Fuzzy Logic and Systems

Why Fuzzy

- Based on intuition and judgment
- No need for a mathematical model
- Provides a smooth transition between members and nonmembers
- Relatively simple, fast and adaptive
- Less sensitive to system fluctuations
- Can implement design objectives, difficult to express mathematically, in linguistic or descriptive rules.

Applications Domain

- Fuzzy Logic
- Fuzzy Control
  - Neuro-Fuzzy System
  - Intelligent Control
  - Hybrid Control
- Fuzzy Pattern Recognition
- Fuzzy Modeling

Some Interesting Applications

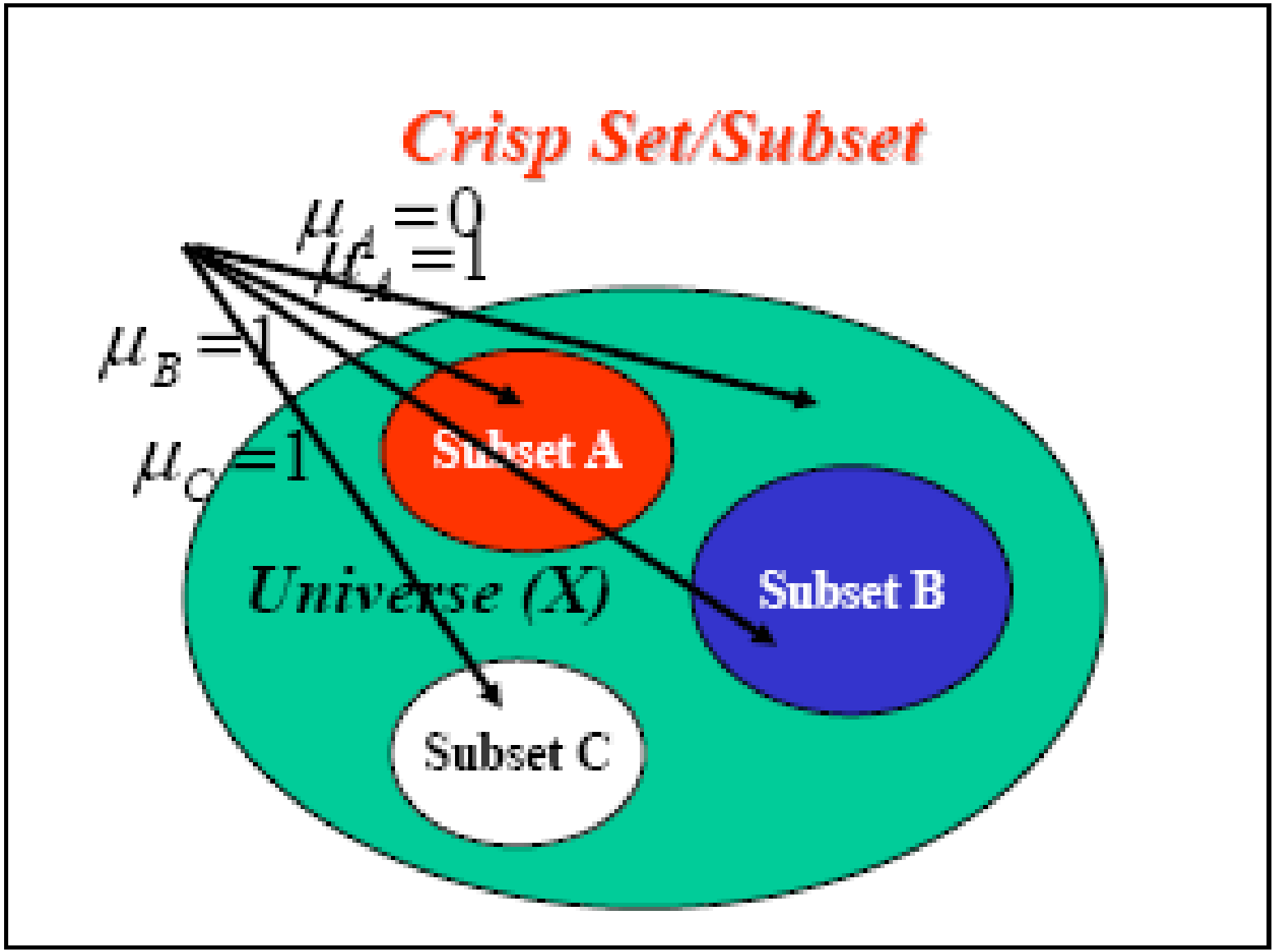
- Ride smoothness control
- Camcorder auto-focus and jiggle control
- Braking systems
- Copier quality control
- Rice cooker temperature control
- High performance drives
- Air-conditioning systems

Crisp Set

Conventional or *crisp* sets are binary. An element either belongs to the set or doesn't.

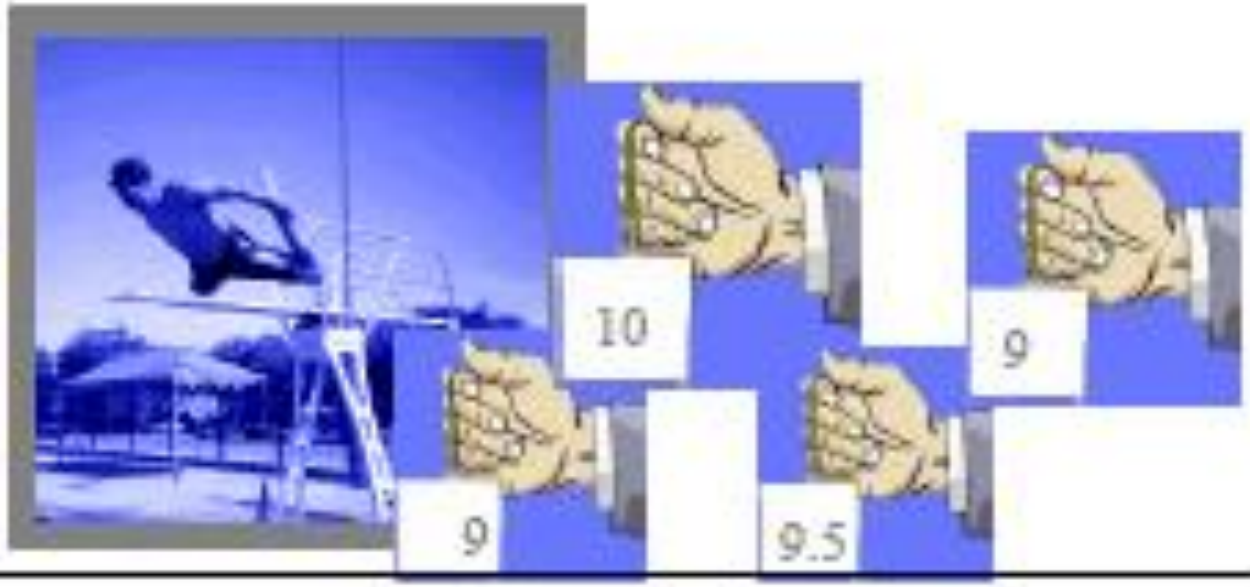
{True, false}

{1, 0}


Crisp Set/Subset
$\mu_A = 0$
$\mu_A = 1$
$\mu_B = 1$
$\mu_C = 1$
Subset A
Universe (X)
Subset B
Subset C

Fuzzy Linguistic Variables

- Examples of fuzzy measures include *close*, *heavy*, *light*, *big*, *small*, *smart*, *fast*, *slow*, *hot*, *cold*, *tall* and *short*.



