(1)

Ex: 1.1 8th Edition 1

# Linear Algebra Bernard Kolman

Q1 $x+2y=8$ —— (1)

$3x-4y=4$ —— (2)

xing equation (1) by 3 and subtring from (2)

$$\begin{array}{r} 3x+6y=24 \\ \underline{\cancel{3x}\mp 4y=\underline{-}4} \\ 10y=20 \Rightarrow \boxed{y=2} \end{array}$$

eq (1) $\Rightarrow x+2(2)=8 \Rightarrow x=8-4=4 \Rightarrow \boxed{x=4}$

Hence $(x,y)=(4,2)$ which is unique solution.

Q3 $3x+2y+z=2$ —— (i)

$4x+2y+2z=8$ —— (ii)

$x-y+z=4$ —— (iii)

Solving eq (i) & (iii)

xing eq (iii) by 3 then subtring from (i)

$$\begin{array}{r} \cancel{3x}-3y+3z=12 \\ \underline{\underline{-}3x\underline{+}2y\underline{+}z=\underline{-}2} \\ -5y+2z=10 \end{array}$$ —— (iv).

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Now solving Equation (iii) & (ii)

xing Equation (iii) by 4 then subtracting from (ii)

$$4x-4y+4z=16$$
$$\underline{\overset{-}{4x}\ \overset{-}{+2y}\ \overset{-}{+2z}=\overset{-}{8}}$$

$$-6y+2z=8 \text{ ——— (iv)}$$

Now solving eq (iv) & (v).

(iv) − (v)

$$-5y+2z=10$$
$$\underline{\overset{+}{-6y}\ \overset{-}{+2z}=\overset{-}{8}}$$

$$\boxed{y=2}$$

eq (v) ⟹ $-5(2)+2z=10$

$$-10+2z=10$$

$$2z=10+10 \implies \cancel{2}z=\overset{10}{\cancel{20}} = \boxed{z=10}$$

Putting the value of y & z in eq ①

$$3x+2(2)+10=2$$

$$3x=2-14=-12$$

$$\frac{\cancel{3}x}{\cancel{3}}=\frac{-\cancel{12}^{4}}{\cancel{3}} = \boxed{x=-4}$$

Hence $(x,y,z)=(-4,2,10)$ which is unique solution.



Q.7 $x+4y-z=12$ ——— (i)

$3x+8y-2z=4$ ——— (ii)

xing Eqn (i) by 3 then sub. from (ii).

$$3x+12y-3z=36$$
$$\underline{\overset{-}{3x}\ \overset{-}{+8y}\ \overset{+}{-2z}=\overset{-}{4}}$$

$$4y-z=32$$

$$\implies 4y=32+z \implies y=8+z/4 \text{ ——— (iii)}$$

when z is any real no then $z=r$.

eq (iii) ⟹ $y=8+r/4$.

eq (i) ⟹ $x=12-4y+z$

$$x=12-4(8+z/4)+z$$

$$x=12-32-\frac{\cancel{4}z}{\cancel{4}}+z$$

$$x=12-32-\cancel{z}+\cancel{z}$$

$$x=-20$$

Hence $x=-20$, $y=\frac{1}{4}r+8$, $z=r$, where r is any real number.

Q9 $x+y+3z=12$ —— (i)
$2x+2y+6z=6$ —— (ii)

Xing eq (i) by (2) then subtr. from eq (ii)

$2x+2y+6z=24$
$\underline{2x+2y+6z=6}$
$0 = 18$

No Solution.

Q11 $2x+y=13$ —— (i)
$x-2y=3$ —— (ii)
$5x+2y=27$ —— (iii)

Xing eq (ii) by (2) and subtr. from (i)

$2x+y=13$
$\underline{2x-4y=6}$
$7y=7 \Rightarrow \boxed{y=1}$

Xing eq (ii) by 5 then subtr. from (iii)

$5x-10y=15$
$\underline{5x+2y=27}$
$-12y=-12 \Rightarrow \boxed{y=1}$

Ex: 1.1



Xing eq (i) by 5 & eq (iii) by 2 then subtr.

$10x+5y=65$
$\underline{10x+4y=54}$
$1y=11 \Rightarrow \boxed{y=1}$

Put y= in (i) then

$2x+3(1)=13 \Rightarrow 2x=13-3=10 \Rightarrow \boxed{x=5}$

Hence $y=1$ & $x=5$ which is unique solution

—— x ——

Q17 without using the method of elimination solve the linear system

$2x+y-2z=-5$ —— (i)
$3y+z=7$ —— (ii)
$z=4$ —— (iii)

Sol. $z=4$ then eq (ii) $\Rightarrow 3y+4=7 \Rightarrow 3y=7-4=3$

$3y=3 \Rightarrow \boxed{y=1}$

If $z=4$ & $y=1$ eq (i) $\Rightarrow$

$2x+1-2(4)=-5$
$2x=-5+7=2 \Rightarrow \cancel{2}x=\cancel{2}$ $\boxed{x=1}$

Hence $x=1$, $y=1$ & $z=4$ Ans

Q19 Is there a value of r so that $x=1, y=2, z=r$ is a solution to the following linear system? If there is, find it

$$2x+3y-z=11 \quad \text{---(i)}$$
$$x-y+2z=-7 \quad \text{---(ii)}$$
$$4x+y-2z=12 \quad \text{---(iii)}$$

Given that $x=1, y=2, z=r$

eq (i) $\Rightarrow 2(1)+3(2)-r=11$
$2+6-r=11 \Rightarrow -r=11-8=3 \Rightarrow \boxed{r=-3}$

eq (ii) $\Rightarrow 1-2+2r=-7$
$-1+2r=-7 \Rightarrow 2r=-7+1=-6$
$\Rightarrow \boxed{r=-3}$

eq (iii) $\Rightarrow 4(1)+2-2r=12$
$-2r=12-6$
$-2r=6$
$\boxed{r=-3}$

Hence $\boxed{r=-3}$ Ans.

Q23 Let $x_1$ denotes lower-sulfur & $x_2$ denotes high-sulfur for each ton

$5x_1+4x_2=3\times 60=180$
$4x_1+2x_2=2\times 60=120$

or

$5x_1+4x_2=180$ ---(i)
$4x_1+2x_2=120$ ---(ii)

xing eq (ii) by 2 then subtr. from (i)

$5x_1+4x_2=180$
$8x_1+4x_2=240$

$-3x_1=-60 \Rightarrow x_1=\frac{\cancel{-60}^{20}}{\cancel{-3}}=20$

$\boxed{x_1=20}$

Put $x_1=20$ in eq (i)

$5\times 20+4x_2=180$
$4x_2=180-100=80$
$4x_2=80$
$x_2=80/4=20$

$\boxed{x_1=x_2=20 \text{ tons}}$

unique solution

(24) Let $x_1$ denotes regular plastic ton
& $x_2$ denotes special plastic ton

Now $2x_1 + 2x_2 = 8$ —— (i)
$5x_1 + 3x_2 = 15$ —— (ii)

Solving (i) & (ii)

$x_1 = 1.5$ tons

& $x_2 = 2.5$ tons

——— x ———

Q25) Let $x_1$ denotes of ounce A
$x_2$ denotes of ounce B
& $x_3$ denotes of ounce C.

Now
$$\begin{array}{llll} & A & B & C \\ & 2x_1 + 3x_2 + 3x_3 = 25 & & \text{—— (i)} \\ & 3x_1 + 2x_2 + 3x_3 = 24 & & \text{—— (ii)} \\ & 4x_1 + x_2 + 2x_3 = 21 & & \text{—— (iii)} \end{array}$$

xing eq (iii) by 2 then subtr: from (ii)

$3x_1 + \cancel{2x_2} + 3x_3 = 24$
$8x_1 + \cancel{2x_2} + 4x_3 = 42$

$-5x_1 - x_3 = -18$

$\Rightarrow 5x_1 + x_3 = 18$ —— (iv).

xing eq (iii) by 3 then subtr: from (i)

$2x_1 + \cancel{3x_2} + 3x_3 = 25$
$12x_1 + \cancel{3x_2} + 6x_3 = 63$

$-10x_1 - 3x_3 = -38$

$\Rightarrow 10x_1 + 3x_3 = 38$ —— (v)

$2 \times 5x_1 + x_3 = 18$
$10x_1 + 3x_3 = 38$
$-x_3 = -2$

xing eq (iv) by 2 then subtr: from (v)

$\cancel{10x_1} + 2x_3 = 36$
$\cancel{10x_1} + 3x_3 = 38$

$-x_3 = -2 \Rightarrow \boxed{x_3 = 2}$

xing eq (iv) by 3 then subtr: from (v)

$15x_1 + \cancel{3x_3} = 54$
$10x_1 + \cancel{3x_3} = 38$

$+5x_1 = 16 \Rightarrow x_1 = 16/5 = 3.2$

$\boxed{x_1 = 3.2}$

Put $x_1 = 3.2$ & $x_3 = 2$ in eq (i)

eq (i) $\Rightarrow 2(3.2) + 3x_2 + 3(2) = 25$

$3x_2 = 25 - 12.5 = 12.6$

$3x_2 = 12.6 \Rightarrow x_2 = 12.6/3 = 4.2$

$\boxed{x_2 = 4.2}$

Q26 let $x_1$ denotes tons of 2 minute
$x_2$ denotes tons of 6-minute
& $x_3$ denotes tons of 9-minute

Now $6x_1+12x_2+12x_3=10\times 60=600$
$24x_1+12x_2+12x_3=16\times 60=960$

or

$6x_1+12x_2+12x_3=600$
$24x_1+12x_2+12x_3=960$

or

$x_1+2x_2+2x_3=100$ —— (i)
$2x_1+x_2+x_3=80$ —— (ii)

xing eq (ii) by 2 then subt: from (i)

$x_1+2x_2+2x_3=100$
$\underline{4x_1+2x_2+2x_3=160}$
$-3x_1=-60 \Rightarrow x_1=\frac{-60}{-3}=20$

$\boxed{x_1=20}$

xing eq (i) by 2 then ~~subt~~ subt: from (ii)

$2x_1+4x_2+4x_3=200$
$\underline{2x_1+x_2+x_3=80}$
$3x_2+3x_3=120$

$\Rightarrow x_2+x_3=40$ —— (iii)



let $x_3=r$ any real no. & $t$ $r$.

then eq (iii) $\Rightarrow$ $x_2+r=40$ $\Rightarrow \boxed{x_2=40-r}$

Hence $\boxed{x_1=20}$, $\boxed{x_2=40-r}$, $\boxed{x_3=r}$ Ans.

Q27 (a) Given that $P_1(x)=$ ~~...~~ $(1,-5)$
$P_2(x)=(-1,1)$
$P_3(x)=(2,7)$

& Parabola $P(x)=ax^2+bx+c$.

3000
130
100
900000
1/2

(a) $P_1(x)=y=ax^2+bx+c$

$\Rightarrow -5=a(1)^2+b(1)+c$

$-5=a+b+c$ —— (i)

$P_2(x)=y=ax^2+bx+c$

$1=a(-1)^2+b(-1)+c$

& $1=a-b+c$ —— (ii)

$P_3(x)=y=a(2)^2+b(2)+c$

$7=4a+2b+c$ —— (iii)

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eq (2) $\Rightarrow Z_1 = 24000 - 3x_1$

$Z_1 = 24000 - 3(7000)$

$Z_1 = 24000 - 21000$

$\boxed{Z_1 = 3000\$}$

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Q1 Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & -5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix}$ and $C = \begin{bmatrix} 7 & 3 & 2 \\ -2 & 3 & 5 \\ 6 & 1 & -1 \end{bmatrix}$.

a) $a_{12} = -3$, $a_{22} = -5$, $a_{23} = 4$

b) $b_{11} = 4$, $b_{31} = 5$

c) $c_{13} = 2$, $c_{31} = 6$, $c_{33} = -1$

Q2 If $\begin{bmatrix} a+b & c+d \\ c-d & a-b \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$ find a, b, c & d.

Sol:

$a+b = 4$ —— (i)

$c+d = 6$ —— (ii)

$c-d = 10$ —— (iii)

$a-b = 2$ —— (iv)

(i) + (iv)

$a + \cancel{b} = 4$

$a - \cancel{b} = 2$

$2a = 6 \Rightarrow \boxed{a = 3}$

eq (i) $\Rightarrow 3 + b = 4 \Rightarrow b = 4 - 3 = 1 \Rightarrow \boxed{b = 1}$

(ii) + (iii)

$c + \cancel{d} = 6$

$c - \cancel{d} = 10$

$2c = 16 \Rightarrow \boxed{c = 8}$

eq (ii) $\Rightarrow 8 + d = 6 \Rightarrow d = 6 - 8 = -2 \Rightarrow \boxed{d = -2}$

Q3 is similarly to Q2.

In Exercise 4 through 7, let

$$A=\begin{bmatrix}1&2&3\\2&1&4\end{bmatrix},\ B=\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix},\ C=\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}$$

$$D=\begin{bmatrix}3&-2\\2&4\end{bmatrix},\ E=\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix},\ F=\begin{bmatrix}-4&5\\2&3\end{bmatrix}$$

and $O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$.

Q4 a) $C+E$ & $E+C$

$$C+E=\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}+\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}=\begin{bmatrix}5&-5&8\\4&2&9\\5&3&4\end{bmatrix}$$

$$\&\ E+C=\begin{bmatrix}2&-4&5\\0&1&4\\3&2&1\end{bmatrix}+\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}=\begin{bmatrix}5&-5&8\\4&2&9\\5&3&4\end{bmatrix}.$$

commutative addition.

(b) $A+B=\begin{bmatrix}1&2&3\\2&1&4\end{bmatrix}+\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix}$ addition is not possible because the order of matrices is not same.





(c) $D-F=\begin{bmatrix}3&-2\\2&4\end{bmatrix}-\begin{bmatrix}-4&5\\2&3\end{bmatrix}=\begin{bmatrix}3+4&-2-5\\2-2&4-3\end{bmatrix}=\begin{bmatrix}7&-7\\0&1\end{bmatrix}$

(d) $-3C+5O=-3\begin{bmatrix}3&-1&3\\4&1&5\\2&1&3\end{bmatrix}+5\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

$$=\begin{bmatrix}-9&3&-9\\-12&-3&-15\\-6&-3&-9\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$=\begin{bmatrix}-9&3&-9\\-12&-3&-15\\-6&-3&-9\end{bmatrix}\text{ Ans.}$$

(f) $2B+F=2\begin{bmatrix}1&0\\2&1\\3&2\end{bmatrix}+\begin{bmatrix}-4&5\\2&3\end{bmatrix}$

$=\begin{bmatrix}2&0\\4&2\\6&4\end{bmatrix}+\begin{bmatrix}-4&5\\2&3\end{bmatrix}$ addition is not possible because the order of matrices is not same.

Q5 If possible compute the indicated linear combination

(a) $3D+2F=3\begin{bmatrix}3&-2\\2&4\end{bmatrix}+2\begin{bmatrix}-4&5\\2&3\end{bmatrix}=\begin{bmatrix}9&-6\\6&12\end{bmatrix}+\begin{bmatrix}-8&10\\4&6\end{bmatrix}$

$$=\begin{bmatrix}1&4\\10&18\end{bmatrix}$$

(b) $3(2A)$ and $6A$

$$3(2A) = 6A = 6\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 12 & 18\\ 12 & 6 & 24\end{bmatrix}$$

(f) $3(B+D) = 3\left(\begin{bmatrix}1 & 0\\ 2 & 1\\ 3 & 1\end{bmatrix} + \begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}\right)$ addition is not possible because the order of matrices is not same.

Q6 If possible, compute

(a) $A^T$ and $(A^t)^t$

$$A = \begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\end{bmatrix}$$

$$A^t = \begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\end{bmatrix}^t = \begin{bmatrix}1 & 2\\ 2 & 1\\ 3 & 4\end{bmatrix}.$$

$$(A^t)^t = \begin{bmatrix}1 & 2\\ 2 & 1\\ 3 & 4\end{bmatrix}^t = \begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 4\end{bmatrix}$$

So $A^t = (A^t)^t$.

(e) $(C+E)^T$ and $C^T + E^T$

$$C+E = \begin{bmatrix}3 & -1 & 3\\ 4 & 1 & 5\\ 2 & 1 & 3\end{bmatrix} + \begin{bmatrix}2 & -4 & 5\\ 0 & 1 & 4\\ 3 & 2 & 1\end{bmatrix} = \begin{bmatrix}5 & -5 & 8\\ 4 & 2 & 9\\ 5 & 3 & 4\end{bmatrix}.$$

$$(C+E)^t = \begin{bmatrix}5 & -5 & 8\\ 4 & 2 & 9\\ 5 & 3 & 4\end{bmatrix}^t = \begin{bmatrix}5 & 4 & 5\\ -5 & 2 & 3\\ 8 & 9 & 4\end{bmatrix}.$$

$$C^t = \begin{bmatrix}3 & 4 & 2\\ -1 & 1 & 1\\ 3 & 5 & 3\end{bmatrix} \text{ \& } E^t = \begin{bmatrix}2 & 0 & 3\\ -4 & 1 & 2\\ 5 & 4 & 1\end{bmatrix}.$$

$$C^t + E^t = \begin{bmatrix}5 & 4 & 5\\ -5 & 2 & 3\\ 8 & 9 & 4\end{bmatrix}.$$

So $(C+E)^t = C^t + E^t$ Ans.

Q7 is similarly to Q6.

Q8 Is the matrix $\begin{bmatrix}4 & 1\\ 0 & -3\end{bmatrix}$ a linear combination of the matrices $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$? Justify your answer.

$$\begin{bmatrix}4 & 1\\ 0 & -3\end{bmatrix} = c_1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + c_2\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} = \begin{bmatrix}c_1 & 0\\ 0 & c_1\end{bmatrix} + \begin{bmatrix}c_2 & 0\\ 0 & 0\end{bmatrix}.$$

$$\begin{bmatrix}4 & 1\\ 0 & -3\end{bmatrix} = \begin{bmatrix}c_1+c_2 & 0\\ 0 & c_1\end{bmatrix}.$$

$c_1+c_2=4$, & $c_1=-3$ $c_2=7$

So $\begin{bmatrix} 4 & 1 \\ 0 & -3 \end{bmatrix} \neq \begin{bmatrix} 4 & 0 \\ 0 & -3 \end{bmatrix}$

So it is not a linear combination.

Q13 let $A=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ bit matrices.

(a) Find B so that $A+B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}-A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$

$=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ bit

(b) Find C so that $A+C=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$C=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}-A$

$C=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$.

Q15 Given that $u=[0\ 1\ 0\ 1]$ & $u+v=[1\ 1\ 1\ 1]$

let $v=[a,b,c,d]$

then $[0\ 1\ 0\ 1]+[a,b,c\ d]=[1\ 1\ 1\ 1]$

$[a\ \ b+1\ \ c\ \ d+1]=[1\ 1\ 1\ 1]$

$\boxed{a=1}$ $b+1=1 \Rightarrow \boxed{b=0}$ $\boxed{c=1}$ $d+1=1 \Rightarrow \boxed{d=0}$

So $v=[a\ b\ c\ d]=[1\ 0\ 1\ 0]$.



Q1 (a) $a=[1,2]$, $b=\begin{bmatrix} 4 \\ -1 \end{bmatrix}$. Find $a\cdot b$

Sol $a\cdot b=[1\ 2]\cdot\begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$=1\times4+2(-1)$

$=4-2=2$ Ans.

Q2 is similarly to Q1

Q3 let $a=[-3,2\ x]$ and $b=\begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$ if $a\cdot b=17$ find $x$.

Sol Given that $a\cdot b=17$ — (i)

$a\cdot b=[-3\ \ 2\ \ x]\cdot\begin{bmatrix} -3 \\ 2 \\ x \end{bmatrix}$

$=-3\times-3+2(2)+x\cdot x$

$=9+4+x^2$

$a\cdot b=13+x^2$ — (ii)

comparing eq (i) & eq (ii)

$17=13+x^2$

$\Rightarrow x^2=17-13=4$

$x^2=4$

$\boxed{x=\pm2}$ A.

?

Q4 Let $W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$. compute $W \cdot W$

Sol: $W \cdot W = \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$

$= \sin^2\theta + \cos^2\theta$

$\boxed{W \cdot W = 1}$ An.

Q5 ~~Sol~~ Find all values of $x$ so that $V \cdot V = 1$ where $V = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$.

Sol: $V \cdot V = 1$

$\begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} \cdot \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix} = 1$

$(\frac{1}{2})(1/2) + (-1/2)(-1/2) + x \cdot x = 1$

$1/4 + 1/4 + x^2 = 1$

$\frac{1+1}{4} - 1 = -x^2$

$\frac{\cancel{2}}{\cancel{4}_2} - 1 = -x^2$

$\frac{1-2}{2} = -x^2$

$\cancel{-}1/2 = \cancel{-}x^2$

$x^2 = 1/2 = \frac{2}{4}$

$\boxed{x = \pm\frac{\sqrt{2}}{2}}$ An.

②

Ex-1.3

12

Q6 Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ if $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ find $x+y$

Sol: Given that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ —(i)

Now $A \cdot B = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$= \begin{bmatrix} y+2x+x \\ 3y-x+2 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix}$ —(ii)

comparing eq (i) & eq (ii)

$\begin{bmatrix} y+3x \\ 3y-x+2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$y+3x = 6$ —(i) $\quad 3y-x+2 = 8$ —(ii)

Xing eq (i) by 3 then subtr from (ii)

$3y+9x = 18$

$\underline{\overset{-}{3y} \overset{+}{-x} = \overset{-}{6}}$

$10x = 12 \Rightarrow \boxed{x = 6/5}$

eq (i) $\Rightarrow y + 3(6/5) = 6$

$y = 6 - 18/5 = \frac{30-18}{5} = 12/5$

$\boxed{y = 12/5}$

In Exercises 7 and 8 let

$A=\begin{bmatrix}1 & 2 & -3\\ 4 & 0 & -2\end{bmatrix}$ $B=\begin{bmatrix}3 & 1\\ 2 & 4\\ -1 & 5\end{bmatrix}$ $C=\begin{bmatrix}2 & 3 & 1\\ 3 & -4 & 5\\ 1 & -1 & -2\end{bmatrix}$.

$D=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$ $E=\begin{bmatrix}1 & 0 & -3\\ -2 & 1 & 5\\ 3 & 4 & 2\end{bmatrix}$ and $F=\begin{bmatrix}2 & -3\\ 4 & 1\end{bmatrix}$.

Q7 If Possible compute

a) AB

if $c_1 = R_1$ then xing (?)

$$AB=\begin{bmatrix}1 & 2 & -3\\ 4 & 0 & -2\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 4\\ -1 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1\times3+2\times2+(-3)(-1) & 1\times1+2\times4+(-3)(5)\\ 4\times3+0\times2+(-2)(-1) & 4\times1+0\times4+(-2)(5)\end{bmatrix}$$

$$=\begin{bmatrix}3+4+3 & 1+8-15\\ 12+0+2 & 4+0-10\end{bmatrix}$$

$$=\begin{bmatrix}10 & -6\\ 14 & -6\end{bmatrix}$$ Ans



Q9 Let $A=\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -1 & 3\\ 1 & 2 & 4\end{bmatrix}$.

(a) The (1,2) entry

Sol (1,2) means = 1st row & 2nd column of AB matrix.

$[2\ 3]\begin{bmatrix}-1\\ 2\end{bmatrix} = [-2+3\times2] = [-2+6] = 4$ Ans.

(b) (2,3) entry.

$[-1\ 4]\begin{bmatrix}3\\ 4\end{bmatrix} = [-3+16] = 13$ Ans

(c) (3,1) entry.

$[0\ 3]\begin{bmatrix}3\\ 1\end{bmatrix} = 0+3 = 3$ Ans

(d) (3,3) entry

$[0\ 3]\begin{bmatrix}3\\ 4\end{bmatrix} = 12$ Ans

Here was my notes
my notes were placed here

Q10 If $I_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $D=\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$ compute $DI_2$ & $I_2D$.

Sol $I_2D = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix} = \begin{bmatrix}2+0 & 3-0\\ 0-1 & 0-2\end{bmatrix} = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$.

& $DI_2 = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\ -1 & -2\end{bmatrix}$.

So $ID = DI_2 = D$ Ans

Q11 Let $A=\begin{bmatrix}1&2\\3&2\end{bmatrix}$ & $B=\begin{bmatrix}2&-1\\-3&4\end{bmatrix}$

Show that $AB\neq BA$

Sol $AB=\begin{bmatrix}1&2\\3&2\end{bmatrix}\begin{bmatrix}2&-1\\-3&4\end{bmatrix}=\begin{bmatrix}2-6&-1+8\\6-6&-3+8\end{bmatrix}=\begin{bmatrix}-4&7\\0&5\end{bmatrix}$

$BA=\begin{bmatrix}2&-1\\-3&4\end{bmatrix}\begin{bmatrix}1&2\\3&2\end{bmatrix}=\begin{bmatrix}2-3&4-2\\-3+12&-6+8\end{bmatrix}=\begin{bmatrix}-1&2\\9&2\end{bmatrix}$

So $AB\neq BA$

Q12 $A=\begin{bmatrix}1&2&-1\\3&1&4\end{bmatrix}_{2\times3}$ $O=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}_{3\times2}$

$AO=\begin{bmatrix}0&0\\0&0\end{bmatrix}_{2\times2}$

Q13 a) The 1st column AB $A=\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}$ $B=\begin{bmatrix}1&0&-1&2\\3&3&-3&4\\4&2&5&1\end{bmatrix}$

Sol $\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}\begin{bmatrix}1\\3\\4\end{bmatrix}=\begin{bmatrix}1-3+8\\3+6+16\\4-6+12\\2+3+20\end{bmatrix}=\begin{bmatrix}6\\25\\10\\25\end{bmatrix}$

b) The 3rd column.

$\begin{bmatrix}1&-3&2\\3&2&4\\4&-2&3\\2&1&5\end{bmatrix}\begin{bmatrix}-1\\-3\\5\end{bmatrix}=\begin{bmatrix}\\ \end{bmatrix}$

Q14 is similarly Q13.

Q15 Let $A=\begin{bmatrix}2&-3&4\\-1&2&3\\5&-1&-2\end{bmatrix}$ and $C=\begin{bmatrix}2\\1\\4\end{bmatrix}$.

$c_1\,col_1A+c_2\,col_2A+c_3\,col_3A$

$=2\begin{bmatrix}2\\-1\\5\end{bmatrix}+1\begin{bmatrix}-3\\2\\-1\end{bmatrix}+4\begin{bmatrix}4\\3\\-2\end{bmatrix}$

Q16 $A=\begin{bmatrix}1&-2&-1\\2&4&3\\3&0&-2\end{bmatrix}$ and $B=\begin{bmatrix}1&-1\\3&2\\2&4\end{bmatrix}$.

$col_1(AB)=A\,col_1(B)$

$=1\begin{bmatrix}1\\2\\3\end{bmatrix}+3\begin{bmatrix}-2\\4\\0\end{bmatrix}+2\begin{bmatrix}-1\\3\\-2\end{bmatrix}$

$col_2(AB)=A\,col_2(B)$

$=-1\begin{bmatrix}1\\2\\3\end{bmatrix}+2\begin{bmatrix}-2\\4\\0\end{bmatrix}+4\begin{bmatrix}-1\\3\\-2\end{bmatrix}$

Q17 $A=\begin{bmatrix}2&-3&1\\1&2&4\end{bmatrix}$ and $B=\begin{bmatrix}3\\5\\2\end{bmatrix}$

a) Verify $AB = 3a_1 + 5a_2 + 2a_3$

$$AB = \begin{bmatrix}2 & -3 & 1\\ 1 & 2 & 4\end{bmatrix}\begin{bmatrix}3\\5\\2\end{bmatrix}$$

$$= \begin{bmatrix}6-15+2\\ 3+10+8\end{bmatrix} = \begin{bmatrix}-7\\21\end{bmatrix} \quad \text{—(1)}$$

Now $3a_1 + 5a_2 + 2a_3 = 3\begin{bmatrix}2\\1\end{bmatrix} + 5\begin{bmatrix}-3\\2\end{bmatrix} + 2\begin{bmatrix}1\\4\end{bmatrix}$

$$= \begin{bmatrix}6\\3\end{bmatrix} + \begin{bmatrix}-15\\10\end{bmatrix} + \begin{bmatrix}2\\8\end{bmatrix} = \begin{bmatrix}6-15+2\\ 3+10+8\end{bmatrix} = \Big[$$

from eq (1) & eq (1) we have

$AB = 3a_1 + 5a_2 + 2a_3$ Hence verified.

(b) Verify that $AB = \begin{bmatrix}(row_1(A))B\\ (row_2(A))B\end{bmatrix}$.

from part a $AB = \begin{bmatrix}-7\\21\end{bmatrix}$ —(1)

$[row_1(A)]B = [2\ -3\ 1]\begin{bmatrix}3\\5\\2\end{bmatrix} = [6-15+2] = -7$.

$[row_2(A)]B = [1\ 2\ 4]\begin{bmatrix}3\\5\\2\end{bmatrix} = [3+10+8] = 21$

Now $\begin{bmatrix}(row_1(A))B\\ (row_2(A))B\end{bmatrix} = \begin{bmatrix}-7\\21\end{bmatrix}$ —(2)

from (2) & (1) Hence verified.

Q18 Sol: $\begin{bmatrix}-2 & 2 & 3\\ 3 & 5 & -1\end{bmatrix}\begin{bmatrix}3\\4\\2\end{bmatrix}$.

Q19 a) $\begin{bmatrix}2 & 0 & 0 & 1\\ 3 & 2 & 3 & 0\\ 2 & 3 & -4 & 0\\ 1 & 0 & 3 & 0\end{bmatrix}$ which is coefficient matrix

b) $\begin{bmatrix}2 & 0 & 0 & 1\\ 3 & 2 & 3 & 0\\ 2 & 3 & -4 & 0\\ 1 & 0 & 3 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix} = \begin{bmatrix}7\\-2\\3\\5\end{bmatrix}$ which is the linear system in matrix form.

c) $\left[\begin{array}{cccc|c}2 & 0 & 0 & 1 & 7\\ 3 & 2 & 3 & 0 & -2\\ 2 & 3 & -4 & 0 & 3\\ 1 & 0 & 3 & 0 & 5\end{array}\right]$ which is augmented matrix.

Q20 a) Sol:

$$-2x - y + 0z + 4w = 5$$
$$-3x + 2y + 7z + 8w = 3$$
$$x + 0y + 0z + 2w = 4$$
$$3x + 0y + z + 3w = 6$$

Q21 similarly Q20

Q22 similarly Q19.

Q23 sol: They are equivalent as third low have zero entry.

Q24. a) $\begin{bmatrix}1 & 2 & 0\\ 2 & 5 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}1\\ 1\end{bmatrix}$

b) $\begin{bmatrix}1 & 2 & 1\\ 1 & 1 & 2\\ 2 & 0 & 2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$

Q25 a) $\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}$

$x\begin{bmatrix}1\\ 2\end{bmatrix}+y\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}$

b) $\begin{bmatrix}2 & -3 & 5\\ 1 & 4 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-2\\ 3\end{bmatrix}$

$x\begin{bmatrix}2\\ 1\end{bmatrix}+y\begin{bmatrix}-3\\ 4\end{bmatrix}+z\begin{bmatrix}5\\ -1\end{bmatrix}=\begin{bmatrix}-2\\ 3\end{bmatrix}$

Q26 a) can say nothing.

b) can say nothing.

