National University of Computer and Emerging Sciences, Lahore Campus

Course Name:	Design and Analysis of Algorithms	Course Code:	CS2009
Degree Program:	BSCS	Semester:	SPRING 2023
Exam Date:	Monday, February 27, 2023	Total Marks:	24 + 12 = 36
Section:	ALL	Page(s):	2
Exam Type:	Mid-Term - I		

Student : Name:_____ **Roll No.**_____ **Section:**_____

Instruction/Notes: Attempt all questions. There are two question, don't forget to check the back side as well.

Question 1: [CLO - 2]   [4+5+3+3+3+(4+2) = 24 Marks]

For each part in this question, you are required to analyze the given functions T(n) and answer the given question.

a) For the following function f(n), find a function g(n), such that f(n) = θ (g(n))

[Hint: For part a you have to provide the constants n_o, c_1, c_2]

$$T(n) = (1000)2^n + 4^n$$

b) Solve the recurrence relation $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{4n}{5}\right) + n$, where $T(1) = 1$

[For part b you are required to use **recursion tree** method]

c) Solve the recurrence relation $T(n) = 25T\left(\frac{n}{5}\right) + n^2$, where $T(1) = 1$

[For part c you can use any method]

d) Prove or disprove the following statement

$$2^{n+12} \text{ is } O(2^n)$$

e) Prove or disprove the following statement

$$4^{12n} \text{ is } O(2^n)$$

f) Consider the following sorting algorithm:

MySort(A, 1, 10)

MergeSort(A, 1, 7)

MergeSort(A, x, y)

MergeSort(A, 1, 7)

i. What should be the minimum value of $y - x + 1$, so that MySort(A, 1, 10) correctly sorts the 100 elements array given to it as input.

ii. Based on your answer (for the minimum value of $y - x + 1$), what are exact values of x and y.

Question 2: [CLO - 1]   [12 Marks]

Let A[1..n] be an array of *n* distinct numbers. If $i < j$ and $A[i] < A[j]$ then the pair (i, j) is called a compatibility of A. Design an algorithm that determines the number of compatibilities in any permutation on *n* elements in $O(n \log n)$ worst case time. (Hint: Modify merge sort.)