Fuzzy Indicators

- **Can you distinguish between American and French person?**
- **Some Rules:**
  - ***If*** speaks English ***then*** American
  - ***If*** speaks French ***then*** French
  - ***If*** loves perfume ***then*** French
  - ***If*** loves outdoors ***then*** American
  - ***If*** good cook ***then*** French
  - ***If*** plays baseball ***then*** American

Fuzzy Indicators

- Rules may give contradictory indicators

  {good cook, loves outdoors, speaks French}
- The right answer is a question of a degree of association
- Fuzzy logic resolves these conflicting indicators
  - Membership of the person in the French set is 0.9
  - Membership of the person in the American set is 0.1

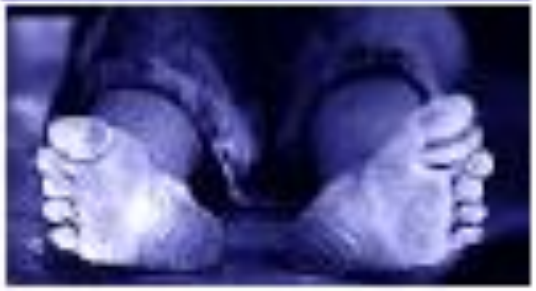
Fuzzy Versus Probability

- Fuzzy ≠ Probability
- Probability deals with uncertainty and likelihood
- Fuzzy logic deals with ambiguity and vagueness

Fuzzy Versus Probability

- Fuzzy ≠ Probability
- Example #1
  - Billy has ten toes. The *probability* Billy has nine toes is zero. The fuzzy membership of Billy in the set of people with nine toes, however, is nonzero.

(cite: Professor Marks)



Fuzzy Versus Probability

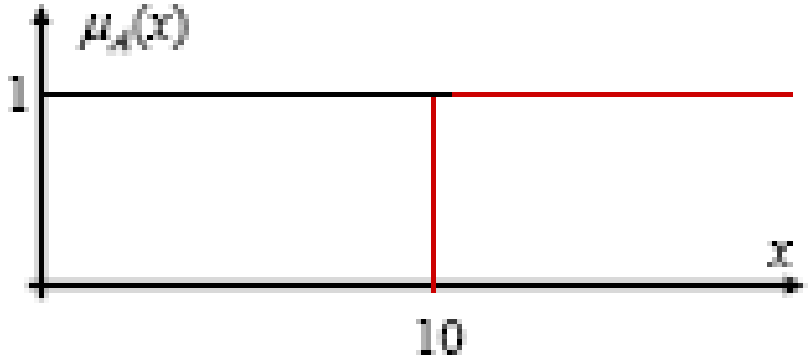
Example #2

- A bottle of liquid has a probability of ½ of being rat poison and ½ of being pure water.
- A second bottle's contents, in the fuzzy set of liquids containing *lots* of rat poison, is ½.
- The meaning of ½ for the two bottles clearly differs significantly and would impact your choice should you be dying of thirst.
- 50% probability means 50% chance that the water is clean.
- 50% fuzzy membership means that the water has poison.

Crisp Membership Functions

- Crisp membership functions (μ) are either one or zero.
- e.g. Numbers greater than 10.

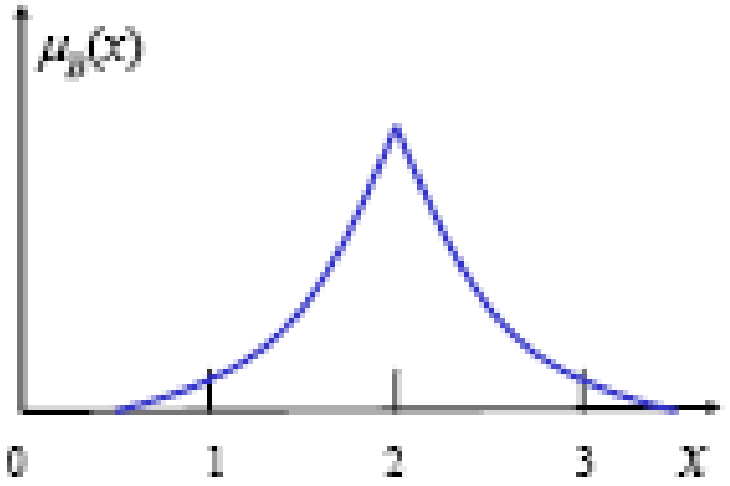
$A = \{x \mid x > 10\}$



Fuzzy Membership Functions

- The set, *B*, of numbers *near* to 2 can be represented by a membership function

$$\mu_B(x) = e^{-|x-2|}$$



Fuzzy Subsets

- A fuzzy set, A, is said to be a subset of B if

$$\mu_A(x) \leq \mu_B(x)$$

- *e.g.* B = far and A=*very* far.
- For example...

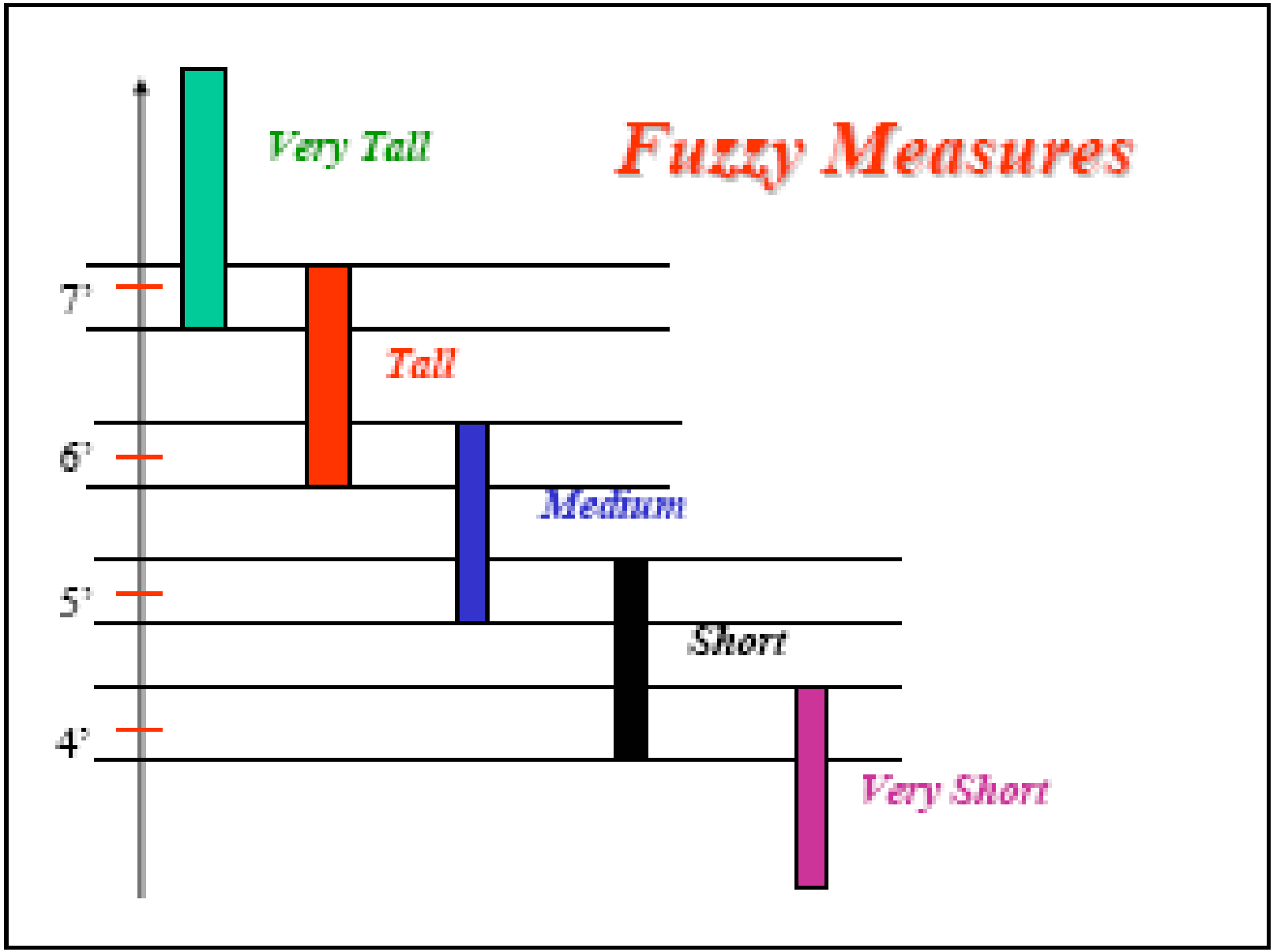
$$\mu_A(x) = \mu_B^2(x)$$

Fuzzy Sets

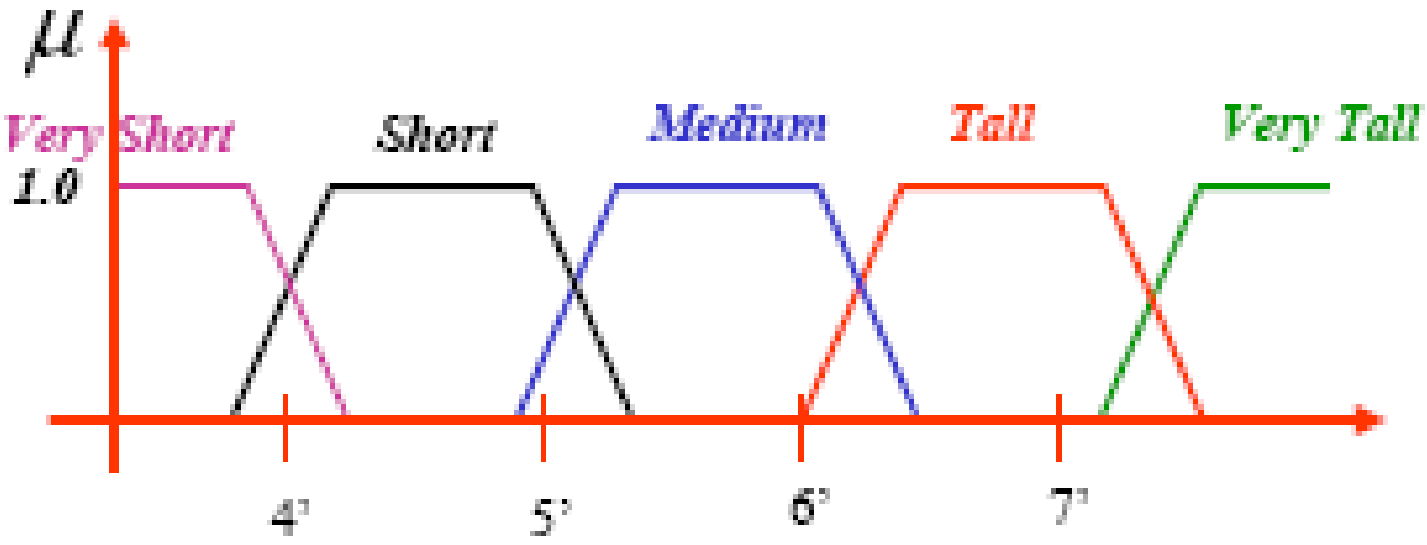
Tall

Tall or Short?

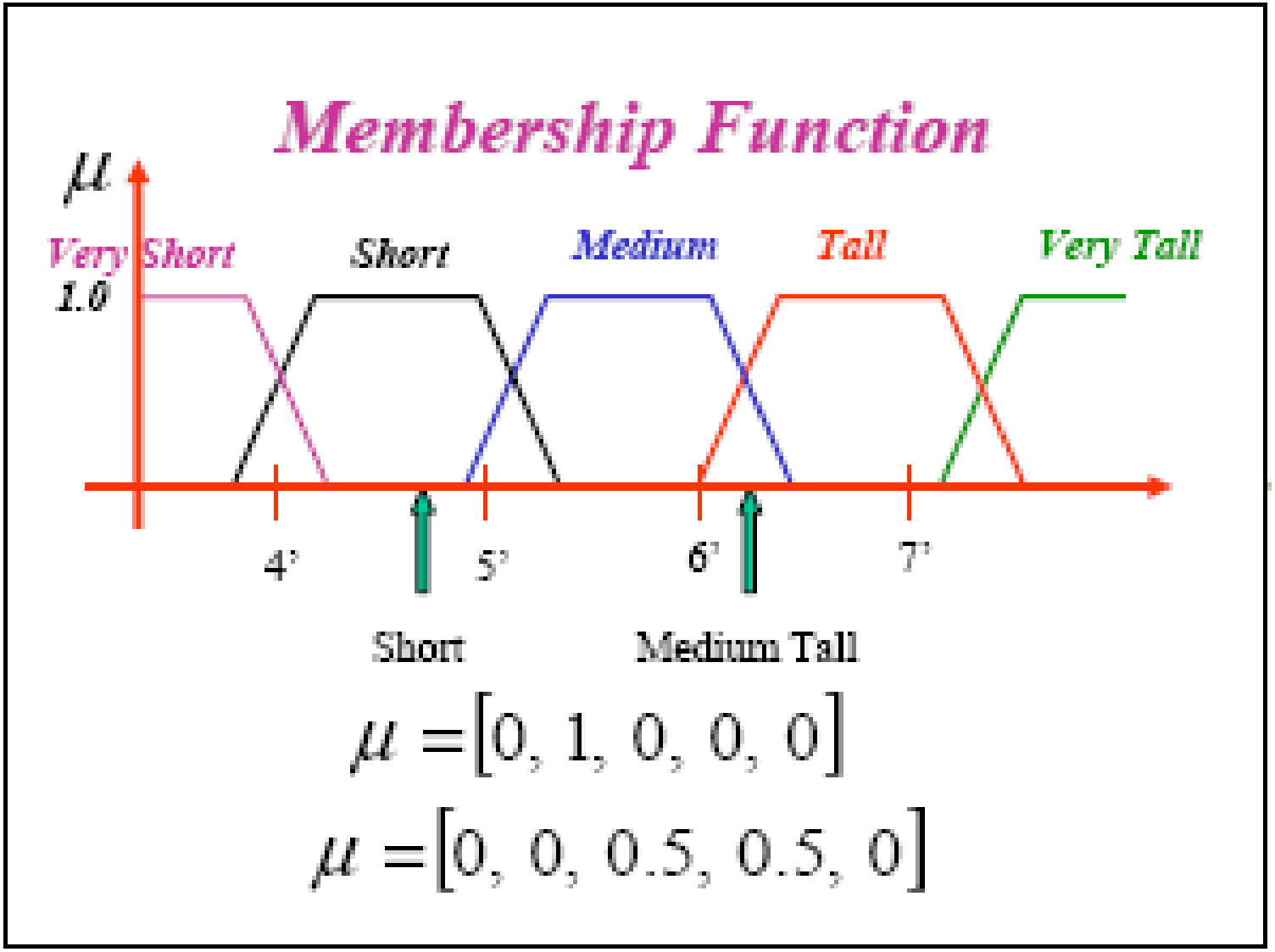
Short


Fuzzy Measures
Very Tall
Tall
Medium
Short
Very Short
7'
6'
5'
4'

Membership Function



$$\mu = [\mu_{vs}, \mu_{s}, \mu_{m}, \mu_{t}, \mu_{vt}]$$


Membership Function
μ
Very Short
Short
Medium
Tall
Very Tall
1.0
4'
5'
6'
7'
Short
Medium Tall
$\mu = [0, 1, 0, 0, 0]$
$\mu = [0, 0, 0.5, 0.5, 0]$

