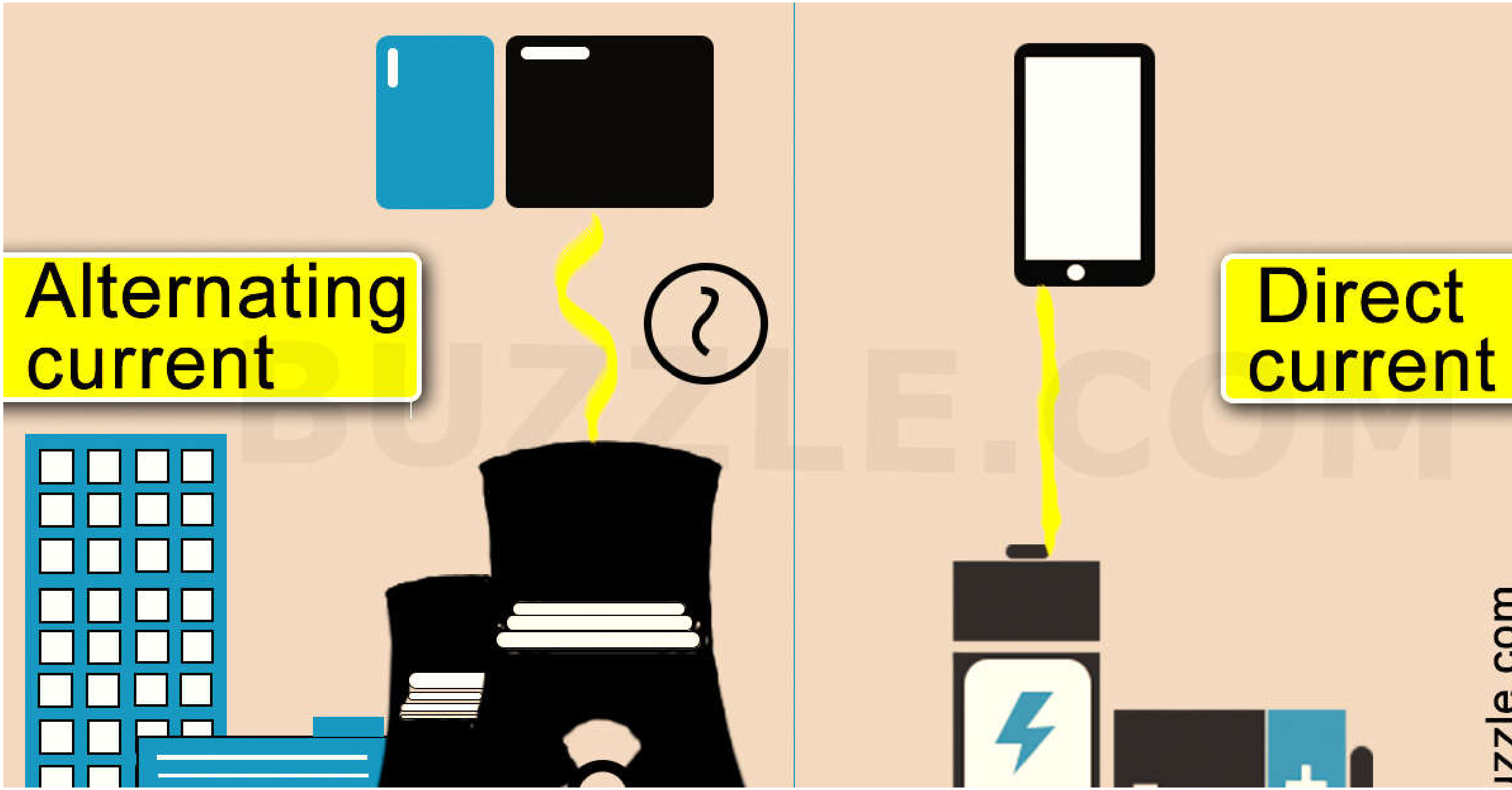
OGUN DIGICLASS

CLASS: SECONDARY SCHOOL

SUBJECT: PHYSICS

TOPIC: SIMPLE ALTERNATING CURRENT CIRCUITS



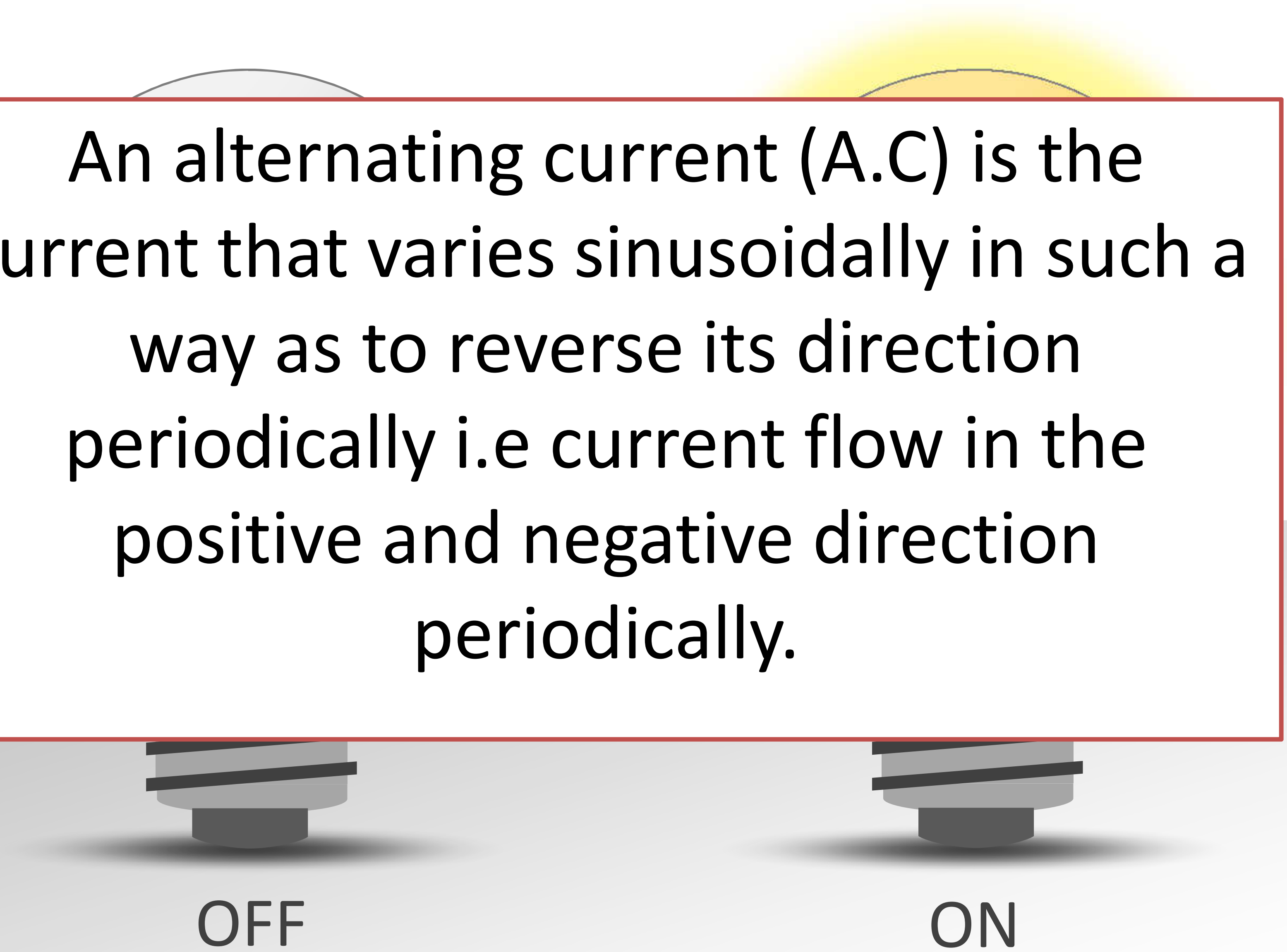
SIMPLE ALTERNATING CURRENT CIRCUITS

Learning Objectives

- Explain the peak and R.M.S values of current and P.D
- Establish the phase relationship between current and P.D in an A.C. Circuit
- Explain reactance and impedance
- Determine current in circuits containing: resistance and inductance; resistance and capacitance; resistance, inductance and capacitance.
