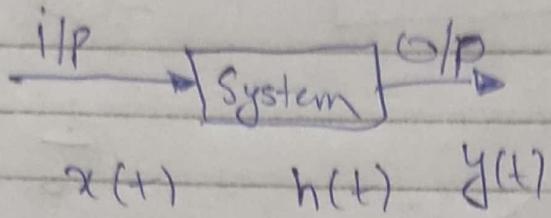


Date: 2/5/2023

S.S

Day: M T W T F S

Systems :-



$$y(t) = x(t) * h(t)$$

$$y(t) = 0.8 x(t)$$

$$x(t) \rightarrow y(t) = x(t - t_d)$$

$$y(t) = x(t)$$

$$y(0) = x(0) \quad \text{Present i/p}$$

$$y(t) = x(t-1)$$

$$t=0$$

$$y(0) = x(-1) \quad \text{Previous}$$

$$y[n] = x[n-1]$$

$$y(t) = x(t+1)$$

$$y(0) = x(1)$$

Memory less System

* Static & Dynamic Systems.

$$y(+)=x(t)$$

$$y(+)=x(+)+x(t-1)$$

$$* y(+) = 2x(t)$$

$t=0$

$$y(0) = 2 \cdot x(0)$$

$$* y(+) = x(2t)$$

$$y(0) = x(0)$$

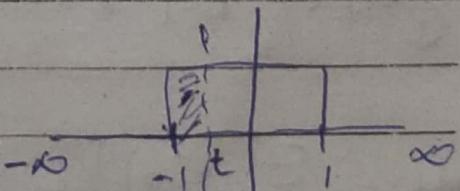
$$y(1) = x(2)] \rightarrow \text{Dynamic}$$

$$* y(t) = x(-t) \rightarrow \text{Dynamic}$$

$$* y(t) = \int_{-\infty}^t x(\tilde{t}) d\tilde{t} \rightarrow$$

$$* y(t) = x(\sin t)$$

$$y(0) = x(0)$$



$$y(\tau) = x(0)$$

If there is a time scaling
time shifting
or integration.
then system is dynamic.

$$y(t) = \operatorname{Re}[x(+)]$$

$$x(+) = a + ib$$

$$x^*(+) = a - ib$$

$$x(+) + x^*(+) = 2a$$

$$\boxed{a = \frac{x(+) + x^*(+)}{2}}$$

$$\text{Now } y(+) = \operatorname{Re}[x(+)] = a = \frac{x(+) + x^*(+)}{2}$$

$$y(t) = E[x(+)]$$

$$y(t) = E[x(+)] = \frac{x(+) + x(-t)}{2}$$

Date:

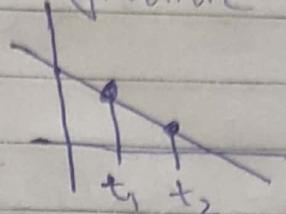
K=3

R=1

Day: M T W Th F S

$$\textcircled{a} \quad y(t) = x(t^3) \rightarrow \text{Dynamic}$$

$$\textcircled{b} \quad y(t) = \frac{d}{dt} x(t) \rightarrow \text{Dynamic}$$



* Causal & Non Causal System

does not depend
upon future values.

$$(i) \quad y(t) = x(t) \rightarrow \text{Causal}$$

$$y(t) = x(t) + x(t+1) \rightarrow \text{Non Causal}$$

$$y(t) = x(t+1) \rightarrow \text{Anti-Causal}$$

Causal :- Past / Present

Non Causal :- Past / Present + Future

Anti-Causal :- Future

$$(i) \quad y(t) = x(t+2)$$

$$(ii) \quad y(t) = x(t) + x(t-1) + x(t+1)$$

Date: _____

Day: M T W T F S

$$\textcircled{1} \quad y(t) = x(3t) \quad \text{Non-Causal}$$

$$\textcircled{2} \quad y(t) = \begin{cases} x(3t) & t < 0 \\ x(t-1) & t \geq 0 \end{cases} \rightarrow \text{Causal System}$$

$$\textcircled{3} \quad y(t) = \sin(t+1) \cdot x(t-1) \quad \text{Causal}$$

$$y(0) = \sin(1) \cdot x(-1)$$

$$\textcircled{4} \quad y(t) = x(e^t) \quad \text{Non-Causal}$$

$$y(0) = x(e^0) = x(1)$$

$$\textcircled{5} \quad y(t) = \int_{-\infty}^t x(\tau) d\tau$$

Causal

$y(t)$ depends upon present and past values.

$$\textcircled{6} \quad y(t) = \int_{-\infty}^{t+1} x(\tau) d\tau \quad \text{Non-Causal}$$

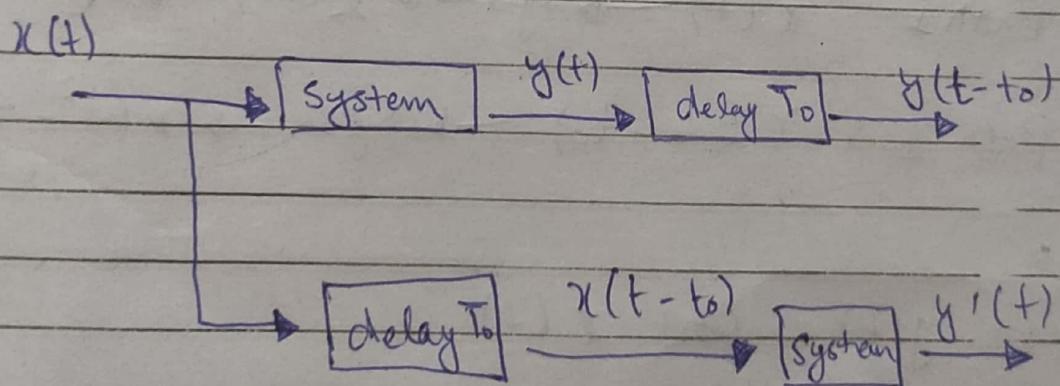
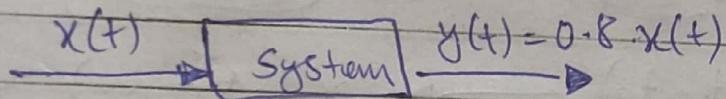
$x(-\infty) \quad x(+\infty)$

$$\textcircled{7} \quad y(t) = \int_{-\infty}^t x(3\tau) d\tau$$

Non-Causal

$$\textcircled{8} \quad y(t) = \frac{d}{dt} \cdot x(t)$$

Time Invariant and Time Variant Systems:-



$$y'(t) = y(t - T_0) \quad \text{TIV}$$

$$y'(t) \neq y(t - T_0) \quad \text{TV}$$

$$\textcircled{1} \quad y(t) = x(\cos(+))$$

Path I : $x(t) \rightarrow \text{System} \rightarrow x(\cos t) \rightarrow x[\cos(t - t_0)]$

Path II : $x(t) \rightarrow x(t - t_0) \rightarrow \text{System} \rightarrow x[\cos(t - t_0)]$

$$\textcircled{2} \quad y(t) = x(t^2)$$

$$\text{Path I} = x(t^2) \rightarrow x((t - t_0)^2)$$

$$\text{Path II} = x(t - t_0) \rightarrow x(t^2 - t_0)$$

Path I : $x(t) \rightarrow \text{sys} \rightarrow x(t^2) \rightarrow x[(t - t_0)^2]$

Path II : $x(t - t_0) \rightarrow \text{sys} \rightarrow x(t^2 - t_0)$

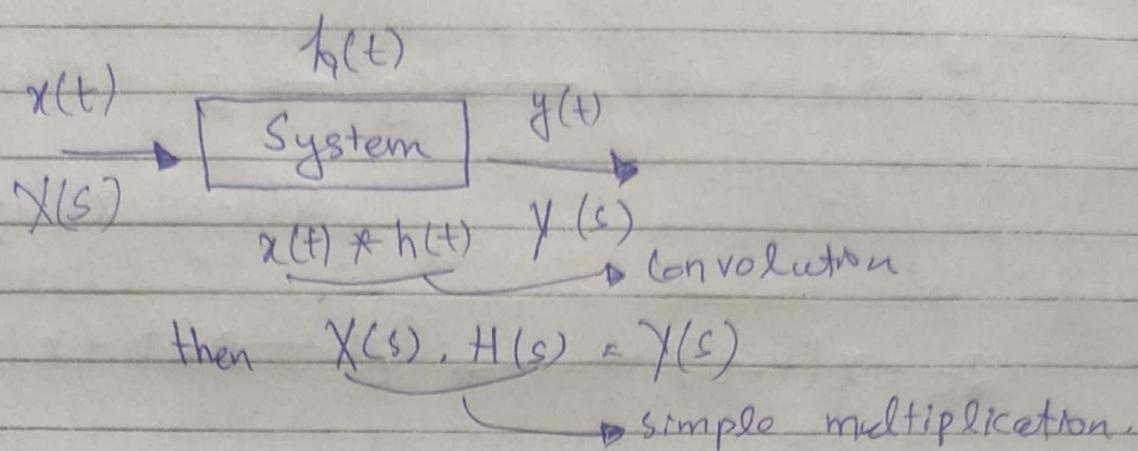
$$\textcircled{3} \quad y(t) = \text{const. } x(t)$$

Path I : $x(t) \rightarrow \text{syst} \rightarrow (\text{const. } x(t)) \rightarrow (\cos(t - t_0) \cdot x(t - t_0))$

Path II : $x(t) \rightarrow (\cancel{\cos(t - t_0)} \cdot \cancel{x(t - t_0)}) \rightarrow (\cos(t - t_0) \cdot x(t - t_0))$

Date: 9/5/2023

Day: M ✓ W T F S


 $x(t) \rightarrow \text{System} \rightarrow \text{Delay} \rightarrow y(t)$
 $x(t) \rightarrow \text{Delay} \rightarrow \text{System} \rightarrow y'(t)$

If $y(t) = y'(t)$ Time Invariant

$$y(t) = x(t^3)$$

$x(t) \rightarrow \text{System} \rightarrow y(t) = x(t^3) \rightarrow \text{delay}$

$(x(t-t_0)^3)$

 x

If the power operator exist in input
then system will be time invariant

$$e^{-t} \cdot x(t) \rightarrow \text{sys} \rightarrow \frac{-t}{e} x(t) \rightarrow e^{-(t-t_0)} \cdot x(t-t_0)$$

$$\underline{e^{-t} \cdot x(t-t_0) \rightarrow \text{sys} \rightarrow e^{-t} \cdot x(t-t_0)}$$