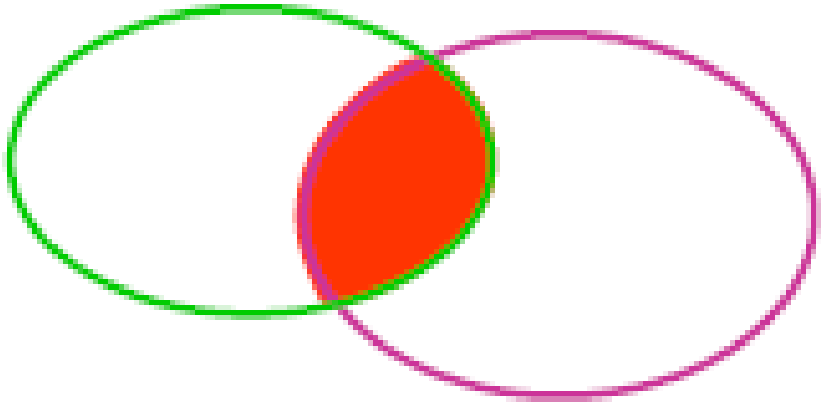
Fuzzy Logic Operations

Fuzzy union operation or fuzzy *OR*

$$\mu_{A+B}(x) = \max\left[\mu_A(x), \mu_B(x)\right]$$

Fuzzy Logic Operations

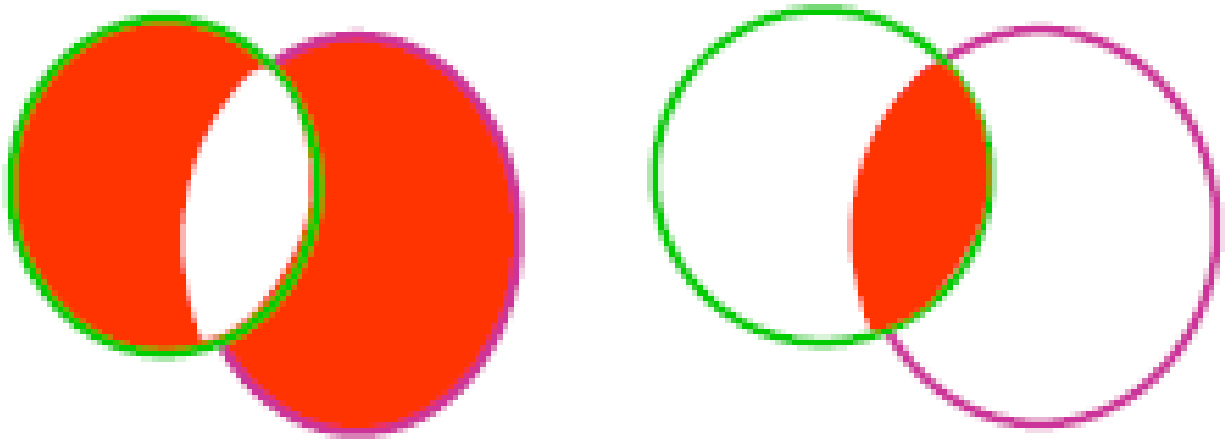
Fuzzy intersection operation or fuzzy *AND*



$$\mu_{A\cdot B}(x) = \min\left[\mu_A(x), \mu_B(x)\right]$$

Fuzzy Logic Operations

Complement operation



$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

Fuzzy Logic Operations

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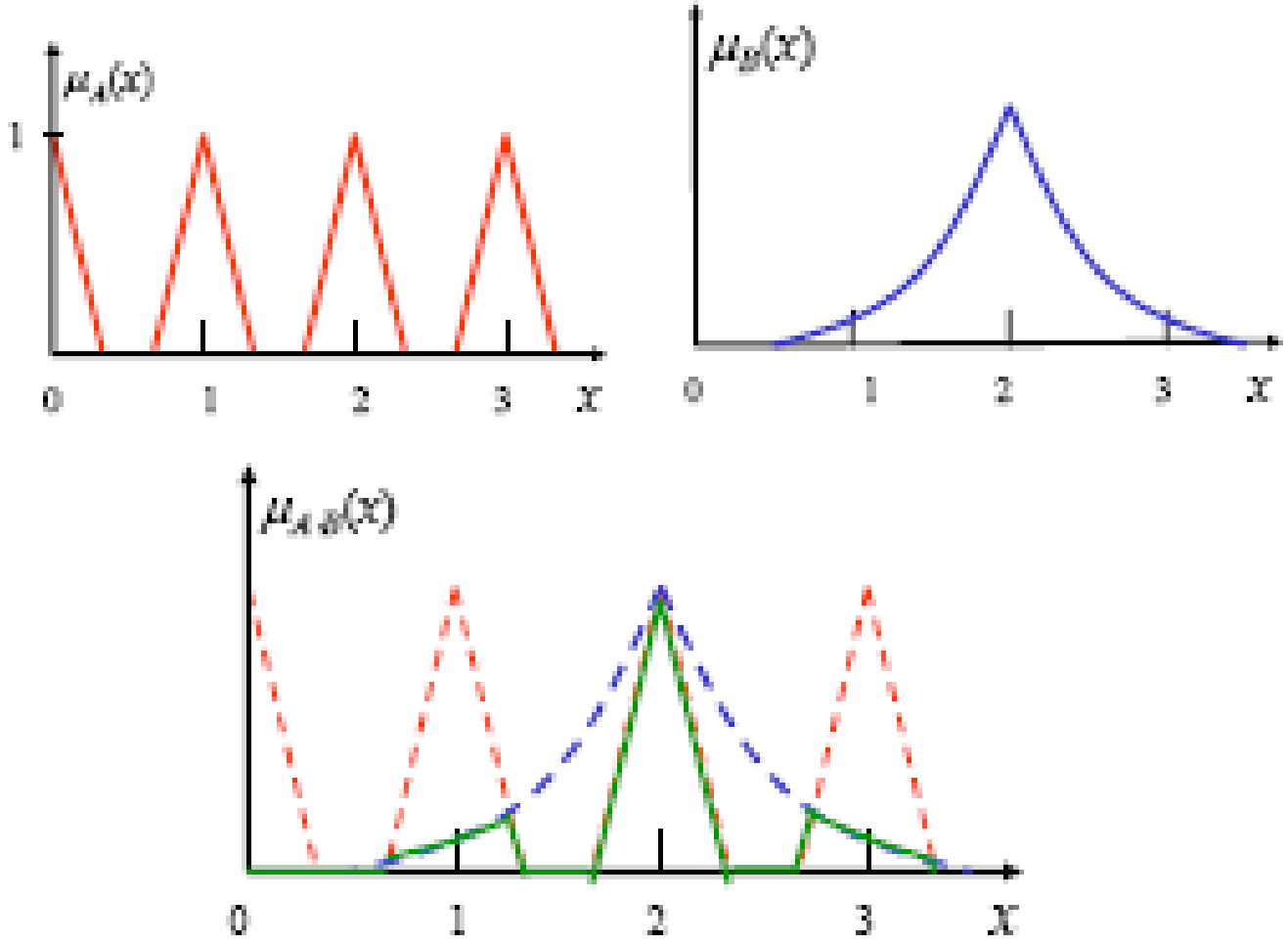