- Determine power in an A.C circuit

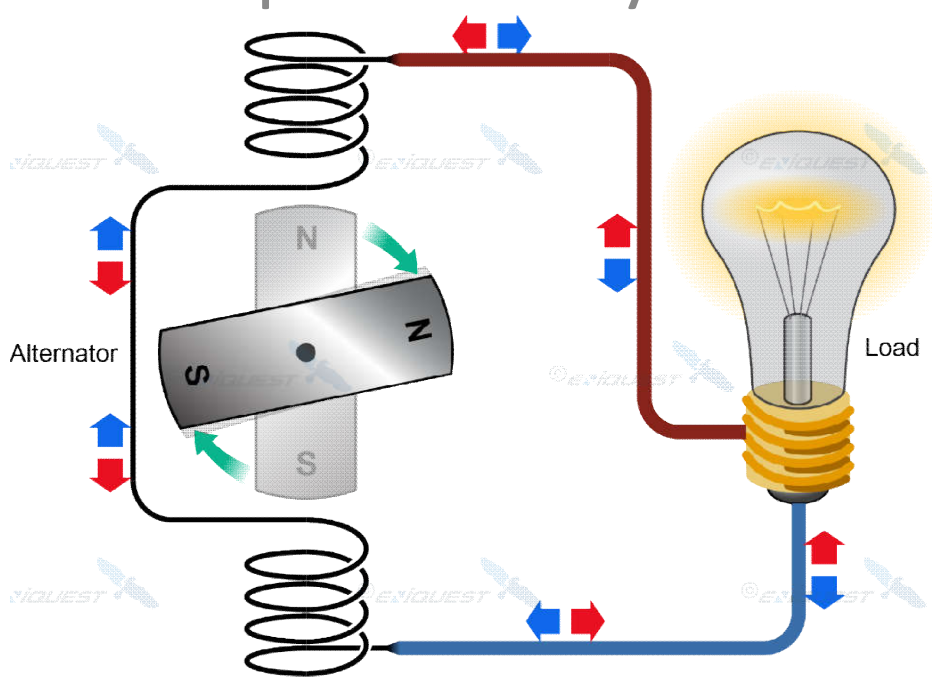
INTRODUCTION

An alternating current (A.C) is the current that varies sinusoidally in such a way as to reverse its direction periodically i.e current flow in the positive and negative direction periodically.

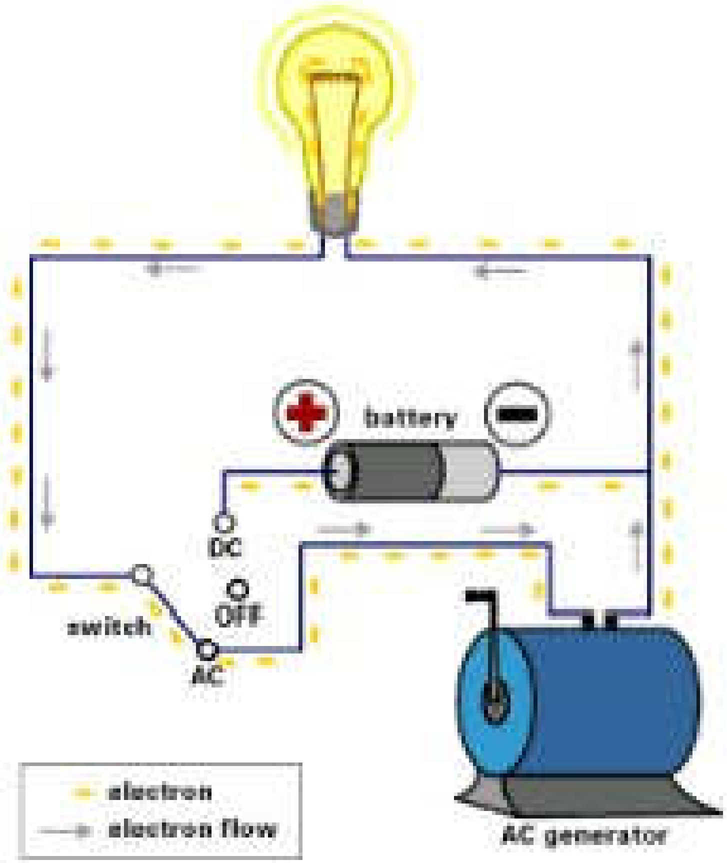


INTRODUCTION

An alternating current (A.C) is the current that varies sinusoidally in such a way as to reverse its direction periodically i.e current flow in the positive and negative direction periodically.



