OGUN DIGICLASS

CLASS: SECONDARY SCHOOL

SUBJECT: MATHEMATICS

TOPIC: COORDINATE GOMETRY(PART 2)

LARNING OBJECTIVE

DERIVE EQUATION OF A LINE

DETERMINE THE PARALLELISM AND PERDICULARIM OF TWO STRAIT LINE

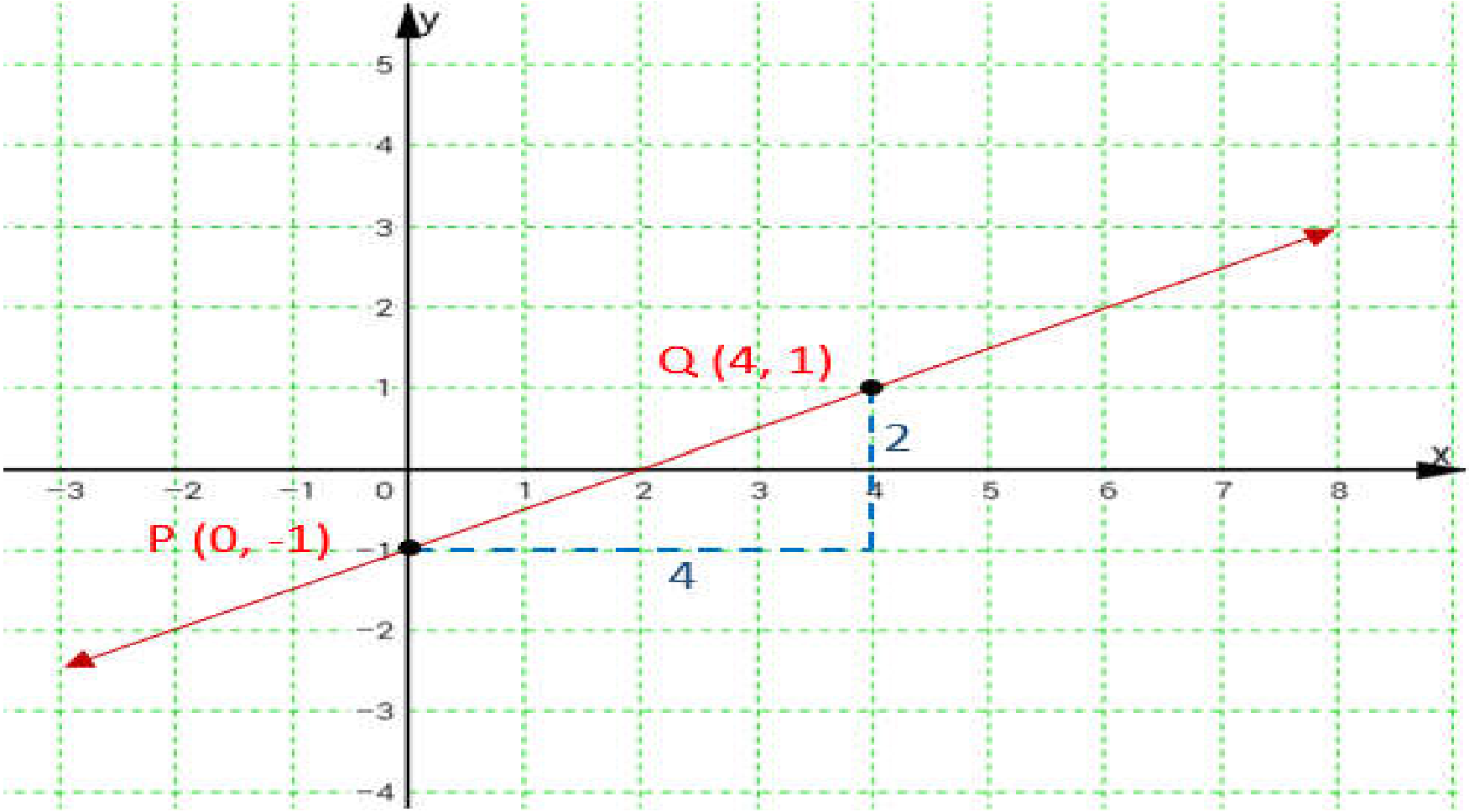
How to find the slope of a line?

