

Public Screen

Faye was babysitting her two year old brother Frank. When Faye wasn't watching, Frank had taken the aquarium and tipped it on its side, so the water level was as shown. When Faye found him, she quickly grabbed the tank and returned it to a horizontal position. What was the depth of the aquarium in cm if the dimensions of the tank are 100cm long, 60cm wide and 40cm high.

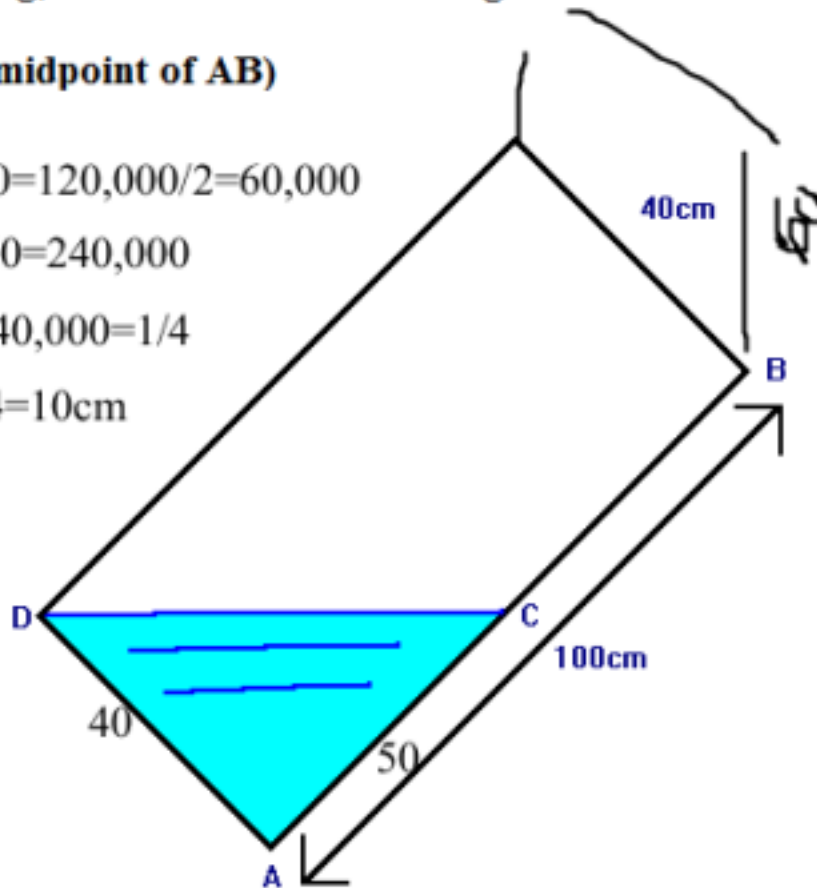
(C is the midpoint of AB)

$$50 \times 40 \times 60 = 120,000 / 2 = 60,000$$

$$100 \times 40 \times 60 = 240,000$$

$$60,000 / 240,000 = 1/4$$

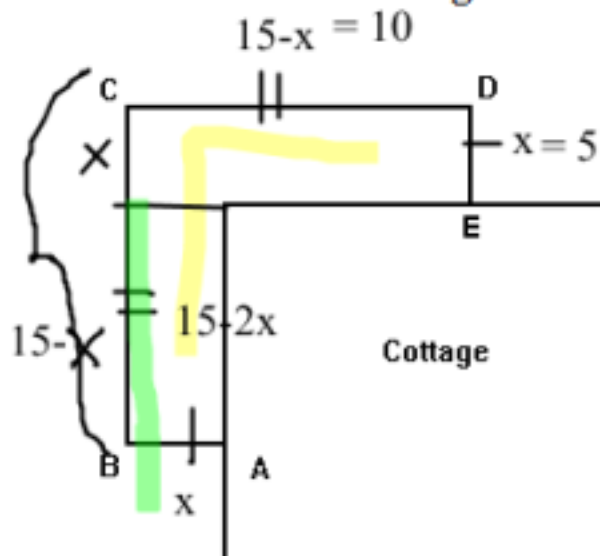
$$40 \text{ cm} \times 1/4 = 10 \text{ cm}$$



Public Screen 2



The Bunbury's want to build a deck on their cottage. The architect drew them a diagram which shows it built on the corner of the cottage. A railing is to be constructed around the four outer edges of the deck. If $AB = DE$, $BC = CD$ and the length of the railing is 30 metres, then what dimensions will give maximum area?



$$AB = DE = x = 10$$

$$x(x-10) = 0$$

$$x = 0$$

$$BC = CD = (30-2x)$$

$$x = 10$$

$$15-x = 5$$

$$x(15-x)$$

$$f(x) = x(30-2x)$$

$$= 30x - 2x^2$$

$$= 7.6$$

$$x = 7.6$$

$$7.4$$

$$x(15-x) + x(15-2x)$$

$$15x - x^2 + 15x - 2x^2$$

$$-3x^2 + 30x =$$

$$x^2 - 10x = 0$$

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Introduction

Patterns in Graphs

How do we organize the many types of functions into groups? One way is to use a "parameter." Parameters are constants that you can change. By using parameters, we can represent whole families of functions. For example, the family of parabolas with the vertex at the origin can be represented by $y = ax^2$.

In this lesson, we'll discuss graph shifting and families of functions. Then we'll look at how to represent those families using parameters. This will tie the concepts of symmetry and parameters into geometry and algebra. Symmetry is somewhat like "reflections" in geometry, while parameters are more like the "translations" and "scale factors."

Objectives

By the end of this lesson, you should be able to:

- Recognize a new function (algebraically and graphically) as an altered form of a familiar function
- Use the rules for shifting and distorting to quickly sketch the graph of one function from the graph of another
- Match a function with a parameter to a given family of functions
- Graph a family of functions when given a function that includes a parameter
- Write an equation with parameters to represent a given family of functions