Date: Whenever there is

Day: M T W T F S

a coefficient then
TV

$$y(t) = 2t + x(t) \rightarrow \text{Sys} \rightarrow 2t + x(t)$$
$$2(t-t_0) + x(t-t_0)$$

$$2t + x(t-t_0) \rightarrow \text{Sys} \rightarrow 2t + x(t-t_0)$$

If any addition or subtraction other than the input and output signal is said to be TV

If is any added constant at all of the instance of time subtracted then the system is TIV

If additive or subtractive term depends upon t , then TV

If there is a time scaling, TIV

$$x(t-1) + x(t+1) \rightarrow x(t-t_0-1) + x(t-t_0+1)$$

$$x(t-t_0-1) + x(t-t_0+1) \rightarrow x(t-t_0-1) + x(t-t_0+1)$$

Date: _____

Day: M T W T F S

$$y(t) = \int_{-\infty}^t x(\tilde{t}) d\tilde{t}$$

$$x(\tilde{t}) \xrightarrow{\text{sys}} \int_{-\infty}^t x(\tilde{t}) dt \xrightarrow{\text{delay}} \int_{-\infty}^t x(\tilde{t}-t_0) d\tilde{t}$$

$$x(\tilde{t}) \xrightarrow{\text{delay}} x(\tilde{t}-t_0) \xrightarrow{\text{sys}} \int_{-\infty}^t x(\tilde{t}-t_0) d\tilde{t}$$

$$y(t) = \frac{d}{dt} x(t)$$

$$\xrightarrow{\text{sys}} \int_{-\infty}^t x(3\tilde{t}) d\tilde{t} \longrightarrow \int_{-\infty}^t x(3(\tilde{t}-t_0)) d\tilde{t}$$

$$\xrightarrow{\text{sys}} x(\tilde{t}-t_0) \longrightarrow \int_{-\infty}^t x(3(\tilde{t}-t_0)) d\tilde{t}$$

$\left\{ \int_{-\infty}^t x(3(\tilde{t}-t_0)) d\tilde{t} \right\}$

Integration of time scaled

TV

Date: _____

Day: M T W T F S

$$y(t) = \int_{-\infty}^t \cos^2 \omega_n x(\tau) d\tau$$

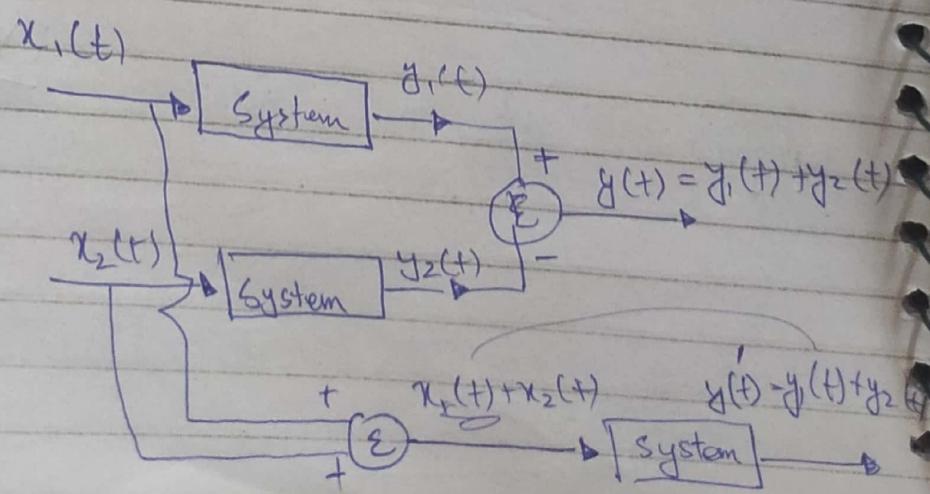
~~Y(t)~~ $x(t) \rightarrow \text{sys} \xrightarrow{\text{delay}} \int_{-\infty}^t \cos(\omega_n(t-\tau)) x(\tau) d\tau$

Linear & Non-Linear Systems :-

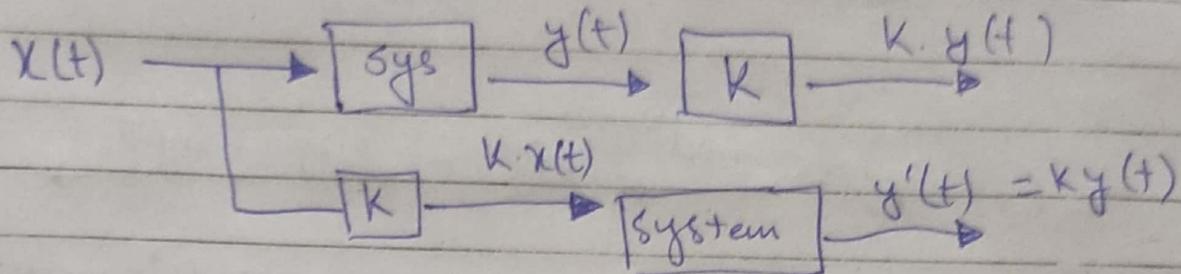
→ which follows the law of superposition
are linear.

- i) Law of additivity LOA
- ii) Law of homogeneity LOH

(i) LOA :-



If $y'(t) = y(t)$ LOA
 $y'(t) \neq y(t)$ LOA X



If $y'(t) = K.y(t)$ LOH
 $\cdot \neq K.y(t)$ LOH

$$y(t) = x(\text{Sint})$$

1st Path:

$$\begin{aligned} x_1(t) &\xrightarrow{\text{Sys}} x_1(\text{Sint}) \\ x_2(t) &\xrightarrow{\text{Sys}} x_2(\text{Sint}) \end{aligned} \quad \left\{ \begin{array}{l} x_1(\text{Sint}) + x_2(\text{Sint}) \\ \vdots \end{array} \right.$$

2nd Path:

$$x_1(t) + x_2(t) \longrightarrow \text{Sys} \longrightarrow x_1(\text{Sint}) + x_2(\text{Sint})$$

$$K.x(\text{Sint})$$

$$K.x(t) \longrightarrow K.x(\text{Sint})$$

$$L(u+v) = L(u) + L(v)$$

$$L(ku) = k \cdot L(u)$$

Date: _____

Day: M T W T F S

$$y(t) = x(t^2)$$

$$\begin{array}{c} x_1(t) \xrightarrow{\text{sys}} x_1(t^2) \\ x_2(t) \xrightarrow{\text{sys}} x_2(t^2) \end{array} \quad \left[\begin{array}{l} x_1(t^2) + x_2(t^2) \end{array} \right]$$

$$\begin{array}{c} x_1(t) + x_2(t) \xrightarrow{\text{add}} x_1(t^2) + x_2(t^2) \xrightarrow{\text{Syst}} \\ x_1(t^2) + x_2(t^2) \end{array}$$

$$k \cdot x(t^2)$$

$$k \cdot x(t) \longrightarrow k \cdot x(t^2)$$

$$y(t) = x(\log t)$$

$$\begin{array}{c} x_1(t) \xrightarrow{\text{sys}} x_1(\log t) \\ x_2(t) \xrightarrow{\text{sys}} x_2(\log t) \end{array} \quad \left[\begin{array}{l} x_1(\log t) + x_2(\log t) \end{array} \right]$$

$$x_1(t) + x_2(t) \xrightarrow{\text{sys}} x_1(\log t) + x_2(\log t)$$

$$k \cdot x(\log t)$$

$$k \cdot x(t) \Rightarrow k \cdot x(\log t)$$

Date: _____

Day: M T W T F S

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$$\star \quad y(t) = \log[x(t)]$$

$$\log[x_1(t)] + \log[x_2(t)]$$

$$\log[x_1(t)] + \log[x_2(t)] \longrightarrow \log[x_1(t) + x_2(t)]$$

$$x_1(t) + x_2(t) \longrightarrow \log[x_1(t) + x_2(t)]$$

$$y(t) = x^2(t)$$

$$x_1^2(t) \quad] \quad x_1^2(t) + x_2^2(t)$$

$$x_2^2(t)$$

$$x_1(t) + x_2(t) \longrightarrow [x_1(t) + x_2(t)]^2$$

$$\star \quad y(t) = \text{Sint. } x(t)$$

$$\text{Sint. } x_1(t) \quad] \quad \text{sint}(x_1(t) + x_2(t))$$

$$\text{Sint. } x_2(t)$$

$$x_1(t) + x_2(t) \xrightarrow{\text{Sys}} \text{Sint. } x_1(t) + \text{Sint. } x_2(t)$$

If any addition / subtraction other than the input signals, the sys is said to be non-linear system.

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

$$\int_{-\infty}^t x_1(\tau) d\tau + \int_{-\infty}^t x_2(\tau) d\tau$$

$$x_1(\tau) + x_2(\tau) = \int_{-\infty}^t (x_1(\tau) d\tau + x_2(\tau) d\tau)$$

Linear

$$\overline{y(t)} = E(x(t))$$

$$E[x(t)] = \frac{x_1(t) + x_1(-t)}{2}$$

$$\left. \begin{aligned} E[x_1(t)] &= \frac{x_1(t) + x_1(-t)}{2} \\ E[x_2(t)] &= \frac{x_2(t) + x_2(-t)}{2} \end{aligned} \right] \rightarrow x_1(t) + x_1(-t)$$

Date:

Day: M T W T F S

$$y(t) = \operatorname{Re}\{x(t)\} \quad \text{NL linear}$$

$$x(t) = a + i b(t)$$

$$a = \frac{x(t) + x^*(t)}{2}$$

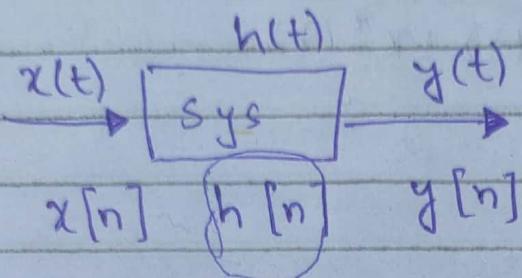
$$\frac{x_1(t) + x_1^*(t)}{2} + \frac{x_2(t) + x_2^*(t)}{2}$$

$$\frac{x_1(t) + x_1^*(t) + x_2(t) + x_2^*(t)}{2}$$

$$x_1(t) + x_2(t) \xrightarrow{\text{System}} \frac{[x_1(t) + x_1^*(t)] + [x_2(t) + x_2^*(t)]}{2}$$

