

Q2: Think of ten (10) 3-variables functions and design circuits for them using sum of minterms technique.

Q. Think of ten (10) 3-variable functions and design a circuit for it using sum of min term technique.

For three variables, there are $2^{2^3} = 256$ possible outputs. One of them is given below.

A	B	C	F_1	min Term.
0	0	0	0	$m_0 = \bar{a}\bar{b}\bar{c}$
0	0	1	0	$m_1 = \bar{a}\bar{b}c$
0	1	0	1	$m_2 = \bar{a}b\bar{c}$
0	1	1	1	$m_3 = \bar{a}bc$
1	0	0	1	$m_4 = a\bar{b}\bar{c}$
1	0	1	0	$m_5 = a\bar{b}c$
1	1	0	1	$m_6 = ab\bar{c}$
1	1	1	0	$m_7 = abc$

From this truth table for F_1 we can write.

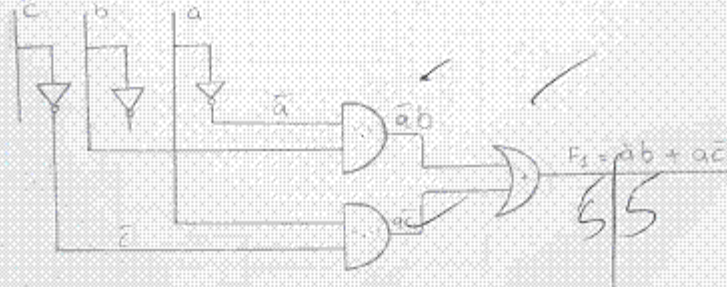
$$F_1(a, b, c) = \sum_m(2, 3, 4, 6)$$

$$= \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} \rightarrow \text{Can form}$$

$$= \bar{a}b(\bar{c} + c) + a\bar{c}(\bar{b} + b)$$

$$= \bar{a}b + a\bar{c} \rightarrow \text{Standard form}$$

Now the circuit for F_1 can be drawn as,



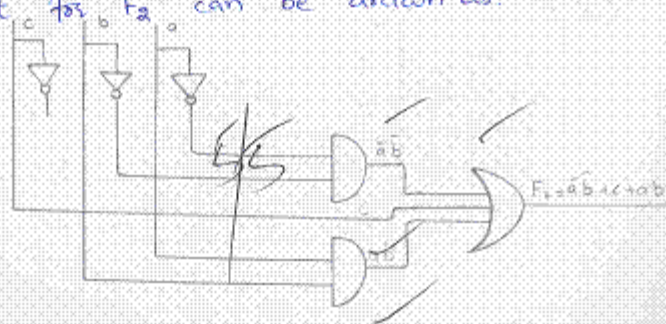
For any function F_2 :

A	B	C	F_2	Min Term
0	0	0	0	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	1	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	1	abc

The function can be written as,

$$\begin{aligned}
 F_2(a, b, c) &= \sum_m(0, 1, 3, 5, 6, 7) \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}\bar{c} + a\bar{b}c \\
 &= \bar{a}\bar{b}(\bar{c} + c) + c(\bar{a}b + a\bar{b}) + ab(\bar{c} + c) \\
 &= \bar{a}\bar{b} + \bar{a}bc + a\bar{b}c + ab \\
 &= \bar{a}(\bar{b} + bc) + a(\bar{b}c + b) \\
 &= \bar{a}(\bar{b} + c) + a(\bar{b}c + b) \\
 &= \bar{a}\bar{b} + \bar{a}c + ac + ab \\
 &= \bar{a}\bar{b} + c(\bar{a} + a) + ab \\
 &= \bar{a}\bar{b} + c + ab \rightarrow \text{standard form}
 \end{aligned}$$

The circuit for F_2 can be drawn as:



For F_3 the truth table is,

A	B	C	F_3	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	0	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	1	abc

function can be written as,

$$F_3(a, b, c) = \sum_m(0, 1, 2, 6, 7)$$

$$= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + ab\bar{c} + abc$$

$$= \bar{a}\bar{b}(\bar{c} + c) + b\bar{c}(\bar{a} + a) + abc$$

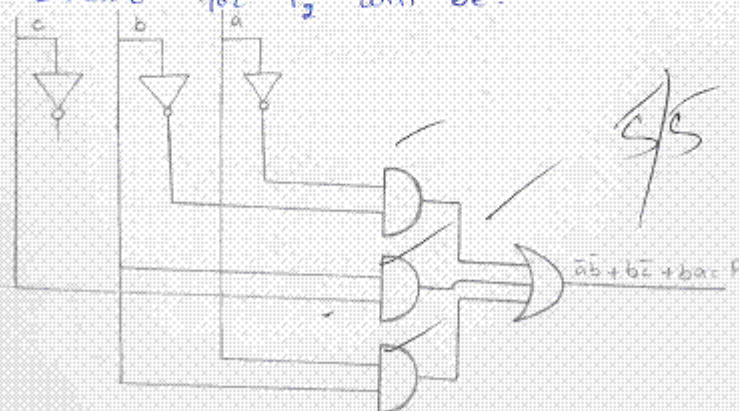
$$= \bar{a}\bar{b} + b\bar{c} + abc$$

$$= \bar{a}\bar{b} + b(\bar{c} + ac)$$

$$= \bar{a}\bar{b} + b(\bar{c} + a)$$

$$= \bar{a}\bar{b} + b\bar{c} + ba$$

The circuit for F_3 will be:



Now for F_4 ;

A	B	C	F_4	in Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	0	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	0	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	0	abc

The function can be written as,

$$\begin{aligned}
 F_4(a, b, c) &= \sum_m(0, 2, 4, 6) \\
 &= \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + ab\bar{c} \\
 &= \bar{a}\bar{c}(\bar{b} + b) + a\bar{c}(\bar{b} + b) \\
 &= \bar{a}\bar{c} + a\bar{c} \\
 &= \bar{c}(\bar{a} + a) \\
 &= \bar{c}
 \end{aligned}$$

Circuit can be drawn as,



For any function F_5 , let the truth table is,

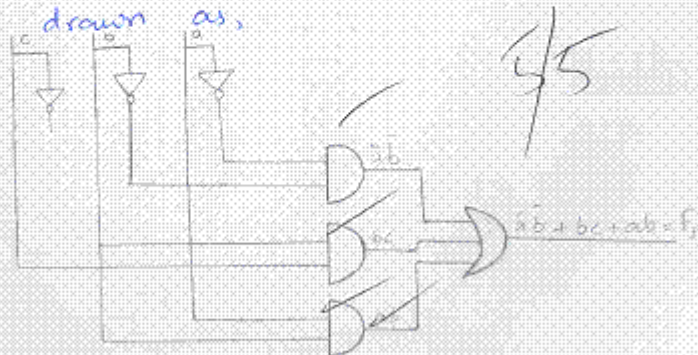
A	B	C	F_5	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	1	$ab\bar{c}$
1	1	1	1	abc

The function can be written as,

$$F_5(a,b,c) = \sum_m(0,1,2,3,7)$$

$$\begin{aligned} F_5 &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + ab\bar{c} + abc \\ &= \bar{a}\bar{b}(\bar{c}+c) + bc(\bar{a}+a) + ab(\bar{c}+c) \\ &= \bar{a}\bar{b} + bc + ab \end{aligned}$$

Now looking at the standard form, the circuit can be drawn as,



for any out put F_c , truth table.

