OGUN DIGICLASS

SUBJECT: MATHEMATICS

TOPIC: VECTORS

Learning Objectives

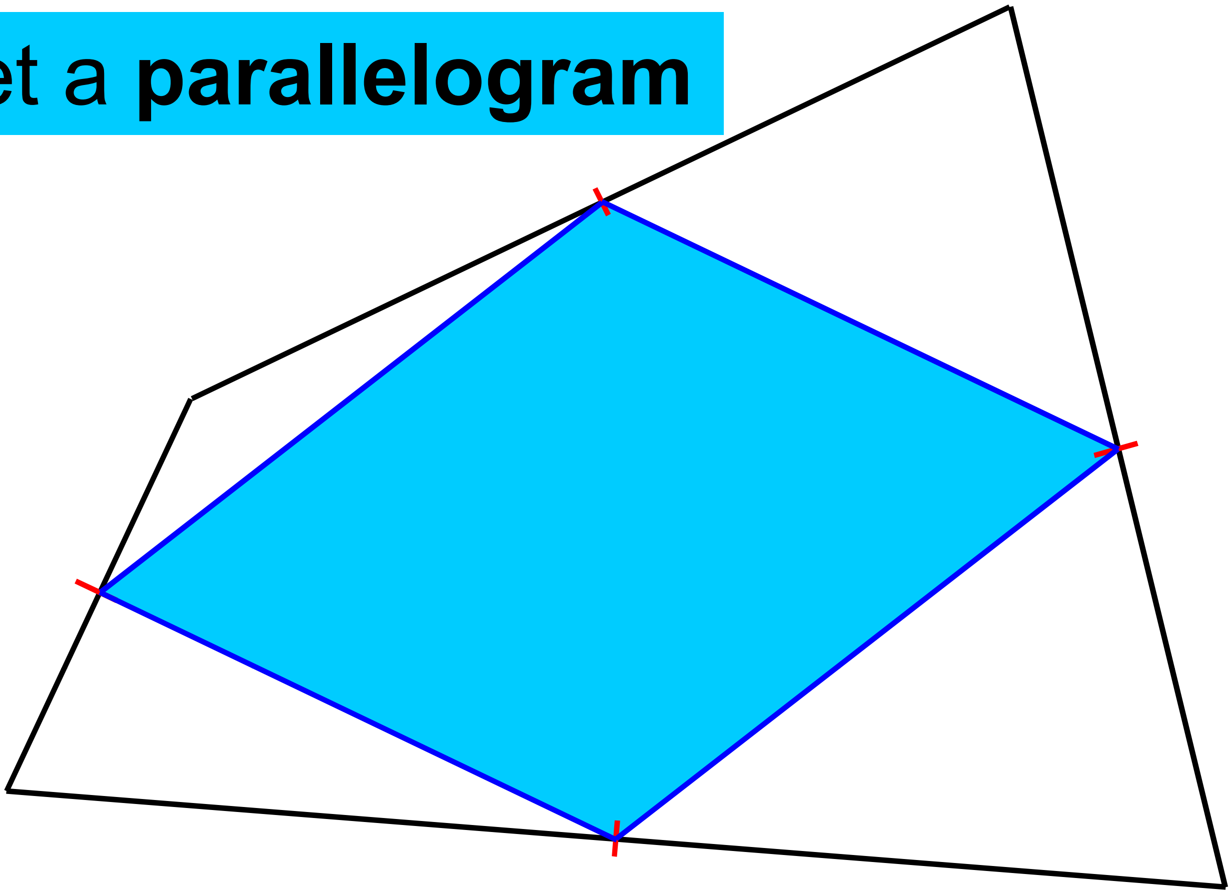
- ❖Define vector
- ❖Draw a column vector
- ❖Sum of vectors
- ❖Determine magnitude of a vector

Draw any quadrilateral

Find the midpoints of each side

Connect them in order…

And you get a **parallelogram**



This is **always** true no matter what quadrilateral you draw…

Geometric problems like this can be understood using **vectors**.

Vectors are also used to understand physical situations, such as the motion of objects acted on by different forces, as you will see further in A-Level **Mechanics**.

What is a vector?

A **vector** describes *direction* and *length*

Eg Translate shape A using the vector:

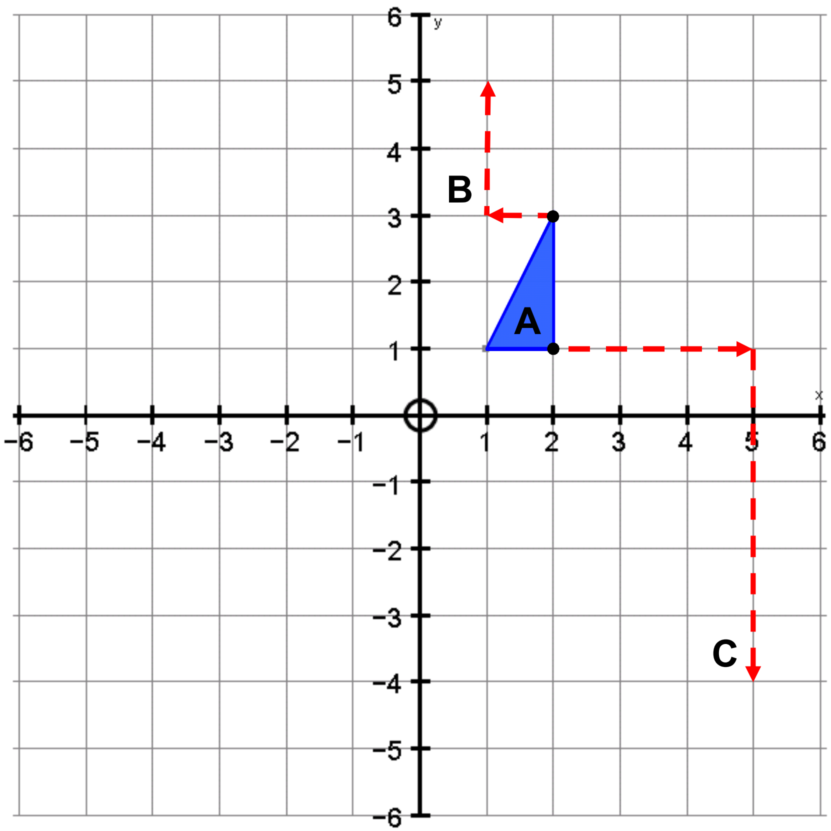
$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ to obtain B — 1 left, 2 up

