OGUN DIGICLASS

CLASS: SECONDARY SCHOOL

SUBJECT: MATHEMATICS

TOPIC: MATRICES

SUBTOPIC: Determinant of Matrix

LEARNING OBJECTIVES

Identify symbols used in representing determinant

Find the determinant of a square matrix like 2X2 and 3x3

Solve related problems on determinant matrices

Determinant of Matrix

The determinant of Matrix **A** is written as |**A**|

→ It is calculated as follows:

If Matrix A is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then the determinant is:

$$|A| = ad - bc$$

And as said earlier, this is used to find the inverse Matrix…

Find the determinant of the following Matrix:

$$\begin{bmatrix} 4 & 3 \\ 9 & 6 \end{bmatrix}$$

$$|A| = ad - bc$$

Sub in values

$$|A| = (4 \times 6) - (3 \times 9)$$

Calculate

$$|A| = -3$$

The determinant of matrices for example matrix **A**, is denoted by det **A** or |**A**| **or** $\triangle$

If $$A = \begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix}$$

then $$|A| = \begin{vmatrix} 1 & 2 \\ 6 & 5 \end{vmatrix}$$

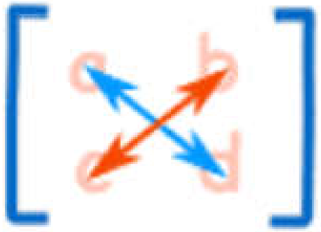
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The determinant is:

$$|A| = ad - bc$$

"The determinant of A equals a times d minus b times c"

It is easy to remember when you think of a cross:



- Blue is positive (+ad),
- Red is negative (−bc)

Determinant of a 2x2 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is given by $\det A = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$

$$= a_{11}\, a_{22} - a_{12}\, a_{21}$$

Example If $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$ find $|A|$

Solution:

$$|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = 9 - (-2) = 9 + 2 = 11$$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 2 Columns)

The determinant of that matrix is (calculations are explained later):

$$3\times6 - 8\times4 = 18 - 32 = \mathbf{-14}$$

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

$$\begin{aligned} |B| &= 4\times8 - 6\times3 \\ &= 32-18 \\ &= 14 \end{aligned}$$

For a 3×3 Matrix

For a 3×3 matrix (3 rows and 3 columns):

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

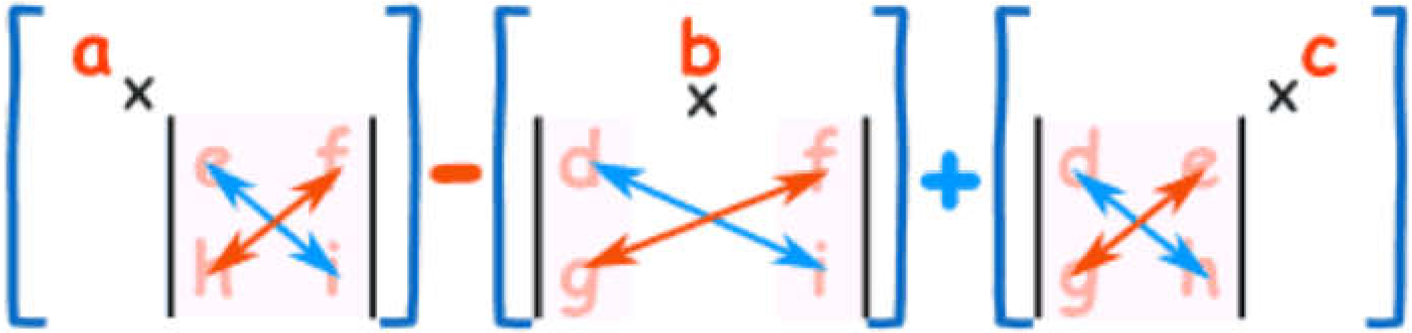
The determinant is:

As a formula *(remember the vertical bars || mean "determinant of")*:

$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

It may look complicated, but **there is a pattern**:



To work out the determinant of a **3×3** matrix:

- Multiply **a** by the **determinant of the 2×2 matrix** that is **not in a**'s row or column.
- Likewise for **b**, and for **c**
- Sum them up, but remember the minus in front of the **b**

Example:

$$C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}$$

$$\begin{aligned} |C| &= 6\times(-2\times7 - 5\times8) - 1\times(4\times7 - 5\times2) + 1\times(4\times8 - (-2\times2)) \\ &= 6\times(-54) - 1\times(18) + 1\times(36) \\ &= \mathbf{-306} \end{aligned}$$

Lets Solve Questions From Past Certificate Examination Questions

1. Solve this simultaneous equation using determinant method

$9x + 4y = 17$

$2x + y = 4$

2. Given that $X = \begin{pmatrix} 2 & 4 \\ 8 & 6 \end{pmatrix}$, $Y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $XY = \begin{pmatrix} 12 & 16 \\ 8 & 10 \end{pmatrix}$, find

a. matrix Y

b. the determinant of Y

Summary

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- For a 2×2 matrix the determinant is **ad - bc**
- For a 3×3 matrix multiply **a** by the **determinant of the 2×2 matrix** that is **not** in **a**'s row or column, likewise for **b** and **c**, but remember that **b** has a negative sign!
- The pattern continues for larger matrices: multiply **a** by the **determinant of the matrix** that is **not** in **a**'s row or column, continue like this across the whole row, but remember the + + pattern.