Q#01 Compute the limit of:

$$f(x) = \begin{cases} x^3+4, & x<1 \\ 7, & x=1 \\ x+6, & x>1 \end{cases}$$

$$\lim_{x \to 1^+} f(x) = x+6$$
$$= 1+6$$
$$= 7$$

$$\lim_{x \to 1^-} f(x) = x^3+4$$
$$= 1+4$$
$$= 5$$

$$\lim_{x \to 1} f(x) = \text{DNE}$$

$$f(7) = 1$$

Q#02

compute the limit of

$$f(x) = \begin{cases} -x^2 & , x < 2 \\ -x-1 & , x \geq 2 \end{cases}$$

$$\lim_{x \to -2} f(x) = -x^2$$
$$= -(-2)^2$$
$$= -4$$

$$\lim_{x \to 2} f(x) = -x-1$$
$$= -2-1$$
$$= -3$$

$$\lim_{x \to 4} f(x) = -x-1$$
$$= -4-1$$
$$= -5$$

$$\cancel{\lim}\ f(-2) = -x^2$$
$$= -4$$

$$f(2) = -x-1$$
$$= -2-1$$
$$= -3$$

$$f(4) = -x-1$$
$$= -5$$

Q#03 Evaluate:

1. $\lim_{x \to 0} \frac{3x+4}{x^2}$

$= +\infty$

2. $\lim_{x \to 3^+} \frac{x^2-9}{\sqrt{x-3}}$

By using L'Hopitals' Rule

$= \frac{2x}{\frac{1}{2\sqrt{x-3}}}$

$= 4x\sqrt{x-3}$

Applying limit.

$= 4 \times 3\sqrt{3-3}$

$= 0.$

3. $\lim_{x \to -2} \frac{x^2 - 4}{2x^2 - x - 6}$

$= \lim_{x \to -2} \left(\frac{2x}{2x-1} \right)$

$= \frac{-4}{-5}$

$= 4/5$

4. $$\lim_{t \to 1} \frac{t^3 + t^2 - 5t + 3}{t^3 - 3t + 2}$$

$$= \lim_{\substack{\cancel{x \to t} \\ t \to 1}} \left(\frac{3t^2 + 2t - 5}{3t^2 - 3} \right)$$

$$= \frac{3(1) + 2(1) - 5}{3(1) - 3} = \frac{0}{0}$$

Again Applying L'Hopital's Rule:

$$= \lim_{t \to 1} \left(\frac{6t + 2}{6t} \right)$$

$$= 8/6 = 4/3$$

5. $$\lim_{x \to 0} \frac{\sqrt{x + 64} - 8}{x}$$

$$= \lim_{x \to 0} \frac{(x + 64)^{1/2} - 8}{x}$$

$$= \lim_{x \to 0} \frac{1}{2\sqrt{x + 64}}$$

$$= \cancel{\lim_{x \to 0}} = \frac{1}{2(8)}$$

$$= 1/16.$$

6. $\lim_{y \to 0} \dfrac{5y^3 + 8y^2}{3y^4 - 16y^2}$

$= \lim_{y \to 0} \dfrac{\cancel{y^2}(5y+8)}{\cancel{y^2}(3y-16)}$

$= \lim_{y \to 0} \dfrac{5y+8}{3y-16}$

$= \dfrac{8}{-16} = -1/2.$

Q#05 Find the derivative of the following:

1. $g(x) = \dfrac{\tan 3x}{(x+7)^4}$

$= (x+7)^4 \frac{d}{dx} \tan 3x - \tan 3x \frac{d}{dx} (x+7)^4 \Big/ (x+7)^{4\times 2}$

$= (x+7)^4 \sec^2 3x \cdot 3 - \tan 3x \, 4(x+7)^3 \cdot 1 \Big/ (x+7)^{4\times 2}$

$= \cancel{(x+7)^3} \left[(3x+21)(\sec^2 3x) - \tan 3x \cdot 4\right] \Big/ (x+7)^{\cancel{8}}$

$= \dfrac{(3x+21)(\sec^2 3x) - 4\tan 3x}{(x+7)^5}$

2. $\frac{dx}{d\theta} = \tan\sqrt{\theta} \sec\left(\frac{1}{\theta}\right)$

$= \tan\sqrt{\theta} \frac{d}{d\theta} \sec\frac{1}{\theta} + \sec\frac{1}{\theta} \frac{d}{d\theta} \tan\sqrt{\theta}.$

$= \dfrac{\tan\sqrt{\theta} \times \sec\frac{1}{\theta} \tan\frac{1}{\theta}}{-\theta^2} + \dfrac{\sec\frac{1}{\theta} \sec^2\sqrt{\theta}}{2\sqrt{\theta}.}$

$= \sec\frac{1}{\theta} \left[\dfrac{\tan\sqrt{\theta} \times \tan 1/\theta}{-\theta^2} + \dfrac{\sec^2\sqrt{\theta}}{2\sqrt{\theta}.} \right]$

$\cancel{4}$. 3. $y = \left[\dfrac{t^2}{t^{3-4t}}\right]^3$

$y = (t^{-1+4t})^3$

$y = t^{12-3}$

$\ln y = \ln t^{12-3}$

$\frac{1}{y}\frac{dy}{dt} = 12t-3 \cdot \ln t$

$\frac{1}{y}\frac{dy}{dt} = \ln t(12) + \frac{12t-3}{t}$

$dy/dt = y\left[\ln t(12) + \frac{12t-3}{t}\right]$

$$\frac{dy}{dt} = t^{12t-3}\left[\ln t(12) + \frac{12t-3}{t}\right].$$

4. $q = \tan\left[\frac{\cos t}{t}\right]$

$= \sec^2\left[\frac{\cos t}{t}\right] \cdot \frac{d}{dt}\left(\frac{\cos t}{t}\right)$

$= \sec^2\left[\frac{\cos t}{t}\right] \cdot \frac{-t\sin t - \cos t}{t^2}$

5. $y = (t^{-3/4}\sin t)^{4/3}$

$= 4/3\,(t^{3/4}\sin t)^{1/3}\ \frac{d}{dx}\ t^{-3/4}\sin t$

$= 4/3\,(t^{-3/4}\sin t)^{1/3} \cdot t^{-3/4}\cos t + \sin t \cdot \frac{3}{4} t^{-7/4}.$

$= 4/3\,(t^{-3/4}\sin t)^{1/3} \cdot \left(\frac{\cos t}{t^{3/4}} - \frac{3\sin t}{4t^{7/4}}\right):$

Q#04:- use the function $y = f(x)$ defined by the graph to find each limit.

$\lim_{x \to 4^-} f(x) = DNE$

$\lim_{x \to -2^-} f(x) = 2$

$\lim_{x \to 1} f(x) = DNE$

$\lim_{x \to 3} f(x) = DNE.$