You may have noticed the letters a.c. on the power pack. This is **alternating current**. The electrons rapidly change direction. Alternating current can be provided by generators. Mains electricity is an alternating current.



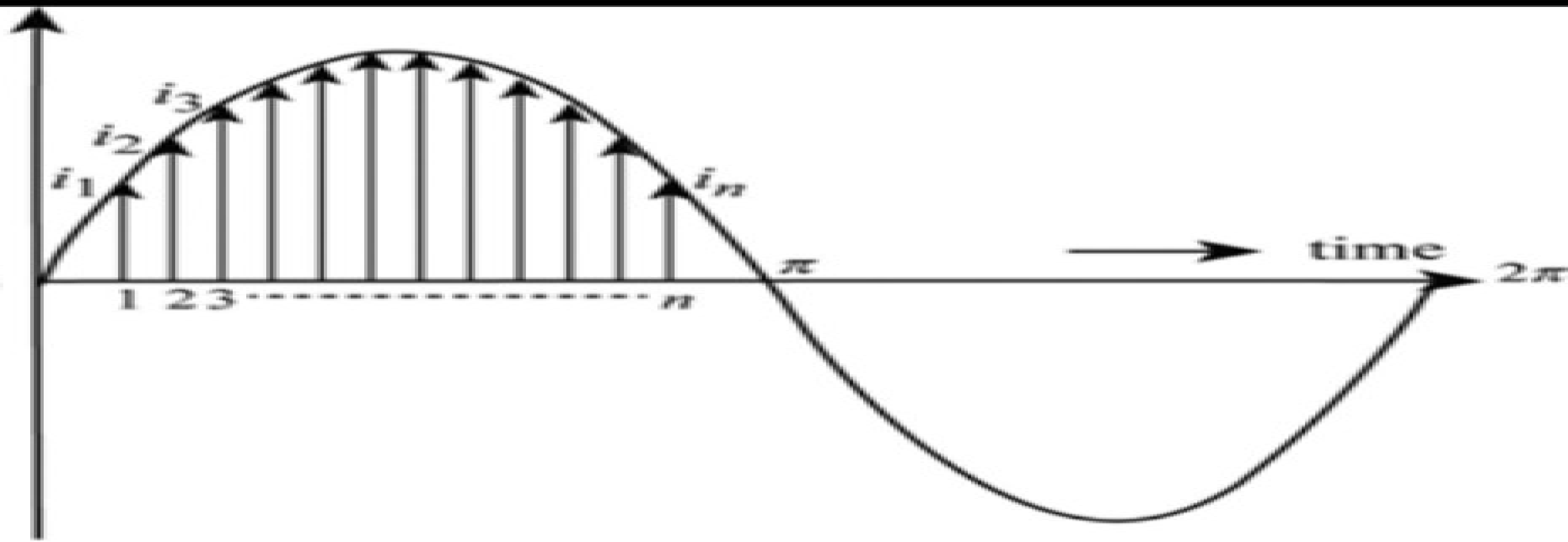


Figure 1

The symbol for A.C is

The simplest a.c voltage V (instantaneous) generated over time by a simple dynamo is represented by the relation

$V= V \ \ \text{Sin}\ \omega t$

where V is the maximum or peak voltage value, ω is the angular velocity $2\pi f$, t is the oscillating time, ωt is the phase angle and f is the oscillating frequency

PEAK AND R.M.S VALUES OF A.C

The amplitude or peak value of the current I (voltage V) is the maximum numerical value of the current (voltage).

The root mean square (r.m.s.) value of the current $I_{r.m.s}$(voltage $V_{r.m.s}$)is the effective value of the current (voltage)

The root mean square value is defined as that value of steady current which will produce heat at the same rate in a given conductor.

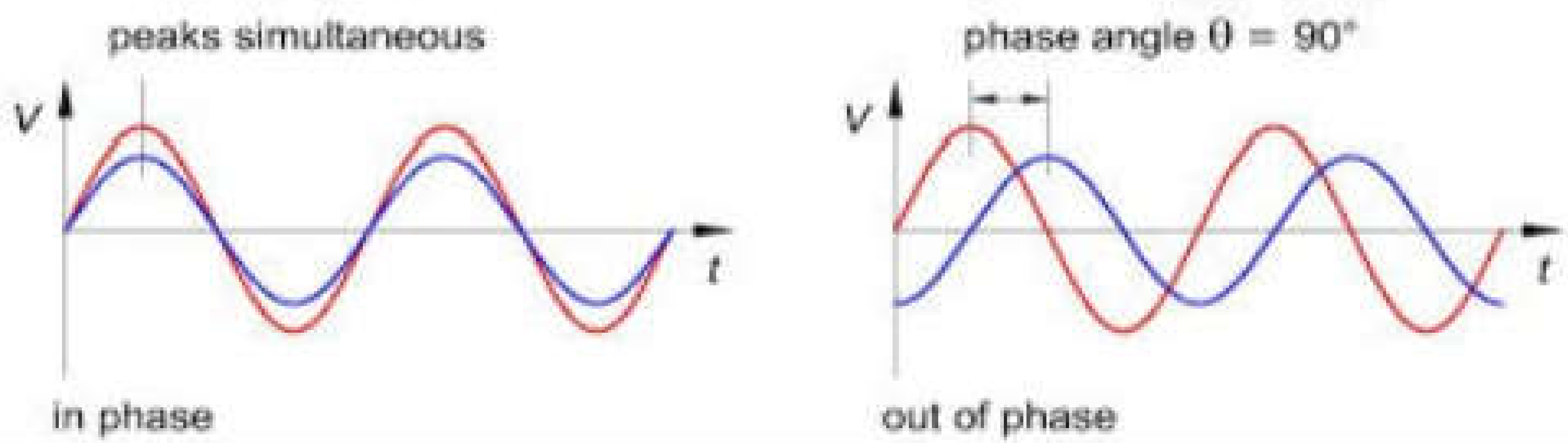
It occurs when I or V flowing in a.c circuit has phase angle of 45 i.e

