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CS 202

HW # 1

15/07/2020

Q1)

Insertion Sort) **Key: Key (Grey), Sorted elements (Black)**

Initial Array:

4	8	3	7	6	2	1	5

1st Swap:

4	8	3	7	6	2	1	5

2nd Swap:

3	4	8	7	6	2	1	5

3rd Swap:

3	4	7	8	6	2	1	5

4th Swap:

3	4	6	7	8	2	1	5

5th Swap:

2	3	4	6	7	8	1	5

6th Swap:

1	2	3	4	6	7	8	5

Sorted Array:

1	2	3	4	5	6	7	8

Selection Sort) **Key: Selected elements (Grey), Sorted elements (Black)**

Initial Array:

4	8	3	7	6	2	1	5

1st Swap:

4	5	3	7	6	2	1	8

2nd Swap:

4	5	3	1	6	2	7	8

3rd Swap:

4	5	3	1	2	6	7	8

4th Swap:

4	2	3	1	5	6	7	8

5th Swap:

1	2	3	4	5	6	7	8

6th Swap:

1	2	3	4	5	6	7	8

7th Swap:

1	2	3	4	5	6	7	8

Sorted Array:

1	2	3	4	5	6	7	8

Bubble Sort) **Key: Selected elements (Grey), Sorted elements (Black)**

Initial Array:

4	8	3	7	6	2	1	5

1st Pass:

4	8	3	7	6	2	1	5

4	8	3	7	6	2	1	5

4	3	8	7	6	2	1	5

4	3	7	8	6	2	1	5

4	3	7	6	8	2	1	5

4	3	7	6	2	8	1	5

4	3	7	6	2	1	8	5

4	3	7	6	2	1	5	8

2nd Pass:

4	3	7	6	2	1	5	8

3	4	7	6	2	1	5	8

3	4	7	6	2	1	5	8

3	4	6	7	2	1	5	8

3	4	6	2	7	1	5	8

3	4	6	2	1	7	5	8

3	4	6	2	1	5	7	8

3rd Pass:

3	4	6	2	1	5	7	8

3	4	6	2	1	5	7	8

3	4	6	2	1	5	7	8

3	4	2	6	1	5	7	8

3	4	2	1	6	5	7	8

3	4	2	1	5	6	7	8

4th Pass:

3	4	2	1	5	6	7	8

3	4	2	1	5	6	7	8

3	2	4	1	5	6	7	8

3	2	1	4	5	6	7	8

3	2	1	4	5	6	7	8

5th Pass:

3	2	1	4	5	6	7	8

2	3	1	4	5	6	7	8

2	1	3	4	5	6	7	8

2	1	3	4	5	6	7	8

6th Pass:

2	1	3	4	5	6	7	8

1	2	3	4	5	6	7	8

Sorted Array:

1	2	3	4	5	6	7	8

Merge Sort)

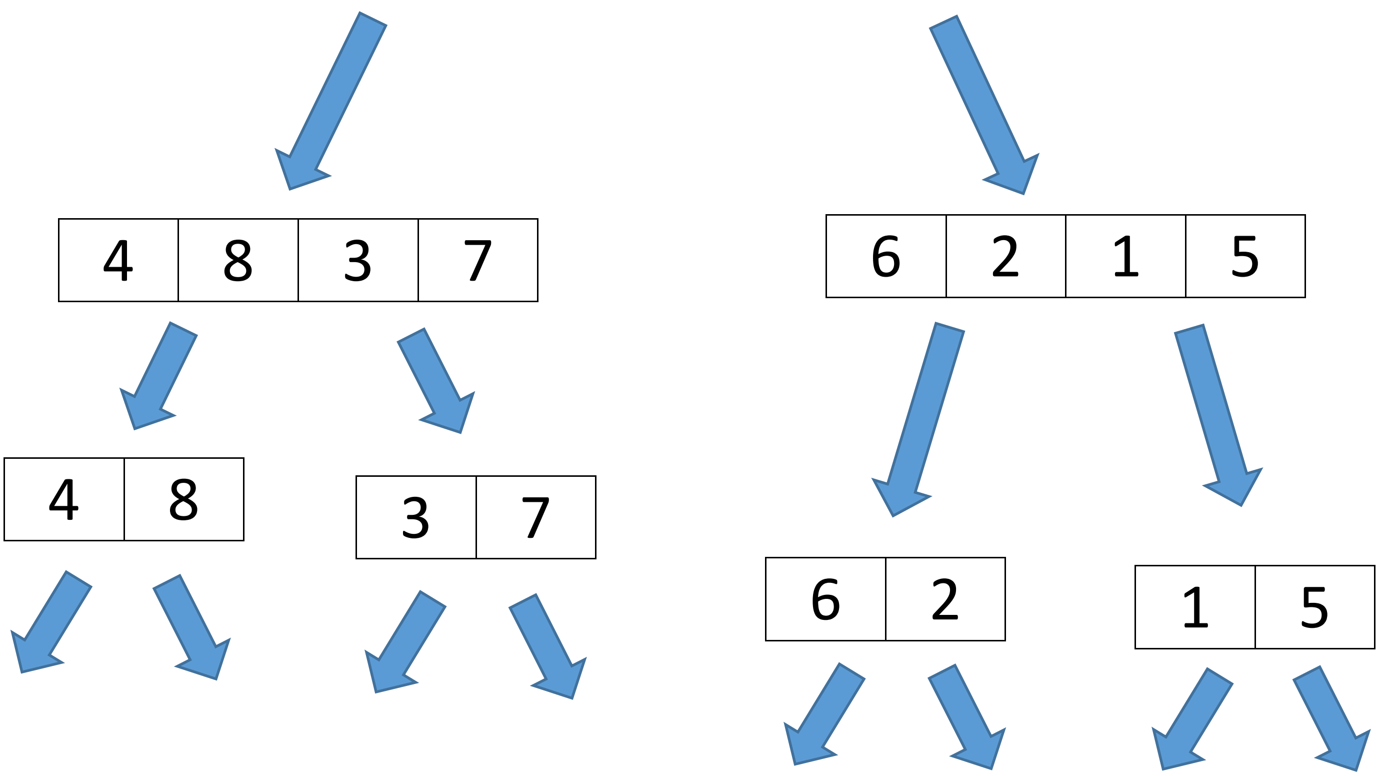
Mergesort calls:

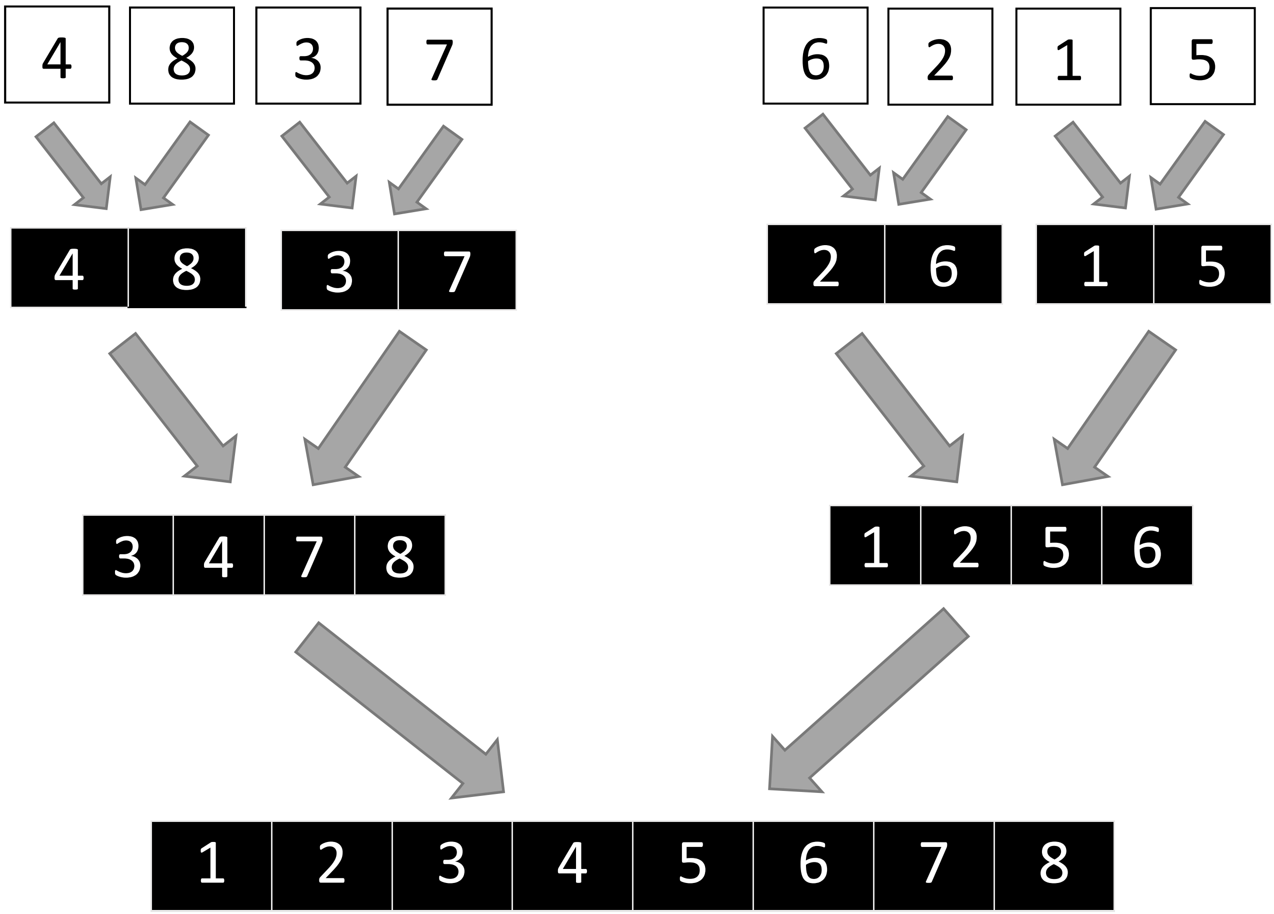
Merge calls:

Sorted: Black

Initial Array:

4	8	3	7	6	2	1	5





Quick Sort)

Key: Unknown (grey), Sorted (black)

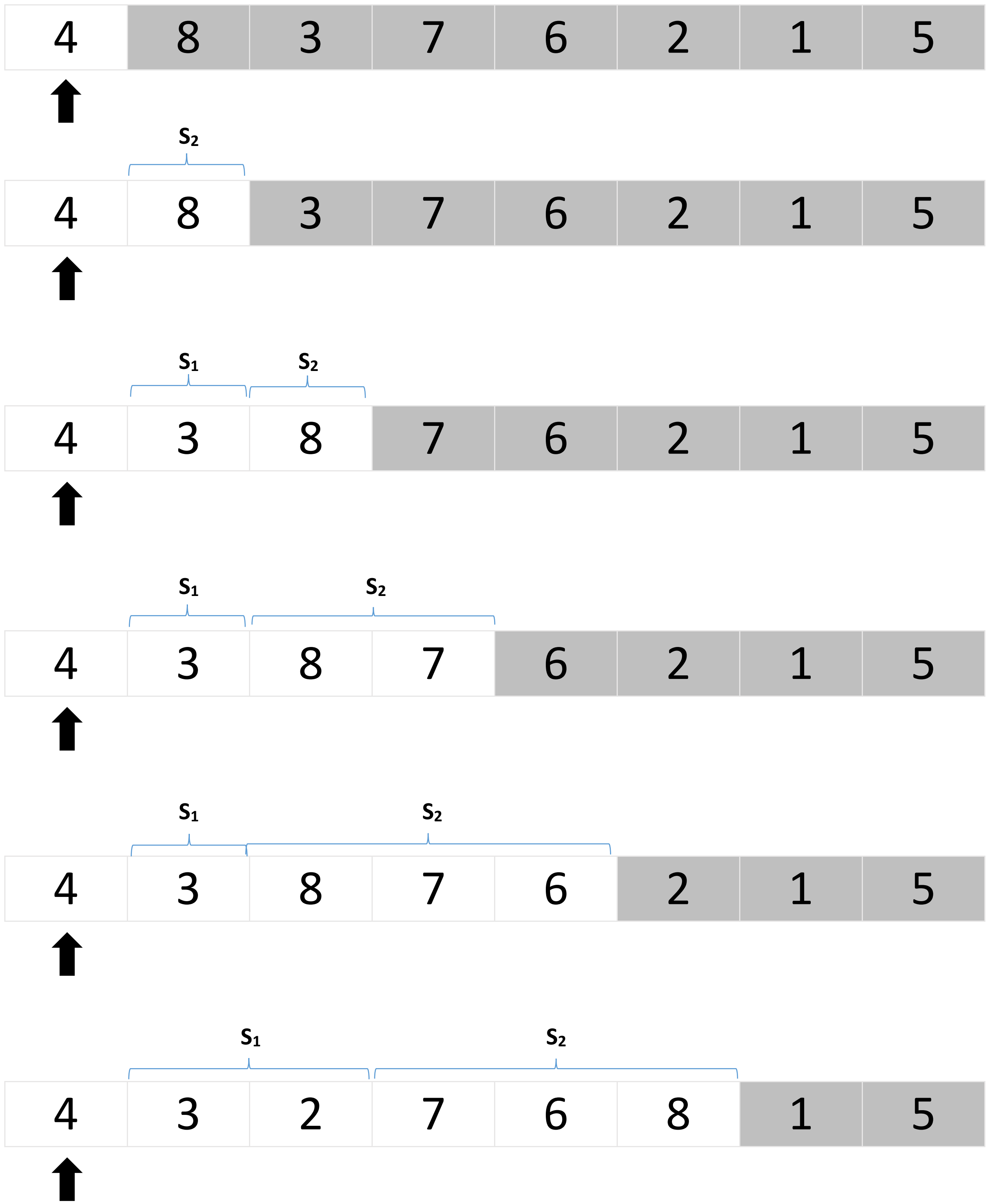
Quicksort calls:

Partition:

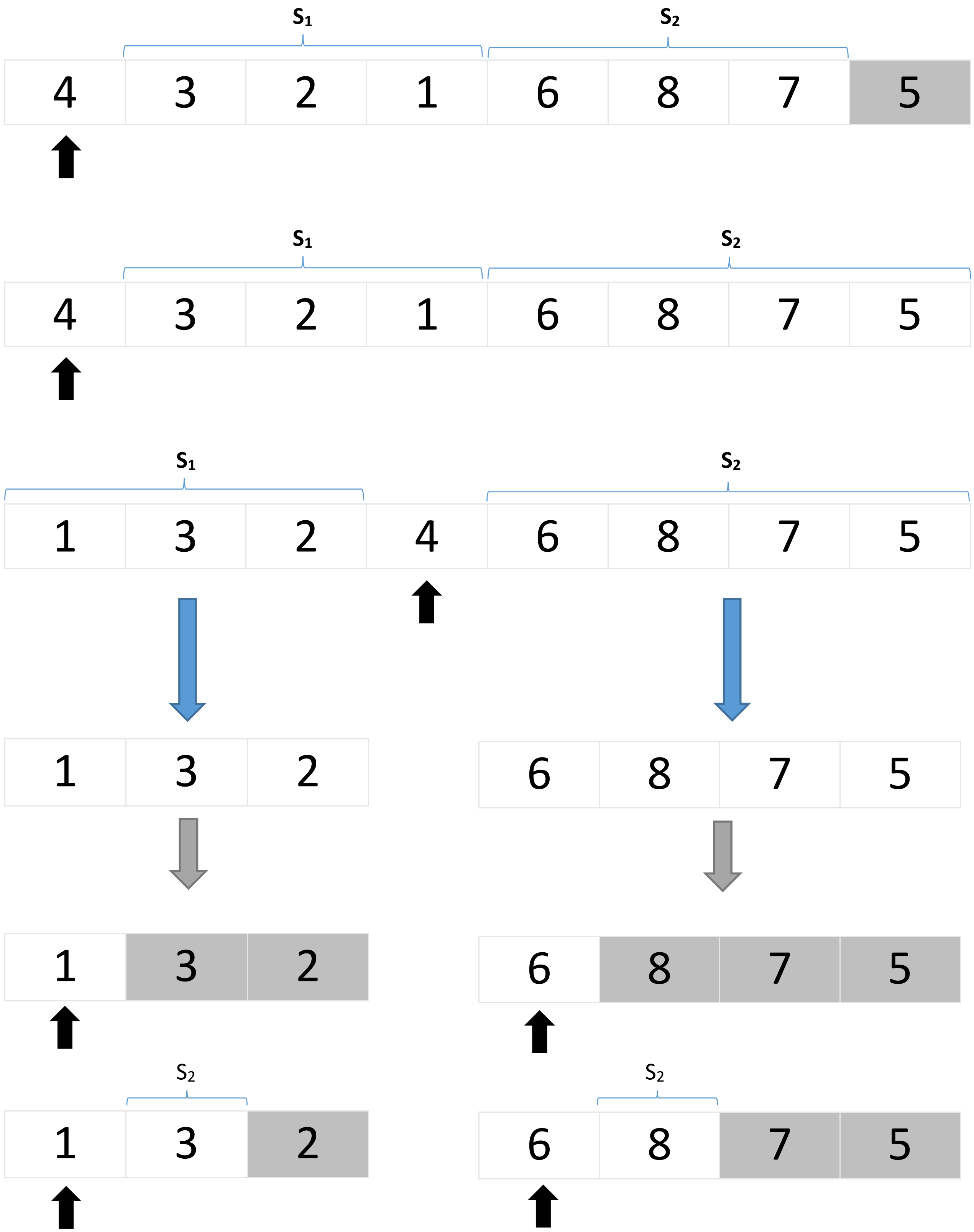
Pivot:

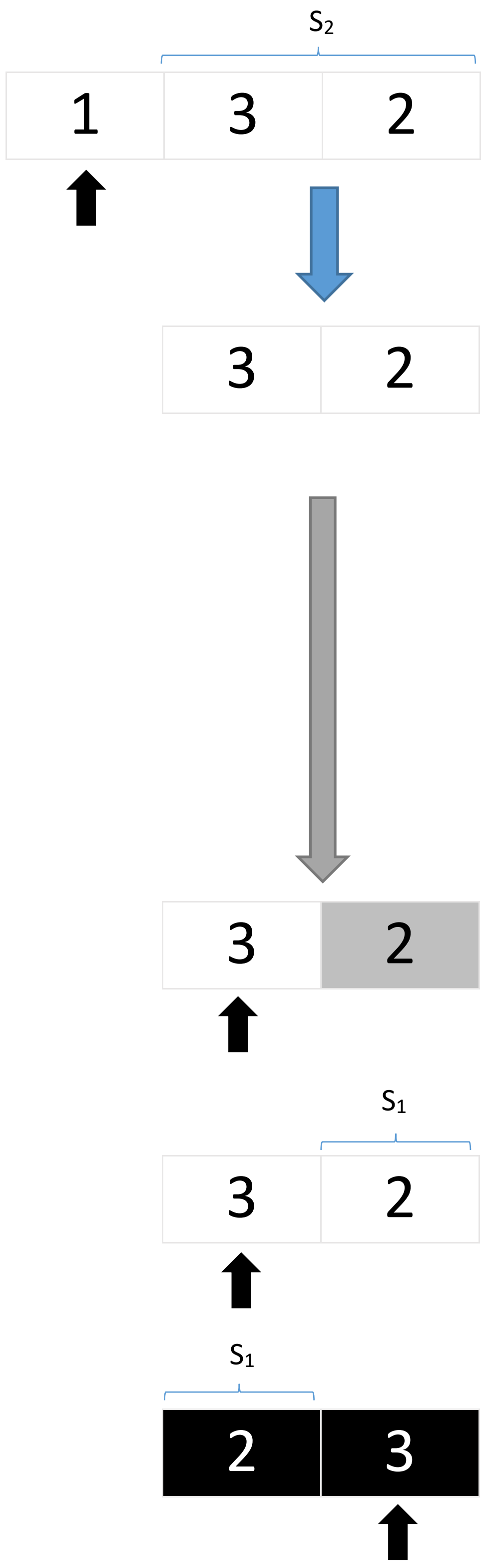
Initial Array:

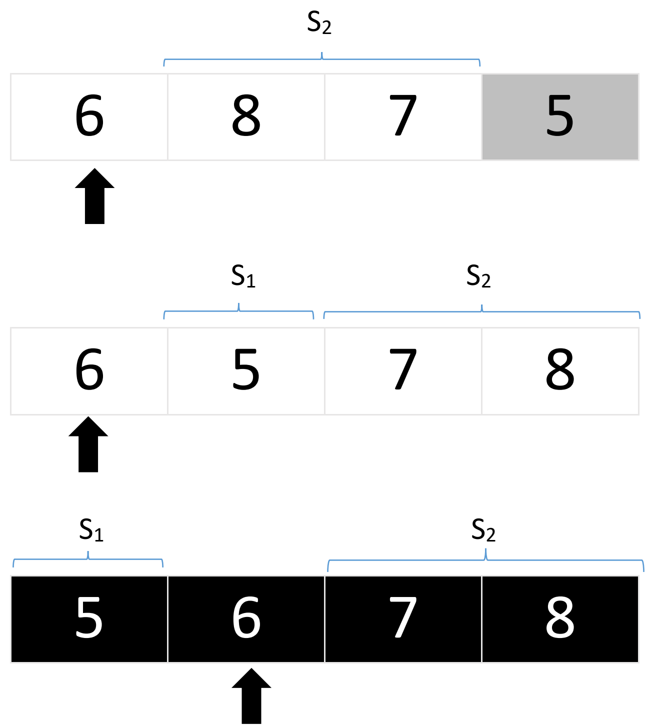
4	8	3	7	6	2	1	5



4 8 3 7 6 2 1 5
S2
4 8 3 7 6 2 1 5
S1 S2
4 3 8 7 6 2 1 5
S1 S2
4 3 8 7 6 2 1 5
S1 S2
4 3 8 7 6 2 1 5
S1 S2
4 3 2 7 6 8 1 5


S1
S2
4 3 2 1 6 8 7 5
S1
S2
4 3 2 1 6 8 7 5
S1
S2
1 3 2 4 6 8 7 5
1 3 2
6 8 7 5
1 3 2
6 8 7 5
S2
1 3 2
S2
6 8 7 5





More recursions to take place but the sub list has been sorted and therefore no use to show that.