Q27 a) $AB^t=0$

$\begin{bmatrix}r & 1 & -2\end{bmatrix}\begin{bmatrix}1\\ 3\\ -1\end{bmatrix}=0$

$1(r)+3(1)-1(-2)=0$

$r+3+2=0$

$\boxed{r=-5}$

Q28 Similarly Q27.

$$A=\begin{array}{c} \begin{array}{cc} A.P & F.P \end{array} \\ \begin{bmatrix}2 & 2\\ 3 & 4\end{bmatrix} \end{array}\begin{array}{l}\text{chair}\\ \text{Table}\end{array} \qquad B=\begin{array}{c} \begin{array}{cc} S.L.C & chicago \end{array} \\ \begin{bmatrix}9 & 10\\ 10 & 12\end{bmatrix} \end{array}\begin{array}{l}A.P\\ F.P\end{array}$$

$$AB=\begin{bmatrix}2 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}9 & 10\\ 10 & 12\end{bmatrix}=\begin{array}{c} \begin{array}{cc} S.L.C & chicago \end{array} \\ \begin{bmatrix}38 & 44\\ 67 & 78\end{bmatrix} \end{array}\begin{array}{l}\text{chair}\\ \text{table}\end{array}$$

where AB gives total cost of Producing each kind of Product in each city.

Q29 $$A=\begin{array}{c} \begin{array}{ccc} S.D & N.O & P.M \end{array} \\ \begin{bmatrix}300 & 100 & 150\\ 200 & 250 & 400\end{bmatrix} \end{array}\begin{array}{l}\text{Product P}\\ \text{Product Q}\end{array}$$

$$B=\begin{array}{c} \begin{array}{cc} \text{Plant X} & \text{Plant Y} \end{array} \\ \begin{bmatrix}8 & 12\\ 7 & 9\\ 15 & 10\end{bmatrix} \end{array}\begin{array}{l}S.D\\ N.O\\ P.M\end{array}$$

$$A\cdot B=\begin{bmatrix}300 & 100 & 150\\ 200 & 250 & 400\end{bmatrix}\begin{bmatrix}8 & 12\\ 7 & 9\\ 15 & 10\end{bmatrix}$$

$$=\begin{array}{c} \begin{array}{cc} \text{Plant X} & \text{Plant Y} \end{array} \\ \begin{bmatrix}5350 & 6000\\ 9350 & 8650\end{bmatrix} \end{array}\begin{array}{l}\text{Product P}\\ \text{" Q.}\end{array}$$

where AB gives each Product quantity in each Plant.

**Q33**

$$A=\begin{array}{c c} \text{Adults} & \text{children} \end{array}$$
$$A=\begin{bmatrix}80 & 120\\ 100 & 200\end{bmatrix}\begin{array}{l}\text{Male}\\ \text{Female}\end{array} \qquad B=\begin{array}{c}\begin{array}{ccc}P & F & C\end{array}\\ \begin{bmatrix}20 & 20 & 20\\ 10 & 20 & 30\end{bmatrix}\end{array}\begin{array}{l}\text{Adult}\\ \text{child}\end{array}$$

a) $\begin{bmatrix}20\\10\end{bmatrix}\begin{bmatrix}80 & 120\end{bmatrix} = 80\times20+10\times120 = 2800g.$

b) $\begin{bmatrix}20\\20\end{bmatrix}\begin{bmatrix}100 & 200\end{bmatrix} = 20\times100+20\times200 = 6000g.$

**Q34** (a) $\begin{bmatrix}220\\330\\120\end{bmatrix}\begin{bmatrix}200 & 150 & 120\end{bmatrix} = 103400$

(b) $\begin{bmatrix}50 & 40 & 25\end{bmatrix}\begin{bmatrix}100\\120\\250\end{bmatrix} = 16050$

**Q35** a) $S_1=\begin{bmatrix}18.95 & 14.75 & 8.98\end{bmatrix}$

$S_2=\begin{bmatrix}17.80 & 13.50 & 10.79\end{bmatrix}$

$$\varphi=\begin{bmatrix}18.95 & 14.75 & 8.98\\ 17.80 & 13.50 & 10.79\end{bmatrix}$$

(b) 20% Reduced.

$$\varphi=\begin{bmatrix}18.95\times80\% & 14.75\times80\% & 8.98\times80\%\\ 17.80\times80\% & 13.50\times80\% & 10.79\times80\%\end{bmatrix}$$

$$=\begin{bmatrix}15.16 & 11.8 & 7.18\\ 14.24 & 10.8 & 8.62\end{bmatrix}$$

**Q36** (i) $a\cdot b=\begin{bmatrix}0 & 1 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}=0$

(ii) $a\cdot b=\begin{bmatrix}1 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}=1$

Q37 similarly to Q36.

**Q38** $a=\begin{bmatrix}1 & x & 0\end{bmatrix}$ & $b=\begin{bmatrix}x\\1\\1\end{bmatrix}$ find $x=?$ & $a\cdot b=0$

$a\cdot b=\begin{bmatrix}1 & x & 0\end{bmatrix}\begin{bmatrix}x\\1\\1\end{bmatrix}=0$

$=1[x]+1[x]+0[1]=0$

$=x+x=0$

$2x=0 \Rightarrow \boxed{x=0}$

**Q39** $AB=\begin{bmatrix}1 & 1 & x\\ 0 & y & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$=\begin{bmatrix}x+0\\ y+1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \boxed{x=0,\ y=-1}$ for Bit M

**Q41** $AB=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}a+c & b+d\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\boxed{c=0},\ \boxed{d=1}$

$a+c=1 \Rightarrow a+0=1 \Rightarrow \boxed{a=1}$

$b+d=0 \Rightarrow$ ~~...~~ $b+1=0 \Rightarrow b=-1=1$ ...

$$B=\begin{bmatrix} 1 & +1 \\ 0 & 1 \end{bmatrix}$$

——— X ———

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Q1 verify theorem 1.1

(a) $A+B=B+A$

$$A+B=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix} \text{ and } B+A=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix}.$$

(b) $A+(B+C)=(A+B)+C.$

$$B+C=\begin{bmatrix} -2 & -6 & 2 \\ 5 & 1 & 5 \end{bmatrix}.$$

$$A+(B+C)=\begin{bmatrix} -1 & -4 & 0 \\ 8 & 5 & 10 \end{bmatrix}.$$

Similarly

$$A+B=\begin{bmatrix} 3 & 2 & -1 \\ 6 & 2 & 10 \end{bmatrix}$$

$$(A+B)+C=\begin{bmatrix} -1 & -4 & 0 \\ 8 & 5 & 10 \end{bmatrix}.$$

Hence $\boxed{A+(B+C)=(A+B)+C}$

**Q2** Verify (a) of theorem 1.2 for

$A(BC) = (AB)C$.

**Sol:**

$$BC = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1+9+2 & -3+4 \\ 1-9+4 & 3+8 \end{bmatrix} = \begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 10 & 1 \\ -4 & 11 \end{bmatrix} = \begin{bmatrix} 10-12 & 1+33 \\ 20+4 & 2-11 \end{bmatrix} = \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 3 & 2 \\ 1 & -3 & 4 \end{bmatrix}.$$

$$= \begin{bmatrix} -1+3 & 3-9 & 2+12 \\ -2-1 & 6+3 & 4-4 \end{bmatrix} = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}.$$

$$(AB)C = \begin{bmatrix} 2 & -6 & 14 \\ -3 & 9 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 34 \\ 24 & -9 \end{bmatrix}.$$

Hence $\boxed{A(BC) = (AB)C}$.

**Q3** Verify (b) of Th: 1.2

$A(B+C) = AB+AC$

S.Y.S.



**Q4** Verify (a), (b) and (c) of theorem 1.3 for $r=6$, $s=-2$.

(a) $r(sA) = (rs)A$

(b) $(r+s)A = rA+sA$

(c) $r(A+B) = rA+rB$

Answer

(a) $r(sA) = (rs)A$

$$sA = -2\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix}.$$

$$r(sA) = 6\begin{bmatrix} -8 & -4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}.$$

$(rs) = (-2)(6) = -12$

$$(rs)A = -12\begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} -48 & -24 \\ -12 & 36 \end{bmatrix}.$$

Hence $\boxed{r(sA) = (rs)A}$ Verified.

$\boxed{\text{(b) \& (c) Similarly to a.}}$

Q5 verify (d) of thm 1.3 for $r = -3$

(d) $A(rB) = r(AB) = (rA)B$

S.Y.S.

Q6 verify (b) + (d) of thm: 1.4 for $r = -4$

(b) $(A+B)^T = A^T + B^T$

(d) $(rA)^T = rA^T$

S.Y.S.

Q7 verify (c) of Thm: 1.4 for

(c) $(AB)^T = B^T A^T$

S.Y.S.

Q10. Q11 S.Y.S

Q12 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ show $A^2 = I_2$

$AA = A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

Hence $A^2 = I_2$

Q13 + Q14 S.Y.S.



Q15 $Ax = rx$

$Ax = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 1+2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$.

$rx = r\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$

given $Ax = rx$

$\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix} \Rightarrow \boxed{r = 3}$.

Q16 $(kA)^T(kA) = 1$

$kA = k\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -2k \\ k \\ -k \end{bmatrix}$.

$(kA)^T = [-2k \ k \ -k]$.

$(kA)^T(kA) = [-2k \ k \ -k]\begin{bmatrix} -2k \\ k \\ -k \end{bmatrix} = 4k^2 + k^2 + k^2 = 1$

$6k^2 = 1$

$k^2 = 1/6$

$\boxed{k = \pm 1/\sqrt{6}}$ Ans.

Q18 a) $Ax_0 = \begin{bmatrix} 1/3 & 2/5 \\ 2/3 & 3/5 \end{bmatrix}\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$.

$Ax_0 \begin{bmatrix} 2/9 + 2/15 \\ 4/9 + 3/15 \end{bmatrix}$.

b) For stable distribution

$Ax_0 = x_0$

$\begin{bmatrix} 1/3 & 2/5 \\ 2/3 & 3/5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$= \begin{bmatrix} 1/3 a + 2/5 b \\ 2/3 a + 3/5 b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$\frac{1}{3}a + \frac{2}{5}b = a$ — (i).

$\frac{2}{3}a + \frac{3}{5}b = b$ — (ii).

eq (i) $\Rightarrow \frac{1}{3}a - a + \frac{2}{5}b = 0 \Rightarrow$

$-\frac{2}{3}a + \frac{2}{5}b = 0$

$-10a + 6b = 0$ ——— (iii)

eq (ii) $\Rightarrow \frac{2}{3}a + \frac{3}{5}b - b = 0$

$\Rightarrow 10a - 6b = 0$ ——— (iv)

Solving eq (iii) & (iv)

$a = 3/8$ & $b = 5/8$   $x_0 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3/8 \\ 5/8 \end{bmatrix}$.



Q19 $A = \begin{array}{c|c} & \begin{array}{cc} M & N \end{array} \\ \begin{array}{c} M \\ N \end{array} & \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix} \end{array}$   $x_0 = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$.

(a) $Ax_0 = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 1/9 + 2/6 \\ 2/9 + 2/6 \end{bmatrix} = \begin{bmatrix} 4/9 \\ 5/9 \end{bmatrix}$.

(b) $Ax_0 = x_0$

$\begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$.

$\frac{1}{3}a + \frac{1}{2}b = a$ —— (i)

$\frac{2}{3}a + \frac{1}{2}b = b$ —— (ii)

S.Y.S

$a = 3/7$ & $b = 4/7$

$x_0 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3/7 \\ 4/7 \end{bmatrix}$.

Q20 (a) $x_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$   $A = \begin{array}{c|c} & \begin{array}{ccc} R & S & T \end{array} \\ \begin{array}{c} R \\ S \\ T \end{array} & \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \end{array}$.

$Ax_0 = \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix}\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 13/36 \\ 17/36 \\ 1/6 \end{bmatrix}$.

after 2 years

$x_2 = Ax_1 = A(Ax_0)$

$$= \begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} 13/36 \\ 17/36 \\ 1/6 \end{bmatrix} = \begin{bmatrix} 43/108 \\ 191/432 \\ 23/144 \end{bmatrix}.$$

(b) $Ax_0 = x_0$, $x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$a+b+c = 1$ —— (1)

$$\begin{bmatrix} 1/3 & 1/2 & 1/4 \\ 2/3 & 1/4 & 1/2 \\ 0 & 1/4 & 1/4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$\frac{1}{3}a + \frac{1}{2}b + \frac{1}{4}c = a$ —— (2)

$\frac{2}{3}a + \frac{1}{4}b + \frac{1}{2}c = b$ —— (3)

$\frac{1}{4}b + \frac{1}{4}c = c$ —— (4)

of (2) $\Rightarrow \frac{4a+6b+3c}{12} = a$

$4a+6b+3c = 12a$

$\Rightarrow -8a+6b+3c$ —— (5)

of (3) $\Rightarrow \frac{8a+3b+6c}{12} = b$

$8a+3b+6c = 12b$

$\Rightarrow 8a-9b+6c = 0$ —— (6)

of (4) $\Rightarrow b+c = 4c$

$\Rightarrow b-3c = 0$ —— (7)

$b = 3c$

of (5) $\Rightarrow -8a+6(3c)+3c = 0$

$\Rightarrow \boxed{a = 21/8\, c}$ —— (8)

of (1) $\Rightarrow 21/8\, c + 3c + c = 1$

$\Rightarrow \boxed{c = 8/53}$

of (8) $\Rightarrow a = \frac{21}{\cancel{8}} \times \frac{\cancel{8}}{53} = \boxed{21/53 = a}$

of (7) $\Rightarrow b = 3 \cdot 8/53 = \boxed{24/53 = b}$

So $x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 21/53 \\ 24/53 \\ 8/53 \end{bmatrix}$ Ans.

12. (a) $x_1 = Ax_0$

$$x_1 = \begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$

$$x_1 = \begin{bmatrix} 4/15 \\ 3/10 \\ 13/30 \end{bmatrix}.$$

After 2 years:

$Ax_1 = A(Ax_0)$

$$= \begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} 4/10 \\ 9/20 \\ 13/20 \end{bmatrix}$$

$$= \begin{bmatrix} 84/200 \\ 97/200 \\ 119/200 \end{bmatrix}.$$

(b) $Ax_0 = x_0 \qquad x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$a+b+c = 1$ ——— ①

Then

$$\begin{bmatrix} 0.4 & 0 & 0.4 \\ 0 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

S.V.S as ① = 0.

$$x_0 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10/37 \\ 12/37 \\ 15/37 \end{bmatrix}.$$



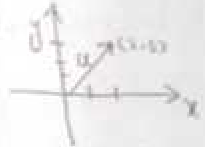
In Exercises 1 through 8, sketch u and its image under the given matrix transformation f.

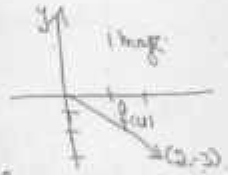
1. $f: R^2 \to R^2$ defined by

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}; \quad u = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Given $u = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

sketch



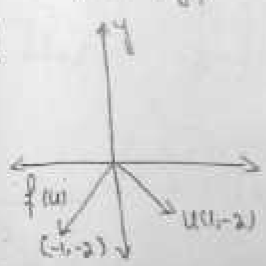

Now $f(u) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

2. $f: R^2 \to R^2$ (reflection w.r.t. y-axis) defined by

$$f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Given $u = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}.$

Sketch




Now $f(u) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$

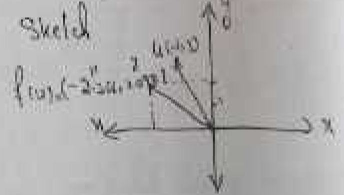
Q3 f: $R^2 \to R^2$ is a counter clockwise rotation through 30°, u(-1,3)

$$f(u) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -\cos 30^\circ - 3\sin 30^\circ \\ -\sin 30^\circ + 3\cos 30^\circ \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} - \frac{3}{2} \\ -\frac{1}{2} + \frac{3\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} -2.366 \\ 2.098 \end{bmatrix}$$

Sketch

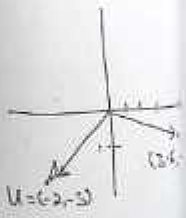



Q4 f: $R^2 \to R^2$ is a counter clockwise rotation through $\frac{2}{3}\pi$ radians $u = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$$f(u) = \begin{bmatrix} \cos\frac{2}{3}\pi & -\sin\frac{2}{3}\pi \\ \sin\frac{2}{3}\pi & \cos\frac{2}{3}\pi \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2\cos\frac{2}{3}\pi + 3\sin\frac{2}{3}\pi \\ -2\sin\frac{2}{3}\pi - 3\cos\frac{2}{3}\pi \end{bmatrix} = \begin{bmatrix} 3.6 \\ -0.23 \end{bmatrix}$$

Sketch

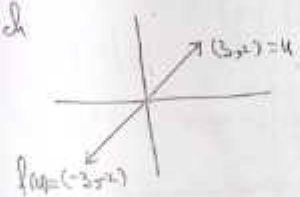



Q5 f: $R^2 \to R^2$ defined by

$$f\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

Sketch



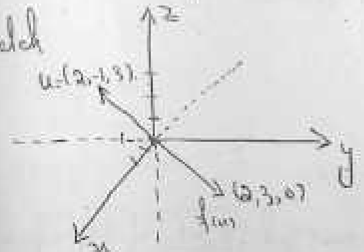

Q6 is similarly to Q2.

Q7 f: $R^3 \to R^2$ defined by

$$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Given $u = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Sketch



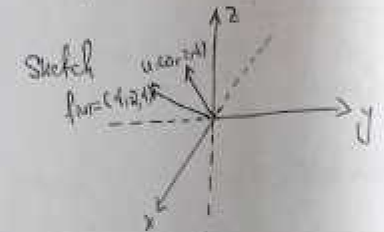

Now $f(u) \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2+1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$.

Q8 $f: R^3 \to R^3$ defined by

$$f\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=\begin{bmatrix}1&0&1\\-1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix};\quad u=\begin{bmatrix}0\\-3\\4\end{bmatrix}$$

$$f(u)=\begin{bmatrix}1&0&1\\-1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0\\-3\\4\end{bmatrix}=\begin{bmatrix}4\\-3\\4\end{bmatrix}.$$

Sketch




Q9 Given that $f(x)=Ax$ & $A=\begin{bmatrix}1&3\\-1&2\end{bmatrix}$ & $W=\begin{bmatrix}7\\3\end{bmatrix}$.

Now $Ax=f(x)=W$

$$\begin{bmatrix}1&3\\-1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}7\\3\end{bmatrix} \text{ —— (A)}$$

$$\begin{bmatrix}x+3y\\-x+2y\end{bmatrix}=\begin{bmatrix}7\\3\end{bmatrix}$$

$x+3y=7$ —— (i)

$-x+2y=3$ —— (ii)

Solving eq (i)&(ii) we get $\boxed{x=1}$ & $\boxed{y=2}$

eq (A) $\Rightarrow$

$$\begin{bmatrix}1&3\\-1&2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}7\\3\end{bmatrix}$$

$$\begin{bmatrix}1+6\\-1+4\end{bmatrix}=\begin{bmatrix}7\\3\end{bmatrix}$$

$$\begin{bmatrix}7\\3\end{bmatrix}=\begin{bmatrix}7\\3\end{bmatrix}$$

Hence $W=\begin{bmatrix}7\\3\end{bmatrix}$ is in range of $f$.

Q10 & Q11 is similarly to Q9

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Q12 $f: R^2 \to R^3$.

Given that $f(x)=Ax$ & $A=\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}$ & $W=\begin{bmatrix}1\\-1\\2\end{bmatrix}$.

Now $Ax=f(x)=W$ where $x=\begin{bmatrix}x\\y\end{bmatrix}$.

$$\begin{bmatrix}1\\-1\\2\end{bmatrix}=\begin{bmatrix}1&2\\0&1\\1&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} \longrightarrow \text{(A)}$$

$1=x+2y$ —— (i)

$-1=+y$ —— (ii)

$2=x+y$ —— (iii)

eq (ii) $\Rightarrow \boxed{y=-1}$ eq (iii) $\Rightarrow x=2+1=3 \Rightarrow \boxed{x=3}$

eq (ii) ⟹

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 3-2 \\ 0-1 \\ 3-1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Hence $w = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ is in the range of $f$.

Q13 Given $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ & $x = \begin{bmatrix} x \\ y \end{bmatrix}$

Now $Ax = f(x) = w$.

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{--- (A)}$$

$$\begin{bmatrix} x+2y \\ 0+y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$x + 2y = 1$ --- (i)

$+y = 1$ --- (ii)

$x + y = 1$ --- (iii)

eq (ii) ⟹ $\boxed{y=1}$ eq (iii) ⟹ $\boxed{x=0}$ & eq (i) ⟹ $\boxed{x=-1}$

$x$ has two values.



eq (A) ⟹

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

w is not in range of f

Again eq (A) ⟹

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1+2 \\ 0+1 \\ -1+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

W is not in range of f

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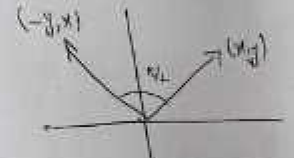
Q15 (a) $A=\begin{bmatrix}-1&0\\0&1\end{bmatrix}$ let $u=\begin{bmatrix}x\\y\end{bmatrix}$

$$Au=\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-x+0y\\0x+y\end{bmatrix}=\begin{bmatrix}-x\\y\end{bmatrix}.$$


Reflection about y-axis.

(b) $A=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$, $u=\begin{bmatrix}x\\y\end{bmatrix}$

$$Au=\begin{bmatrix}0&-1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0-y\\x+0\end{bmatrix}=\begin{bmatrix}-y\\x\end{bmatrix}$$

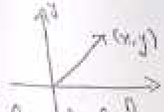



Rotate counter clockwise through $\pi/2$.

$x=-y$
$y=x=-$
$=(x,y)=($

16(a) $A=\begin{bmatrix}0&1\\1&0\end{bmatrix}$. let $u=\begin{bmatrix}x\\y\end{bmatrix}$.

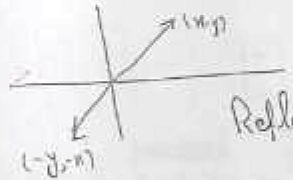
$$Au=\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}y\\x\end{bmatrix}.$$




Reflection about line $y=x$.

(b) $A=\begin{bmatrix}0&-1\\-1&0\end{bmatrix}$ let $u=\begin{bmatrix}x\\y\end{bmatrix}$.

$$Au=\begin{bmatrix}0&-1\\-1&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-y\\-x\end{bmatrix}.$$




Reflection about line $y=-x$.

Q17 (a) $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$. let $u=\begin{bmatrix}x\\y\end{bmatrix}$ $Au=\begin{bmatrix}x\\0\end{bmatrix}$.

Projection onto x-axis.

(b) $A=\begin{bmatrix}0&0\\0&1\end{bmatrix}$, $u=\begin{bmatrix}x\\y\end{bmatrix}$ then $Au=\begin{bmatrix}0\\y\end{bmatrix}$.

Projection onto y-axis.

Q18(a) $A=\begin{bmatrix}1&2&0\\0&1&-1\end{bmatrix}$, $w=\begin{bmatrix}0\\-1\end{bmatrix}$

$Ax=w$

$\begin{bmatrix}1&2&0\\0&1&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\-1\end{bmatrix}$

$x+2y=0$

$y-z=-1$

$x=-2y$ —(i) & $y+1=z$ —(ii)

let $z=r$ be real no.

let $z=0$ then $\boxed{y=-1}$ & $\boxed{x=2}$

let $z=1$, $y=0$, $x=0$

So $u=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $v=\begin{bmatrix}2\\-1\\0\end{bmatrix}$.

(b) $A=\begin{bmatrix}2&1&0\\0&2&-1\end{bmatrix}$, $w=\begin{bmatrix}4\\4\end{bmatrix}$

$Ax=w$

$\begin{bmatrix}2&1&0\\0&2&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\4\end{bmatrix} \Rightarrow 2x+y=4 \Rightarrow \boxed{x=4-y/2}$

$2y-z=4 \Rightarrow \boxed{y=\frac{4+z}{2}}$

let $z=4$, $y=4$, $x=0$

let $z=0$ $y=2$, $x=1$

So $u=\begin{bmatrix}0\\4\\4\end{bmatrix}$, $v=\begin{bmatrix}1\\2\\0\end{bmatrix}$



Q1 Reduced row echelon form.

Q2 Neither Q3 Reduced row echelon form

Q4 Neither Q5 Row echelon form Q6 Neither

Q7 Neither Q8 Neither

Q9 $A=\begin{bmatrix}1&0&3\\-3&1&4\\4&2&2\\5&-1&5\end{bmatrix}$.

a) Interchanging the second and fourth rows.

$A=\begin{bmatrix}1&0&3\\5&-1&5\\4&2&2\\-3&1&4\end{bmatrix}$.

b) Multiplying the third row by 3.

$A=\begin{bmatrix}1&0&3\\-3&1&4\\12&6&6\\5&-1&5\end{bmatrix}$

(c) Adding (-3) times the 1st row to the fourth row

R.W

-3 0 -9 (-3

+with $R_4$

-3 0 -9

5 -1 5

$\boxed{2 \ -1 \ -4}$

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 1 & 4 \\ 4 & 2 & 2 \\ 2 & -1 & -4 \end{bmatrix}$$

Q10 is similarly to Q9.

Q11 Find three matrices that are row equivalent to

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix}$$

1st Possibility

(a) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 5 & 2 & -3 & 4 \\ 0 & 1 & 2 & -1 \end{bmatrix} \underset{\sim}{R_{23}}$

2nd Possibility

(b) $\begin{bmatrix} 4 & -2 & 6 & 8 \\ 0 & 1 & 2 & -1 \\ 5 & 2 & -3 & 4 \end{bmatrix} \underset{\sim}{2R_1}$

3rd Possibility

(c) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 7 & 1 & 0 & 8 \end{bmatrix} \underset{\sim}{R_{1+3}}$

Q13 In Exercise 13 through 16, find a row echelon form of the given matrix.

Q13

$$\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 2 & 3 & 4 & 5 \\ 0 & -1 & 2 & 3 \\ 3 & 2 & 4 & 1 \end{bmatrix} \underset{\sim}{R_{13}} \text{ row operation}$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -3 & 6 & 1 \\ 0 & -1 & 2 & 3 \\ 0 & -7 & 7 & -5 \end{bmatrix} \begin{matrix} R_2 - 2R_1 \\ R_4 - 3R_1 \end{matrix}$$

$R_4 - 3R_1$

3 2 4 1

-3 -9 +3 -6

0 -7 7 -5

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix} \underset{\sim}{R_{23}}$$

$$\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & 1 \\ 0 & -7 & 7 & -5 \end{bmatrix} (-1)R_2$$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & -8 \\ 0 & 0 & -7 & -26 \end{bmatrix} \begin{matrix} R_3+3R_2 \\ R_4+7R_2 \end{matrix}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -7 & -26 \\ 0 & 0 & 0 & -8 \end{bmatrix} R_{34}$$

$$= \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 26/7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} R_3/-7 \\ R_4/-8 \end{matrix}$$

Q4

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & -3 & -1 & 5 \\ 1 & 3 & 2 & 5 \\ 1 & 1 & 0 & 2 \\ 2 & -6 & -2 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & 2 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & -2 & -2 & -3 \end{bmatrix} \begin{matrix} R_2-2R_1 \\ R_3-R_1 \\ R_4-R_1 \\ R_5-R_1 \end{matrix}.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 7 & -2 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 4 & 1 \end{bmatrix} \begin{matrix} R_3-5R_2 \\ R_4-3R_2 \\ R_5+2R_2 \end{matrix}.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 4 & 1 \end{bmatrix} \tfrac{1}{7}R_3.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & -15/7 \\ 0 & 0 & 0 & 15/7 \end{bmatrix} \begin{matrix} R_4-3R_3 \\ R_5-4R_3 \end{matrix}.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 15/7 \end{bmatrix} \left(-\tfrac{7}{15}\right)R_4.$$

$$= \begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_5-\frac{15}{7}R_4$$

Q18 Let $A=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 2 & 1 & -2\end{bmatrix}$ & $Ax=b$.

(a) $x=\begin{bmatrix}1\\2\\3\end{bmatrix}$; $b=0$

Now $Ax=b$

$$\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 2 & 1 & -2\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1+4+3\\ -1+2+6\\ 2+2-6\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}8\\7\\-2\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$$ No solution.

(b) $x=\begin{bmatrix}0\\0\\0\end{bmatrix}$; $b=0$

$Ax=b$

$$\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 2 & 1 & -2\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0+0+0\\ 0+0+0\\ 0+0+0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Solution exist. / Qc & Qd similarly.

In Exercises 20 through 22 find all solutions of the given linear system

Q20 (a) $x+y+2z=-1$
$x-2y+z=-5$
$3x+y+z=3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -3 & -1 & -4 \\ 0 & -2 & -5 & 6 \end{array}\right] \begin{array}{l} R_2-R_1 \\ R_3-3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 3 & 1 & 4 \\ 0 & -2 & -5 & 6 \end{array}\right] (-1)R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & -2 & -5 & 6 \end{array}\right] R_2+R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & -4 & 10 \\ 0 & 0 & -13 & 26 \end{array}\right] R_3+2R_2.$$



$x+y+2z=-1$ —— (i)
$y-4z=10$ —— (ii)
$-13z=26$ —— (iii)

By (iii) $\Rightarrow z=\frac{26}{-13} \Rightarrow \boxed{z=-2}$

By (ii) $\Rightarrow y-4(-2)=10$
$y=10-8$
$\boxed{y=2}$

By (i) $\Rightarrow x+(2)+2(-2)=-1$
$x+2-4=-1 \Rightarrow \boxed{x=1}$.

Qb, Qc, Qd similarly

Q21 (a) $x+y+2z+3w=13$
$x-2y+z+w=8$
$3x+y+z-w=1$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 1 & -2 & 1 & 1 & 8 \\ 3 & 1 & 1 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & -3 & -1 & -2 & -5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] \begin{array}{l} R_2-R_1 \\ R_3-3R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 3 & 1 & 2 & 5 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] (-1)R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & -2 & -5 & -10 & -38 \end{array}\right] R_2+R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 13 \\ 0 & 1 & -4 & -8 & -33 \\ 0 & 0 & -13 & -26 & -104 \end{array}\right] R_3+2R_2$$

$x+y+2z+3w=13$ — (i)

$y-4z-8w=-33$ — (ii)

$-13z-26w=-104$ — (iii)

Let $\boxed{w=r}$

By (iii) $\Rightarrow -13z-26r=-104$.

$-13(z+2r)=-104$

$z+2r=8$

$\boxed{z=8-2r}$

By (ii) $\Rightarrow$

$y-4(8-2r)-8r=-33$

$y-32+\cancel{8r}-\cancel{8r}=-33$

$y=-33+32=-1 \Rightarrow \boxed{y=-1}$

By (i) $\Rightarrow x+(-1)+2(8-2r)+3r=13$

$x-1+16-4r+3r=13$

$x+15-r=13$

$x=13-15+r$

$\boxed{x=r-2}$

Ob. Ol. Similarly

Q21(c) $\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 1 \\ 3 & -2 & 1 & -6 & -2 \\ 1 & 1 & -1 & -1 & -1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right]$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 3 & -2 & 1 & -6 & -2 \\ 2 & 1 & 1 & -2 & 1 \\ 6 & 0 & 1 & -9 & -2 \\ 5 & -1 & 2 & -8 & 3 \end{array}\right] R_{13}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -1 & 3 & 0 & 3 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right]\begin{array}{l} R_2-3R_1 \\ R_3-2R_1 \\ R_4-6R_1 \\ R_5-5R_1 \end{array}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & -5 & 4 & -3 & 1 \\ 0 & -6 & 7 & -3 & 4 \\ 0 & -6 & 7 & -3 & 8 \end{array}\right]\begin{array}{l} -R_3 \\ R_{32} \end{array}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & -11 & -3 & 1 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & -11 & -3 & -10 \end{array}\right]\begin{array}{l} R_3+5R_2 \\ R_4+6R_2 \\ R_5+6R_2 \end{array}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & 3/11 & -1/11 \\ 0 & 0 & -11 & -3 & -14 \\ 0 & 0 & -11 & -3 & -10 \end{array}\right] -\frac{1}{11}(R_3)$$

$$\sim\left[\begin{array}{cccc|c} 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & 3/11 & -1/11 \\ 0 & 0 & 0 & 0 & -15 \\ 0 & 0 & 0 & 0 & -11 \end{array}\right]\begin{array}{l} R_4+11R_3 \\ R_5+11R_3 \end{array}$$

No solution.