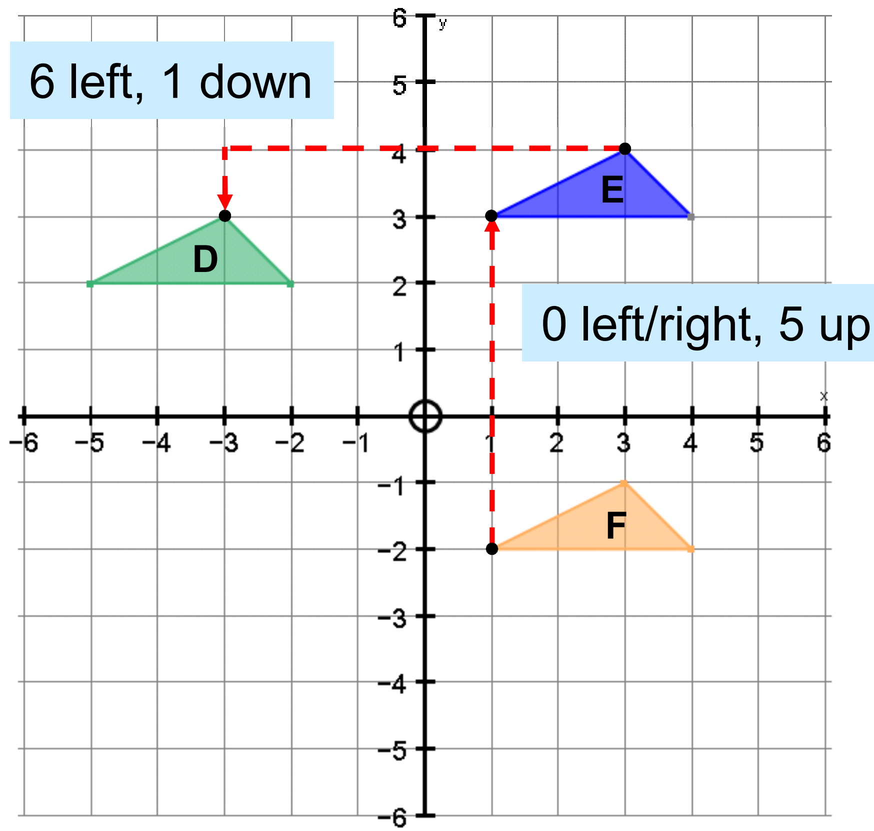
$\begin{pmatrix} 3 \\ -5 \end{pmatrix}$ to obtain C — 3 right, 5 down

Eg Write down the vector needed to translate from:

F to E $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

E to D $\begin{pmatrix} -6 \\ -1 \end{pmatrix}$





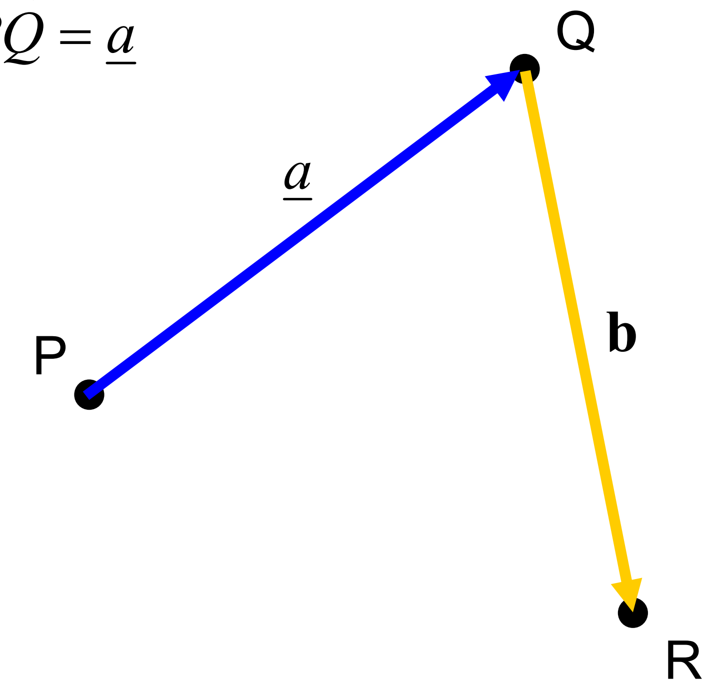
Vector notation

A **vector** describes the *direction* and *length* of a movement

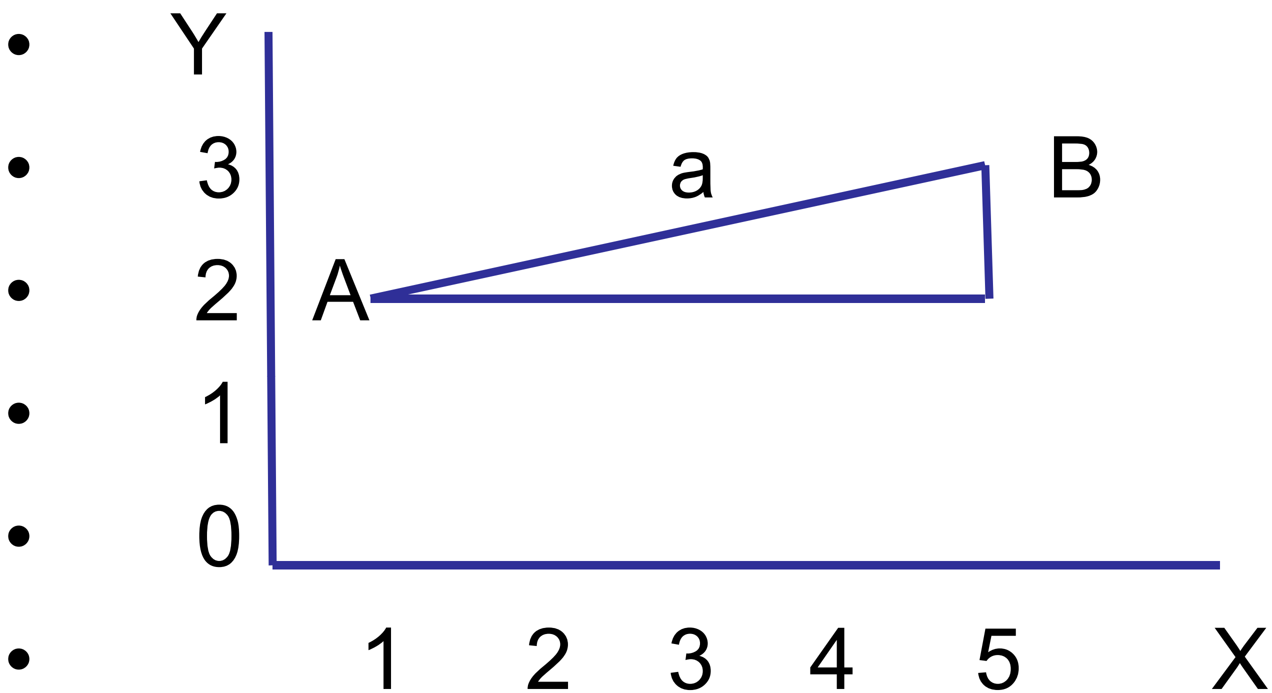
Example: displacement, velocity, force, acceleration …

Vectors can also be represented by a single letter in **bold** or <u>underlined</u> or by the letters at the start and finish with an arrow above them

$$\overrightarrow{PQ} = \underline{a}$$



$$\overrightarrow{QR} = \mathbf{b}$$


Y
3
2
1
0
a
A
B
1
2
3
4
5
X

Column vectors

1. Draw the following column vectors:

a) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

b) $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

c) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$

d) $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$

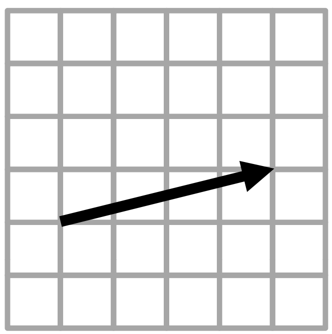
e) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

f) $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

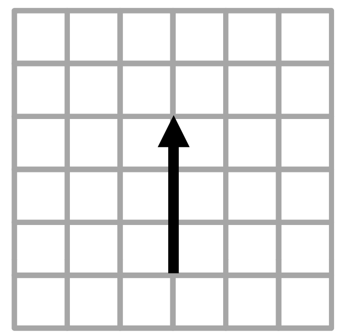
Column vectors

1. Draw the following column vectors:

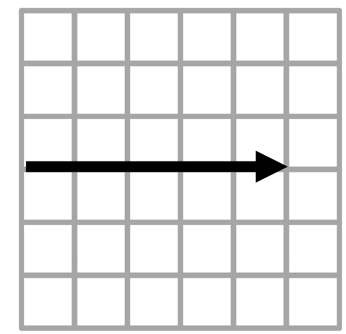
a) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$



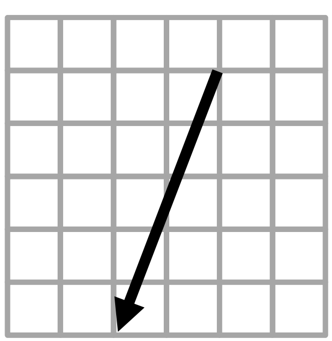
b) $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$



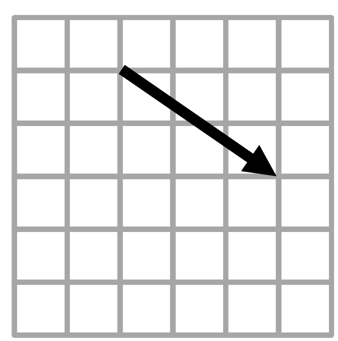
c) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$



