QUANTUM COMPUTING

A Journey From Qubit to Quantum Computation

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Session Breakdown

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- Definitions.
- The need for Quantum Computers.
- Quantum bit and Quantum states.
- Quantum Gates, Pauli Operators.
- Deutsch's Algorithm.

CONTENTS

- The focus in this session would be on:
- Introducing and defining terms related to the quantum domain.
- A basic idea about how quantum computing arises from quantum mechanics and its links to quantum information processing
- A crude definition of *buzzwords* superposition, entanglement, quantum advantage and quantum supremacy.
- Introduce quantum gates and a simple application demonstrating quantum speedup.

Famous but misunderstood ☹️

Quantum/Q domain/Q Scale/Q Tech/Q Regime are new and fancy terms widely used now a days but many do not understand what it really means.

Q Reading, Q Therapy, Q. Astrology, etc. are Just named QUANTUM!

Q = QUANTUM !

Let us Q - (Quantum Computation?)

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Time to replace Moore's law.

As the name suggests, it operates at "quantum scale".

Quantum Computing is the next step towards the future of computing!

Quanta what? Moore who?

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What is the Moore's Law?

What is Quantum Scale?

What Quantum effects are you talking about?

Wait…. What is Quantum in the 1st Place?

In fact, what are you talking about :D

A Mathematical Framework

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- It is a mathematical model first used to describe the physics at Q.scale.
- Q.scale refers to small small things (approx. nano scale).
- Not very small small (stuff > Qscale) == Classical Domain
- Very small small (stuff < Qscale) == Quantum Domain requires Q.Mechanics where classical explanations fail.

Fein, Y.Y., Geyer, P., Zwick, P. *et al.* Quantum superposition of molecules beyond 25 kDa. *Nat. Phys.* 15, 1242–1245 (2019). https://doi.org/10.1038/s41567-019-0663-9

So,,, quantum is small?

Does the “smallness” and “largeness” of stuff separate Quantum from Classical?

BLURRED LINES !!

SMALL + SMALL WILL EVENTUALLY BE TOO LARGE!

Why large things loose quantumness?

Big quantum molecules can exist.

A quantum effect, superposition, for Joseph Junctions (Solid state Physics)

[Y. Y. Fein, P. Geyer, P. Zwick, F. Kiałka, S. Pedalino, M. Mayor, S. Gerlich and M. Arndt, Nature Physics, 15, 1242 (2019)]

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Nature of the theory

- Quantum theory is probabilistic at its core!
- *"GOD does not pay dice"*
- *Yet, "you can't tell GOD what to do!"*

Vaidman, L. Quantum theory and determinism. *Quantum Stud.: Math. Found.* **1,** 5–38 (2014). https://doi.org/10.1007/s40509-014-0008-4

What is so super about superposition?

- **Q.Superposition** The property of a quantum body to exist in all positions at the same time (wave nature) before we look for it and force it in a certain way and place (particle nature).

- **Q.Entanglement** A weird connection b/w two Q.bodies that exists without any physical attachments (non-local).

- **Q.Discord** A weird and more robust connection other than entanglement that can exist b/w Q.entities.

In short?

Quantum Mechanics is a mathematical theory that completes the picture.

Quantum Computing is easier than Quantum Mechanics.

We use quantum effects for Quantum Computing.

QC : QIP = Physics + Maths + QIS + CS + Engr + Philosophy.

QC is comparatively easier than QM

A basic knowledge of quantum mechanics, its mathematical machinery and formalism is more than sufficient to get started with quantum computation.

QC makes use of the quantum phenomenon (that have their theoretical basis in QM) to perform computation tasks.

QC is closely linked (overlaps) with, Q.I.P, QML, Q.A.I, O.Q.S, etc.

Let us Q - (Quantum Computation?)

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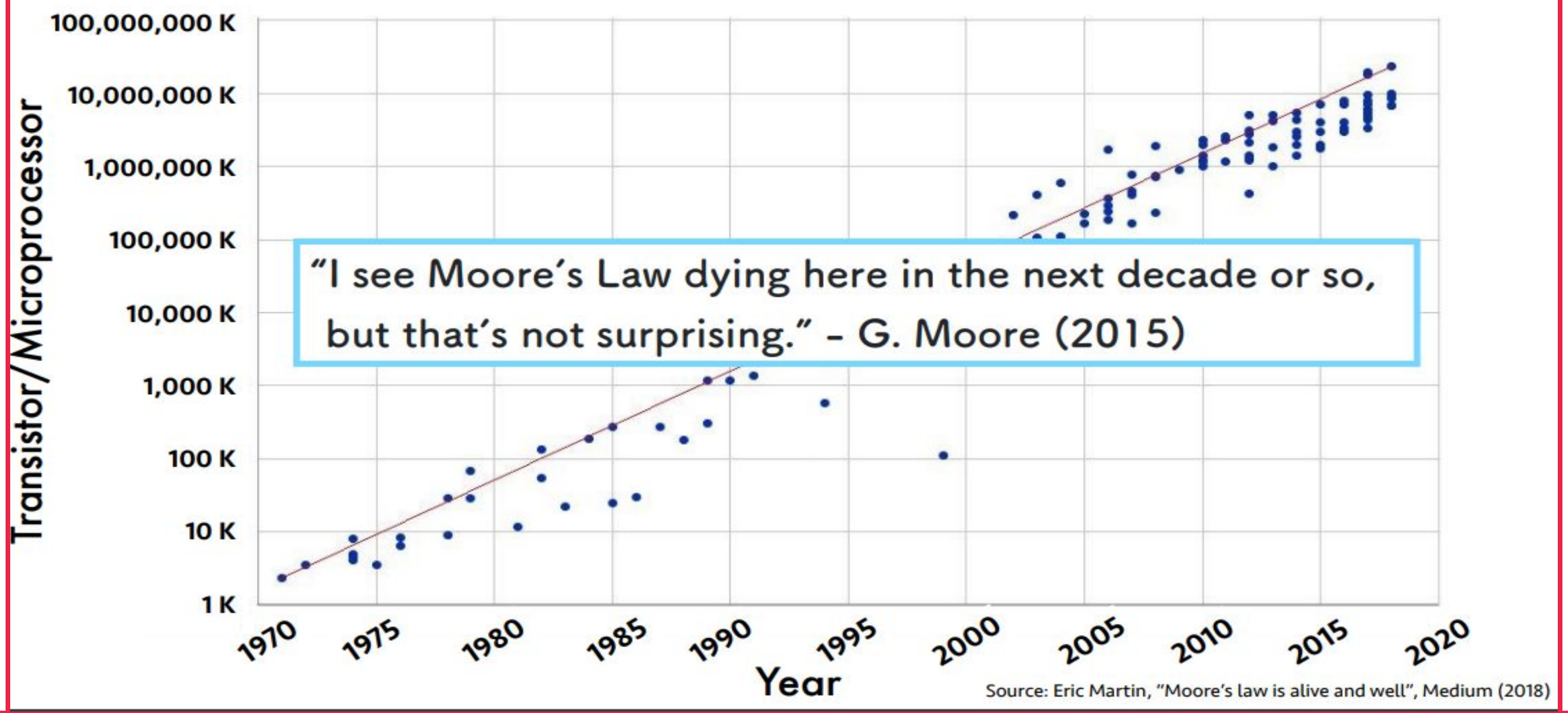
Time to replace Moore's law.