Q3
$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2-5)z = a$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]\begin{array}{l} R_2-R_1 \\ R_3-R_1 \end{array}$$

a) If $a=-2$ no solution

(b) If $a \neq \pm 2$ then unique solution

(c) If $a=2$ Infinitely many solution

Next $a^2-4=0 \Rightarrow a^2=4 \Rightarrow a=\pm 2$

If $a=-2$: $a^2-4 = a-2$

$\cancel{4}-\cancel{4} = -2-2 \Rightarrow 0=-4$ not soln

If $a=2$: $4-4=2-2 \Rightarrow 0=0$ Infinite sol.

If $a \neq \pm 2$ unique sol.

Q24 $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & (a^2-1) & a+1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-2R_1 \end{array}$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right]$

a) $a = \pm\sqrt{3}$ (b) $a \neq \pm\sqrt{3}$, (c) None.

Q25 $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & a^2-5 & a \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a^2-6 & a-2 \end{array}\right] \begin{array}{l} R_2-R_1 \\ R_3-R_1 \end{array}$

a) $a = \pm\sqrt{6}$, (b) $a \neq \pm\sqrt{6}$ (c) None.

Q26 $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & a^2-8 & a \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & a^2-9 & a-3 \end{array}\right] R_2-R_1$

(a) $a=-3$, (b) $a \neq \pm 3$, (c) $a=3$.

Q27 to Q30 Similarly to Q20 ~~extra~~

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Q31 Sol:- $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ —— (i)

& $f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$ —— (ii)

Comparing eq (i) & (ii) we get

$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ -1 \end{bmatrix}$.

By augmented matrix

$\left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 2 & 2 & 0 & -1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 4 & 1 & 3 & 4 \\ 2 & -1 & 3 & 5 \\ 1 & 1 & 0 & -1/2 \end{array}\right] \frac{R_3}{2}$

$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 2 & -1 & 3 & 5 \\ 4 & 1 & 3 & 4 \end{array}\right] R_{13}$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 0 & -3 & 3 & 6 \\ 0 & -3 & 3 & 6 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 0 & 1 & -1 & -2 \\ 0 & -3 & 3 & 6 \end{array}\right] -R_2/3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & -1/2 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3+3R_2$$

Now $x+y=-1/2$ — (i)

$y-z=-2$ — (ii)

Let $z=t$

$\Rightarrow$ (ii) $\Rightarrow \boxed{y=-2+t}$

$\Rightarrow$ (i) $\Rightarrow x+(-2+t)=-1/2$

$x=-1/2+2-t$

$\boxed{x=3/2-t}$

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33 Sol:-

$$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

A.M

$$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \left[\begin{array}{ccc|c} 4 & 1 & 3 & a \\ 2 & -1 & 3 & b \\ 2 & 2 & 0 & c \end{array}\right]$$

~~Augmented~~

$$\sim \left[\begin{array}{ccc|c} 2 & 2 & 0 & c \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right] R_{13}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 2 & -1 & 3 & b \\ 4 & 1 & 3 & a \end{array}\right] (1/2)R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & -3 & 3 & b-c \\ 0 & -3 & 3 & a-2c \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & 1 & -1 & \frac{(-b+c)}{3} \\ 0 & -3 & 3 & a-2c \end{array}\right] -R_2/3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & c/2 \\ 0 & 1 & -1 & -b+c/3 \\ 0 & 0 & 0 & a-c-b \end{array}\right] R_3+3R_2$$

solution is possible if

~~[illegible]~~ $a-c-b=0$

$\boxed{-a+c+b=0}$

Q34 is similarly to Q33

<u>Q35</u> $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$, $b_1=\begin{bmatrix}1\\ -8\end{bmatrix}$, $b_2=\begin{bmatrix}5\\ -5\end{bmatrix}$.

$Ax=b_1$

$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ -8\end{bmatrix}$.

$\begin{bmatrix}x-y\\ 2x+3y\end{bmatrix}=\begin{bmatrix}1\\ -8\end{bmatrix}$.

New Augmented matrix

$\left[\begin{array}{cc:c}1 & -1 & 1\\ 2 & 3 & -8\end{array}\right] \Rightarrow \sim \left[\begin{array}{cc:c}1 & -1 & 1\\ 0 & 5 & -10\end{array}\right] R_2-2R_1$

$\Rightarrow 5y=-10 \Rightarrow \boxed{y=-2}$

$x-y=1 \Rightarrow x=-2+1=\boxed{x=-1}$

$\boxed{x=-1,\ y=-2}$

<u>2nd Part</u> $Ax=b_2$

$\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}. \Rightarrow \begin{bmatrix}x-y\\ 2x+3y\end{bmatrix}=\begin{bmatrix}5\\ -5\end{bmatrix}$.

A.M

$\left[\begin{array}{cc:c}1 & -1 & 5\\ 2 & 3 & -5\end{array}\right]$



$\sim\left[\begin{array}{cc:c}1 & -1 & 5\\ 0 & 5 & -15\end{array}\right] R_2-2R_1$

$5y=-15 \Rightarrow \boxed{y=-3}$

$x-y=5 \Rightarrow x=5-3 \Rightarrow \boxed{x=2}$

$\boxed{x=2,\ y=-3}$

<u>2nd Part</u> $[A \vdots b_1\ b_2]$

$\left[\begin{array}{cc:cc}1 & -1 & 1 & 5\\ 2 & 3 & -8 & -5\end{array}\right]$

$\sim\left[\begin{array}{cc:cc}1 & -1 & 1 & 5\\ 0 & 5 & -10 & -15\end{array}\right] R_2-2R_1$

Same results

o

Q36 is similarly to Q35.

Q37 $A=\begin{bmatrix}1&0&5\\1&1&1\\0&1&-4\end{bmatrix}$ & $(-4I_3-A)X=0$

$I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$-4I_3=\begin{bmatrix}-4&0&0\\0&-4&0\\0&0&-4\end{bmatrix}$.

$-4I_3-A=\begin{bmatrix}-4&0&0\\0&-4&0\\0&0&-4\end{bmatrix}-\begin{bmatrix}1&0&5\\1&1&1\\0&1&-4\end{bmatrix}=\begin{bmatrix}-5&0&-5\\-1&-5&-1\\0&-1&0\end{bmatrix}$

Now $(-4I_3-A)X=0$

$\Rightarrow \begin{bmatrix}-5&0&-5\\-1&-5&-1\\0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}-5x+0y-5z\\-1x-5y-1z\\0x-1y+0z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

A.M.

$\left[\begin{array}{ccc|c}-5&0&-5&0\\-1&-5&-1&0\\0&-1&0&0\end{array}\right]$

$\sim\left[\begin{array}{ccc|c}-1&-5&-1&0\\-5&0&-5&0\\0&-1&0&0\end{array}\right] R_{21}$

$\sim\left[\begin{array}{ccc|c}1&5&1&0\\-5&0&-5&0\\0&-1&0&0\end{array}\right] -R_1$

$\sim\left[\begin{array}{ccc|c}1&5&1&0\\0&25&0&0\\0&-1&0&0\end{array}\right] R_2+5R_1$

$\sim\left[\begin{array}{ccc|c}1&5&1&0\\0&1&0&0\\0&-1&0&0\end{array}\right] R_2/25$

$\sim\left[\begin{array}{ccc|c}1&5&1&0\\0&1&0&0\\0&0&0&0\end{array}\right] R_3+R_2$

$x+5y+z=0$ —— (i)

$y=0$ —— (2)

let $z=r$ (real no.)

by (i) $\Rightarrow x+0+r=0 \Rightarrow \boxed{x=-r}$

$\boxed{x=-r,\ y=0,\ z=r}$

Q38 is similarly to Q37

Q39 Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & 1 & -9 & 2a-b \\ 0 & 0 & 0 & c-3a-b \end{array}\right] -R_2+R_3, -R_2$$

$c-3a-b=0$

$\Rightarrow \boxed{3a-b+c=0}$ Ans.

Q40 is similarly to Q39

Q41 $A=\begin{bmatrix}4 & 1\\0 & 2\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$

$Ax=\begin{bmatrix}4 & 1\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=4\begin{bmatrix}x\\y\end{bmatrix}=4x.$

Now $4x-Ax=0$

$(4I_2-A)X=0$

$\left(\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}-\begin{bmatrix}4 & 1\\0 & 2\end{bmatrix}\right)\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$

$\begin{bmatrix}4-4 & 0-1\\0-0 & 4-2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}0 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}0x-y\\0x+2y\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

A·M

$\left[\begin{array}{cc|c} 0 & -1 & 0 \\ 0 & 2 & 0 \end{array}\right].$

$\sim \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 2 & 0 \end{array}\right] -R_1$

$\sim \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2-2R_1$

$y=0$, let $x=r$ (real no.)

$X=\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}r\\0\end{bmatrix}.$



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Q42 $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ & $(3I_2 - A)X = 0$

Now $3X - AX = 0$

$(3I_2 - A)X = 0$

$\left(3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left(\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3-2 & 0-1 \\ 0-1 & 3-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

A.M

$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ -1 & 1 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2 + R_1$

$x - y = 0$ let $x = r$.

$y = r$

$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$ Ans

Q43 $(3I_3 - A)X = 0$

$\left(\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$\begin{bmatrix} 2 & -2 & 1 \\ -1 & 3 & -1 \\ -4 & 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2x-2y+z \\ -1x+3y-z \\ -4x+4y-2z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

A.M

$\left[\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ -1 & 3 & -1 & 0 \\ -4 & 4 & -2 & 0 \end{array}\right]$

$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 2 & -2 & 1 & 0 \\ -4 & 4 & -2 & 0 \end{array}\right] \begin{array}{l} -R_2 \\ R_{21} \end{array}$

$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & -8 & 2 & 0 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 + 4R_1 \end{array}$

$\sim \left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -1/4 & 0 \\ 0 & -8 & 2 & 0 \end{array}\right] R_2/4$

$$\sim\left[\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3+8R_2$$

$x-3y+z=0$ — (i)

$y-\frac{1}{4}z=0$ — (2)

let $\boxed{z=r}$ (real no any)

(2) $\Rightarrow$ $\boxed{y=\frac{1}{4}r}$

eq (i) $\Rightarrow$ $x-\frac{3}{4}r+r=0$

$x=\frac{-3r+4r}{4}=0 \Rightarrow x+r/4=0 \Rightarrow \boxed{x=-}$

Hence $\boxed{x=-r/4,\ y=\frac{1}{4}r,\ z=r}$

Q44 is similarly to Q43

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**Q45** Given $X=X_p+X_h$,

where $X_p$ is particular solution

& $X_h$ is a solution to the associated homog: system.

A.M

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 2 & 1 & -2 & 3 & 2 \\ 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & -4 & -1 & 6 \end{array}\right]$$

$$\sim\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 0 & -3 & 0 & 7 & -2 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & -3 & 0 & 7 & -2 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-R_1 \\ R_4-4R_1 \end{array}$$

$$\sim\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 0 & 1 & 0 & -7/3 & 2/3 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & -3 & 0 & 7 & -2 \end{array}\right] -R_2/3$$

$$\sim\left[\begin{array}{cccc|c} 1 & 2 & -1 & -2 & 2 \\ 0 & 1 & 0 & -7/3 & 2/3 \\ 0 & 0 & 4 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_4+3R_2$$

$x+2y-z-2w=2$ — (i)

$y-\frac{7}{3}w=\frac{2}{3}$ — (ii)

$4z+6w=3$ — (iii)

let $w=r$ (any real no.)

$q(iii) \Rightarrow 4z + 6Y = 3$

$4z = 3 - 6Y$

$z = 3/4 - \frac{6}{4}Y \Rightarrow \boxed{z = 3/4 - 3/2\, Y}$

$q(ii) \Rightarrow \boxed{y = 3/2 + 7/3\, Y}$

$q(i) \Rightarrow x + 2(3/2 + 7/3\, Y) - (3/4 - 3/2\, Y) - 2Y = 2$

$x = 17/12 - \frac{50}{12}Y$

$x = 17/12 - 25/6\, Y$

$X = X_p + X_h$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 17/12 \\ 3/2 \\ 3/4 \\ 0 \end{bmatrix} + \begin{bmatrix} -25/6\, Y \\ 7/3\, Y \\ -3/2\, Y \\ Y \end{bmatrix}$$ Ans:-

Q46 is similarly to Q45

Q47 $(1,2)(3,3)(5,8)$.

Since quadratic polynomials $a_2x^2 + a_1x + a_0 = y$ ~~form~~ ⊛

(1,2) Point

$q \circledast \Rightarrow y \quad 2 = a_2 + a_1 + a_0$ —— (i)

(3,3)

$q \circledast \Rightarrow 3 = 9a_2 + 3a_2 + a_0$ —— (ii)

For Point (5,8)

$q \circledast \Rightarrow 8 = 25a_2 + 5a_1 + a_0$ —— (iii)

in ~~an~~ A.M

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 9 & 3 & 1 & 3 \\ 25 & 5 & 1 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -6 & -8 & -15 \\ 0 & -20 & -24 & -42 \end{array}\right].$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4/3 & 5/2 \\ 0 & -20 & -24 & -42 \end{array}\right] -\tfrac{1}{6}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 4/3 & 5/2 \\ 0 & 0 & 8/3 & 8 \end{array}\right] R_3+20R_2$$

$a_2+a_1+a_0=2$ —— (a)

$a_1+\frac{4}{3}a_0=\frac{5}{2}$ —— (b)

$\frac{8}{3}a_0=8$ —— (c)

by (c) $\Rightarrow a_0=\frac{8\times 3}{8} \Rightarrow \boxed{a_0=3}$

by (b) $\Rightarrow a_1+\frac{4}{3}(3)=\frac{5}{2}$

$a_1=\frac{5}{2}-4=\frac{5-8}{2}=-\frac{3}{2}$

$\boxed{a_1=-\frac{3}{2}}$

by (a) $\Rightarrow a_2+(-\frac{3}{2})+(3)=2$

$a_2=2-3+\frac{3}{2}=\frac{4-6+3}{2}=\frac{1}{2}$

$\therefore \boxed{a_2=\frac{1}{2}}$

by (1) $\Rightarrow \boxed{y=\frac{1}{2}x^2-\frac{3}{2}x+3}$ Ans.

Q48, Q49, Q50 similarly to Q47

Q51 $10x_1+12x_2+15x_3=16\times 60=960$

$6x_1+8x_2+12x_3=11\times 60=660$

$12x_1+12x_2+18x_3=18\times 60=1080$

Aug: M

$$\left[\begin{array}{ccc|c} 10 & 12 & 15 & 960 \\ 6 & 8 & 12 & 660 \\ 12 & 12 & 18 & 1080 \end{array}\right]$$ S.Y.S.

Q52 $x_1 + 2x_2 + 3x_3 = 6\times 60 = 360$

$2x_1 + 4x_2 + 5x_3 = 11\times 60 = 660$

Aug: Matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 2 & 4 & 5 & 660 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 360 \\ 0 & 0 & -1 & -60 \end{array}\right] R_2 - 2R_1$$

$\boxed{x_3 = 60}$ (deluxe binding)

$x_1 + 2x_2 + 3x_3 = 360$ — (i)

let $\boxed{x_2 = \gamma}$ (any real no.)

eq (i) $\Rightarrow x_1 + 2\gamma + 3(60) = 360$

$x_1 = 360 - 180 - 2\gamma$

$\boxed{x_1 = 180 - 2\gamma}$

$\boxed{x_1 = 180 - 2\gamma, \; x_2 = \gamma, \; x_3 = 60}$

Q53 Given $P(x) = ax^2 + bx + c$ — (A)

$P(0) = f(0)$, $P'(0) = f'(0)$, & $P''(0) = f''(0)$ & $f(x) = e^{2x}$.

$f(x) = e^{2x}$ — (i)

Put $x = 0$

$f(0) = e^0 = 1 \Rightarrow \boxed{f(0) = 1}$

eq (A) $\Rightarrow P(0) = a0 + b0 + c$

$\boxed{P(0) = c}$

But Given $P(0) = f(0)$

$\boxed{c = 1.}$

eq (A) $\Rightarrow P'(x) = 2ax + b$

$P'(0) = 2a(0) + b \Rightarrow P'(0) = b.$

eq (i) $\Rightarrow f'(x) = 2e^{2x}$

$f'(0) = 2e^0 = 2$

But Given $f'(0) = P'(0)$

$\boxed{2 = b}$

Similarly $\boxed{a = 2}$

eq ⊛ $\Rightarrow$ $\boxed{P(x) = 2x^2+2x+1}$

~~~ o ~~~ o ~~~

Q54 is similarly to Q53

~~~ o ~~~ o ~~~

Ex 1.6
Q55

$$T_1 = \frac{30^\circ + 50^\circ + T_2 + T_3}{4}$$

$$4T_1 = 80^\circ + T_2 + T_3$$

$$4T_1 - T_2 - T_3 = 80^\circ \text{ —— (i)}$$

$$T_2 = \frac{30 + 50^\circ + T_1 + T_4}{4}$$

$$4T_2 - T_1 - T_4 = 80^\circ \text{ —— (ii)}$$

$$T_3 = \frac{T_1 + T_4 + 50^\circ + 0^\circ}{4}$$

$$4T_3 - T_1 - T_4 = 50^\circ \text{ —— (iii)}$$

$$T_4 = \frac{T_3 + T_2 + 50^\circ + 0}{4}$$

$$4T_4 - T_3 - T_2 = 50^\circ \text{ —— (iv)}$$

New Aug: Matrix

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 80 \\ -1 & 4 & 0 & -1 & 80 \\ -1 & 0 & 4 & -1 & 50 \\ 0 & -1 & -1 & 4 & 50 \end{array}\right] \text{ S.Y.S}$$



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BIT Matrices (optional) P-17

Table ①

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

Table

| × | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

Ex) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A+B = \begin{bmatrix} 1+1 & 0+1 \\ 1+0 & 1+1 \\ 0+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

Note:- We have observed that $1+1=0$. Thus the additive inverse of 0 is 0 and the additive inverse of 1 is 1. Hence to compute the difference of bit matrices A and B we proceed as follows

$A-B = A + (\text{inverse of } B) = A+B = A+B$

$A-B = A+B$

Note:- A bit is a binary digit i.e. either 0 or 1
A bit matrix is also called a boolean matrix.



Ans: M

Q17(a)

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right].$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{array}\right] R_2+R_1$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] R_3+R_2$$

$x+y+w=0$ —— (i)

$y+z=1$ —— (ii)

$\boxed{w=0}$

Eq (ii) $\Rightarrow y = 1-z$

Eq (i) $x=-y$

$x=-(1-z)$

let $z=0$

$y=1-0=1$

$y=1.$

$x=-(1-0)$

$x=1$

let $z=1$

$y=1-1=0$

$x=-(1-1)$

$x=0$

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ Ans.

(b) $\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_2+R_1 \\ R_3+R_1 \end{array}$

$\sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] R_3+R_2$

$x+y=0 \Rightarrow x=-y$

$z=1$   let $y=0$

$w=1$   $x=0$

let $y=1$   $x=1$

$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ &

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Q5 (a) $Ax = C$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x+y+0z \\ 0x+y+0z \\ x+y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$Ay = M$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] R_3+R$$

$x+y=0 \Rightarrow x=-y \Rightarrow x=1$

$y=1$

$z=0$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Q) Show that $\begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ is nonsingular.

Sol: Let $A = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$ To find $A^{-1}$, we let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ —(*)

Then we must have

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So that

$$\begin{bmatrix} 2a+c & 2b+d \\ -2a+3c & -2b+3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$2a+c = 1$ —(1) $\quad -2a+3c = 0$ —(2)

$2b+d = 0$ —(3) $\quad -2b+3d = 1$ —(4)

(1) + (2)

$4c = 1 \Rightarrow \boxed{c = 1/4}$

eq(1) $\Rightarrow 2a = 1 - 1/4 = 3/4 \Rightarrow \boxed{a = 3/8}$

(3) + (4)

$4d = 1 \Rightarrow \boxed{d = 1/4}$

eq(3) $\Rightarrow 2b = -1/4 \Rightarrow \boxed{b = -1/8}$

eq(*) $\Rightarrow A^{-1} = \begin{bmatrix} 3/8 & -1/8 \\ 1/4 & 1/4 \end{bmatrix}$ we conclude $A^{-1}$ is exist

So A is nonsingular.

**Q2** let $A=\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}$, let $A^{-1}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$.

Now $AA^{-1}=I_2$

$$\begin{bmatrix}2 & 1\\ -4 & -2\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

$$=\begin{bmatrix}2a+c & 2b+d\\ -4a-2c & -4b-2d\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

$2a+c=1$ —(1)
$-4a-2c=0$ —(2)
$2b+d=0$ —(3)
$-4b-2d=1$ —(4)

x-ing eq (1) by (2) + adding eq (2)

$$\begin{aligned}4a+2c&=2\\ -4a-2c&=0\\ \hline 0&=2\end{aligned}$$ impossible

$A^{-1}$ doesn't exist so it is singular

Q3 is similarly to Q2 + Q1.

**Q4** $A=\begin{bmatrix}1 & 2 & -1\\ 3 & 2 & 3\\ 2 & 2 & 1\end{bmatrix}$ let $A^{-1}=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$

Now $AA^{-1}=I_3$

$$\begin{bmatrix}1 & 2 & -1\\ 3 & 2 & 3\\ 2 & 2 & 1\end{bmatrix}\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

$$=\begin{bmatrix}a+2d-g & b+2e-h & c+2f-i\\ 3a+2d+3g & 3b+2e+3h & 3c+2f+3i\\ 2a+2d+g & 2b+2e+h & 2c+2f+i\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$a+2d-g=1$ —(1) $\quad b+2e-h=0$ —(4) $\quad c-2f-i=0$ —(7)
$3a+2d+3g=0$ —(2) $\quad 3b+2e+3h=1$ —(5) $\quad 3c+2f+3i=0$ —(8)
$2a+2d+g=0$ —(3) $\quad 2b+2e+h=0$ —(6) $\quad 2c+2f+i=1$ —(9)

(1) − (2)
$$\begin{aligned}a+2d-g&=1\\ 3a+2d+3g&=0\\ \hline -2a-4g&=1\end{aligned}$$ —(10)

(1) − (3)
$$\begin{aligned}a+2d-g&=1\\ 2a+2d+g&=0\\ \hline -a-2g&=1\end{aligned}$$ —(11)

x-ing 2 by eq (11) then sub. from 10

$$\begin{aligned}-2a-4g&=2\\ -2a-4g&=1\\ \hline 0&=1\end{aligned}$$ impossible. $A^{-1}$ doesn't exist so it is singular.

Q5(a) $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$. let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. — (*)

Now $AA^{-1} = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} a-2b & 3a+6b \\ c-2d & 3c+6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$a-2b=1$ — (1) $\quad 3a+6b=0$ — (3)

$c-2d=0$ — (2) $\quad 3c+6d=1$ — (4)

solving the above equation we get

$\boxed{a=1/2}$ $\boxed{b=-1/4}$ $\boxed{c=1/6}$ $\boxed{d=1/12}$

eq (*) $\Rightarrow A^{-1} = \begin{bmatrix} 1/2 & 1/4 \\ 1/6 & 1/12 \end{bmatrix}$.



(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

Now $[A : I_3]$ is

$$[A : I_3] = \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

We now compute the reduced row echelon form of the matrix. To find $A^{-1}$. we proceed as follows.

$$\begin{array}{c} \quad A \qquad\qquad I_3 \\ \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] R_2 - R_1$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] -R_2$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & -1 & 2 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{array}{l} R_3 - R_2 \\ R_1 - 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right] \begin{matrix} R_1 - R_3 \\ R_1 - R_2 \end{matrix}$$

So $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

Q6 — to — Q10 R similarly to Q5

Q11 Which of the following linear system have a non-trivial solution?

(b) $2x + y - z = 0$

$x - 2y - 3z = 0$

$-3x - y + 2z = 0$

**Method 1** A.M

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & -2 & -3 & 0 \\ -3 & -1 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 2 & 1 & -1 & 0 \\ -3 & -1 & 2 & 0 \end{array}\right] R_{12}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & -7 & -7 & 0 \end{array}\right] \begin{matrix} R_2 - 2R_1 \\ R_3 + 3R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -7 & -7 & 0 \end{array}\right] \tfrac{1}{5}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R_3 + 7R_2$$

As infinite solution exists so it is non-trivial.

**Method 2** $[A : I_3]$

$$A : I_3 \Rightarrow \begin{array}{c} \begin{matrix} \quad A & \qquad\qquad I_3 \end{matrix} \\ \left[\begin{array}{ccc|ccc} 2 & 1 & -1 & 1 & 0 & 0 \\ 1 & -2 & -3 & 0 & 1 & 0 \\ -3 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1/2 & -1/2 & 1/2 & 0 & 0 \\ 1 & -2 & -3 & 0 & 1 & 0 \\ -3 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \tfrac{1}{2}R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.5 & -0.5 & 0.5 & 0 & 0 \\ 0 & -2.5 & -2.5 & -0.5 & 1 & 0 \\ 0 & 0.5 & 0.5 & 1.5 & 0 & 1 \end{array}\right] \begin{matrix} R_2 - R_1 \\ R_3 + 3R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.5 & -0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 1 & 0.2 & 0.4 & 0 \\ 0 & 0.5 & 0.5 & 1.5 & 0 & 1 \end{array}\right] \frac{-R_2}{2.5}$$

$$\sim \begin{bmatrix} 1 & 0.5 & -0.5 & : & 0.5 & 0 & 0 \\ 1 & 1 & 1 & : & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & : & 1.5 & 0 & 1 \end{bmatrix} R_3 - 0.5R_2$$

So no inverse exist hence it's non-trivial

Q12 is similarly to Q11

Q13 $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ find $A$ let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ — (*)

Since $A^{-1}A = I_2$

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 2a+3c & 2b+3d \\ a+4c & b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$2a+3c=1$ — ① $\quad 2b+3d=0$ — ③

$a+4c=0$ — ② $\quad b+4d=1$ — ④

Solving the above equation we get

$\boxed{a = 4/5}$, $\boxed{b = -3/5}$ $\boxed{c = -1/5}$ $\boxed{d = 2/5}$

eq (*) $\Rightarrow A = \begin{bmatrix} 4/5 & -3/5 \\ -1/5 & 2/5 \end{bmatrix}$.

Q14 is similarly to Q13.

Q15 Given $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ $A^{-1} = ?$ let $A^{-1} = \begin{bmatrix} b & c & d \\ e & f & g \\ h & i & j \end{bmatrix}$ — (*)

Since $AA^{-1} = I_3$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}\begin{bmatrix} b & c & d \\ e & f & g \\ h & i & j \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} b+e & c+f & d+g \\ b & c & d \\ b+2e+ha & c+2f+ai & d+2g+ja \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$b+e=1$ — ① $\quad b=0$ — ④ $\quad b+2e+ha=0$ — ⑦

$c+f=0$ — ② $\quad c=1$ — ⑤ $\quad c+2f+ai=0$ — ⑧

$d+g=0$ — ③ $\quad d=0$ — ⑥ $\quad d+2g+ja=1$ — ⑨

eq ① $\Rightarrow 0+e=1 \Rightarrow \boxed{e=1}$

eq ② $\Rightarrow 1+f=0 \Rightarrow \boxed{f=-1}$

eq ③ $\Rightarrow 0+g=0 \Rightarrow \boxed{g=0}$

eq ⑦ $\Rightarrow 0+2+ha=0 \Rightarrow \boxed{h=-2/a}$

eq ⑧ $\Rightarrow 1+2(-1)+ai \Rightarrow \boxed{i = 1/a}$

eq ⑨ $\Rightarrow 0+2(0)+ja=1 \Rightarrow \boxed{j = 1/a}$

eq (*) $\Rightarrow A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -2/a & 1/a & 1/a \end{bmatrix}$ when $a \neq 0$.

**Q17** Let $x$ be ... $x = A^{-1}b$ given and find ...

... find $A^{-1}$. Let $A^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ — ④ & $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

Now $AA^{-1} = I_3$

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a+d+3g & 2b+e+3h & 2c+f+3i \\ 3a+2d-g & 3b+2e-h & 3c+2f-i \\ 2a+d+g & 2b+e+h & 2c+f+i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$2a+d+3g=1$ —① $\quad 2b+e+3h=0$ —④ $\quad 2c+f+3i=0$ —⑦

$3a+2d-g=0$ —② $\quad 3b+2e-h=1$ —⑤ $\quad 3c+2f-i=0$ —⑧

$2a+d+g=0$ —③ $\quad 2b+e+h=0$ —⑥ $\quad 2c+f+i=1$ —⑨

③ + ② are added

$3a+2d-g=0$
$2a+d+g=0$

$5a+3d=0$ —(2)'

add 3 times ② to ①

$2a+d+3g=1$
$9a+6d-3g=0$

$11a+7d=1$ —(3)'

Solving (2)' & (3)'

$5a=-3d \Rightarrow a=-3/5\,d$

$11(-3/5\,d)+7d=1$

$-33d+35d=5 \Rightarrow \boxed{d=5/2}$

$\boxed{a=-3/2}$

Put values in ①

$2a+d+3g=1$

$2(-3/2)+(5/2)+3g=1$

$-3+5/2+3g=1 \Rightarrow \boxed{g=1/2}$

⑤ + ⑥

$3b+2e-\cancel{h}=1$
$2b+e+\cancel{h}=0$

$5b+3e=1$ —(5)'

Adding 3 times ⑤ to ④

$2b+e+\cancel{3h}=0$
$9b+6e-\cancel{3h}=3$

$11b+7e=3$ —(6)'

Solving (5)' & (6)'

$5b = 1 - 3e \Rightarrow b = \frac{1-3e}{5}$

$11\left(\frac{1-3e}{5}\right) + 7e = 3$

$\Rightarrow \boxed{e = 2}$

$b = \frac{1-3(2)}{5} = -5/5 = -1$

$\boxed{b = -1}$

eq ① $\Rightarrow$ $2b + e + 3h = 0$

$2(-1) + 2 + 3h = 0$

$\Rightarrow \boxed{h = 0}$

⑧ + ⑨

$3c + 2f - i = 0$

$2c + f + i = 1$

$5c + 3f = 1$ —— (8)'

Add (3) times ⑧ to ⑦

$2c + f + 3i = 0$

$9c + 6f - 3i = 0$

$11c + 7f = 0$ —— (9)'

Solving (8)' & (9)'

$11c = -7f \Rightarrow c = -7/11 f$

$5(-7/11)f + 3f = 1 \Rightarrow \boxed{f = -11/2}$

$c = -7/11 \times -11/2 = 7/2$

$\boxed{c = 7/2}$

$3c + 2f - i = 0$

$3(7/2) + 2(-11/2) - i = 0$

$\Rightarrow \boxed{i = -1/2}$

eq ⑧ $\Rightarrow$ $A = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$

$X = A^{-1}b, \quad b = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$X = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$

$X = \begin{bmatrix} -30 \\ 60 \\ 10 \end{bmatrix}$ A

17(b) $x = A^{-1}b, \quad b = \begin{bmatrix} 12 \\ 8 \\ 14 \end{bmatrix}$

$x = \begin{bmatrix} -3/2 & -1 & 7/2 \\ 5/2 & 2 & -11/2 \\ 1/2 & 0 & -1/2 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \\ 14 \end{bmatrix} = \begin{bmatrix} 23 \\ -31 \\ -1 \end{bmatrix}$ A

Q16 $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

(a) $A^{-1}$

Now $AA^{-1} = I_2$ let $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ —(A)

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} a+3c & b+3d \\ 2a+7c & 2b+7d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a+3c = 1$ —(1) $\quad b+3d = 0$ —(2)

$2a+7c = 0$ —(3) $\quad 2b+7d = 1$ —(4)

Solving the above equations we get

$\boxed{a=7}$ $\boxed{b=-3}$ $\boxed{c=-2}$ $\boxed{d=1}$

by (A) $\Rightarrow A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$ —(a)

(b) $(A^T)^{-1} \Rightarrow$

$A^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ let $(A^T)^{-1} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ —(B)

$(A^T)(A^T)^{-1} = I_2$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+2z & y+2w \\ 3x+7z & 3y+7w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$x+2z = 1$ —(1) $\quad y+2w = 0$ —(2)

$3x+7z = 0$ —(3) $\quad 3y+7w = 1$ —(4)

Solving the above equations we get

$\boxed{x=7}$ $\boxed{y=-2}$ $\boxed{z=-3}$ $\boxed{w=1}$

by (B) $\Rightarrow (A^T)^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$ Ans.

by (a) $\Rightarrow (A^{-1})^T = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

So $(A^{-1})^T = (A^T)^{-1}$.

Q17 (a) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Then $(A+B)^{-1}$ exists but $A^{-1}$ & $B^{-1}$ do not exist.

(b) Yes for $A$ non-singular & $c \neq 0$

$$(cA)^{-1} = \frac{1}{c} A^{-1}$$

$$I_n = cA(\tfrac{1}{c}A)^{-1}$$

$$I_n = c(1/c) AA^{-1}$$

$$I_n = I_n$$

Q21 $(\lambda-1)x + 2y = 0$ —(1)

$2x + (\lambda-1)y = 0$ —(2)

from (2) $x = \frac{(1-\lambda)y}{2}$

eq (1) ⟹ $\frac{(\lambda-1)(1-\lambda)y}{2} + 2y = 0$

$\Rightarrow -\frac{(1-\lambda)(1-\lambda)}{2}y + 2y = 0$

$= \frac{-(1-\lambda)^2 y + 4y}{2} = 0$

$= (-(1-\lambda)^2 + 4)y = 0$

$= -(1+\lambda^2-2\lambda) + 4 = 0$

$= +\lambda^2 - 2\lambda - 3 = 0 \quad \lambda = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2\pm 4}{2}$

$\boxed{\lambda = 3, -1}$

Q23 Similarly to Q5

Q24 $A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$ & $B^{-1} = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix}$

find $(AB)^{-1}$

Since $(AB)^{-1} = (B^{-1})(A^{-1}) = \begin{bmatrix} 2 & 5 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 6+5 & 4+15 \\ 9-2 & 6-6 \end{bmatrix} = \begin{bmatrix} 11 & 19 \\ 7 & 0 \end{bmatrix}$

Q25 $x = A^{-1}b$

$x = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 10+9 \\ 20+3 \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$.

Q27
$$\begin{array}{c} \quad A \qquad\qquad I_3 \\ \left[\begin{array}{ccc|ccc} 5 & 2 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_1/5$$

$$\sim\left[\begin{array}{ccc:ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 0 & -0.2 & 0 & -0.6 & 1 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_2-3R_1$$

$$\sim\left[\begin{array}{ccc:ccc} 1 & 0.4 & 0 & 0.2 & 0 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] -\frac{R_2}{0.2}$$

$$\sim\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & -4 & 0 & 0 & 1 \end{array}\right] R_1-0.4R_2$$

$$\sim\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 3 & -5 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/4 \end{array}\right] -\frac{R_3}{4}$$

So $A^{-1}=\begin{bmatrix} -1 & 2 & 0 \\ 3 & -5 & 0 \\ 0 & 0 & -1/4 \end{bmatrix}$

~ 0 ~

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Q29(c)

$$\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_2+R_1$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_{32}$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_4+R_3$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_2+R_4$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_2+R_3$$

$$\sim\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right] R_1+R_2$$

So $A^{-1} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.

Q30 is similarly to Q29.

Q32 is similarly to Q31.

Q31 (a) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_2+R_1$

$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right] R_3+R_2$

it's nontrivial so matrix is singular.

(b) $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$.

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \begin{matrix} R_2+R_1 \\ R_3+R_1 \end{matrix}$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] R_2+R_3$ As inverse exist so it's trivial



Q1 $A = \begin{bmatrix} 2 & 8 & 0 \\ 2 & 2 & -3 \\ 1 & 2 & 7 \end{bmatrix}$; $b = \begin{bmatrix} 18 \\ 3 \\ 12 \end{bmatrix}$, $L = \begin{bmatrix} 2 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & -1 & 4 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

Now $UX = Z$ —(i) $LZ = b$ —(2)

eq(2) => $LZ = b$

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 18 \\ 3 \\ 12 \end{bmatrix}$$

$2z_1 = 18$ —(i) $2z_1 - 3z_2 = 3$ —(2)

$z_1 - z_2 + 4z_3 = 12$ —(3)

solving the above equation we get

$\boxed{z_1 = 9}$ $\boxed{z_2 = 5}$ $\boxed{z_3 = 2}$

eq(1) => $UX = Z$

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$x_1 + 4x_2 = 9$ —(1)

$2x_2 + x_3 = 5$ —(2)

$2x_3 = 2$ —(3)

From above equation we get

$\boxed{x_1=1}$ $\boxed{x_2=2}$ $\boxed{x_3=1}$

$\therefore X=\begin{bmatrix}1\\2\\1\end{bmatrix}$ Ans.

Q2 & Q1 is similarly to Q1

Q5 $A=\begin{bmatrix}2 & 3 & 4\\4 & 5 & 10\\4 & 8 & 2\end{bmatrix}$ ; $b=\begin{bmatrix}6\\16\\2\end{bmatrix}$.

For matrix U (upper triangular)

$\sim\begin{bmatrix}2 & 3 & 4\\0 & -1 & 2\\0 & 2 & -6\end{bmatrix}$ $R_2-2R_1$, $R_3-2R_1$

$\sim\begin{bmatrix}2 & 3 & 4\\0 & -1 & 2\\0 & 0 & -2\end{bmatrix}$ $R_3+2R_2$



For matrix L (lower triangular)

$L=\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\2 & -2 & 1\end{bmatrix}$

Now $LZ=b$

$\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\2 & -2 & 1\end{bmatrix}\begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}=\begin{bmatrix}6\\16\\2\end{bmatrix}$

$z_1=6$ — (1)

$2z_1+z_2=16$ — (2)

$2z_1-2z_2+z_3=2$ — (3)

Solving we get

$\boxed{z_1=6}$ $\boxed{z_2=4}$ $\boxed{z_3=-2}$

Now solving

$UX=Z$

$\begin{bmatrix}2 & 3 & 4\\0 & -1 & 2\\0 & 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}6\\4\\-2\end{bmatrix}$

$2x_1+3x_2+4x_3=6$ — (1)

$-x_2+2x_3=4$ — (2)

$-2x_3=-2$ — (3)

Now we get

$\boxed{x_1 = 4}$ $\boxed{x_2 = -2}$ $\boxed{x_3 = 1}$

$$X = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

Q6 — to Q10 similarly to Q5

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Electrical Engg:





Q1 $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ & $V = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix}$. (1,1) (2,1) (1,3) (2,3)

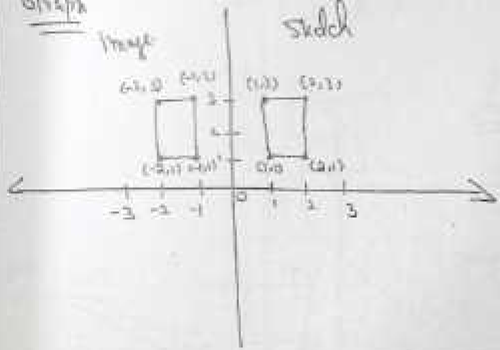
Given $f(v) = AV$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 3 & 3 \end{bmatrix}.$$

$$= \begin{bmatrix} -1\times1+0\times1 & -1\times2+0\times1 & -1\times1+0\times3 & -1\times2+0\times3 \\ 0\times1+1\times1 & 0\times2+1\times1 & 0\times1+1\times3 & 0\times2+1\times3 \end{bmatrix}$$

$$f(v) = \begin{bmatrix} -1 & -2 & -1 & -2 \\ 1 & 1 & 3 & 3 \end{bmatrix} \text{ image.}$$

Graph

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Q2 Let $f$ be the shear in the x-direction

$$f(v) = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} V$$

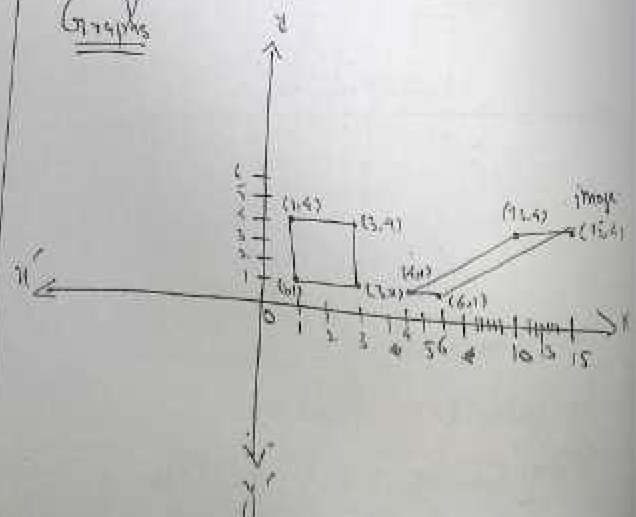
when $k = 3$ & $V = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

$$f(v) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} V$$

$$\Rightarrow f(v) = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 & 6 & 15 \\ 1 & 4 & 1 & 4 \end{bmatrix} \text{ Image}$$

Graphs



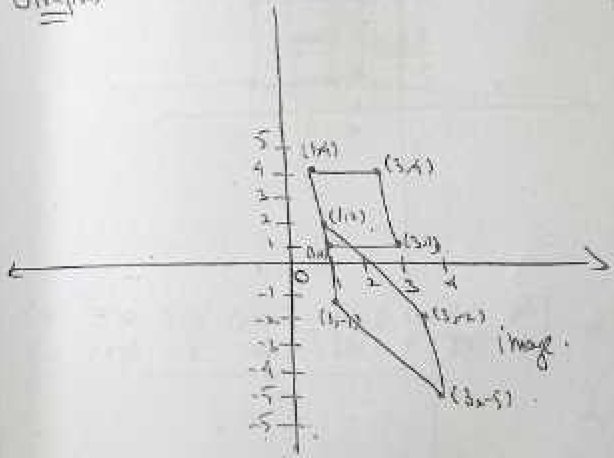

Q3 $A = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ & $k = -2$ then $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$.

& $V = \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

Now $f(v) = AV = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 & 3 \\ 1 & 4 & 1 & 4 \end{bmatrix}$.

$$= \begin{bmatrix} 1 & 1 & 3 & 3 \\ -1 & 2 & -5 & -2 \end{bmatrix} \text{ Image.}$$

Graphs



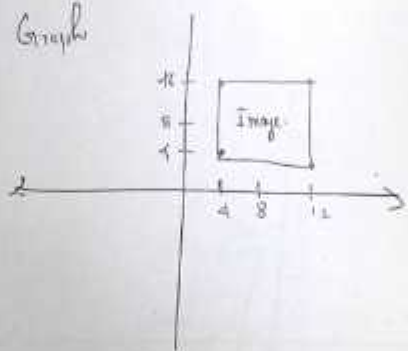
Q4(a) $A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$. $k=4$

$V=\begin{bmatrix}1 & 1 & 3 & 3\\ 1 & 4 & 1 & 4\end{bmatrix}$.

$f(v)=AV=\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & 1 & 3 & 3\\ 1 & 4 & 1 & 4\end{bmatrix}$

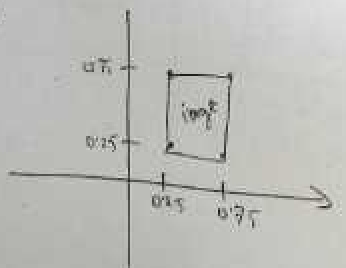
$=\begin{bmatrix}4 & 4 & 12 & 12\\ 4 & 16 & 4 & 16\end{bmatrix}$ image.

Graph




(b) $k=1/4$ $\begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix}\begin{bmatrix}1 & 1 & 3 & 3\\ 1 & 4 & 1 & 4\end{bmatrix}=\begin{bmatrix}0.25 & 0.25 & 0.75 & 0.75\\ 0.25 & 1 & 0.25 & 1\end{bmatrix}$

Graph



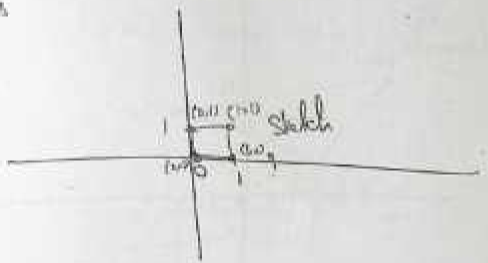

Q5 $k=2$. $A=\begin{bmatrix}k & 0\\ 0 & 1\end{bmatrix} \Rightarrow A=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$.

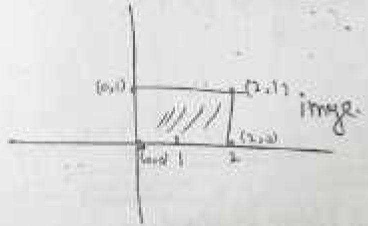
$V=\begin{bmatrix}0 & 0 & 1 & 1\\ 0 & 1 & 0 & 1\end{bmatrix}$

$f(v)=AV=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 1\\ 0 & 1 & 0 & 1\end{bmatrix}$

$=\begin{bmatrix}0 & 0 & 2 & 2\\ 0 & 1 & 0 & 1\end{bmatrix}$. image

Graphs




Q6 is similarly to Q5

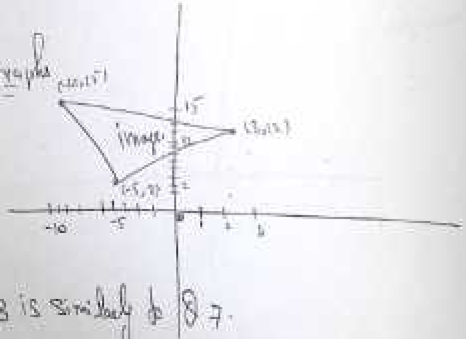
**Q7** $f(v) = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix} V$

$T = V = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & -1 \end{bmatrix}$.

$f(v) = \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} -10 & 3 & -5 \\ 15 & 12 & 2 \end{bmatrix}$.

Graph




Q8 is similarly to Q7.



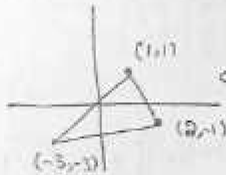
**Q9** $T = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$ & $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ & $\theta = 60°$

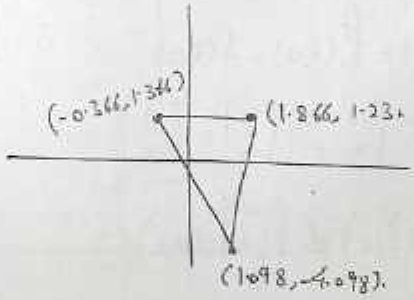
$A = \begin{bmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 \\ 0.866 & 0.5 \end{bmatrix}$ (Exp 9 Ex 1.5)

$f(T) = AT = \begin{bmatrix} 0.5 & -0.866 \\ 0.866 & 0.5 \end{bmatrix}\begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$

$= \begin{bmatrix} -0.366 & 1.098 & 1.866 \\ 1.366 & -4.098 & 1.232 \end{bmatrix}$

Graph



Q10 (Assig.)

$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (refl. y-axis) & $u = \begin{bmatrix} x \\ y \end{bmatrix}$.

$f_2(u) = Au = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$

$f_1(f_2(u)) = f_1(Au) = B(Au)$

$B = \begin{bmatrix} \cos\pi/2 & \sin\pi/2 \\ -\sin\pi/2 & \cos\pi/2 \end{bmatrix}$ (for y-axis).

$B(Au) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$

Now

$f_1(u) = \begin{bmatrix} \cos\pi/2 & \sin\pi/2 \\ -\sin\pi/2 & \cos\pi/2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$

$f_2(f_1(u)) = f_2(Bu) = A(Bu)$

$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} y \\ -x \end{bmatrix} = \begin{bmatrix} -y \\ -x \end{bmatrix}$.

Hence $f_1(f_2(u)) \neq f_2(f_1(u))$.



Q11 $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $T = \begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$.

$f(v) = Av = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & -3 & 2 \\ 1 & -3 & -1 \end{bmatrix}$.

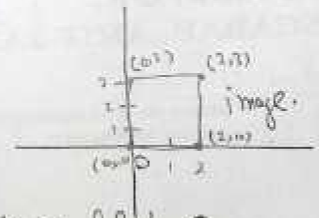
$= \begin{bmatrix} 3 & -9 & 0 \\ 6 & -18 & 0 \end{bmatrix}$.

Q12 $A = \begin{bmatrix} h & 0 \\ 0 & k \end{bmatrix}$ & $h=2,\ k=3$ (Given)

$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$, $V = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

$AV = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$.

$= \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{bmatrix}$.




Q13 is similarly to Q12.

The transition Probabilities arranged in $n \times n$ matrix $T = [t_{ij}]$ is called transition matrix of markov chain

where

$$t_{1j} + t_{2j} + t_{3j} + \cdots + t_{nj} = 1 \quad \text{—— (1)}$$

entries in each column of T are non-negative & added upto 1 as in (1)

(a) not transition b/c $a_{11} + a_{21} = 0.3 + 0.4 \neq 1$

(b) transition as
$0.2 + 0.8 + 0.0 = 1$
$0.3 + 0.5 + 0.2 = 1$
$0.1 + 0.7 + 0.2 = 1.$

(c) transition because
$0.55 + 0.45 = 1$
$0.33 + 0.67 = 1.$

(d) not transition because All each column are not equal to 1.

<u>Probability vector</u>

The vector $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is called Probability vector if

$u_1 + u_2 + \cdots u_n = 1$

Q2 (a) $\frac{1}{2}+\frac{1}{3}+\frac{2}{3} = \frac{3+2+4}{6} = 9/6 \neq$ not P.V.

(b) $0+1+0=1$ Prob. value

(c) $\frac{1}{4}+\frac{1}{6}+\frac{1}{3}+1/4 = \frac{3+2+4+3}{12} = \cancel{12}/\cancel{12} = 1$ Prob.

(d) $\frac{1}{5}+\frac{2}{5}+\frac{1}{10}+\frac{2}{10} = \frac{2+4+1+2}{10} = \frac{9}{10} \neq 1$ not P.V

Q3 $\begin{bmatrix} \square & 0.4 & 0.3 \\ 0.3 & \square & 0.5 \\ \square & 0.2 & \square \end{bmatrix}$

For j=1

$a_{11}+a_{21}+a_{31}=1$

$\square + 0.3 + \square = 1$

$a_{11}+a_{31} = 1-0.3 = 0.7$

$a_{11}+a_{31}=0.7$ So there are more than Possible values $(0.5+0.2)$ etc ).

For j=2

$a_{12}+a_{22}+a_{32}=1$

$0.4 + \boxed{a_{22}} + 0.2 = 1$

$a_{22} = 1-0.2-0.4 = 0.4 \quad = \boxed{a_{22}=0.4}$

For j=3

$a_{13}+a_{23}+a_{33}=1$

$0.3+0.5+a_{33}=1$

$a_{33}=1-0.3-0.5=0.2$

$\boxed{a_{33}=0.2}$

Q4 j=1

$0.2+0.3+a_{13}=1$

$a_{13}=1-0.2-0.3=0.5$

$\boxed{a_{13}=0.5}$

j=2 $0.1+a_{22}+a_{32}=1$

$a_{22}+a_{32}=1-0.1=0.9$ (So there are more than possible values $\boxed{a_{22}+a_{32}=0.9}$

j=3

$0.3+0.5+a_{33}=1$

$a_{33}=1-0.3-0.5=0.2$

$\boxed{a_{33}=0.2}$

Q5 $T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}$.

(a) If $x^{(0)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$x^{(1)} = Tx^{(0)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix}.$$

$$x^{(2)} = Tx^{(1)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.61 \\ 0.39 \end{bmatrix}.$$

$$x^{(3)} = Tx^{(2)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.61 \\ 0.39 \end{bmatrix} = \begin{bmatrix} 0.583 \\ 0.417 \end{bmatrix}.$$

(b) Show that T is regular.

Now $T^2 = T \cdot T = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.61 & 0.52 \\ 0.39 & 0.48 \end{bmatrix}$.

As all entries are non-zero so it's regular.

T is regular if all entries in some Power of T are Positive

Steady state vector

$0.575 + 0.425 = 1$

$$x^{(4)} = Tx^{(3)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.583 \\ 0.417 \end{bmatrix} = \begin{bmatrix} 0.575 \\ 0.425 \end{bmatrix}$$

$$x^{(5)} = Tx^{(4)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.575 \\ 0.425 \end{bmatrix} = \begin{bmatrix} 0.573 \\ 0.428 \end{bmatrix}$$

$$x^{(6)} = Tx^{(5)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.573 \\ 0.428 \end{bmatrix} = \begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix}.$$

$$x^{(7)} = Tx^{(6)} = \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix}\begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix} = \begin{bmatrix} 0.572 \\ 0.429 \end{bmatrix}$$ same

It's steady state vector.

Q6 is similarly to Q5

Q7 (a) $T^2 = \begin{bmatrix} 0 & 1/2 \\ 1 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1/2 \\ 1 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/4 \\ 1/2 & 3/4 \end{bmatrix}$.

So no entry is 0 in $T^2$ so it's regular

(b) $T^2 = T \cdot T = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 1/2 \\ 1/2 & 0 & 1/2 \end{bmatrix}$

$$= \begin{bmatrix} 1/4 & 0 & 0 \\ 1/4 & 1 & 3/4 \\ 1/2 & 0 & 1/4 \end{bmatrix}$$

So all entries in $T^2$ are not non-zero so its not regular.

* Q (c) & d Part are Similarly to a & b Part *

Ex 2.5



Q9 (a) $T^2 = T \cdot T = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1/2 & 0 \\ 3/4 & 1 \end{bmatrix}$.

it's not regular because $t_{12} = 0$.

(b) let $x^{(0)} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$ (standard)

$x^{(1)} = Tx^{(0)} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$.

$x^{(2)} = Tx^{(1)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}$

$x^{(3)} = Tx^{(2)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix} = \begin{bmatrix} 0.025 \\ 0.975 \end{bmatrix}$

$x^{(4)} = Tx^{(3)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.025 \\ 0.975 \end{bmatrix} = \begin{bmatrix} 0.0125 \\ 0.9875 \end{bmatrix}$

$x^{(5)} = Tx^{(4)} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 0.013 \\ 0.99 \end{bmatrix} \rightarrow \begin{bmatrix} 0.00 \\ 1 \end{bmatrix}$

Q10 (a) $T = \begin{bmatrix} 1/3 & 1/2 \\ 2/3 & 1/2 \end{bmatrix}$

$T^2 = \begin{bmatrix} 0.333 & 0.5 \\ 0.6667 & 0.5 \end{bmatrix}$

$T^3 = \begin{bmatrix} 0.4444 & 0.4166 \\ 0.5555 & 0.5833 \end{bmatrix}$

$T^4 = \begin{bmatrix} 0.4259 & 0.4305 \\ 0.574 & 0.5694 \end{bmatrix}$

$T^5 = \begin{bmatrix} 0.429 & 0.4282 \\ 0.5709 & 0.5717 \end{bmatrix}$

$T^6 = \begin{bmatrix} 0.4284 & 0.4286 \\ 0.5715 & 0.5719 \end{bmatrix}$

$$\left.\begin{array}{l} T^7 = \begin{bmatrix} 0.4285 & 0.4285 \\ 0.5714 & 0.5714 \end{bmatrix} \\ T^8 = \begin{bmatrix} 0.4285 & 0.4285 \\ 0.5714 & 0.5714 \end{bmatrix} \end{array}\right\} \text{same}$$

Steady state.

Q3 $\vec{PQ} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ where $P = (3,2)$ $Q = ?$ let $Q(a,b)$

Then $\vec{PQ} = (a-3, b-2) = (-2,5)$

$a-3 = -2 \Rightarrow \boxed{a=1}$ & $b-2 = 5 \Rightarrow \boxed{b=7}$

Q4 is similarly to Q3.

Q5 Find $u+v$, $u-v$, $2u$ & $3u-3v$ if

(a) $u = (2,3)$, $v = (-2,5)$.

$u+v = (2-2, 3+5) = (0,8)$

$u-v = (2+2, 3-5) = (4,-2)$

$2u = 2[2,3] = (4,6)$.

$3u-2v = 3[2,3] - 2[-2,5] = (6,9) - (-4,10)$

$= (6+4, 9-10)$

$= (10,-1)$.

(b) & (c) Part similarly to Part(a).

Q6 is similarly to Q5.

Q7 $u = (1,2)$, $v = (-3,4)$, $w = (w_1, 4)$ & $x = (-2, x_2)$.

(a) $w = 2u$

$(w_1, 4) = 2(1,2)$

$(w_1, 4) = (2, 4)$

$\Rightarrow w_1 = 2$

(b) $\frac{3}{2}x = v$

$\frac{3}{2}(-2, x_2) = (-3, 4)$

$(-3, \frac{3}{2}x_2) = (-3, 4)$

$\Rightarrow \frac{3}{2}x_2 = 4 \Rightarrow \boxed{x_2 = 8/3}$

Q8 is similarly to Q7.

Q9 (a) $(1, 2)$

$\|u\| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$.

b, c, d is similarly to a

Q10 is similarly to Q9.

Q11 (a) $\|u\| = \sqrt{(3-2)^2 + (4-3)^2} = \sqrt{1+1} = \sqrt{2}$.

Q12 is similarly to Q11.



Q13 $(-5, 6) = c_1(1, 2) + c_2(3, 4)$ — (*)

$= (c_1 + 2c_1) + (c_2 3 + c_2 4)$.

$(-5, 6) = (c_1 + 3c_2, 2c_1 + 4c_2)$

$c_1 + 3c_2 = -5$ — ①

$2c_1 + 4c_2 = 6$

$\div 2$

$\Rightarrow c_1 + 2c_2 = 3$ — ②

① − ②

$\boxed{c_2 = -8}$

eq ① $\Rightarrow c_1 - 24 = -5 \Rightarrow \boxed{c_1 = 19}$

eq (*) $\Rightarrow (-5, 6) = 19(1, 2) - 8(3, 4)$

Q14 $c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}3\\4\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1\\2c_1\end{bmatrix} + \begin{bmatrix}3c_2\\4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}c_1 + 3c_2\\2c_1 + 4c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}\begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

Augmented form

$$\left[\begin{array}{cc:c} 1 & 3 & 0 \\ 2 & 4 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc:c} 1 & 3 & 0 \\ 0 & -2 & 0 \end{array}\right] R_2 - 2R_1$$

$-2x_2 = 0 \Rightarrow \boxed{x_2 = 0}$

So it is impossible

Q15

$$= \frac{1}{2}\left|\det\left(\begin{bmatrix} 3 & 3 & 1 \\ -1 & -1 & 1 \\ 4 & 1 & 1 \end{bmatrix}\right)\right|$$

$$= \frac{1}{2}\left|3\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|3(-1-1) + 1(3-1) + 4(3+1)\right|$$

$$= \frac{1}{2}\left|3(-2) + 2 + 4(4)\right|$$

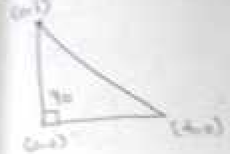
$$= \frac{1}{2}\left|-6 + 2 + 16\right|$$

$$= \frac{1}{2}\left|18 - 6\right|$$

$$= \frac{1}{2}(12) = 6 \text{ Ans}$$



Q16



$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(4)(3) = 6.$$

or

$$= \frac{1}{2}\left|\det\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 1 \\ 4 & 0 & 1 \end{bmatrix}\right)\right|$$

$$= \frac{1}{2}\left|4\begin{pmatrix} 0 & 1 \\ 3 & 1 \end{pmatrix}\right|$$

$$= \frac{1}{2}\left|4(0-3)\right| = \frac{1}{2}\left|-12\right| = \frac{1}{2}(12) = 6 \text{ Ans}$$

Q17 Area $= \left|\det\left(\begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}\right)\right|$ (P-221).

$$= \left|2\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} - 5\begin{vmatrix} 3 & 1 \\ 5 & 1 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix}\right|$$

$$= \left|2(3-5) - 5(3-5) + 4(3-3)\right|$$

$$= \left|-4 + 10\right| = |6|$$

$$= 6.$$

Q19 (a) $x=(3,4)$

$= \frac{3,4}{\sqrt{3^2+4^2}} = \frac{3,4}{\sqrt{25}} = \frac{3,4}{5} = \left(\frac{3}{5}, \frac{4}{5}\right)$

b & c part similarly to part "a"

Q20 is similarly to Q19.

Q21 (a) $\cos\theta = \frac{u\cdot v}{\|u\|\|v\|}$ — ① $u=(1,2)$, $v=(2,-3)$

where $\|u\| = \sqrt{1^2+2^2} = \sqrt{5}$

& $\|v\| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \sqrt{13}$.

$u\cdot v = (1,2)\cdot(2,-3) = (1)(2)+(2)(-3) = 2-6=-4$

eq ① $\Rightarrow \cos\theta = \frac{-4}{\sqrt{5}\sqrt{13}}$

Q(b) (c) & (d) is similarly to Part (a).

Q22 is similarly to Q21.

Q23 is S.Y.S

Q24 (a) For orthogonal? $u\cdot v=0$

$u_1\cdot u_6 = (i+2j)\cdot(-6i+3j) = (-6+6) = 0$

Sm $u_1\cdot u_4 = (i+2j)\cdot(-2i+j) = (-2+2) = 0$

$u_3\cdot u_4 = (-2i-4j)\cdot(-2i+j) = (4-4) = 0$

$u_3\cdot u_6 = (-2i-4j)\cdot(-6i+3j) = (12-12) = 0$

$u_4\cdot u_5 = (-2i+j)\cdot(2i+4j) = (-4+4) = 0$

$u_5\cdot u_6 = (2i+4j)\cdot(-6i+3j) = (-12+12) = 0$

(b) $u_1$ & $u_5$ because $u_5 = 2u_1$

$u_4$ & $u_6$ because $u_6 = 3u_4$

(c) $u_1$ & $u_3$ because $u_3 = -2u_1$

$u_3$ & $u_5$ because $u_3 = -u_5$

Q25 Find slope $m_1$ = slope $m_2$ (vector are parallel if slopes are equal).

$m_1 = 4/a$, $m_2 = 5/2$

i.e $4/a = 5/2$

$5a = 8$

$\boxed{a = 8/5}$

Scanned by CamScanner

**Q26** vectors are orthogonal if $\cos\theta = 0$.

$(a, 2)\cdot(a, -2) = 0$

$a^2 + (-4) = 0$

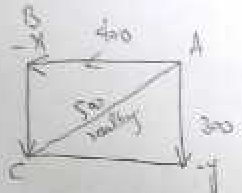
$a^2 = 4$

$\boxed{a = \pm 2}$

**Q27** (a) $i + 3j$, (b) $-2i - 3j$ (c) $-2i$ (d) $3j$

**Q28** (a) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$, (b) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$.