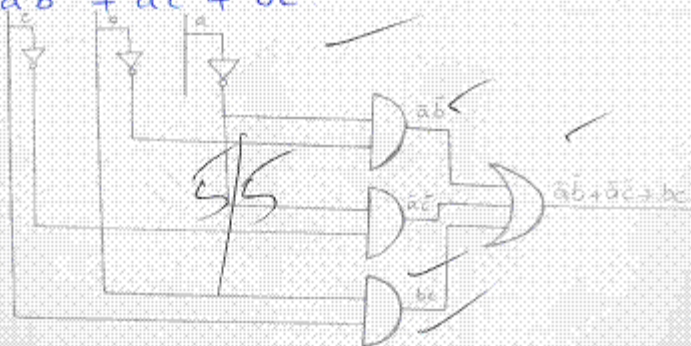
A	B	C	F_c	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	0	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	1	abc

The function can be written as,

$$F_c(a, b, c) = \sum_m(0, 1, 2, 3, 7)$$

$$\begin{aligned}
 F_c &= \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + abc \\
 &= \bar{a}\bar{b}(\bar{c} + c) + \bar{a}b\bar{c} + bc(\bar{a} + a) \\
 &= \bar{a}\bar{b} + \bar{a}b\bar{c} + bc \\
 &= \bar{a}(\bar{b} + b\bar{c}) + bc \\
 &= \bar{a}(\bar{b} + \bar{c}) + bc \\
 &= \bar{a}\bar{b} + \bar{a}\bar{c} + bc
 \end{aligned}$$

Circuit:



for any F_7 , the truth table,

A	B	C	F_7	Min Terms
0	0	0 ✓	0 ✓	$\bar{a} \bar{b} \bar{c}$
0	0	1	1 ✓	$\bar{a} \bar{b} c$
0	1	0 ✓	1 ✓	$\bar{a} b \bar{c}$
0	1	1	1 ✓	$\bar{a} b c$
1	0	0 ✓	0 ✓	$a \bar{b} \bar{c}$
1	0	1	1 ✓	$a \bar{b} c$
1	1	0 ✓	0 ✓	$ab \bar{c}$
1	1	1	0 ✓	abc

function can be written as,

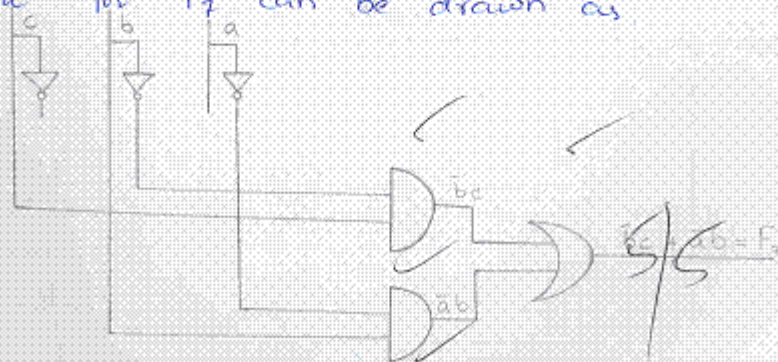
$$F_7(a, b, c) = \sum_m(0, 2, 3, 5) ✓$$

$$F_7 = \bar{a} \bar{b} c + \bar{a} b \bar{c} + \bar{a} b c + a \bar{b} c ✓$$

$$= \bar{b} c (\bar{a} + a) + \bar{a} b (\bar{c} + c) ✓$$

$$= \bar{b} c + \bar{a} b ✓$$

Circuit for F_7 can be drawn as



For any function F_g ,

A	B	C	F_g	min Terms
0	0	0	0	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	1	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	0	$a\bar{b}\bar{c}$
1	0	1	1	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	0	abc

