Adversarial Search

Chapter 6

Section 1 – 4

Warm Up

- Let’s play some games!

Outline

- Optimal decisions
- Imperfect, real-time decisions
- α-β pruning

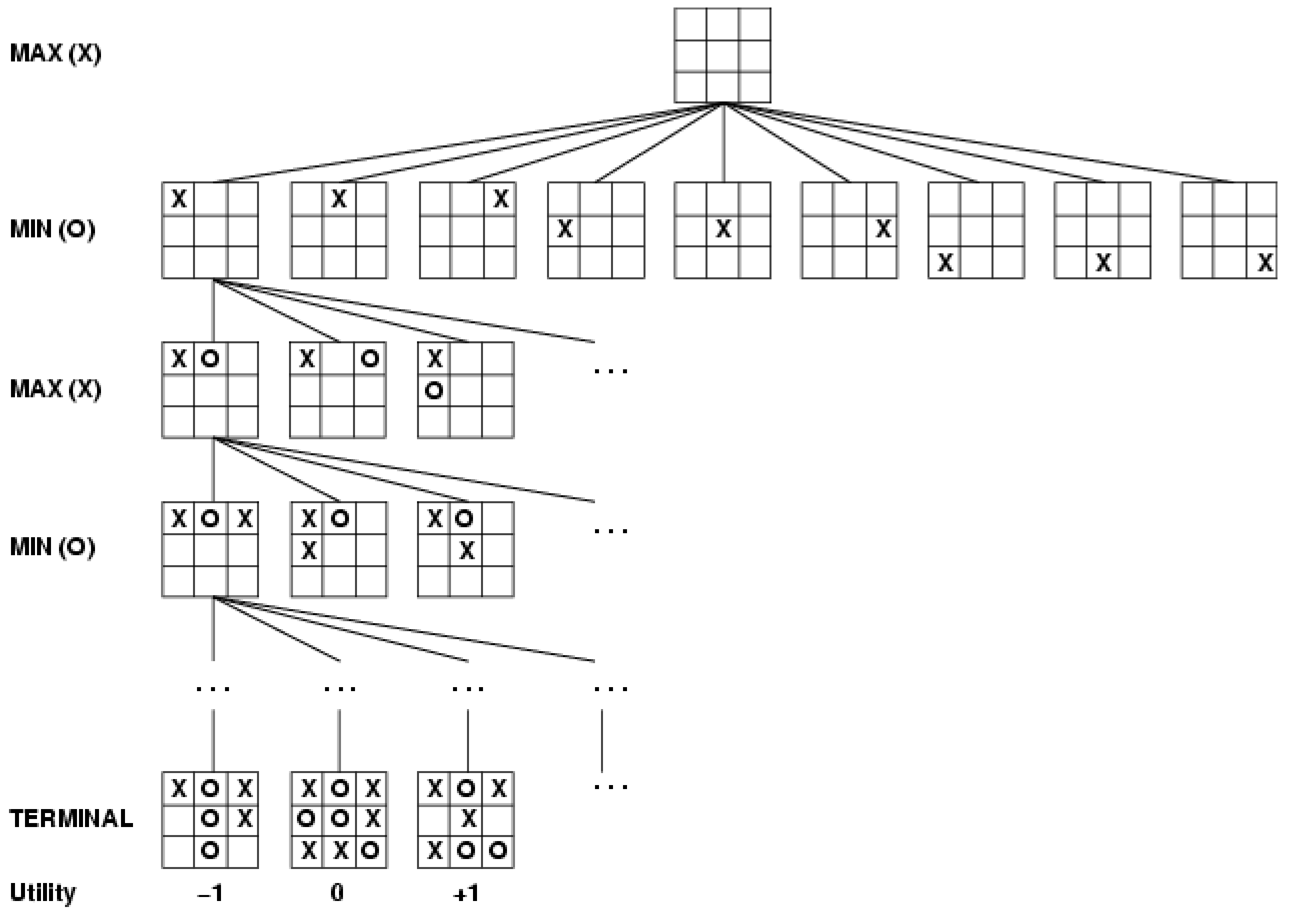
Games vs. search problems

- "Unpredictable" opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

Minimax Search

- Core of many computer games
- Pertains primarily to:
  - Turn based games
  - Two players
  - Players with “perfect knowledge”

Game tree (2-player, deterministic, turns)



Game Tree

- Nodes are states
- Edges are decisions
- Levels are called “plys”

Naïve Approach

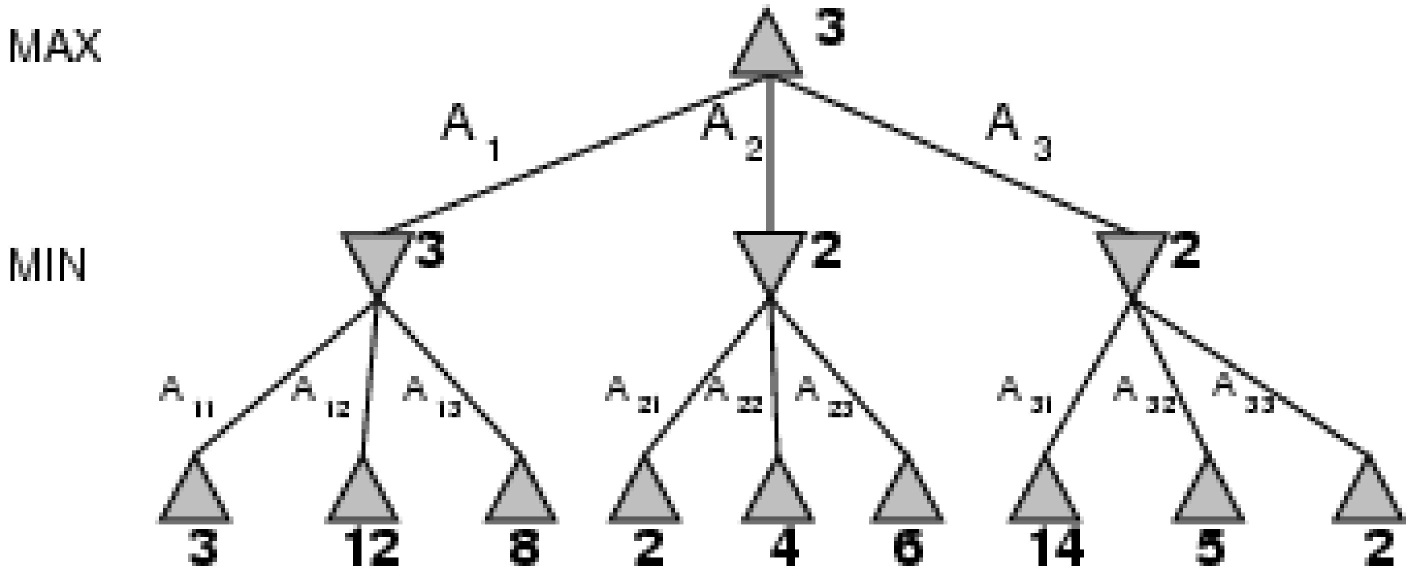
- Given a game tree, what would be the most straightforward playing approach?
- Any potential problems?

Minimax

- **Mini**mizing the **max**imum possible loss
- Choose move which results in best state
  - Select highest expected score for you
- Assume opponent is playing optimally too
  - Will choose lowest expected score for you

Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

Properties of minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)

- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  → exact solution completely infeasible

Resource limits

Suppose we have 100 secs, explore 10^4 nodes/sec
→ 10^6 nodes per move

Standard approach:

- cutoff test:

  e.g., depth limit (perhaps add quiescence search)

- evaluation function

  = estimated desirability of position

Evaluation Functions

- Assign a utility score to a state
  - Different for players?
- Usually a range of integers
  - [-1000,+1000]
- +infinity for win
- -infinity for loss

Cutting Off Search

- How to score a game before it ends?
  - You have to fudge it!
- Use a **heuristic** function to approximate state's utility

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

$b^m = 10^6$, $b=35 \rightarrow m=4$

4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov

(A computer program which evaluates no further than its own legal moves plus the legal responses to those moves is searching to a depth of two-ply.)