**Q29**




$u = 0i - 300j$ & $v = -400i + 0j$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$= \vec{v} + \vec{u}$

$= (-400i + 0j) + (0i - 300j) = -400i - 300j$

$|\overrightarrow{AC}| = \sqrt{(400)^2 + (300)^2} = 500$



**Q1** (a) $u = (1, 2, -3)$, $v = (0, 1, -2)$

$u + v = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+0 \\ 2+1 \\ -3-2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$

$u - v = \begin{bmatrix} 1-0 \\ 2-1 \\ -3+2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

$2u = 2\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$

$3u - 2v = 3\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} - 2\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix} - \begin{bmatrix} 0 \\ 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 3-0 \\ 6-2 \\ -9+4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$

Part b is same as "a"

Q2 as same as Q1

**Q3** $u = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} a \\ -1 \\ b \end{bmatrix}$, $x = \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix}$.

Find a, b & c so that

(a) $w = \frac{1}{2}u \Rightarrow \begin{bmatrix} a \\ -1 \\ b \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1 \\ 3/2 \end{bmatrix}$

$\Rightarrow a = 1/2$

$b = 3/2$.

(b) $w+v=u$

$\Rightarrow \begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} a-3 \\ -2 \\ b+3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

$a-3=1 \Rightarrow a=1+3=4 \Rightarrow \boxed{a=4}$

$b+3=3 \Rightarrow \boxed{b=0}$

(c) $w+x=v$

$\begin{bmatrix} a \\ -1 \\ b \end{bmatrix} + \begin{bmatrix} 3 \\ c \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+3 \\ c-1 \\ b+2 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix} \Rightarrow a+3=-3=-3-3=-6 \Rightarrow \boxed{a=-6}$

$c-1=-1 \Rightarrow \boxed{c=0}$ $b+2=3 \Rightarrow \boxed{b=1}$.

Q4 as same as Q3.



Q5 $u=(4,5,-2,3)$ & $v=(3,-2,0,1)$, $c=2$, $d=3$

(a) $u+v=v+u$

$u+v=(4+3, 5-2, -2+0, 3+1)=(7,3,-2,4)$

$v+u=(3+4, -2+5, 0-2, 1+3)=(7,3,-2,4)$.

(b) $u+(v+w)=(u+v)+w$.

L.H.S $u+(v+w)$

$(4,5,-2,3)+[(3,-2,0,1)+(-3,2,-5,3)]$.

$(4,5,-2,3)+(0,0,-5,4)$

$(4,5,-7,7)$.

R.H.S $(u+v)+w$

$[(4,5,-2,3)+(3,-2,0,1)]+(-3,2,-5,3)$.

$(7,3,-2,4)+(-3,2,-5,3)$

$(4,5,-7,7)$.

Hence $u+(v+w)=(u+v)+w$

(c) $u+0=0+u$.

L.H.S $=(4,5,-2,3)+(0,0,0,0)=(4,5,-2,3)$

R.H.S $(0,0,0,0)+(4,5,-2,3)=(4,5,-2,3)$.

(d) $u+(-u)=0$

$(4,5,-2,3)+(-(4,5,-2,3))$

$(4,5,-2,3)+(-4,-5,+2,-3)$

$(4-4,5-5,-2+2,3-3)$

$(0,0,0,0)$.

(e) $c(u+v)=cu+cv$

$L.H.S = 2[(4,5,-2,3)+(3,-2,0,1)]$

$=2(7,3,-2,4)$

$=(14,6,-4,8)$

$R.H.S \quad 2(4,5,-2,3)+2(3,-2,0,1)$

$=(8,10,-4,6)+(6,-4,0,2)$

$=(14,6,-4,8)$

(f) $(c+d)u=cu+du$

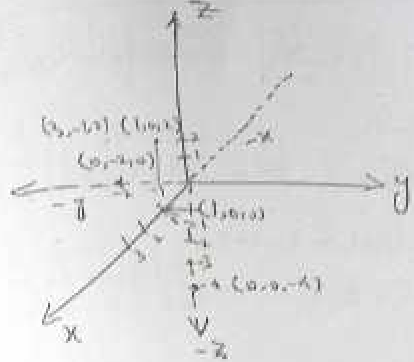
$L.H.S \quad (2+3)(4,5,-2,3)=5(4,5,-2,3)=(20,25,-10,15)$

$R.H.S \quad 2(4,5,-2,3)+3(4,5,-2,3)$

$(8,10,-4,6)+(12,15,-6,9)$

$=(20,25,-10,15)$

Q6




Q7 Same as Q.6

Q8 (a) P(1st Point), Q(Second Point).

$\overrightarrow{PQ}=\vec{Q}-\vec{P}=(0,0,2)-(2,3,-1)$

$=(-2,-3,3)$

$\overrightarrow{PQ}=\begin{bmatrix}-2\\-3\\3\end{bmatrix}$.

Part b, c, d Same as Part a.

Q9 Find $Q = ?$ $P = (1,-2,3)$ $\vec{PQ}$ (vector) $= (3,4,-1)$.

$\vec{PQ} = \vec{Q} - \vec{P}$

$\Rightarrow \vec{Q} = \vec{PQ} + \vec{P} = (3,4,-1) + (1,-2,3)$

$\vec{Q} = (3+1, 4-2, -1+3)$

$\vec{Q} = (4, 2, 2)$.

Q10 (a) $\|u\| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{1+4+9} = \sqrt{14} = \sqrt{14}$.

Part b, c, d same as part a.

Q11 Same Q10.

Q12 (a) $(1,-1,2)$, $(3,0,2)$.

$\|u - v\| = \sqrt{(1-3)^2 + (-1+0)^2 + (2-2)^2}$.

$= \sqrt{4+1+0}$

$= \sqrt{5}$.

Part b, c, d Same as part a.

Q13 Same as Q12.

Q14 $a\begin{bmatrix}1\\2\\-3\end{bmatrix} + b\begin{bmatrix}-1\\1\\1\end{bmatrix} + c\begin{bmatrix}-1\\4\\-1\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$\begin{bmatrix}a\\2a\\-3a\end{bmatrix} + \begin{bmatrix}-b\\b\\b\end{bmatrix} + \begin{bmatrix}-c\\4c\\-c\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$\begin{bmatrix}a-b-c\\2a+b+4c\\-3a+b-c\end{bmatrix} = \begin{bmatrix}2\\-2\\3\end{bmatrix}$.

$a - b - c = 2$ —— (1)

$2a + b + 4c = -2$ —— (2)

$-3a + b - c = 3$ —— (3)

add (-2) time first to (2)

$-\cancel{2a} + 2b + 2c = -4$

$\cancel{2a} + b + 4c = -2$

$3b + 6c = -6$

$b + 2c = -2$ —— (2)'

add (3) times (1) to (3).

$\cancel{3a} - 3b - 3c = 6$

$-\cancel{3a} + b - c = 3$

$-2b - 4c = 9$

$2b + 4c = -9$ —— (3)'

add (-2) times (2)' to (3)'

$-\cancel{2b}-\cancel{4c}=4$
$\cancel{2b}+\cancel{4c}=-9$

$0=-5$

which is impossible.

Q15 $c_1\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_2\begin{bmatrix}1\\3\\-2\end{bmatrix}+c_3\begin{bmatrix}3\\7\\-4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}c_1\\2c_1\\-c_1\end{bmatrix}+\begin{bmatrix}c_2\\3c_2\\-2c_2\end{bmatrix}+\begin{bmatrix}3c_3\\7c_3\\-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\begin{bmatrix}c_1+c_2+3c_3\\2c_1+3c_2+7c_3\\-c_1-2c_2-4c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$c_1+c_2+3c_3=0$ — ①
$2c_1+3c_2+7c_3=0$ — ②
$-c_1-2c_2-4c_3=0$ — ③

add (-2) times ① to ②

$-\cancel{2c_1}-2c_2-6c_3=0$
$\cancel{2c_1}+3c_2+7c_3=0$

$c_2+c_3=0$

$\boxed{c_2=-c_3}$

add (+2) times ① to ③

$2c_1+\cancel{2c_2}+6c_3=0$
$-c_1-\cancel{2c_2}-4c_3=0$

$c_1+2c_3=0$

$\boxed{c_1=-2c_3}$

let $c_3=r$

then $c_2=-r$ & $c_1=-2r$

For possible solution put $r=1$ (unit value)

$c_3=1$, $c_2=-1$, $c_1=-2(1)=-2$

then $\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}-2\\-1\\1\end{bmatrix}$

Q16 $\|(1,a,-3,2)\|=5$

$\sqrt{(1)^2+(a)^2+(-3)^2+(2)^2}=5$

$\sqrt{14+a^2}=5$

Sq: B.S

$14+a^2=25$
$a^2=25-14=11 \Rightarrow \boxed{a=\pm\sqrt{11}}$

$Q7$ $u=(a,2,1,a)$ & $v=(a,-1,-2,-3)$.

& $u\cdot v=0$.

$(a,2,1,a)\cdot(a,-1,-2,-3)=0$

$(a^2-2,-2-3a)=0$

$a^2-3a-4=0$

$a=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}$

$a=\frac{3\pm\sqrt{9+16}}{2}$

$a=\frac{3\pm\sqrt{25}}{2}=\frac{3\pm5}{2}=\frac{8}{2},-\frac{2}{2}$

$\boxed{a=4,-1}$

$Q8$ $c=3$, $u=(1,2,3)$, $v=(1,2,-4)$, $w=(1,0,2)$.

(a) $u\cdot u\geq 0$

$(1,2,3)\cdot(1,2,3)\geq 0$

$1+4+9\geq 0$

$14\geq 0$.



(iv) $u\cdot v=-7=v\cdot u$.

(v) $(u+v)\cdot w=u\cdot w+v\cdot w$.

L.H.S $[(1,2,3)+(1,2,-4)]\cdot(1,0,2)$

$=(2,4,-1)\cdot(1,0,2)$

$=\cancel{(2+0-2)}$

$=(2-2)=0$.

R.H.S $(1,2,3)\cdot(1,0,2)+(1,2,-4)(1,0,2)$

$(1+6)+(1-8)$

$(7-7)=0$.

(vi) $|u\cdot v|\leq\|u\|\|v\|$

$|u\cdot v|=|(1,2,3)\cdot(1,2,-4)|$

$=|1^2+2^2+3(-4)|=|1+4-12|=|-7|=7$.

$\|u\|=\sqrt{u\cdot u}=\sqrt{(1,2,3)\cdot(1,2,3)}=\sqrt{1^2+4+9}=\sqrt{14}$

$\|v\|=\sqrt{v\cdot v}=\sqrt{(1,2,-4)(1,2,-4)}=\sqrt{1+4+16}=\sqrt{21}$

$\|u\|\|v\|=\sqrt{14}\sqrt{21}=\sqrt{14\times21}=7\sqrt{6}$.

So $7\leq7\sqrt{6}$ varified.

Q20 (a) $U=(1,2,3)$, $V=(-4,4,5)$.

$$\cos\theta = \frac{U\cdot V}{\|U\|\,\|V\|} \quad \text{—①}$$

$U\cdot V = (1,2,3)\cdot(-4,4,5) = (-4+8+15) = 19.$

$\|U\| = \sqrt{1+2^2+3^2} = \sqrt{14}$

$\|V\| = \sqrt{(-4)^2+(4)^2+5^2} = \sqrt{57}$.

Eq ① ⇒ $\cos\theta = \dfrac{19}{\sqrt{14}\sqrt{57}} = 0.673.$

(b) $U=(0,2,3,1)$ $V=(-3,1,-2,0)$

$$\cos\theta = \frac{U\cdot V}{\|U\|\cdot\|V\|} \quad \text{—①}.$$

$U\cdot V = (0,2,3,1)\cdot(-3,1,-2,0) = (0+2-6+0) = -4.$

$\|U\| = \sqrt{4+9+1} = \sqrt{14}$ $\|V\| = \sqrt{9+4+1} = \sqrt{14}$.

Eq ① ⇒ $\cos\theta = \dfrac{-4}{\sqrt{14}\sqrt{14}} = \dfrac{-4}{14} = \dfrac{-2}{7}$

$\boxed{\cos\theta = -2/7}$.

c+d part similarly.

Q21 is similarly to Q20.

Q22 S.Y.S

Q23 is similarly Q24 (Ex 4.1).

Q24 For orthogonal $V\cdot W = 0$.

$(2,c,3)\cdot(1,-2,1) = 0$

$2-2c+3 = 0$

$-2c = -5$

$\boxed{c = 5/2}$

Q25 $V=(a,b,c)$, $W=(1,2,1)$ $X=(1,-1,1)$

$V\cdot W = (a,b,c)\cdot(1,2,1) = (a+2b+c) = 0$ —①

$V\cdot X = (a,b,c)\cdot(1,-1,1) = (a-b+c) = 0$ —②

① − ②

$\cancel{a}+2b+\cancel{c} = 0$

$\cancel{a}-b+\cancel{c} = 0$

$3b = 0$ ⇒ $\boxed{b=0}$.

eq ① ⇒ $a+c=0 \Rightarrow a=-c$

eq ② ⇒ $a+c=0 \Rightarrow a=-c$.

Let $c=r$ then $a=-r$, $b=0$

For possible solution put $r=1$

$c=1$, $a=-1$, $b=0$.

$\boxed{a=-1, b=0, c=1}$

Q.. triangle inequality $\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

$u=(1,2,3,-1)$ & $v=(1,0,-2,3)$

$u+v=(2,2,1,2)$

$\|u+v\| = \sqrt{(2)^2+(2)^2+(1)^2+(2)^2} = \sqrt{4+4+1+4} = \sqrt{13}$

$\|u+v\|^2 = (\sqrt{13})^2 = 13$ — ①

$\|u\| = \sqrt{1+4+9+1} = \sqrt{15}$

$\|v\| = \sqrt{(1)^2+(0)^2+(-2)^2+(3)^2} = \sqrt{1+4+9} = \sqrt{14}$

Now $(\|u\|+\|v\|)^2 = (\sqrt{15}+\sqrt{14})^2 = 57.98 = 58$ — ②

From ① & ② we

$\|u+v\|^2 \le (\|u\|+\|v\|)^2$.

Q.. $X=(2,-1,3)$

unit vector in the direction of $X$

then $u = \frac{X}{\|X\|}$ — ①

$\|X\| = \sqrt{4+1+9} = \sqrt{14}$

eq ① ⇒ $u = \frac{(2,-1,3)}{\sqrt{14}} = \left(\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

Part b, c, d are same as part (a).

Q28 is same as Q27.

Q.. (a) $(1,2,-3) = i+2j-3k$.

Part b, c, d are same as part (a).

Q.. (a) $2i+3j-4k$

$2i = 2I_1 = 2\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\0\\0\end{bmatrix}$

$3j = 3I_2 = 3\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}0\\3\\0\end{bmatrix}$

$-4k = -4I_3 = -4\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\-4\end{bmatrix}$

3×1 matrix $\begin{bmatrix}2\\0\\0\end{bmatrix}+\begin{bmatrix}0\\3\\0\end{bmatrix}+\begin{bmatrix}0\\0\\-4\end{bmatrix} = \begin{bmatrix}2\\3\\-4\end{bmatrix}_{3\times1}$.

(Part b, c, d same as part a)

Q3. For isosceles triangle two sides are equal.

$\|P_1 - P_2\| = \sqrt{[illegible]} = \sqrt{41}$

$\|P_2 - P_3\| = \sqrt{[illegible]} = \sqrt{100} = 10$

$\|P_3 - P_1\| = \sqrt{[illegible]} = \sqrt{41}.$

From above we have

$\|P_1 - P_2\| = \|P_3 - P_1\|$ verified.

—— x ——

Q. For right triangle angle b/w two sides are 90° &

$\cos\theta = 0$

$\vec{u} = \overrightarrow{P_1P_2} = (3-2, 1-3, 2+4) = (1, -2, 6)$

$\vec{v} = \overrightarrow{P_1P_3} = (7-2, 0-3, 1+4) = (5, -3, 5)$

$\vec{w} = \overrightarrow{P_2P_3} = (7-3, 0-1, 1-2) = (4, -1, -1)$

$\vec{u}\cdot\vec{v} = (5+6+30) = 41$

$\vec{v}\cdot\vec{w} = (5+6+30) = 41$

$\vec{u}\cdot\vec{w} = (4+2-6) = 0.$





Q. $U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{2000} \end{bmatrix}$

8 % increase

$$U = \begin{bmatrix} u_1 + 0.08u_1 \\ u_2 + 0.08u_2 \\ \vdots \\ u_{2000} + 0.08u_{2000} \end{bmatrix} = \begin{bmatrix} (1+0.08)u_1 \\ (1+0.08)u_2 \\ \vdots \\ (1+0.08)u_{2000} \end{bmatrix}$$

$$U = \begin{bmatrix} 1.08u_1 \\ 1.08u_2 \\ \vdots \\ 1.08u_{2000} \end{bmatrix} = 1.08U.$$

—— x ——

Q. U.V will show price of total CD players speakers and cassette recorders.

—— x ——

Q. $\frac{1}{2}(t+b)$ (average).

Q36 $U=(1,1,0,0)$ $U+V=0$ Let $V=\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$.

Now $U+V=0$

$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a+1 \\ b+1 \\ c \\ d \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow a=1,\ b=1,\ c=0,\ d=0$

$V=(1,1,0,0)$.

Q37 Same as Q36.

Q38 $U=(1,0,1)$ & Let $V=(a,b,c)$

$U.V=0$

$(1,0,1).(a,b,c)=0$

$a+0+c=0$

$a+c=0$. —①

For bit matrix $b=0,\ b=1$.

If ① $\Rightarrow b=0$ then $a=c=0$

if $b=1$ then $a=c=1$

So $V=(0,0,0),(0,1,0),(1,0,1),(1,1,1)$.

Q38 Same as Q39.



Linear transformation:- A linear transformation $L$ of $R^n$ into $R^m$ is a function assigning a unique vector $L(u)$ in $R^m$ to each $u$ in $R^n$ such that

(i) $L(u+v)=L(u)+L(v)$, for every $u$ and $v$ in $R^n$

(ii) $L(ku)=kL(u)$, for every $u$ in $R^n$ and every scalar $k$.

Q1 Which of the following are linear transformation?

(a) $L(x,y)=(x+1,y,x+y)$.

Let $\vec{u}=(x_1,y_1)$ and $\vec{v}=(x_2,y_2)$ be the two vectors in $R^2$

Then $L(\vec{u})=L(x_1,y_1)=(x_1+1,y_1,x_1+y_1)$.

& $L(\vec{v})=L(x_2,y_2)=(x_2+1,y_2,x_2+y_2)$

$L(\vec{u})+L(\vec{v})=(x_1+1,y_1,x_1+y_1)+(x_2+1,y_2,x_2+y_2)$.

$=(x_1+x_2+2,y_1+y_2,x_1+y_1+x_2+y_2)$ —①

Now $\vec{u}+\vec{v}=(x_1,y_1)+(x_2,y_2)$

$=(x_1+x_2,y_1+y_2)$.

$L(\vec{u}+\vec{v}) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$ —— ②

From eq ① & eq ② we have

$L(\vec{u}+\vec{v}) \neq L(\vec{u})+L(\vec{v})$.

it's not a linear transformation.

(b) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y \\ x-z \end{bmatrix}$

Sol: $L: R^3 \to R^3$

Let $\vec{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ be the two vectors in $R^3$.

then

$L(\vec{u}) = L\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix}$

and

$L(\vec{v}) = L\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$.

Now $L(\vec{u})+L(\vec{v}) = \begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ y_2 \\ x_2-z_2 \end{bmatrix}$

$= \begin{bmatrix} (x_1+y_1)+(x_2+y_2) \\ y_1+y_2 \\ (x_1+x_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ①$

Now

$(\vec{u}+\vec{v}) = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}$.

$L(\vec{u}+\vec{v}) = L\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix} = \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ y_1+y_2 \\ (x_1+x_2)-(z_1+z_2) \end{bmatrix} \longrightarrow ②$

From eq ② & eq ① we have.

$L(\vec{u}+\vec{v}) = L(\vec{u})+L(\vec{v})$ —— (A)

checking 2nd Property:

$L(k\vec{u}) = k(L\vec{u})$.

$k\vec{u} = k\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$.

$L(k\vec{u}) = L\begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix} = \begin{bmatrix} kx_1+ky_1 \\ ky_1 \\ kx_1-kz_1 \end{bmatrix}$.

$= k\begin{bmatrix} x_1+y_1 \\ y_1 \\ x_1-z_1 \end{bmatrix}$

$\Rightarrow L(k\vec{u}) = k(L\vec{u})$ —— (B)

From eq (A) & eq (B) it's clear that it's a linear transformation from $R^3$ to $R^3$.

(c) $L(x,y) = (x^2+x,\ y-y^2)$

Sol. Let $\vec{u} = (x_1, y_1)$ & $\vec{v} = (x_2, y_2)$ be the two vector in $R^2$

Then $L(\vec{u}) = L(x_1, y_1) = (x_1^2+x_1,\ y_1-y_1^2)$

& $L(\vec{v}) = L(x_2, y_2) = (x_2^2+x_2,\ y_2-y_2^2)$.

$L(\vec{u})+L(\vec{v}) = (x_1^2+x_1, y_1-y_1^2)+(x_2^2+x_2, y_2-y_2^2)$

$= [(x_1^2+x_1)+(x_2^2+x_2), (y_1-y_1^2)+(y_2-y_2^2)]$ — ①

Now $\vec{u}+\vec{v} = (x_1, y_1)+(x_2, y_2)$

$= (x_1+x_2, y_1+y_2)$

$L(\vec{u}+\vec{v}) = L(x_1+x_2, y_1+y_2)$

$= (x_1+x_2)^2+(x_1+x_2), (y_1+y_2)-(y_1+y_2)^2$ — ②

From eq ① & eq ② we have

$L(\vec{u})+L(\vec{v}) \neq L(\vec{u}+\vec{v})$

it's not a linear transformation.



$Q_2$ & $Q_3$ is similarly to $Q_1$

Q(a) $L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}\right) = \begin{bmatrix} u_1 \\ u_1+u_2 \\ u_1-u_3 \end{bmatrix}$.

Let $\vec{U} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ be the two vector in $R^4$ then

$L(\vec{U}) = L\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_1+u_2 \\ u_1-u_3 \end{bmatrix}$.

$L(\vec{V}) = L\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_1+v_2 \\ v_1-v_3 \end{bmatrix}$.

Now $L(\vec{U})+L(\vec{V}) = \begin{bmatrix} u_1 \\ u_1+u_2 \\ u_1-u_3 \end{bmatrix}+\begin{bmatrix} v_1 \\ v_1+v_2 \\ v_1-v_3 \end{bmatrix}$

$= \begin{bmatrix} u_1+v_1 \\ (u_1+u_2)+(v_1+v_2) \\ (u_1-u_3)+(v_1-v_3) \end{bmatrix} \longrightarrow ①$

Again now

$\vec{U}+\vec{V} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}+\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \\ u_4+v_3 \end{bmatrix}$.

$$L(\vec{u}+\vec{v}) = L\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \\ u_4+v_4 \end{bmatrix}\right) = \begin{bmatrix} u_1+v_1 \\ (u_1+v_1)+(u_2+v_2) \\ (u_1+v_1)-(u_3+v_3) \end{bmatrix} \longrightarrow (2)$$

From eq (1) & eq (2) we have

$$L(\vec{u}+\vec{v}) \neq L(\vec{u})+L(\vec{v})$$

Is not a linear transformation.

(iv) $$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} x+y+0z \\ 0x-y+2z \\ x+y-z \end{bmatrix} = \begin{bmatrix} x+y \\ -y+2z \\ x+y-z \end{bmatrix}$$

Let $\vec{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ be the two vectors in $R^3$, then

$$L(\vec{u}) = L\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ -y_1+2z_1 \\ x_1+y_1-z_1 \end{bmatrix}.$$

$$\& \; L(\vec{v}) = L\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2+y_2 \\ -y_2+2z_2 \\ x_2+y_2-z_2 \end{bmatrix}.$$

$$L(\vec{u})+L(\vec{v}) = \begin{bmatrix} x_1+y_1 \\ -y_1+2z_1 \\ x_1+y_1-z_1 \end{bmatrix} + \begin{bmatrix} x_2+y_2 \\ -y_2+2z_2 \\ x_2+y_2-z_2 \end{bmatrix}$$

$$= \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ -(y_1+y_2)+2(z_1+z_2) \\ (x_1+x_2)+(y_1+y_2)-(z_1+z_2) \end{bmatrix} \longrightarrow (1)$$

Now $$\vec{u}+\vec{v} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}.$$

$$L(\vec{u}+\vec{v}) = L\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} (x_1+x_2)+(y_1+y_2) \\ -(y_1+y_2)+2(z_1+z_2) \\ (x_1+x_2)+(y_1+y_2)-(z_1+z_2) \end{bmatrix} \longrightarrow (2)$$

From eq (1) & eq (2) we have.

$$L(\vec{u})+L(\vec{v}) = L(\vec{u}+\vec{v}) \text{ —— (A)}$$

checking 2nd Property

$L(k\vec{u}) = k(L\vec{u})$ where $k \in R$ then

$$k\vec{u} = k\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$$

$$L(k\vec{u}) = L\begin{bmatrix} kx_1 \\ ky_1 \\ kz_1 \end{bmatrix}$$

$$= \begin{bmatrix} kx_1 + ky_1 \\ -ky_1 + 2kz_1 \\ kx_1 + ky_1 - kz_1 \end{bmatrix} = k\begin{bmatrix} x_1 + y_1 \\ -y_1 + 2z_1 \\ x_1 + y_1 - z_1 \end{bmatrix}.$$

$$\Rightarrow L(k\vec{u}) = k(L\vec{u}) \longrightarrow (B)$$

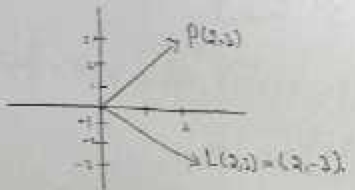
From eq. (A) & eq. (B) it is clear that it's a linear transformation from $R^3$ to $R^3$.

(C) Same as

In Exercises 5–12, sketch the image of the given point P or vector u under the given linear transformation L.

(5) $L(x, y) = (x, -y)$ : $P(2, 3)$

$L(2, 3) = (2, -3)$.



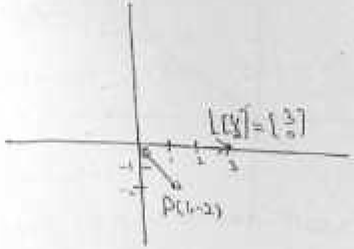



(6) $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ ; $u = (1, -2)$

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + y \end{bmatrix}$$

$$L\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 1 - (-2) \\ 2(1) + (-2) \end{bmatrix} = \begin{bmatrix} 1 + 2 \\ 2 - 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$$

$$L\left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$




(7) $L = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ $\theta = 30°$ $P = (-1, 3) = u$

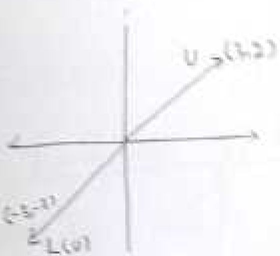
$$LU = \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}.$$

$$= \begin{bmatrix} -\cos 30° - 3\sin 30° \\ -\sin 30° + 3\cos 30° \end{bmatrix} = \begin{bmatrix} -2.366, 2.098 \end{bmatrix}.$$

Q8 is similarly to Q7.

Q9 $L(v) = -v$ , $v = (3,2)$

$L(3,2) = -(3,2) = (-3,-2)$




Q10 is similarly to Q9.

Q11 $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x-y \end{bmatrix}$ $u = (2,-1,3)$

$L(u) = L\left(\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 2-(-1) \end{bmatrix} = \begin{bmatrix} 2 \\ 2+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Q12 is similarly to Q11

~o~



Q13 (a) $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$.

Since $L(\vec{u}) = \vec{w}$

$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\Rightarrow x+z = 1$ —(i) $y+z = -1$ —(ii), $x+2y+2z = 0$ —(iii)

$x = 1-z$ ——(iv)

& $y = -1-z$ ——(v)

eq (iii) $\Rightarrow (1-z) + 2(-1-z) + 2z = 0$

$1 - z - 2 - \cancel{2z} + \cancel{2z} = 0$

$-z - 1 = 0 \Rightarrow \boxed{z = -1}$

eq (iv) $\Rightarrow x = 1-(-1) \Rightarrow \boxed{x = 2}$

eq (v) $\Rightarrow y = -1-(-1) = -1+1 = 0 \Rightarrow \boxed{y = 0}$

eq (iii) $\Rightarrow 2+2(0)+2(-1) = 0$

$\cancel{2} + 0 - \cancel{2} = 0$

$0 = 0$

$\therefore \vec{U} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$ This shows the image and so it's the range L.

(b) $L(\vec{v}) = w$

$$\begin{bmatrix} x+z \\ y+z \\ x+2y+2z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

$x+z = 2$ —— (i)

$y+z = -1$ —— (ii)

$x+2y+2z = 3$ —— (iii)

eq (i) $\Rightarrow x = 2-z$ —— (iv)

eq (ii) $\Rightarrow y = -1-z$ —— (v)

eq (iii) $\Rightarrow (2-z)+2(-1-z)+2z = 3$

$\cancel{2}-z+(\cancel{-2}-\cancel{2z})+\cancel{2z} = 3$

$-z = 3 \Rightarrow \boxed{z=-3}$

eq (iv) $\Rightarrow x = 2+3 = 5 \Rightarrow \boxed{x=5}$

eq (v) $\Rightarrow y = -1+3 \Rightarrow \boxed{y=2}$

eq (iii) $\Rightarrow 5+2(2)+2(-3) = 3$

$5+4-6 = 3$

$9-6 = 3$

$3 = 3$

$\therefore \vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$ This show the image so it's the range L.



Q14 is similarly to Q13.

Q15 Let $\vec{U} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ the such that

$L\vec{U} = \vec{W}$

$L\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$

$$\begin{bmatrix} 4 & 1 & 3 \\ 2 & -1 & 3 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$\begin{bmatrix} 4x+y+3z \\ 2x-y+3z \\ 2x+2y \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$\Rightarrow 4x+y+3z = a$ —— (i)

$2x-y+3z = b$ —— (ii)

$2x+2y = c$ —— (iii).

Xeq (ii) by (2) then subt. from (i)

$4x+y+3z = a$

$\underline{\cancel{4x}-2y+6z = 2b}$

$3y-3z = a-2b$ —— (iv).

$\cancel{4x+y+3z=a}$

eq(2) − eq(3)

$2x - y + 3z = b$

$2x + 2y \quad = c$

$-3y + 3z = b - c$ ——— (5)

eq(4) + eq(5)

$\cancel{3y} - \cancel{3z} = a - 2b$

$\cancel{-3y} + \cancel{3z} = b - c$

$0 = a - 2b + b - c$

$a - b - c = 0$

$\Rightarrow c - a + b = 0$ Ans.

~o~o~

Q16 is similarly to Q15





Q2 As $\begin{bmatrix}4\\-3\end{bmatrix} = 4i - 3j$ where $L(i) = \begin{bmatrix}2\\3\end{bmatrix}$ & $L(j) = \begin{bmatrix}-1\\2\end{bmatrix}$

$\Rightarrow L\begin{bmatrix}4\\-3\end{bmatrix} = L(4i - 3j)$

$= 4L(i) - 3L(j)$

$= 4\begin{bmatrix}2\\3\end{bmatrix} - 3\begin{bmatrix}-1\\2\end{bmatrix}$

$= \begin{bmatrix}8\\12\end{bmatrix} + \begin{bmatrix}3\\-6\end{bmatrix}$

$= \begin{bmatrix}8+3\\12-6\end{bmatrix} = \begin{bmatrix}11\\6\end{bmatrix}$

$\Rightarrow L\left(\begin{bmatrix}4\\-3\end{bmatrix}\right) = \begin{bmatrix}11\\6\end{bmatrix}$

Q3 As $\begin{bmatrix}2\\-1\\3\end{bmatrix} = 2i - j + 3k$

$\Rightarrow L\begin{bmatrix}2\\-1\\3\end{bmatrix} = L(2i - j + 3k) = 2L(i) - L(j) + 3L(k)$

$= 2\begin{bmatrix}1\\2\\-1\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix} + 3\begin{bmatrix}1\\1\\3\end{bmatrix}$

$= \begin{bmatrix}2\\4\\-2\end{bmatrix} - \begin{bmatrix}1\\0\\2\end{bmatrix} + \begin{bmatrix}3\\3\\9\end{bmatrix} = \begin{bmatrix}2-1+3\\4-0+3\\-2-2+9\end{bmatrix}$

$L\begin{bmatrix}2\\-1\\3\end{bmatrix} = \begin{bmatrix}4\\7\\5\end{bmatrix}$ Ans.