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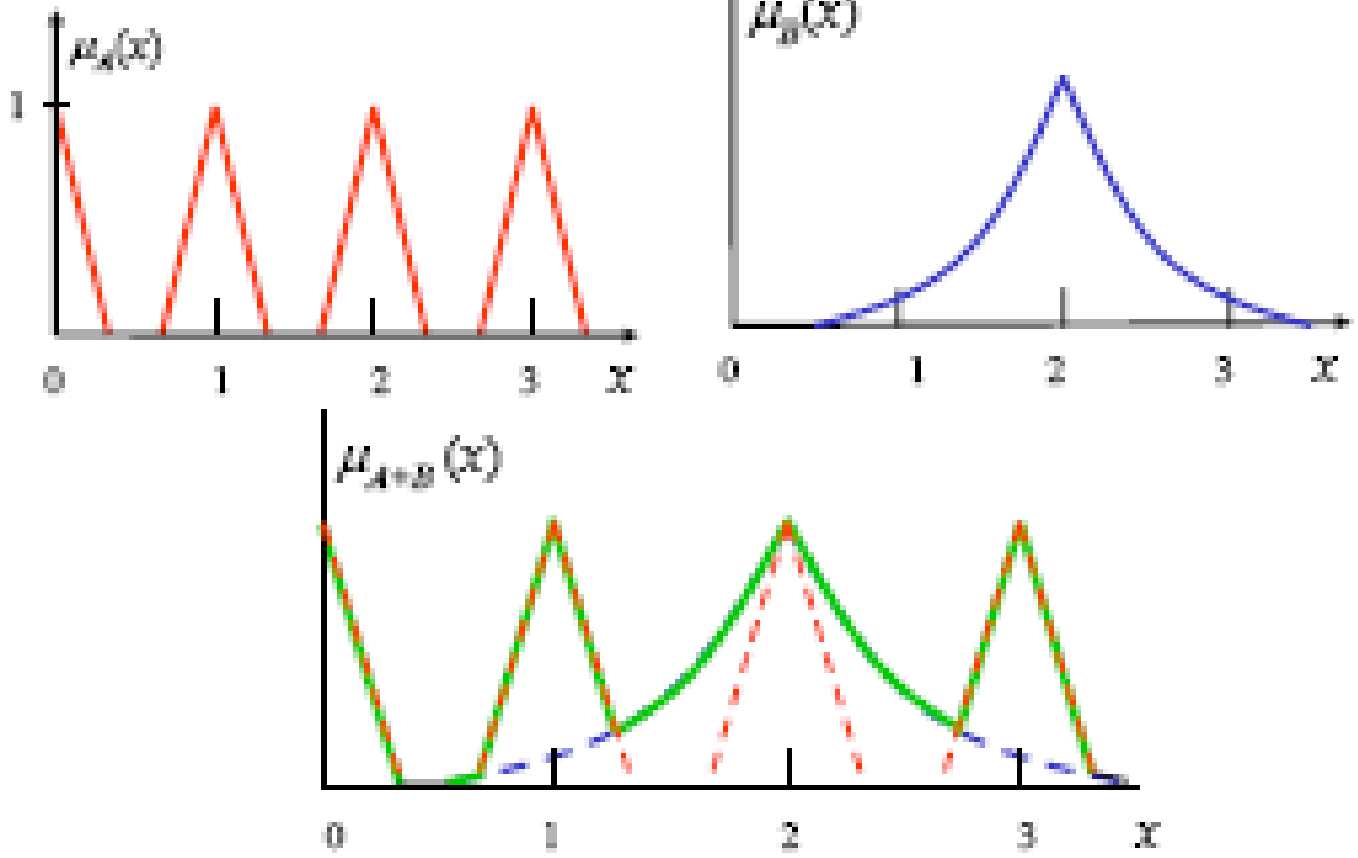
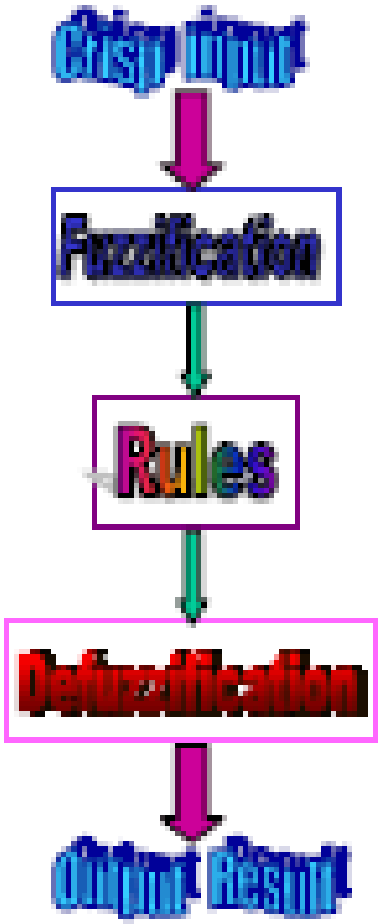


Fuzzy System

Basic Components



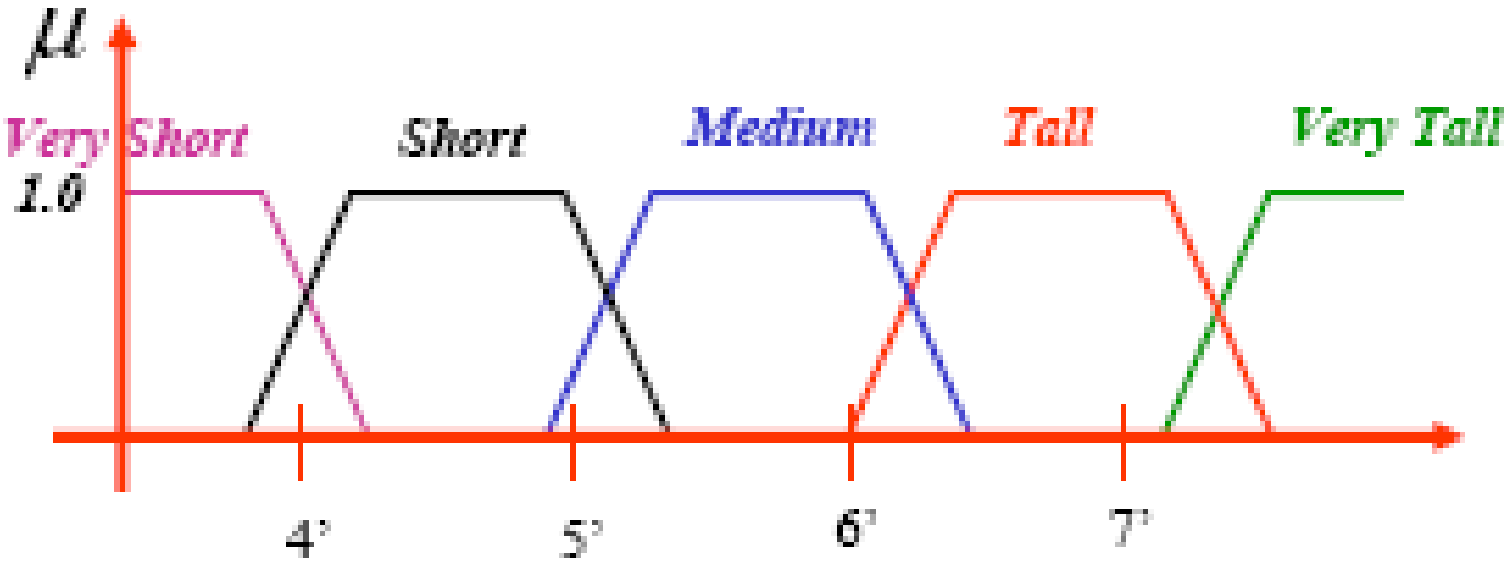
Step 1: Fuzzification

- Fuzzifier converts a crisp input into a fuzzy variable.
- Definition of the membership functions must
  - reflects the designer's knowledge
  - provides smooth transition between member and nonmembers of a fuzzy set
  - simple to calculate
- Typical shapes of the membership function are Gaussian, trapezoidal and triangular.

Example 1

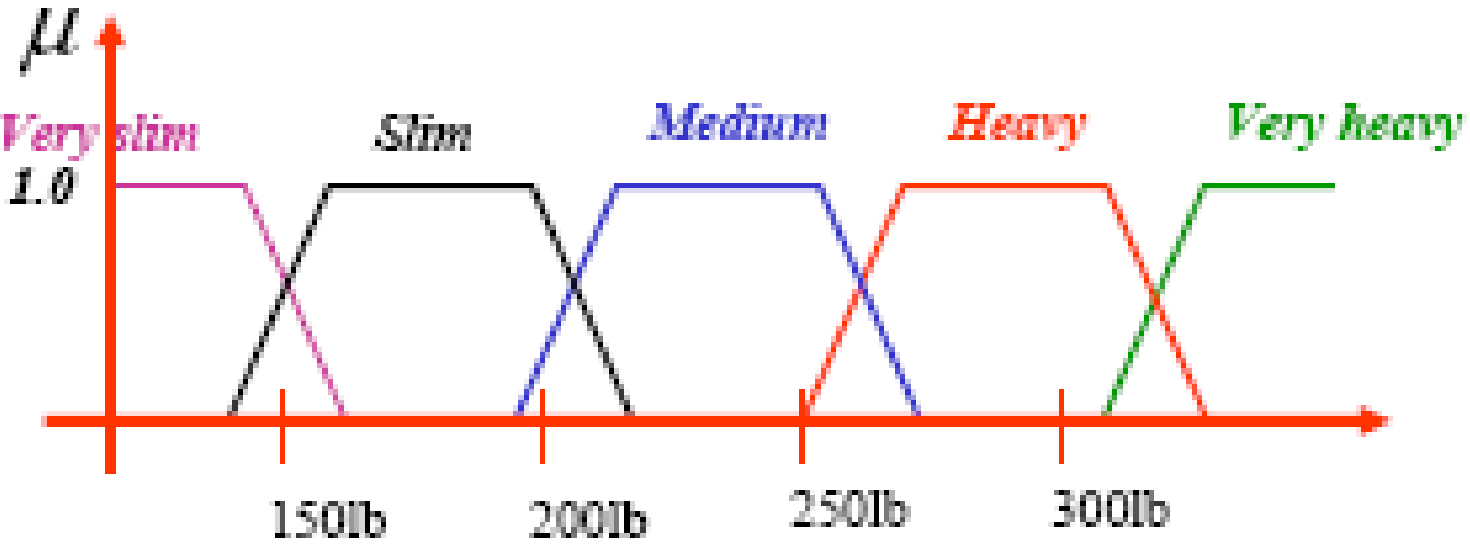
- Assume we want to evaluate the health of a person based on his height and weight.
- The input variables are the crisp numbers of the person's height and weight.
- Fuzzification is a process by which the numbers are changes into linguistic words

Fuzzification of Height



VS = very short
S = Short
M = Medium
etc.