$V_{rms} = V \sin(\omega t) = V \sin 45$

$V_{rms} = v/\sqrt{2}$ also

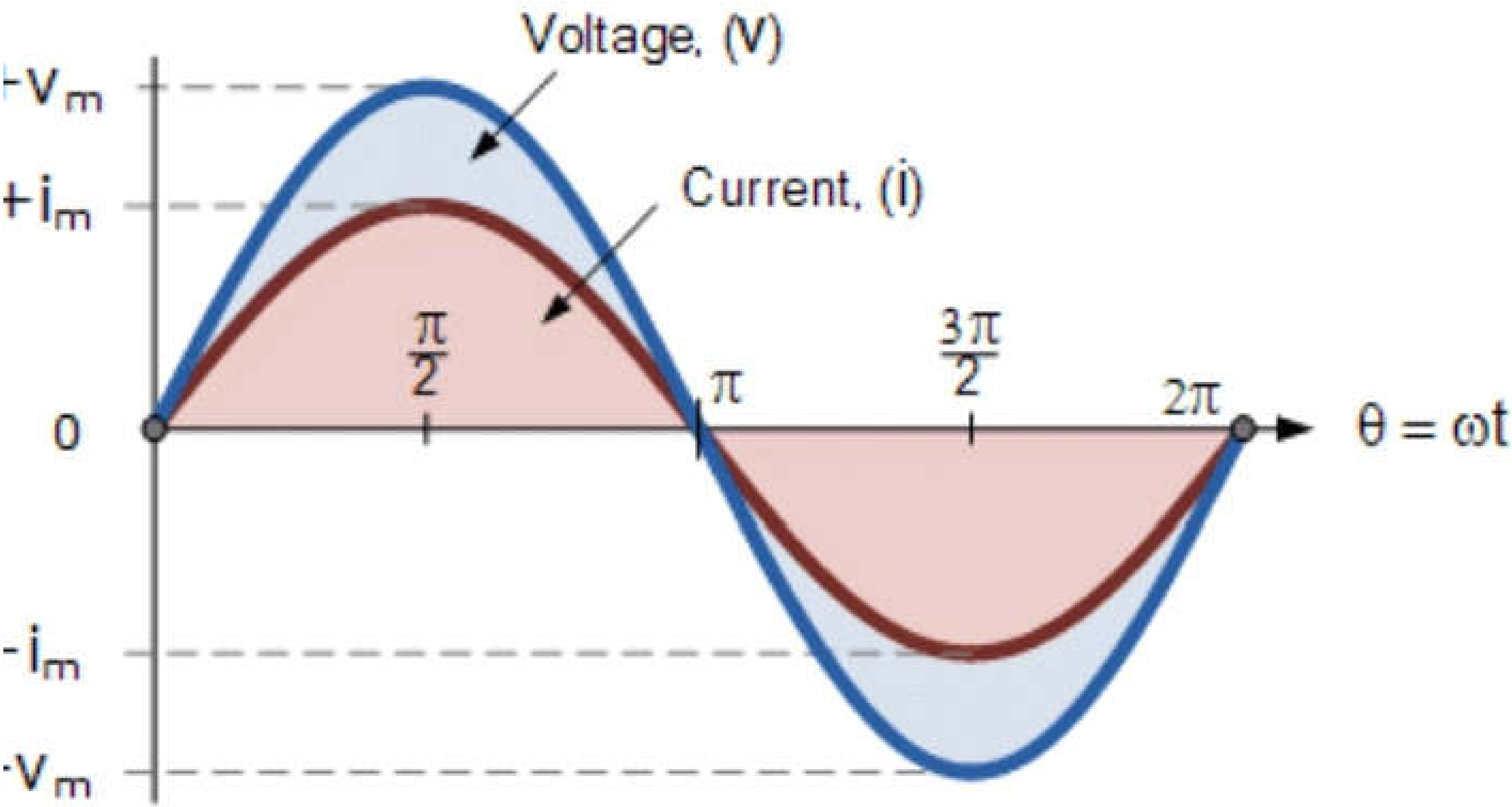
$I_{rms} = I \sin(\omega t) = I \sin 45$

$I_{rms} = I/\sqrt{2}$


peaks simultaneous
V
t
in phase
phase angle θ = 90°
V
t
out of phase

PHASE RELATION IN A.C

In resistance, I and V are in phase.
In capacitor, I leads V by π/2 while in an inductor, I lags V by π/2. Hence, from ohm's law, $I = V/R : IR = Vrms/R$ for the capacitor $I = I \sin \omega t + \pi/2$ and $V = V \sin\omega t$; for the inductor $I = I \sin \omega t - \pi/2$ and $V = V \sin \omega t$



Phase Difference and Phase Shift in an AC Circuit

EXAMPLE

The current I in an a.c circuit is given by the equation $i=30\sin(100\pi t)$where t is the time in seconds Deduce the following from this equation

(i) Frequency of the current
(ii) Peak value of the current
(iii) rms value of the current

SOLUTION

(i)Frequency of the current: $\omega=2\pi f=100\pi$

$f= 100\pi/2\pi= 50Hz$

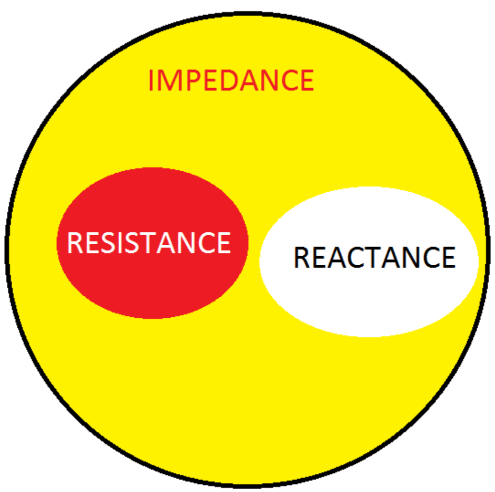
(ii)Peak value of the current: $I=30A$

(iii)rms value of the current: $I_{rms}=I/\sqrt{2}= 30/1.414 = 21.22A$.