Let $\vec{U} = (x_1, y_1)$ & $\vec{V} = (x_2, y_2)$. be the two vector in $R^2$, then

$L(\vec{U}) = L(x_1, y_1) = (x_1 + y_1 + 1, x_1 - y_1)$

$L(\vec{V}) = L(x_2, y_2) = (x_2 + y_2 + 1, x_2 - y_2)$.

$L(\vec{U}) + L(\vec{V}) = (x_1 + y_1 + 1, x_1 - y_1) + (x_2 + y_2 + 1, x_2 - y_2)$

$= ((x_1 + y_1) + (x_2 + y_2) + 2, (x_1 + x_2) - (y_1 + y_2)) \longrightarrow (1)$

Now $\vec{U} + \vec{V} = (x_1, y_1) + (x_2 + y_2)$

$= (x_1 + x_2, y_1 + y_2)$

$L(\vec{U} + \vec{V}) = L(x_1 + x_2, y_1 + y_2)$

$= (x_1 + x_2 + y_1 + y_2 + 1, x_1 + x_2 - y_1 - y_2) \text{ ——— } (2)$

From eq (1) & eq (2) it's clearly that

$L(\vec{U} + \vec{V}) \neq L(\vec{U}) + L(\vec{V})$.

$\therefore$ it's not a linear transformation.

Sol: let $\vec{U} = (x_1, y_1)$ and $\vec{V} = (x_2, y_2)$ be the two vectors in $R_2$, then

$\vec{U} + \vec{V} = (x_1 + y_1) + (x_2 + y_2) = (x_1 + x_2, y_1 + y_2)$.

$L(\vec{U} + \vec{V}) = L(x_1 + x_2, y_1 + y_2)$

$L(\vec{U} + \vec{V}) = (\sin(x_1 + x_2), \sin(y_1 + y_2))$ —— ①

Now $L(\vec{U}) = L(x_1, y_1)$

$= \sin x_1 + \sin y_1$

& $L(\vec{V}) = L(x_2, y_2)$

$= \sin x_2 + \sin y_2$.

$\Rightarrow L(\vec{U}) + L(\vec{V}) = (\sin x_1 + \sin y_1) + (\sin x_2 + \sin y_2)$

$= (\sin(x_1 + x_2), \sin(y_1 + y_2))$ —— ②

From eq ① & eq ② we have

$L(\vec{U} + \vec{V}) = L(\vec{U}) + L(\vec{V})$. —— Ⓐ

Now let $k \in R$ then $k\vec{U} = k(x_1, y_1) = (kx_1, ky_1)$

$L(k\vec{U}) = L(kx_1, ky_1) = (k\sin x_1 + k\sin y_1) = k(\sin x_1 + \sin y_1)$

$\Rightarrow L(k\vec{U}) = k(L\vec{U})$ —— Ⓑ

From eq Ⓐ & eq Ⓑ, we can say that the given function is a linear transformation.



Theorem: If $L: R^n \to R^m$ is a L.T. then

$L(c_1\vec{U_1} + c_2\vec{U_2} + \dots + c_k\vec{U_k})$

$= c_1 L(\vec{U_1}) + c_2 L(\vec{U_2}) + \dots + c_k L(\vec{U_k})$.

for any vector $\vec{U_1}, \vec{U_2}, \dots \vec{U_k}$ in $R^n$ and any scalar $c_1, c_2 \dots c_k$.

For $R^2$ $\vec{i} = \vec{e_1} = (1, 0) = \begin{bmatrix}1\\0\end{bmatrix}$.

$\vec{j} = \vec{e_2} = (0, 1) = \begin{bmatrix}0\\1\end{bmatrix}$.

For $R^3$ $\vec{i} = \vec{e_1} = (1, 0, 0) = \begin{bmatrix}1\\0\\0\end{bmatrix}$.

$\vec{j} = \vec{e_2} = (0, 1, 0) = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$\vec{k} = \vec{e_3} = (0, 0, 1) = \begin{bmatrix}0\\0\\1\end{bmatrix}$.

For $R^n$ $\vec{i} = \vec{e_1} = (1, 0, 0, \dots 0)$

$\vdots$

$\vec{e_n} = (0, 0, 0, \dots 1)$.

Q26 $L: R^2 \to R^2$ $L\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x-y\\x+y\end{bmatrix}$

For $R^2$ $e_1=(1,0)=\begin{bmatrix}1\\0\end{bmatrix}$ & $e_2=(0,1)=\begin{bmatrix}0\\1\end{bmatrix}$.

$L(e_1)=L\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1-0\\1+0\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$.

& $L(e_2)=L\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0-1\\0+1\end{bmatrix}=\begin{bmatrix}-1\\1\end{bmatrix}$.

then $A=\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$. Ans

Q28 $\theta = 60^\circ$

$L(v)=\begin{bmatrix}\cos 60 & -\sin 60\\ \sin 60 & \cos 60\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}\frac{x}{2} & -\frac{y\sqrt{3}}{2}\\ \frac{x\sqrt{3}}{2} & y/2\end{bmatrix}$.

For $R^2$ $e_1=\begin{bmatrix}1\\0\end{bmatrix}$ & $e_2=\begin{bmatrix}0\\1\end{bmatrix}$

$L(e_1)=L\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1/2 & 0\\ \sqrt{3}/2 & 0\end{bmatrix}=\begin{bmatrix}1/2\\ \sqrt{3}/2\end{bmatrix}$.

$L(e_2)=L\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0 & -\sqrt{3}/2\\ 0 & 1/2\end{bmatrix}=\begin{bmatrix}-\sqrt{3}/2\\ 1/2\end{bmatrix}$.

$A=\begin{bmatrix}1/2 & -\sqrt{3}/2\\ \sqrt{3}/2 & 1/2\end{bmatrix}$.

Q27 is same as Q28

Q29 $L\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=\begin{bmatrix}x-y\\x+z\\y-z\end{bmatrix}$.

For $R^3$ $e_1=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $e_2=\begin{bmatrix}0\\1\\0\end{bmatrix}$ & $e_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$.

$L(e_1)=L\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1-0\\1+0\\0-0\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$.

$L(e_2)=L\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0-1\\0+0\\1-0\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}$.

$L(e_3)=L\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0-0\\0+1\\0-1\end{bmatrix}=\begin{bmatrix}0\\1\\-1\end{bmatrix}$.

Thus $A=\begin{bmatrix}1 & -1 & 0\\1 & 0 & 1\\0 & 1 & -1\end{bmatrix}$.

Q30 $L(u)=-2u=-2\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-2x\\-2y\\-2z\end{bmatrix}$.

$L(e_1)=L\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}-2\\0\\0\end{bmatrix}$

$L(e_2)=L\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\-2\\0\end{bmatrix}$

$L(e_3)=L\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}0\\0\\-2\end{bmatrix}$.

Thus $A=\begin{bmatrix}-2 & 0 & 0\\0 & -2 & 0\\0 & 0 & -2\end{bmatrix}$.

Q(a) SEND HIM MONEY
19 5 14 4 8 9 13 13 15 14 5 25

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$.

Breaking message into 4 vectors

$$\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} \begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix}.$$

Now $L(x) = AX$

$$Ax = A\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}$$

$$A\begin{bmatrix} 19 \\ 5 \\ 14 \end{bmatrix} = \begin{bmatrix} 71 \\ 52 \\ 33 \end{bmatrix}.$$

$$Ax = A\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}$$

$$A\begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 47 \\ 30 \\ 26 \end{bmatrix}$$

$$Ax = A\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 84 \\ 56 \\ 43 \end{bmatrix}$$

$$A\begin{bmatrix} 13 \\ 13 \\ 15 \end{bmatrix} = \begin{bmatrix} 84 \\ 56 \\ 43 \end{bmatrix}.$$

$$Ax = A\begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 14 \\ 5 \\ 25 \end{bmatrix} = \begin{bmatrix} 99 \\ 69 \\ 56 \end{bmatrix}.$$



So the message is 71 52 33 47 30 26 84 56 43 99 69 56.

(b) 67 44 41 49 39 19 113 76 62 104 69 55

Breaking into vectors in $R^3$

$$\begin{array}{cccc} L(x_1) & L(x_2) & L(x_3) & L(x_4) \\ \begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix} & \begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} & \begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix} & \begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix}. \end{array}$$

find $A^{-1} = ?$ $\quad L(x) = Ax \therefore x = A^{-1}L(x)$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}.$$

$$x_1 = A^{-1}L(x_1) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 67 \\ 44 \\ 41 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 18 \end{bmatrix}.$$

$$x_2 = A^{-1}L(x_2) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 49 \\ 39 \\ 19 \end{bmatrix} = \begin{bmatrix} 20 \\ 1 \\ 9 \end{bmatrix}$$

$$x_3 = A^{-1}L(x_3) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 113 \\ 76 \\ 62 \end{bmatrix} = \begin{bmatrix} 14 \\ 12 \\ 25 \end{bmatrix}.$$

$$x_4 = A^{-1}L(x_4) = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 104 \\ 69 \\ 55 \end{bmatrix} = \begin{bmatrix} 14 \\ 15 \\ 20 \end{bmatrix}$$

So the message is
3 5 18 20 1 9 14 12 25 14 15 20
C E R T A I N L Y N O T
certainly not.

Q32 is same as Q31.

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In exercises 1 and 2 compute $u \times v$

Q(a) $u = 2i+3j+4k$ , $v = -i+3j-k$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$$i\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 2 & 4 \\ -1 & -1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ -1 & 3 \end{vmatrix}$$

$i(-3-12) - j(-2+4) + k(6+3)$

$i(-15) - j(2) + k(9)$

$= -15i - 2j + 9k.$

(b) $u = (1,0,1)$, $v = (2,3,-1)$

$\vec{u} = i+0j+k$ $\vec{v} = 2i+3j-k$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 2 & 3 & -1 \end{vmatrix} = i(0-3) - j(-1-2) + k(3)$$
$$= -3i + 3j + 3k.$$

(c) Same as part (a).

(d) $u = (2,-1,1)$ , $v = -2u = -2(2,-1,1) = (-4,2,-2)$

$\vec{u} = 2i - j + k$ & $\vec{v} = -4i + 2j - 2k$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ -4 & 2 & -2 \end{vmatrix} = i(2-2) - j(-4+4) + k(4-4)$$
$$= 0i + 0j + 0k.$$

Q2 is same as Q1.

Q3 Let $U = i+2j-3k$, $V = 2i+3j+k$, $W = 2i-j+2k$, & $C=-3$

(i) $U \times V = -(V \times U)$.

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 2 & 3 & 1 \end{vmatrix}$$

$= 11i - 7j - k$ —— ①

$$-(V \times U) = -\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$= -(-11i + 7j + k) = 11i - 7j - k$ —— ②

From eq ① & eq ② we have

$U \times V = -(V \times U)$ Proved

Part b, c, d same as part (a)



Q4 $U = 2i - j + 3k$

$V = 3i + j - k$

$W = 3i + j + 2k$.

(a) verify equation ① $(U \times V) \cdot W = U \cdot (V \times W)$.

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{vmatrix}$$

$= -2i + 11j + 5k$

$(U \times V) \cdot W = (-2i + 11j + 5k) \cdot (3i + j + 2k)$

$= -6 + 11 + 10 = 15$ —— ①

Now

$$V \times W = \begin{vmatrix} i & j & k \\ 3 & 1 & -1 \\ 3 & 1 & +2 \end{vmatrix} = 3i - 9j + 0k$$

$U \cdot (V \times W) = (2i - j + 3k) \cdot (3i - 9j + 0k)$

$= 6 + 9 + 0 = 15$ —— ②

From eq ① & eq ② we have

$(U \times V) \cdot W = U \cdot (V \times W)$ Proved

Part (b) is same as Part (a).

Q5 is same as Q4.

Orthogonal: If $u \times v$ is orthogonal to both $u$ & $v$ i.e

$(u \times v) \cdot u = 0$

& $(u \times v) \cdot v = 0$

Q6 Exercise a —— to —— d.

(a) $\vec{U} = 2i + 3j + 4k$ & $\vec{V} = -i + 3j - k$

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 3 & -1 \end{vmatrix}$$

$= -15i - 2j + 9k$

Now $(U \times V) \cdot U = (-15i - 2j + 9k) \cdot (2i + 3j + 4k)$

$= -30 - 6 + 36$

$= \cancel{-36} + \cancel{36} = 0$

$\therefore (U \times V) \cdot U = 0$

Also $(U \times V) \cdot V = (-15i - 2j + 9k) \cdot (-i + 3j - k)$

$= 15 - 6 - 9 \Rightarrow \cancel{15} - \cancel{15} = 0$

$(U \times V) \cdot V = 0$

which is orthogonal.



Q7 is same as Q6

Q8 Area of triangle $= \frac{1}{2}|\vec{U} \times \vec{V}|$ —— ①

Let $\vec{U} = P_1P_2 = (-3-1, 1+2, 4-3)$

$= (-4, 3, 1)$

& $\vec{V} = P_1P_3 = (0-1, 4+2, 3-3)$

$= (-1, 6, 0)$

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ -4 & 3 & 1 \\ -1 & 6 & 0 \end{vmatrix}$$

$\vec{U} \times \vec{V} = -6i - j - 21k$

Now $|\vec{U} \times \vec{V}| = \sqrt{(-6)^2 + (-1)^2 + (-21)^2} = \sqrt{478}$

eq ① $\Rightarrow$ Area of triangle $= \frac{1}{2}|\vec{U} \times \vec{V}|$

$= \frac{1}{2}\sqrt{478}$ (unit)$^2$.

Q10 is same as Q9.

**Q11** Area of //gm $= |\vec{U}\times\vec{V}|$ — ⓐ

Now $\vec{U}\times\vec{V} = \begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 3 & -1 & -4 \end{vmatrix}$

$= -5i - 5j - 10k$

Now $|\vec{U}\times\vec{V}| = \sqrt{(-5)^2+(-5)^2+(-10)^2} = \sqrt{150}$

ans ⇒ Area of //gm $= \sqrt{150}$ (unit)$^2$.

**Q12** $U = 2i - j$, $V = i - 2j - 2k$ & $W = 3i - j + k$.

Volume of //Prd $= \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$= \begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & -2 \\ 3 & -1 & 1 \end{vmatrix}$

$= 2(-2-2) + 1(1+6) + 0$

$= 2(-4) + 7$

$= -8 + 1 = |-1|$

Vol //P $= 1$ (unit)$^3$

Q13 is same as Q12



EX 5.2

**Q1** (a) Let $P_1(x_1, y_1) = (-2, -3)$ & $P_2(x_2, y_2) = (3, 4)$.

Now $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x & y & 1 \\ -2 & -3 & 1 \\ 3 & 4 & 1 \end{vmatrix} = 0$

$x(-3-4) - y(-2-3) + 1(-8+9) = 0$

$-7x + 5y + 1 = 0$

This is the required equation of line for the given Points.

Part (b), (c), (d) same as Part (a)

Q2 is same as Q1.

**Q3** $x = 3 + 2t \Rightarrow t = \frac{x-3}{2}$

$y = -2 + 3t \Rightarrow t = \frac{y+2}{3}$

$z = 4 - 3t \Rightarrow t = \frac{z-4}{-3}$.

Now $\frac{x-3}{2} = \frac{y+2}{3} = \frac{z-4}{-3}$ —①

(a) (1,1,1)

eq① => $\frac{1-3}{2} = \frac{1+2}{3} = \frac{1-4}{-3}$

$\frac{-2}{2} = \frac{3}{3} = \frac{-3}{-3}$

$-1 \neq 1 \neq 1$

Hence all the value of t are not same. Therefore pts are not lies on the lines.

Part (b) & (c) Same as part (a)

(d) $(4, -\frac{1}{2}, \frac{5}{2})$

eq① => $\frac{4-3}{2} = \frac{-\frac{1}{2}+2}{3} = \frac{\frac{5}{2}-4}{-3}$

$\frac{1}{2} = \frac{1}{2} = \frac{1}{2}$

Hence all the value of t are same so it's lie on line

Q4 is same as Q3

② Ex 5..



Q5 (a) $P_0 = (\overset{x_0}{3}, \overset{y_0}{4}, \overset{z_0}{-2})$, $u = (\overset{a}{4}, \overset{b}{-5}, \overset{c}{2})$. $-\infty < t < \infty$

$x = x_0 + at \Rightarrow x = 3+4t$

$y = y_0 + bt \Rightarrow y = 4-5t$

$z = z_0 + ct \Rightarrow z = -2+2t$.

Part b, c, d same as part (a)

Q6 (a) $P_0 = (2,-3,1)$ & $P_1 = (4,2,5)$

$\vec{v} = \overrightarrow{P_0P_1} = (4-2, 2+3, 5-1) = (2,5,4)$

Now $P_0 = (\overset{x_0}{2}, \overset{y_0}{-3}, \overset{z_0}{1})$ & $v = (\overset{a}{2}, \overset{b}{5}, \overset{c}{4})$

$x = x_0 + at \Rightarrow 2+2t$

$y = y_0 + bt \Rightarrow y = -3+5t$

$z = z_0 + ct \Rightarrow z = 1+4t$

Part (b), (c) & (d) same as part (a).

Q7(a) $P_0 = \overset{x_0\ y_0\ z_0}{(2,-3,1)}$ & $v = \overset{a\ b\ c}{(2,5,4)}$

For symmetric form

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

$$\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-1}{4}$$

(b), (c), & (d) Part as same as Part (a)

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Q8 $3(x-2)+2(y+3)-4(z-1)=0$ —①

(a) $(0,-2,3)$

eq ① $\Rightarrow 3(0-2)+2(-2+3)-4(3-1)=0$

$0=0$

So it's lie on plane.

(b), (c), & (d) Same as Part (a).



Q9 (d) $P_0 = (x_0, y_0, z_0) = (5, 2, 3)$

$n = (a,b,c) = (-1,-2,4)$

Equation of plane is

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$-1(x-5)-2(y-2)+4(z-3)=0$

$-x-2y+4z-3=0$

Part (a) (b) & (c) Same as Part (d)

Q10(b) $P_1(2,3,-2)$, $P_2(-1,-2,3)$ $P_3(-5,-4,2)$

Let $U = \overrightarrow{P_1P_2} = (-3,-5,-1)$

$V = \overrightarrow{P_1P_3} = (-7,-7,-2)$

Now $\vec{U}\times\vec{V} = \begin{vmatrix} x & y & z \\ -3 & -5 & -1 \\ -7 & -7 & -2 \end{vmatrix}$

$\Rightarrow x(10-7)-y(6-7)+z(21-35)=0$

$\Rightarrow 3x+y-14z=0$

Which is the required equation of Plane.

others Parts are Same as.

Q11 (h) $3x-2y-5z+4=0$ —(1)

$2x+3y+4z+8=0$ —(2)

x'ing eq (1) by '3' & eq (2) by '2'

$9x-\cancel{6y}-15z+12=0$

$4x+\cancel{6y}+8z+16=0$

$13x-7z+28=0$

$\frac{13x+28}{7}=z$ —(3)

x'ing eq (1) by '2' & eq (2) by '3' & subtracting

$\cancel{6x}-4y-10z+8=0$

$\cancel{6x}+9y+12z+24=0$

$-13y-22z-16=0$

$\frac{-13y-16}{22}=z$ —(4)

comparing eq (3) & eq (4)

$$\frac{13x+28}{7}=\frac{-13y-16}{22}=z=t \text{ (say)}.$$



Q14 Let $P_1(x_1,y_1,z_1)=(-2,4,2)$

$P_2(x_2,y_2,z_2)=(3,5,1)$

$P_3(x_3,y_3,z_3)=(4,2,-1)$.

Now
$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}=0$$

$$\begin{vmatrix} -2 & 4 & 2 \\ 3 & 5 & 1 \\ 4 & 2 & -1 \end{vmatrix}=0$$

$-2(-5-2)-4(-3-4)+2(6-20)=0$

$14\neq 0$

These points are not on the same plane.

Q13 is same as Q14.

Q5 $x = 2-3s \quad x = 5+2t$

$y = 3+2s$ and $y = 1-3t$

$z = 4+2s \quad z = 2+t$

$2-3s = 5+2t$ —— (1)

$3+2s = 1-3t$ —— (2)

$4+2s = 2+t$ —— (3)

Subtracting (3) from eq (2), we get

$$\begin{array}{r} 3+2s = 1-3t \\ 4+2s = 2+t \\ \hline -1 = -1-4t \end{array}$$

$-4t = 0$

$\boxed{t=0}$

eq (1) $\Rightarrow 2-3s = 5+2(0) \Rightarrow \boxed{s=-1}$

eq (2) $\Rightarrow 3+2s = 1-3(0)$

$3+2s = 1 \Rightarrow 2s = -2 \Rightarrow \boxed{s=-1}$

$\therefore t = 0,\ s = 1$

Now

$x = 2-3s$ | $x = 5+2t$

$x = 2-3(-1)$ | $x = 5+2(0)$

$\boxed{x=5}$ | $\boxed{x=5}$

Now $\dfrac{13x+28}{7} = t$

$\Rightarrow x = \dfrac{7t-28}{13} \longrightarrow$ (a)

& $\dfrac{-13y-16}{22} = t$

$\Rightarrow y = \dfrac{22t+16}{-13} \longrightarrow$ (b)

Also $z = t \longrightarrow$ (c)

Equation (a), (b) & (c) are the required parametric equations of the line of intersection of the given planes.

Q7 (b) $\dfrac{x-2}{-2} = \dfrac{y-3}{4} = \dfrac{z+4}{3}$

Now $\dfrac{x-2}{-2} = \dfrac{y-3}{4}$

$\Rightarrow 4x+2y-14=0$ — which is 1st equation of plane

And $\dfrac{x-2}{-2} = \dfrac{z+4}{3}$

$3x+2z+2=0$ which is 2nd equation of plane.

(c) $x = 4t \longrightarrow (a)$

$y = 1+5t \longrightarrow (b)$

$z = 2-t \longrightarrow (c)$

eq (a) $\Rightarrow \frac{x}{4} = t$ —— (i)

eq (b) $\Rightarrow \frac{y-1}{5} = t$ —— (ii)

eq (c) $\Rightarrow 2-z = t$ —— (iii)

From eq (i), (ii) & eq (iii)

$$\frac{x}{4} = \frac{y-1}{5} = 2-z = t$$

Now $x/4 = y-1/5$

$\Rightarrow 5x-4y+4=0$ —— ① which is 1st eq of Plane.

& $y-1/5 = 2-z$

$y+5z-11=0$ —— ② which is 2nd eq: of Plane.

~o~o

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$y = 3+2s$

$y = 3+2(-1)$

$y = 3-2 = 1$

$\boxed{y=1}$

$z = 4+2s$

$z = 4+2(-1)$

$z = 4-2$

$\boxed{z=2}$

$y = 1-3t$

$y = 1-3(0)$

$\boxed{y=1}$

$z = 2+t$

$z = 2+0$

$\boxed{z=2}$

Therefore the Point of intersection of the given line are $P(x,y,z) = P(5,1,2)$.

~o~o~o

Q.9 (a) $x = 2+2t$, $y = -3-3t$, $z = 4+4t$ and $x = 2+t$, $y = 4-t$, $z = 5-t$

Let $\vec{u} = (2,-3,4)$

& $\vec{v} = (1,-1,-1)$

Now $\vec{U}\cdot\vec{V} = (2,-3,4)\cdot(1,-1,-1)$.

$= 2+3-4 = 1$

$\vec{U}\cdot\vec{V} \neq 0$

The given lines are not $\perp$.

$\boxed{\text{Note } \vec{U}\cdot\vec{V} = 0 \text{ then } \perp}$



Ex 6.1



# Real Vector Spaces

Definition:- A real vector space is a set of elements $V$ together with two operation $\oplus$ and $\odot$ satisfying the following properties

(1) If $u$ and $v$ are any elements of $V$ then $u\oplus v$ is in $V$ (i.e $V$ is closed under the operation $\oplus$).

(a) $u\oplus v = v\oplus u$ for $u$ and $v$ in $V$

(b) $u\oplus(v\oplus w) = (u\oplus v)\oplus w$ for $u, v$ and $w$ in $V$.

(c) There is an element $0$ in $V$ such that

$u\oplus 0 = 0\oplus u = u$ for all $u$ in $V$

(d) For each $u$ in $V$ there is an element $-u$ in $V$ such that $u\oplus -u = 0$.

(3) If $u$ is any element of $V$ and $c$ is any real number then $c\odot u$ is in $V$ (i.e $V$ is closed under the operation $\odot$).

(e) $c\odot(u\oplus v) = c\odot u \oplus c\odot v$ for all real number $c$ and all $u$ and $v$ in $V$

(f) $(c+d)\odot u = c\odot u \oplus d\odot u$ for all real number $c$ and $d$ and all $u$ in $V$

(g) $c\odot(d\odot u) = (cd)\odot u$ for all real number $c$ and $d$ and all $u$ in $V$.

(h) $1\odot u = u$ for all $u$ in $V$.

The elements of $V$ are called vector. The real numbers are called scalars. The operation $\oplus$ is called vector addition. The operation $\odot$ is called scalar.



Example 2. Consider the set $V$ of all ordered triples of real numbers of the form $(x,y,0)$ and define the operations $\oplus$ and $\odot$ by.

$$(x,y,0)\oplus(x',y',0) = (x+x', y+y', 0)$$

$$c\odot(x,y,0) = (cx, cy, 0)$$

Sol. Checking the Properties to show that it is a vector space or not

let $u = (x,y,0)$ & $v = (x',y',0)$

(a) $u\oplus v = v\oplus u$

$\Rightarrow u\oplus v = (x,y,0)\oplus(x',y',0)$.

$= (x+x', y+y', 0) \longrightarrow (i)$

& $v\oplus u = (x',y',0)\oplus(x,y,0)$

$= (x'+x, y'+y, 0)$ — (ii)

From (i) & (ii) we have

Hence $u\oplus v = v\oplus u$.

(b) $u\oplus(v\oplus w) = (u\oplus v)\oplus w$. let $w = (a,b,c)$.

$U\oplus(V\oplus W) = U\oplus[(x',y',0)\oplus(a,b,c)]$

$= U\oplus[x'+a, y'+b, 0+c]$

$= (x,y,0)\oplus(x'+a, y'+b+c)$

$= (x+x'+a, y+y'+b, c)$ —— (i)

& $(U\oplus V)\oplus W = [(x,y,0)\oplus(x',y',0)]\oplus(a,b,c)$

$= (x+x', y+y', 0)\oplus(a,b,c)$

$= (x+x'+a, y+y'+b, c)$ —— (ii)

From eq (i) & eq (ii) we have

$U\oplus(V\oplus W) = (U\oplus V)\oplus W$.

(c) $U\oplus 0 = 0\oplus U = U$.

$U\oplus 0 = (x,y,0)\oplus 0 = (x+0, y+0, 0+0) = (x,y,0) = U$

& $0\oplus U = (0,0,0)\oplus(x,y,0) = (0+x, 0+y, 0+0) = (x,y,0) = U$

Hence $U\oplus 0 = 0\oplus U = U$.

(d) $U\oplus -U = 0$.

$(x,y,0)\oplus -(x,y,0) = (x,y,0)\oplus(-x,-y,0)$

$(x-x, y-y, 0) = (0,0,0) = 0$

Hence $U\oplus -U = 0$.



Now checking the properties for scalar multiplication

(e) $c\odot(U\oplus V) = c\odot((x,y,0)\oplus(x',y',0))$

$= c\odot(x+x', y+y', 0)$

$= (c(x+x'), c(y+y'), 0)$ —— (i)

& $c\odot U\oplus c\odot V = c\odot(x,y,0)\oplus c\odot(x',y',0)$

$= (cx, cy, 0)\oplus(cx', cy', 0)$

$= (cx+cx', cy+cy', 0)$

$= (c(x+x'), c(y+y'), 0)$ —— (ii)

From eq (i) & eq (ii) we have

$c\odot(U\oplus V) = c\odot U\oplus c\odot V$

(f) $(c+d)\odot U = c\odot U\oplus d\odot U$

$(c+d)\odot U = (c+d)\odot(x,y,0)$

$= (cx, cy, 0 + dx, dy, 0)$

$= (cx+dx, cy+dy, 0)$ —— (1)

& $c\odot U\oplus d\odot U = c\odot(x,y,0)\oplus d\odot(x,y,0)$

$= (cx, cy, 0)\oplus(dx, dy, 0)$

$= (cx + dx, cy + dy, 0)$ — (ii)

From eq (i) & eq (ii) we have

$(c+d) \odot u = c \odot u \oplus d \odot u$.

(g) $c \odot (d \odot u) = (cd) \odot u$

$c \odot (d \odot u) = c \odot (d \odot (x, y, 0))$
$= c \odot (dx, dy, 0)$
$= (cdx, cdy, 0)$ — (i)

& $(cd) \odot u = (cd) \odot (x, y, 0)$
$= (cdx, cdy, 0)$ — (ii)

From eq (i) & eq (ii) we have

$c \odot (d \odot u) = (cd) \odot u$.

(h) $1 \odot u = u$

$1 \odot (x, y, 0) = (1 \cdot x, 1 \cdot y, 1 \cdot 0)$
$= (x, y, 0)$
$1 \odot (x, y, 0) = u$

Hence it satisfies all the properties Therefore V is a vector space.





(b) V is the set of all ordered pairs of real number $(x, y)$ where $x > 0$ & $y > 0$

$(x, y) \oplus (x', y') = (x + x', y + y')$.

& $c \odot (x, y) = (cx, cy)$.

Sol: Checking the properties of vector addition $\oplus$

let $u = (x, y)$ & $v = (x', y')$

(a) $u \oplus v = v \oplus u$

$u \oplus v = (x, y) \oplus (x', y')$
$= (x + x', y + y')$ — (i)

& $v \oplus u = (x', y') \oplus (x, y)$ —
$= (x' + x, y' + y)$ — (ii)

From eq (i) & eq (ii) we have

$u \oplus v = v \oplus u$.

(b) $u \oplus (v \oplus w) = (u \oplus v) \oplus w$ let $w = (a, b)$

$u \oplus (v \oplus w) = u \oplus ((x', y') \oplus (a, b))$

$= (x, y) \oplus (x'+a, y'+b)$

$= (x + x' + a, y + y' + b)$ — (i)

& $(u \oplus v) \oplus w = ((x, y) \oplus (x', y')) \oplus (a, b)$

$= (x+x', y+y') \oplus (a, b)$

$= (x + x' + a, y + y' + b)$ — (ii)

From eq (i) & eq (ii) we have

$u \oplus (v \oplus w) = (u \oplus v) + w$

(c) $u \oplus 0 = 0 \oplus u = u$

$u \oplus 0 = (x, y) \oplus (0, 0) = (x+0, y+0) = (x, y) = u$

$0 \oplus u = (0, 0) \oplus (x, y) = (0+x, 0+y) = (x, y) = u$

Hence $u \oplus 0 = 0 \oplus u = u$.