Sorted Array:

1	2	3	4	5	6	7	8

Q2)

Merge Sort recurrence relation:

$$\left.\begin{array}{l} T(1) = O(1) \\ \\ T(n) = 2T(n/2) + O(n) \end{array}\right\} \text{Base Recurrence Equations}$$

We can write $O(n)$ as simply n and $O(1)$ simply as 1.

First, we find $T(n/2)$.

$T(n/2) = 2T(n/2^2) + n/2$ putting into base recurrence equation.

$T(n) = 2[2T(n/2^2) + n/2] + n$ simplifying the equation

$T(n) = 2^2T(n/2^2) + n + n$ we then find $T(n/2^2)$ and put into this equation

$T(n/2^2) = 2T(n/2^3) + n/2^2$

$T(n) = 2^2[2T(n/2^3) + n/2^2] +n + n$ simplifying again

$T(n) = 2^3T(n/2^3) + n + n + n$ we can see a pattern emerging, so we write it in that form

Assuming we do the substitution k times, we have

$T(n) = 2^kT(n/2^k) + kn$

Now, to get the base case, we equate $T(n/2^k) = T(1)$

$n/2^k = 1 \Rightarrow n = 2^k \Rightarrow k = \log_2 n$ using this value of k in the general equation

$T(n) = 2^{\log_2 n} * 1 + n\log_2 n \Rightarrow n + n\log_2 n$

Therefore, we have **$O(n\log_2 n)$** by solving the recurrence relation.

Quick Sort Recurrence Relation:

$$\left.\begin{aligned} &T(0) = T(1) = 0 \\ &T(n) = T(n - 1) + O(n) \end{aligned}\right\} \text{Base Recurrence Equations}$$

We can write O(n) simply as n

We find T(n - 1),

$T(n - 1) = T(n - 2) + n – 1$	Substituting into base equation
$T(n) = [T(n - 2) + n - 1] + n$	Simplifying
$T(n) = T(n - 2) + n + n – 1$	Finding T(n - 2)
$T(n - 2) = T(n - 3) + n – 2$	putting into equation of T(n)
$T(n) = [T(n - 3) + n - 2] + n + n - 1$	Simplifying
$T(n) = T(n - 3) + n + n + n - 1 – 2$	If we do this k times, we have the general form
$T(n) = T(n - k) + kn - \sum_{i=0}^{k-1} i$	Solving for base case, we have
$n – k = 0 \Rightarrow n = k$	Putting in general equation
$T(n) = 0 + n^2 – [n - 1(n – 1 + 1)/2]$	Simplifying

$T(n) = n^2 – (n^2 – n)/2 \Rightarrow T(n) = (n^2 + n) / 2$

This gives us **O(n^2)** for the worst case using the recurrence relation.

Q3)

Sample Output:

"C:\Users\ABDUL MANNAN\Desktop\CS 202\HW1\bin\Debug\HW1.exe"

```
-------------------insertion sort------------------------
Execution took 335 milliseconds.
compCount = 19999
moveCount = 19999
-------------------merge sort------------------------
Execution took 16 milliseconds.
compCount = 260835
moveCount = 574464
-------------------quick sort------------------------
Execution took 0 milliseconds.
compCount = 353662
moveCount = 525074

Process returned 0 (0x0)   execution time : 2.442 s
Press any key to continue.
```

Order	Size (N)	Insertion Sort			Merge Sort			Quick Sort		
		Time (s)	Comparisons	Moves	Time (s)	Comparisons	Moves	Time (s)	Comparisons	Moves
Random	20000	0.369	99919076	99939068	0.011	260835	574464	0.004	353662	525074
	22000	0.443	120819528	120841520	0.014	290045	638464	0.004	395300	591768
	24000	0.528	144090640	144114632	0.015	319233	702464	0.003	427413	647435
	26000	0.653	169302620	169328612	0.014	348960	766464	0.004	474481	735140
	28000	0.733	196315760	196343752	0.015	378604	830464	0.003	544823	795201
Descending	20000	0.762	199990000	200009999	0.009	139216	574464	0.900	199990000	300079996
	22000	0.948	241989000	242010999	0.011	154208	638464	1.097	241989000	363087996
	24000	1.052	287988000	288011999	0.012	170624	702464	1.296	287988000	432095996
	26000	1.278	337987000	338012999	0.012	186160	766464	1.518	337987000	507103996
	28000	1.496	391986000	392013999	0.014	202512	830464	1.797	391986000	588111996
Ascending	20000	0.00007	19999	19999	0.010	148016	574464	0.449	199990000	79996
	22000	0.00009	21999	21999	0.011	165024	638464	0.547	241989000	87996
	24000	0.00009	23999	23999	0.012	180608	702464	0.656	287988000	95996
	26000	0.00009	25999	25999	0.013	197072	766464	0.766	337987000	103996
	28000	0.00012	27999	27999	0.015	212720	830464	0.938	391986000	111996

Graphs for Insertion Sort:

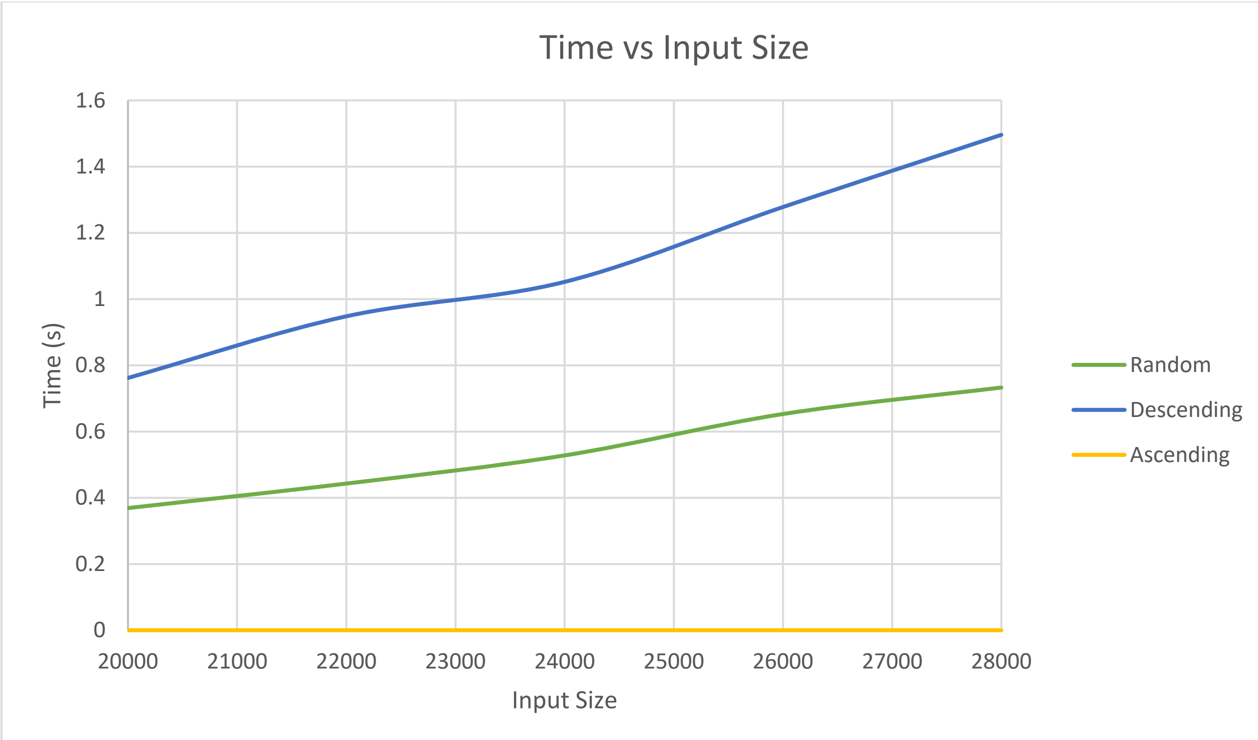


Figure 1: Insertion Sort 1

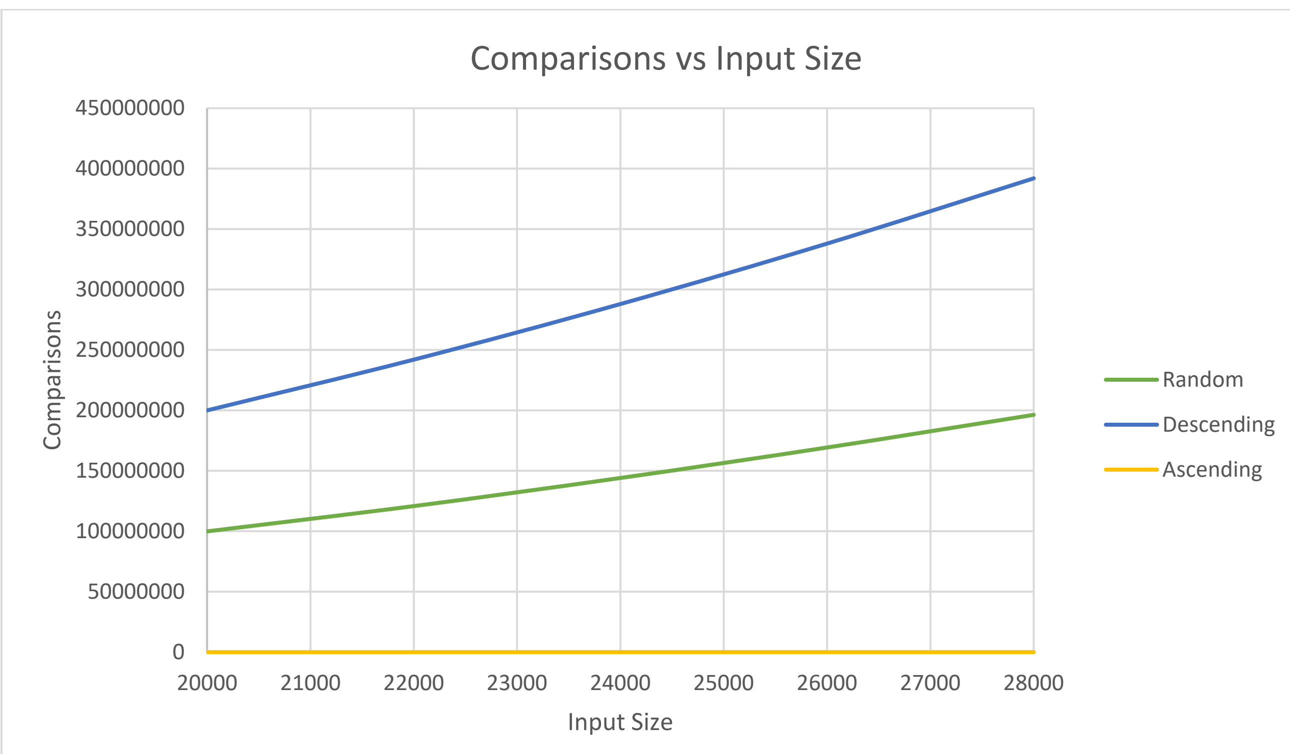


Figure 2: Insertion Sort 2

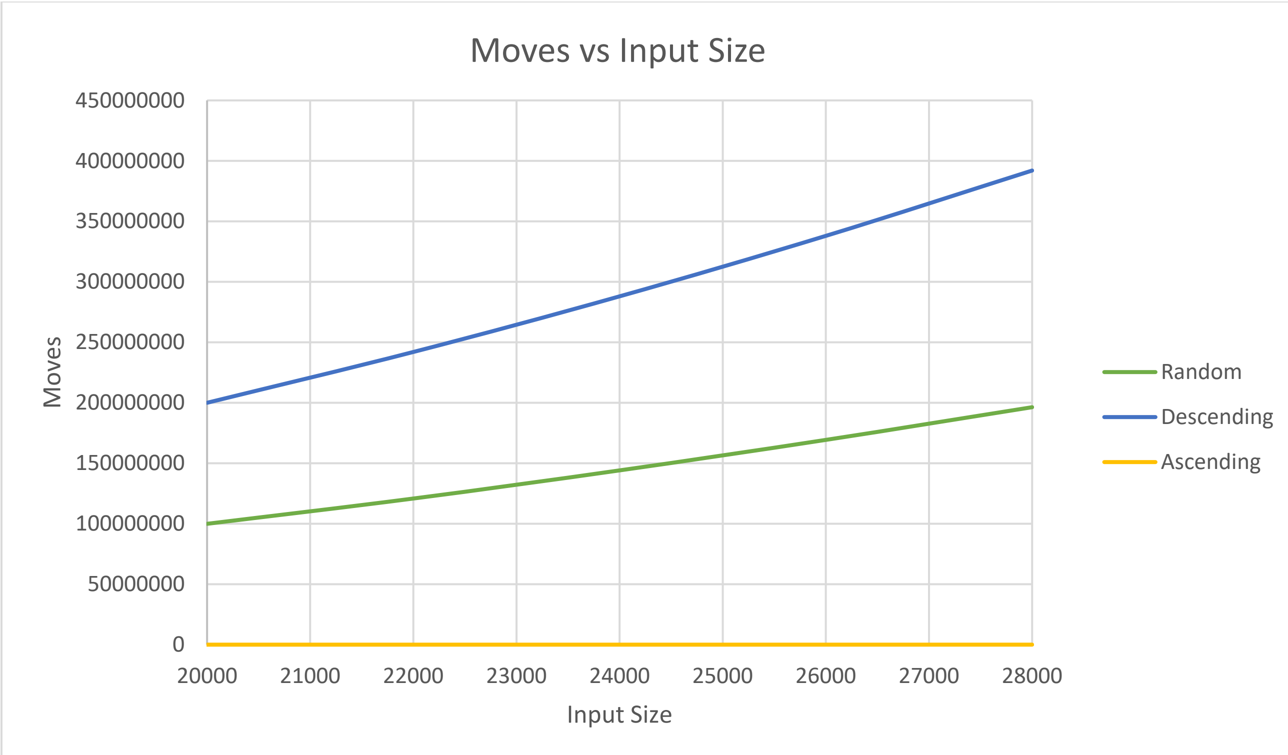


Figure 3: Insertion Sort 3

Graphs for MergeSort:

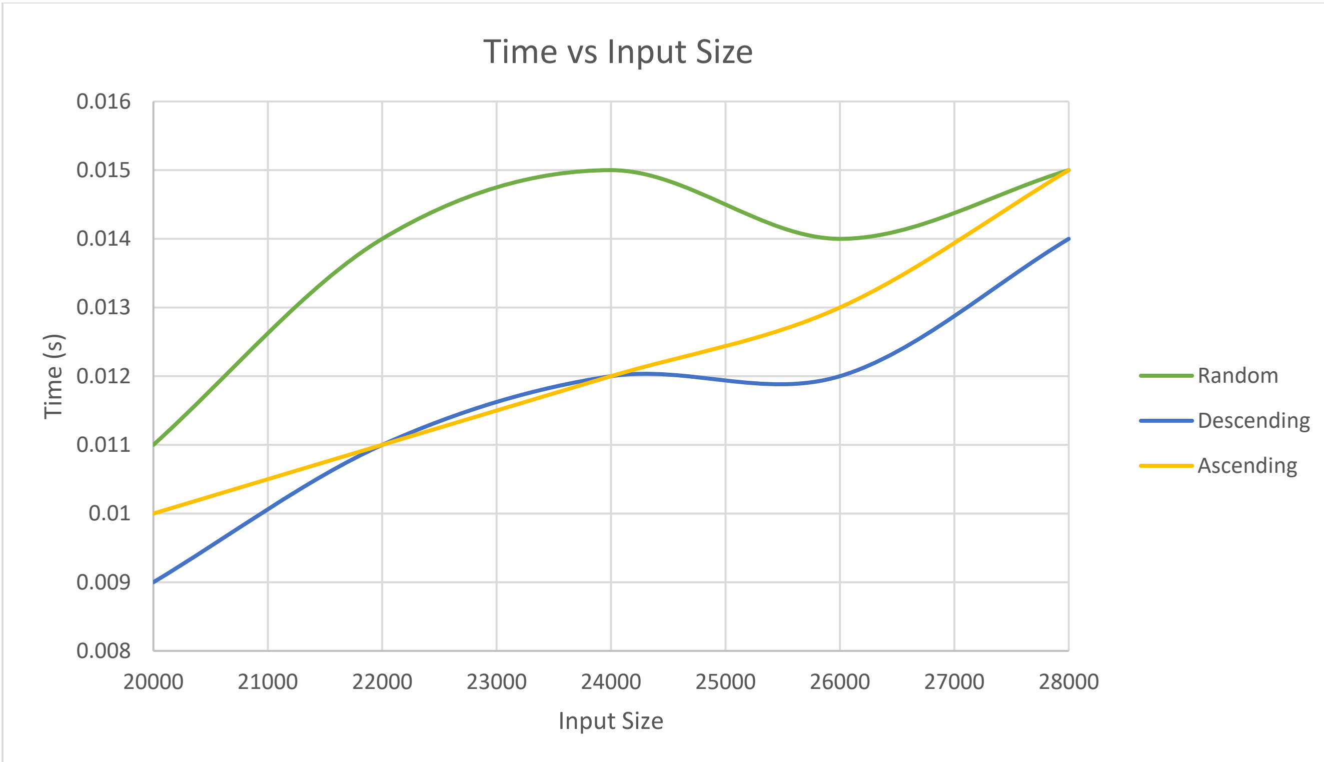


Figure 4: Merge Sort 1

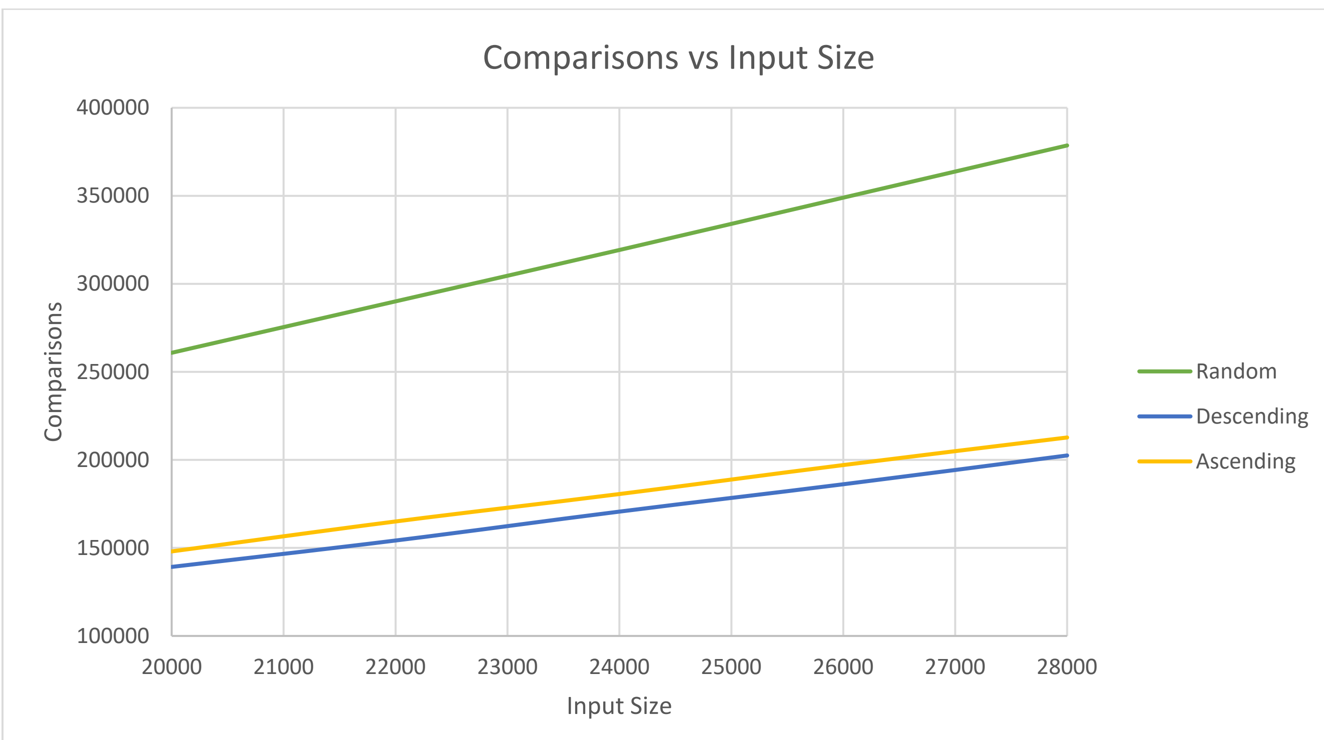


Figure 5: Merge Sort 2

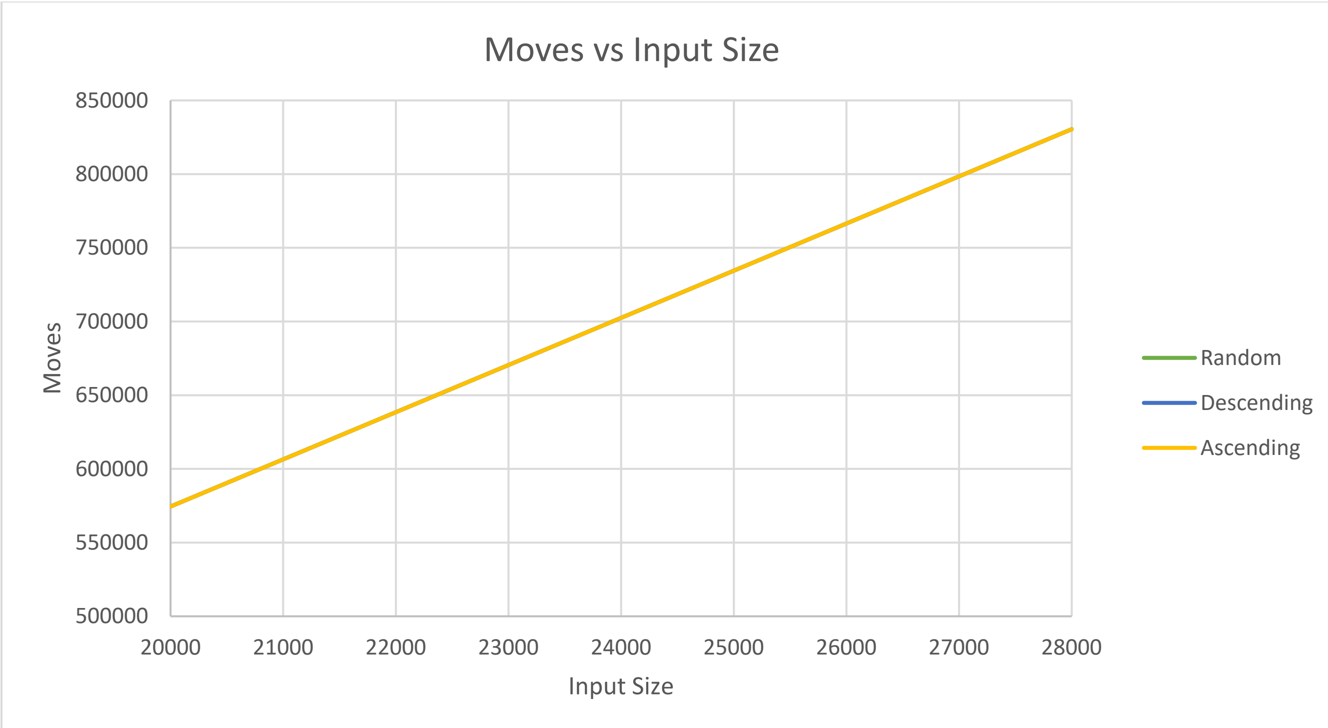


Figure 6: Merge Sort 3

Graphs for Quick Sort:

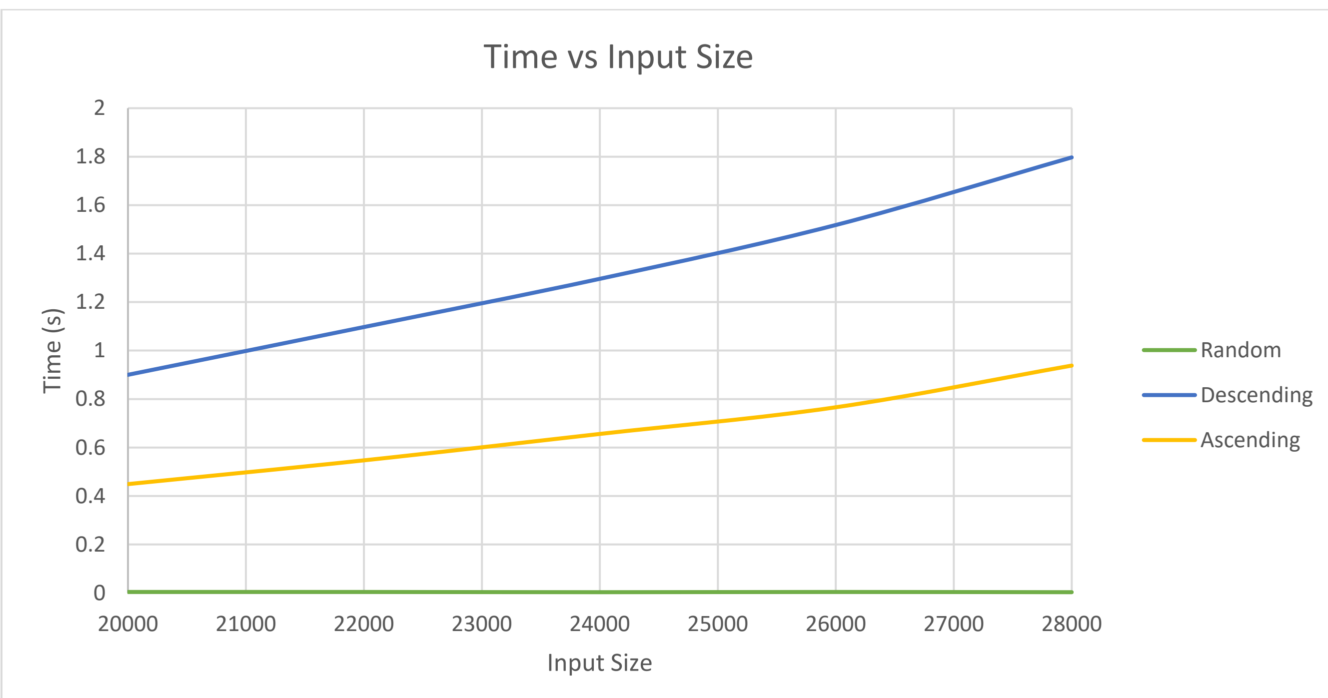


Figure 7: Quick Sort 1

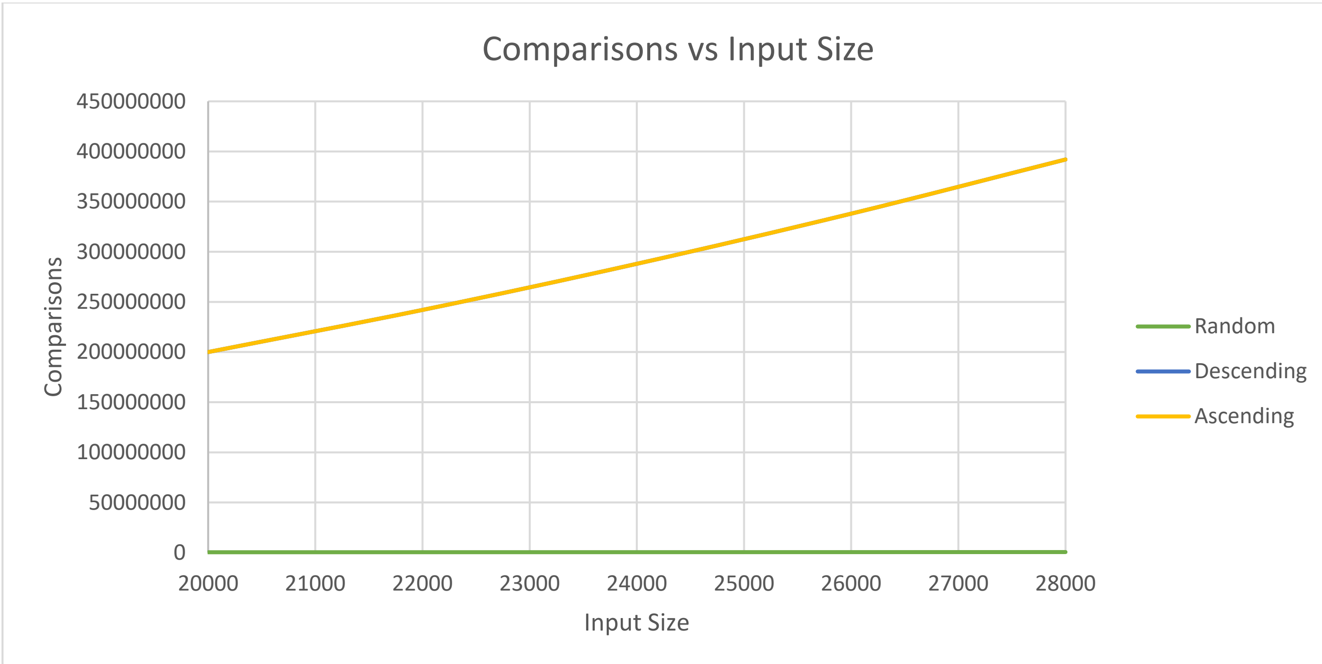
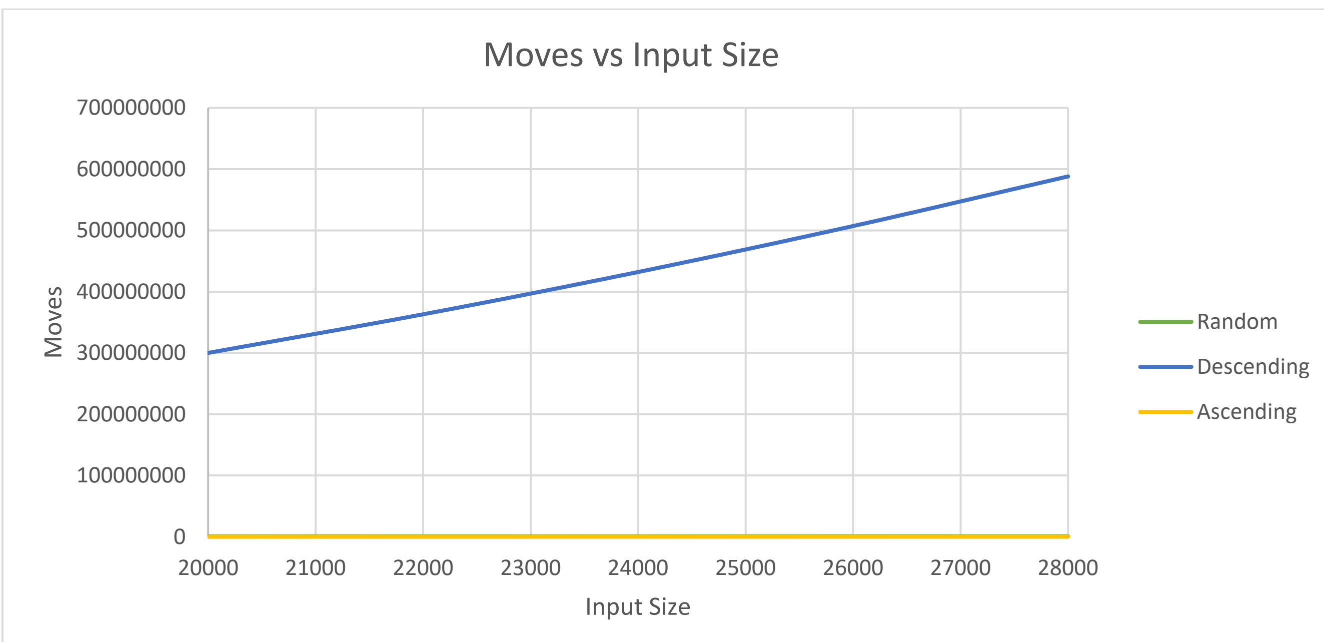


Figure 8: Quick Sort 2



Interpretation:

First, we examine the case of the insertion sort algorithm. The insertion sort algorithm performed best when our array was in an ascending order, in this case, we do not have to execute the inner loop of the algorithm and therefore we are saved a lot of moves and comparisons which results in us saving time. The best case growth rate for insertion sort is **O(n)**. The algorithm performed even worse when the array was in a descending order, which is the worst case scenario for this algorithm as the inner loop executes the maximum number of times. The worst rate growth rate of the insertion sort algorithm is **O(n^2)**. As we can see from the graph, when the array is in a random order, our insertion sort algorithm performs at somewhat of an average between the best and worst case. The average case for insertion sort is still **O(n^2)**. Obviously as the size of the array grew, so did the time taken, comparison and moves made grow. The graph shows a trend similar to that of a parabola which shows an exponential growth and but fails to show the trend of a linear growth rate for the best case because it is so little compared to the other two trends and therefore looks more like a straight line.

Next, we examine the case of the merge sort algorithm. The merge sort algorithm performed extremely well with respect to time in all 3 cases, whether the array was random, in ascending order, or descending order. This is because its worst and average case are both **O($n\log_2 n$)**. While the algorithm is slower than the insertion sort algorithm for the array in an ascending order. It is still better to use as it does better in cases that closely resemble real life, that is, we wouldn't want to sort an already sorted array again. One thing of note is that the number of moves remained the same across all 3 arrays, this is because merge sort first breaks down the arrays completely and then starts building them back up, which means that the number of comparisons may differ but not the number of moves since data has to be transferred to a temp array first and then back to the original. The graphs corroborate our theoretical expectations, that is, the time for all 3 algorithm