Key Terms & Concepts

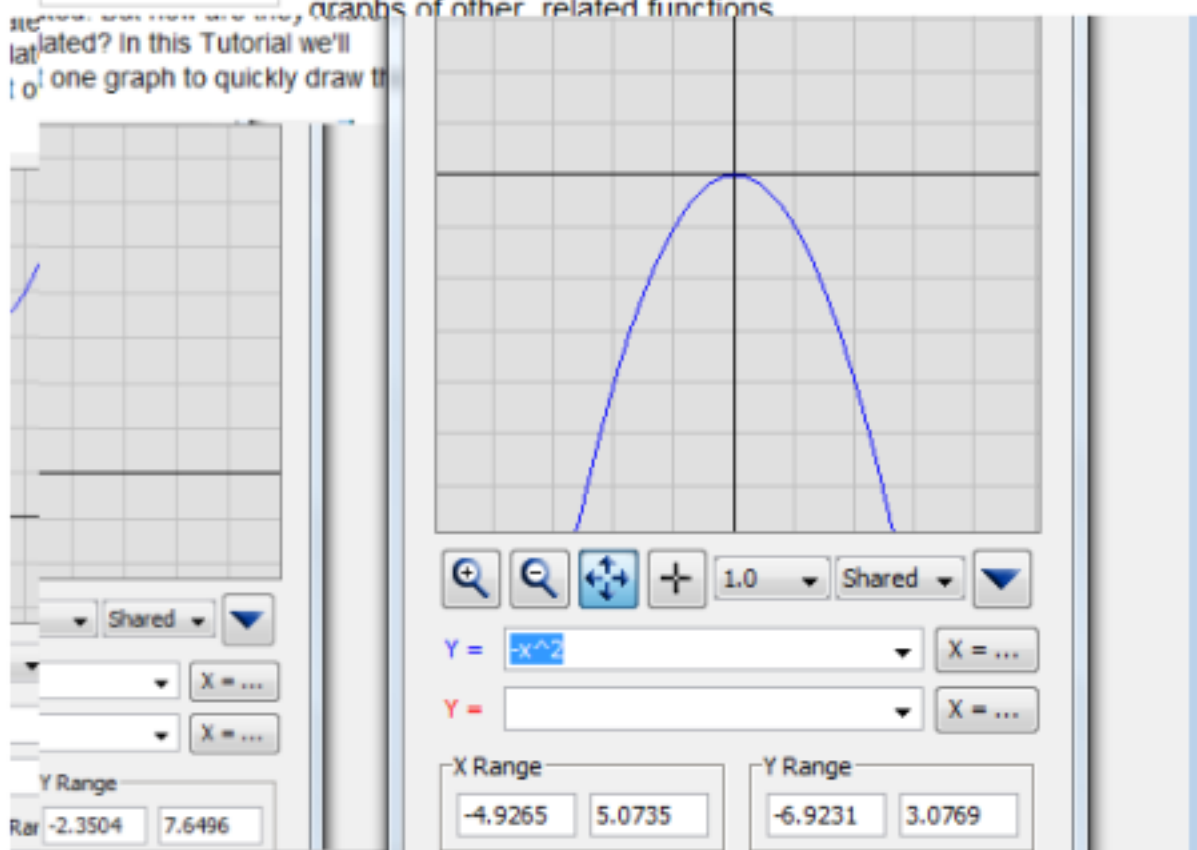
You'll need to know the definitions and significance of the terms and concepts covered in this lesson. To view the Key Terms and their definitions, click the Key Terms button.

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Introduction



You can tell just by looking at their equations that $y = x^2$, $y = 2x^2$, $y = -x^2$, $y = x^2 + 4$, and $y = (x - 1)^2$ are all related. But how are they related? In particular, how are their graphs related? In this Tutorial we'll explore how to use information about one graph to quickly draw the graphs of other related functions.



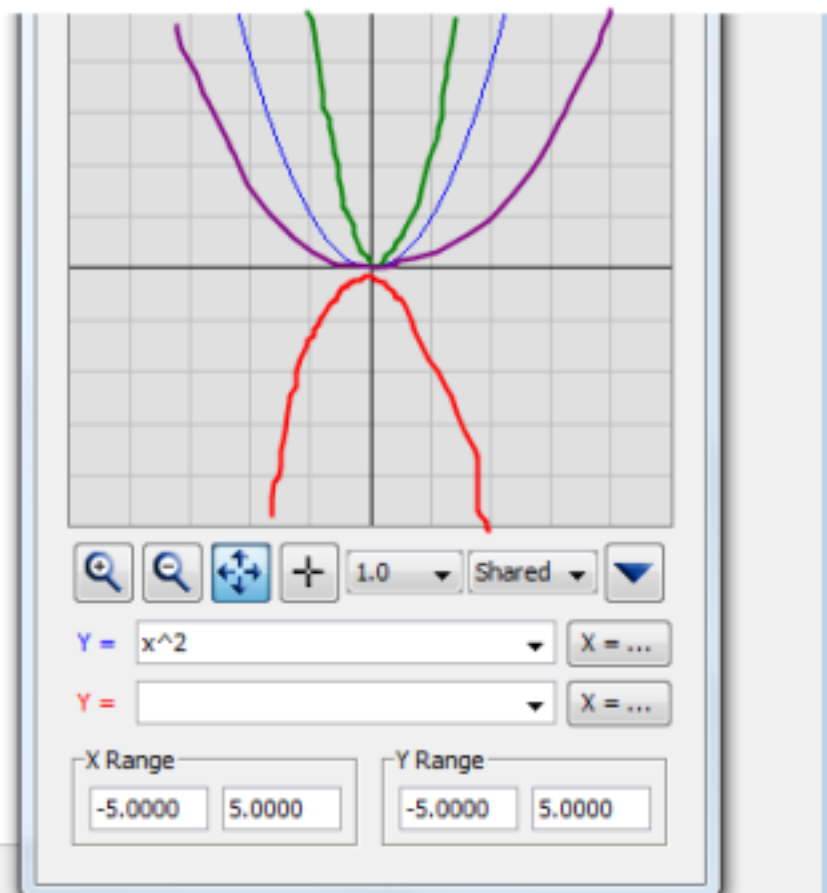
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$$y = x^2$$