Linear Time Invariant Systems

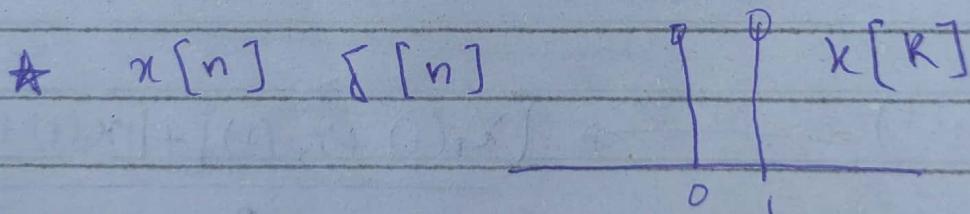
★ LTI got a fundamental role



If $x(t) = \delta(t)$

then response of sys will be
unit impulse response.

$$\delta(t) \xrightarrow{\text{LTI}} y(t) = h(t)$$

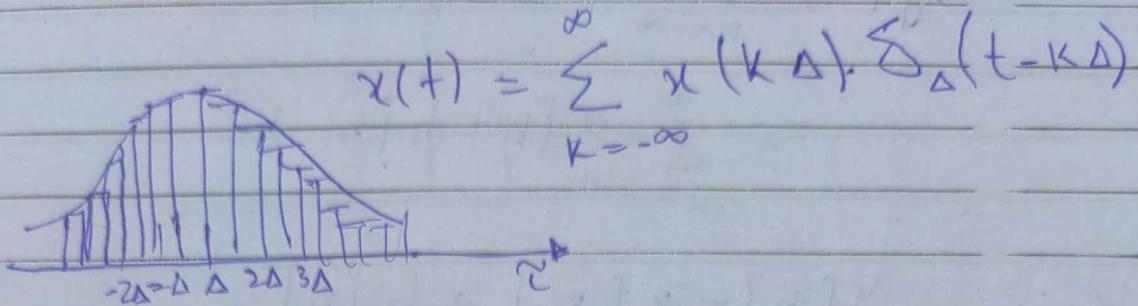


$$\sum_{k=-\infty}^{\infty} x[k] \cdot \delta(n-k)$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta(n-k)$$

$$x[0] \cdot \delta[n] + x[1] \delta[n-1]$$

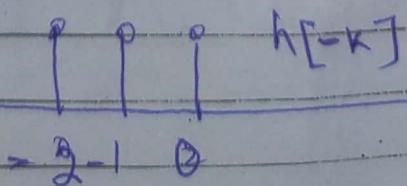
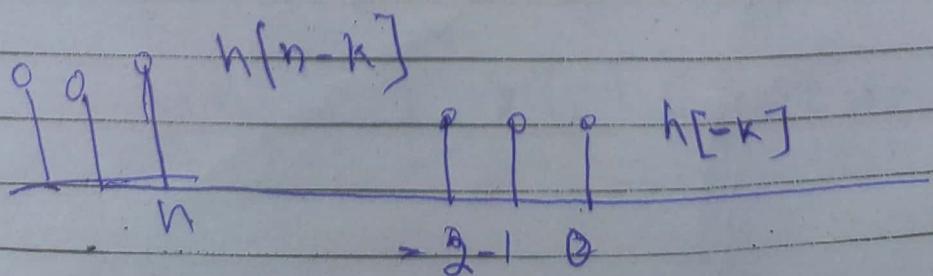
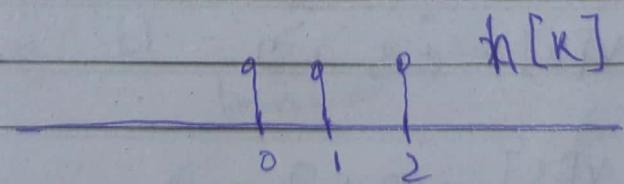
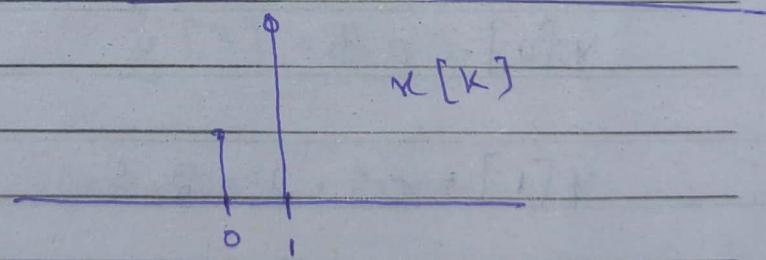
★ Continuous time Signals :-



Let $k\Delta = \tilde{\tau}$

$$\lim_{\Delta \rightarrow 0} x(t) = \sum_{n=-\infty}^{\infty} x(\tilde{\tau}) \underbrace{\delta_{\Delta}(t-\tilde{\tau})}_{\substack{\text{height} \\ \Delta}} \cdot \Delta$$

$$x(t) = \int_{-\infty}^{\infty} x(\tilde{\tau}) \cdot \delta_{\Delta}(t-\tilde{\tau}) d\tilde{\tau}$$



$$y[n] = \sum_{k=-\infty}^{\infty} x[n] \cdot h_k[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$y[n] = x[n] * h[n]$$

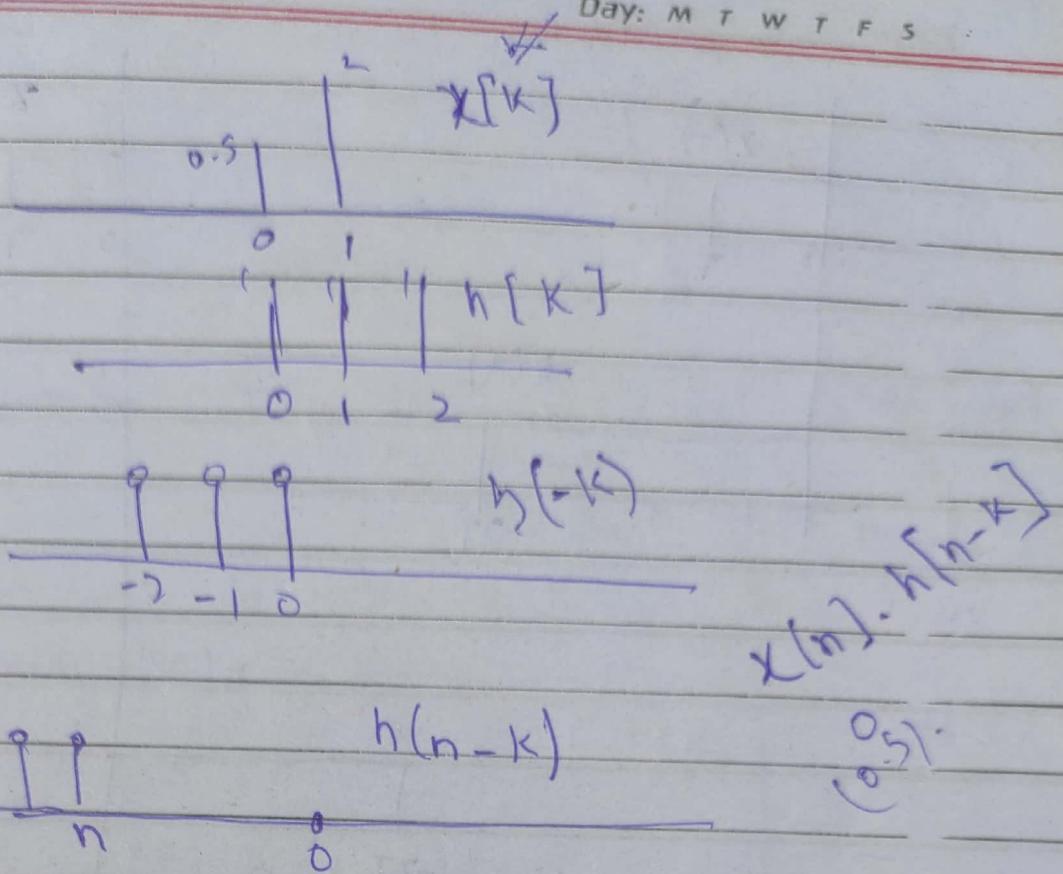
$$y[n] = 0.5(1) + \boxed{2(1)} \rightarrow \text{nullify}$$