function can be written as,

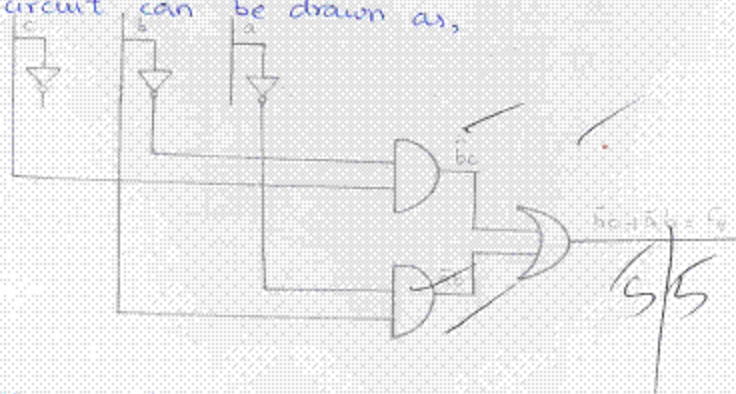
$$F_g(a, b, c) = \sum_m(1, 2, 3, 5)$$

$$F_g = \bar{a}\bar{b}c + \bar{a}b\bar{c} + \bar{a}bc + a\bar{b}c$$

$$= \bar{b}c(\bar{a} + a) + \bar{a}b(\bar{c} + c)$$

$$= \bar{b}c + \bar{a}b$$

The circuit can be drawn as,



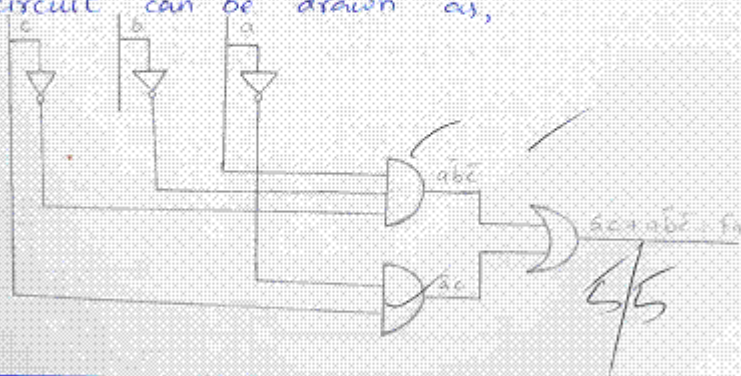
Let F_a be any output for which the truth table is,

A	B	C	F_a	min Terms
0	0	0 ✓	0 ✓	$\bar{a}\bar{b}\bar{c}$
0	0	1 ✓	1 ✓	$\bar{a}\bar{b}c$
0	1	0 ✓	0 ✓	$\bar{a}b\bar{c}$
0	1	1 ✓	1 ✓	$\bar{a}bc$
1	0	0 ✓	1 ✓	$a\bar{b}\bar{c}$
1	0	1 ✓	0 ✓	$a\bar{b}c$
1	1	0 ✓	0 ✓	$ab\bar{c}$
1	1	1 ✓	0 ✓	abc

function can be written as, $f_a(a,b,c) = \sum_m(1,3,4)$

$$\begin{aligned}
 F_a &= \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} \\
 &= \bar{a}c(\bar{b}+b) + a\bar{b}\bar{c} \\
 &= \bar{a}c + a\bar{b}\bar{c}
 \end{aligned}$$

The circuit can be drawn as,



Let F_{10} be any output for which the function truth table is,

A	B	C	F_{10}	Min Terms
0	0	0	1	$\bar{a}\bar{b}\bar{c}$
0	0	1	1	$\bar{a}\bar{b}c$
0	1	0	0	$\bar{a}b\bar{c}$
0	1	1	1	$\bar{a}bc$
1	0	0	1	$a\bar{b}\bar{c}$
1	0	1	1	$a\bar{b}c$
1	1	0	0	$ab\bar{c}$
1	1	1	0	abc

The function can be written as,

$$F_{10}(a, b, c) = \sum_m(0, 1, 3, 4, 5)$$

$$F_{10} = \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + a\bar{b}c$$

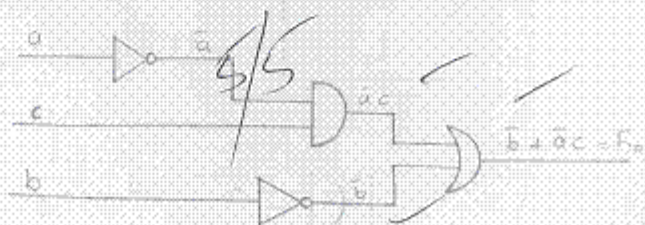
$$= \bar{a}\bar{b}(\bar{c} + c) + \bar{a}b\bar{c} + a\bar{b}(\bar{c} + c)$$

$$= \bar{a}\bar{b} + \bar{a}b\bar{c} + a\bar{b} = \bar{a}(\bar{b} + b\bar{c}) + a\bar{b}$$

$$= \bar{a}(\bar{b} + c) + a\bar{b} = \bar{a}\bar{b} + \bar{a}c + a\bar{b}$$

$$= \bar{b}(\bar{a} + a) + \bar{a}c = \bar{b} + \bar{a}c$$

Circuit:



Q3: Think of five (05) 4-variables functions and design circuits for them using product of maxterms technique.

BOOLEAN ALGEBRA and LOGIC GATES

Pg: 1

Q.No.1: Think of 5, 4-variables functions and design circuits for them using product of Sum (POS) technique.

Ans: As we have 4-variables, so by formula 2^n the total combinations are $2^4 = 16$.

Let F_1, F_2, F_3, F_4 and F_5 are the desired functions.

Now the truth table will be as:

Where A, B, C and D are the required inputs.

A	B	C	D	F_1	F_2	F_3	F_4	F_5	Max Terms
0	0	0	0	0	0	0	1	0	$A+B+C+D$
0	0	0	1	0	0	1	1	0	$A+B+C+\bar{D}$
0	0	1	0	1	0	1	1	1	$A+B+\bar{C}+D$
0	0	1	1	1	1	1	1	1	$A+B+\bar{C}+\bar{D}$
0	1	0	0	1	1	1	1	1	$A+\bar{B}+C+D$
0	1	0	1	0	0	1	1	1	$A+\bar{B}+C+\bar{D}$
0	1	1	0	0	1	0	0	1	$A+\bar{B}+\bar{C}+D$
0	1	1	1	1	1	0	0	1	$A+\bar{B}+\bar{C}+\bar{D}$
1	0	0	0	1	1	0	1	1	$\bar{A}+B+C+D$
1	0	0	1	0	0	1	0	0	$\bar{A}+B+C+\bar{D}$
1	0	1	0	1	1	1	1	0	$\bar{A}+B+\bar{C}+D$
1	0	1	1	1	1	1	1	0	$\bar{A}+B+\bar{C}+\bar{D}$
1	1	0	0	1	1	1	1	1	$\bar{A}+\bar{B}+C+D$
1	1	0	1	1	1	0	0	1	$\bar{A}+\bar{B}+C+\bar{D}$
1	1	1	0	1	1	0	0	1	$\bar{A}+\bar{B}+\bar{C}+D$
1	1	1	1	1	1	1	1	1	$\bar{A}+\bar{B}+\bar{C}+\bar{D}$

Now we take the product of these minterms whose output is zero as,

BOOLEAN EXPRESSIONS:

$$F_1 = (A+B+C+D)(A+B+C+\bar{D})(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D)(\bar{A}+B+C+\bar{D})$$

$$F_2 = (A+B+C+D)(A+\bar{B}+C+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})$$