↓
 $y = 2x^2$

↓
 $y = \frac{1}{2} x^2$

$$y = -x^2$$



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$$y = x^2$$

moves up or down
on y-axis

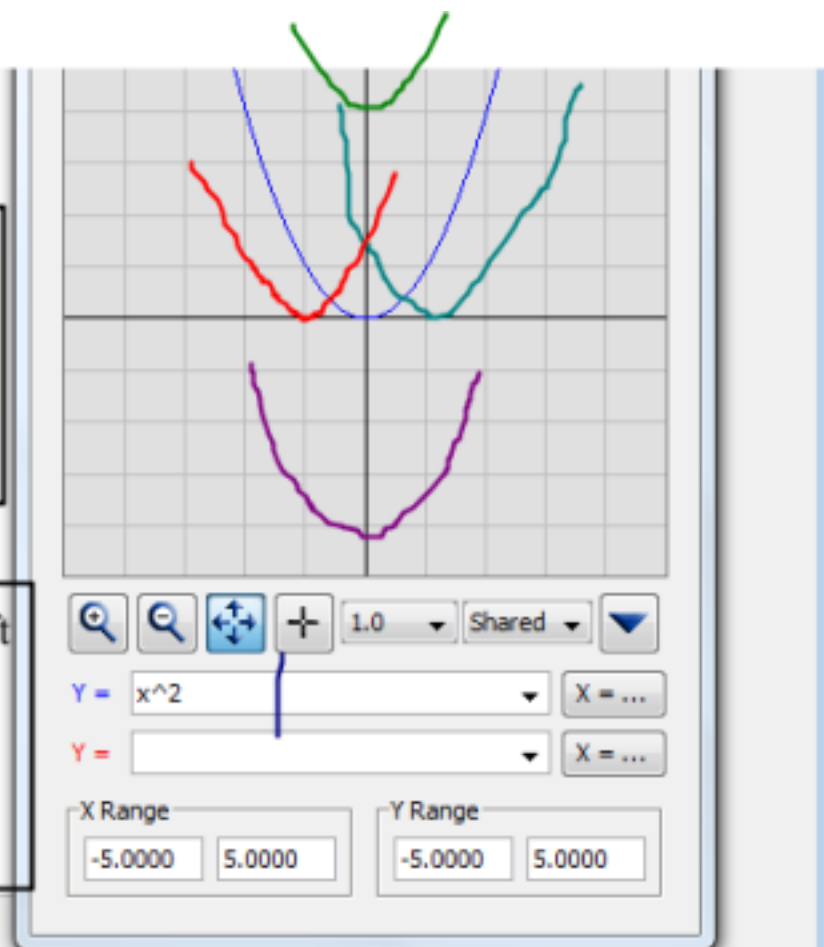
$$y = x^2 + 4$$

$$y = x^2 - 4$$

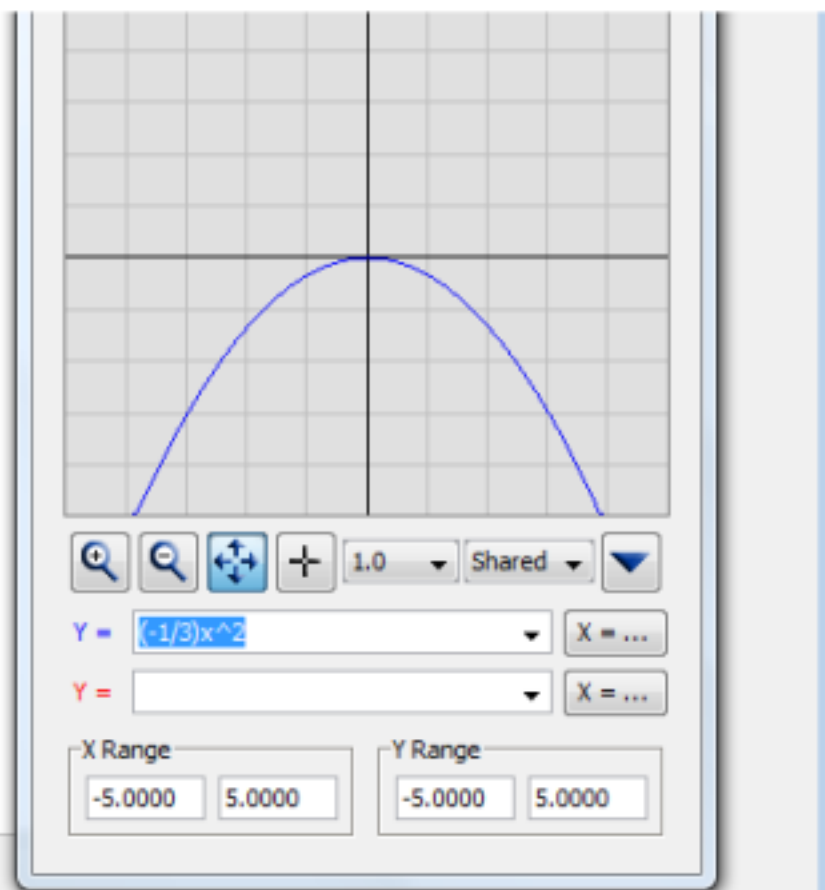
moves right or left
on x-axis

$$y = (x-1)^2$$

$$y = (x+1)^2$$

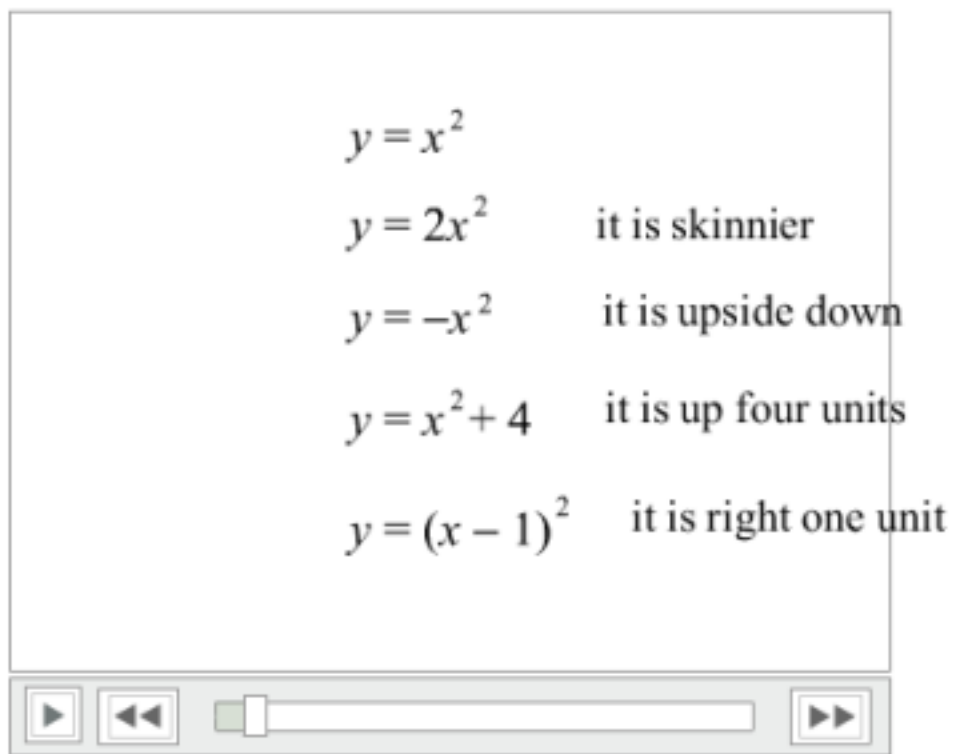


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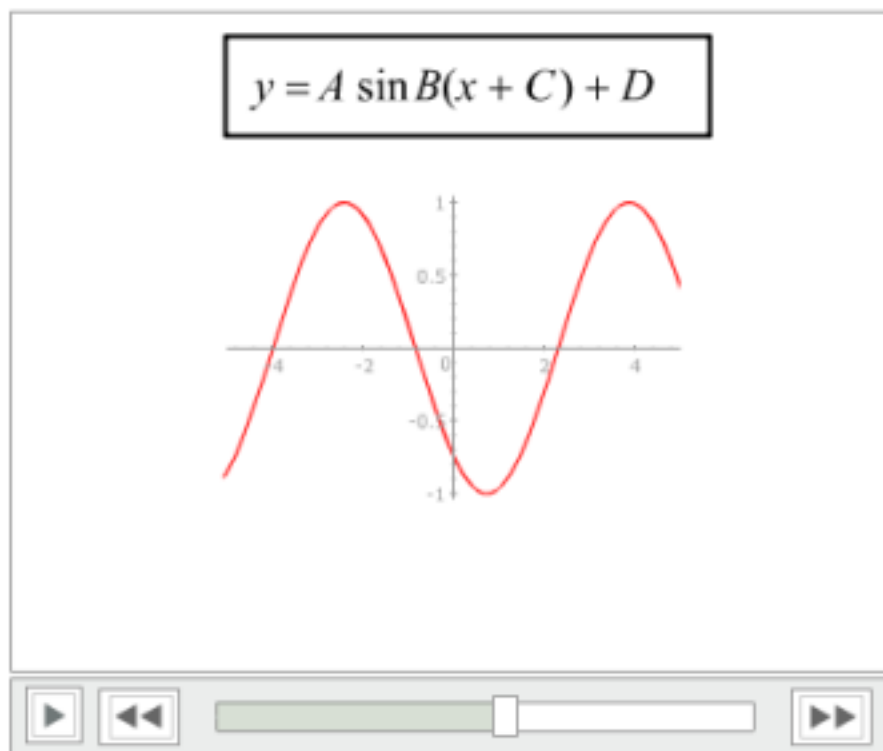
Families



$y = x^2$
 $y = 2x^2$ it is skinnier
 $y = -x^2$ it is upside down
 $y = x^2 + 4$ it is up four units
 $y = (x - 1)^2$ it is right one unit