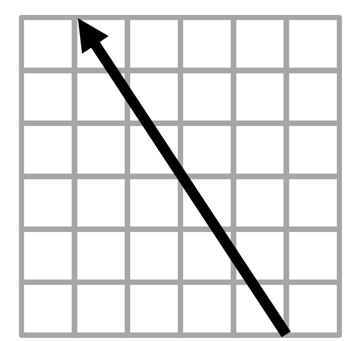
d) $\begin{pmatrix} -2 \\ -5 \end{pmatrix}$



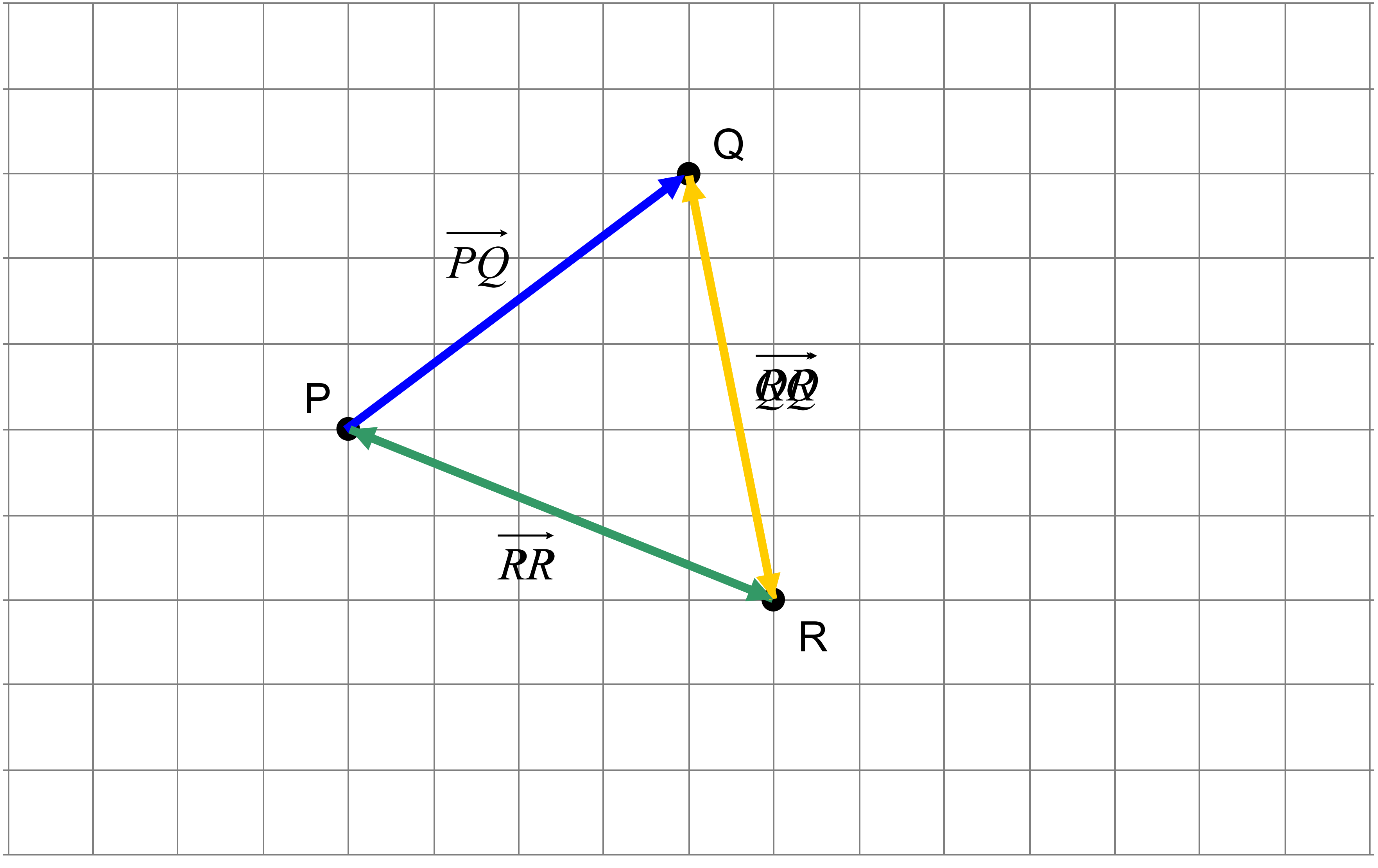
e) $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$



f) $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$



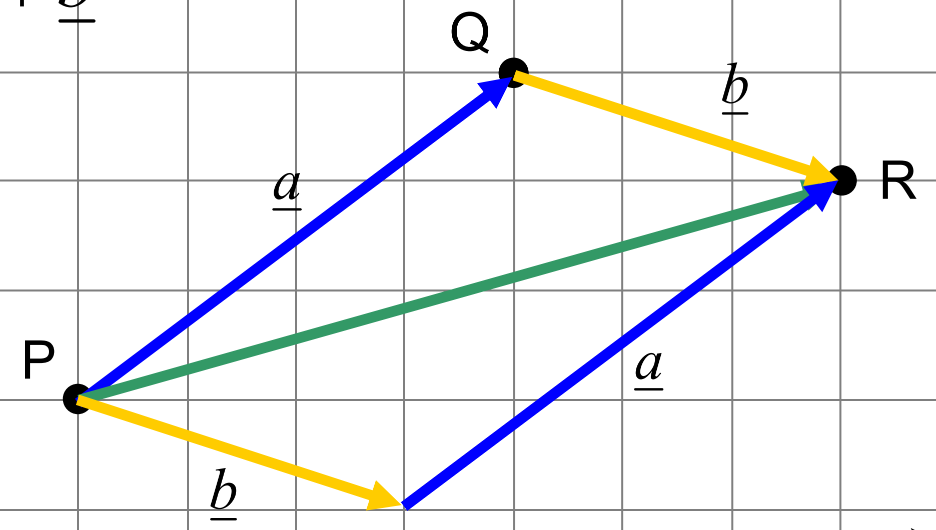
Column vectors



On your white board, write down column vectors for:

Vector addition

$$\overrightarrow{PR} = \underline{a} + \underline{b}$$

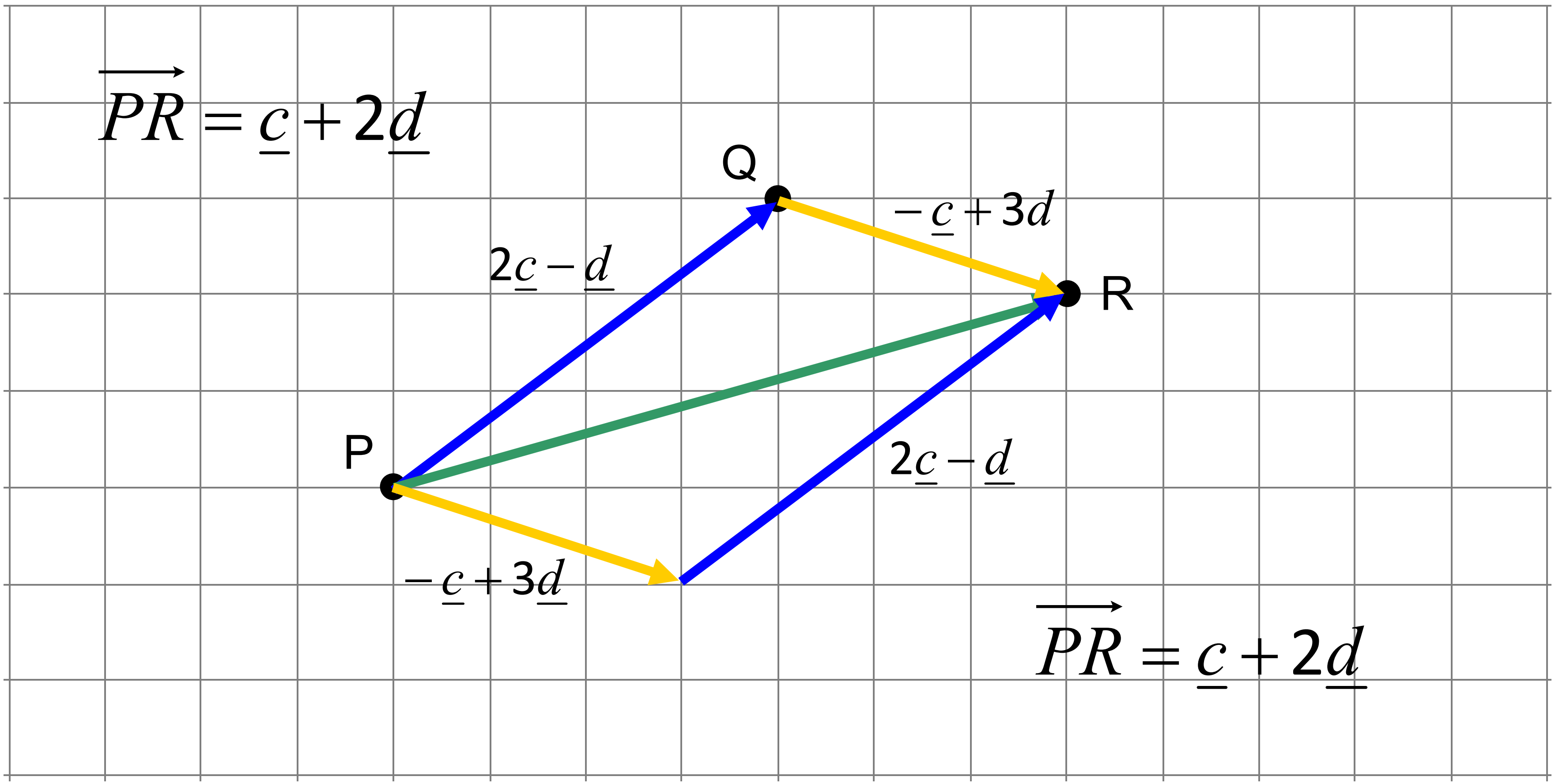


$$\overrightarrow{PR} = \underline{b} + \underline{a}$$

Like normal addition, order doesn't matter when adding vectors

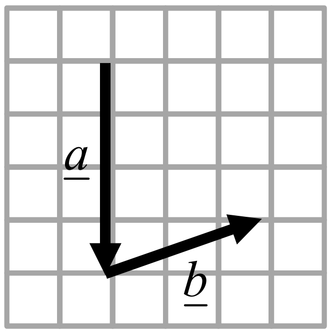
$$\underline{a} + \underline{b} = \underline{b} + \underline{a}$$

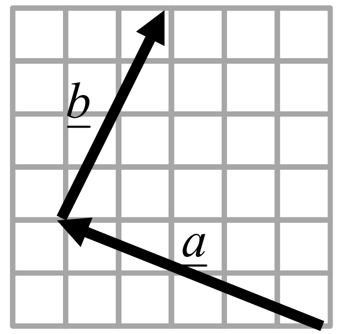
Vector addition

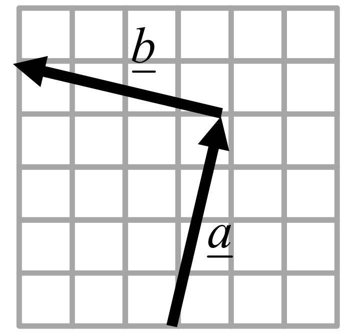


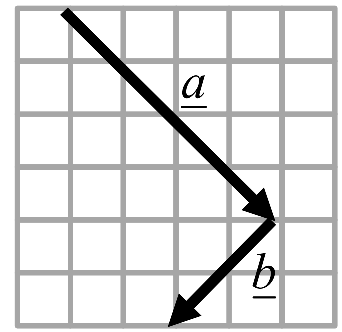
Like normal addition, order doesn't matter when adding vectors

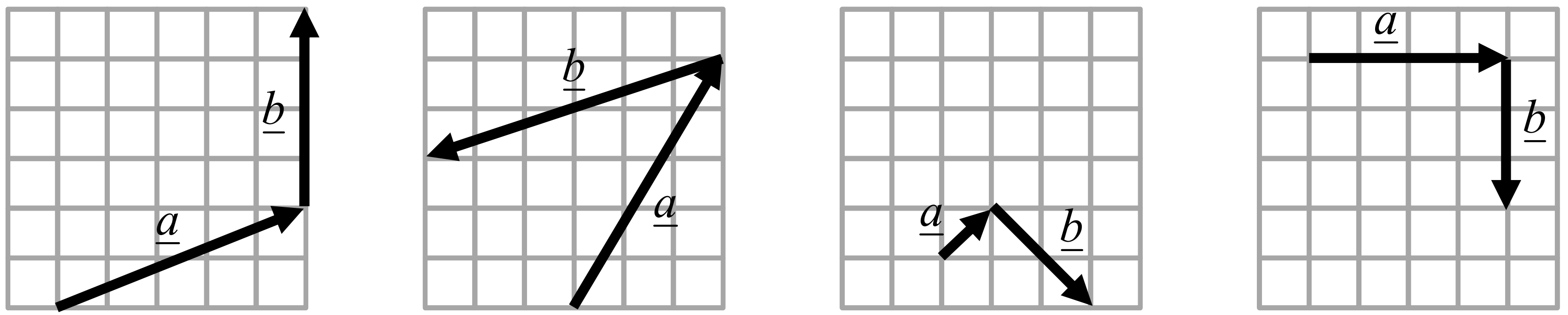
Cut out and match up each diagram, pair of vectors a and b, and resultant vector a+b