$$y[0] = 0.5$$

$$y[1] = 0.5 + 2 = 2.5$$

$$y[2] = 0.5 + 2$$

$$y[3] = 0 + 2 = 2$$



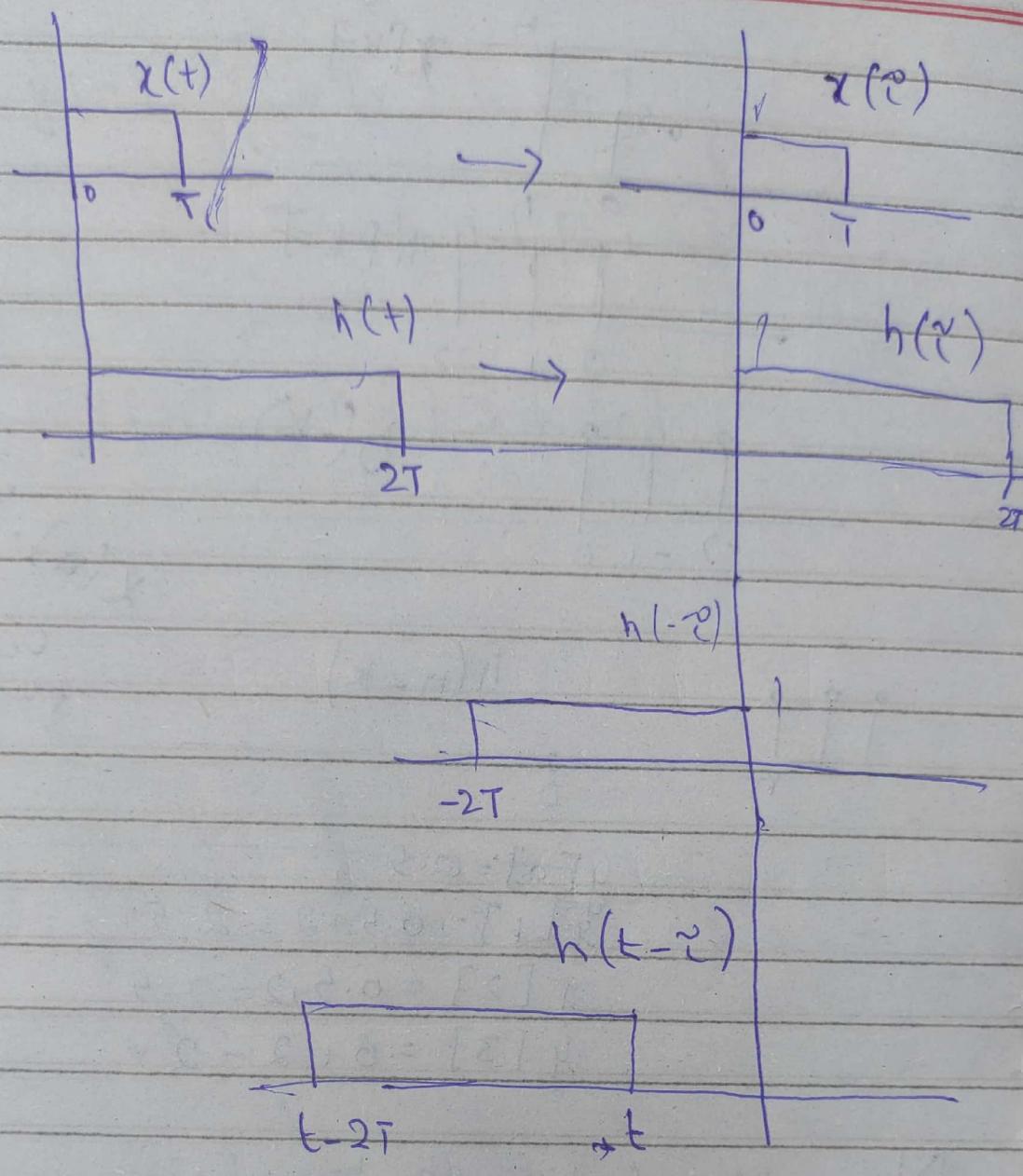
$$\begin{aligned}
 y[0] &= 0.5 \\
 y[1] &= 0.5 + 2 = 2.5 \\
 y[2] &= 0.5 \cdot 2 = 2.5 \\
 y[3] &= 0 + 2 = 2
 \end{aligned}$$

Now in continuous time

$$y(t) = \lim_{\Delta \rightarrow 0} \hat{y}(t) = \sum_{\substack{k=0 \\ \lim n = -\infty}}^{\infty} x(k\Delta) \cdot h(t - k\Delta) \Delta$$

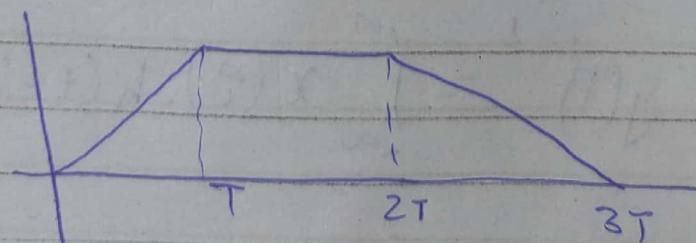
$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t - \tau) d\tau$$

$$y(t) = x(t) * h(t)$$



$$x(t) * h(t) = \int_{-\infty}^t x(\tau) \cdot h(t-\tau) d\tau$$

$\rightarrow \infty \text{ to } 0$



Date:

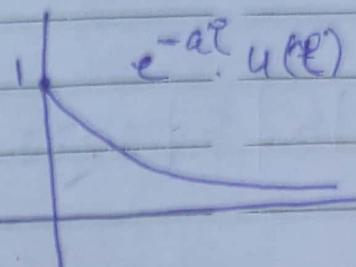
Day: M T W T F S

Prob

$$x(t) = e^{-at} \cdot u(t), a > 0$$

$$h(t) = u(t)$$

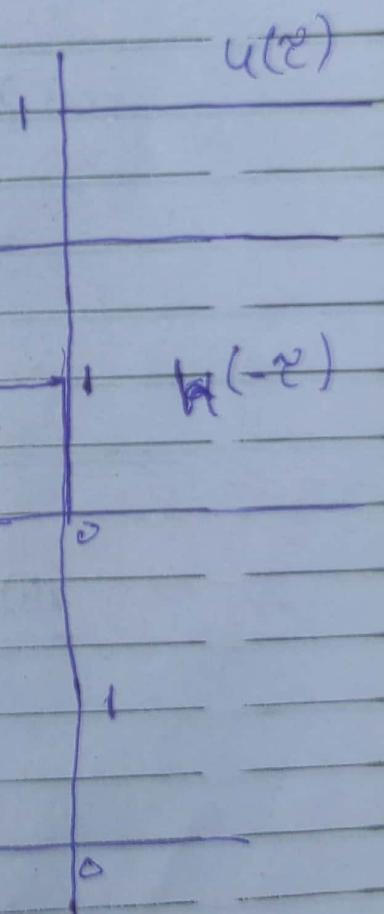
$$x(\tau) = e^{-a\tau} \cdot u(\tau)$$



from $-\infty$ to 0, it is
disjoint,

$$\text{So } y(t) = \int_0^t x(\tau) \cdot h(t-\tau) d\tau$$

$$y(t) = x(t) * h(t)$$



$$y(t) = \int_0^t e^{-a\tau} d\tau$$

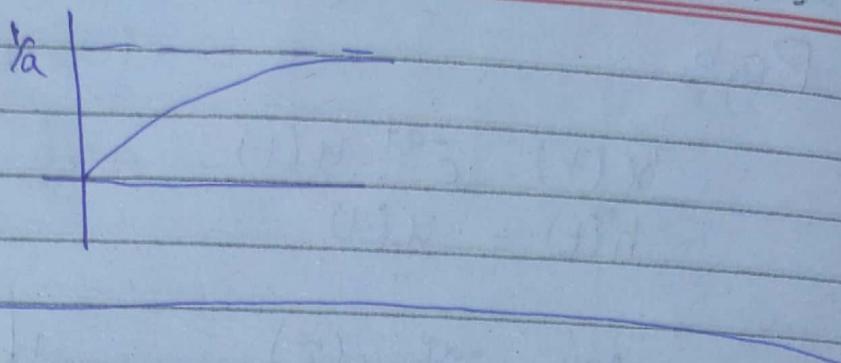
$$y(t) = \frac{e^{-at}}{-a} \Big|_0^t$$

$$y(t) = \frac{e^{-at} - e^0}{-a}$$

$$y(t) = \frac{1}{a} (1 - e^{-at})$$

Date: _____

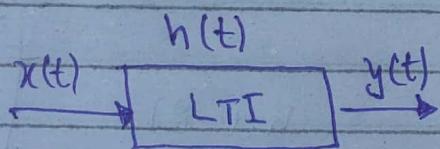
Day: M T W T F S



16 | 5 | 2023

$$x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot s[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$



$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$X(\omega) = \underbrace{|X(\omega)|}_{\text{Amp.}} \cdot e^{j\theta_x(\omega)}$$

Phase angle

$$Y(\omega) = |Y(\omega)| \cdot e^{j\theta_y(\omega)}$$

Date: _____

Day: M T W T F S

$$H(\omega) = |H(\omega)| e^{j\theta_h(\omega)}$$

$$|Y(\omega)| \cdot e^{j\theta_y(\omega)} = |X(\omega)| \cdot e^{j\theta_x(\omega)} \\ |H(\omega)| \cdot e^{j\theta_h(\omega)}$$

$$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$$

$$\theta_y(\omega) = \theta_x(\omega) + \theta_h(\omega)$$

$$|H(\omega)| = \frac{|Y(\omega)|}{|X(\omega)|}$$

$$y(t) = k \cdot x(t-t_d)$$

$$|Y(\omega)| \cdot e^{j\theta_y(\omega)} = k \cdot |X(\omega)| \cdot e^{-j\omega t_d}$$

$$|H(\omega)| = k$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{Rj\omega C + 1}$$

$$H(\omega) =$$

Date:

Day: M T W T F S

$$H(\omega) = \frac{1/j\omega RC}{1 + 1/j\omega RC}$$

Let $R_C = a$

$$H(\omega) = \frac{1/j\omega a}{1 + 1/j\omega a} \times \frac{j\omega a}{j\omega a}$$

$$H(\omega) = \frac{1}{1 + j\omega a} \quad \text{OR}$$

$$H(\omega) = \frac{a}{a + j\omega}$$

$$H(\omega) = \frac{a}{a + j\omega} \times \frac{a - j\omega}{a - j\omega}$$

$$H(\omega) = \frac{a^2 - j\omega a}{a^2 + \omega^2}$$

$$|H(\omega)| = \sqrt{\left(\frac{a^2}{a^2 + \omega^2}\right)^2 + \left(\frac{a\omega}{a^2 + \omega^2}\right)^2}$$

$$|H| = \sqrt{\frac{a^4 + a^2\omega^2}{(a^2 + \omega^2)^2}}$$

$$H(\omega) = \frac{a^2 (a^2 + \omega^2)}{(a^2 + \omega^2)^2}$$

$$= a \cdot \frac{\sqrt{a^2 + \omega^2}}{(a^2 + \omega^2)^2} = \boxed{\frac{a}{\sqrt{a^2 + \omega^2}}}$$

$$\Theta_h(\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

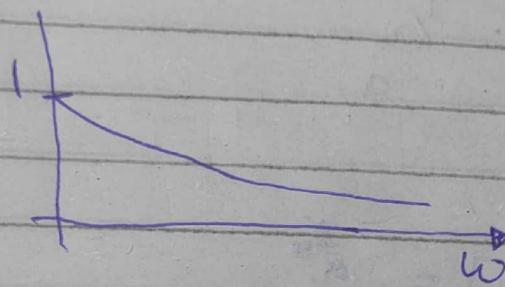
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Day: M T W T F S

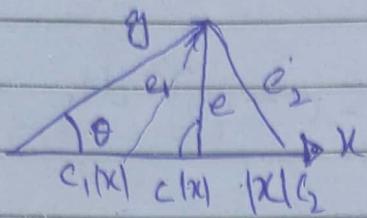
$$|H(\omega)| = \frac{a}{\sqrt{a^2 + \omega^2}}$$

$$a = \frac{1}{RC}$$

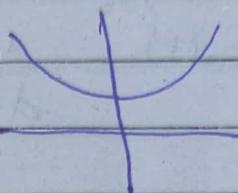


Vectors & Signals :-

Signals are vectors.



$$g = e_1 |x| + e_2$$



$$|g| \leq c|x|$$

$$\cos \theta = \frac{c \cdot |x|}{|g|}$$

$$c \cdot |x| = |g| \cos \theta$$

$$c \cdot |x| \cdot |x| = |g| \cdot |x| \cdot \cos \theta$$

$$c = \frac{g \cdot x}{|x|^2}$$

$$g(t) \approx c \cdot x(t)$$

$$e(t) = g(t) - c \cdot x(t)$$

$$g(t) = c \cdot x(t) + e(t)$$

Strength of signal in time domain will be represented in energy.