On the coordinate plane, the slant of a line is called the Slope, Gradient or Tangent. Slope is the ratio of the change in the y-value over the change in the x-value, also called rise over run.
Given any two points on a line, you can calculate the slope of the line by using this formula:

$$\text{SLOPE(GRADIENT)} = \frac{CHANGER\ IN\ Y}{CHANGE\ IN\ X}$$

For example: Given two points, P = (0, -1) and Q = (4,1), on the line we can calculate the slope of the line.

Calculate Slope of Line



$$\text{slope} = \frac{\text{change in y value}}{\text{change in x value}} = \frac{1-(-1)}{4-0} = \frac{2}{4} = \frac{1}{2}$$

COORDINATE GEOMETRY

Straight Lines

The equations of straight lines come in two forms:

1. $y = mx + c$, where m is the gradient and c is the y-intercept.

2. $ax + by + c = 0$, where a, c and c are integers.

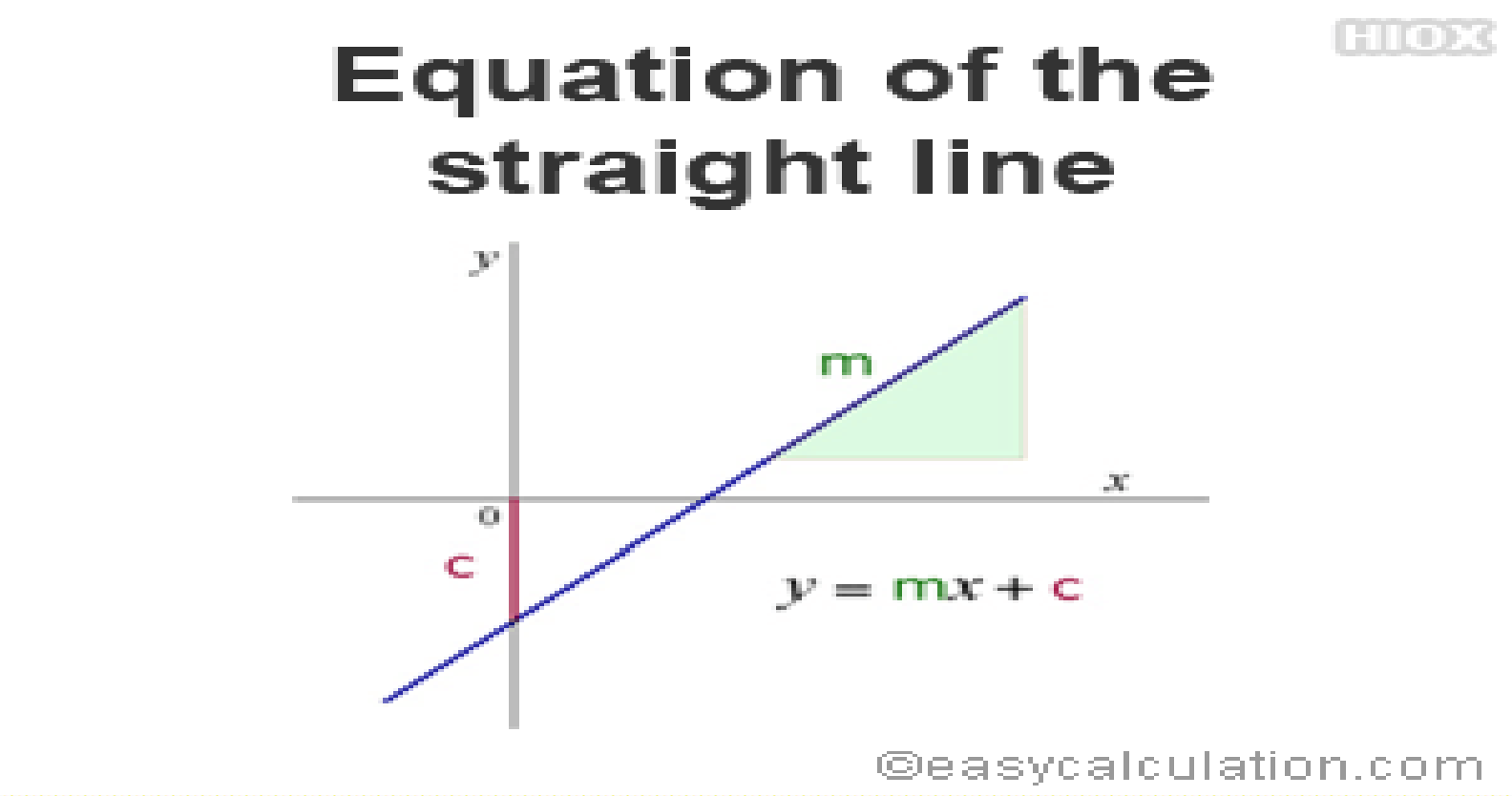
GENERAL FORMS OF EQUATION OF A STRAIGHT LINE