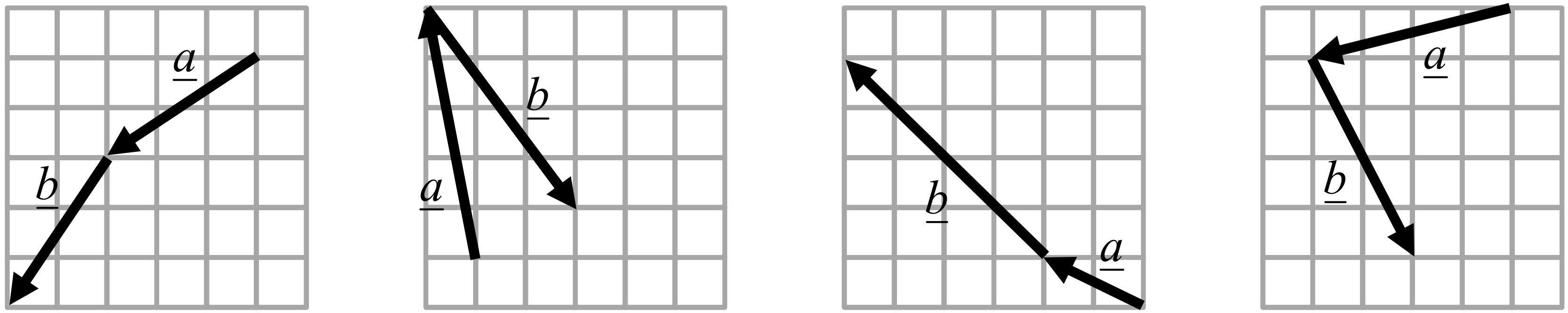




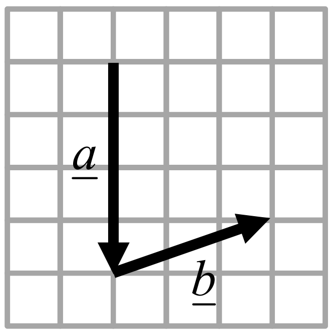


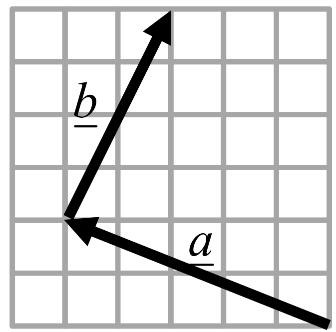


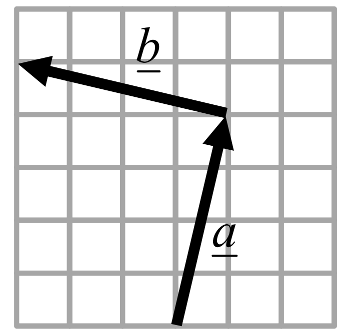
a
b
b
a
a
b
a
b

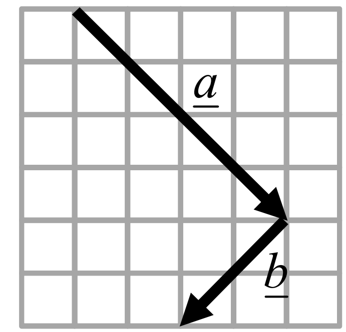

a
b
b
a
b
a
a
b

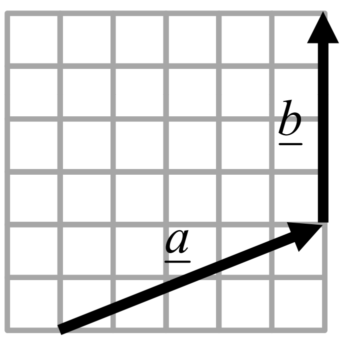
Cut out and match up each diagram, pair of vectors a and b, and resultant vector a+b
Can you spot the quick way of working out the resultant vectors?

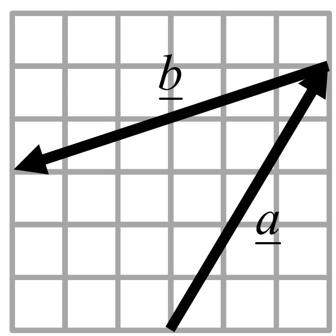


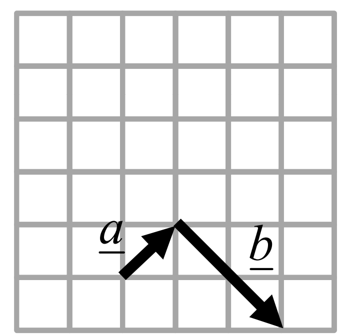


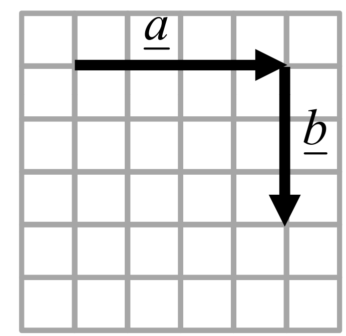


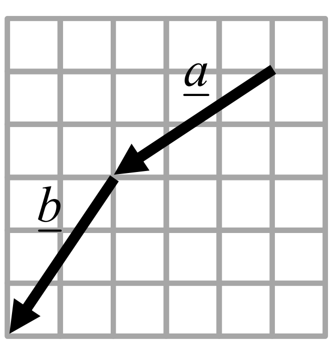


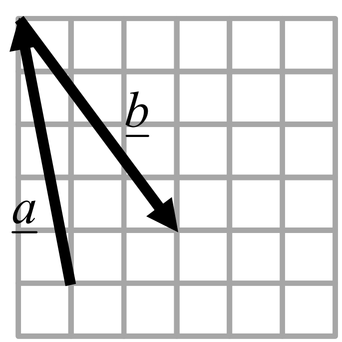


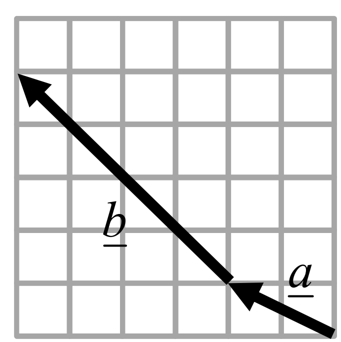


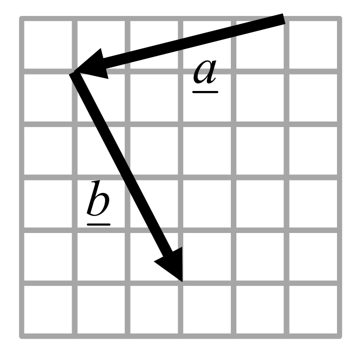










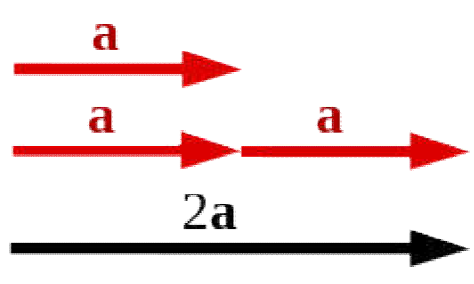


$\underline{a} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -6 \\ -2 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
$\underline{a} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$
$\underline{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$	$\underline{a} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$

$\underline{a}+\underline{b}=\begin{pmatrix}-3\\3\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}-2\\-5\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}3\\-1\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}-6\\5\end{pmatrix}$
$\underline{a}+\underline{b}=\begin{pmatrix}-3\\6\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}-3\\5\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}2\\-6\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}5\\6\end{pmatrix}$
$\underline{a}+\underline{b}=\begin{pmatrix}3\\-3\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}4\\-3\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}-5\\-5\end{pmatrix}$	$\underline{a}+\underline{b}=\begin{pmatrix}2\\1\end{pmatrix}$

Scalar multiplication

- If a vector a is multiplied by a positive scalar k, the result is a new vector ka which is in the same direction as a and k times as big. When k is negative, the new vector ka is in the opposite direction to a.