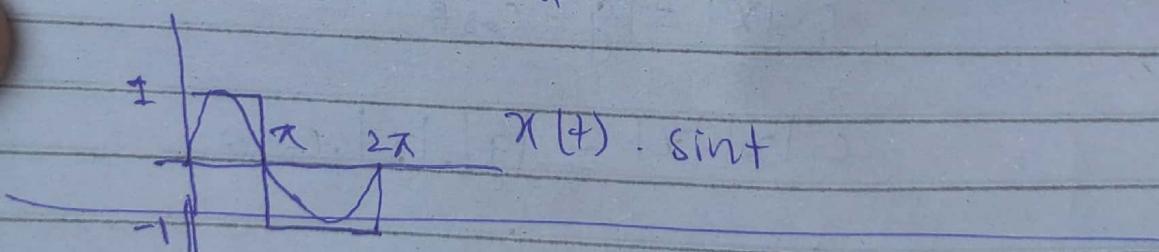
$$\bar{E}_e(t) = \int_{t_1}^{t_2} e^2(t) dt$$

$$E_e(t) = \int_{t_1}^{t_2} [g(t) - c \cdot x(t)]^2 dt$$

$$\frac{d E_e(t)}{dc} = \frac{d}{dc} \left\{ \int_{t_1}^{t_2} g^2(t) dt + c^2 \int_{t_1}^{t_2} x^2(t) dt \right\} -$$

$$2c \int_{t_1}^{t_2} g(t) \cdot x(t) dt = 0$$

$$c = \frac{\int_{t_1}^{t_2} g(t) \cdot x(t) dt}{\int_{t_1}^{t_2} x^2(t) dt}$$



$$\bar{E}_x = \int_0^{2\pi} \left(1 - \frac{\cos 2t}{2} \right) dt$$

$$= \frac{1}{2} \left[\int_0^{2\pi} 1 dt - \int_0^{2\pi} \cos 2t dt \right]$$

$$= \frac{1}{2} \left[t \Big|_0^{2\pi} - 0 \right] = \frac{1}{2} [2\pi - 0] = \pi$$

$$C = \frac{\int_{t_1}^{t_2} g(t) \cdot x(t) dt}{\sqrt{\int_{t_1}^{t_2} x^2(t) dt}} = \frac{U_g}{U_x}$$

When $C=0$, the signals are said to be orthogonal.

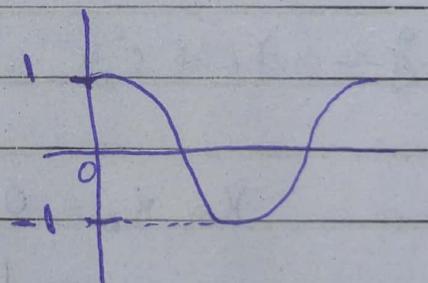
Correlation :- \rightarrow direct multiplication.

$$C_n = \cos \theta$$

$$g \cdot x = |g| |x| \cos \theta$$

$$C_n = \cos \theta = \frac{g \cdot x}{|g| |x|}$$

$$-1 \leq C_n \leq 1$$

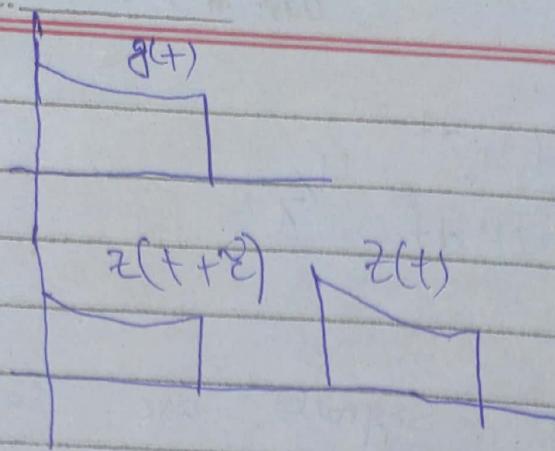


$$C_n = \frac{\int_{t_1}^{t_2} g(t) \cdot x(t) dt}{\sqrt{E_g \cdot E_x}}$$

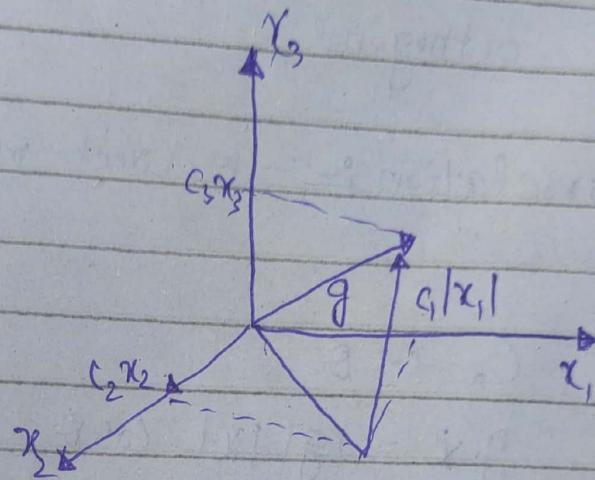
$$\text{Q} = \Psi_{gx}(t) = \int_{-\infty}^{\infty} g(t) \cdot x(t+\tau) dt$$

Date:

Day: M T W T F S



$$\Psi_{g,z}(t) = \int_{-\infty}^{\infty} g(t) \cdot z(t+T) dt$$



$$g \approx c_1 x_1 + c_2 x_2$$

$$e = g - [c_1 x_1 + c_2 x_2]$$

$$g = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$$x_m \cdot x_n = 0 \quad m \neq n \\ = 1 \quad m = n$$

$$g(t) = c_1 \cdot x_1(t) + c_2 \cdot x_2(t) + c_3 \cdot x_3(t) + \dots + x_N(t)$$

$$g(t) = \sum_{n=1}^{\infty} c_n \cdot x_n(t)$$

Date:

Day: M T W T F S

Trigonometric Fourier Series.

$\{ \cos \omega_0 t, \cos 2\omega_0 t, \dots, \cos n\omega_0 t, \sin \omega_0 t, \sin 2\omega_0 t, \dots, \sin n\omega_0 t \}$

$$\int_{T_0} \cos m\omega_0 t \cdot \cos n\omega_0 t dt = 0 \quad n \neq m \\ = T_0/2 \quad m = n$$

$$g(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cdot \cos n\omega_0 t + b_n \cdot \sin n\omega_0 t]$$

$$g(t) = \sum_{n=0}^{\infty} [a_n \cdot \cos n\omega_0 t + b_n \cdot \sin n\omega_0 t]$$

$$a_n \cos n\omega_0 t + C_n \cdot \cos(n\omega_0 t + \theta_n)$$

$$b_n \cdot \sin n\omega_0 t =$$

$$= C_n \cdot \cos n\omega_0 t \cdot \cos \theta_n - C_n \cdot \sin n\omega_0 t$$

$$\sin \theta_n$$

Iff

$$a_n = C_n \cdot \cos \theta_n, \quad b_n = -C_n \cdot \sin \theta_n$$

$$a_n^2 + b_n^2 = C_n^2 \cdot \cos^2 \theta_n + C_n^2 \cdot \sin^2 \theta_n$$

$$\sqrt{a_n^2 + b_n^2} = \sqrt{C_n^2}$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

Date: _____

Day: M T W T F S

$$-\frac{c_n \cdot \sin \theta_n}{c_n \cdot \cos \theta_n} = \frac{b_n}{a_n}$$

$$\tan \theta_n = -\frac{b_n}{a_n}$$

$$\boxed{\theta_n = \tan^{-1} \left(-\frac{b_n}{a_n} \right)}$$

∴ Compact form of trig. Fourier Series.

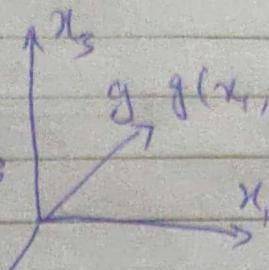
$$g(t) = \sum_{n=0}^{\infty} c_n \cdot \cos(n\omega_0 t + \theta_n)$$

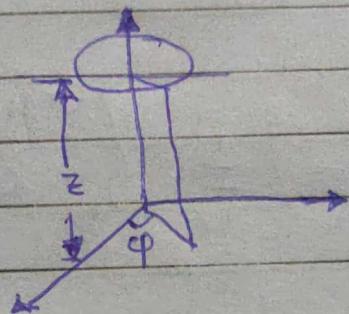
Date: 29/5/2023

Day: M T W T F S

$$g = c_1 x_1 + c_2 x_2 + c_3 x_3$$

$x_m \cdot x_n = 0$
 $= 1 \quad m+n$
 $m=n$





$$x(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \dots a_n \cos n\omega_0 t$$
$$b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \dots b_n \sin n\omega_0 t$$

$$x(t) = a_0 + \sum_{n=-\infty}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$c_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \tan^{-1} \left(\frac{-b_n}{a_n} \right)$$

$$c = \underline{g \cdot x}$$

Date: _____

Day: M T W T F S

$$a_n = \frac{\int_{t_1}^{t_1+T_0} g(t) \cdot \cos n\omega_0 t dt}{\int_{t_1}^{t_1+T_0} \cos^2 n\omega_0 t dt}$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cdot \sin n\omega_0 t dt$$

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t) dt$$

Exponential form: of Fourier Series

$$C_n \cdot \cos(n\omega_0 t + \theta_n) = C_n \left[e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)} \right]$$

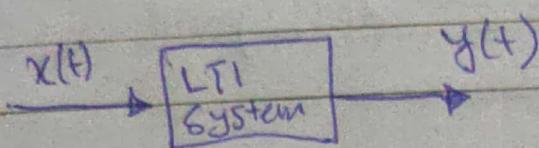
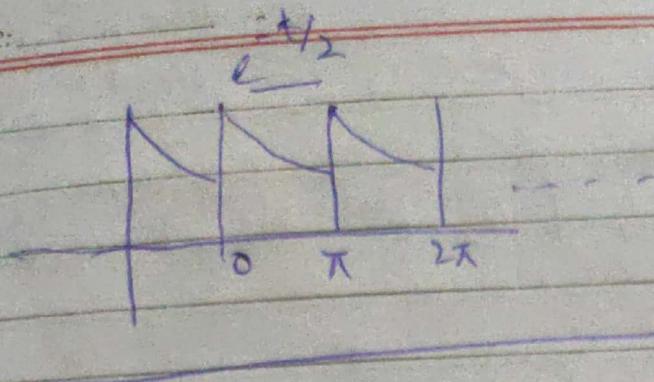
$$\begin{aligned} &= \left(\frac{C_n e^{j\theta_n}}{2} \right) e^{jn\omega_0 t} + \left(\frac{C_n e^{-j\theta_n}}{2} \right) e^{-jn\omega_0 t} \\ &\text{Magnitude} \end{aligned}$$

$$11 = D_n e^{jn\omega_0 t} + P_n e^{-jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n \cdot e^{jn\omega_0 t}$$

Date:

Day: M T W T F S



$$y(t) = x(t) * h(t)$$

$$y(t) = h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t - \tau) d\tau$$

$$\text{Let } x(t) = e^{j\omega_0 t}$$

$$x(t - \tau) = e^{j\omega_0(t - \tau)}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot e^{j\omega_0(t - \tau)} d\tau$$

$$y(t) = e^{j\omega_0 t} \underbrace{\int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega_0 \tau} d\tau}_{H(j\omega_0)} s$$

$$y(t) = e^{j\omega_0 t} \cdot H(s)$$

Response to the sum is
the sum of responses.

Date:

Day: M T W T F S

$$x(t) = a_1 e^{j\omega_1 t} + a_2 e^{j\omega_2 t} + a_3 e^{j\omega_3 t}$$

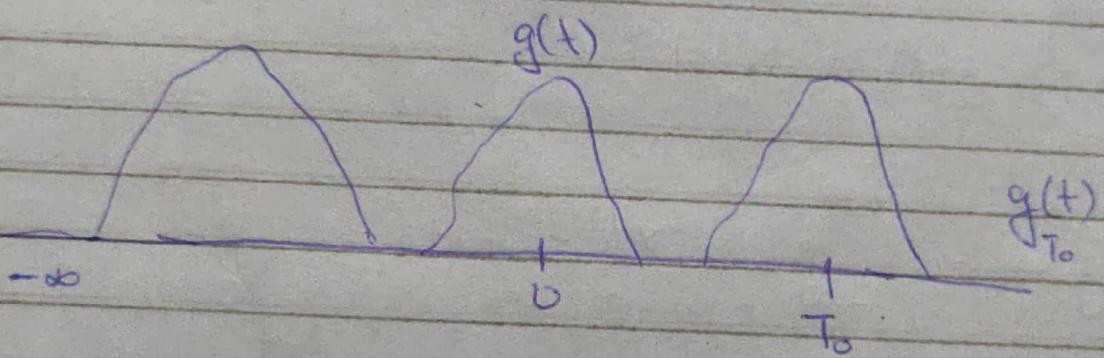
$x(t)$ Passed through LTI system

$$y(t) = a_1 e^{j\omega_1 t} H(s_1) + a_2 e^{j\omega_2 t} H(s_2) + a_3 e^{j\omega_3 t} H(s_3)$$

If n components

$$y(t) = \sum_{n=0}^{\infty} a_n e^{j\omega_n t} H(s_n)$$

Fourier Series for Aperiodic Signals:-



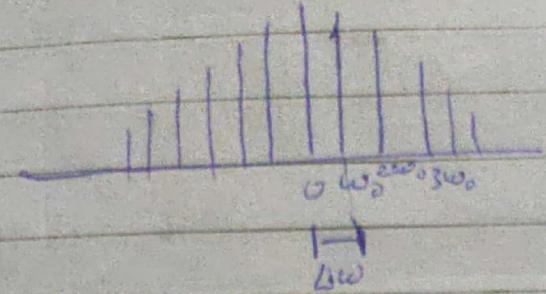
If $T_0 \rightarrow \infty$ then

$$\lim_{T_0 \rightarrow \infty} g_{T_0}(t) = g(t)$$

$$D_n = \frac{1}{T_0} \int_0^{T_0} g(t) \cdot e^{-jn\omega_0 t} dt$$

$$D_n = \frac{G(n\omega_0)}{T_0}$$

Date: _____



$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} \frac{G_s(n\Delta\omega)}{T_0} e^{-jn\omega_0 t}$$

$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} G_s(n\Delta\omega) \cdot \frac{e^{-jn\omega_0 t}}{T_0}$$

If $T_0 \rightarrow \infty$
 $\Delta\omega \rightarrow 0$

$$\lim_{T_0 \rightarrow \infty} g_{T_0}(t) = \lim_{\Delta\omega \rightarrow 0} \sum_{n=-\infty}^{\infty} G_s(n\Delta\omega) \cdot \frac{e^{+jn\omega_0 t}}{T_0}$$

$$T_0 = \frac{2\pi}{\omega_0}$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_s(\omega_0) \cdot e^{+j\omega_0 t} \cdot d\omega$$

Date: _____

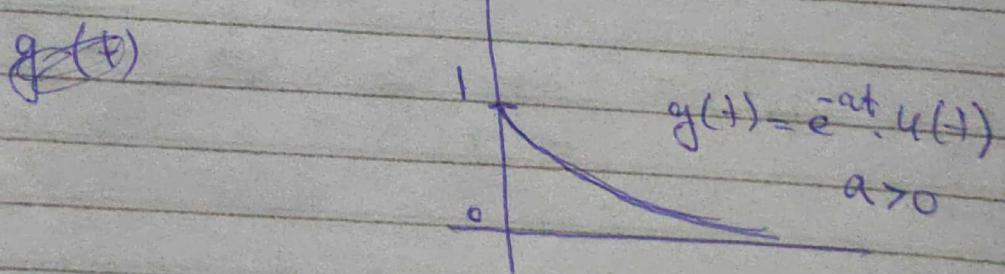
Day: M T W R F S

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

Fourier Transform of the signal.

$$g(t) \rightarrow \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt \quad G(\omega)$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} dt$$



Duichset Condition

* Value of $g(t)$ must

$$G(\omega) = \int_{-\infty}^{\infty} \cdot e^{-at} u(t) \cdot e^{-j\omega t} dt \quad \text{be finite}$$

$$G(\omega) = \int_0^{\infty} \cdot e^{-at} \cdot e^{-j\omega t} dt$$

Date:

$$G(\omega) = \int_0^{\infty} e^{-t(a+j\omega)} dt$$

$$= \left. \frac{e^{-(a+j\omega)t}}{-a-j\omega} \right|_0^{\infty}$$

$$= \frac{e^{-\infty} - e^{-0}}{-(a+j\omega)} = \frac{0-1}{-(a+j\omega)}$$

$$G(\omega) = \frac{1}{a+j\omega} \times \frac{a-j\omega}{a-j\omega} = \frac{a-j\omega}{a^2+\omega^2}$$

$$G_{-}(\omega) = |G(\omega)| \cdot e^{j \theta_g(\omega)}$$

$$|G(\omega)| = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}}$$

$$= \sqrt{\frac{a^2 + \omega^2}{(a^2 + \omega^2)^2}}$$

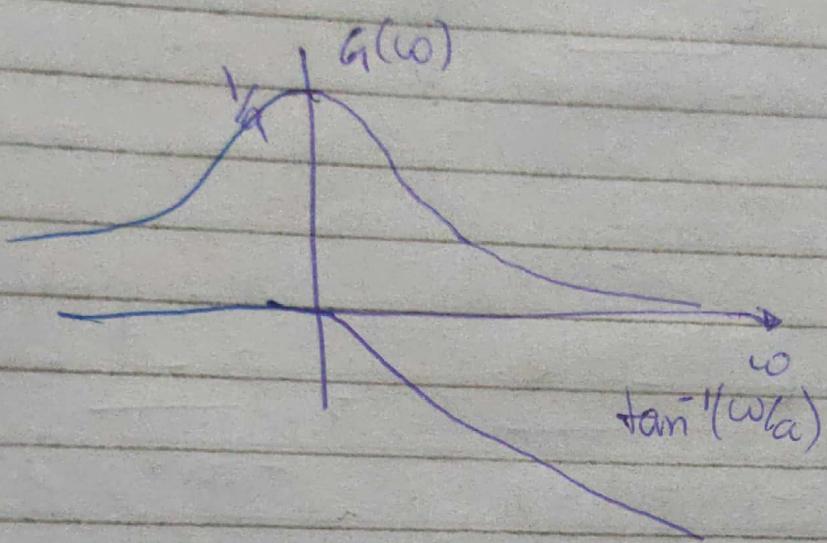
$$= \sqrt{\frac{a^2 + \cancel{\omega^2}}{a^2 + \omega^2}} = \frac{1}{\sqrt{a^2 + \omega^2}}$$

Date: _____

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Day: M T W T F S

$$\Theta_g(\omega) = -\tan^{-1}(\omega/a)$$



$$|G(-\omega)| = |G(\omega)| \text{ even}$$

$$\therefore \Theta_g(-\omega) = -\Theta_g(\omega) \text{ odd}$$

Date: 30/5/2023

Day: M T W T F S

Linearity:-

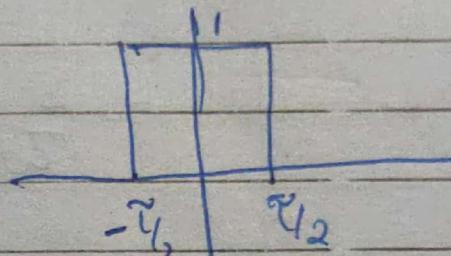
$$a_1 g_1(t) \Leftrightarrow a_1 G_1(\omega)$$

$$a_2 g_2(t) \Leftrightarrow a_2 G_2(\omega)$$

$$a_1 g_1(t) + a_2 g_2(t) \Leftrightarrow a_1 G_1(\omega) + a_2 G_2(\omega)$$

★ Unit Gate function:

$$\text{rect}\left(\frac{t}{\tau_2}\right)$$



$$\text{sinc } x = \frac{\sin x}{x}$$

$$G(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau_2}\right) \cdot e^{-j\omega t} dt$$

$$U = \int_{-\tau_2}^{\tau_2} 1 \cdot e^{-j\omega t} dt$$

$$U = \frac{e^{-j\omega \tau_2}}{-j\omega} \Big|_{-\tau_2}^{\tau_2}$$

$$U = \frac{e^{-j\omega \tau_2} - e^{j\omega \tau_2}}{-j\omega}$$

$$G_7(\omega) \approx e^{\frac{j\omega\gamma_2}{2}} - e^{-\frac{j\omega\gamma_2}{2}} \times \frac{2}{j\omega}$$

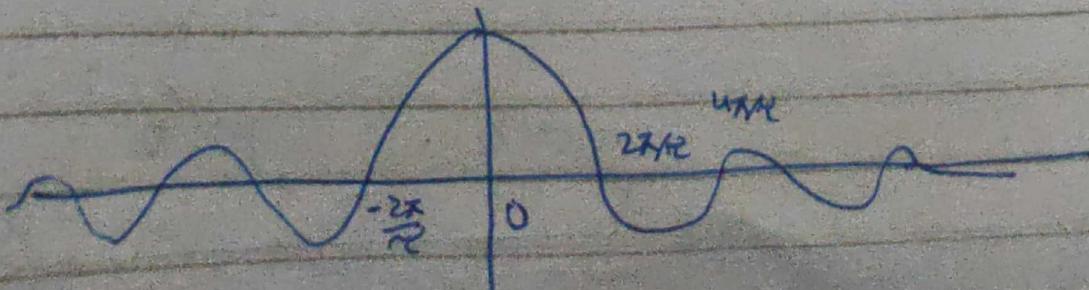
$$G(\omega) = \frac{2}{\omega} \cdot \left[\frac{1}{2} (e^{\frac{j\omega\gamma_2}{2}} - e^{-\frac{j\omega\gamma_2}{2}}) \right]$$

$$= \frac{2}{\omega} \sin(\omega\gamma_2)$$

$$= \frac{\sin(\omega\gamma_2)}{\omega/2} \times \frac{\omega}{\gamma}$$

$$= \gamma \cdot \frac{\sin(\omega\gamma_2)}{\omega\gamma/2}$$

$$= \gamma \cdot \text{sinc}\left(\frac{\omega\gamma}{2}\right)$$



Date: _____

Day: M T W T F S

$$\frac{\omega^2}{2} = \pm n\pi$$

$$\omega = \pm \frac{2\pi}{\sqrt{2}}$$

$$\text{rect}\left(\frac{t}{\tau}\right) \iff \tilde{\mathcal{C}} \cdot \sin\left(\frac{\omega^2}{2}\right)$$

1MBPs

$$\tau = 1 \mu\text{sec}$$

2MBPs

$$\tau = 0.5 \mu\text{sec}$$

Keep the width of rect function small in order to get more bandwidth.

$$g(t) = \delta(t)$$

$$G(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} dt = e^{\frac{j\omega}{2}}$$

$$\approx 1$$

$$G(\omega) = \delta(\omega - \omega_0)$$

$$g(t) = ?$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) \cdot e^{j\omega t} dt$$

$$g(t) = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$2\pi \times \frac{1}{2\pi} \cdot e^{j\omega_0 t} \iff \delta(\omega - \omega_0) \times 2\pi$$

$$\left\{ e^{j\omega_0 t} \iff \delta(\omega - \omega_0) \cdot 2\pi \right.$$

$$\left. e^{-j\omega_0 t} \iff 2\pi \cdot \delta(\omega + \omega_0) \right.$$

$$g(t) = \cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

~~G(t)~~

Date:

$$= \int_{-\infty}^{\infty} e^{j\omega_0 t} \cdot e^{-j\omega t} dt = \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-j(\omega_0 - \omega)t} dt \right] = \frac{1}{2} \delta(\omega - \omega_0)$$

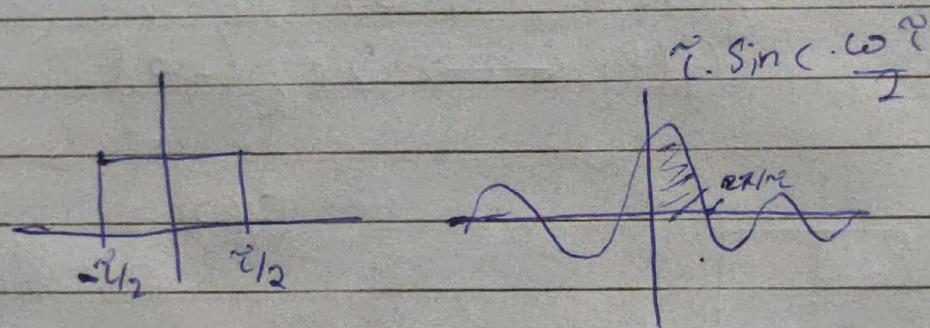
using above results.

$$G_I(\omega) = \Re [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

5/6/2023

$$g(t) \leftrightarrow G(\omega)$$

$$\mathcal{F}\{g(t)\} = G_I(\omega)$$

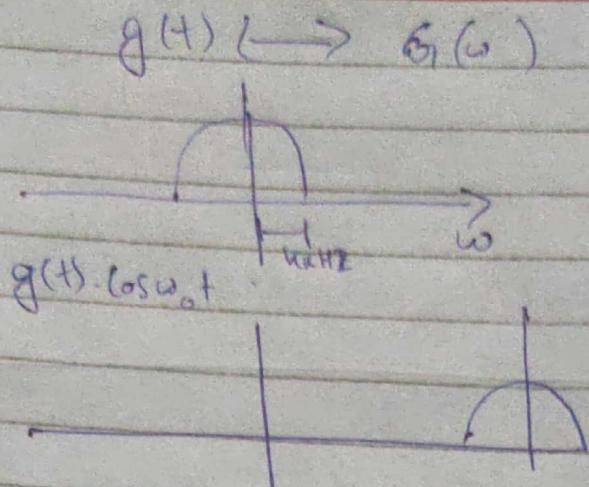


$$= \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$$

$$\delta(t) \rightarrow 1$$



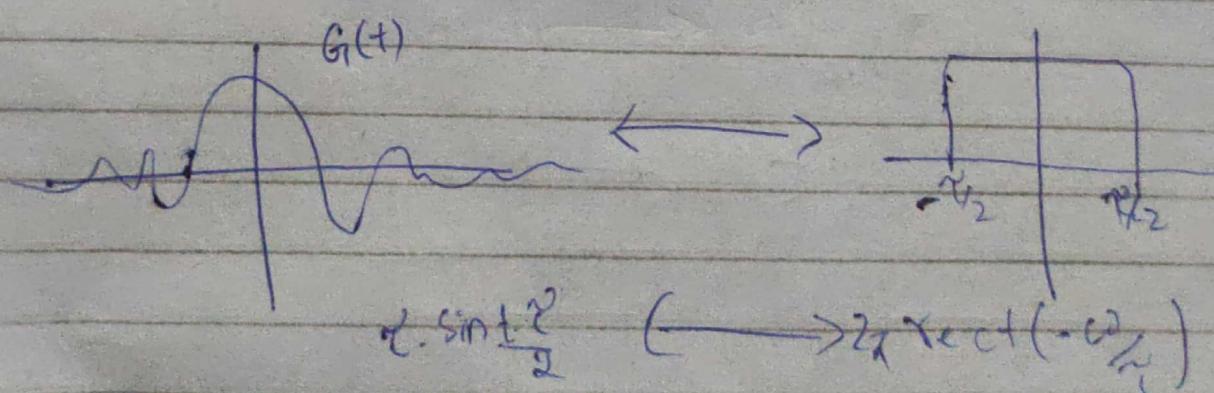
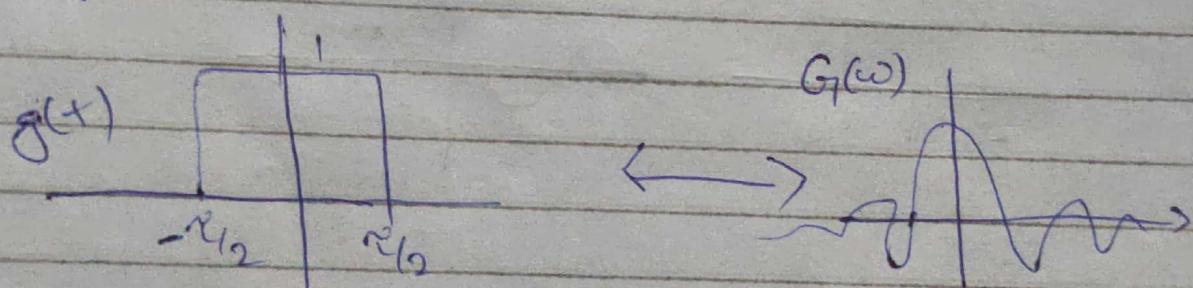
$$\mathcal{F}^{-1}[\delta(\omega - \omega_0)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega - \omega_0) e^{j\omega t} d\omega$$



Props of F.T :-

* Symmetry

e.g.



