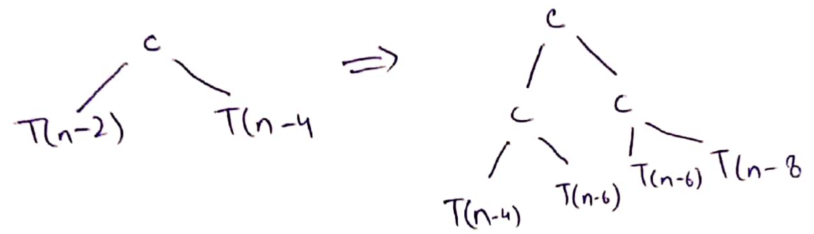
Ans $k_1 = \frac{1}{1000}$, $k_2 = 50$, $n_0 = 5000000$

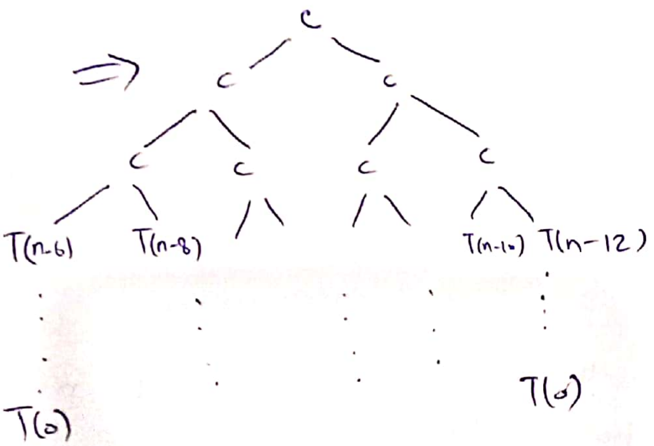
pro tip
In log changing value of n from 1000 to 1001 will show no difference.
Change it by a factor of 10 i.e 1000 to 10000

Q3

$T(n) = T(n-2) + T(n-4) + c$

By Tree Method





$T(n-2)$ will give the longest branch. Size of subproblems at different levels:

$n, n-2, n-4, n-6, \dots 0$

$\boxed{\text{Total levels} = \frac{n}{2} + 1}$

Work/cost at different levels

c
$2c$
$4c$
$8c$
$\vdots$

At some point when the smallest branch ends, cost will not follow this series form, but for simplicity we assume that it does.

$= (c + 2c + 4c + 8c + \dots + 2^k c)$

$= c(1 + 2^1 + 2^2 + \dots + 2^k)$

Total terms of this geometric series will be $\frac{n}{2} + 1$.

$= \frac{c(2^{\frac{n}{2}+1} - 1)}{2-1} = c\,2 \cdot 2^{n/2} - c = c_1 2^{n/2} - c$

$\boxed{T(n) = O(2^{n/2})}$

OR $2^{n/2} = (2^n)^{1/2} = (2^{1/2})^n = (\sqrt{2})^n = (1.414)^n$

$\boxed{T(n) = O(1.4^n)}$