Example Evaluation Function

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g., $w_1 = 9$ with

  $f_1(s)$ = (number of white queens) – (number of black queens), etc.

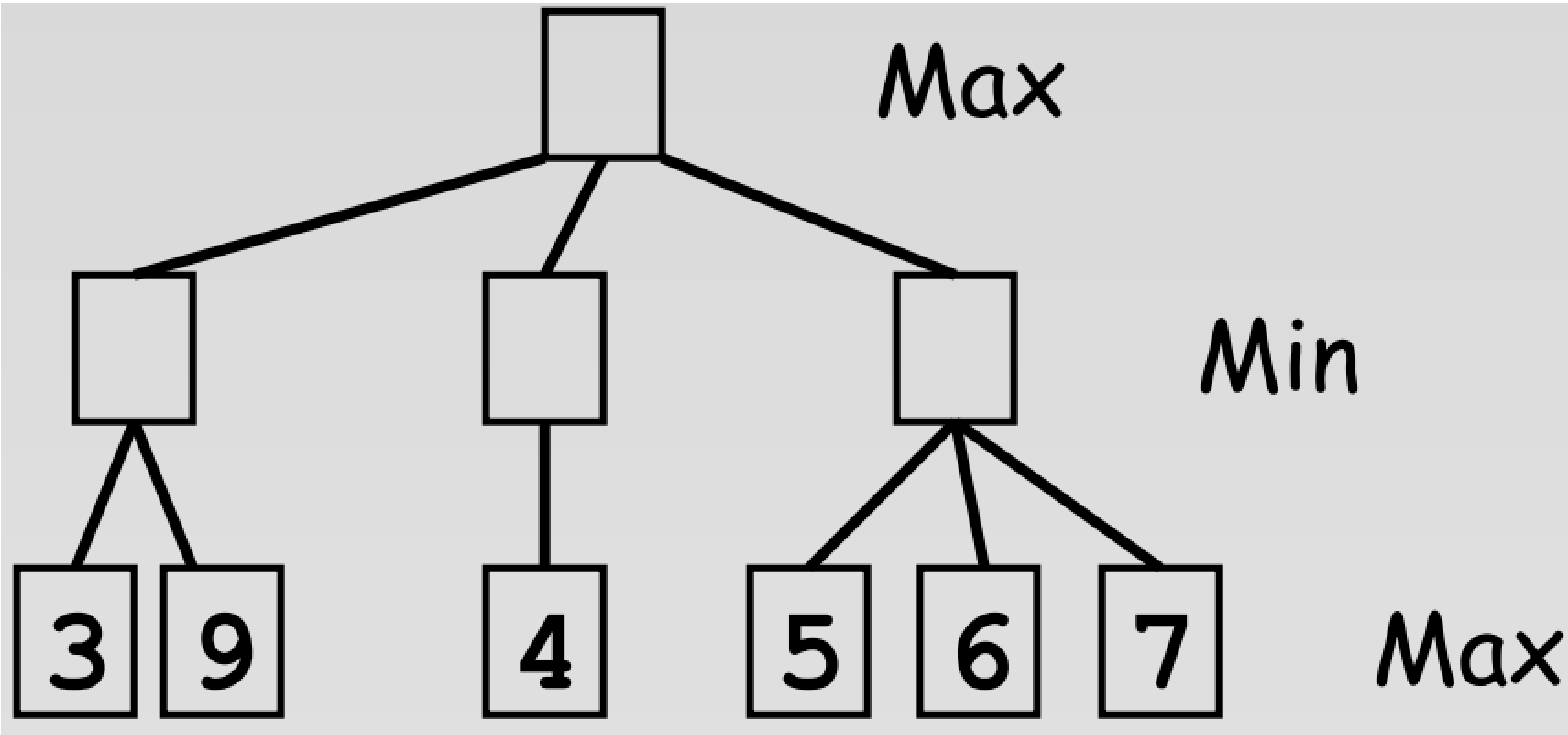
Evaluating States

- Assuming an ideal evaluation function, how would you make a move?
- Is this a good strategy with a bad function?

Look Ahead

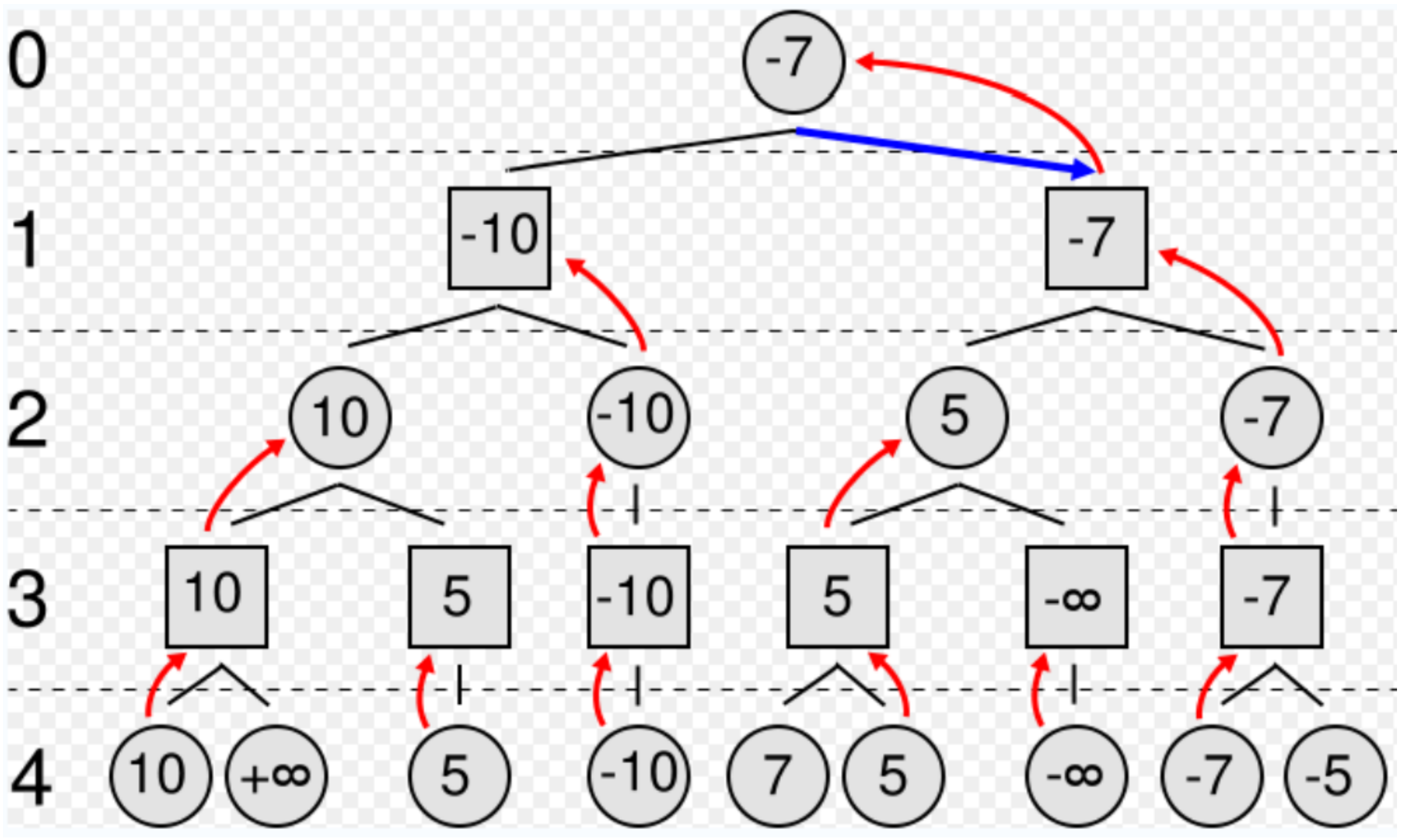
- Instead of only evaluating immediate future, look as far ahead as possible

