Theory of Automata

Homework 1

Problem#1: Consider $\Sigma = \{a, b\}$

a. $L_1 = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3$ What is the cardinality of L_1.

b. $L_2 = \{w\ over\ \Sigma \mid |w| > 5\ and\ |w| \leq 10\ and\ |w|\ is\ odd\ \}$What is the cardinality of L_2.

c. $L_3 = \{w\ over\ \Sigma \mid |w| > 5\ and\ |w| \leq 10\ and\ |w|\ is\ even\ \}$What is the cardinality of L_3.

d. $L_4 = \{w\ over\ \Sigma \mid |w| > 5\ and\ |w| \leq 10\ \}$What is the cardinality of L_4.

e. $L_2 \cap L_3$ and $|L_2 \cap L_3| = ?$

f. $L_2 \cap L_4$ and $|L_2 \cap L_4| = ?$

g. $L_3 \cap L_4$ and $|L_3 \cap L_4| = ?$

h. $L_2 - L_4\ and\ |L_2 - L_4|$ = ?

i. $L_3 - L_4\ and\ |L_3 - L_4|$ = ?

Problem#2: Consider $\Sigma = \{0, 1, 2\}$

a. $L_1 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ are\ \geq 5\ and\ \leq 10\}$

b. $L_2 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 2\ \}$

c. $L_3 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 3\ \}$

d. $L_4 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 2\ or\ divisible\ by\ 3\ \}$

e. $L_5 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 2\ and\ divisible\ by\ 3\ \}$

f. $L_6 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 2\ but\ not\ divisible\ by\ 3\ \}$

g. $L_7 = \{s\ over\ \Sigma \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 3\ but\ not\ divisible\ by\ 2\ \}$

h. $L_8 = \{s\ over\ \Sigma - \{2\} \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 3\}$

i. $L_9 = \{s\ over\ \Sigma - \{2\} \mid |s| \leq 4\ and\ sum\ of\ digits\ in\ s\ is\ divisible\ by\ 2\}$