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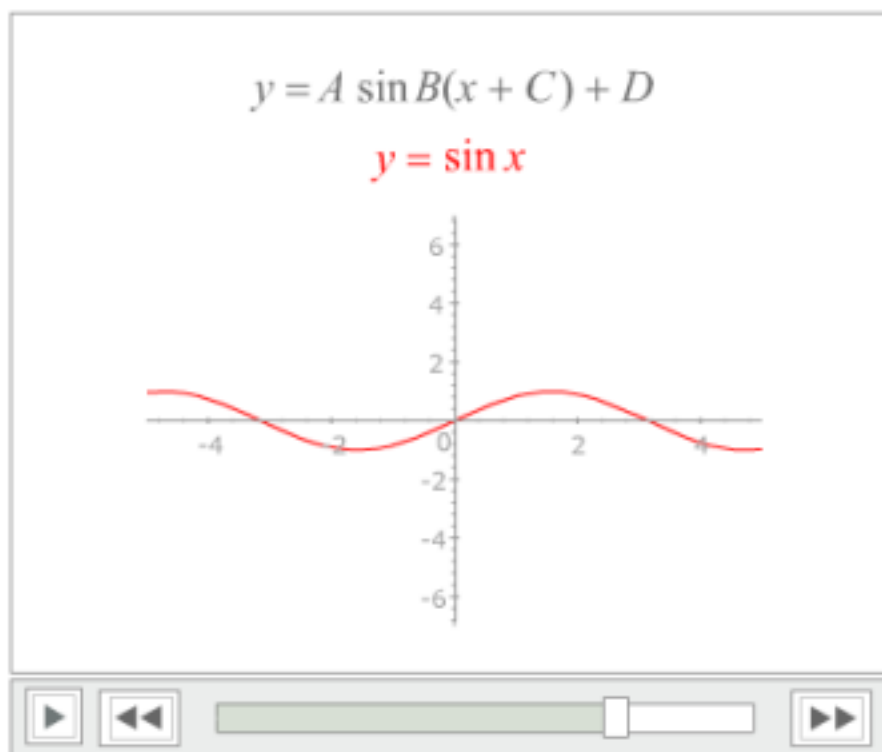
Families



A = amplitude (height of the graph) if it is negative the graph goes down first

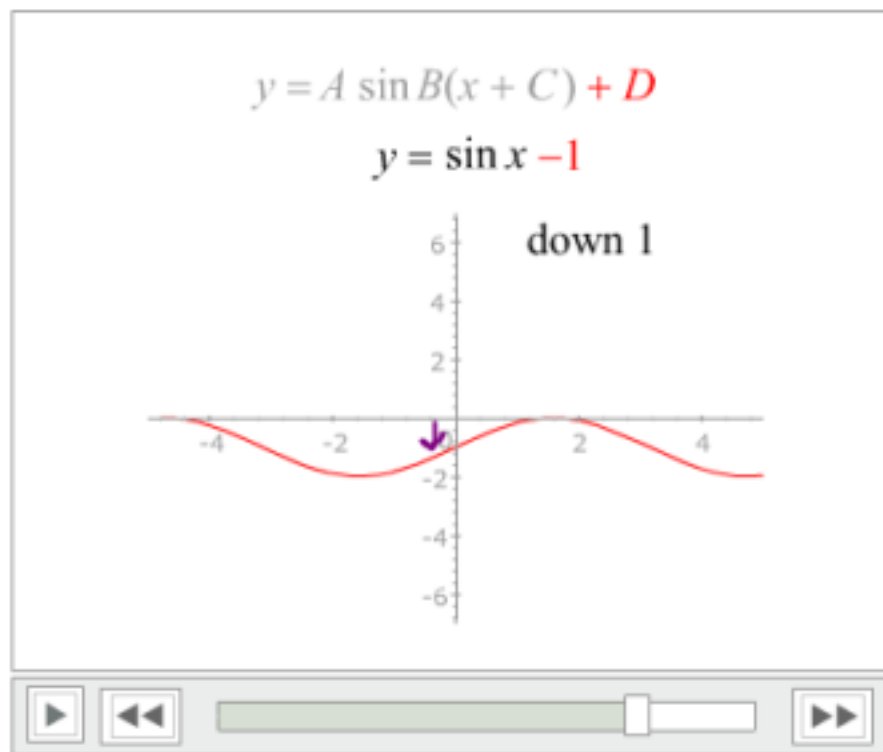
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Families



Public Screen 78

Families



D = moves it up or down on the y-axis

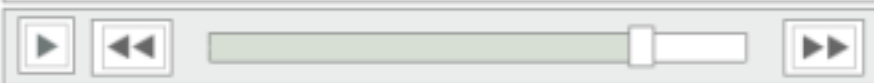
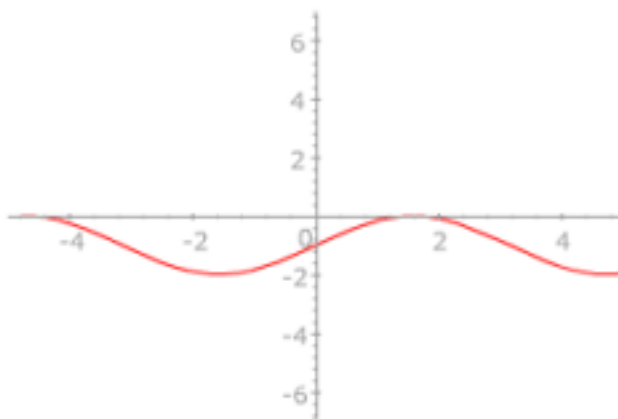
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Families

Vertical Shift

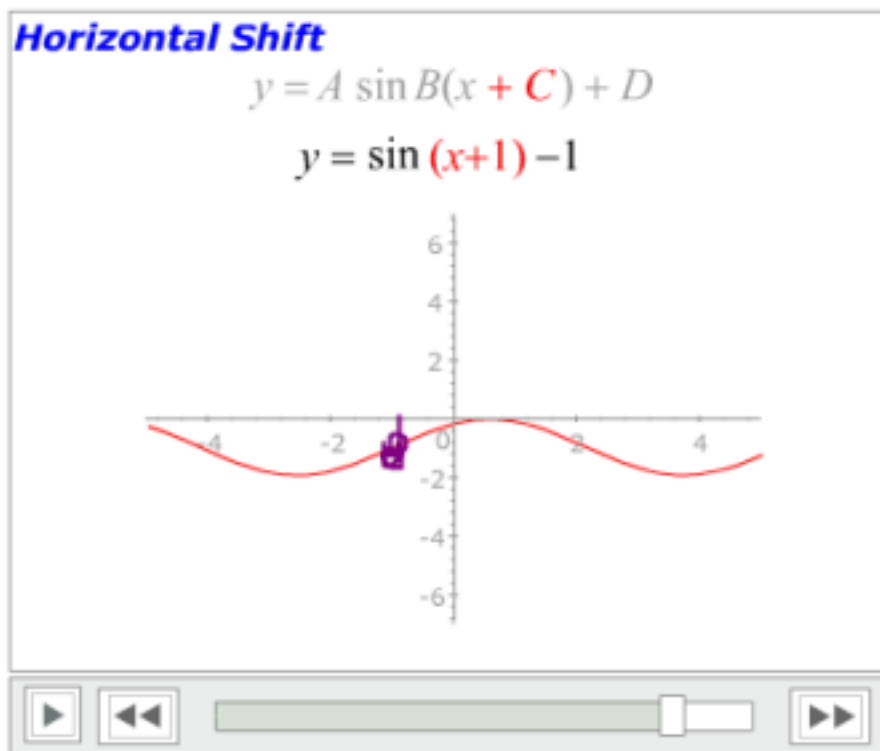
$$y = A \sin B(x + C) + D$$

$$y = \sin x - 1$$



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Families



C = movement on the x-axis, left or right (opposite of what you think)

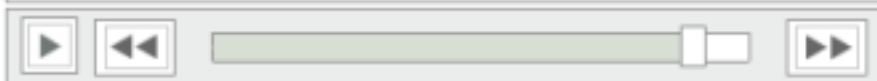
Public Screen 81

Families

Vertical Stretch

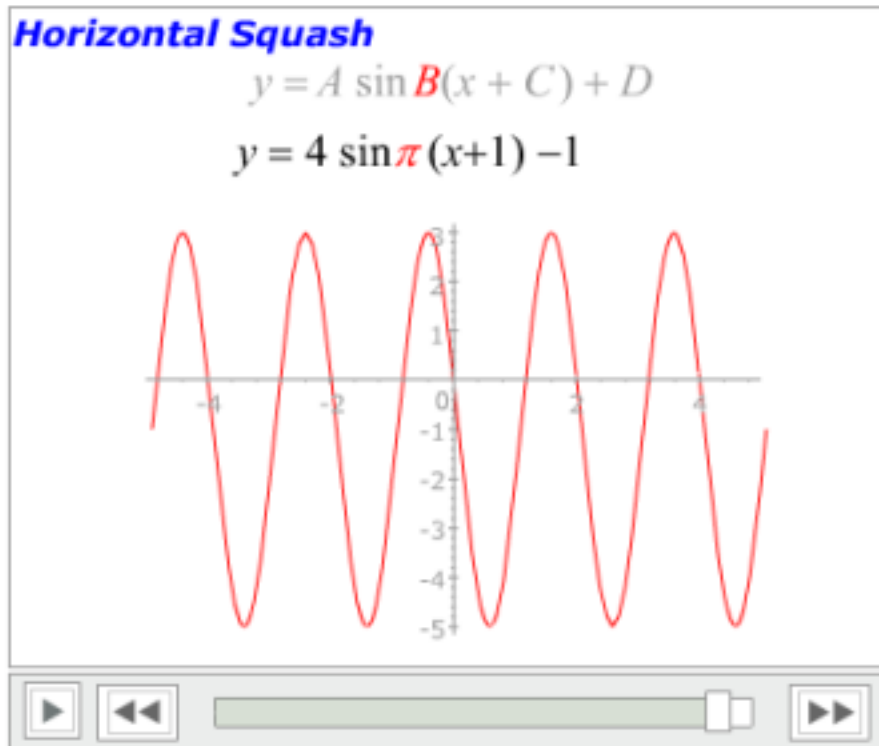
$$y = A \sin B(x + C) + D$$

$$y = 4 \sin (x+1) - 1$$



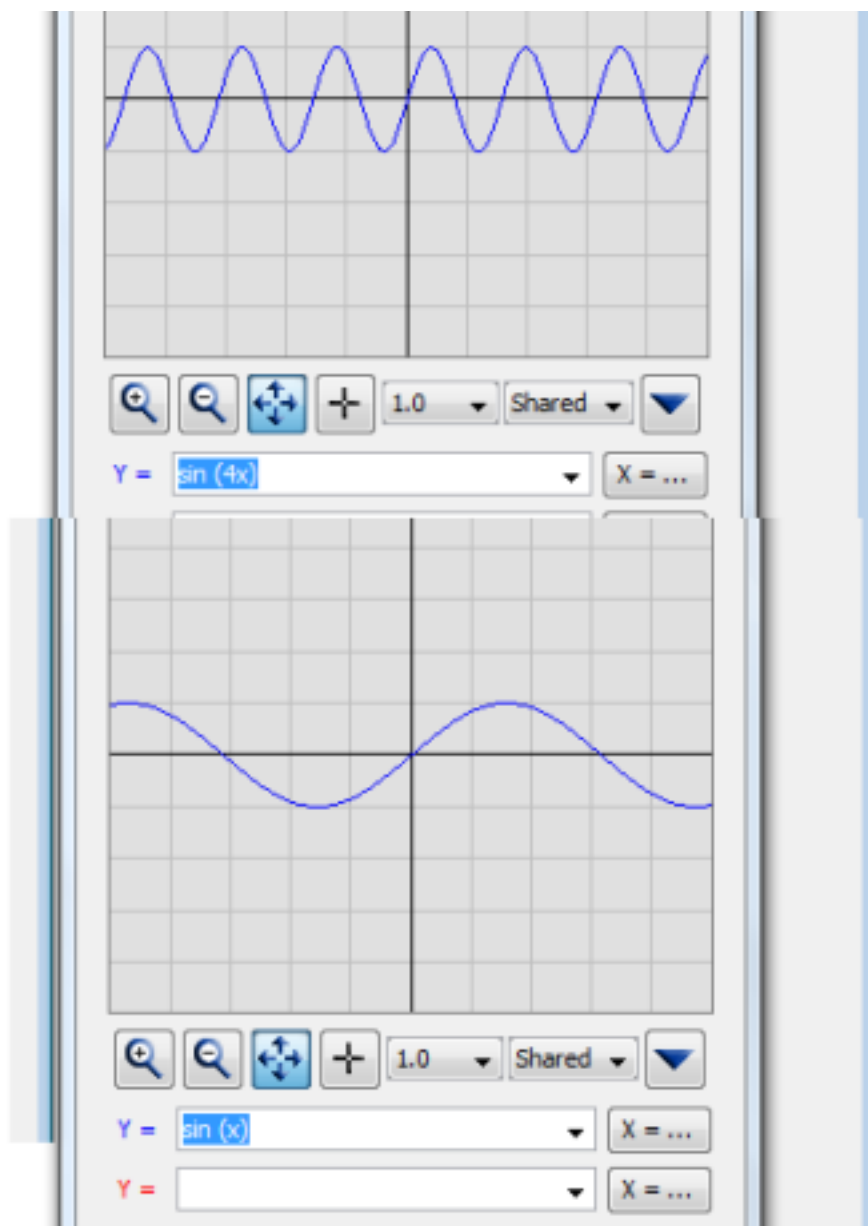
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Families



B = period, means how long it takes for one wave

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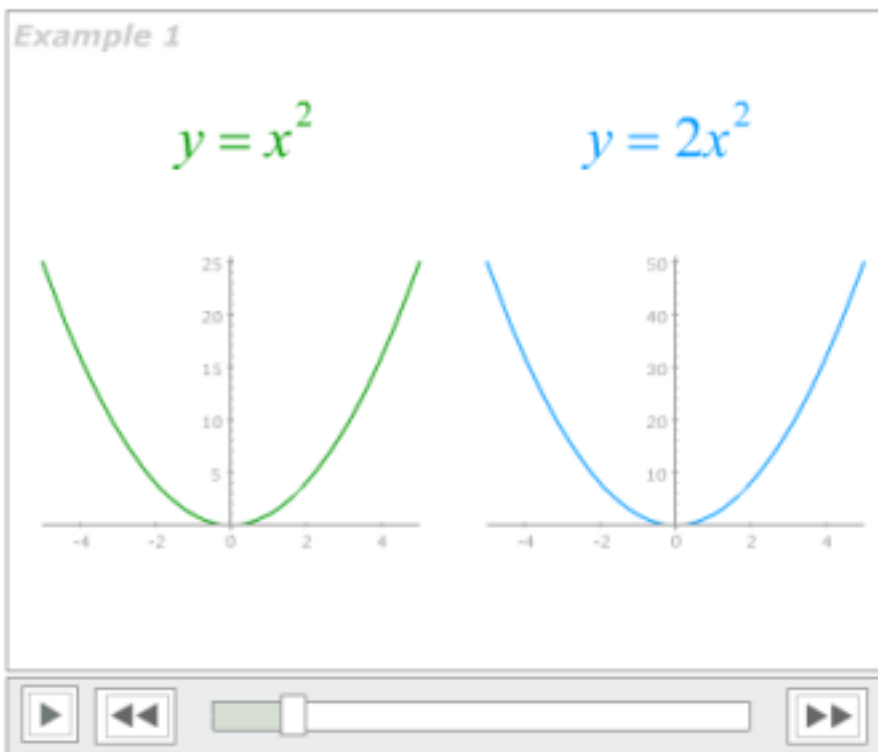
Rules

Rule: ***y*** ***Affects the Vertical***



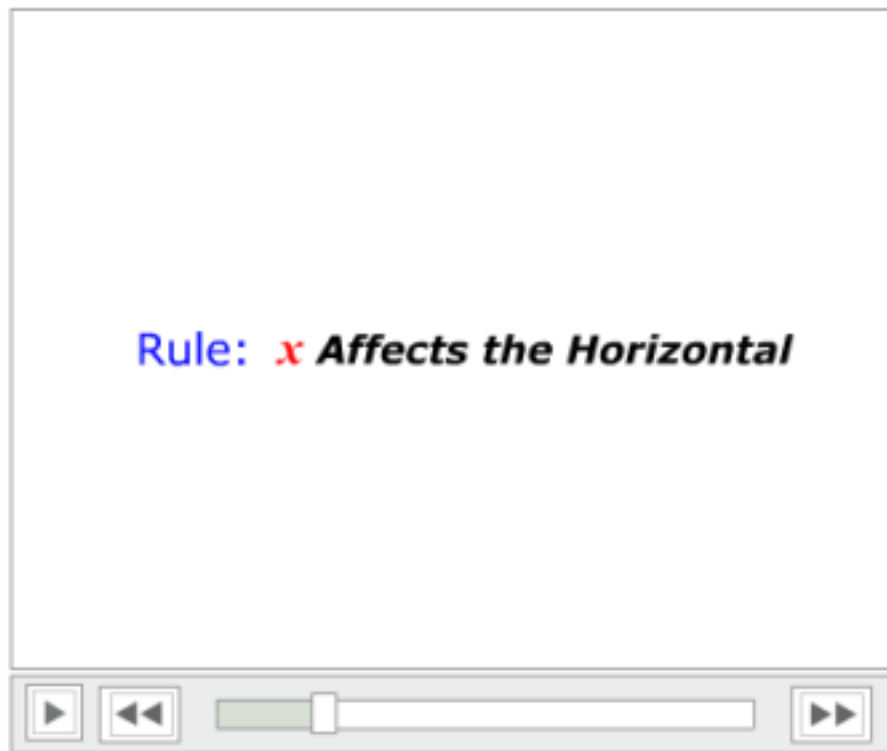
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Rules