REACTANCE (X) AND IMPEDANCE(Z)

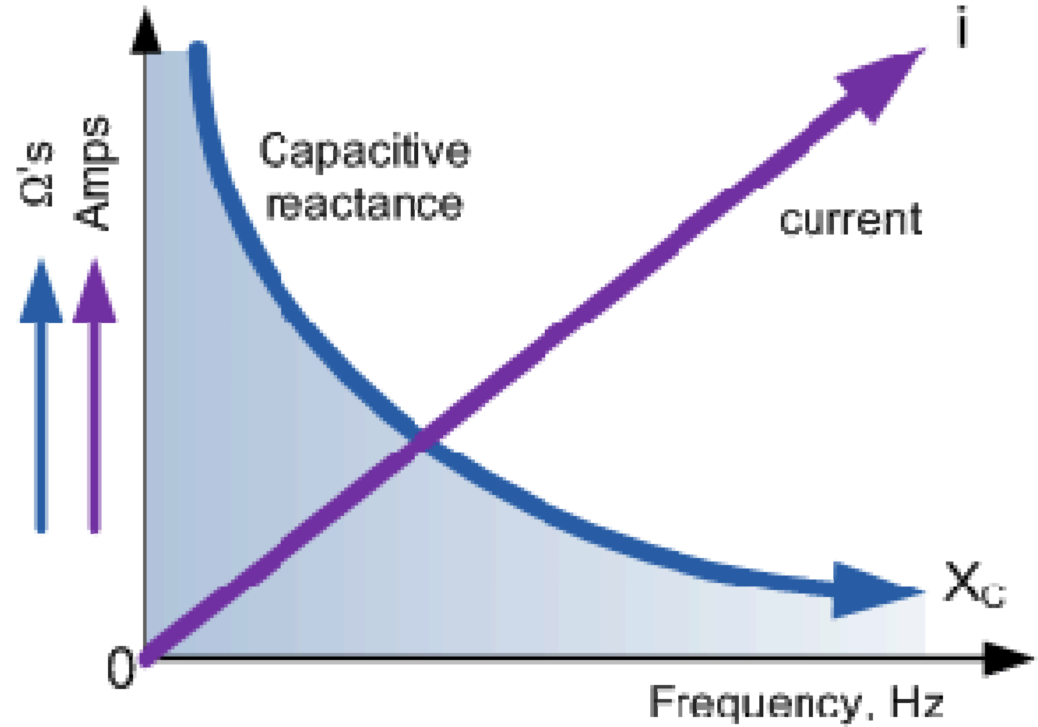
Reactance is the opposition to the flow of a.c offered by a capacitor or an inductor or both.

Impedance is the overall opposition(effective opposition)of a mixed circuit containing a resistor, an inductor and/ or a capacitor. It is measured in ohms.



Capacitive reactance to an a.c

The opposition to the flow of current offered by the capacitor is known as capacitive reactance Xc and it is given as Xc = 1/ωC =1/2πfC where C is the capacitance measured in Farads(F).



EXAMPLE

A series RLC circuit comprises a 100ohm resistor, a 3H inductor and a 4μF capacitor. The a.c source of the circuit has an emf of 100V and frequency of 160/πHz. Determine the capacitive reactance.

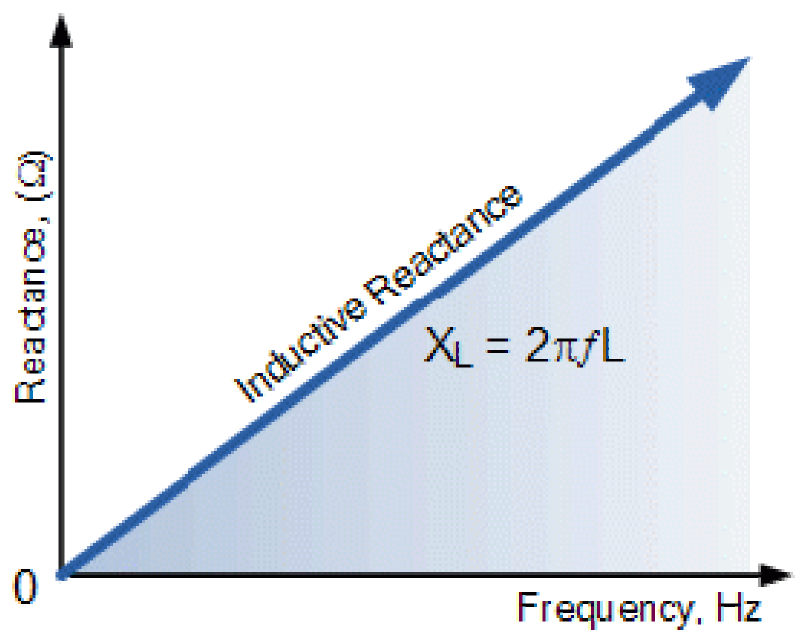
SOLUTION

$X_c = 1/2\pi fC = 1/2\pi x160/ \pi x4x10$

$= 781.25$ohm

REACTANCE OF AN INDUCTOR

XL is the resistance that is opposing the flow of current in the inductor. It is called inductive reactance and it is given as $X_L = \omega L = 2\pi f L$ where Lis the inductance measured in Henry (H)



EXAMPLE

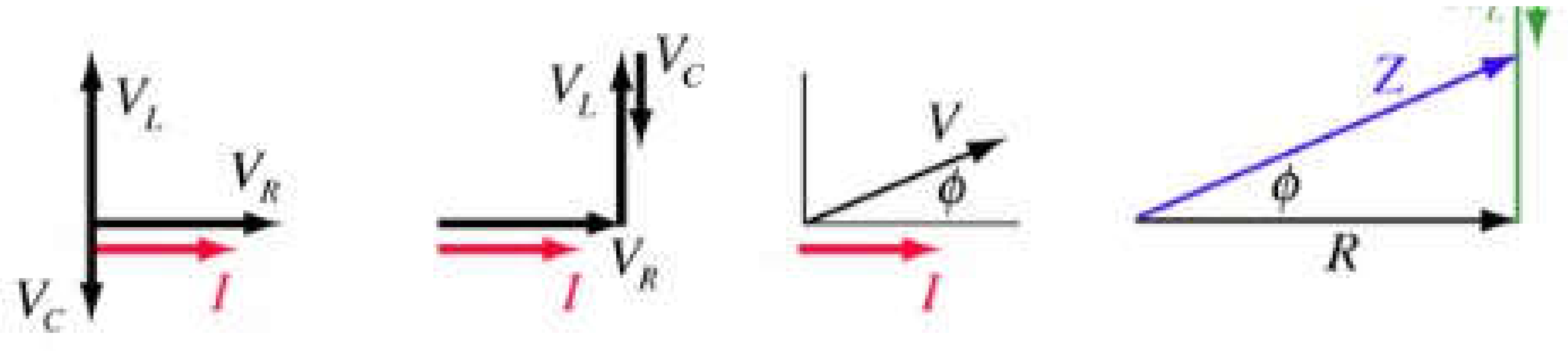
A series RLC circuit comprises a 100ohm resistor, a 3H inductor and a 4μF capacitor. The a.c source of the circuit has an emf of 100V and frequency of 160/πHz. Determine the inductive reactance.