(d) $u \oplus -u = 0$

$(x, y) \oplus -(x, y) = (x, y) \oplus (-x, y)$

$(x - x, y - y) = (0, 0) = 0$

Hence $u \oplus -u = 0$. closed under $\oplus$



Now checking the properties for scalar multiplication

(e) $c \odot (u \oplus v) = c \odot u \oplus c \odot v$

$c \odot (u \oplus v) = c \odot ((x, y) \oplus (x', y'))$

$= c \odot (x+x', y+y')$

$= c(x+x'), c(y+y')$ — (i)

& $c \odot u \oplus c \odot v = c \odot (x, y) \oplus c \odot (x', y')$

$= (cx, cy) \oplus (cx', cy')$

$= (cx + cx', cy + cy')$

$= c(x+x'), c(y+y')$ — (ii)

From eq (i) & eq (ii) we have

$c \odot (u \oplus v) = c \odot u \oplus c \odot v$

But if $c < 0$ then $(x+x') + (y+y') < 0$ so it

not closed under $\odot$

Q2 V is the set of all ordered triples of real numbers of the form $(0, y, z)$.

$$(0,y,z) \oplus (0,y',z') = (0, y+y', z+z')$$

& $c \odot (0,y,z) = (0, 0, cz)$.

Sol. let $U = (0,y,z)$ & $V = (0,y',z')$

checking the Properties for vector addition $(+)$

(a) $U \oplus V = V \oplus U$

$\Rightarrow U \oplus V = (0,y,z) \oplus (0,y',z')$
$= (0+0, y+y', z+z')$
$= (0, y+y', z+z')$. — (i)

& $V \oplus U = (0,y',z') \oplus (0,y,z)$
$= (0+0, y'+y, z'+z)$.
$= (0, y'+y, z'+z)$ — (ii)

From eq (i) & eq (ii) we have.
$U \oplus V = V \oplus U$.

(b) $U \oplus (V \oplus W) = (U \oplus V) \oplus W$ & let $W = (a,b,c)$

$\Rightarrow U \oplus (V \oplus W) = U \oplus [(0,y',z') \oplus (a,b,c)]$
$= U \oplus (0+a, y'+b, z'+c)$
$= (0,y,z) \oplus (a, y'+b, z'+c)$
$= (a, y+y'+b, z+z'+c)$ — (i)

& $(U \oplus V) \oplus W = [(0,y,z) \oplus (0,y',z')] \oplus W$
$= (0, y+y', z+z') \oplus W$
$= (0, y+y', z+z') \oplus (a,b,c)$
$= (a, y+y'+b, z+z'+c)$ — (ii)

From eq (i) & eq (ii) we have.
$U \oplus (V \oplus W) = (U \oplus V) \oplus W$

(c) $U \oplus 0 = 0 \oplus U = U$

$U \oplus 0 = (0,y,z) \oplus (0,0,0) = (0+0, y+0, z+0) = (0,y,z) = U$

& $0 \oplus U = (0,0,0) \oplus (0,y,z) = (0+0, 0+y, 0+z) = (0,y,z) = U$

Hence $U \oplus 0 = 0 \oplus U = U$.

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(d) $U \oplus -U = 0$

$(0,y,z) \oplus -(0,y,z)$

$(0,y,z) \oplus (0,-y,-z) = (0, y-y, z-z) = (0,0,0) = 0$

Hence $U \oplus -U = 0$

Inverse under $\oplus$.

Now checking the properties for scalar multiplication

(e) $c \odot (U+V) = (c \odot U) \oplus (c \odot V)$

$c \odot (U+V) = c \odot [(0,y,z) \oplus (0,y',z')]$

$= c \odot [0+0, y+y', z+z']$

$= c \odot (0, y+y', z+z')$

$= (0, c(y+y'), c(z+z'))$ —— (i)

$(c \odot U) \oplus (c \odot V) = [c \odot (0,y,z)] \oplus [c \odot (0,y',z')]$

$= (0, cy, cz) \oplus (0, cy', cz')$

$= (0+0, cy+cy', cz+cz')$

$= (0, c(y+y'), c(z+z'))$ —— (ii)

From eq (i) & eq (ii) we have.

$c \odot (U+V) = (c \odot U) \oplus (c \odot V)$.



(f) $(c+d) \odot U = c \odot U \oplus d \odot U$

$(c+d) \odot U = (c+d) \odot (0,y,z)$

$= (0, cy, cz + 0, dy, dz)$

$= (0, cy+dy, cz+dz)$ —— (i).

$c \odot U \oplus d \odot U = c \odot (0,y,z) \oplus d \odot (0,y,z)$

$= (0, cy, cz) \oplus (0, dy, dz)$

$= (0, cy+dy, cz+dz)$ —— (ii)

From eq (i) & eq (ii) we have.

$(c+d) \odot U = c \odot U \oplus d \odot U$

(g) $c \odot (d \odot U) = (cd) \odot U$

$\Rightarrow c \odot (d \odot U) = c \odot (d \odot (0,y,z))$

$= c \odot (0, dy, dz)$

$= (0, cdy, cdz)$ —— (i)

& $(cd) \odot U = (cd) \odot (0,y,z)$

$= (0, cdy, cdz)$ —— (ii)

From eq (i) & eq (ii) we have.

$c \odot (d \odot U) = (cd) \odot U$

(h) $1 \odot U = U$

$1 \odot (a, y, z) = (a, y, z) = U$

Hence it satisfies all the Properties. Therefore V is a vector space.

Q3 V is the set of all polynomials of the form $at^2+bt+c$ where a, b and c are real numbers with $b = a+1$

$(a_1t^2+bt+c_1) \oplus (a_2t^2+b_2t+c_2)$.

$= (a_1+a_2)t^2+(b_1+b_2)t+(c_1+c_2)$

& $r \odot (at^2+bt+c) = (ra)t^2+(rb)t+rc$.

Sol:- let $u = a_1t^2+bt+c_1$ & $V = a_2t^2+b_2t+c_2$.

(a) $U \oplus V = V \oplus U$

$\Rightarrow U \oplus V = (a_1t^2+b_1t+c_1) \oplus (a_2t^2+b_2t+c_2)$

$U \oplus V = (a_1+a_2)t^2+(b_1+b_2)t+(c_1+c_2)$

Put $b_1 = a_1+1$, $b_2 = a_2+1$

$U \oplus V = (a_1+a_2)t^2+(a_1+1+a_2+1)t+(c_1+c_2)$

$= (a_1+a_2)t^2+(a_1+a_2+2)t+(c_1+c_2)$



Not closed under $\oplus$ here we get 2 instead of 1. as given $(at^2+bt+c)$ & $b = a+1$.

Now checking the Properties of scalar multiplication

$r \odot (a_1t^2+b_1t+c_1) = ra_1t^2+rb_1t+rc_1$

$= ra_1t^2+r(a_1+1)t+rc_1$

$= ra_1t^2+(ra_1+r)t+rc_1$

Not closed under $\odot$ b/c we get r instead of 1

~ o ~ o ~

Hint

Q4 $U = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ & $V = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$.

Q5 let $U = (x, y)$, $V = (x', y')$, $w = (a, b)$

Q6 $U = (x, y, z)$, $V = (x', y', z')$, $w = (a, b, c)$.

Q11, Q12 & Q13 same as Q2.

Q16 same as Q1.


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**Subspaces:** Let V be a vector space, then W will be subspace of V if

(a) $U \oplus V$ is in W

(b) $k \in R$, $k \odot U$ is in W.

Q1 Which of the following subsets of $R^3$ are subspaces of $R^3$? the set of all vectors of the form

(a) $(a, b, 2)$

Let $\vec{U} = (a, b, 2)$ & $\vec{V} = (a', b', 2)$ are two vectors in W then

$$\vec{U} \oplus \vec{V} = (a, b, 2) \oplus (a', b', 2)$$

$$= (a+a', b+b', 4) \notin W$$

Which is not in W. Hence W is not a subspace.

(b) Let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are two vectors in W then $a+b=c$, $a'+b'=c'$

(a) $\vec{U} \oplus \vec{V} = (a, b, c) \oplus (a', b', c')$

$$= (a+a', b+b', c+c') \in W$$

or

$\vec{U} \oplus \vec{V} = (a+a', b+b', a+b+a'+b')$

$= (a+a', b+b', (a+a')+(b+b')) \in W$.

(b) $k \odot U = k \odot (a, b, c)$

$= k \odot (a, b, a+b)$

$= (k \odot a, k \odot b, k \odot (a+b)) \in W$

$= (k \odot a, k \odot b, k \odot c) \in W$

Hence W is the subspace of $R^3$



(a) let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are the two vector in W then $a = c = 0$, $a' = c' = 0$

(a) $U \oplus V = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c')$

$= (0, b+b', +0) \in W$

(b) $k \odot U = k \odot (a, b, c)$

$= (k \odot a, k \odot b, k \odot c)$

$= (k \odot 0, k \odot b, k \odot 0)$

$= (0, k \odot b, 0) \in W$

$= (0, kb, 0) \in W$

Hence W is the subspace of $R^3$.

(b) $(a, b, c)$ where $a = -c$

let $\vec{U} = (a, b, c)$ & $\vec{V} = (a', b', c')$ are two vector in W then $a = -c$, $a' = -c$

(a) $U \oplus V = (a, b, c) \oplus (a', b', c')$

$= (a+a', b+b', c+c')$

$= (-c + -c', b+b', c+c')$

$\vec{U}\oplus\vec{V} = (-(c+c') + (b+b') + (c+c') \in W$

(b) $k\odot\vec{U} = k\odot(a,b,c)$

$= k\odot a, k\odot b, k\odot c$

$= k\odot(-c), k\odot b, k\odot c$

$= (-kc, kb, kc) \notin W$

if $k<0$ & $kc>0$

(c) let $\vec{U}=(a,b,c)$ & $\vec{V}=(a',b',c')$ be two vector in W

then $b=2a+1$, $b'=2a'+1$

$\vec{U}\oplus\vec{V} = (a,b,c)\oplus(a',b',c')$

$= (a+a', b+b', c+c')$

$= (a+a', (2a+1)(2a'+1), c+c')$

$= (a+a', 2(a+a')+2, c+c') \notin W$

So W is not a subspace of $R^3$.



Q (a) let $\vec{U}=(a,b,c,d)$ & $\vec{V}=(a',b',c',d')$ be two vector in $R^4$ then $a-b=2$ & $a'-b'=2$

$a=2+b$ & $a'=2+b'$

$\vec{U}\oplus\vec{V} = (a,b,c,d)\oplus(a',b',c',d')$

$= (a+a', b+b', c+c', d+d')$

$= (b+2+b'+2, b+b', c+c', d+d')$

$= (b+b'+4, b+b', c+c', d+d') \notin W$

so W is not a subspace of $R^4$.

Part (b), (c) & Q8 is similarly to Q7

Q (a) $a_2t^2+a_1t+a_0$ where $a_1=0$ & $a_0=0$

let $\vec{U}=a_2t^2+a_1t+a_0$ & $\vec{V}=a_2't^2+a_1't+a_0'$

(a) $\vec{U}\oplus\vec{V} = (a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$= (a_2+a_2')t^2 + (a_1+a_1')t + (a_0+a_0') \in W$

(b) $k\odot U = k\odot(a_2t^2+a_1t+a_0)$

$= (ka_2t^2 + ka_1t + ka_0) \in W$ subspace.

(b) $a_2t^2+a_1t+a_0$ where $a_1=2a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ & $\vec{V}=a_2't^2+a_1't+a_0'$

(a) $\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(a_2+a_2')t^2+2(a_0+a_0')t+(a_0+a_0')\in W$

(b) $k\odot U=k\odot(a_2t^2+a_1t+a_0)$

$=ka_2t^2+ka_1t+ka_0$

$=ka_2t^2+2ka_0t+ka_0\in W$ subspace.

(c) $a_2t^2+a_1t+a_0$, where $a_2+a_1+a_0=2$ ∴ $a_2=2-a_1-a_0$

let $\vec{U}=a_2t^2+a_1t+a_0$ + $\vec{V}=a_2't^2+a_1't+a_0'$

$\vec{U}\oplus\vec{V}=(a_2t^2+a_1t+a_0)\oplus(a_2't^2+a_1't+a_0')$

$=(a_2+a_2')t^2+(a_1+a_1')t+(a_0+a_0')$

$=(2-a_1-a_0+2-a_1'-a_0')t^2+(a_1+a_1')t+(a_0+a_0')\notin W$

not a subspace.

Q9 is similarly to Q10



let $w_1=a_1u+b_1v$

& $w_2=a_2u+b_2v$

(a) $w_1\oplus w_2=(a_1u+b_1v)\oplus(a_2u+b_2v)$

$=a_1u+a_2u+b_1v+b_2v$

$=(a_1+a_2)u+(b_1+b_2)v\in W$

(b) $k\odot w_1=k\odot(a_1u+b_1v)$

$=(ka_1)u+(kb_1)v\in W$

So $W$ is a subspace of $R^3$.

Q15 is as same as Q14.

Q16 let $\vec{U}=\begin{bmatrix} a & b & c\\ d & 0 & 0\end{bmatrix}$ + $\vec{V}=\begin{bmatrix} a_1 & b_1 & c_1\\ d_1 & 0 & 0\end{bmatrix}$, $b=a+c$, $b_1=a_1+c_1$

(a) $\vec{U}\oplus\vec{V}=\begin{bmatrix} a & b & c\\ d & 0 & 0\end{bmatrix}\oplus\begin{bmatrix} a_1 & b_1 & c_1\\ d_1 & 0 & 0\end{bmatrix}$

$=\begin{bmatrix} a+a_1 & b+b_1 & c+c_1\\ d+d_1 & 0 & 0\end{bmatrix}$

$$U \oplus V = \begin{bmatrix} a+a_1 & (a+c)+(a_1+c_1) & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$$

(ii) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$

$$= k \odot \begin{bmatrix} a & a+c & c \\ d & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} ka & k(a+c) & kc \\ kd & 0 & 0 \end{bmatrix} \in W$$

Subspace.

(h) $\vec{U} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$ & $c > 0$

(a) $\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} \oplus \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & 0 & 0 \end{bmatrix} \in W$$

(b) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & 0 & 0 \end{bmatrix} = \begin{bmatrix} ka & kb & kc \\ kd & 0 & 0 \end{bmatrix}$

If $k<0$ $ka<0$ so $\notin W$ not subspace.



(i) $\vec{U} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ & $\vec{V} = \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$ & $a = -2c$, $f = 2e+d$.

(a) $\vec{U} \oplus \vec{V} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \oplus \begin{bmatrix} a_1 & b_1 & c_1 \\ d_1 & e_1 & f_1 \end{bmatrix}$

$$= \begin{bmatrix} a+a_1 & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & f+f_1 \end{bmatrix}$$

$$= \begin{bmatrix} -2(c+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & (2e+d)+(2e_1+d_1) \end{bmatrix}$$

$$= \begin{bmatrix} -2(c+c_1) & b+b_1 & c+c_1 \\ d+d_1 & e+e_1 & 2(e+e_1)+(d+d_1) \end{bmatrix} \in W$$

(b) $k \odot U = k \odot \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$= \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$$

$$= \begin{bmatrix} -2ck & kb & kc \\ kd & ke & k(2e+d) \end{bmatrix} \in W$$

Which is subspace.

Q17 is similarly Q16.

**Definition**: If $S = \{v_1, v_2, \dots v_k\}$ is a set of vector in a vector space $V$ then the set of all vectors in $V$ that are linear combination of the vectors in $S$ is denoted by $\text{span} S$ or $\text{span}\{v_1, v_2 \dots v_k\}$.

**Q.5** $v_1 = (1,0,0,1)$ $v_2 = (1,-1,0,0)$, $v_3 = (0,1,2,1)$

(a) $v = (-1,4,2,2)$

let $c_1v_1 + c_2v_2 + c_3v_3 = v$ then $v_1, v_2, v_3 \in V$

$c_1(1,0,0,1) + c_2(1,-1,0,0) + c_3(0,1,2,1) = (-1,4,2,2)$

$(c_1, 0, 0, c_1) + (c_2, -c_2, 0, 0) + (0, c_3, 2c_3, c_3) = (-1,4,2,2)$

$c_1 + c_2 = -1$ — (i)

$-c_2 + c_3 = 4$ — (ii)

$2c_3 = 2$ — (iii)

$c_1 + c_3 = 2$ — (iv)

eq (iii) $\Rightarrow 2c_3 = 2 \Rightarrow \boxed{c_3 = 1}$

eq (ii) $\Rightarrow -c_2 + 1 = 4 \Rightarrow -c_2 = 4-1 = 3 \Rightarrow \boxed{c_2 = -3}$

eq (iv) $\Rightarrow c_1 + 1 = 2 \Rightarrow c_1 = 2-1 = 1 \Rightarrow \boxed{c_1 = 1}$

eq (i) $\Rightarrow c_1 + (-3) = -1 \Rightarrow c_1 = -1+3 = 2 \Rightarrow \boxed{c_1 = 2}$

So solution is not possible so $v$ is not span of $(v_1, v_2, v_3)$.



Q.6 (b), (c), (d) is same as part (a).

$A_1 = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, $A_3 = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$.

$A = \begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix}$.

$\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix} = c_1\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} + c_2\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + c_3\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} c_1 & -c_1 \\ 0 & 3c_1 \end{bmatrix} + \begin{bmatrix} c_2 & c_2 \\ 0 & 2c_2 \end{bmatrix} + \begin{bmatrix} 2c_3 & 2c_3 \\ -c_3 & c_3 \end{bmatrix}$

$\begin{bmatrix} 5 & 1 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} c_1+c_2+2c_3 & -c_1+c_2+2c_3 \\ -c_3 & 3c_1+2c_2+c_3 \end{bmatrix}$

$c_1 + c_2 + 2c_3 = 5$ — (1)

$-c_1 + c_2 + 2c_3 = 1$ — (2)

$-c_3 = -1$ — (3)

$3c_1 + 2c_2 + c_3 = 9$ — (4)

eq (3) $\Rightarrow -c_3 = -1 \Rightarrow \boxed{c_3 = 1}$

Put $c_3$ in eq (1) & eq (2)

(1) $\Rightarrow c_1 + c_2 + 2 = 5 \Rightarrow c_1 + c_2 = 3$ — (5)

(2) $\Rightarrow -c_1 + c_2 + 2 = 1 \Rightarrow -c_1 + c_2 = -1$ — (6)

Put $c_3 = 1$ in eq (4)

$3c_1 + 2c_2 + 1 = 9$

$3c_1 + 2c_2 = 8$ — (7)

(5) + (6)

$$\begin{aligned} c_1 + c_2 &= 3 \\ -c_1 + c_2 &= -1 \\ \hline 2c_2 &= 2 \end{aligned}$$

$\boxed{c_2 = 1}$

x'ing eq (6) by (3) & add with (7)

$$\begin{aligned} -\cancel{3c_1} + 3c_2 &= -3 \\ \cancel{3c_1} + 2c_2 &= 8 \\ \hline 5c_2 &= 5 \end{aligned}$$

$\boxed{c_2 = 1}$

eq (7) $\Rightarrow$ $3c_1 + 2(1) = 8$

$3c_1 = 8 - 2 = 6$

$\boxed{c_1 = 2}$

So $\begin{bmatrix} 8 \\ -9 \end{bmatrix}$ belongs to space if $c_1 = 2$, $c_2 = 1$, $c_3 = 1$.

Part (b), (c) & (d) same as part (a).



$W = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. bit matrix

Let $W_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $W_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $W_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$W_1 + W_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = W_3$

$W_1 + W_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = W_2$

$W_2 + W_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = W_1$

$W_1 + W_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \notin W$ So not a subspace.

$W = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Let $w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $w_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $w_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$w_1 + w_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = w_3$

$w_1 + w_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w_2$

$w_1 + w_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = w_1$

$w_1 + w_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_2 + w_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_3 + w_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_4 + w_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = w_4$

$w_2 + w_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = w_1$

$w_2 + w_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = w_2$

$w_3 + w_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = w_3$

it's subspace.

**Q** Let $V = B^4$. Determine if $W$ the set of all vector in $V$ with first entry zero is a subspace of $V$.

$$W = \left\{ \begin{array}{cccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\ \begin{bmatrix}0\\0\\0\\0\end{bmatrix} & \begin{bmatrix}0\\0\\0\\1\end{bmatrix} & \begin{bmatrix}0\\0\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\1\\1\end{bmatrix} & \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\0\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \end{array} \right\}$$

$$w_1 + w_2 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} = w_2$$

$$w_1 + w_3 = \begin{bmatrix}0\\0\\1\\1\end{bmatrix} = w_3$$

$$w_1 + w_4 = \begin{bmatrix}0\\1\\1\\1\end{bmatrix} = w_4$$

$$w_1 + w_5 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} = w_5$$

$w_1 + w_6 = w_6$

$w_1 + w_7 = w_7$

$w_1 + w_8 = w_8$

$w_1 + w_1 = w_1$

$$w_2 + w_2 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = w_1$$

Similarly others also hold.

**Q** Let $V = B^4$. Determine if $W$, the set of all vector in $V$ with second entry one is a subspace of $V$

$$W = \left\{ \begin{array}{cccccccc} w_1 & w_2 & w_3 & w_4 & w_5 & w_6 & w_7 & w_8 \\ \begin{bmatrix}0\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\1\end{bmatrix} & \begin{bmatrix}1\\1\\1\\1\end{bmatrix} & \begin{bmatrix}1\\1\\1\\0\end{bmatrix} & \begin{bmatrix}1\\1\\0\\1\end{bmatrix} & \begin{bmatrix}1\\1\\0\\0\end{bmatrix} & \begin{bmatrix}0\\1\\1\\0\end{bmatrix} & \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \end{array} \right\}$$

Not vector space as $w_1 + w_8 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix} \notin W$

**Q** $u \in S$ if

$u = c_1 v_1 + c_2 v_2 + c_3 v_3$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1\\0\\0\end{bmatrix} + \begin{bmatrix}c_2\\c_2\\0\end{bmatrix} + \begin{bmatrix}c_3\\c_3\\c_3\end{bmatrix}$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1 + c_2 + c_3\\c_2 + c_3\\c_3\end{bmatrix}$$

$c_1 + c_2 + c_3 = 1$ —— ①

$c_2 + c_3 = 0$ —— ②

$c_3 = 1$ —— ③

$\boxed{c_3 = 1}$

eq ② $\Rightarrow c_2 + 1 = 0$ bit mod

$\boxed{c_2 = -1}$ or $\boxed{c_2 = 1}$

eq ① $= c_1 + 1 + 1 = 1$

$c_1 = 1 - 2 = -1 = 1$ (from b.M)

$\boxed{c_1 = 1}$

It belong to when $c_1 = c_2 = c_3 = 1$.

Q33 is Same as Q 32.

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Which of the following vectors span $R^2$?

$(1,2), (-1,1)$

Let $V = (a,b)$ & $V_1 = (1,2)$, $V_2 = (-1,1)$

Now $V = c_1V_1 + c_2V_2$

$(a,b) = c_1(1,2) + c_2(-1,1)$

$(a,b) = (c_1, 2c_1) + (-c_2, c_2)$

$(a,b) = (c_1 - c_2, 2c_1 + c_2)$

$c_1 - c_2 = a$ — (i)

$2c_1 + c_2 = b$ — (ii)

(i)+(ii)

$$c_1 - \cancel{c_2} = a$$
$$2c_1 + \cancel{c_2} = b$$
$$3c_1 = a+b \Rightarrow \boxed{c_1 = \frac{a+b}{3}}$$

eq (i) $\Rightarrow \frac{a+b}{3} - c_2 = a$

$$c_2 = \frac{a+b}{3} - a = \frac{a+b-3a}{3} = \frac{-2a+b}{3}$$

$$\boxed{c_2 = \frac{-2a+b}{3}}$$

So solution exist so span $R^2$

~~[illegible]~~

(b) $(0,0), (1,1), (-2,-2)$

let $V=(a,b)$ $V_1=(0,0)$, $V_2=(1,1)$ $V_3=(-2,-2)$

Now $V = c_1V_1 + c_2V_2 + c_3V_3$

$(a,b) = c_1(0,0) + c_2(1,1) + c_3(-2,-2)$

$(a,b) = (0,0) + (c_2, c_2) + (-2c_3, -2c_3)$

$(a,b) = (0 + c_2 - 2c_3, 0 + c_2 - 2c_3)$

$0 + c_2 - 2c_3 = a$ — ①

$0 + c_2 - 2c_3 = b$ — ②

① − ②

$0 + \cancel{c_2} - \cancel{2c_3} = a$

$0 \mathop{+}\limits_{-} \cancel{c_2} \mathop{-}\limits_{+} \cancel{2c_3} = \mathop{b}\limits_{-}$

$0 \neq a - b$

solution don't exist so $V_1, V_2, V_3$ don't span $R^2$

Part c same Part b & (d) same (c)



Ex 6.3

which of the following sets of vector span $R^3$?

$(1,-1,2), (0,1,1)$

let $V=(a,b,c)$, $V_1=(1,-1,2)$ $V_2=(0,1,1)$.

Now $V = c_1V_1 + c_2V_2$

$(a,b,c) = c_1(1,-1,2) + c_2(0,1,1)$

$(a,b,c) = (c_1, -c_1, 2c_1) + (0, c_2, c_2)$

$(a,b,c) = (c_1, -c_1 + c_2, 2c_1 + c_2)$

$c_1 = a$ — ①

$-c_1 + c_2 = b$ — ②

$2c_1 + c_2 = c$ — ③

② − ③

$-c_1 + \cancel{c_2} = b$

$\mathop{2c_1}\limits_{-} \mathop{+}\limits_{-} \cancel{c_2} = \mathop{c}\limits_{-}$

$-3c_1 = b - c$

$c_1 = -\left(\frac{b-c}{3}\right) = \boxed{\frac{c-b}{3} = c_1}$

There are different values of $c_1$ so solution does not exist Not a span.

Q3 Which of the following vectors span $R^4$?

(a) $(1,0,0,1)$
$(0,1,0,0)$
$(1,1,1,1)$
$(1,1,1,0)$

Let $v=(a,b,c,d)$, $v_1=(1,0,0,1)$, $v_2=(0,1,0,0)$, $v_3=(1,1,1,1)$, $v_4=(1,1,1,0)$

Let $v = c_1v_1+c_2v_2+c_3v_3+c_4v_4$

$(a,b,c,d)=c_1(1,0,0,1)+c_2(0,1,0,0)+c_3(1,1,1,1)+c_4(1,1,1,0)$

$(a,b,c,d)=(c_1,0,0,c_1)+(0,c_2,0,0)+(c_3,c_3,c_3,c_3)+(c_4,c_4,c_4,0)$

$(a,b,c,d)=(c_1+c_3+c_4,\ c_2+c_3+c_4,\ c_3+c_4,\ c_1+c_3)$

$c_1+c_3+c_4=a$ —— (1)
$c_2+c_3+c_4=b$ —— (2)
$c_3+c_4=c$ —— (3)
$c_1+c_3=d$ —— (4)

Put $c_1+c_3=d$ in eq (1)

eq (1) $\Rightarrow c_4+d=a \Rightarrow \boxed{c_4=a-d}$

eq (3) $\Rightarrow c_3+(a-d)=c \Rightarrow \boxed{c_3=c-a+d}$

eq (4) $\Rightarrow c_1+(c-a+d)=d \Rightarrow c_1=d-(c-a+d)$

$c_1=\cancel{d}-c+a-\cancel{d}=-c+a$

$$\boxed{c_1 = a - c}$$

eq (2) ⟹ $c_2 + (c - a + d) + (a - d) = b$

$c_2 + c \cancel{-a} \cancel{+d} \cancel{+a} \cancel{-d} = b$

$$\boxed{c_2 = b - c}$$

Hence solution exist so spans $R^4$

other Parts are same as Part a

Q4 (a) $\{t^2+1, t^2+t, t+1\}$

Let $P = (a, b, c)$, $P_1 = (t^2+1)$, $P_2 = (t^2+t)$, $P_3 = (t+1)$

$P = c_1P_1 + c_2P_2 + c_3P_3$

$(a,b,c) = c_1(t^2+1) + c_2(t^2+t) + c_3(t+1)$

$= (c_1t^2 + c_1) + (c_2t^2 + c_2t) + (c_3t + c_3)$

$= \cancel{\quad}$

$(a,b,c) = (c_1+c_2)t^2 + (c_2+c_3)t + (c_1+c_3)$

$c_1 + c_2 = a$ — (1)

$c_3 + c_2 = b$ — (2)

$c_1 + c_3 = c$ — (3)

eq (1) ⟹ $c_1 = a - c_2$

eq (3) ⟹ $a - c_2 + c_3 = c$

$-c_2 + c_3 = c - a$ — (4)

(2) + (4)

$c_3 + \cancel{c_2} = b$

$c_3 - \cancel{c_2} = c - a$

$2c_3 = c - a + b$

$$\boxed{c_3 = \frac{c-a+b}{2}}$$

eq (2) $c_2 = b - c_3$

$c_2 = b - \left(\frac{c-a+b}{2}\right)$

$= \frac{2b-c+a-b}{2} = \frac{b-c+a}{2}$

$$\boxed{c_2 = \frac{b-c+a}{2}}$$

eq (1) ⟹ $c_1 = a - c_2$

$= a - \left(\frac{b+a-c}{2}\right) = \frac{2a-b-a+c}{2}$

$$\boxed{c_1 = \frac{a-b+c}{2}}$$

So the solution exist hence a polynomial span $P_2$

Q5 is same as Q4.

Q6 Augmented form

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 \\ 2 & 1 & 3 & 1 & 0 \\ 1 & 1 & 2 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_2-R_1 \\ -2R_2-2R_1 \\ R_3-R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \end{array}\right] \begin{array}{l} R_3-1/2R_2 \\ R_4-1/2R_2 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] R_4-R_3$$

$\frac{1}{2}c_4=0 \Rightarrow c_4=0$

$2c_2+2c_3+c_4=0$

$2c_2+2c_3+0=0$

$2c_2+2c_3=0$

$2(c_2+c_3)=0$

$c_2+c_3=0$

$c_2=-c_3$ let $c_3=r$ (any real no.)

$c_2=-r$

$c_1+c_3=0$

$c_1=-c_3$

$c_1=-r$

So span exist has infinite many solution

Q7 is same Q6

—o—

Q9 $x_1=\begin{bmatrix}1\\2\\0\\1\end{bmatrix}$, $x_2=\begin{bmatrix}1\\0\\-1\\1\end{bmatrix}$, $x_3=\begin{bmatrix}1\\6\\2\\0\end{bmatrix}$, $x=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

Now

$X=c_1x_1+c_2x_2+c_3x_3$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=c_1\begin{bmatrix}1\\2\\0\\1\end{bmatrix}+c_2\begin{bmatrix}1\\0\\-1\\1\end{bmatrix}+c_3\begin{bmatrix}1\\6\\2\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix}=\begin{bmatrix}c_1+c_2+c_3\\2c_1+0+6c_3\\0-c_2+2c_3\\c_1+c_2+0\end{bmatrix}$$

$c_1+c_2+c_3=0$ — (1)
$2c_1+6c_3=0$ — (2)
$-c_2+2c_3=0$ — (3)
$c_1+c_2=0$ — (4)

Put $c_1+c_2$ value in eq(1)

eq (1) $\Rightarrow 0+c_3=0 \Rightarrow \boxed{c_3=0}$

eq(3) $\Rightarrow -c_2+2(0)=0 \Rightarrow \boxed{c_2=0}$

eq(4) $\Rightarrow c_1+0=0 \Rightarrow \boxed{c_1=0}$

Hence solution exist so $[x_1, x_2, x_3]$ is L.I.

———— x ————

Q8 is same as Q9.