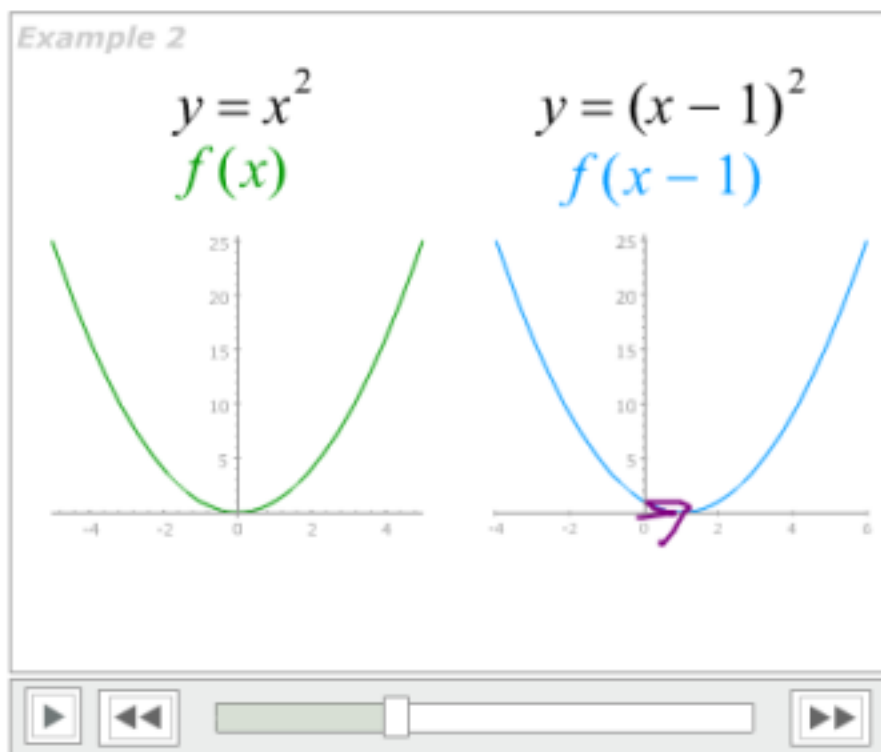
Public Screen 85

Rules



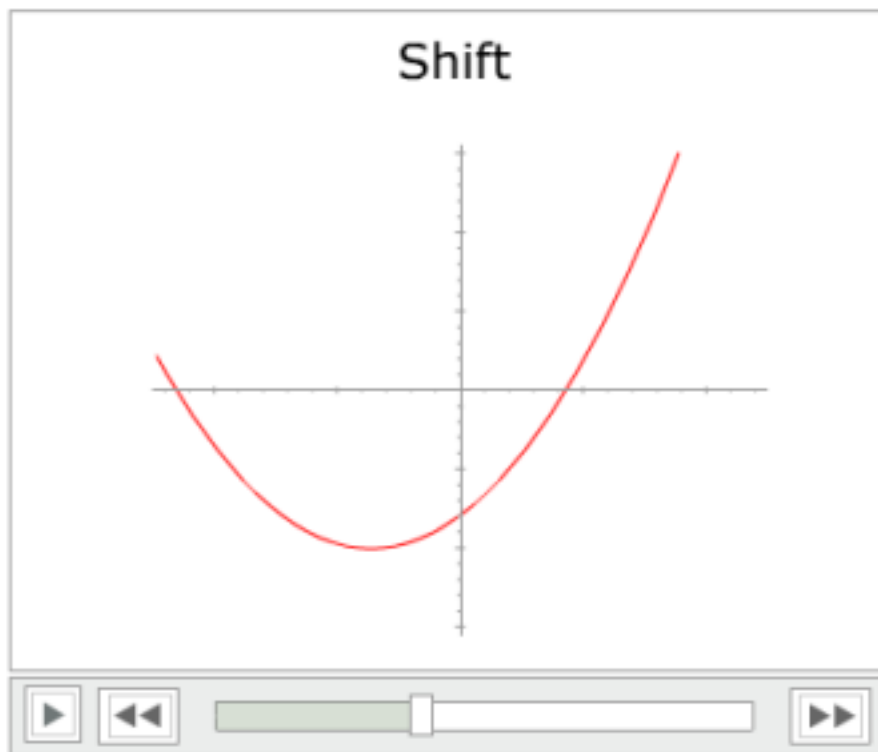
Public Screen 86

Rules



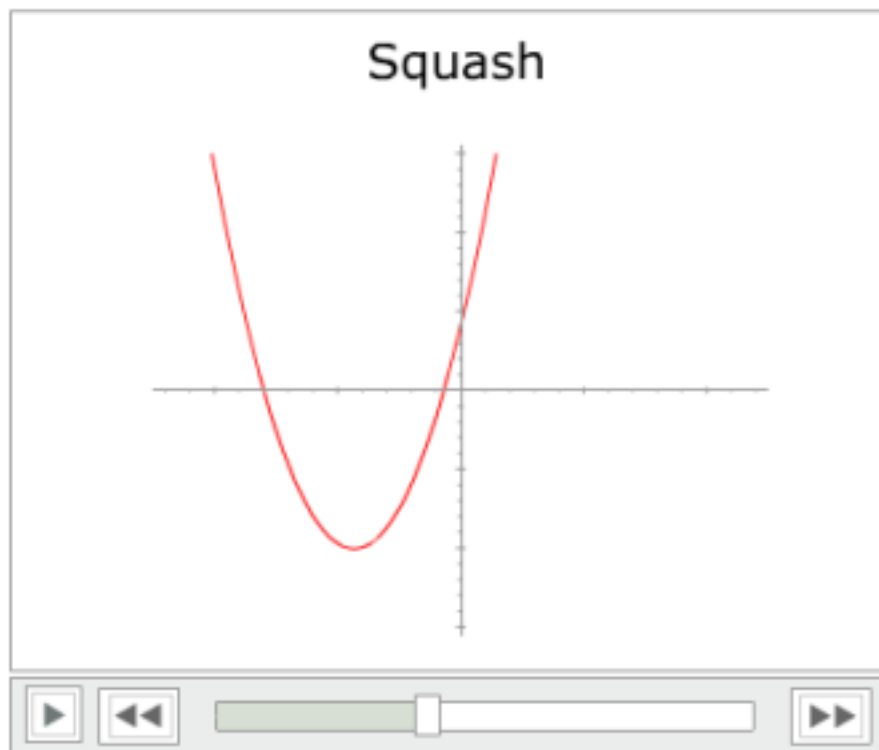
Public Screen 87

Rules



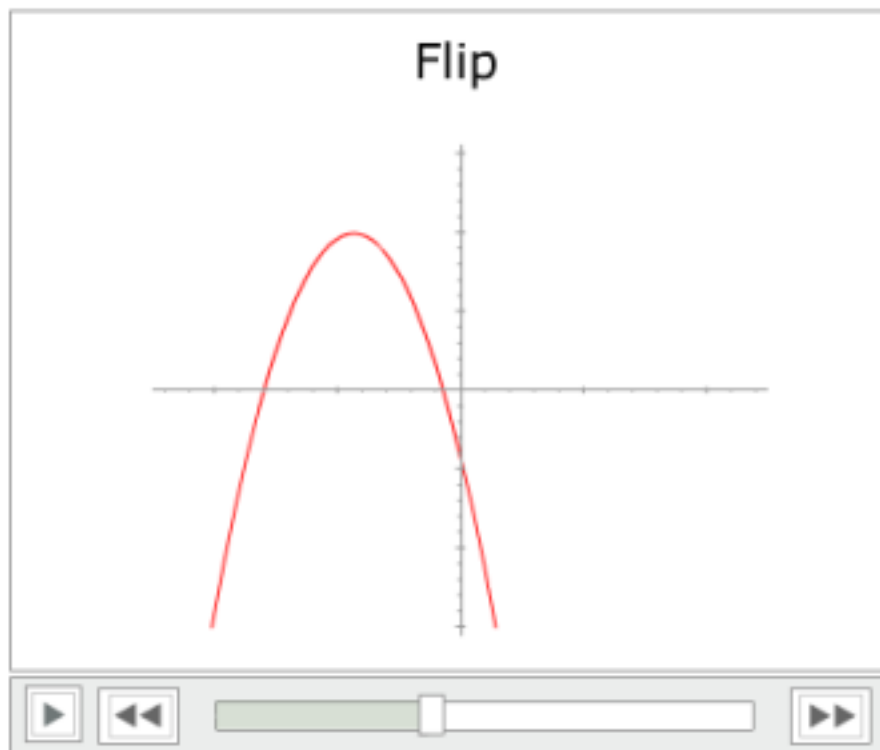
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Rules



Public Screen 89

Rules



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Rules

$y + a = f(x)$ or $y = f(x) - a$	is shifted down a units.
$y = f(x + a)$	is shifted to the left a units.
$ay = f(x)$ or $y = \frac{1}{a}f(x)$	is $\frac{1}{a}$ times as tall.
$y = f(ax)$	is $\frac{1}{a}$ times as wide.
$-y = f(x)$ or $y = -f(x)$	is upside down.
$y = f(-x)$	is mirror image, left to right.

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Rules

$y + a = f(x)$ or $y = f(x) - a$	is shifted down a units.	vertical
$y = f(x + a)$	is shifted to the left a units.	horizontal
$ay = f(x)$ or $y = \frac{1}{a}f(x)$	is $\frac{1}{a}$ times as tall.	squash (fat)
$y = f(ax)$	is $\frac{1}{a}$ times as wide.	stretch (skinnier)
$-y = f(x)$ or $y = -f(x)$	is upside down.	flip over x-axis
$y = f(-x)$	is mirror image, left to right.	flip over y-axis

1st: Shift
2nd: Squash or Stretch
3rd: Flip

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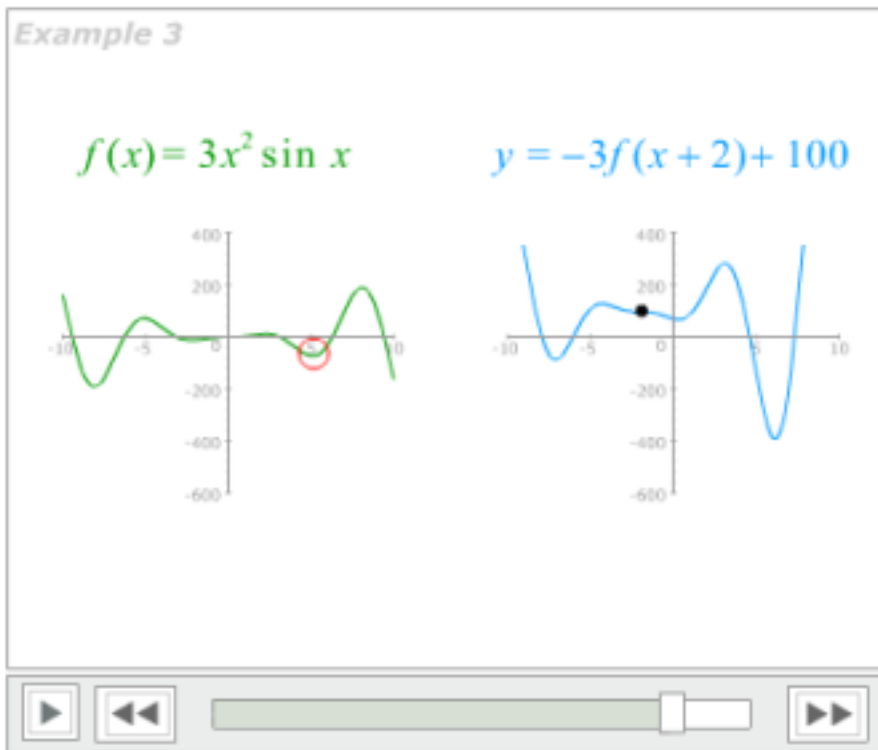
Rules

$y + a = f(x)$ or $y = f(x) - a$	is shifted down a units.
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$y = f(ax)$	is $\frac{1}{a}$ times as wide.
$-y = f(x)$ or $y = -f(x)$	is upside down.
$y = f(-x)$	is mirror image, left to right.