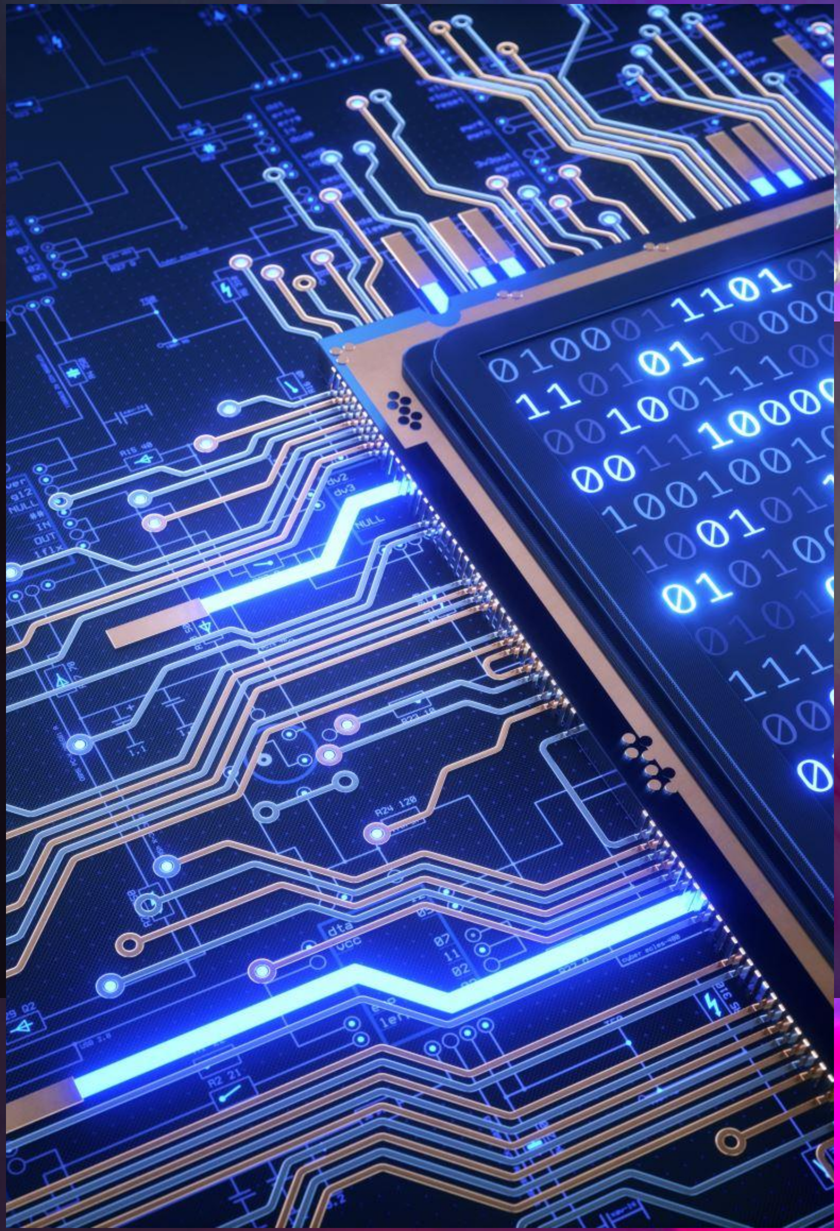
As the name suggests, it operates at "quantum scale".

Quantum Computing is the next step towards the future of computing!

As size of devices approach nano scale, we may cater for quantum effects



Can we simulate a quantum computer on a classical machine ?



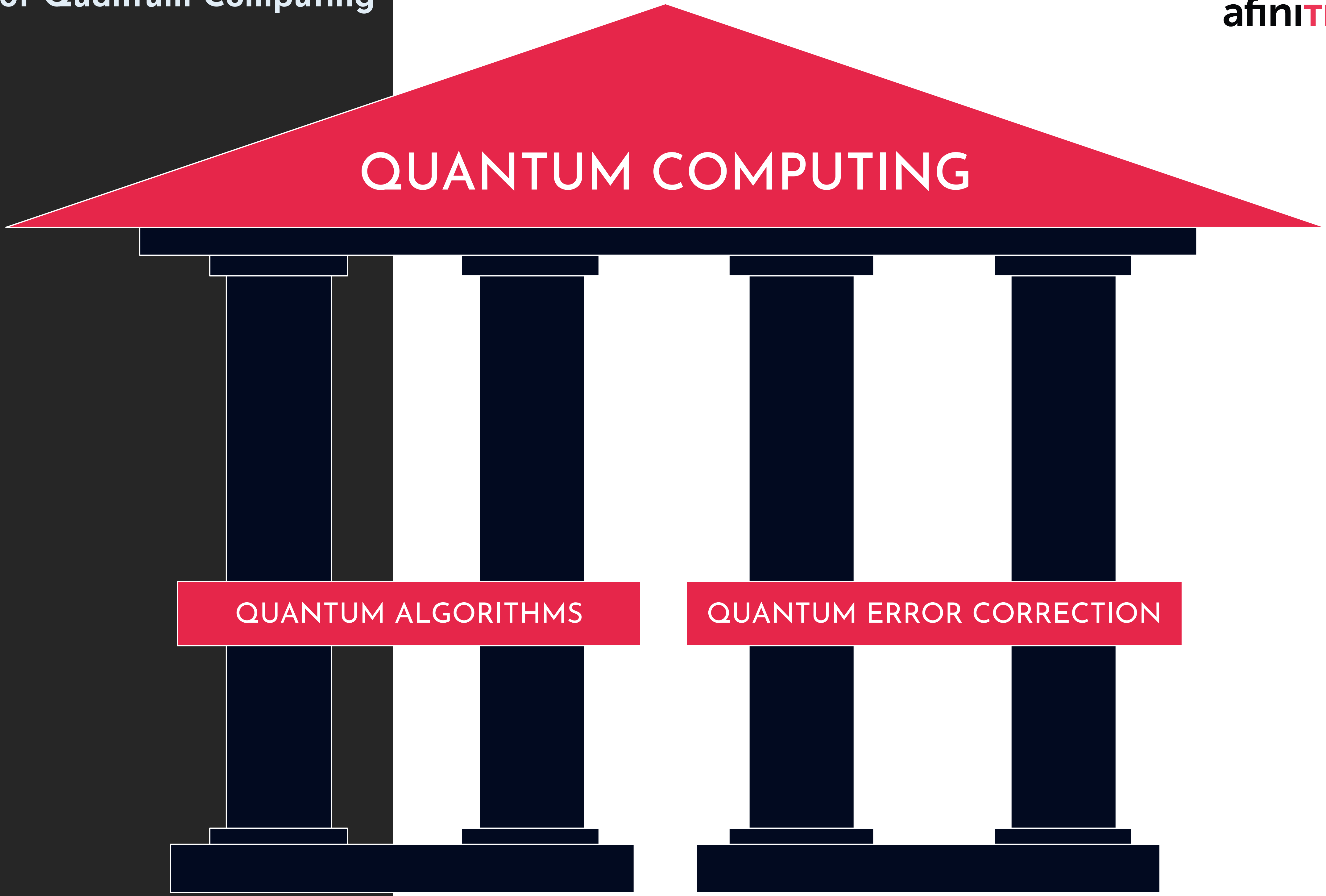
Not even with a Probabilistic Turing Machine.