$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \cdot e^{j\omega t} d\omega$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$$

A Scaling

$$g(t) \xrightarrow{\text{ }} G_1(\omega)$$

$$g(at) \xrightarrow{\text{ }} ?$$

$$F\{g(at)\} = \int_{-\infty}^{\infty} g(at) \cdot e^{-j\omega t} dt$$

$$\text{Let } at = x$$

$$t = x/a \Rightarrow dt = \frac{dx}{a}$$

$$\therefore F\{g(at)\} = \int_{-\infty}^{\infty} g(x) \cdot e^{-j\omega \frac{x}{a}} \cdot \frac{dx}{a}$$

From $G_1(\omega) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j\omega t} dt$

$$\cancel{F\{g(at)\}} =$$

$$\frac{1}{a} G_1\left(\frac{\omega}{a}\right)$$

Compress in time domain, expand in frequency domain.

$$\therefore g(at) \xleftrightarrow{\text{ }} \frac{1}{a} G_1\left(\frac{\omega}{a}\right)$$

* Shifting Property

$$g(t-t_d) \longleftrightarrow ?$$

$$\int_{-\infty}^{\infty} g(t-t_d) \cdot e^{-j\omega t} dt$$

$$\text{Let } t-t_d = x$$

$$\begin{aligned} t &= x + t_d \\ dx &= dt \end{aligned}$$

$$\int_{-\infty}^{\infty} g(x) \cdot e^{-j\omega(x+t_d)} dx$$

$$e^{-j\omega t_d} \int_{-\infty}^{\infty} g(x) \cdot e^{-j\omega x} dx = \boxed{G(\omega) \cdot e^{-j\omega t_d}}$$

Now,

$$g(t-t_d) \longleftrightarrow G(\omega) \cdot e^{-j\omega t_d}$$

* Convolution:-

$$F[g_1(t) * g_2(t)] = \int_{-\infty}^{\infty} [g_1(t) * g_2(t)] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} (g_1(\tau) g_2(t-\tau)) d\tau \right) e^{-j\omega t} dt$$

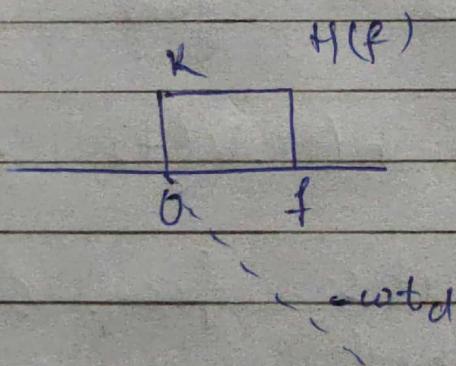
$$\int_{-\infty}^{\infty} g_1(\tau) \left[\int_{-\infty}^{\infty} g_2(t-\tau) \cdot e^{-j\omega t} dt \right] d\tau$$

$$\int_{-\infty}^{\infty} g_1(\tau) \cdot G_2(\omega) \cdot e^{-j\omega \tau} d\tau$$

$$G_2(\omega) \left(\int_{-\infty}^{\infty} g_1(\tau) \cdot e^{-j\omega \tau} d\tau \right)$$

$$G_1(\omega) \cdot G_2(\omega)$$

Ideal Filter :-



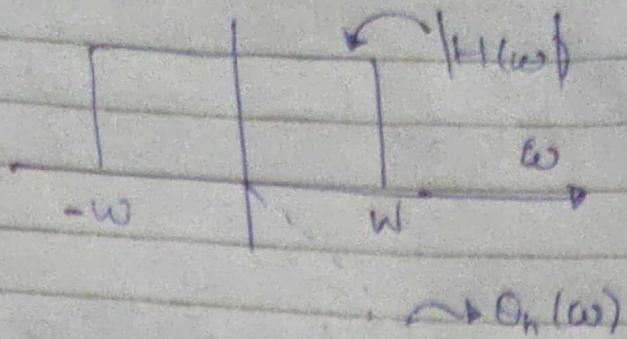
$$y(t) = g(t - t_d)$$

$$Y(\omega) = G_1(\omega) \cdot e^{-j\omega t_d}$$

$$H(\omega) = 1 \cdot e^{-j\omega t_d}$$

$$H(\omega) = \text{rect} \frac{\omega}{2W} e^{-j\omega t_d}$$

→ Ideal Filter
ka proper
Response



$$h(t) = \frac{\omega_0}{\pi} \cdot \text{Sinc. } \omega_0 (t + d)$$

4
a is not realizable as it is
non-causal signal.

so $\hat{h}(t) = h(t) u(t)$

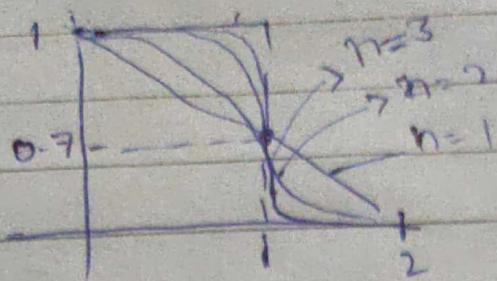
But now it's freq. domain is
also disturbed.



Butterworth
filter

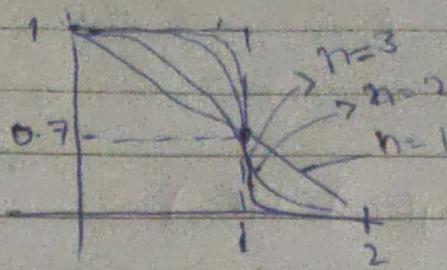
$$|H(\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega}{2\pi B})^{2n}}}$$

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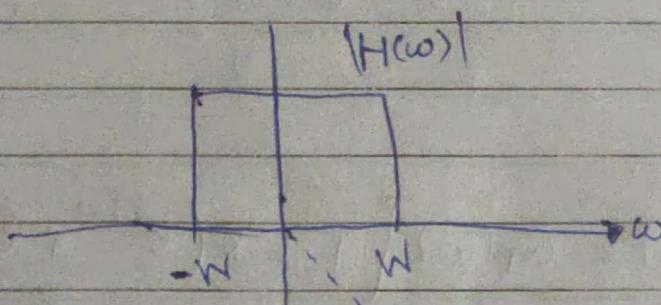


Date: _____

Day: M T W T F S

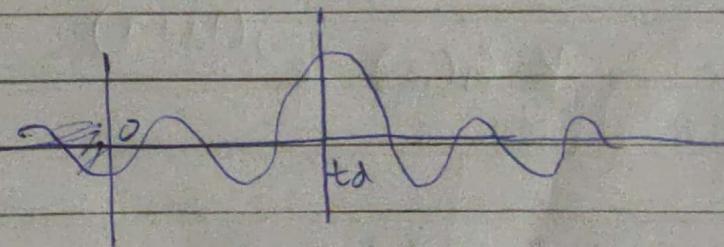


Date 6/6/2023

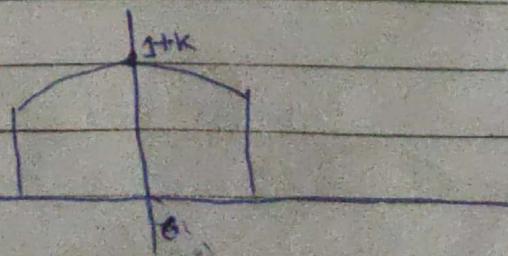


$$\phi_n(\omega) = -\omega \cdot t_d$$

$$X e^{j\omega t} + \left(\frac{\omega}{2W} \right) \cdot e^{-j\omega t_d}$$



$$H(\omega) = \begin{cases} \left(1 + K \cdot \cos T_w \right) e^{-j\omega t_d} & |\omega| < 2\pi B \\ 0 & |\omega| > 2\pi B \end{cases}$$



$$\phi_n(\omega) = -\omega \cdot t_d$$

$$Y(t) = ?$$

$$Y(\omega) = G_1(\omega) \cdot H(\omega)$$

$$Y(\omega) = G_1(\omega) \left[(1 + K \cos \tau \omega) e^{j\omega t_d} \right]$$

$$\cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

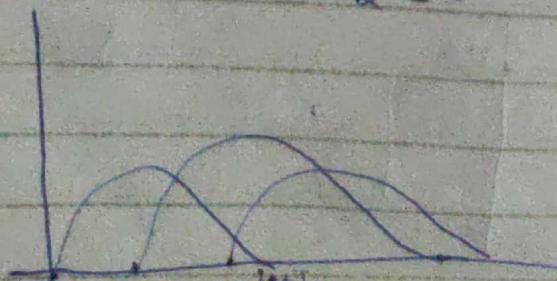
$$Y(\omega) = G_1(\omega) \left[1 + K \left(\frac{e^{j\tau\omega} + e^{-j\tau\omega}}{2} \right) \right] e^{-j\omega t_d}$$

$$Y(\omega) = G_1(\omega) \cdot e^{-j\omega t_d} + \frac{K}{2} G_1(\omega) \cdot e^{-j(t_d - \tau)\omega} + \frac{K}{2} G_1(\omega) \cdot e^{-j(t_d + \tau)}$$

$$g(t) \rightarrow G_1(\omega)$$

$$g(t-t_d) \rightarrow G_1(\omega) \cdot e^{j\omega t_d}$$

$$y(t) = g(t-t_d) + \frac{K}{2} g[t - (t_d - \tau)] + \frac{K}{2} g[t - (t_d + \tau)]$$



Date:

Day: M T W T F S

Fourier Transform for DTS:-

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$x[n] = a^n u[n]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} a^n u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (ae^{-j\omega n})^n \quad \text{Power Series}$$

$$\left[\sum_{n=0}^{\infty} x^n - \frac{1}{1-x} \right]$$

$$= \frac{1}{1 - ae^{-j\omega}}$$

$$= \frac{1}{1 - a[\cos\omega - j\sin\omega]}$$

$$= \frac{1}{1 - a\cos\omega + a j\sin\omega}$$

Date: _____

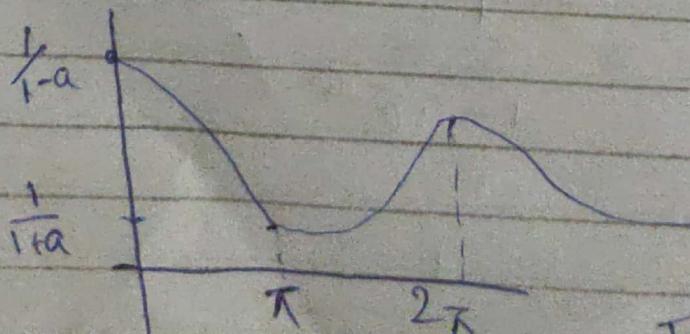
Day: M T W T F S

$$= \frac{1}{(1-a\cos\omega) + j(a\sin\omega)} \times \frac{(1-a\cos\omega) - j(a\sin\omega)}{(1-a\cos\omega) - j(a\sin\omega)}$$

$$= \frac{1-a\cos\omega - ja\sin\omega}{(1-a\cos\omega)^2 + a^2\sin^2\omega}$$

$$\begin{aligned} &= \frac{1-a\cos\omega - ja\sin\omega}{1+a^2\cos^2\omega - 2a\cos\omega + a^2\sin^2\omega} \\ &= \frac{1-a\cos\omega - ja\sin\omega}{1-2a\cos\omega + a^2} \end{aligned}$$

$$|\chi(e^{j\omega})| = \frac{1}{\sqrt{(1-a\cos\omega)^2 + a^2\sin^2\omega}}$$



Find Phase Angle:

$$\tan^{-1}\left(\frac{-a\sin\omega}{1-a\cos\omega}\right)$$