1st: Shift
2nd: Squash or Stretch
3rd: Flip

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Rules



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Graded Assignment

MTH500A: AP Ca

Multiple Choice, Section 1 (no calculator)

You may not use your calculator on this section of the Quiz.

1. The domain of $f(x) = \frac{1}{\sqrt{x^2 - 6x - 7}}$ is:

A. $(1, 7)$

B. $[-1, 7]$

C. $x > -1$ or $x < 7$

D. $\{x < -1\} \cup \{x > 7\}$

$x < -1$ and $x > 7$

E. $(-\infty, -1] \cup [7, \infty)$

denominator cannot
equal zero

square root cannot
be negative

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$x \neq 7 \text{ and } x \neq -1$$



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2. In which of the following is y a function of x ?

I. $y^2 = 9 - x^2$

II. ~~$|y| = x$~~

III. $y = \sqrt[3]{x^2}$

$$y = \sqrt{9-x^2}$$

- A. I only
- B. II only
- C. III only
- ☒ D. I and III only
- E. I, II, and III

3. You would probably use calculus to determine the area for which of the following shapes

A. 

B. 

C. 

D. 

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and Odd

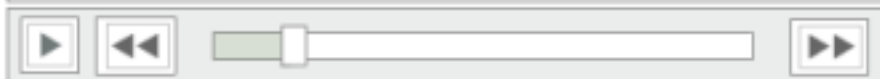
Even Functions

Algebraic Definition:

A function $y = f(x)$ is called an even function if $f(-x) = f(x)$.

$y = x^2$ is an even function because
 $(-x)^2 = x^2$.

$y = x^{-2} + 5$ is an even function because
 $(-x)^{-2} + 5 = x^{-2} + 5$.



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and Odd

Odd Functions

Algebraic Definition:

A function $y = f(x)$ is called an odd function if $f(-x) = -f(x)$.

$y = x^3$ is an odd function because
 $(-x)^3 = -(x^3)$.

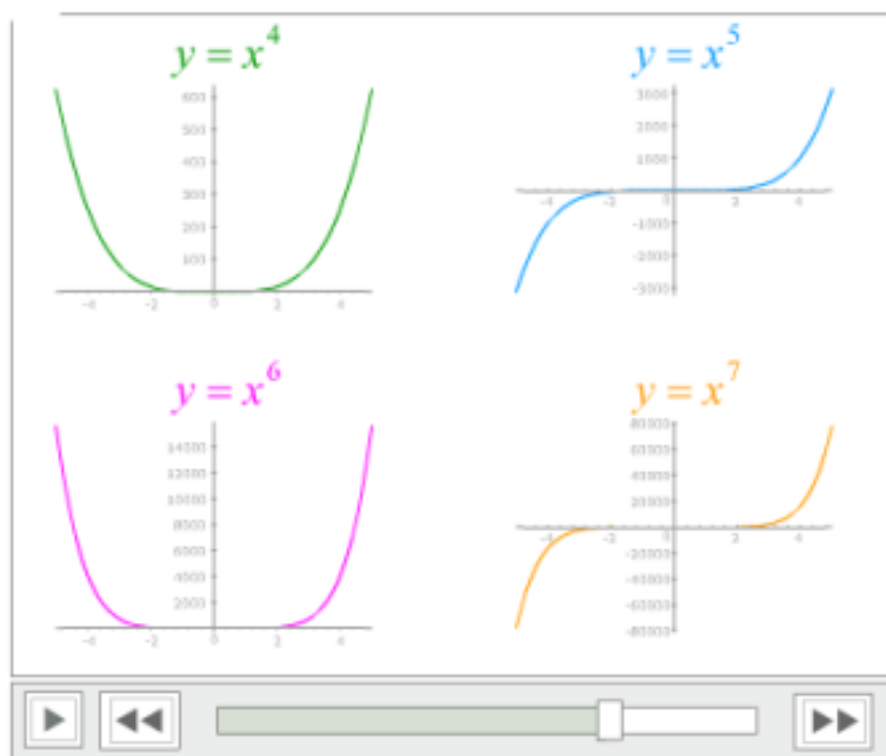
$y = x^9 - x^5$ is an odd function because
 $(-x)^9 - (-x)^5 = -x^9 + x^5 = -(x^9 - x^5)$.



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Even

Odd



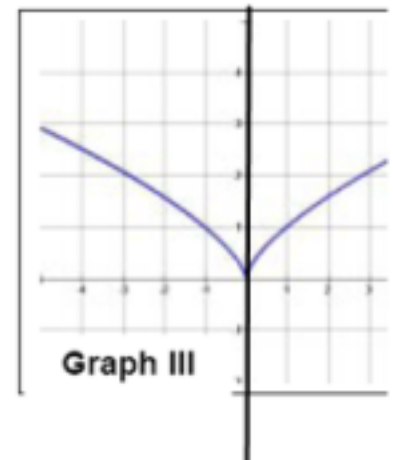
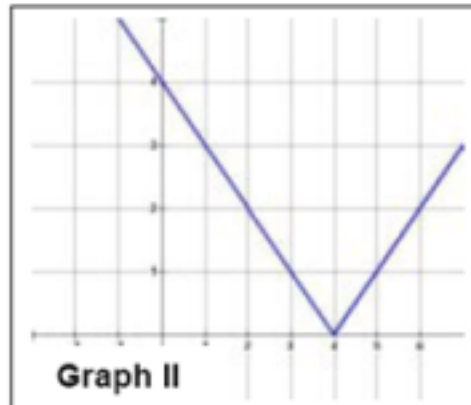
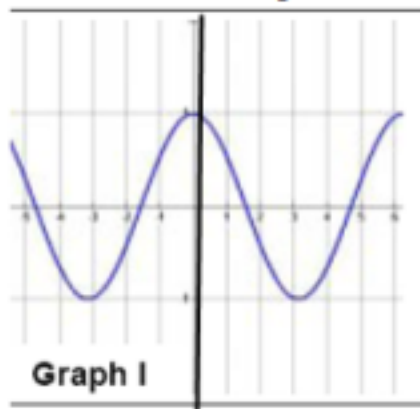
are not one to one

one to one

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Graded Assignment MTH500A: AP Calculus AB| Unit 2

Which of the following is an even function?



- A. I only
- B. II only
- C. III only
- ☒ D. I and III only
- E. None of these

symmetry across the y-axis

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$$f(x) = \sqrt{9 - x^2}$$

If $g(x) = 3x - 1$ and

then which of the following will have a domain of: $[-3, 1/3) \cup$

$(1/3, 3]$

- A. $(g + f)(x)$
- B. $(f/g)(x)$
- C. $(g - f)(x)$
- D. $g(f(x))$
- E. $f(g(x))$

denominator cannot equal zero

A. $3x-1 + \sqrt{9-x^2}$

B. $\frac{\sqrt{9-x^2}}{3x-1}$

C. $3x-1 - \sqrt{9-x^2}$

D. $3(\sqrt{9-x^2}) - 1$

E. $\sqrt{9-(3x-1)^2}$