1. GIVEN GRADIENT(m) AND Y-INTERCEPT(c)

2. GIVEN GRADIENT(m) AND A POINT(X_1,Y_1) ON THE LINE

3. GIVERN TWO POINTS (X_1,Y_1) AND (X_2,Y_2) ON THE LINE

1. GIVEN GRADIERNT(m) AND Y-INTERCERPT(c)



Y = mx + c

Where m is the gradient/slope

C is the y-intercept i.e where the line cut/pass through y-axis

EXAMPLE

Find the equation of a line which has

i) gradient of 4 and passes through -3 on y-axis

gradient of $\frac{-2}{3}$ and passes through 5 on y-axis

2. GIVEN GRADIENT(m) AND A POINT(X_1, Y_1) ON THE LINE

Equation of Straight Line

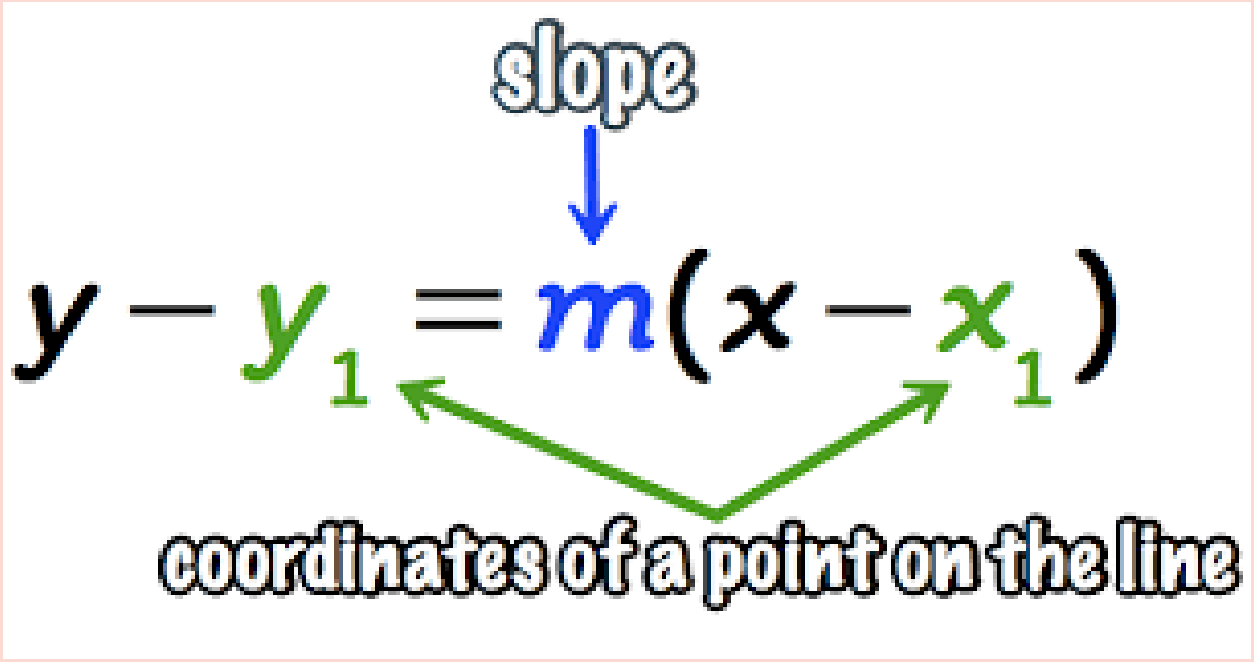
Equation of a straight line (gradient-intercept form):

$y = mx + c$ where *m is the gradient* and *c is the y-intercept*.