2. $\underline{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ Write down as a column vector:

a) $2\underline{p}$	b) $-\underline{q}$	c) $\frac{1}{2}\underline{p}$
d) $\underline{p} + \underline{q}$	e) $\underline{p} - \underline{q}$	f) $3\underline{p} + 5\underline{q}$

2. $\underline{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ Write down as a column vector:

a) $2\underline{p}$ $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$	b) $-\underline{q}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	c) $\frac{1}{2}\underline{p}$ $\begin{pmatrix} 1 \\ \frac{3}{2} \end{pmatrix}$
d) $\underline{p} + \underline{q}$ $\begin{pmatrix} 2-1 \\ 3+1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$	e) $\underline{p} - \underline{q}$ $\begin{pmatrix} 2+1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$	f) $3\underline{p} + 5\underline{q}$ $\begin{pmatrix} 6-5 \\ 9+5 \end{pmatrix} = \begin{pmatrix} 1 \\ 14 \end{pmatrix}$

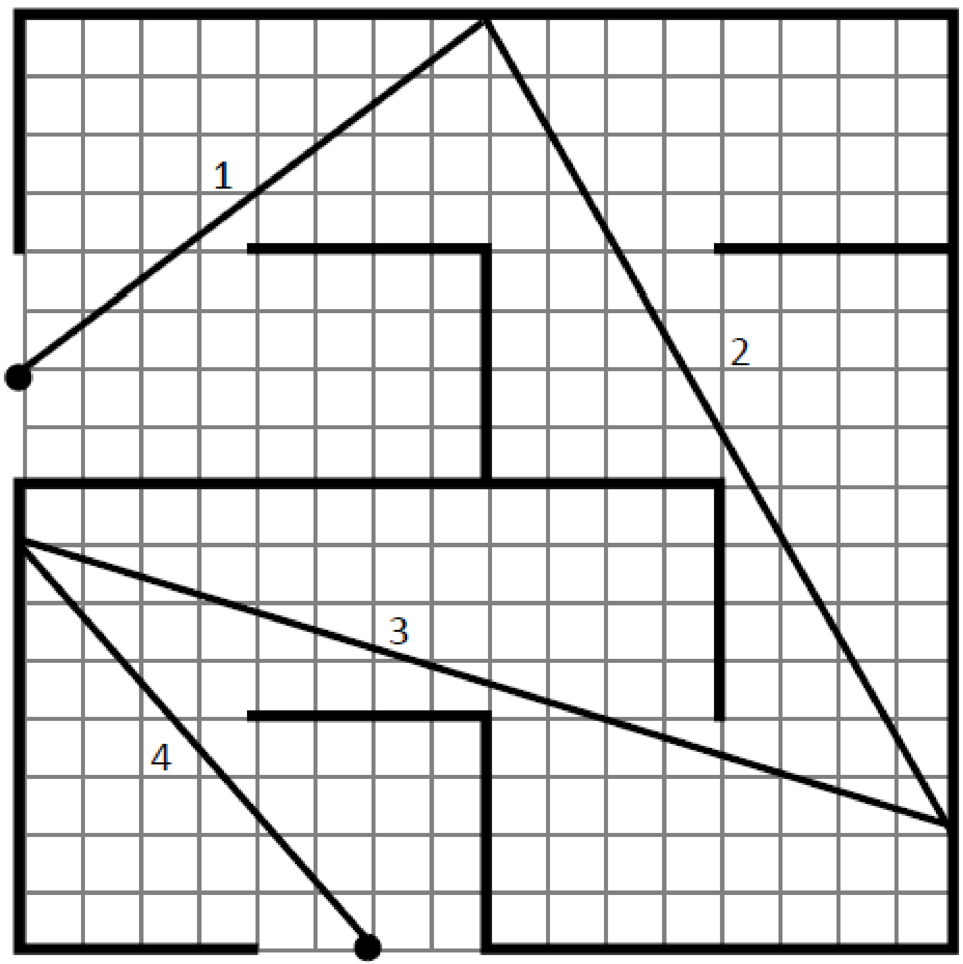
Magnitude of a Vector

- `   if AB = $\binom{4}{1}$, the magnitude or size of AB is the length of the line segment AB, It is also called modulus of AB.
- We use Pythagoras rule to determine the magnitude of a vector.
- The magnitude of a vector is always given as a positive number of units

- Y
- 3 a B
- 2 A
- 1
- 0
- 1 2 3 4 5 X

Vector mazes

Use vectors to get through the maze. Record your choices in the table, using Pythagoras's theorem to work out the exact length of your route.
Play against an opponent – the person with the smallest route length wins.



Vector used	Exact length of vector
$\begin{pmatrix} 8 \\ 6 \end{pmatrix}$	$\sqrt{8^2+6^2}=10$
$\begin{pmatrix} 8 \\ -14 \end{pmatrix}$	$\sqrt{8^2+14^2}=2\sqrt{65}$
$\begin{pmatrix} -16 \\ 5 \end{pmatrix}$	$\sqrt{16^2+5^2}=\sqrt{281}$
$\begin{pmatrix} 6 \\ -7 \end{pmatrix}$	$\sqrt{6^2+7^2}=\sqrt{85}$
Total (1dp)	52.1

Class work

- If x = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, y = $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and z = $\begin{pmatrix} -4 \\ 13 \end{pmatrix}$, find scalar p and q such that px + qy = z

Assignment

In triangle ABC, $\overline{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$ and $\overline{AC} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}$.
If p is the mid point of $\overline{AB}$, express $\overline{CP}$ as a column vector.