Look Ahead



Bubbling Up

- Looking ahead allows utility values to "bubble up" to root of search tree



Minimax Algorithm

- BESTMOVE function
- Inputs:
  - Board state
  - Depth bound
- Explores search tree to specified depth
- Output:
  - Best move

Minimax Algorithm

```
BESTMOVE = proc (posn, depth)
  begin  (movelist, bestscore, bestm, try, tryscore) %local variables

         % posn: is the current BOARD CONFIGURATION FROM WHICH A MOVE
         %       MUST BE CHOSEN BY OUR INTELLIGENT AGENT COMPUTER

         % depth: MAXIMUM NUMBER OF PLIES TO LOOK AHEAD. THIS IS DETERMINED
         %       BY SPEED CONSTRAINTS AND COMPLEXITY OF THE GAME
         %       (i.e. branching factor b)

         % movelist: the list of all possible MOVES from the current posn
         % bestscore: the best "backed up" score found so far as we iterate
         %            thru the movelist
         % bestm:     the best move found so far that results in the bestscore
         % try:       just a tmp to hold returned values from recursive calls
         % tryscore:  just a tmp to hold returned scores from recursive calls
```

Minimax Algorithm

```
if depth = 0 then return list(EVALUATE(posn), nil) fi;
 %This ends the recursion at the bottom of the search space
 %Note a two element list of values is returned.

Movelist = POSSIBLE-MOVES(posn);
 %According to the rules of the game, we generate all possible
 %moves from the given board position.

Bestscore = -infinity;  %initializations
Bestm = nil;
```

Minimax Algorithm

```
%We are now ready to scan the movelist and select the
%best move. Note how we initialized Bestscore and Bestm.

while ( Movelist <> nil )
 repeat,
   try = BESTMOVE( NEWPOSITION(posn, first(Movelist)), depth-1);
         %Here is the main recursive call that expands and searches
         %the state space from the selected move.

   tryscore = - first(try); %recall BESTMOVE returns two values

   %Now we determine how well this current move did and whether
   %it should be selected as our best move found so far.

   if tryscore > Bestscore then
    do,
     Bestscore = tryscore;
     Bestm = first(Movelist);
    od;

   %Now we continue scanning down the list of moves to see
   %if we can find a better move then found so far.

   Movelist = rest(Movelist);
 taeper;
```

Minimax Algorithm

```
     %After scanning the entire Movelist, we have our best move so:
    return( list(Bestscore, Bestm) );
nigeb; %End of procedure min-max.
```

Minimax Algorithm

- Did you notice anything missing?

Minimax Algorithm

- Did you notice anything missing?
- Where were Max-Value and Min-Value?

Minimax Algorithm

- Did you notice anything missing?
- Where were Max-Value and Min-Value?
- What is going on here?

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tryscore = - first(try); %recall BESTMOVE returns two values
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Minimax Algorithm

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Be Careful!

- Things to worry about?

Complexity

- What is the space complexity of depth-bounded Minimax?

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  - Board size s
  - Depth d
  - Possible moves m

Complexity

- What is the space complexity of depth-bounded Minimax?
  - Board size s
  - Depth d
  - Possible moves m
- O(ds+m)
- Board positions can be released as bubble up

Minimax Algorithm

- Did I just do your project for you?

Minimax Algorithm

- Did I just do your project for you?
- No!

Minimax Algorithm

- Did I just do your project for you?
- No!
- You need to create:
  - Evaluation function
  - Move generator
  - did_i_win? function

Isolation Clarification

- Standalone game clients
  - Opponent moves entered manually
  - Output your move on stdout
- Assignment is out of 100
- Tournament is single elimination
  - But there will be food!

Recap

- What is a zero sum game?

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  - Will this work for all games?

Recap

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- What is its space complexity?
- How can the Minimax algorithm be simplified?
  - Will this work for all games?

Next Up

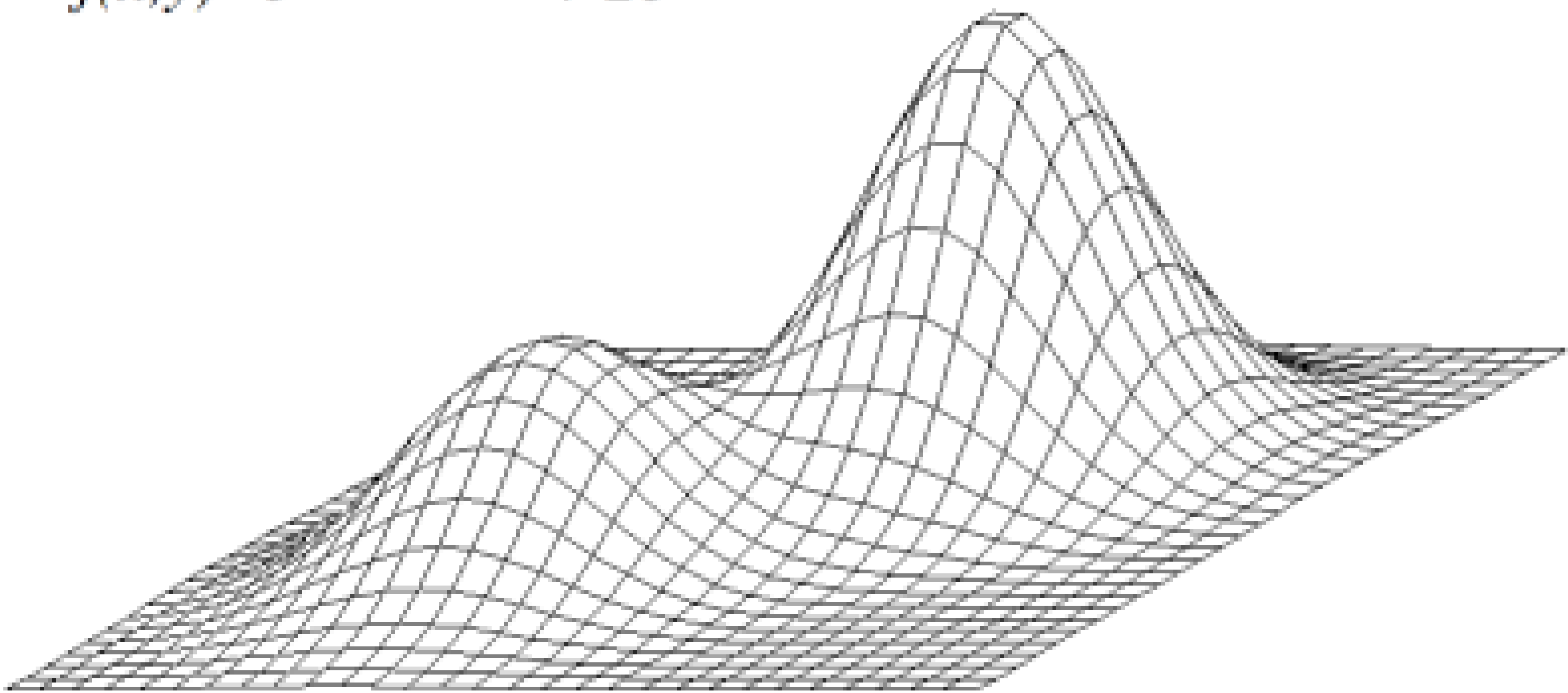
- Recall that minimax will produce optimal play against an optimal opponent if entire tree is searched
- Is the same true if a cutoff is used?