Feynman: *"To simulate nature, you must use quantum mechanics".*

Physics and Information Science

- Information is stored physically to be processed.
- Physics determines the processing machine.
- Since quantum theory is a better theory than classical mechanics, an Argument can be made for Quantum computers being better than classical computers.
- Naturally, a quantum computer will be based on quantum mechanics.

Theoretical Pillars of Quantum Computing



What are Q.Advantage and Q.Supremacy?

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Marketing this speedup has given hype to keywords like:

Q.Advantage For a particular computation a QC can perform:

1. significantly faster than even the best *classical computer*.
2. computations which no classical computer can perform at all *(quantum supremacy)*

Q.supremacy A QC can compute a solution to a particular problem when no CC is able to do so at all or in any reasonable amount of time.

These don't suggest either an advantage or supremacy for any other problems beyond the specific, or a closely related, problem.

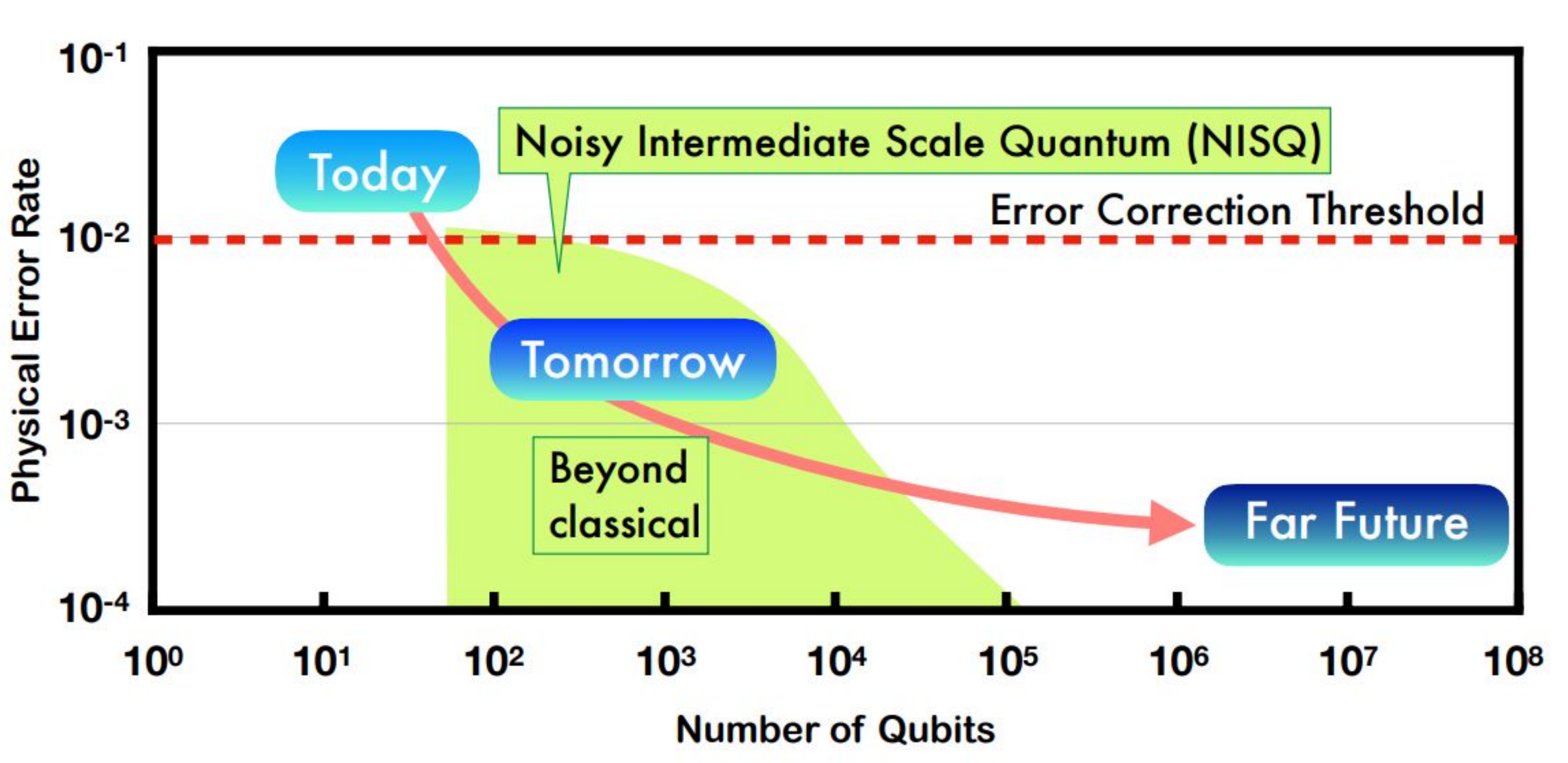
Exponential / Polynomial speed-up for computational tasks like:

Integer Factoring

Solving Systems of Linear Equations

Eigenvalue & Eigenvector finding problems

Principal Component Analysis



Noisy Intermediate Scale Quantum (NISQ)
Today
Error Correction Threshold
Tomorrow
Beyond classical
Far Future
Physical Error Rate
10-1
10-2
10-3
10-4
10⁰ 10¹ 10² 10³ 10⁴ 10⁵ 10⁶ 10⁷ 10⁸
Number of Qubits

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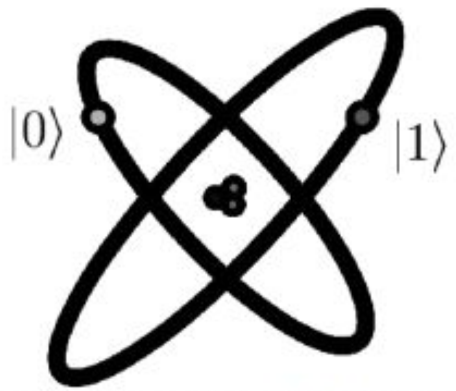
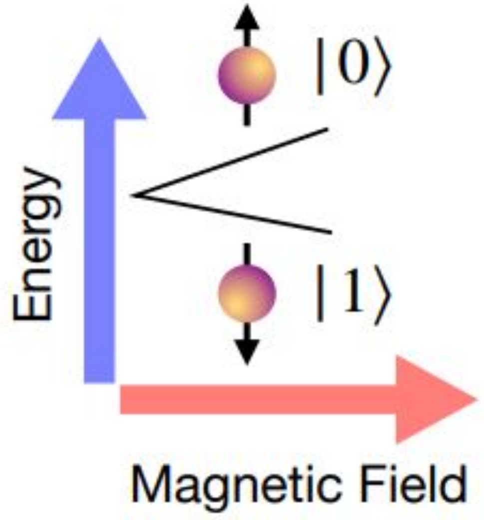
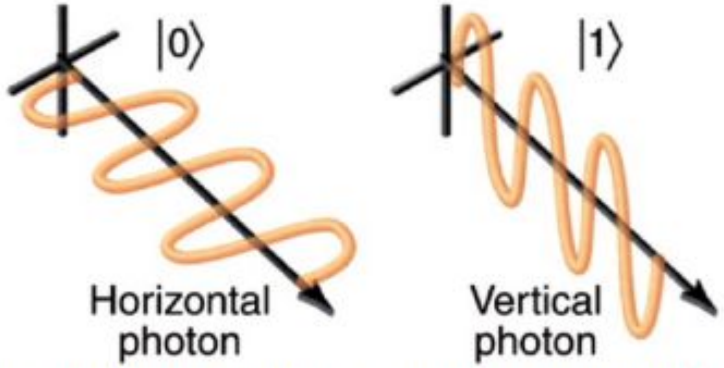
QUBIT

- Just as a ***Bit*** for classical computers is the smallest unit of data that stores information i.e. 0 or 1.
- For a Bipartite state we may write the quantum bit in terms of quantum states, e.g in computational basis, $|0\rangle$, $|1\rangle$.
- $|\Psi\rangle = C_0\,|0\rangle + C_1\,|1\rangle \longrightarrow |C_0|^2 + |C_1|^2 = 1$
- $|\Psi\rangle = \cos\theta/2\,|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$
- $|\Psi\rangle = C_0\,|00\rangle + C_1\,|11\rangle$
- For n qubits we have $2^{n+1}-2$ quantum states

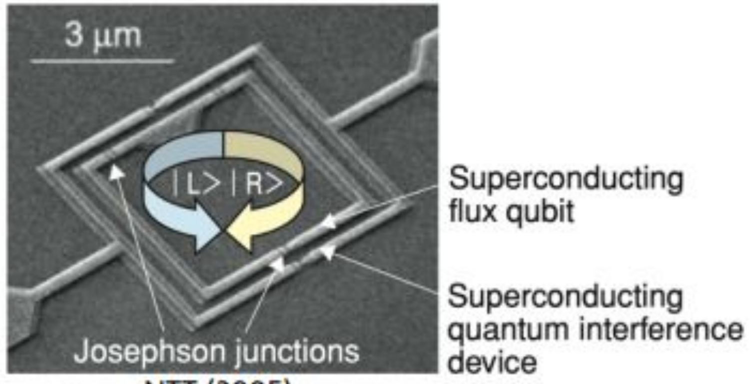


Quantum Computation and Quantum Information (2000)

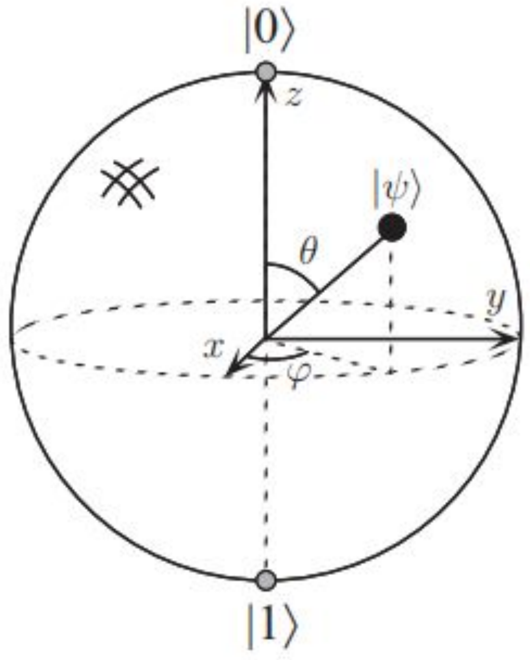




Optical Quantum Computing, Science, **318**, 1567-1570 (2007)



NTT (2005)

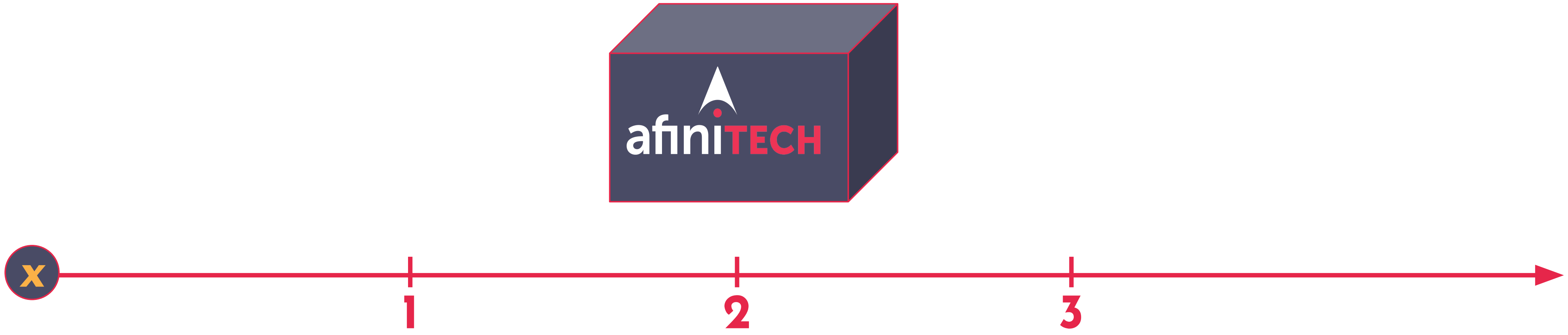


QUBIT REPRESENTATION

Any 2 level atom would suffice for a basic |0⟩, |1⟩ qubit representation, while the bloch sphere is a 3d unit sphere representation of all the qubit states for such a system parameterized by θ and ϕ

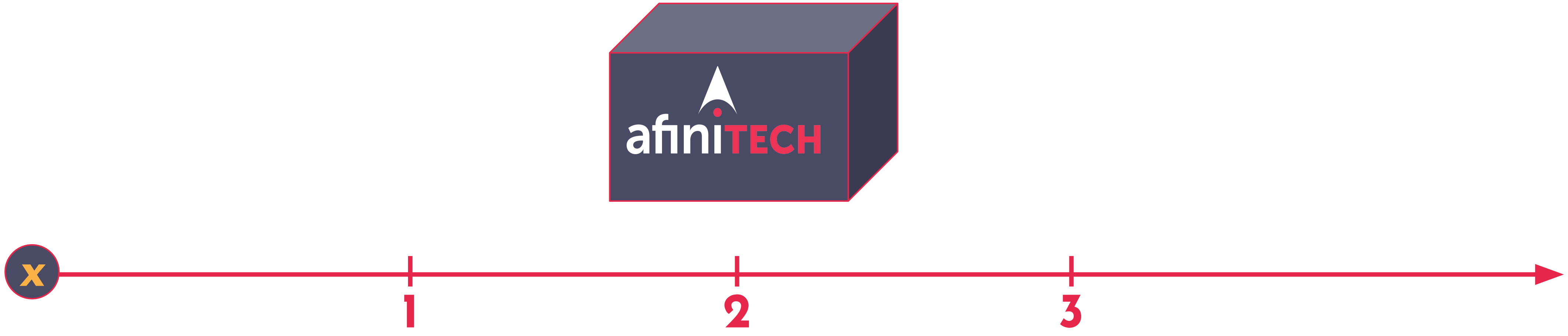
From Nielsen and Chuang

Quantum State Vectors



Classical Track | Location of center of box, ***x* = 2**

Quantum State Vectors

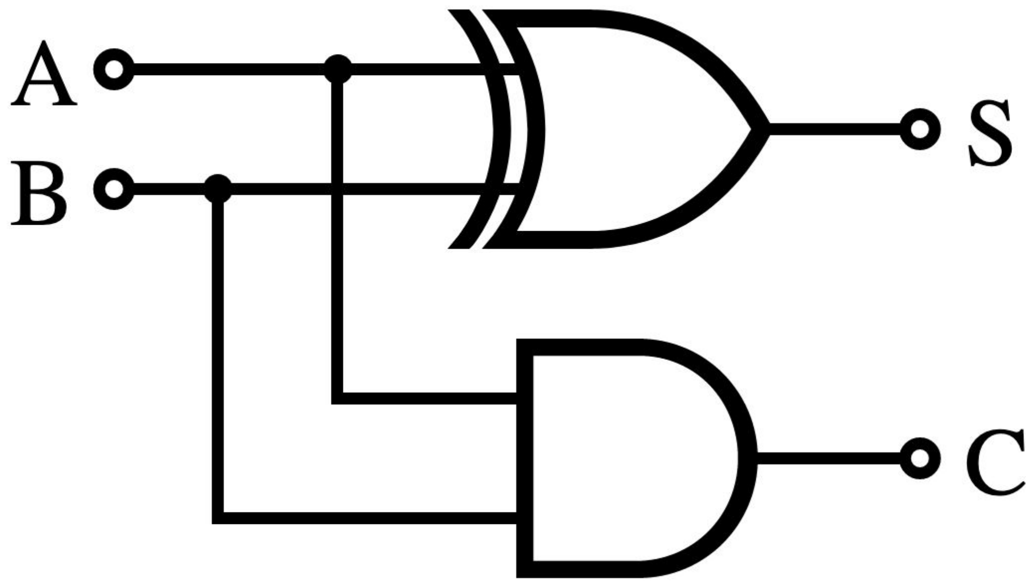


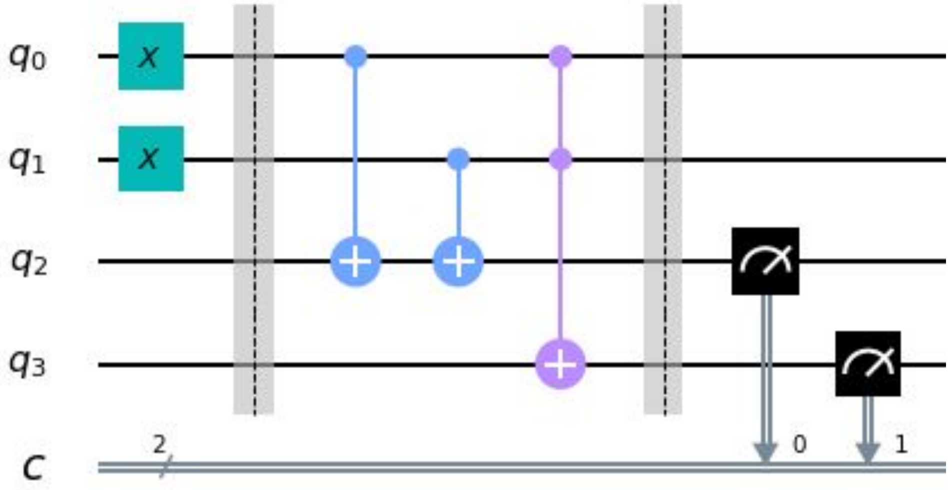
Probability of finding the box at position 2 is 100%

$$\begin{array}{c c} Pr(0) \longrightarrow & \\ Pr(1) \longrightarrow & \\ Pr(2) \longrightarrow & \\ Pr(3) \longrightarrow & \end{array} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Classical Bit	Quantum Bit
0 1	$\lvert 0 \rangle$ $\lvert 1 \rangle$
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Vector Representation	$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ $\alpha, \beta \in \mathbb{C}$ Vector Representation
–	$\Pr(0) = \lvert\alpha\rvert^2$ $\Pr(1) = \lvert\beta\rvert^2$
Deterministic Evolution	Unitary Evolution
Well-defined	Well-defined only after measurement

Logic Gates VS Q.Circuits





- The top image is a classical logic gate whereas on the bottom is a quantum circuit (qiskit) representation.
- Can you read the quantum circuit?

Single Qubit Gates

$|\Psi\rangle = \cos\theta/2\ |0\rangle + e^{i\phi} \sin\theta/2\ |1\rangle$

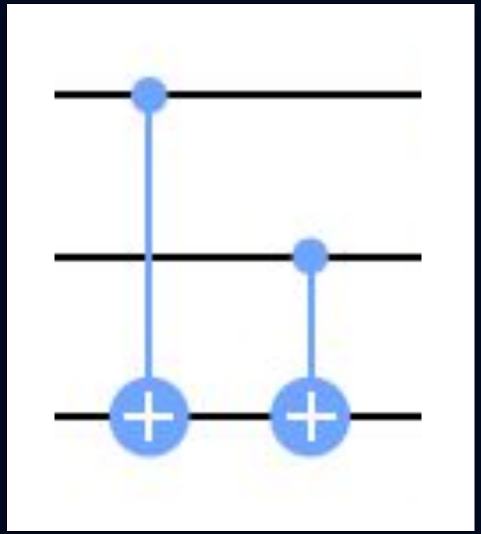
Pauli Gates X,Y,Z & Hadamard Gate:

$X\|0\rangle = \|1\rangle$	$X\|1\rangle = \|0\rangle$	NOT - *gate*
$Y\|0\rangle = i\|1\rangle$	$Y\|1\rangle = -i\|0\rangle$	*pi rotation along y-axis*
$Z\|0\rangle = \|0\rangle$	$Z\|1\rangle = -\|1\rangle$	*z-base / computational base*
$H\|0\rangle = \|+\rangle$	$H\|1\rangle = \|-\rangle$	*from z to x base*

$\sqrt{2}|+\rangle = |0\rangle + |1\rangle$ *x-basis*

$\sqrt{2}|-\rangle = |0\rangle - |1\rangle$ *x-basis*

Controlled NOT - C-NOT Gate



CNOT $|00\rangle$ = $|00\rangle$

CNOT $|01\rangle$ = $|01\rangle$

CNOT $|10\rangle$ = $|11\rangle$

CNOT $|11\rangle$ = $|10\rangle$

The ***control*** qubits remain unchanged and only perform Pauli-X on the ***target*** qubits if control itself is in state $|1\rangle$

DEUTSCH'S ALGO: An Example of Quantum Speedup

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	CONSTANT		BALANCED	
f(x) : Inputs	A	B	C	D
0	0	1	0	1
1	0	1	1	0
	CONST. 0	CONST. 1	IDENTITY	NOT

Given a Binary function f(x) - How many runs does it take at best to tell if its balanced or constant?

DEUTSCH'S ALGO: An Example of Quantum Speedup

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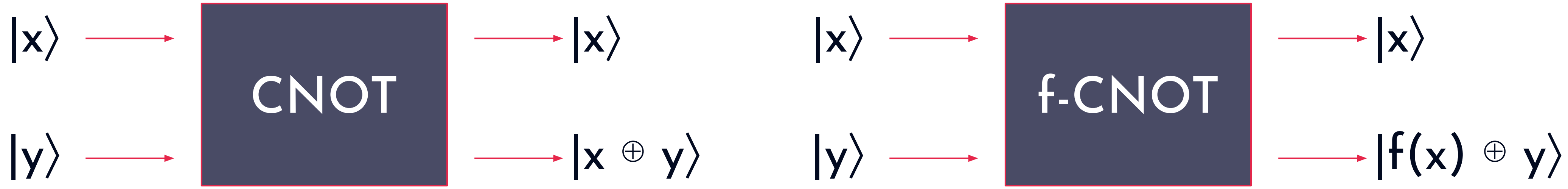
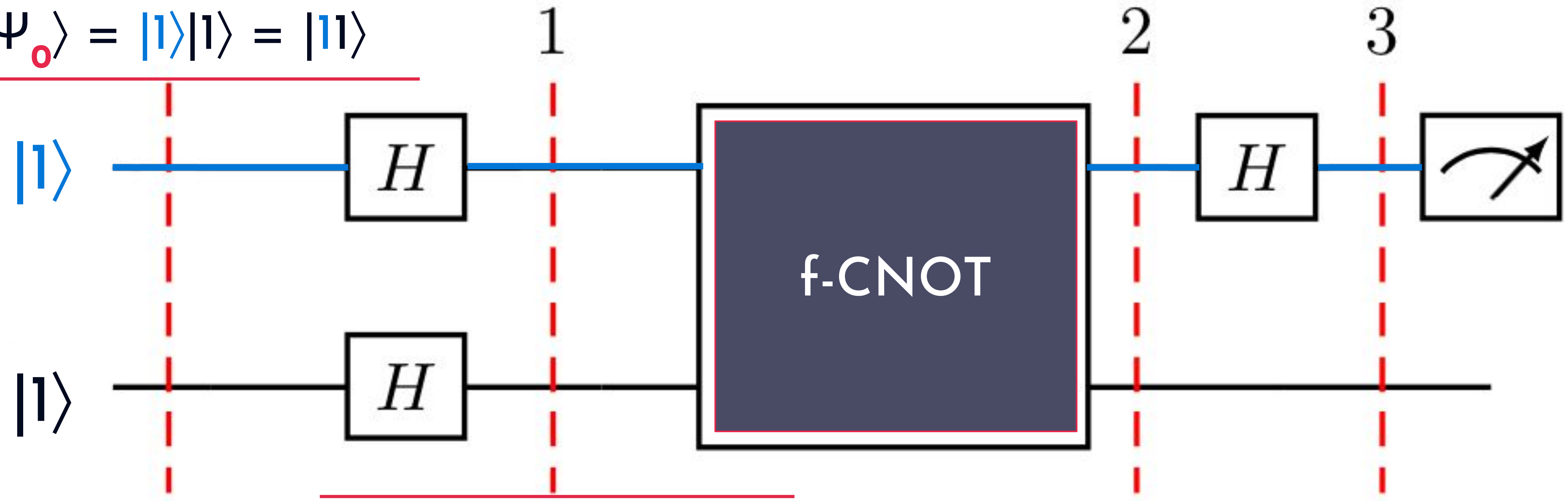
Classical Answer: Run f(x) at least twice.

$f(0) = 0$		$f(0) = 1$	
$f(1) = 0$		$f(1) = 0$	
CONST. 0		**BALANCED**	

Quantum Solution



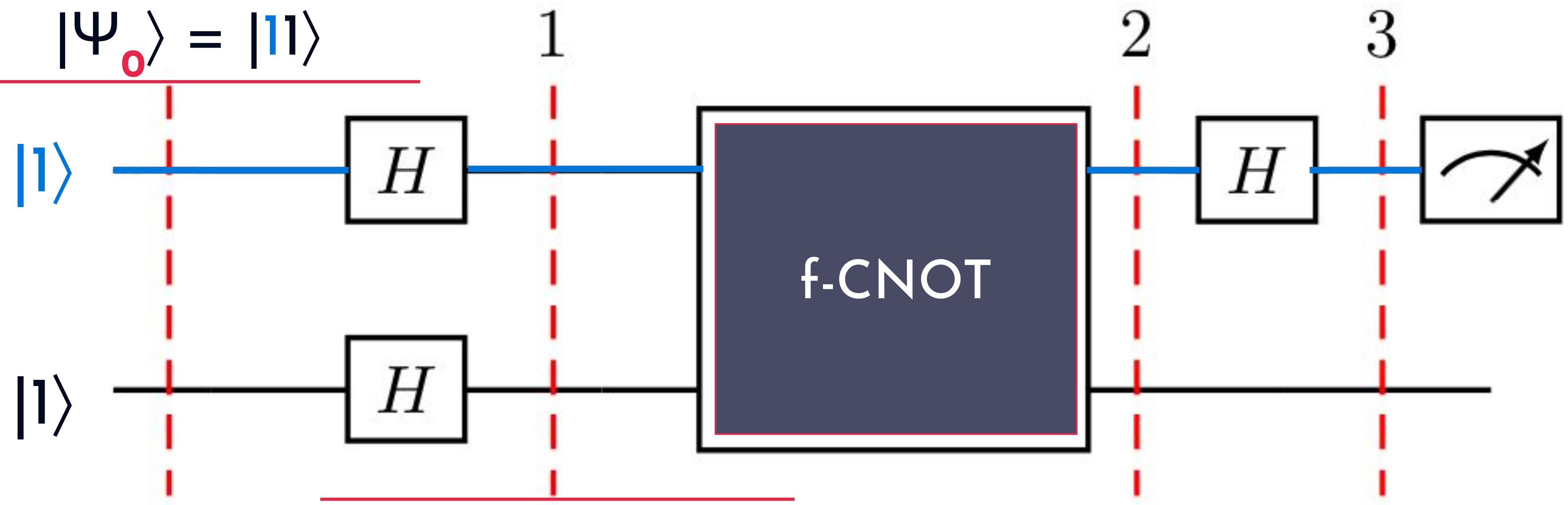
$|\Psi_0\rangle = |1\rangle|1\rangle = |11\rangle$



$|\Psi_1\rangle = |-\rangle|-\rangle = |--\rangle$

Quantum Solution

$$|\Psi_0\rangle = |11\rangle$$



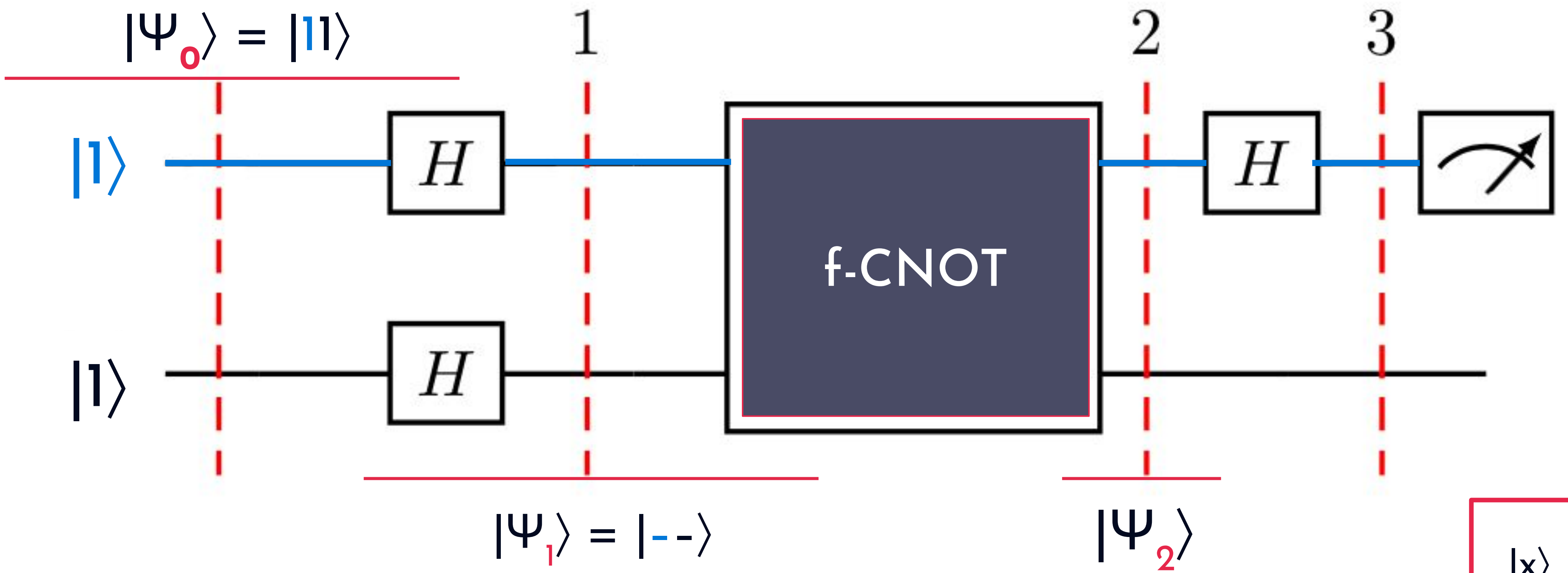
$$|\Psi_1\rangle = |-\rangle|-\rangle = |--\rangle$$