———— x ————

Q10 $V_1=(1,2,-1)$ & $V_2=(3,2,5)$

Now $c_1V_1+c_2V_2=0$
$c_1(1,2,-1)+c_2(3,2,5)=0$
$c_1+3c_2=0$ — (1)
$2c_1+2c_2=0$ — (2)
$-c_1+5c_2=0$ — (3)



(1) - (3)
$\cancel{c_1}+3c_2=0$
$-\cancel{c_1}+5c_2=0$
$8c_2=0$
$\boxed{c_2=0}$

eq(1) $\Rightarrow c_1+3(0)=0 \Rightarrow \boxed{c_1=0}$

Hence linearly independent.

(b), (c) & (d) is similarly.

Q11 is similarly to Q10

———— o ———— o ————

Q12 (a) $P_1=t^2+1,\ P_2=t-2,\ P_3=t+3$

Now $c_1P_1+c_2P_2+c_3P_3=0$
$c_1(t^2+1)+c_2(t-2)+c_3(t+3)=0$
$(c_1t^2+c_1)+(c_2t-c_2)+(c_3t+c_3)=0$
$(c_1)t^2+(c_2+c_3)t+(c_1-c_2+c_3)=0$
$c_1=0$ — (1) $\Rightarrow \boxed{c_1=0}$
$c_2+c_3=0$ — (2)
$c_1-c_2+c_3=0$ — (3)

Put $c_1 = 0$ in eq(3)

eq(3) $\Rightarrow 0 - c_2 + c_3 = 0$

$-c_2 + c_3 = 0$ —— (4)

(2) + (4)

$$\begin{array}{r} \cancel{c_2} + c_3 = 0 \\ -\cancel{c_2} + c_3 = 0 \\ \hline 2c_3 = 0 \end{array}$$

$\boxed{c_3 = 0}$

eq(2) $\Rightarrow c_2 + 0 = 0$

$\boxed{c_2 = 0}$

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$ so linearly independent.

Part b, c, d is similarly



Jan Laser Photostate
Matric to M.A.
Notes are available



Q13 Consider the vector space $M_{22}$. Follow the direction of Exercise 10

(c) $\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \right\}$

Now $c_1 V_1 + c_2 V_2 + c_3 V_3 + c_4 V_4 = 0$

$$c_1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + c_3 \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + c_4 \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = 0$$

$c_1 + 2c_2 + 3c_3 + 2c_4 = 0$ —— (1)

$c_1 + 3c_2 + c_3 + 2c_4 = 0$ —— (2)

$c_1 + c_2 + 2c_3 + c_4 = 0$ —— (3)

$c_1 + 2c_2 + c_3 + c_4 = 0$ —— (4)

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & -3 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] R_3 + R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \\ 0 & 0 & -2 & -1 & 0 \end{array}\right] -\tfrac{1}{3}R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & -1/3 & 0 \end{array}\right] 2R_3+R_4$$

$-\frac{1}{3}c_4 = \boxed{c_4 = 0}$

$c_3 + \frac{1}{3}c_4 = 0$

$c_3 + \frac{1}{3}(0) = 0$

$\boxed{c_3 = 0}$

$c_2 - 2c_3 + 0c_4 = 0$

$c_2 - 2(0) + 0 = 0$

$\boxed{c_2 = 0}$

$c_1 + 2c_2 + 3c_3 + 2c_4 = 0$

$c_1 + 2(0) + 3(0) + 2(0) = 0$

$\boxed{c_1 = 0}$

$c_1 = c_2 = c_3 = c_4 = 0$

So it's linearly independent.



Q8 (a) $\{\cos t, \sin t, e^t\}$.

$v_1 = \cos t$, $v_2 = \sin t$, $v_3 = e^t$

Now $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1 \cos t + c_2 \sin t + c_3 e^t = 0$

$c_1 = c_2 = c_3 = 0$ linearly Independent.

(b) $\{t, e^t, \sin t\}$

$v_1 = t$, $v_2 = e^t$, $v_3 = \sin t$

Now $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1 t + c_2 e^t + c_3 \sin t = 0$

$c_1 = c_2 = c_3 = 0$ L. Independent.

(d) $\{\cos^2 t, \sin^2 t, \cos 2t\}$

$v_1 = \cos^2 t$, $v_2 = \sin^2 t$, $v_3 = \cos 2t = \cos^2 t - \sin^2 t$

Now $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$c_1 \cos^2 t + c_2 \sin^2 t + c_3(\cos^2 t - \sin^2 t) = 0$

$(c_1 + c_3)\cos^2 t + (c_2 - c_3)\sin^2 t = 0$

$c_1 + c_3 = 0$ — ①

$c_2 - c_3 = 0$ — ②

let $\boxed{c_3 = r}$

eq ② $\Rightarrow c_2 - r = 0 \Rightarrow \boxed{c_2 = +r}$

eq ③ $\Rightarrow c_1 + r = 0 \Rightarrow \boxed{c_1 = -r}$

Which is linearly dependent or non-trivial.

Q5 let

$V_1 = (-1, 0, -1)$

$V_2 = (2, 1, 2)$

$V_3 = (1, 1, c)$

Now $c_1V_1 + c_2V_2 + c_3V_3 = 0$

$c_1(-1,0,-1) + c_2(2,1,2) + c_3(1,1,c) = 0$

$-c_1 + 2c_2 + c_3 = 0$ — ①

$+c_2 + c_3 = 0$ — ②

$-c_1 + 2c_2 + c\,c_3 = 0$ — ③

given L-Dependent so the solution is non-trivial then det $= 0$

$$\begin{vmatrix} -1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 2 & c \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 1 & 1 \\ 2 & c \end{vmatrix} - 2\begin{vmatrix} 0 & 1 \\ -1 & c \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 0$$

$-1(c-2) - 2(0+1) + 1(0+1) = 0$

$-c + \cancel{2} - \cancel{2} + 1 = 0$

$-c + 1 = 0$

$\Rightarrow \boxed{c = 1}$

Q6 let $V_1 = t + 3$

$V_2 = 2t + \lambda^2 + 2$

Now $c_1V_1 + c_2V_2 = 0$

$c_1(t+3) + c_2(2t + \lambda^2 + 2) = 0$

$c_1t + 3c_1 + 2c_2t + c_2(\lambda^2 + 2) = 0$

$c_1t + 2c_2t = 0$ — ①

$3c_1 + c_2(\lambda^2 + 2) = 0$ — ②

Det $= 0$ for non-trivial solution

$$\begin{vmatrix} 1 & 2 \\ 3 & \lambda^2 + 2 \end{vmatrix} = 0 \qquad \lambda^2 + 2 - 6 = 0$$

$\lambda^2 - 4 = 0 \Rightarrow \lambda^2 = 4 \Rightarrow \boxed{\lambda = \pm 2}$

Q17 $v_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}, v_2 = \begin{bmatrix}1\\0\\1\end{bmatrix}, v_3 = \begin{bmatrix}1\\1\\1\end{bmatrix}$ & $v = \begin{bmatrix}a\\b\\c\end{bmatrix}$.

$c_1v_1 + c_2v_2 + c_3v_3 = v$

$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}1\\0\\1\end{bmatrix} + c_3\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\begin{bmatrix}c_1\\c_1\\0\end{bmatrix} + \begin{bmatrix}c_2\\0\\c_2\end{bmatrix} + \begin{bmatrix}c_3\\c_3\\c_3\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$\begin{bmatrix}c_1 + c_2 + c_3\\c_1 + 0 + c_3\\0 + c_2 + c_3\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

$c_1 + c_2 + c_3 = a$ —— ①
$c_1 + 0 + c_3 = b$ —— ②
$0 + c_2 + c_3 = c$ —— ③

① − ②

$c_1 + c_2 + c_3 = a$
$c_1 + 0 + c_3 = b$

$c_2 = a - b \Rightarrow \boxed{c_2 = a - b} = \boxed{c_2 = a + b}$ For Q.M

① − ③

$c_1 + c_2 + c_3 = a$
$0 + c_2 + c_3 = c$

$c_1 = a - c \Rightarrow \boxed{c_1 = a - c} \Rightarrow \boxed{c_1 = a + c}$ For Q.M

eq ① ⇒ $a + c + a + b + c_3 = a$

$(1+1)a + c_3 = a + c + b = 0(a) + c_3 = a + c + b$ For Q.M

$c_3 = a + c + b - a = \boxed{c + b + a = c_3}$

$c_3 = a + c + b$

$c_3 = c_1 + b$

$c_3 - c_1 = b$

$c_3 + c_1 = b$ (For Q.M)

$c + b + a + a + c = b$

$c(1+1) + (1+1)a + b = b$

$c(0) + (0)a + b = b$

$0 + 0 + b = b$

$b = b$ verified.

So it span $R^3$

Q18 is same as Q17.

Q19 is same

Q20 is same as Q3

Q21 is similarly

Q22 " "

Let $S=\{v_1, v_2, v_3, v_4, v_5\}$ where

$v_1=(1,2,3)$

$v_2=(2,1,4)$

$v_3=(-1,-1,2)$

$v_4=(0,1,2)$ & $v_5=(1,1,1)$

Note that V is the row space of matrix A whose rows are the given vectors

$$A=\begin{bmatrix}1&2&3\\2&1&4\\-1&-1&2\\0&1&2\\1&1&1\end{bmatrix}.$$

$$\sim\begin{bmatrix}1&2&3\\0&-3&-2\\0&1&5\\0&1&2\\0&-1&-2\end{bmatrix}\begin{matrix}\\-2R_1+R_2\\R_1+R_3\\ \\-R_1+R_5\end{matrix}$$

$$\sim\begin{bmatrix}1&2&3\\0&1&5\\0&-3&-2\\0&1&2\\0&-1&-2\end{bmatrix}R_{23}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \begin{matrix} -2R_2+R_1 \\ 3R_4+R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} -R_2+R_4 \\ R_4+R_5 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} R_3/4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} -5R_3+R_2 \\ 7R_3+R_1 \\ 3R_3+R_4 \end{matrix}$$

Which is reduced row echelon form. ~~the vectors~~

~~Now~~ Now $w_1 = (1,0,0)$
$w_2 = (0,1,0)$
$w_3 = (0,0,1)$ form a basis for V.



$Q_2$ is Similarly do $Q_1$

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix} \quad V_3 = \begin{bmatrix} 3 \\ 2 \\ 3 \\ 2 \end{bmatrix} \quad V_5 = \begin{bmatrix} 5 \\ 3 \\ 5 \\ 3 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \quad V_4 = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$$

Now

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 3 & 2 & 3 & 2 \\ 3 & 3 & 3 & 3 \\ 5 & 3 & 5 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -3 & 0 & -3 \\ 0 & -4 & 0 & -4 \\ 0 & -3 & 0 & -3 \\ 0 & -7 & 0 & -7 \end{bmatrix} \begin{matrix} -2R_1+R_2 \\ -3R_1+R_3 \\ -3R_1+R_4 \\ -5R_1+R_5 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & -4 & 0 & -4 \\ 0 & -3 & 0 & -3 \\ 0 & -7 & 0 & -7 \end{bmatrix} -\tfrac{1}{3}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} -2R_2+R_1 \\ 4R_2+R_3 \\ 3R_2+R_4 \\ 7R_2+R_5 \end{matrix}$$

Hence Possible solution $\left\{ \begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\1\end{bmatrix} \right\}$

Q4 $A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 1 & 3 & 2 \\ 0 & 2 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 5 & 0 & 0 & -1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -3 & 1 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & -4 & -2 & 1 \\ 0 & -10 & -5 & -6 \end{bmatrix}$ $R_2-2R_1$, $R_4-3R_1$, $R_5-5R_1$

$\sim \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 2 & 1 & 2 \\ 0 & -4 & -2 & 1 \\ 0 & -10 & -5 & -6 \end{bmatrix}$ $-(R_2+R_3)$

$\sim \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & -10 & -7 \\ 0 & 0 & -25 & -26 \end{bmatrix}$ $R_1-2R_2$, $R_3-2R_2$, $R_4+4R_2$, $R_5+10R_1$

$\sim \begin{bmatrix} 1 & 0 & 5 & 5 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & -10 & -7 \\ 0 & 0 & -25 & -26 \end{bmatrix}$ $R_3/5$



$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $R_4+10R_3$, $R_5+25R_3$, $R_2+2R_3$, $R_1-5R_3$

$\sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2/5 \\ 0 & 0 & 1 & 6/5 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $\frac{1}{5}R_4$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_5-4R_4$, $R_3-\frac{6}{5}R_4$, $R_2-\frac{2}{5}R_4$, $R_1+R_4$

So $w_1 = (1,0,0,0)$
$w_2 = (0,1,0,0)$
$w_3 = (0,0,1,0)$
$w_4 = (0,0,0,1)$ form a basis for V

R5 (a) consisting of vector that are not row vectors of A

$$A=\begin{bmatrix}1 & 2 & -1\\ 1 & 9 & -1\\ -3 & 8 & 3\\ -2 & 3 & 2\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 2 & -1\\ 0 & 7 & 0\\ 0 & 14 & 0\\ 0 & 7 & 0\end{bmatrix}\begin{matrix}-R_1+R_2\\ 3R_1+R_3\\ 2R_1+R_4\end{matrix}$$

$$\sim\begin{bmatrix}1 & 2 & -1\\ 0 & 1 & 0\\ 0 & 14 & 0\\ 0 & 7 & 0\end{bmatrix}\ \tfrac{1}{7}R_2$$

$$\sim\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{matrix}-2R_2+R_1\\ -14R_2+R_3\\ -7R_2+R_4\end{matrix}$$

Then $w_1=(1,0,-1)$
$w_2=(0,1,0)$ is basis for A.
$w_3=$

(b) consisting of vector that are row vector of A





whose augmented matrix is

$$\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 2 & 9 & 8 & 3 & 0\\ -1 & -1 & 3 & 2 & 0\end{array}\right]=[A^T:0].$$

i.e. the coefficient matrix is $A^T$. Transforming the augmented matrix $[A^T:0]$ is to reduced row echelon form we obtain

$$\sim\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 0 & 7 & 14 & 7 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]\begin{matrix}-2R_1+R_2\\ R_1+R_3\end{matrix}$$

$$\sim\left[\begin{array}{cccc|c}1 & 1 & -3 & -2 & 0\\ 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]\ R_2/7$$

$$\sim\left[\begin{array}{cccc|c}1 & 0 & -5 & -3 & 0\\ 0 & 1 & 2 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right]-R_2+R_1$$

Since the leading 1s in column 1, 2, we conclude that the 1st two rows of A form a basis for the row space A that is $\{(1,2,-1),(1,9,-1)\}$.

Q6 is similarly to Q5.

**Q7** $A = \begin{bmatrix} 1 & -2 & 7 & 0 \\ 1 & -1 & 4 & 0 \\ 3 & 2 & -3 & 5 \\ 2 & 1 & -1 & 3 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 3 & 2 \\ -2 & -1 & 2 & 1 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 7 & 4 & -3 & -1 \\ 0 & 0 & 5 & 3 \end{bmatrix} 2R_1+R_2$

$\sim \begin{bmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 8 & 5 \\ 0 & -3 & -24 & -15 \\ 0 & 0 & 5 & 3 \end{bmatrix} -7R_1+R_3$

$\sim \begin{bmatrix} 1 & 0 & -5 & -3 \\ 0 & 1 & 8 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} \begin{array}{l} -R_2+R_1 \\ 3R_2+R_3 \\ 1/5\, R_4 \end{array}$

$\sim \begin{bmatrix} 1 & 0 & -5 & -3 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} -8R_4+R_2$



$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3/5 \end{bmatrix} 5R_4+R_1$

Hence $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1/5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3/5 \end{bmatrix} \right\}$ Ans.

is the basis for the column space A.

(b) the augmented matrix is

$\left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 1 & -1 & 4 & 0 & 0 \\ 3 & 2 & -3 & 5 & 0 \\ 2 & 1 & -1 & 3 & 0 \end{array}\right] = [A^T : 0]$

i.e. the coefficient matrix is $A^T$. Transforming the augmented matrix $[A^T : 0]$ is on to reduced row echelon form we obtain

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 7 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 8 & -24 & 5 & 0 \\ 0 & 5 & -15 & 3 & 0 \end{array}\right] \begin{array}{l} -R_1+R_2 \\ -3R_1+R_3 \\ -2R_1+R_4 \end{array}$

$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right] \begin{array}{l} -8R_2+R_3 \\ -5R_2+R_4 \end{array}$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \tfrac{1}{3}R_4$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] -5R_4+R_3$$

Since the leading 1's in column 1, 2 & 4 we conclude that the 1st two rows of A form a basis for the row space A that is

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix} \right\}$$

Q8 is similarly do Q7

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Ex 6.6      134

$$A = \begin{bmatrix} 1 & -2 & 5 \\ 2 & 3 & 2 \\ 0 & -7 & 8 \end{bmatrix}$$

(a) basis for row space of A

$$\sim \begin{bmatrix} 1 & -2 & 5 \\ 0 & 7 & -8 \\ 0 & -7 & 8 \end{bmatrix} R_2-2R_1$$

$$\sim \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -8/7 \\ 0 & -7 & 8 \end{bmatrix} \tfrac{1}{7}R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 19/7 \\ 0 & 1 & -8/7 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1+2R_2 \\ R_3+7R_2 \end{matrix}$$

Basis for row space of A

$$A = \{[1, 0, 19/7], [0, 1, -8/7]\}$$

(b) Basis for column space of A

$$A^T = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & -7 \\ 5 & 2 & 8 \end{bmatrix}$$

Augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 5 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & -7 & 8 & 0 \end{array}\right] [A:0]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 19/7 & 0 \\ 0 & 1 & -8/7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ (From 1st part (a))}$$

As leading 1s occur in col 1 & 2 so 1st & 2nd col of $A^T$ form the basis

$$\begin{bmatrix} 1 \\ 0 \\ 19/7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 8/7 \end{bmatrix}$$

(c) basis for row space of $A^T$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ -2 & 3 & -7 & 0 \\ 5 & 2 & 8 & 0 \end{array}\right] \quad [A^T : 0]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right] \begin{array}{l} \\ R_2+2R_1 \\ R_3-5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right] \tfrac{1}{7}R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1-2R_2 \\ R_3+8R_2 \\ \end{array}$$

$\{[1\ 0\ 2], [0\ 1\ -1]\}$ form Basis



(d) column space of A

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} : [A:0]$$

Basis for col space of A

$\left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$ form Basis

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Q10 is similarly to Q9.

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Q11 $A = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}$

augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 2 & 1 & 8 & 0 \\ 3 & -5 & -1 & 0 \\ 2 & -2 & 2 & 0 \\ 1 & 1 & 5 & 0 \end{array}\right] = [A^T | 0]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 0 & -5 & -6 & 0 \\ 0 & -14 & -22 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \begin{array}{l} \\ -2R_1+R_2 \\ -3R_1+R_3 \\ -2R_1+R_4 \\ -R_1+R_5 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 3 & 7 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & -14 & -22 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] -\tfrac{1}{5}R_2$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & -8 & -12 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] \begin{array}{l} -3R_2+R_1 \\ -7R_2+R_3 \end{array}$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & -2 & -2 & 0 \end{array}\right] -4R_3+R_4$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 7/5 & 0 \end{array}\right] 2R_2+R_5$$

$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 17/5 & 0 \\ 0 & 1 & 6/5 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \tfrac{5}{7}R_5$$



$$\sim\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} 4R_3+R_4 \\ 8R_3+R_3 \\ -\tfrac{4}{5}R_5+R_4 \\ -\tfrac{17}{5}R_5+R_1 \end{array}$$

Since the non-zero row is 3 so rank = 3.

(b) where augmented matrix is $[A|0]$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 3 & 1 & -5 & -2 & 1 & 0 \\ 7 & 8 & -1 & 2 & 5 & 0 \end{array}\right] = [A|0]$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 0 & -5 & -14 & -8 & -2 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] \begin{array}{l} -3R_1+R_2 \\ -7R_1+R_3 \end{array}$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 1 & 0 \\ 0 & 1 & 14/5 & 8/5 & 2/5 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] -\tfrac{1}{5}R_2$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 14/5 & 8/5 & 2/5 & 0 \\ 0 & -6 & -22 & -12 & -2 & 0 \end{array}\right] -2R_2+R_1$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & 0 & -48/5 & -12/5 & 2/5 & 0 \end{array}\right] \quad 6R_2+R_3$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 0 & -7/5 & -6/5 & 1/5 & 0 \\ 0 & 1 & 11/5 & 8/5 & 2/5 & 0 \\ 0 & 0 & 1 & 12/45 & -2/45 & 0 \end{array}\right] \quad -\tfrac{5}{45}R_3$$

$$\sim\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -62/75 & 31/225 & 0 \\ 0 & 1 & 0 & 76/75 & 68/225 & 0 \\ 0 & 0 & 1 & 12/45 & -2/45 & 0 \end{array}\right] \quad \begin{array}{l} -\frac{11}{5}R_3+R_2 \\ \frac{7}{5}R_3+R_1 \end{array}$$

So non-zero row = 3 rank = 3

Row rank = column rank = 3.

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Q12 is similarly to Q11

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$$A=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 4 \\ 0 & -5 & -7 \end{bmatrix} \quad \begin{array}{l} R_2+R_1 \\ R_3-3R_1 \end{array}$$

$$\sim\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & -5 & -7 \end{bmatrix} \quad \tfrac{1}{4}R_2$$

$$\sim\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad R_3+5R_2$$

$$\sim\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad R_1-2R_2$$

Rank of A = No: of non-row = 3

So rank = 3.

For nullity $Ax=0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right]$$

$-2x_3=0 \Rightarrow \boxed{x_3=0}$

$x_2+x_3=0 \Rightarrow \boxed{x_2=0}$

$x_1+3x_3=0 \Rightarrow \boxed{x_1=0}$

Q13, Q14, Q16, Q17 is similarly to Q15

Q26 If Rank A = n = 3
then there is unique solution

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 8 & -7 \\ 3 & -2 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 8 & -7 \\ 0 & -8 & 7 \end{bmatrix} R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7/8 \\ 0 & -8 & 7 \end{bmatrix} R_2/8$$

$$\sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7/8 \\ 0 & 0 & 0 \end{bmatrix} R_3 + 8R_2$$

Non-zero rows is 2

So rank = 2

No unique solution

Q27 is same as Q26



Q28 Linearly independent holds when det ≠ 0

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 3 & 2 & 3 \end{vmatrix}$$

$2(0-2) - 1(6-3) + 0(4-0)$

$-4 - (3) + 0$

$-4 - 3$

$-7 \neq 0$ so linearly independent

Q29 is same as Q28

Q30 Non-trivial solution occurs when rank < n.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} 2R_2 + R_3$$

Rank A < 3 so non-trivial.

Q30 & Q32 is same as Q31

0 6

33 solution exist when

Rank A = Rank of [A:b]

$$\left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 2 & 3 & -2 & 4 & 0 \\ 5 & 1 & 0 & 2 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 0 & -1 & -12 & 8 & 0 \\ 0 & -9 & -25 & 12 & 0 \end{array}\right] \begin{array}{l} R_2-2R_1 \\ R_3-5R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & 5 & -2 & 0 \\ 0 & 1 & 12 & -8 & 0 \\ 0 & -9 & -25 & 12 & 0 \end{array}\right] -R_2$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & -19 & 14 & 0 \\ 0 & 1 & 12 & -8 & 0 \\ 0 & 0 & 83 & -60 & 0 \end{array}\right] \begin{array}{l} R_3+9R_2 \\ R_1-2R_2 \end{array}$$

Rank of A = Rank of [A:0]

$\Rightarrow$ so it has solution



solution exist when

Rank of A = Rank of [A:b]

$$\left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 4 & -1 & -5 & 6 & 2 \\ 2 & 3 & 1 & -2 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 0 & 7 & 7 & -10 & -2 \\ 0 & 7 & 7 & -10 & 0 \end{array}\right] \begin{array}{l} R_2-4R_1 \\ R_3-2R_1 \end{array}$$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & -3 & 4 & 1 \\ 0 & 7 & 7 & -10 & -2 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right] R_2-R_1$$

Rank of A = 3 & Rank of [A:b] = 2

Rank of A $\neq$ Rank of [A:b]

$\Rightarrow$ so it's has no solution

Q34, Q36 is similarly

6

bit matrix

Q: $\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\end{bmatrix} \sim \begin{bmatrix}1&0&1\\0&1&1\\0&1&1\end{bmatrix} R_3+R_1$

$\sim \begin{bmatrix}1&0&1\\0&1&1\\0&0&0\end{bmatrix} R_3+R_2$

Non-zero row = 2

So rank = 2.

Q: $\begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&1&0\\1&1&0&0\end{bmatrix}$

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&1&1&0\\0&1&1&0\end{bmatrix} R_4+R_1$

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&1&1\\0&0&1&1\end{bmatrix} \begin{matrix}R_3+R_2\\R_4+R_2\end{matrix}$

$\sim \begin{bmatrix}1&0&1&0\\0&1&0&1\\0&0&1&1\\0&0&0&0\end{bmatrix} R_4+R_3$

non-zero row = 3 $\Rightarrow$ rank = 3

Q39 & Q40 similarly.





Let $A=\begin{bmatrix}3&-1\\-2&2\end{bmatrix}$.

For eigenvalues

$\det(\lambda I_n - A)=0$

$\det\left(\lambda\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\right)=0$

$\det\left(\begin{bmatrix}\lambda&0\\0&\lambda\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\right)=0$

$\det\left(\begin{bmatrix}\lambda-3&1\\2&\lambda-2\end{bmatrix}\right)=0$

$(\lambda-3)(\lambda-2)-2=0$

$\lambda^2-2\lambda-3\lambda+6-2=0$

$\lambda^2-5\lambda+4=0$

$\boxed{\lambda=1}$ $\boxed{\lambda=4}$

For eigenvector

$(\lambda I_n - A)X=0$

For $\lambda=1$

$1\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}3&-1\\-2&2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=0$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}3 & -1\\-2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$$\begin{bmatrix}-2 & 1\\2 & -1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$$-2x_1+x_2=0$$
$$2x_1-x_2=0$$

$x_1=\frac{1}{2}x_2$

$x_2=r$ (any real no's) $\quad X=\begin{bmatrix}\frac{1}{2}r\\r\end{bmatrix}$.

$\boxed{\text{For } \lambda=4}$

$$4\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}3 & -1\\-2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$$\begin{bmatrix}4 & 0\\0 & 4\end{bmatrix}-\begin{bmatrix}3 & -1\\-2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$$\begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

$$x_1+x_2=0$$
$$2x_1+2x_2=0$$

$x_1=-x_2$

$x_2=r$ (any real no's)

$$X=\begin{bmatrix}-r\\r\end{bmatrix}$$



Let $A=\begin{bmatrix}2 & 2 & 3\\1 & 2 & 1\\2 & -2 & 1\end{bmatrix}$.

For eigenvalues

$$\det(\lambda I_n - A)=0$$

$$\det\left(\lambda\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}-\begin{bmatrix}2 & 2 & 3\\1 & 2 & 1\\2 & -2 & 1\end{bmatrix}\right)=0$$

$$\det\left(\begin{bmatrix}\lambda & 0 & 0\\0 & \lambda & 0\\0 & 0 & \lambda\end{bmatrix}-\begin{bmatrix}2 & 2 & 3\\1 & 2 & 1\\2 & -2 & 1\end{bmatrix}\right)=0$$

$$\det\left(\begin{bmatrix}\lambda-2 & -2 & -3\\-1 & \lambda-2 & -1\\-2 & 2 & \lambda-1\end{bmatrix}\right)=0$$

$$\lambda-2\left[(\lambda-1)(\lambda-2)+2\right]+2\left[-(\lambda-1)-2\right]-3\left[-2+2(\lambda-2)\right]=0$$

$$=(\lambda-2)(\lambda^2-3\lambda+4)+(-2\lambda-2)+(-6\lambda+18)=0$$

$$\lambda^3-5\lambda^2+2\lambda+8=0$$

$$\lambda=-1$$

S.D
$$\begin{array}{r|rrrr} -1 & 1 & -5 & 2 & 8\\ & & -1 & 6 & -8\\ \hline & 1 & -6 & 8 & \boxed{0} \end{array}$$

$$(\lambda+1)(\lambda^2-6\lambda+8)=0$$

$\lambda+1=0$

$\lambda=-1$

$\lambda^2-6\lambda+8=0$

$\lambda^2-4\lambda-2\lambda+8=0$

$\lambda(\lambda-4)-2(\lambda-4)=0$

$(\lambda-2)(\lambda-4)=0$

$\lambda-2=0,\ \lambda-4=0$

$\lambda=2,\ \lambda=4.$

Now $\lambda=-1,\ \lambda=2,\ \lambda=4.$

For eigenvector

$(\lambda I_n - A)X=0$

$\lambda=-1$

$$=\left(-1\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}2&2&3\\1&2&1\\2&-2&1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$=\left(\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}-\begin{bmatrix}2&2&3\\1&2&1\\2&-2&1\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$=\begin{bmatrix}-3&-2&-3\\-1&-3&-1\\-2&2&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$



$$(A/b)=\left[\begin{array}{ccc|c}-3&-2&-3&0\\-1&-3&-1&0\\-2&2&-2&0\end{array}\right]$$

$$\overset{R}{\sim}\left[\begin{array}{ccc|c}1&3&1&0\\0&1&0&0\\0&0&-1&0\end{array}\right]$$

$x_3=-1$

$x_2=0$

$x_1=-3x_2-x_3$

$x_1=-3(0)-(-1)=+1$

$$X=\begin{bmatrix}1\\0\\-1\end{bmatrix}.$$

$\lambda=2,\ \lambda=4$ S.Y.S



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Q3 Let $A=\begin{bmatrix}1&2&1\\0&1&2\\-1&3&2\end{bmatrix}$

The characteristic Poly: is

$P(\lambda)=\det(\lambda I_n - A)$

$=\det\left(\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}-\begin{bmatrix}1&2&1\\0&1&2\\-1&3&2\end{bmatrix}\right)$

$=\det\left(\begin{bmatrix}\lambda-1&-2&-1\\0&\lambda-1&-2\\1&-3&\lambda-2\end{bmatrix}\right)$

$=(\lambda-1)[(\lambda-1)(\lambda-2)-6]+2(0+2)-1(0+1-\lambda)$

$=(\lambda-1)(\lambda^2-3\lambda-4)+4+\lambda-1$

$P(\lambda)=\lambda^3-4\lambda^2+7$ which is required ch. Poly:

Q4, Q5, Q6, Q7 is similarly to Q3







$A=\begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}$

The characteristic Poly: of matrix A is given by

$P(\lambda)=\det(\lambda I_3 - A)$

$=\det\left(\begin{bmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{bmatrix}-\begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}\right)$

$=\det\left(\begin{bmatrix}\lambda&-1&-2\\0&\lambda&-3\\0&0&\lambda\end{bmatrix}\right)$

$=\lambda(\lambda^2+0)+1(0+0)-2(0+0)$

$P(\lambda)=\lambda^3$

Now $P(\lambda)=\lambda^3=0$

$\lambda=0$ Eigen value

For Eigen vector

$(\lambda I_3 - A)x=0$

$\left(\lambda\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\left(0\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\left(0 - \begin{bmatrix}0&1&2\\0&0&3\\0&0&0\end{bmatrix}\right)\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0&-1&-2\\0&0&-3\\0&0&-0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$(A/b)\quad \left[\begin{array}{ccc|c}0&-1&-2&0\\0&0&-3&0\\0&0&0&0\end{array}\right]$$

$$\boxed{x_3 = 0}$$

$$-x_2 - 2x_3 = 0$$

$$-x_2 = 2x_3$$

$$-x_2 = 2(0)$$

$$-x_2 = 0$$

$$\boxed{x_2 = 0}$$

Let $x_1 = r$

$$X = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}r\\0\\0\end{bmatrix}.$$

Q9, Q10, Q11, Q12, Q13, Q14, Q15 Similarly.