Horizon Effect

- Your algorithm searches to depth n
- What happens if:
  - Evaluation(s) at depth n is very positive
  - Evaluation(s) at depth n+1 is very negative
- Or:
  - Evaluation(s) at depth n is very negative
  - Evaluation(s) at depth n+1 is very positive
- Will this ever happen in practice?

Local Maxima Problem

$$f(x,y)=e^{-(x^2+y^2)} + 2e^{-((x-1.7)^2+(y-1.7)^2)}$$



Search Limitation Mitigation

- Sometimes it is useful to look deeper into game tree
- We could peak past the horizon…
- But how can you decide what nodes to explore?
  - Quiescence search

Quiescence Search

- Human players have some intuition about move quality
  - “Interesting vs “boring”
  - “Promising” vs “dead end”
  - “Noisy” vs “quiet”
- Expand horizon for potential high impact moves
- Quiescence search adds this to Minimax

Quiescence Search

- Additional search performed on leaf nodes
- if looks_interesting(leaf_node):

  extend_search_depth(leaf_node)

  else:

  normal_evaluation(leaf_node)

Quiescence Search

- What constitutes an “interesting” state?
  - Moves that substantially alter game state
  - Moves that cause large fluctuations in evaluation function output
- Chess example: capture moves
- Must be careful to prevent indefinite extension of search depth
  - Chess: checks vs captures

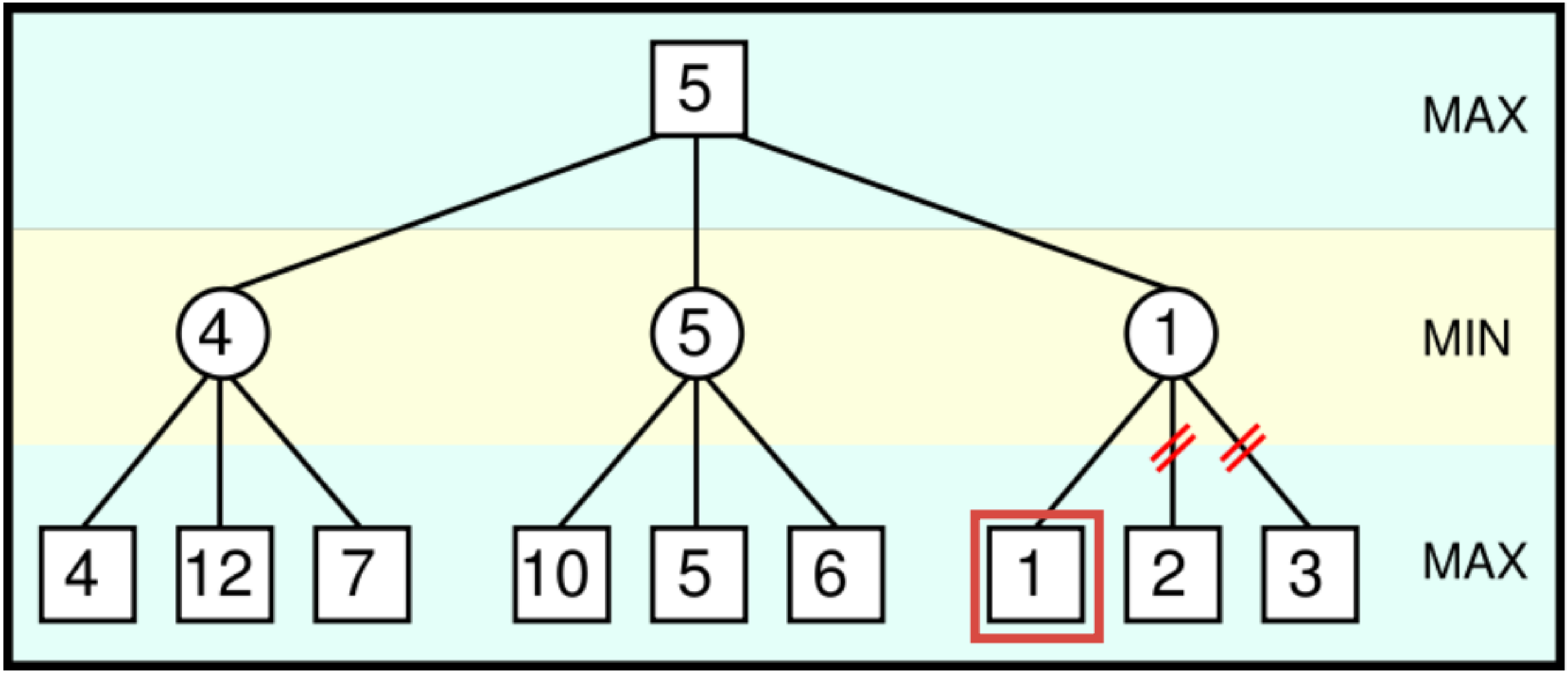
Search Limitation Mitigation

- Do you always need to search the entire tree?
  - No!
- Sometimes it is useful to look *less deeply* into tree
- But how can you decide what branches to ignore?
  - Tree pruning

Tree Pruning

- Moves chosen under assumption of optimal adversary
- You know the best move so far
- If you find a branch with a worse move, is there any point in looking further?
- Thought experiment: bag game

Pruning Example



Alpha-Beta Pruning

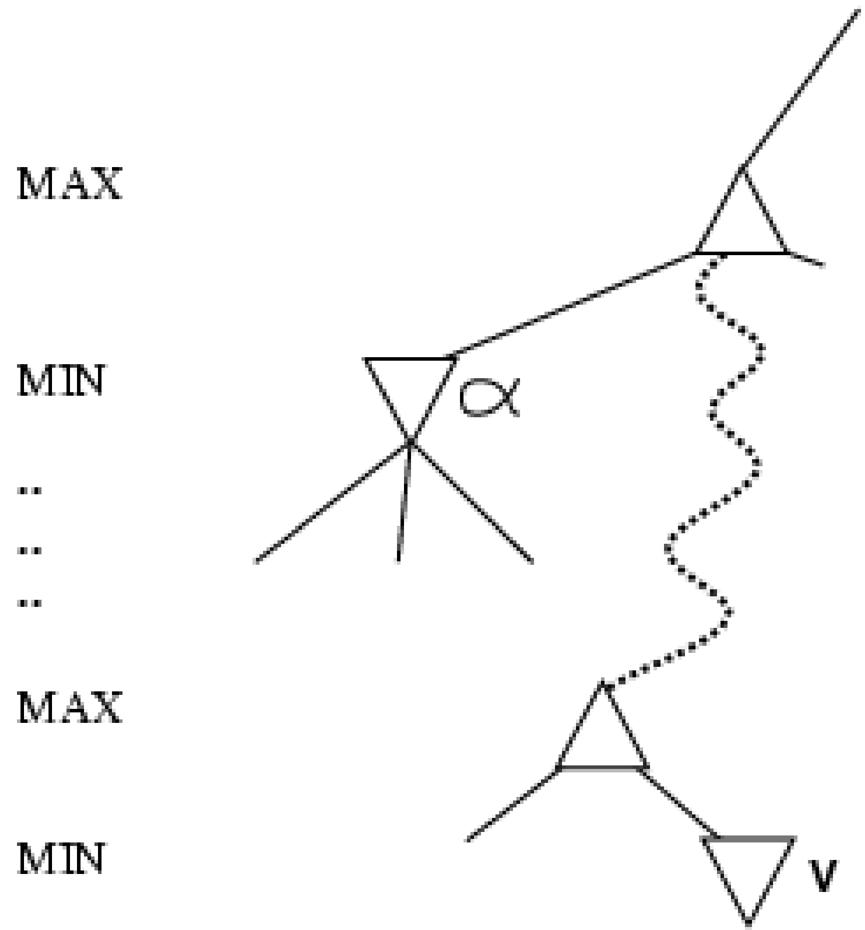
- During Minimax, keep track of two additional values
- Alpha
  - Your best score via any path
- Beta
  - Opponent’s best score via any path