Equation of a straight line (given gradient and 1 point):

$$y - y_1 = m(x - x_1)$$

$$M = \frac{(Y - Y_1)}{(X - X_1)}$$



Example

Determine the equation of a line that passes through point (2,-15) with -2 as the gradient.

Example

Determine the equation of a line that passes through point (2,--15) with –2 as gradiernt

$$y - y_1 = m(x - x_1)$$

$$y - (-15) = -2(x - 2)$$

$$y + 15 = -2(x - 2)$$

$$y + 15 = -2(x - 2)$$

$$y + 5 = -2x + 4$$

$$y + 5 - 15 = -2x + 4 - 15$$

$$y = -2x - 11$$

3. GIVERN TWO POINTS (X_1, Y_1) AND (X_2, Y_2) ON THE LINE

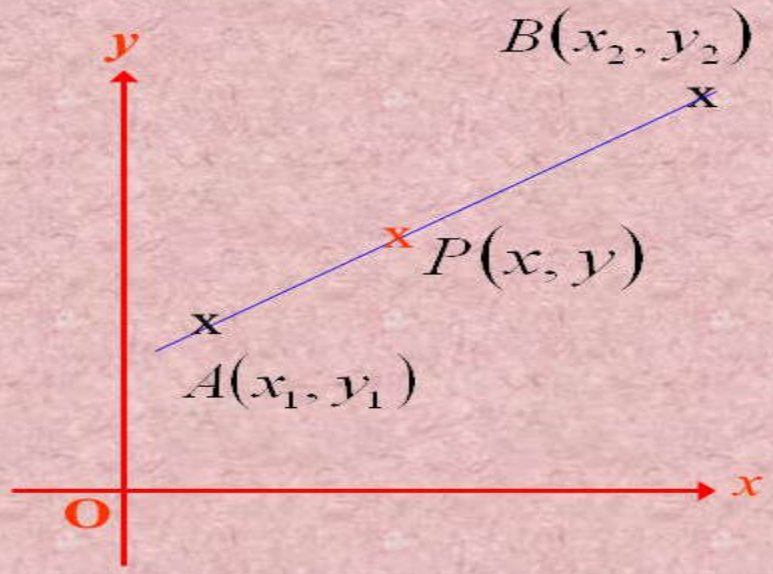
Two-Point Form

Given a straight line which passes through the points A and B, then

$$\text{Slope of } AB = \frac{y_2 - y_1}{x_2 - x_1}$$

If P(x, y) is any point on the line AB, then

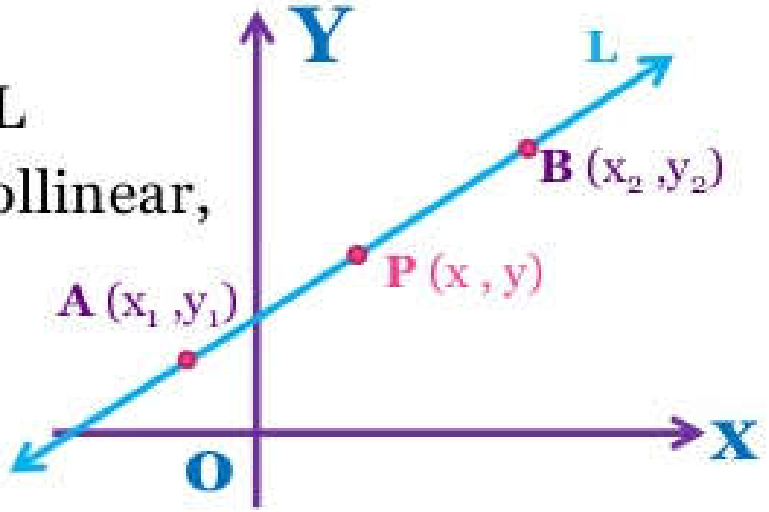
$$\text{Slope of } PA = \frac{y - y_1}{x - x_1}$$



THEREFORE: $\dfrac{(Y - Y_1)}{(X - X_1)} = \dfrac{(Y_2 - Y_1)}{(X_2 - X_1)}$

Two-point form

- Let the line L passes through two given points A (x_1, y_1) and B (x_2, y_2).
- Let P (x, y) be a general point on L
- The three points A, B and P are collinear,



therefore, we have

slope of AP = slope of BP

i.e.,

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Thus, equation of the line passing through the points A (x_1, y_1) and B (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

