IE 203 PS 1

2026-02-11

Q1. Assignment Problem

a) Given a set of n workers and n tasks, where c_{ij} represents the cost of assigning worker i to task j, formulate a Binary Integer Program (BIP) to minimize total cost such that each worker is assigned to exactly one task and each task is assigned to exactly one worker.

b) Consider 4 specialists and 4 surgical procedures with the following estimated completion times (minutes):

	Proc 1	Proc 2	Proc 3	Proc 4
Dr. A	45	60	50	55
Dr. B	50	55	45	60
Dr. C	40	70	50	45
Dr. D	55	50	60	40

Instantiate the model from part (a) using this data.

c) Dr. A cannot perform Procedure 2 due to a hand injury. Modify the existing constraints to prevent this assignment.

d) To ensure variety in the workplace, Dr. B cannot be assigned to either Procedure 1 or Procedure 3. Modify the existing constraints to enforce this restriction.

e) If Dr. C is assigned to Procedure 4, then Dr. D must be assigned to Procedure 2. Formulate this conditional dependency.

Q2. Generalized Assignment Problem

a) Given m agents with resource capacities b_i and n tasks that each require a_{ij} resources when assigned to agent i, formulate a BIP to minimize total cost c_{ij}. Each task must be assigned to exactly one agent.

b) Two cloud servers (S_1, S_2) must handle 5 data processing jobs. Server capacities are $S_1 = 100$ GB and $S_2 = 120$ GB. Job sizes are: $J_1 : 30$ GB, $J_2 : 50$ GB, $J_3 : 40$ GB, $J_4 : 20$ GB, $J_5 : 60$ GB.

Processing costs (dollars per job) are:

	Job 1	Job 2	Job 3	Job 4	Job 5
Server 1	8	12	10	6	15
Server 2	10	9	11	8	13

Write the complete model using this data. Each job must be processed by exactly one server.

c) For system redundancy, Job 1 and Job 2 cannot be hosted on the same server. Add the appropriate constraint(s).

d) To prevent CPU overheating, no single server can be assigned more than 3 jobs, regardless of their storage size. Formulate this constraint.

e) If Server 1 processes Job 3, it requires a specialized operating system that consumes an additional 10 GB of its total capacity. Modify the capacity constraint to reflect this conditional overhead.

f) To justify the cost of the more expensive hardware, Server 2 must be utilized to at least 70% of its total capacity. Formulate this minimum utilization requirement.

Q3. Traveling Salesperson Problem

a) Formulate an Integer Program to minimize the total distance traveled by a salesperson who must visit all cities exactly once and return to the starting city.

b) A technician must visit 5 locations (1 to 5). Location 1 is the depot. The symmetric distance matrix (in miles) is:

$$D = \begin{bmatrix} \infty & 10 & 15 & 20 & 25 \\ 10 & \infty & 12 & 18 & 22 \\ 15 & 12 & \infty & 8 & 16 \\ 20 & 18 & 8 & \infty & 15 \\ 25 & 22 & 16 & 15 & \infty \end{bmatrix}$$

Write the complete model.

c) Due to tool delivery requirements, the technician must visit Location 2 immediately before Location 3. Add the necessary constraint.

d) Location 5 is high priority and must be either the first or the second stop made after leaving the depot (Location 1). Formulate this requirement using position variables.

e) For safety regulations, direct travel between locations 1 and 2, and between locations 3 and 4, cannot both be included in the same route. Add the mutual exclusion constraint.

Q4. Vehicle Routing Problem

a) Consider m vehicles with capacity Q serving n customers with demands q_j. The distance between location i and location j is d_{ij}. All vehicles start and end at a central depot (node 0). Formulate a model to minimize total distance traveled such that every customer is visited exactly once by exactly one vehicle and no vehicle exceeds its capacity.

b) Consider the following scenario: 2 trucks with 40-unit capacity each must serve 5 customers. Customer demands are: $q_1 = 15$ units, $q_2 = 25$ units, $q_3 = 10$ units, $q_4 = 20$ units, $q_5 = 5$ units.

The distance matrix (in miles) from depot (0) and customers (1-5) is:

$$D = \begin{bmatrix} \infty & 10 & 15 & 12 & 18 & 20 \\ 10 & \infty & 8 & 14 & 16 & 22 \\ 15 & 8 & \infty & 10 & 12 & 18 \\ 12 & 14 & 10 & \infty & 9 & 15 \\ 18 & 16 & 12 & 9 & \infty & 11 \\ 20 & 22 & 18 & 15 & 11 & \infty \end{bmatrix}$$

Write the complete model.

c) Customer 2 requires a refrigerated truck. If only Truck 1 has refrigeration, formulate the assignment constraint.

d) Locations 3 and 4 are in a restricted "Low Emission Zone." Only one of the two trucks (either truck, but not both) is allowed to enter this zone to service these locations. Formulate this restriction.

e) Due to driver union rules, Truck 2 cannot exceed a total travel distance of 200 miles. Formulate this distance limit.

f) If Truck 1 is chosen to service the most distant customer (Customer 5), then Truck 2 must be the one to service the heaviest customer (Customer 2). Formulate this cross-vehicle conditional dependency.

Q5. Set Covering Problem

a) Formulate a Binary Integer Program for the Set Covering Problem. Given a set of elements (e.g., neighborhoods) that must be covered and a collection of subsets (e.g., fire station coverage areas), each with an associated cost, the goal is to select a minimum-cost collection of subsets such that every element is covered by at least one selected subset.

b) A city needs to locate fire stations to cover 5 neighborhoods. There are 4 potential station locations. Station j costs f_j thousand dollars to open. The coverage matrix is shown below ($a_{ij} = 1$ if station j covers neighborhood i):

Neighborhood	Station 1	Station 2	Station 3	Station 4
1	1	1	0	0
2	1	0	1	0
3	0	1	1	0
4	0	0	1	1
5	0	0	0	1

Station costs: $f_1 = 30$, $f_2 = 25$, $f_3 = 35$, $f_4 = 20$ (thousand dollars)

Write the complete model.

c) Add a constraint: If station 2 is opened, then station 4 must also be opened.

d) Modify the model from part (b): Neighborhood 3 must be covered by at least 2 stations for redundancy.

Q6. Facility Location Problem

a) Formulate a capacitated Facility Location Problem. Given a set of potential facility locations with fixed opening costs and capacities, and a set of customers with known demands, the goal is to decide which facilities to open and how to assign customer demand to facilities to minimize total cost (fixed costs plus transportation costs). Customer demand can be split among multiple facilities if needed.

b) A company considers 4 warehouse locations to serve 5 customers. Write the complete model using the following data:

Data:

- Fixed costs: $f_1 = 120$, $f_2 = 100$, $f_3 = 140$, $f_4 = 110$ (thousand dollars)
- Capacities: $K_1 = 200$, $K_2 = 180$, $K_3 = 220$, $K_4 = 190$ (units)
- Customer demands: $d_1 = 60$, $d_2 = 50$, $d_3 = 70$, $d_4 = 55$, $d_5 = 65$ (units)

Transportation cost matrix (dollars/unit):

$$C = \begin{bmatrix} 6 & 8 & 5 & 7 & 9 \\ 7 & 5 & 9 & 6 & 8 \\ 8 & 7 & 6 & 8 & 5 \\ 5 & 9 & 7 & 5 & 6 \end{bmatrix}$$

c) Add the following constraints to the model from part (b):

- At least one of warehouses 1 and 3 must be opened
- Exactly 2 warehouses must be opened
- Customer 2 must receive all its demand from a single warehouse