Alpha-Beta Pruning

- Max player (you) will never make a move that could lead to a worse score for you
- Min player (opponent) will never make a move that could lead to a better score for you
- Stop evaluating a branch whenever:
  - A value greater than beta is found
  - A value less than alpha is found

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid it
  → prune that branch
- Define β similarly for *min*

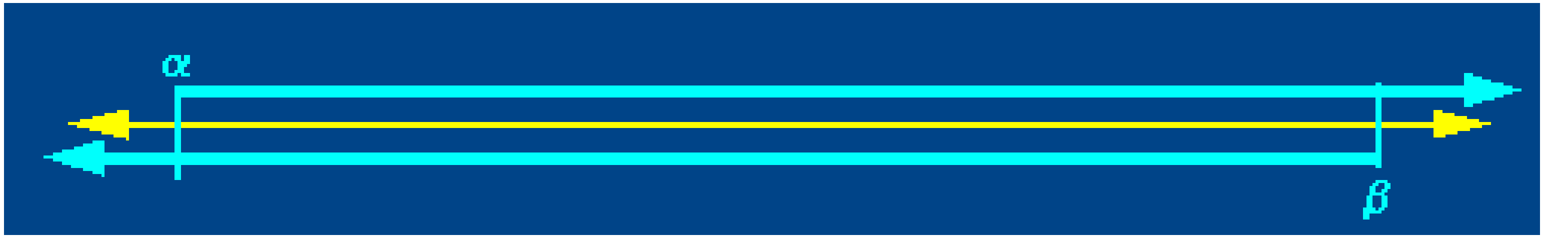


Alpha-Beta Pruning

- Based on observation that for all viable paths utility value n will be **α <= n <= β**

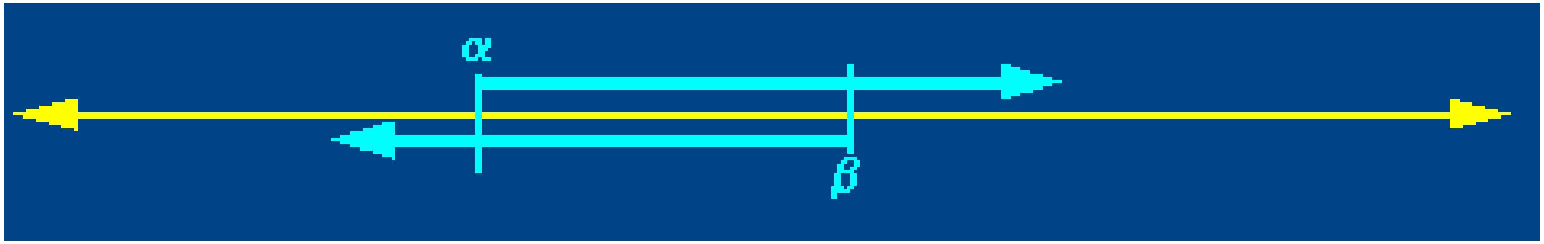
Alpha-Beta Pruning

- Initially, α = -infinity, β=infinity



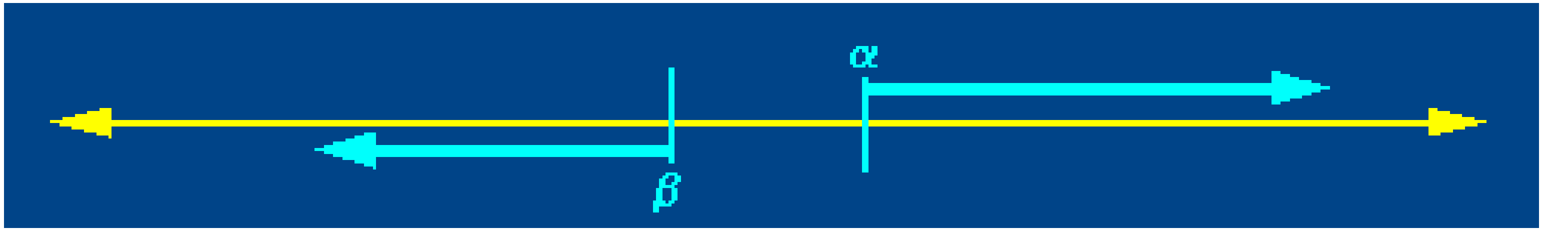
Alpha-Beta Pruning

- As the search tree is traversed, the possible utility value window shrinks as
  - Alpha increases
  - Beta decreases



Alpha-Beta Pruning

- Once there is no longer any overlap in the possible ranges of alpha and beta, it is safe to conclude that the current node is a dead end



Minimax algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*

    $v \leftarrow$ MAX-VALUE(*state*)
    return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*) **returns** *a utility value*

    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    $v \leftarrow -\infty$
    for a, s in SUCCESSORS(*state*) **do**
        $v \leftarrow$ MAX(v, MIN-VALUE(s))
    return v

---

function MIN-VALUE(*state*) **returns** *a utility value*

    if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
    $v \leftarrow \infty$
    for a, s in SUCCESSORS(*state*) **do**
        $v \leftarrow$ MIN(v, MAX-VALUE(s))
    return v

The α-β algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

The α-β algorithm

function MIN-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
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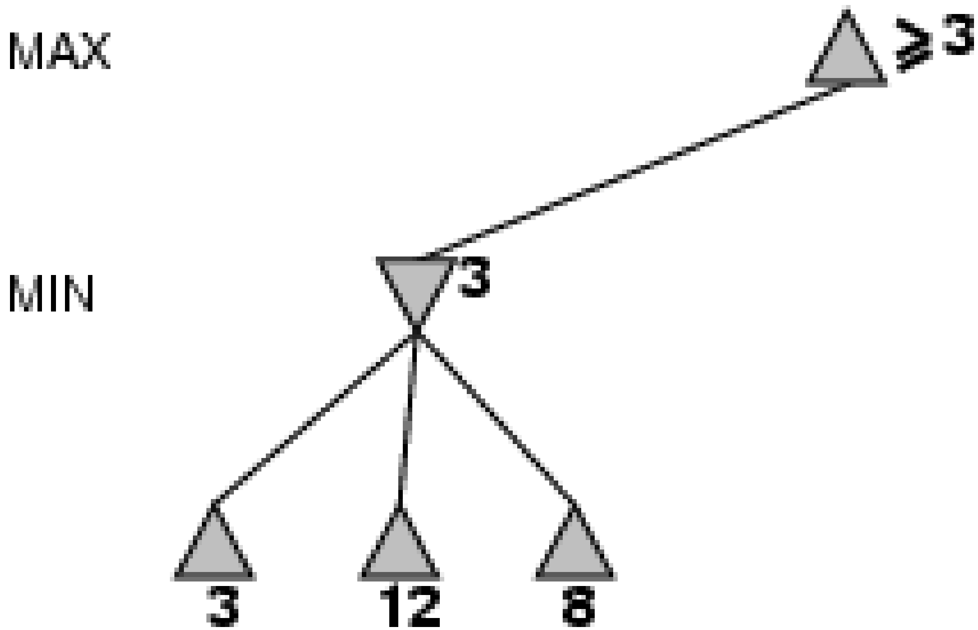
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s, α, β))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