EXAMPLE

THE EQUATION OF A LINE THROUGH THE POINTS (4,2) AND (-8,- 2) IS 3y = Px + q. FIND THE VALUE OF p AND q

SOLUTION

$$\frac{(Y - Y_1)}{(X - X_1)} = \frac{(Y_2 - Y_1)}{(X_2 - X_1)}$$

HOW TO DETERMINE THE GRADIERNT FROM A LINERAR EQUATION

$$4y - 3x + 3 = 0$$

$\Rightarrow$ $4y - 3x = -3$ added -3 to both sides

$\Rightarrow$ $4y = 3x - 3$ added $3x$ to both sides

$\Rightarrow$ $y = \frac{3x - 3}{4}$ divided both sides by 4

$\Rightarrow$ $y = \frac{3}{4}x - \frac{3}{4}$

THE GRADIENT IS THE COEFFIENT OF X WHEN Y IS MADE THE SUBJECT OF THE FORMULA

GRADIENT $= \frac{3}{4}$

DETERRMINER THE SLOPE OF THER FOLLOWING EQUATIONS

5) $y = 5 + x$

6) $-2x + y = 5$

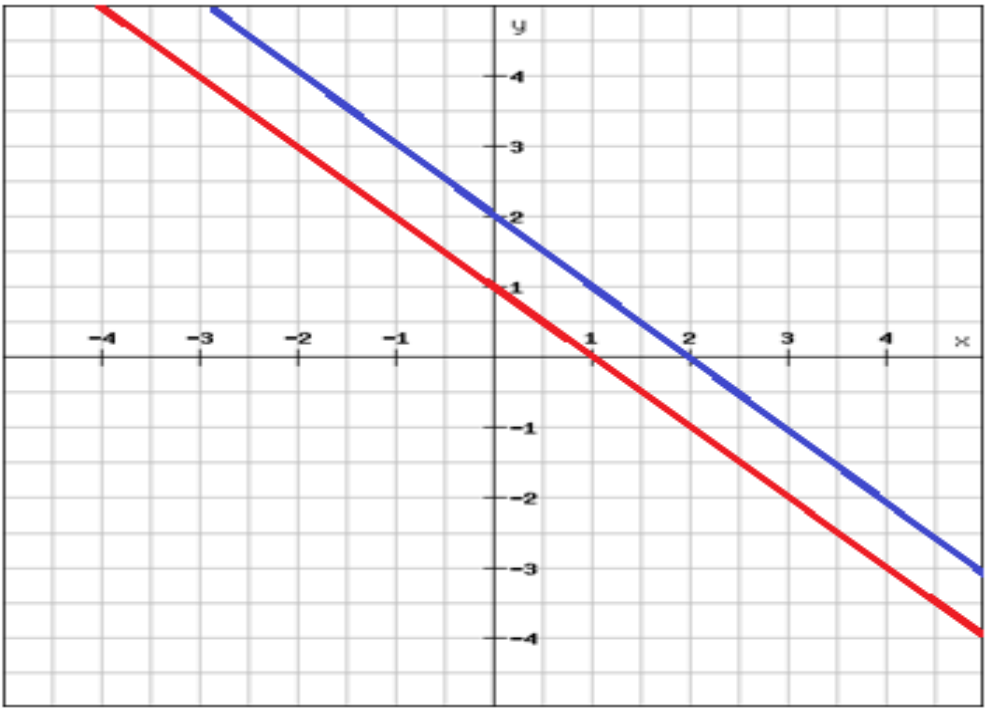
7) $7x - y = 4$

8) $4x - 2y = 12$

9) $4x + 3y = 11$

10) $5x + 2y = -7$

Parallel and perpendicular lines



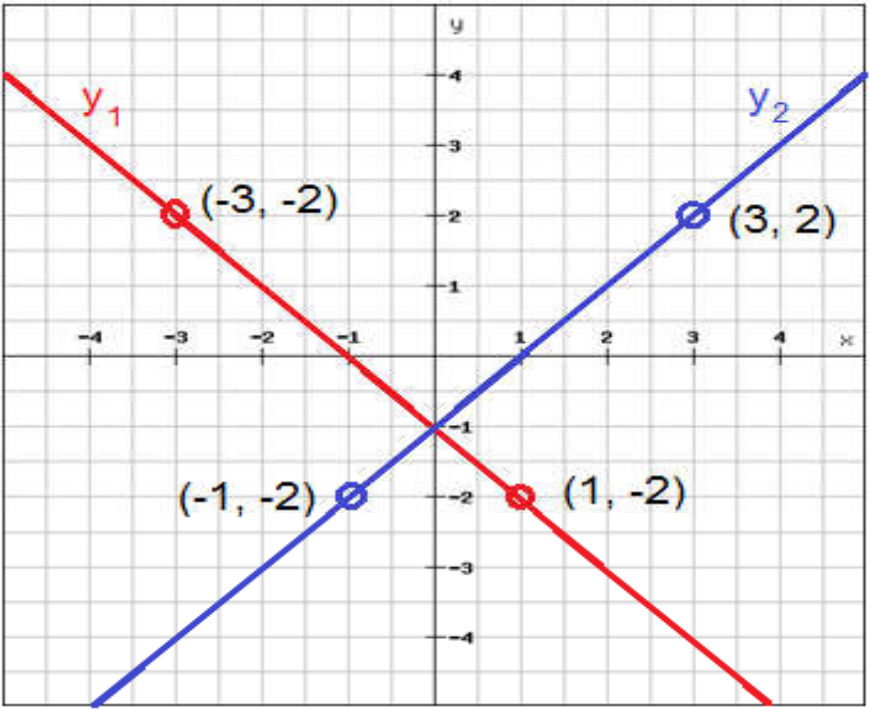
If two non-vertical lines that are in the same plane has the same slope, then they are said to be parallel. Two parallel lines won't ever intersect

Y = m_1x + b.

Y = m_2x + c

i.e, $m_1 = m_2$

CONDITION FOR PERPENDICULAR



If two non-vertical lines in the same plane intersect at a right angle then they are said to be perpendicular. Horizontal and vertical lines are perpendicular to each other i.e. the axes of the coordinate plane.

Y = m_1x + b.

Y = $\frac{-1}{m_2}$ x + c

i.e, $m_1 = \frac{-1}{m_2}$

EXAMPLE