Solution

$X_L = \omega L = 2\pi f L = 2\pi \times 160/\pi \times 3$

$= 960$ohm

VECTOR DIAGRAMS



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1}\frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\frac{X_L - X_C}{R}$$

Phase Relationships in AC Circuits

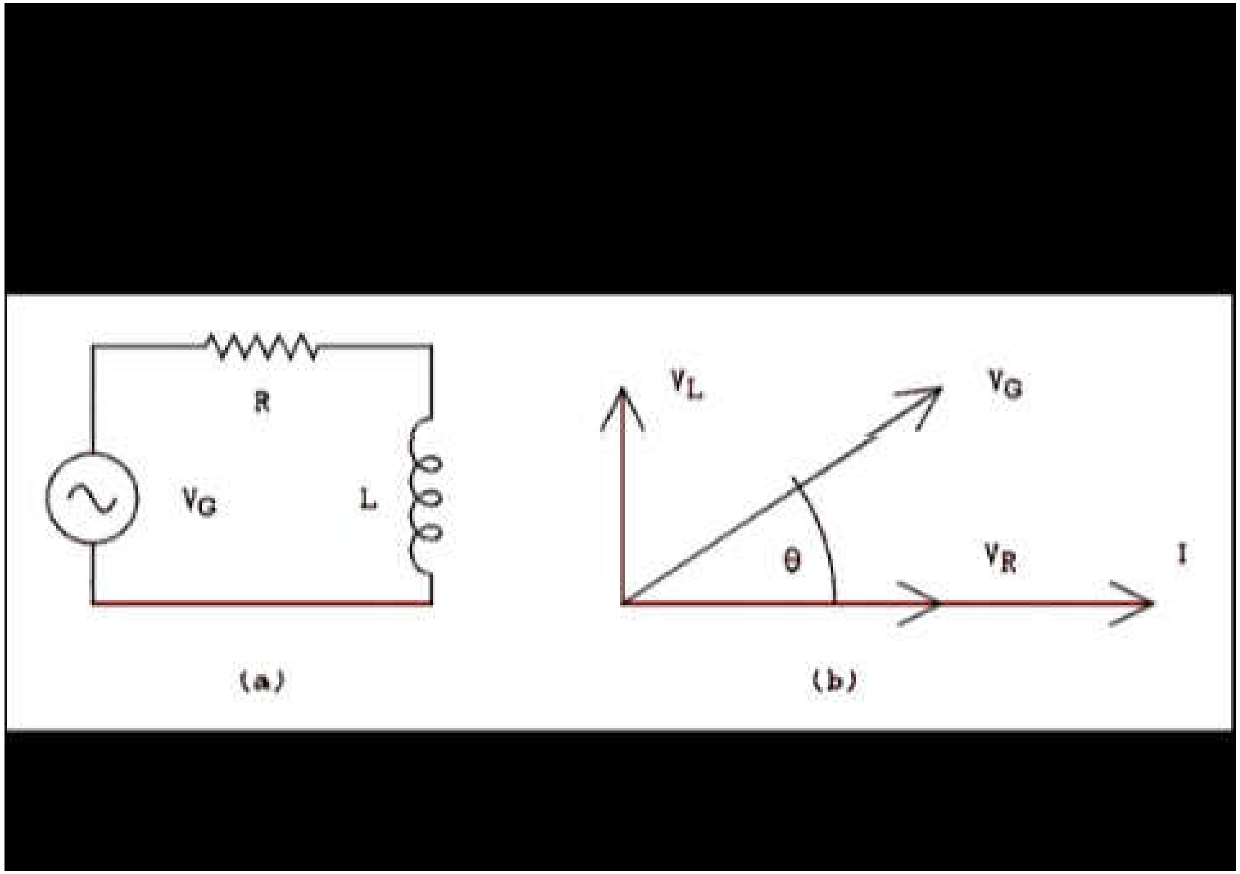
EXAMPLE

A series RLC circuit comprises a 100ohm resistor, a 3H inductor and a 4μF capacitor. The a.c source of the circuit has an emf of 100V and frequency of 160/πHz. Determine the impedance of the circuit.

$Z = \sqrt{R^2 + (X_L - X_c)^2} = \sqrt{100^2 + (960 - 781.25)^2}$

$Z = \sqrt{100^2 + 178.25^2} = 204.82$ohm

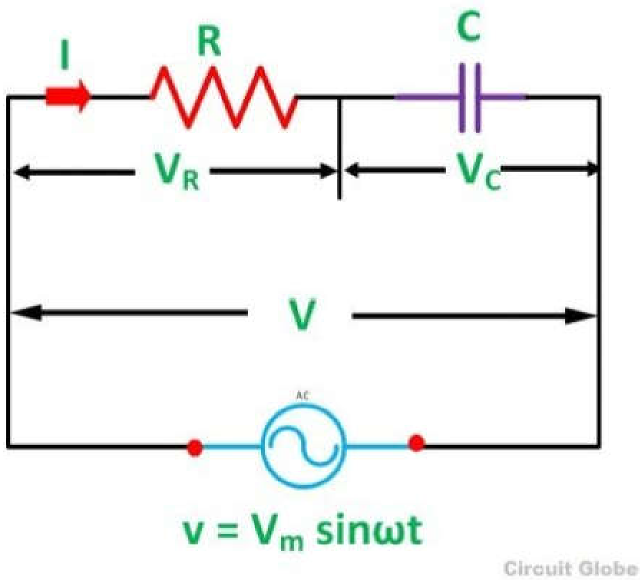
IMPEDANCE OF L AND R IN AN L-R SERIES CIRCUIT