α-β pruning example



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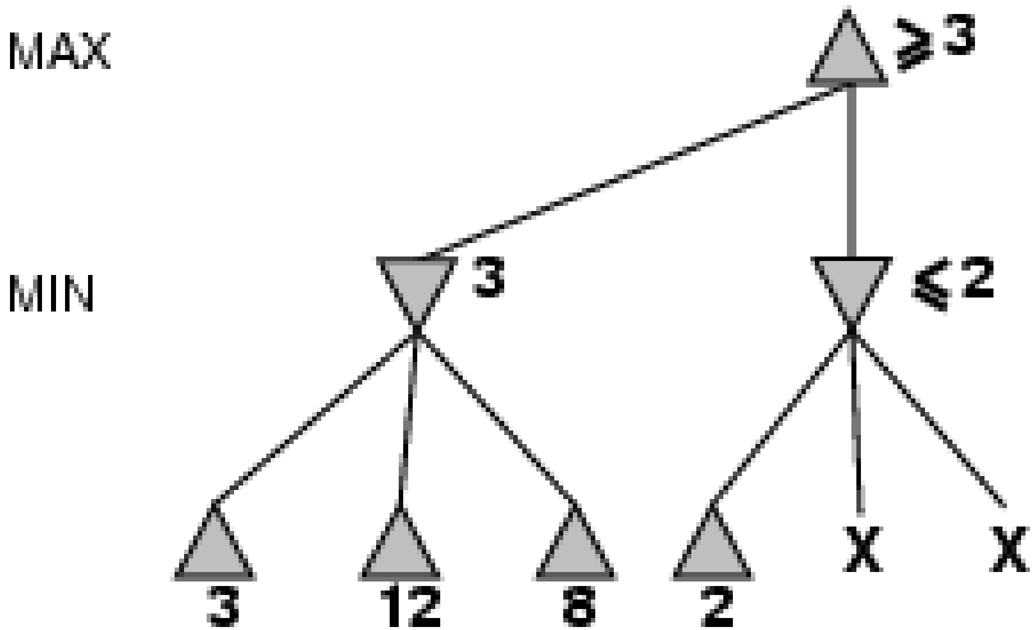

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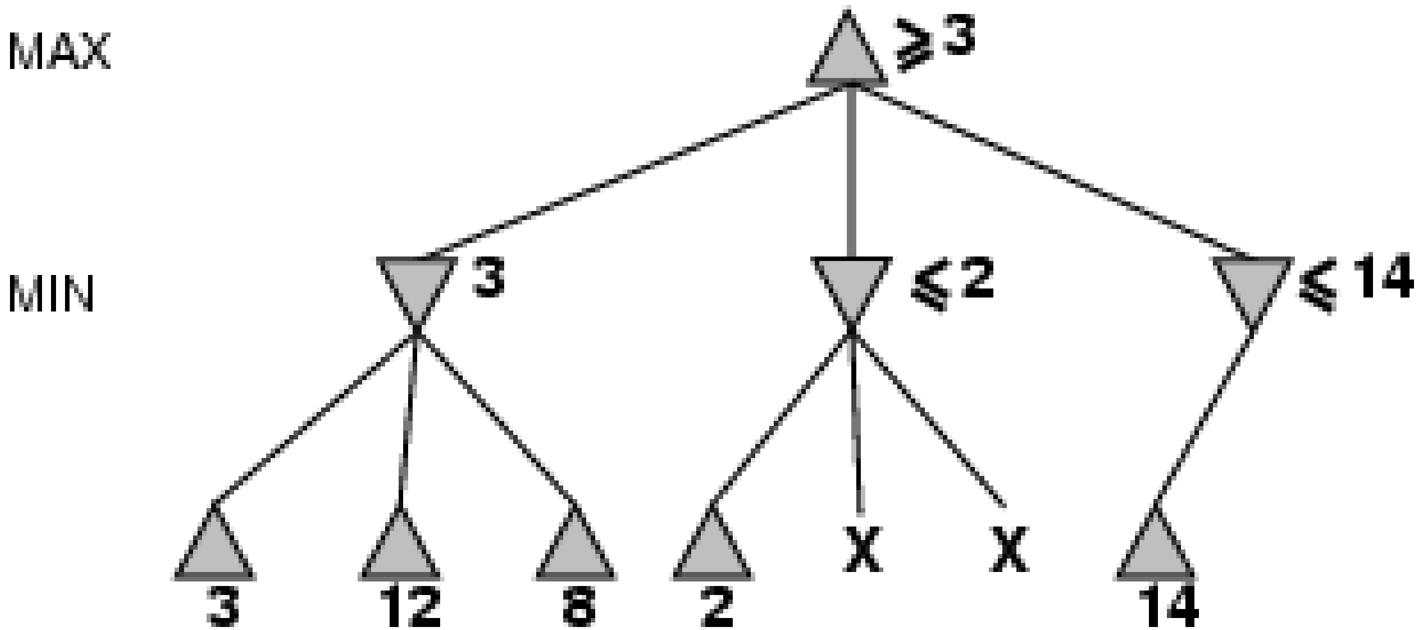

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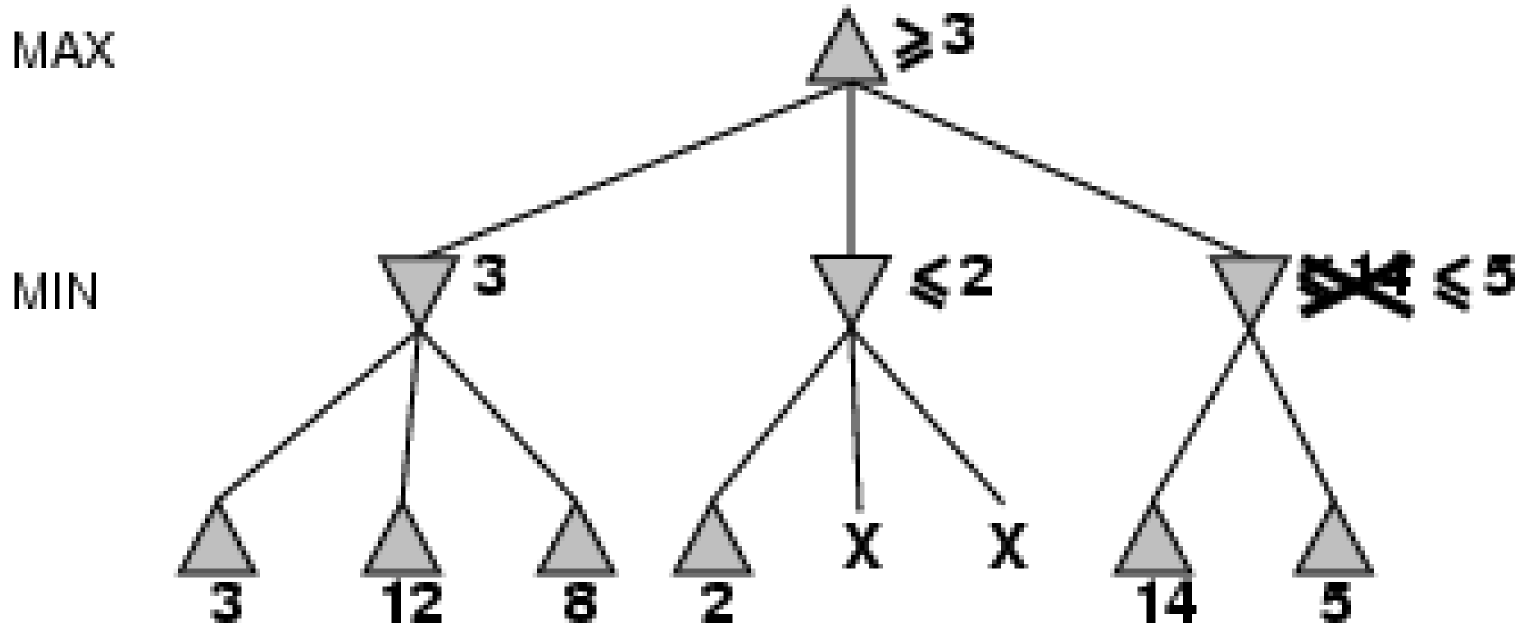

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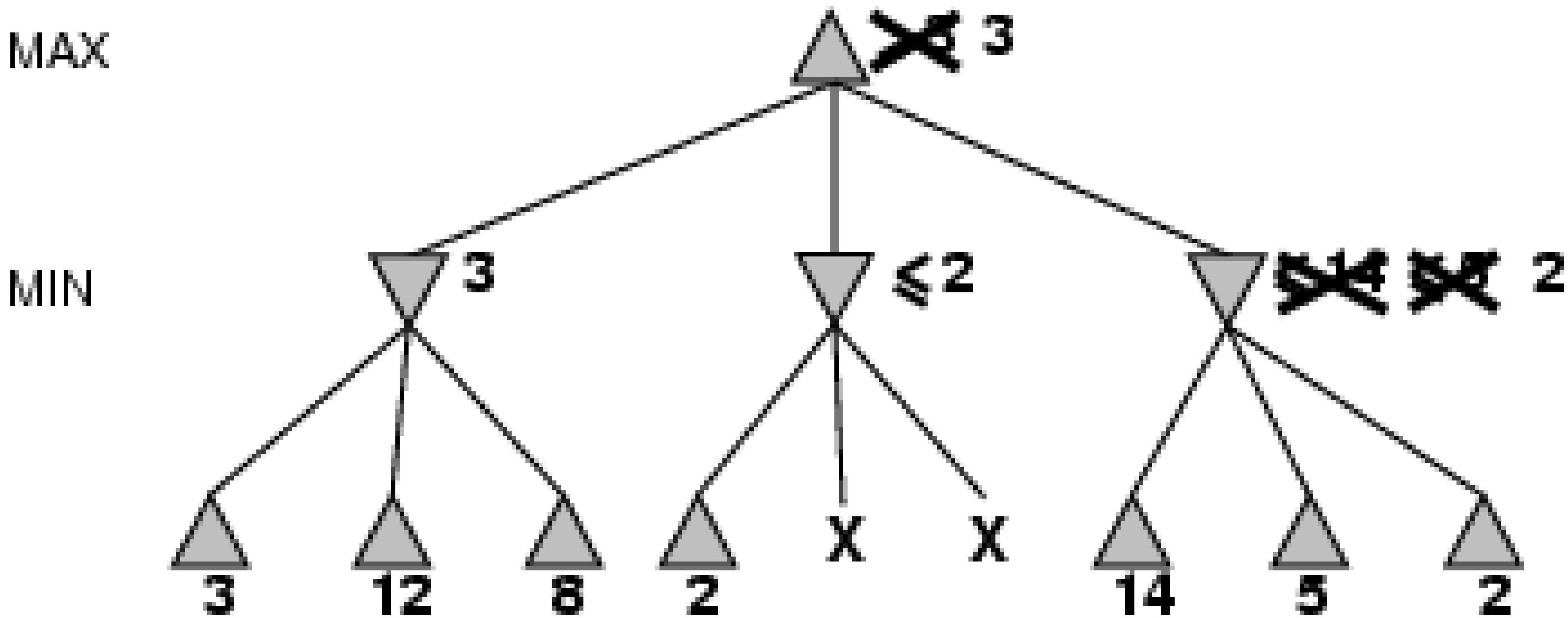
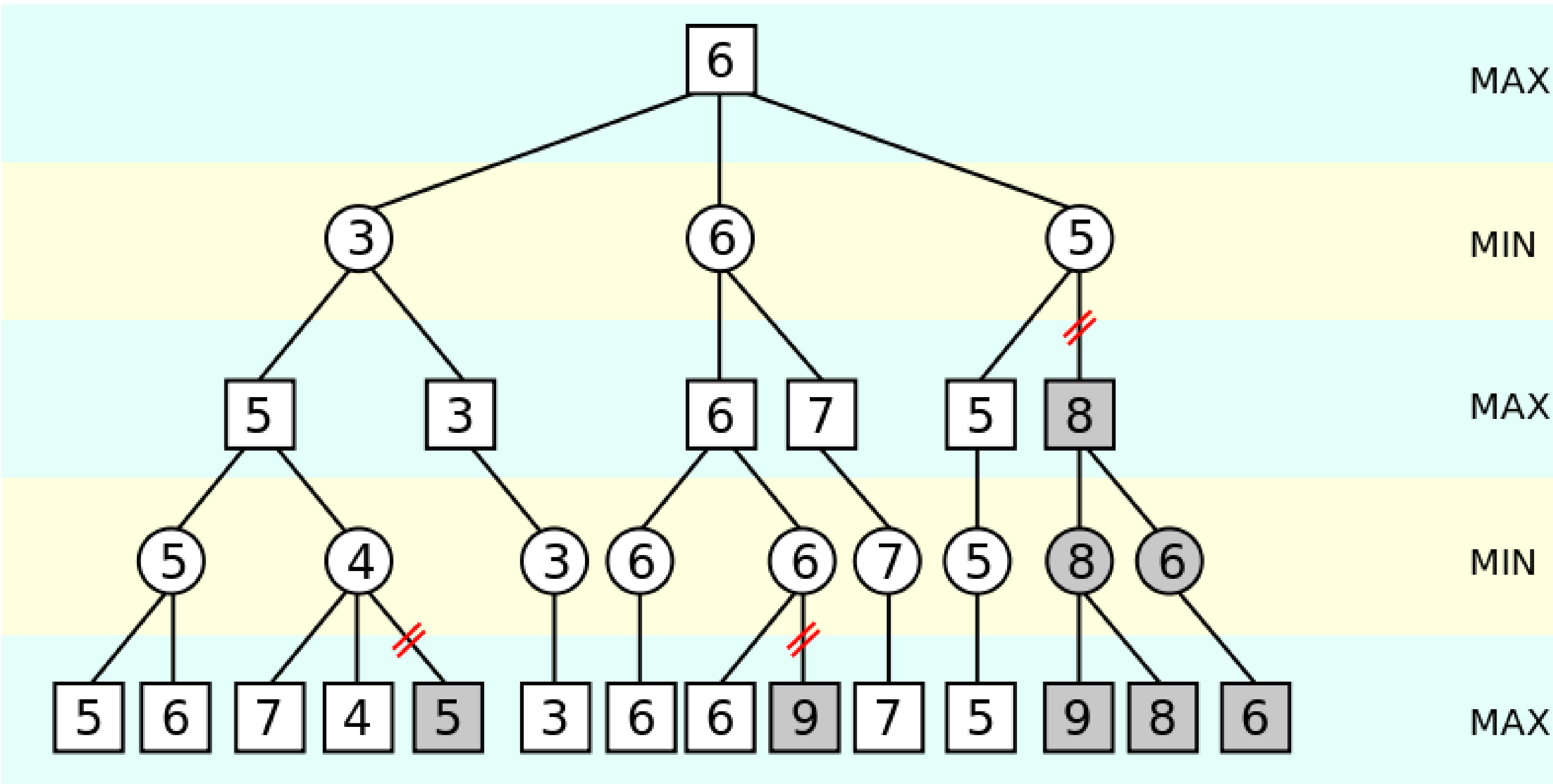