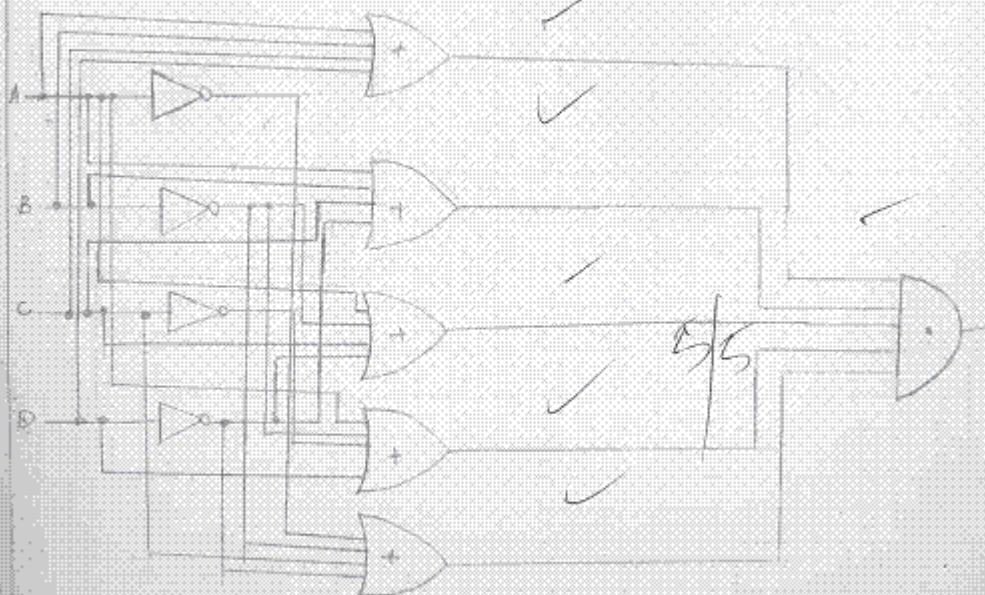
$$F_3 = (A+B+C+D)(A+\bar{B}+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)$$

$$F_4 = (A+\bar{B}+\bar{C}+D)(A+\bar{B}+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})(\bar{A}+\bar{B}+\bar{C}+D)$$

$$F_5 = (A+B+C+D)(A+B+C+\bar{D})(\bar{A}+B+C+\bar{D})(\bar{A}+B+C+D)(\bar{A}+B+\bar{C}+D)(\bar{A}+B+\bar{C}+\bar{D})$$

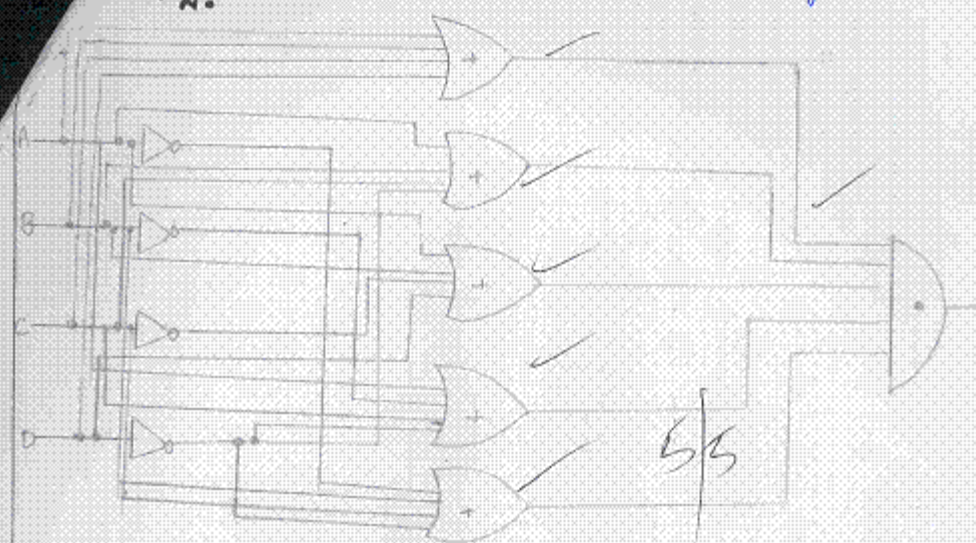
Circuits :

F₁:



F₂:

Pg:



F₃:

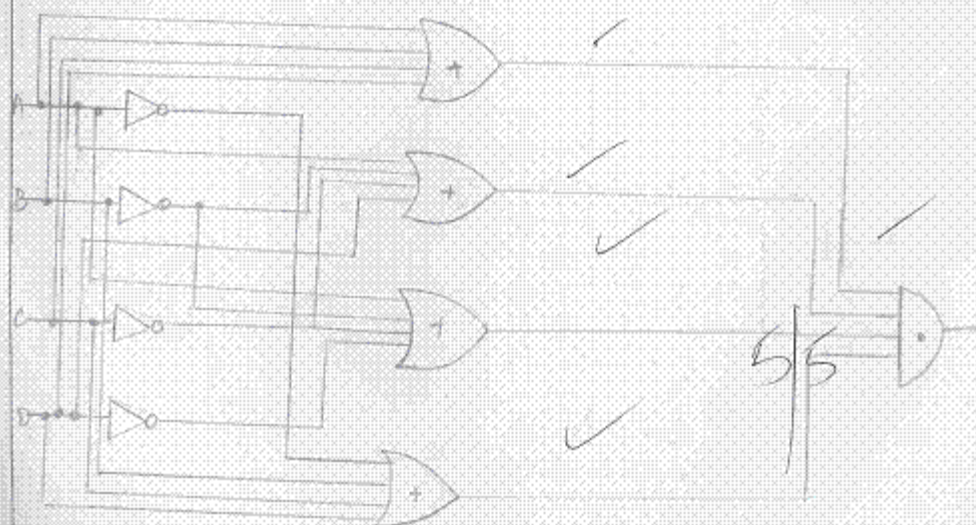
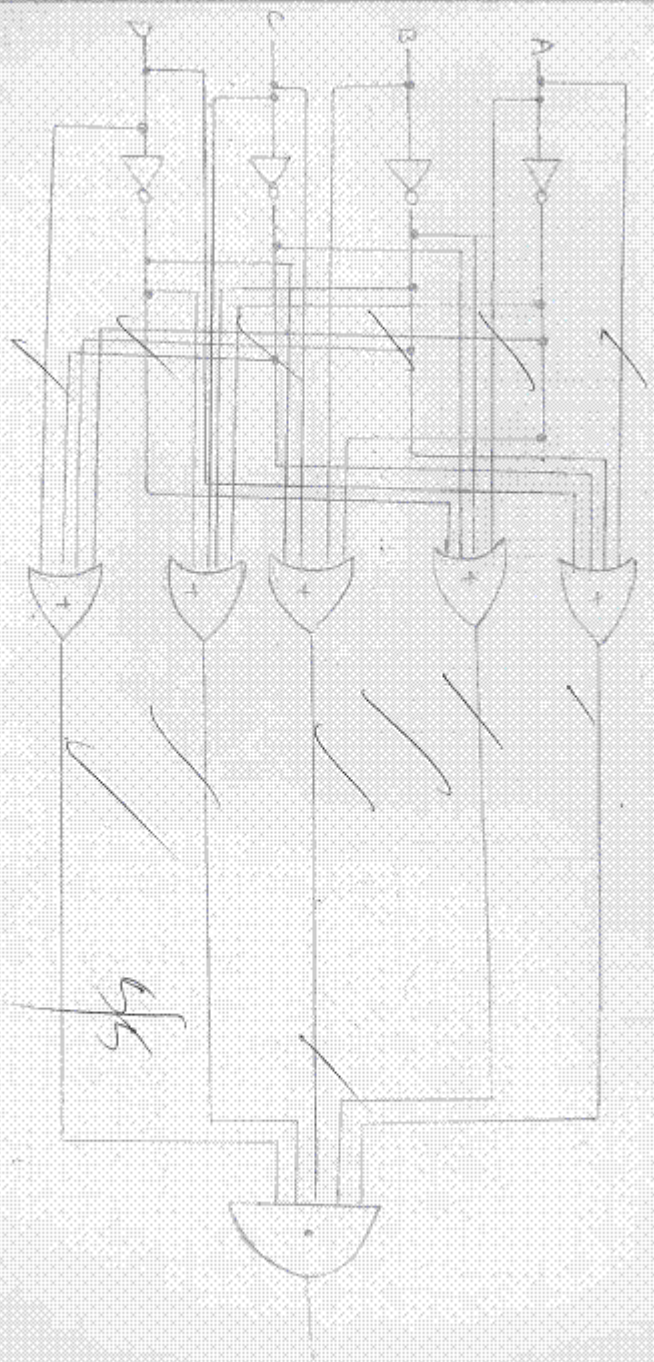


Fig. 2

Fig. 2



FS:

