

OGUN DIGICLASS

CLASS: SECONDARY SCHOOL

SUBJECT: MATHEMATICS

TOPIC: MATRICES

SUBTOPIC: Multiplication and Determinant of
Matrices



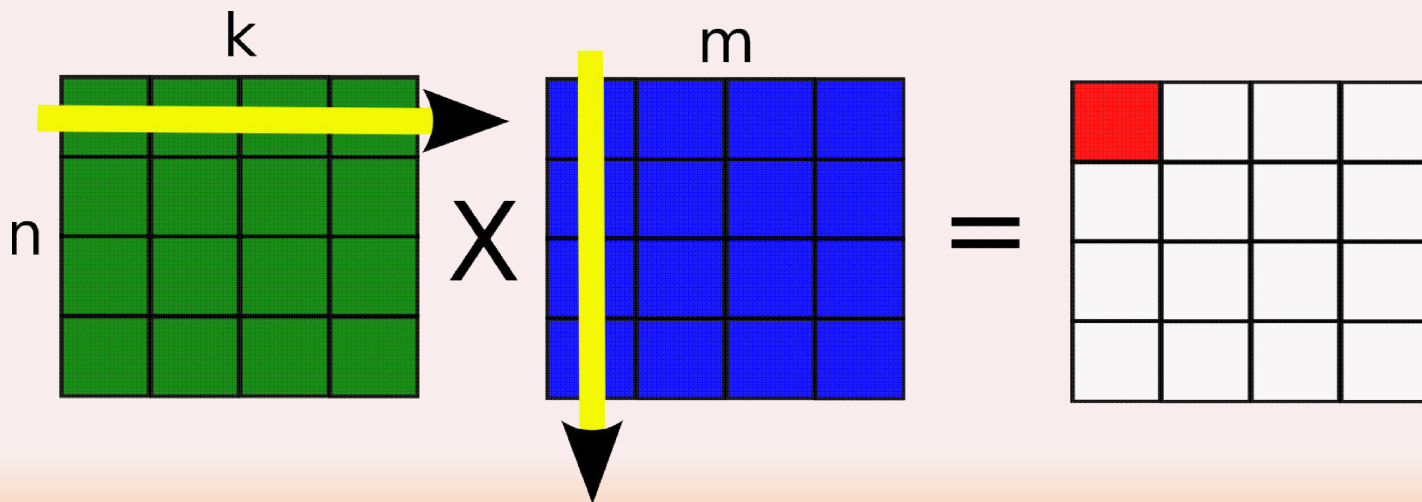
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Learning Objectives

- Multiplying two matrices.
- Relating it to past questions from WASSCE.

MULTIPLICATION / PRODUCT OF MATRIX

- Multiply the terms or elements in rows by the corresponding terms or elements in the column.
- Then calculate the sum of these multiplication



Matrix Algebra

Handwritten notes showing matrix multiplication and scalar multiplication. The first part shows the multiplication of two 2x2 matrices: $\begin{bmatrix} 1 & -3 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & -9 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 10 + (-3)(12) & (-9) + (-3)(-2) \\ 7(10) + 5(12) & (-9) + 5(-2) \end{bmatrix} = \begin{bmatrix} -26 & -15 \\ 110 & -19 \end{bmatrix}$. The second part shows scalar multiplication: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 2 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+2 \cdot 7 & 1+2 \cdot 8 \\ 3 \cdot 5 & 4 \cdot 4 \end{bmatrix} = \begin{bmatrix} 19 & 17 \\ 15 & 16 \end{bmatrix}$.

You need to be able to multiply a matrix by a number, as well as another matrix

$$A = \begin{bmatrix} 5 & 2 \\ -4 & 0 \end{bmatrix}$$

Calculate:

a) $2A$

b) $-3A$

a) $A = \begin{bmatrix} 5 & 2 \\ -4 & 0 \end{bmatrix}$

$$2A = \begin{bmatrix} 10 & 4 \\ -8 & 0 \end{bmatrix}$$

Just multiply each part by 2

b) $A = \begin{bmatrix} 5 & 2 \\ -4 & 0 \end{bmatrix}$

$$-3A = \begin{bmatrix} -15 & -6 \\ 12 & 0 \end{bmatrix}$$

Just multiply each part by -3

So to multiply a matrix by a number, you just multiply each part in the matrix separately

Matrix Algebra (2)

Handwritten examples of matrix operations:

$$\begin{bmatrix} 2 & -3 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & -9 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 2 \cdot 10 + (-3) \cdot 12 & 2 \cdot (-9) + (-3) \cdot (-2) \\ 7 \cdot 10 + 5 \cdot 12 & 7 \cdot (-9) + 5 \cdot (-2) \end{bmatrix} = \begin{bmatrix} -16 & 15 \\ 130 & -61 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 \cdot 5 + 2 \cdot 7 & 1 \cdot 6 + 2 \cdot 8 \\ 3 \cdot 5 + 4 \cdot 7 & 3 \cdot 6 + 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 35 & 46 \end{bmatrix}$$

You need to be able to multiply a matrix by a number, as well as another matrix

To multiply matrices together, multiply each ROW in the first, by each COLUMN in the second (like in the starter)

→ Remember for each row and column pair, you need to sum the answers!

a) Calculate the following

$$\begin{bmatrix} 2 & 5 & 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$$

→ Multiply each number in the row with the corresponding number in the column

$$(2 \times 4) + (5 \times 6) + (3 \times 1)$$

$$= 41$$

Show workings like these - it is essential to have a good routine in place when we move onto bigger Matrices!