Fuzzification of Weight



VS = very slim
S = Slim
M = Medium
etc.

Step 2: Rules

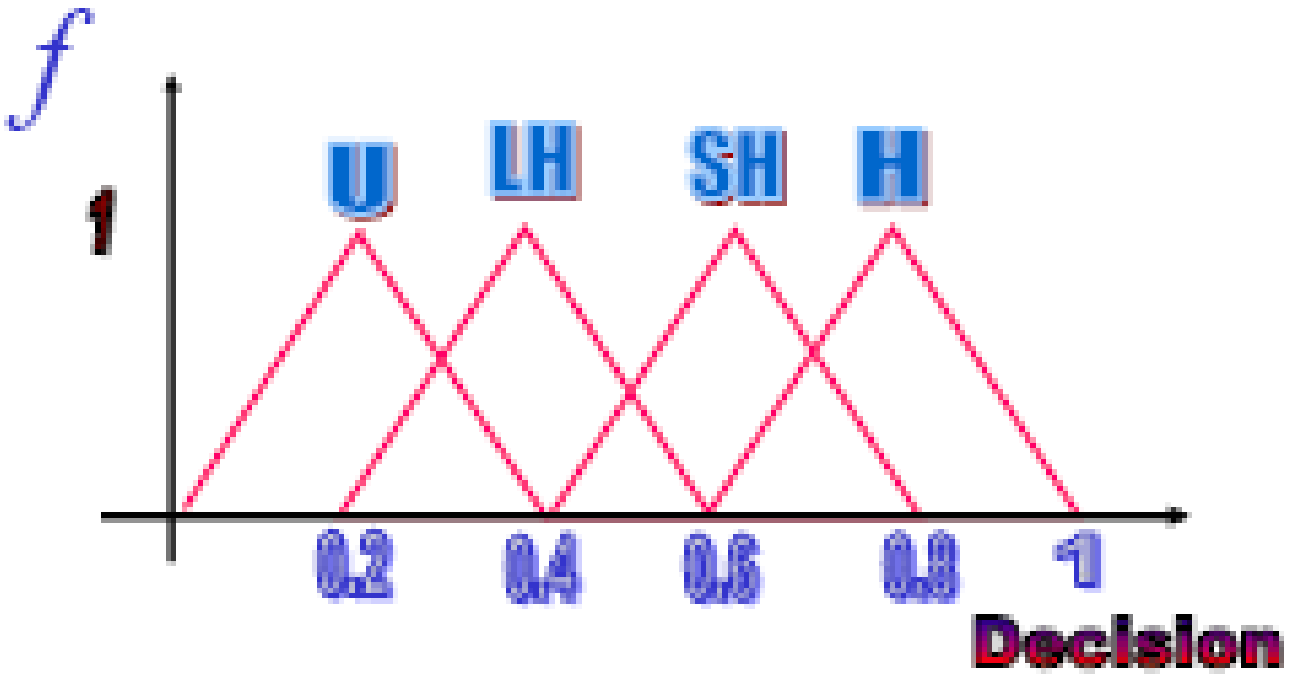
- Rules reflect experts decisions.
- Rules are tabulated as fuzzy words
- Rules can be grouped in subsets
- Rules can be redundant
- Rules can be adjusted to match desired results

Rules Function

- Rules are tabulated as fuzzy words
  - Healthy (H)
  - Somewhat healthy (SH)
  - Less Healthy (LH)
  - Unhealthy (U)
- Rule function *f*

$$f = \{U, LH, SH, H\}$$

Fuzzified Decision



$$f = \{U, LH, SH, H\}$$

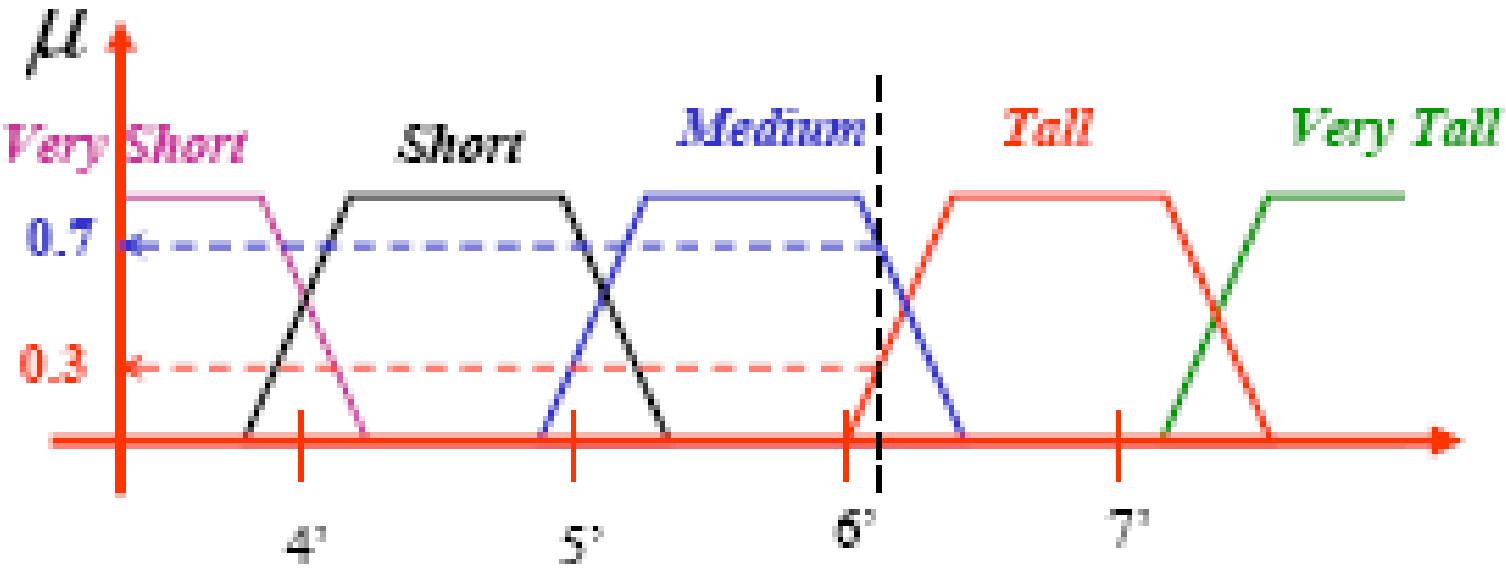
Fuzzy Rules Table

Height	Weight					
		Very Slim	Slim	Medium	Heavy	Very Heavy
	Very Short	H	SH	LH	U	U
	Short	SH	H	SH	LH	U
	Medium	LH	H	H	LH	U
	Tall	U	SH	H	SH	U
	Very Tall	U	LH	H	SH	LH

Step 3: Calculate

- For a given person, compute the membership of his/her weight and height
- Example:
  - Assume that a person height is 6' 1"
  - Assume that the person's weight is 140 lb