types is similar. The number of moves are the same and lastly, the number of comparisons are less for the arrays in ascending and descending orders because if we have 2 sub lists, we only need to compare the first element of one sub list to all the elements of the other because of the distribution, all others would either be smaller or larger. As such, we would quickly get to the end of one sub list quicker and can then just move the data in the other sub list to the temp array. If the distribution is random, the comparisons increase because all data from both sub lists needs to be compared to each other before we reach the end of either and proceed to move all the data to the temp array.

Lastly, we examine the quick sort algorithm. The quick sort algorithm performs best when the array is random. This fact is also proven by our experimental results. It can perform even better if we try to apply some sort of algorithm to choose our pivot better. Its best and average case growth rate is **O($n\log_2 n$)**. The algorithm performs way worse in its worst case, when the array is already sorted and we choose the first element as our pivot. In this case its growth rate is **O(n^2)**. This is again proven by out graphs. This is because the number of comparisons increase dramatically. In a random array, the array is divided into smaller arrays for sorting, these smaller arrays are a lot smaller than the parent array because one has elements bigger than the pivot and one has elements smaller than the pivot, as such since these arrays are separate they do not have to make comparisons with each other but only within themselves, this reduces the comparisons by a large margin. In sorted arrays that have the first element as a pivot, the smaller array that is made has only one element less that the parent array, which was the pivot. In this smaller array, all the data is going to be compared with each other which makes the number of comparisons in this equal to the number of elements in an array – 1 factorial. This is why both the ascending and descending array have the same number of comparisons. The ascending array still performs faster because it saves time on not having to move elements behind the pivot.

Overall, the quick sort and merge sort are both some of the best algorithms to use while the insertion sort does okay where the number of elements is small.