Matrix Algebra (2)

Handwritten example of matrix multiplication:

$$\begin{bmatrix} 1 & -3 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} 10 & -4 \\ 12 & -2 \end{bmatrix} = \begin{bmatrix} 10 + (-3)(12) & (-4) + (-3)(-2) \\ 70 + 5(12) & (-2) + 5(-2) \end{bmatrix} = \begin{bmatrix} -26 & 2 \\ 130 & -12 \end{bmatrix}$$

You need to be able to multiply a matrix by a number, as well as another matrix

b) Calculate the following:

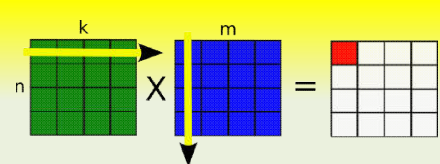
$$\begin{bmatrix} -3 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \\ 1 \\ 5 \end{bmatrix}$$

→ Multiply each number in the row with the corresponding number in the column

$$(-3 \times 4) + (0 \times -2) + (1 \times 1) + (2 \times 5) = -1$$

→ Remember for each row and column pair, you need to sum the answers!

Show workings like these - it is essential to have a good routine in place when we move onto bigger Matrices!



Matrix Algebra

Multiplying Matrices together

→ Matrices can only be multiplied if the number of columns in the first is the same as the number of rows in the second.

$$\begin{bmatrix} 6 & 5 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 8 \end{bmatrix} = [39]$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 5 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 6 & 1 & 5 & -4 \\ 2 & 3 & 7 & 3 \end{bmatrix} = \begin{bmatrix} 22 & 9 & 29 & -6 \\ 22 & 17 & 45 & 7 \\ 34 & 3 & 23 & -27 \end{bmatrix}$$

1 × 3 3 × 1 1 × 1

These numbers
have to be the
same!

These numbers give the
dimensions of the final matrix!

3 × 2 2 × 4 3 × 4

These numbers
have to be the
same!

These numbers give the
dimensions of the final matrix!

Example 1: Calculate the product of;

$$\begin{aligned} (2 \quad 3) \quad \begin{bmatrix} 5 \\ 6 \end{bmatrix} &= (2 \times 5 + 3 \times 6) \\ &= 10 + 18 \\ &= 28 \end{aligned}$$

Example 2: Find the value of x

$$(x^2 \quad 1) \begin{pmatrix} 3 \\ 9 \end{pmatrix} = 84$$

$$x^2 \times 3 + 1 \times 9 = 84$$

$$3x^2 + 9 = 84$$

$$3x^2 = 84 - 9$$

$$3x^2/3 = 75/3$$

$$x^2 = 25$$

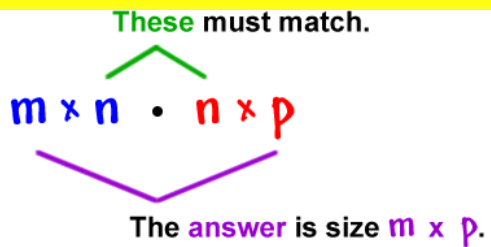
$$x = \pm 5$$

Example3: If $M = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$

Find the matrix product of MN

Solution: $MN = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$$\begin{pmatrix} 3 \times 2 + 2 \times 0 & 3 \times -1 + 2 \times 3 \\ 1 \times 2 + 4 \times 0 & 1 \times -1 + 4 \times 3 \end{pmatrix} = \begin{pmatrix} 6+0 & -3+6 \\ 2+0 & -1+12 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 2 & 1 \end{pmatrix}$$



Plenary

The values of x and y in these pairs of Matrices are the same.
Calculate what x and y must be!

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = [20] \qquad \begin{bmatrix} y & -2 \end{bmatrix} \begin{bmatrix} 2 \\ x \end{bmatrix} = [-24]$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = [20] \xrightarrow{\text{As an equation}} 5x + 3y = 20 \xrightarrow{\text{Multiply by 2}} 10x + 6y = 40$$

$$\begin{bmatrix} y & -2 \end{bmatrix} \begin{bmatrix} 2 \\ x \end{bmatrix} = [-24] \xrightarrow{\text{As an equation}} 2y - 2x = -24 \xrightarrow{\text{Multiply by 5}} 10y - 10x = -120$$

Add the two equations together

$$\begin{aligned} 16y &= -80 \\ y &= -5 \\ x &= 7 \end{aligned} \quad \begin{array}{l} \text{Divide by} \\ 16 \\ \text{Then find } x \end{array}$$

EVALUATION

WASSCE QUESTION ON MULTIPLICATION OF MATRIX

Given that $M = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ $N = \begin{pmatrix} m & x \\ n & y \end{pmatrix}$ and $MN = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$

Find the matrix N

ASSIGNMENT

1. If $A = \begin{pmatrix} 6 & 0 \\ -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ 3 & 5 \end{pmatrix}$ find BA

2. Find the product

$$\begin{pmatrix} 7 & 3 \\ 2 & 5 \\ 6 & 8 \\ 9 & 0 \end{pmatrix} \begin{pmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{pmatrix}$$