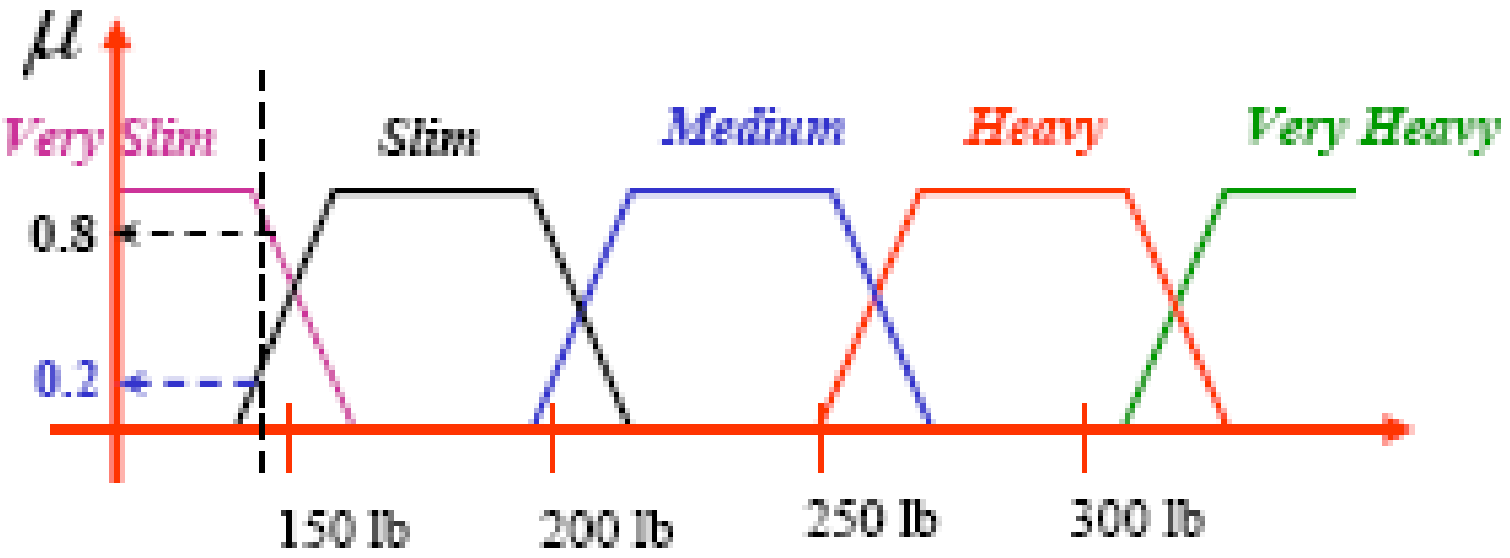
Membership of Height



$$\mu_{height} = \{\mu_{VS} \quad \mu_S \quad \mu_M \quad \mu_T \quad \mu_{VT}\}$$

$$\mu_{height} = \{0 \quad 0 \quad 0.7 \quad 0.3 \quad 0\}$$

Membership of Weight



$$\mu_{Weight} = \{\mu_{VS} \quad \mu_S \quad \mu_M \quad \mu_H \quad \mu_{VH}\}$$

$$\mu_{Weight} = \{0.8 \quad 0.2 \quad 0 \quad 0 \quad 0\}$$

Step 4: Activate Rules

Height \ Weight	Very Slim	Slim	Medium	Heavy	Very Heavy
Very Short	H	SH	LH	U	U
Short	SH	H	SH	LH	U
Medium			H	LH	U
Tall			H	SH	U
Very Tall	U	LH	H	SH	LH

Substitute Membership Values

Height	Weight	0.8	0.2	Medium	Heavy	Very Heavy
	Very Short	H	SH	LH	U	U
	Short	SH	H	SH	LH	U
	0.7	LH	H	H	LH	U
	0.3	U	SH	H	SH	U
	Very Tall	U	LH	H	SH	LH

Perform min operation

Height		Weight: 0.8	Weight: 0.2	Weight: Medium (0)	Weight: Heavy (0)	Weight: V.Heavy (0)
	V. Short (0)	0	0	0	0	0
	Short (0)	0	0	0	0	0
0.7		0.7	0.2	0	0	0
0.3		0.3	0.2	0	0	0
	V. Tall (0)	0	0	0	0	0

Step 5: Compute Decision Function

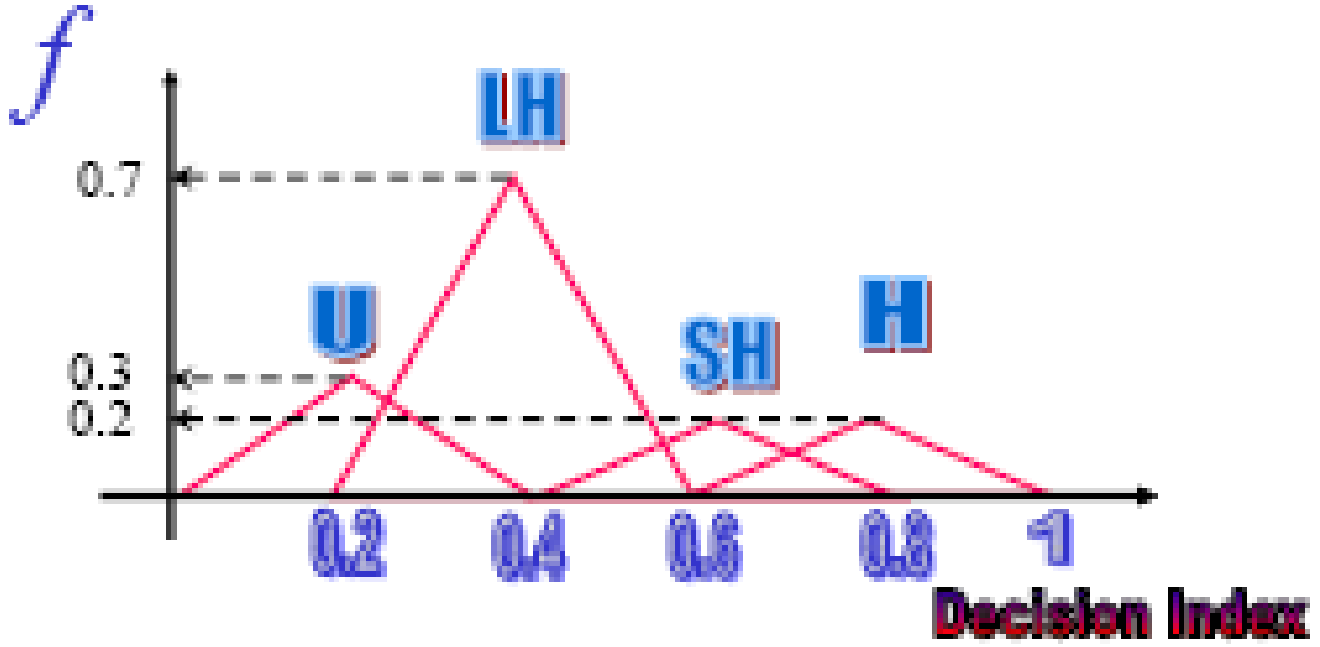
	Weight		
		0.8	0.2
	V. Short (S)	0	0
	Short (S)	0	0
	0.7	0.7	0.2
	0.3	0.3	0.2
	V. Tall (S)	0	0

	Weight		
		0.8	0.2
	Very Short	H	SH
	Short	SH	H
	0.7	LH	H
	0.3	U	SH
	Very Tall	U	LH

$$f = \{U, LH, SH, H\}$$

$$f = \{0.3, 0.7, 0.2, 0.2\}$$

Scaled Fuzzified Decision



$$f = \{U, LH, SH, H\}$$

$$f = \{0.3, 0.7, 0.2, 0.2\}$$

Step 6: Compute Final Decision

- *Use the fuzzified rules to compute the final decision.*
- *Two methods are often used.*
  - *Maximum Method (not often used)*
  - *Centroid*