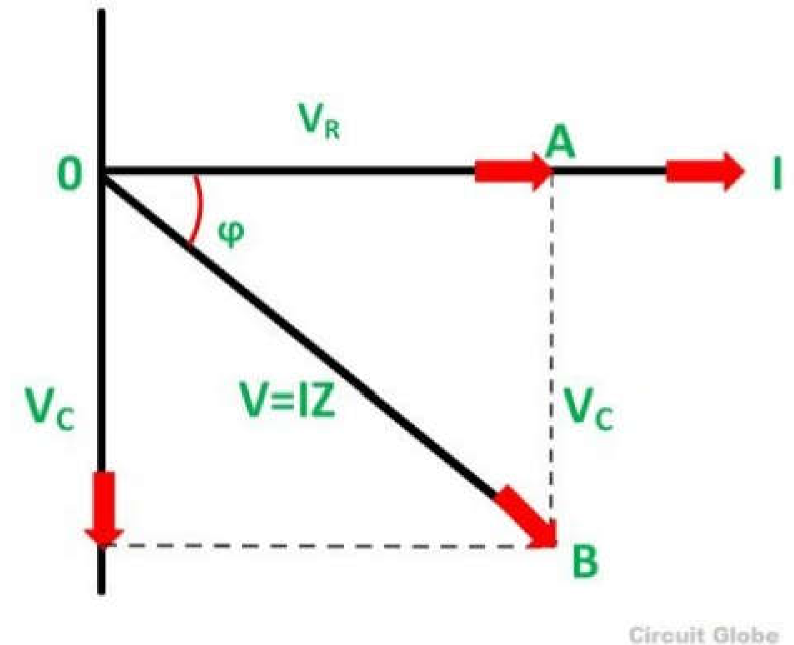
Impedance $Z = \sqrt{X_L^2 + R^2}$ while $\tan\theta = X_L/R$

IMPEDANCE OF C AND R IN AN R-C SERIES CIRCUIT

[redacted]

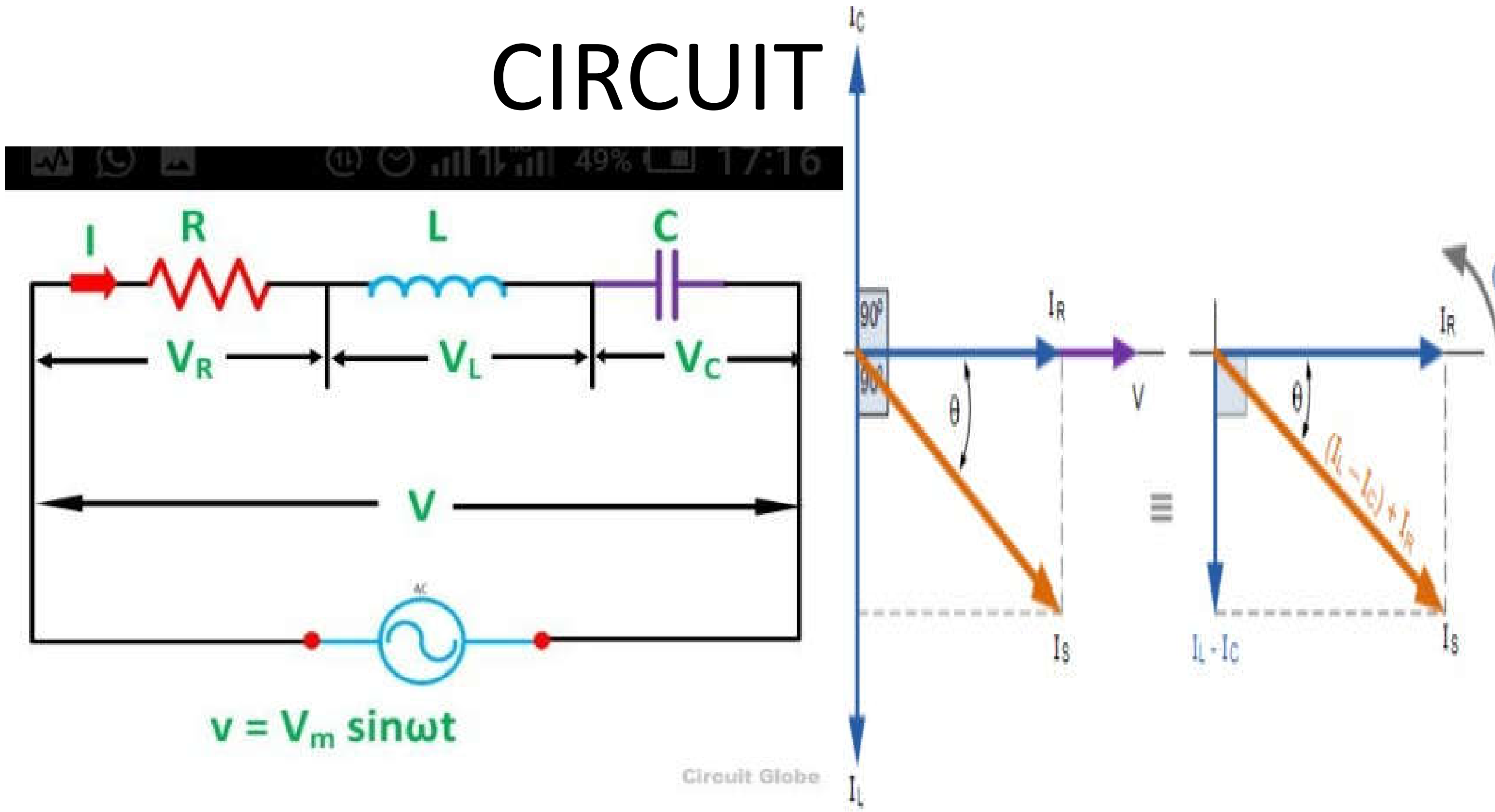


[redacted]



Impedance $Z = \sqrt{X_C^2 + R^2}$ while $\tan\varphi = X_C/R$

IMPEDANCE OF AN L,R,C SERIES CIRCUIT

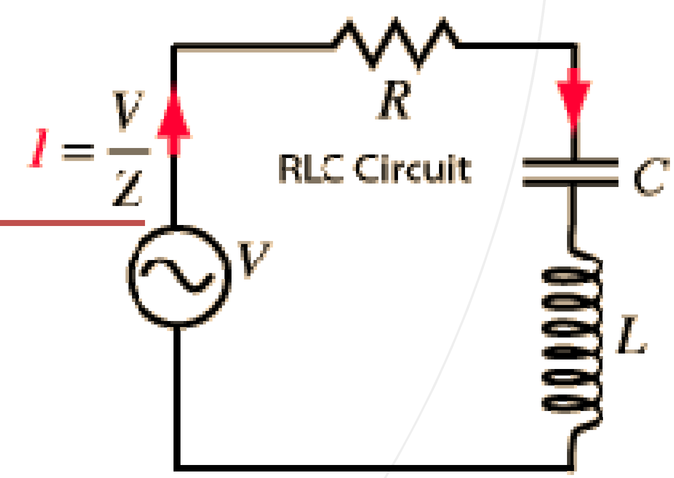


The impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

while $\tan\theta = (X_L - X_C)/R$

POWER IN AN A.C CIRCUIT

The average power in an a.c circuit is given by P=IVcosɵ where I,V are the rms values of the current and voltage respectively and ɵ is the angle of lead or lag between them.