Another α-β Pruning Example



Minimax Psuedocode

```
BESTMOVE = proc (posn, depth)
  begin  (movelist, bestscore, bestm, try, tryscore) %local variables

         % posn: is the current BOARD CONFIGURATION FROM WHICH A MOVE
         %       MUST BE CHOSEN BY OUR INTELLIGENT AGENT COMPUTER

         % depth: MAXIMUM NUMBER OF PLIES TO LOOK AHEAD. THIS IS DETERMINED
         %       BY SPEED CONSTRAINTS AND COMPLEXITY OF THE GAME
         %       (i.e. branching factor b)

         % movelist: the list of all possible MOVES from the current posn
         % bestscore: the best "backed up" score found so far as we iterate
         %            thru the movelist
         % bestm:     the best move found so far that results in the bestscore
         % try:       just a tmp to hold returned values from recursive calls
         % tryscore:  just a tmp to hold returned scores from recursive calls
```

Alpha-Beta Psuedocode

```
BESTMOVE = proc (Posn, Depth, Mybest, Herbest)
         %Note we added two additional parameters. Mybest should be
         %initialized to -infinity, while herbest is initialized to
         %+infinity when calling this version of BESTMOVE.

  begin  (Movelist, Bestscore, Bestm, Try, Tryscore) %local variables
```

Minimax Psuedocode

```
if depth = 0 then return list(EVALUATE(posn), nil) fi;
 %This ends the recursion at the bottom of the search space
 %Note a two element list of values is returned.


Movelist = POSSIBLE-MOVES(posn);
 %According to the rules of the game, we generate all possible
 %moves from the given board position.


Bestscore = -infinity;  %initializations
Bestm = nil;
```

Alpha-Beta Psuedocode

```
if Depth = 0 then return list(EVALUATE(Posn), nil) fi;
 %This ends the recursion at the bottom of the search space
 %Note a two element list of values is returned.

Movelist = POSSIBLE-MOVES(Posn);
 %According to the rules of the game, we generate all possible
 %moves from the given board position.

Bestscore = Mybest  % note the change to initializations here
Bestm = nil;
```

Minimax Psuedocode

```
%We are now ready to scan the movelist and select the
%best move. Note how we initialized Bestscore and Bestm.

while ( Movelist <> nil )
 repeat,
   try = BESTMOVE( NEWPOSITION(posn, first(Movelist)), depth-1);
         %Here is the main recursive call that expands and searches
         %the state space from the selected move.

   tryscore = - first(try); %recall BESTMOVE returns two values

   %Now we determine how well this current move did and whether
   %it should be selected as our best move found so far.

   if tryscore > Bestscore then
    do,
     Bestscore = tryscore;
     Bestm = first(Movelist);
    od;

   %Now we continue scanning down the list of moves to see
   %if we can find a better move then found so far.

   Movelist = rest(Movelist);
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```

Alpha-Beta Psuedocode

```
%We are now ready to scan the movelist and select the
%best move. Note how we initialized Bestscore and Bestm.

while ( Movelist <> nil )
 repeat,
   Try =
    BESTMOVE( NEWPOSITION(posn, first(Movelist)),
              Depth-1,
              -Herbest,
              -Bestscore );
         %Here is the main recursive call that expands and searches
         %the state space from the selected move. But notice we
         %added the two additional parameters here. This makes
         %sense when we view the following conditional expressions.

   Tryscore = - first(Try); %recall BESTMOVE returns two values

   %Now we determine how well this current move did and whether
   %it should be selected as our best move found so far.

   if Tryscore > Bestscore then
    do,
     Bestscore = Tryscore;
     Bestm = first(Movelist);
    od;
```

Minimax Algorithm

```
     %After scanning the entire Movelist, we have our best move so:
    return( list(Bestscore, Bestm) );
nigeb; %End of procedure min-max.
```

Alpha-Beta Psuedocode

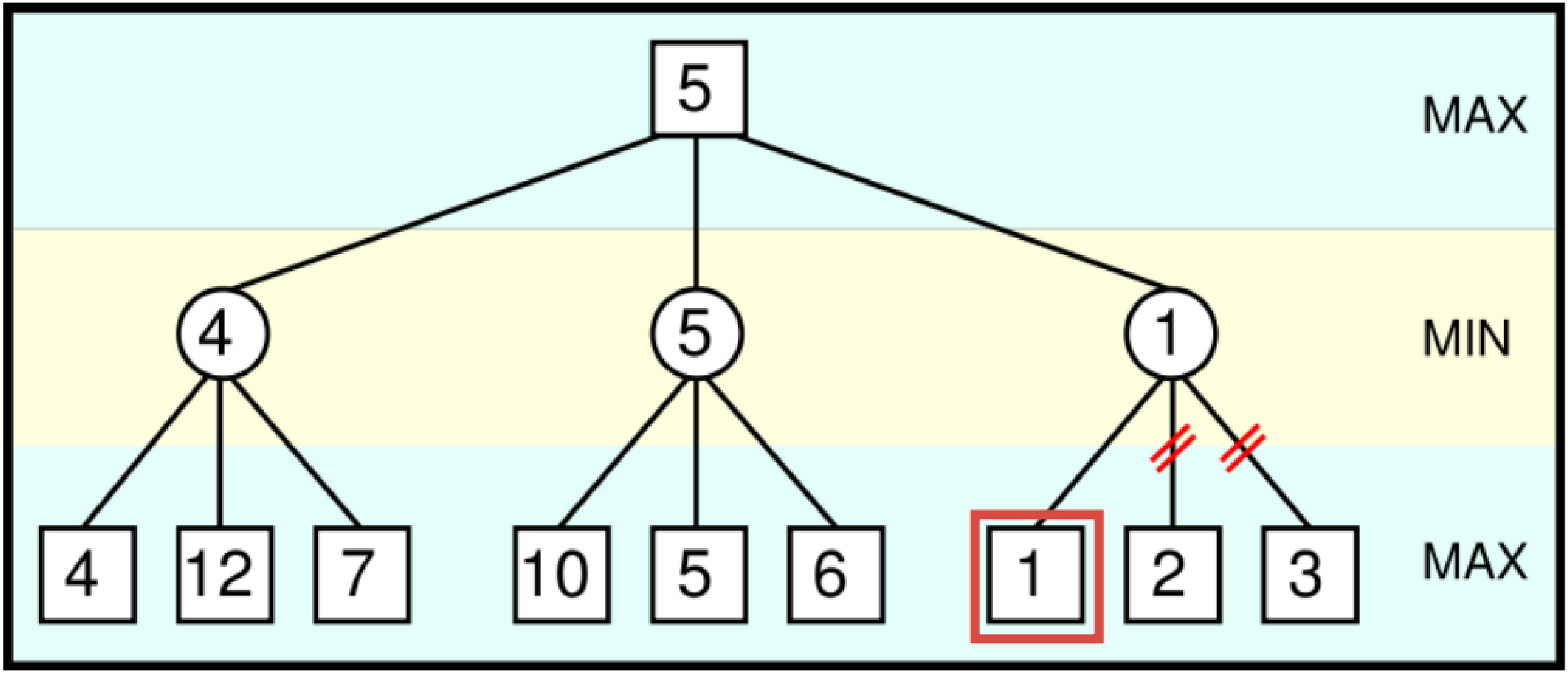
```
        if Bestscore > Herbest then return ( list(Bestscore, Bestm));

        %Now we continue scanning down the list of moves to see
        %if we can find a better move then found so far.

        Movelist = rest(Movelist);
      taeper;

     %After scanning the entire Movelist, we have our best move so:

    return( list(Bestscore, Bestm) );

nigeb; %End of procedure alpha-beta.
```

Tree Pruning vs Heuristics

- Search depth cut off may affect outcome of algorithm
- How about pruning?

Move Ordering

- Does the order in which moves are listed have any impact of alpha-beta?



Move Ordering

- Techniques for improving move ordering
- Apply evaluation function to nodes prior to expanding children
  – Search in descending order
  – But sacrifices search depth
- Cache results of previous algorithm

Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a pre-computed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

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- Othello: human champions refuse to compete against computers, who are too good.

- Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about