Find the equation of the line that is perpendicular to Y = 3x - 1 and goes through point (2, 5).

SOLUTION

If Y = 3x - 1 the gradient of the graph is 3. This means the gradient of the perpendicular graph is $\frac{-1}{3}$ (by finding the negative reciprocal).

Forms for the Equation of a Line		
Slope-Intercept	$y = mx + b$	m is the slope b is the y-intercept
Point-Slope	$y - y_1 = m(x - x_1)$	m is the slope (x_1, y_1) is a point on the line
Standard Form	$ax + by = c$	a is positive
Intercept Form	$\frac{x}{a} + \frac{y}{b} = 1$	a is the x-intercept b is the y-intercept
Vertical	$x = a$	Vertical line with a as the x-intercept
Horizontal	$y = b$	Horizontal line with b as the y-intercept

ASSIGNMENT

1).FIND THER EQUATION OF THE LINE PASSING THROUGH THE POINTS (2, 5) AND (-4, -7). (WASSCE 2018)

2).FIND THE EQUATION OF A STRAIGHT LINE WHICH PASSES THROUGH THE POINT (2,-3) AND IS PARALLEL TO THE LINE $2x + Y = 6$. (WASSCE 2016)

Forms of Equation of a Line

FORM	NAME	EXAMPLE
$ax + by + c = 0$	General form	$2x + 4y + 8 = 0$
$ax + by = c$	Standard form	$2x + 4y = -8$
$y - y_1 = m(x - x_1)$	Point-slope form	$y - 4 = -\frac{1}{2}(x-12)$
$y = mx + b$	Slope-intercept	$y = -\frac{1}{2}x - 2$
$\frac{x}{b} + \frac{y}{a} = 1$	Intercept form	$\frac{x}{4} + \frac{y}{2} = -1$
$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} = m$	Two point form	$\frac{3-y_1}{10-x_1} = \frac{4-y_1}{12-x_1} = -\frac{1}{2}$