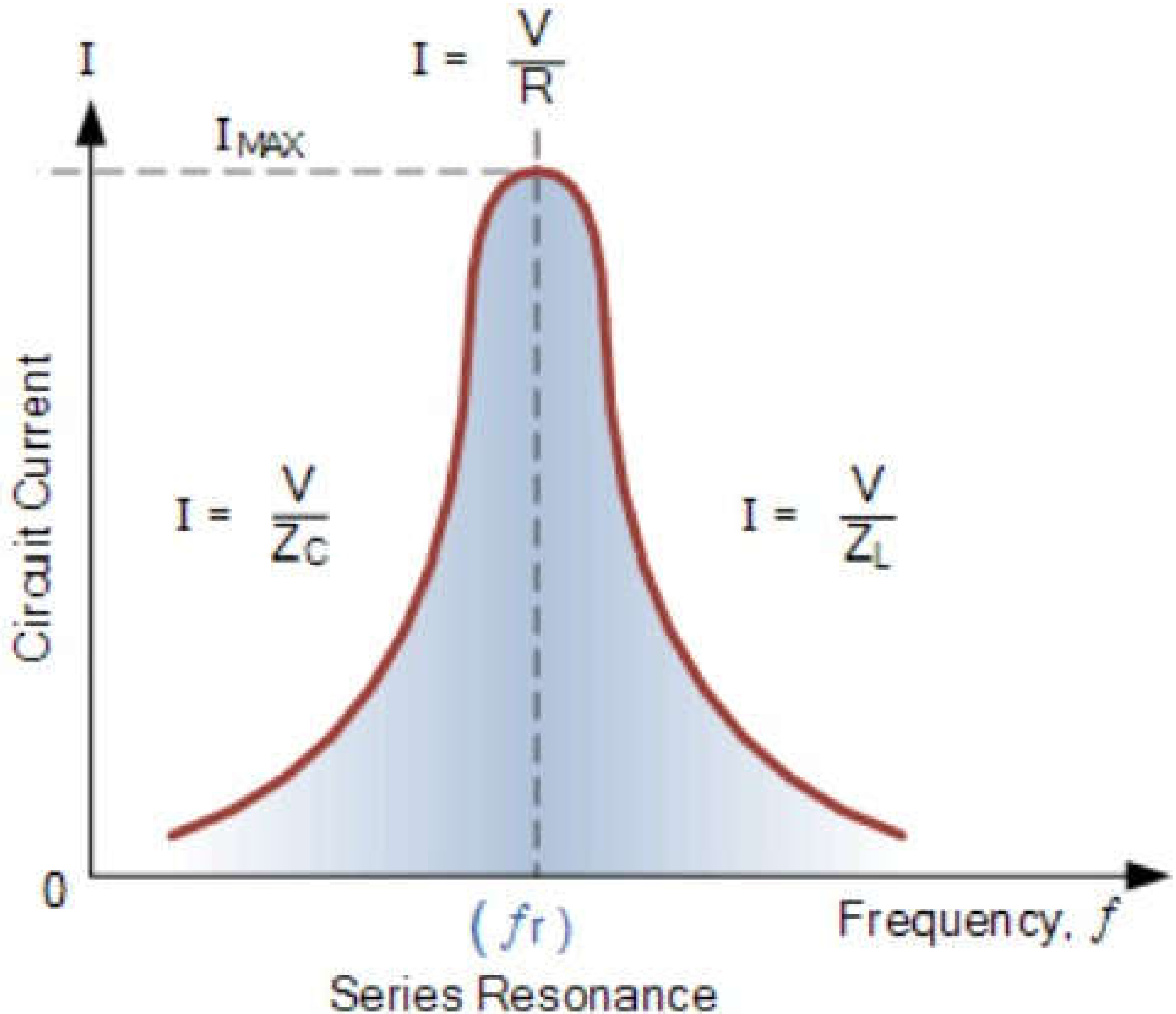


The power factor cos ɵ = R/Z (ratio of resistance to impedance).It can have a value between zero and one for ɵ varying from 90° to 0°.

A power factor of zero means that the device is either a pure reactance, inductance or capacitance.

RESONANCE IN RLC SERIES CIRCUIT

Resonance is said to occur in an a.c series circuit when the maximum current is obtained from such circuit.
From $I = V/Z$, the current I is maximum when impedance Z is minimum. This happens when $X_L = X_c$ i.e $2\pi fL = 1/2\pi fC$.
The frequency at which this resonance occurs is called resonance frequency f_0.


I
I = V/R
I MAX
Circuit Current
I = V/Zc
I = V/ZL
0
(fr)
Frequency, f
Series Resonance

To solve for resonance frequency f0:

$2\pi f0L = 1/ 2\pi f0c$

$f0= 1/ 2\pi\sqrt{LC}$

EXAMPLE

A series RLC circuit comprises a 100ohm resistor, a 3H inductor and a 4μF capacitor. The a.c source of the circuit has an emf of 100V and frequency of 160/πHz.What is the resonance frequency?

SOLUTION

$f_0 = 1/\ 2\pi\sqrt{LC} = 1/\ 2\pi \times \sqrt{3 \times 4 \times 10} \quad = 45.94Hz$

TAKE HOME

(i) The instantaneous current of an a.c power supply is given as $I = 8 \sin(10\pi t)$. Calculate the value of (a) Peak current (b) rms current (c) angular frequency (d) frequency of the instrument.

(ii) A capacitor of 2.0×10^{12}F and an inductor are joined in series. What is the value of the inductance that will give the circuit a resonant frequency of 200kHz.

(iii) What is the value of the angle of lead or lag in alternating current.

(iv) calculate the peak voltage of a mains supply of 240Vrms.