$$\sqrt{2}\,|-\rangle = |0\rangle - |1\rangle$$

$$2\,|\Psi_1\rangle = (\,|0\rangle - |1\rangle\,)\,(\,|0\rangle - |1\rangle\,)$$

$$= (\,|00\rangle - |10\rangle - |01\rangle + |11\rangle\,)$$

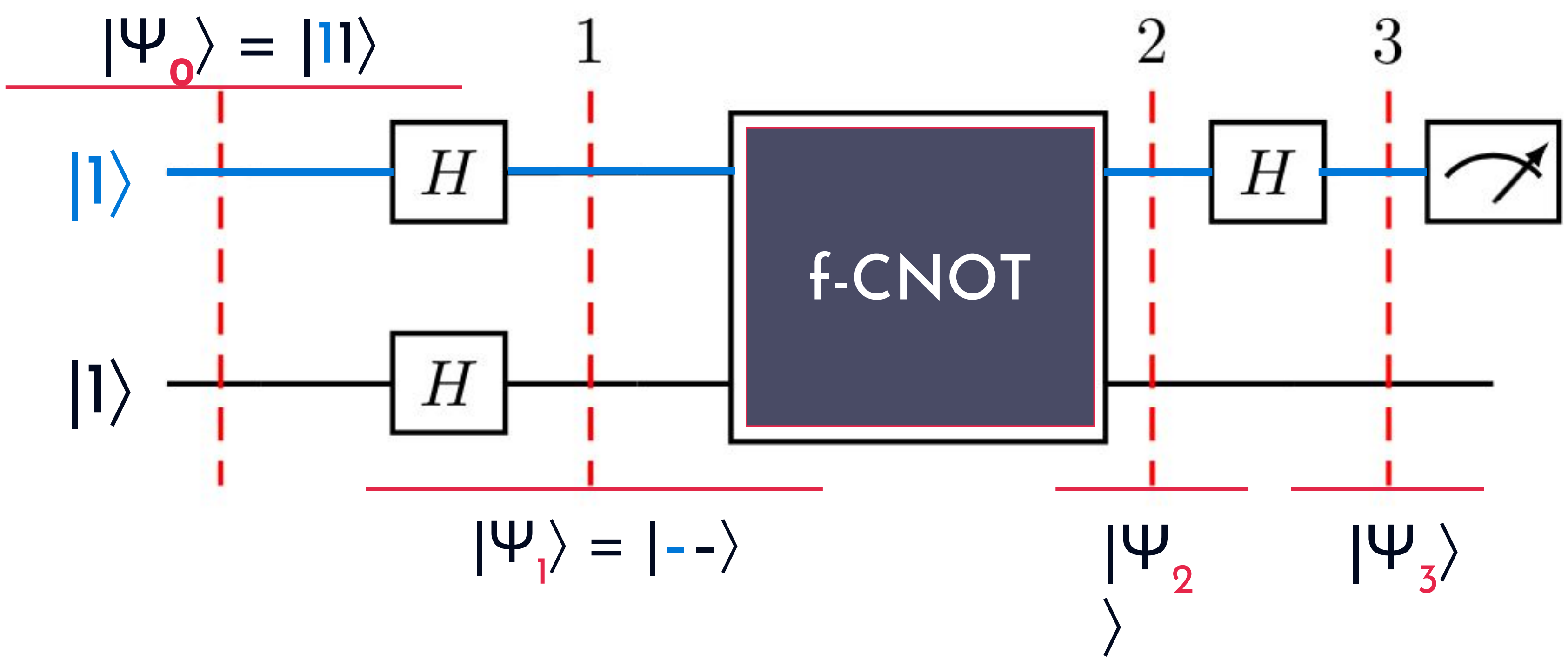
Quantum Solution



$$2\,|\Psi_1\rangle = |0\,0\rangle - |1\,0\rangle - |0\,1\rangle + |1\,1\rangle$$

$$2\,|\Psi_2\rangle = |\,0\ f(0)\rangle - |\,1\ f(1)\rangle - |\,0\ f'(0)\rangle + |\,1\ f'(1)\rangle$$

Quantum Solution



$\sqrt{2}\,|-\rangle = |0\rangle - |1\rangle$

$\sqrt{2}\,|+\rangle = |0\rangle + |1\rangle$

$$2\,|\Psi_2\rangle = |0\ f(0)\rangle - |1\ f(1)\rangle - |0\ f'(0)\rangle + |1\ f'(1)\rangle$$

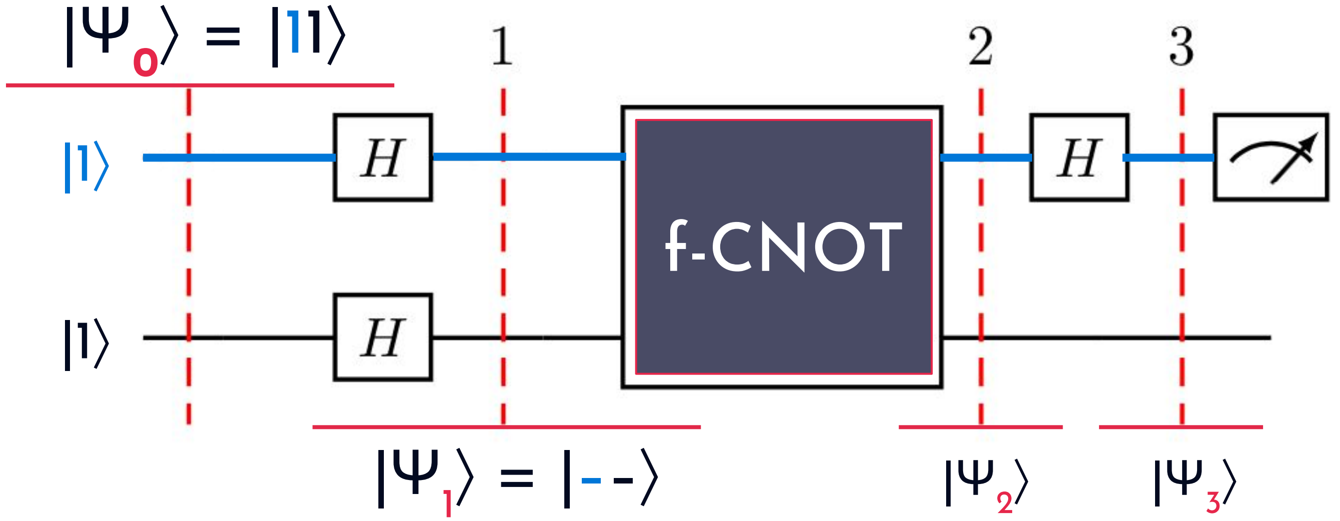
if f is constant then f(0) = f(1)

$$2\,|\Psi_2\rangle = \underline{(\ |0\rangle - |1\rangle\ )}\ (\ |f'(0)\rangle - |f'(0)\rangle\ )$$

if f is balanced then f(0) = f'(1)

$$2\,|\Psi_2\rangle = \underline{(\ |0\rangle + |1\rangle\ )}\ (\ |f'(0)\rangle - |f'(0)\rangle\ )$$

Quantum Solution



if f is constant then f(0) = f(1)

$$2|\Psi_2\rangle = \sqrt{2}\,|-\rangle\,(\,|f'(0)\rangle - |f'(0)\rangle\,)$$

$$2|\Psi_3\rangle = |1\rangle\,(\,|f'(0)\rangle - |f'(0)\rangle\,)$$

if f is balanced then f(0) = f'(1)

$$2|\Psi_2\rangle = \sqrt{2}\,|+\rangle\,(\,|f'(0)\rangle - |f'(0)\rangle\,)$$

$$2|\Psi_3\rangle = |0\rangle\,(\,|f'(0)\rangle - |f'(0)\rangle\,)$$

$_1|\Psi_3\rangle \longrightarrow$ *nature of f*

THANK YOU

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