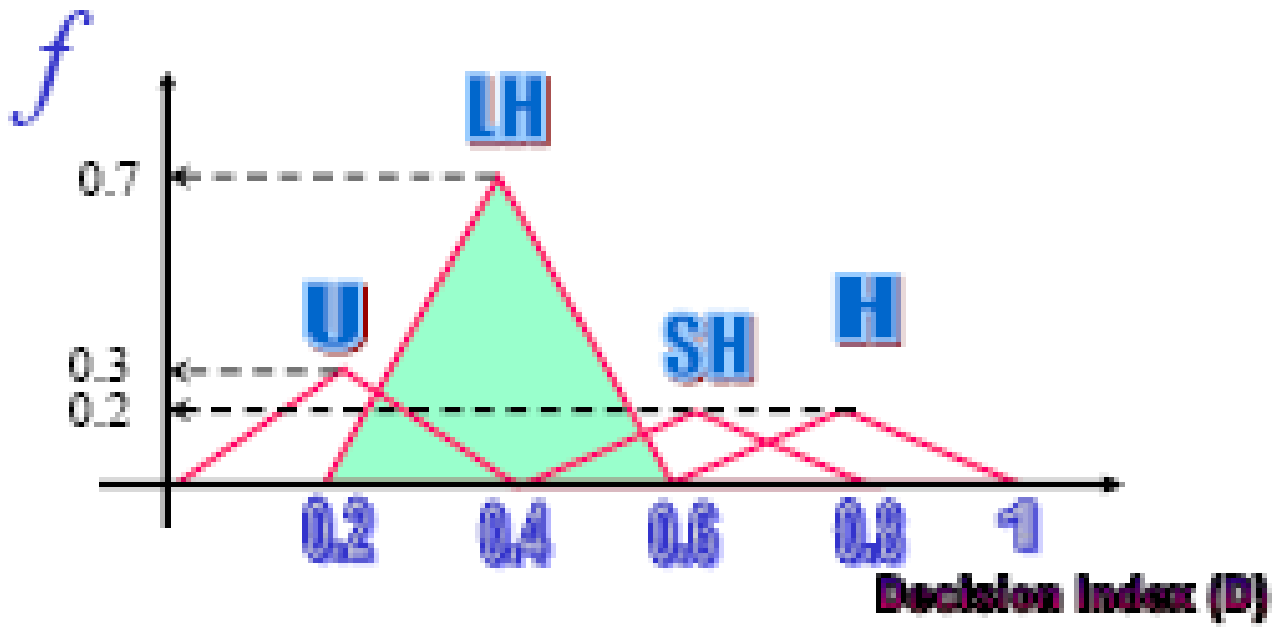
Max Method

- Fuzzy set with the largest membership value is selected.
- Fuzzy decision:

$$f = \{U, LH, SH, H\}$$
$$f = \{0.3, 0.7, 0.2, 0.2\}$$

- Final Decision (FD) = Less Healthy
- If two decisions have same membership max, use the average of the two.

Max Method



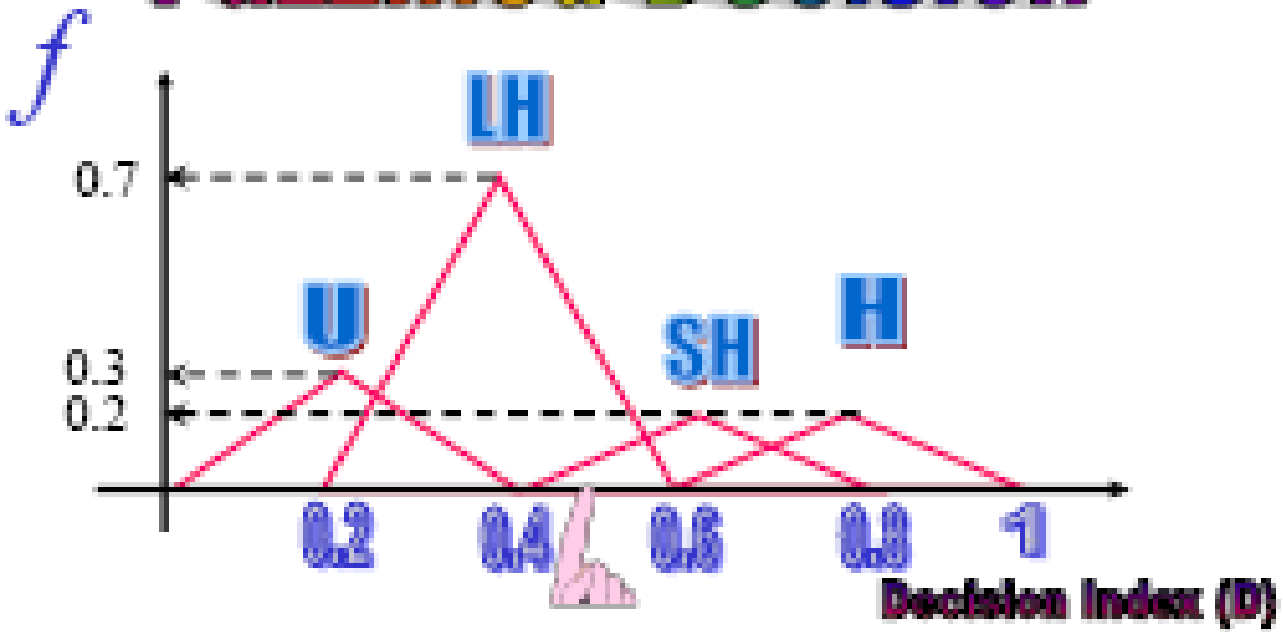
Centroid Method

$$FD = \frac{\sum \mu D}{\sum \mu} = \frac{\mu_U D_U + \mu_{LH} D_{LH} + \ldots\ldots}{\mu_U + \mu_{LH} + \ldots\ldots}$$

$$FD = \frac{0.3 \times 0.2 + 0.7 \times 0.4 + 0.2 \times 0.6 + 0.2 \times 0.8}{0.3 + 0.7 + 0.2 + 0.2} = 0.4429$$

Crisp Decision Index (D) = 0.4429

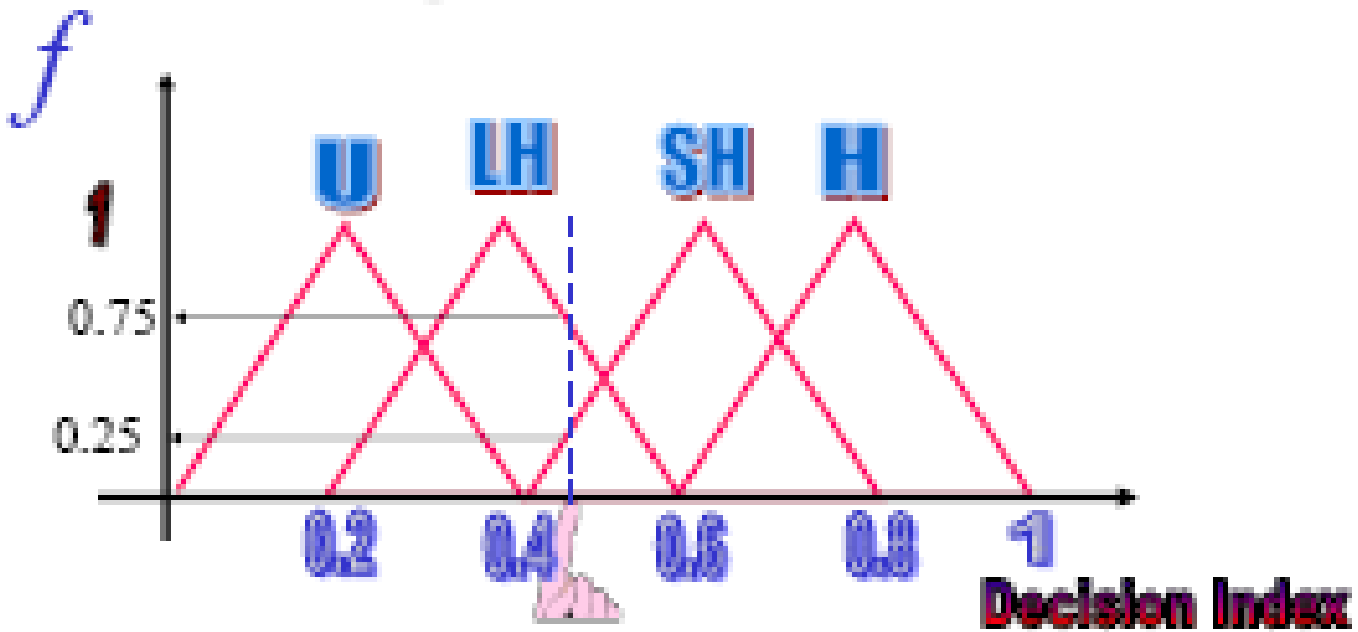
Fuzzified Decision



Crisp Decision Index (D) is the centroid

$D = 0.4429$

Fuzzy Decision Index



Fuzzy Decision Index (D)

75% in Less Healthy group

25% in Somewhat Healthy group