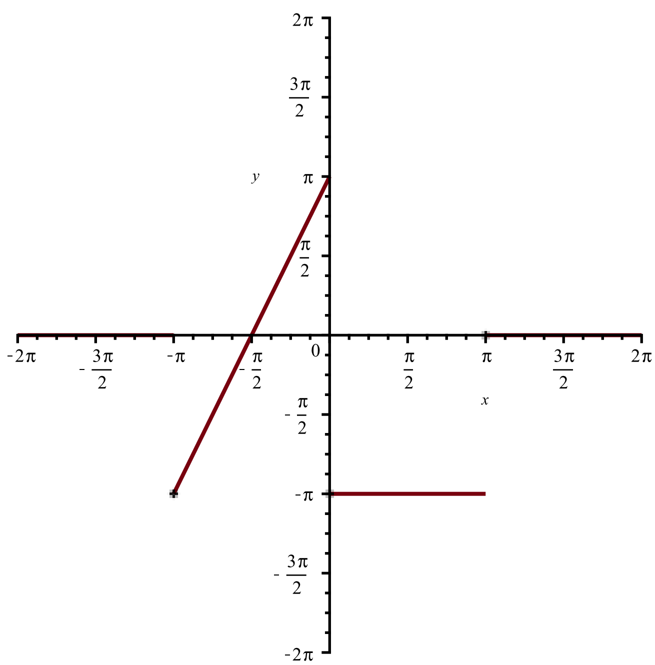
\>

\> *#Лабораторная работа 2*
#Вариант 1
#Выполнил Абдулов Александр, гр. 353503
#Задание 1.
#Получить разложение в тригонометрический ряд Фурье.
#Построить в одной системе координат графики частичных сумм $S_1(x)$, $S_3(x)$, $S_7(x)$ *ряда* и *его суммы* $S(x)$.

\> $f := x \rightarrow piecewise(-\pi \le x < 0, \pi + 2x, 0 \le x < \pi, -\pi);$

$$f := x \mapsto \begin{cases} \pi + 2\cdot x & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases} \tag{1}$$

\> $plot(f(x), x = -2\pi..2\pi, y = -2\pi..2\pi, discont = true);$



\>

$a0 := simplify\left(\frac{1}{\pi}\cdot int(f(x), x = -\pi..\pi)\right);$

$$a0 := -\pi \tag{2}$$

\> $an := simplify\left(\frac{1}{\pi}\cdot int(f(x)\cdot\cos(n\cdot x), x = -\pi..\pi)\right)$ assuming $n :: posint;$

$$an := \frac{-2\,(-1)^n + 2}{\pi\, n^2} \tag{3}$$

\> $bn := simplify\left(\frac{1}{\pi}\cdot int(f(x)\cdot\sin(n\cdot x), x = -\pi..\pi)\right)$ assuming $n :: posint;$

(4)

$$bn := -\frac{2}{n} \tag{4}$$

>

>

```
> GetFourier := proc(f_, m_)
  local a0, an, bn, n;
  a0 := simplify(1/π·int(f_(x), x=-π..π));
  an := simplify(1/π·int(f_(x)·cos(n·x), x=-π..π)) assuming n :: posint;
  bn := simplify(1/π·int(f_(x)·sin(n·x), x=-π..π)) assuming n :: posint;
  return a0/2 + sum(an·cos(n·x) + bn·sin(n·x), n=1..m_);
  end proc:
```

> $S := GetFourier(f, \text{infinity});$

$$S := -\frac{\pi}{2} + \sum_{n=1}^{\infty}\left(\frac{\left(-2\,(-1)^n + 2\right)\cos(n\,x)}{\pi\,n^2} - \frac{2\sin(n\,x)}{n}\right) \tag{5}$$

>

> $S1 := GetFourier(f, 1);$

$$S1 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x) \tag{6}$$

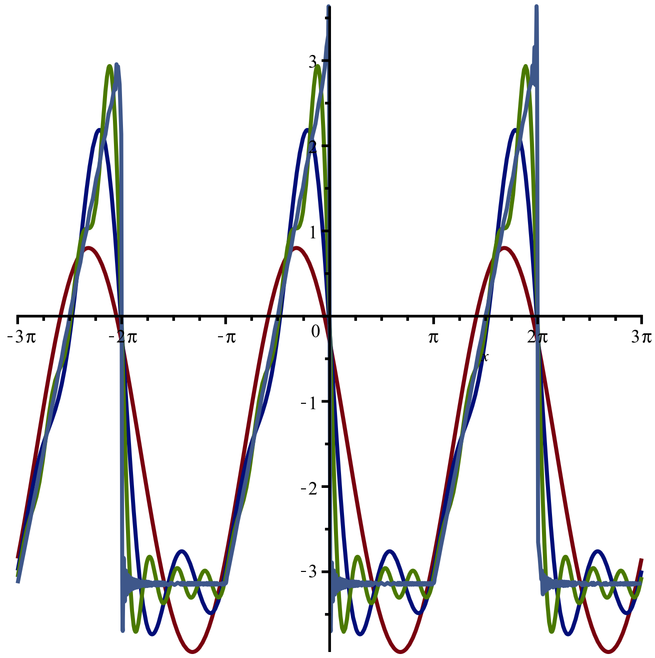
> $S3 := GetFourier(f, 3);$

$$S3 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2\,x) + \frac{4\cos(3\,x)}{9\,\pi} - \frac{2\sin(3\,x)}{3} \tag{7}$$

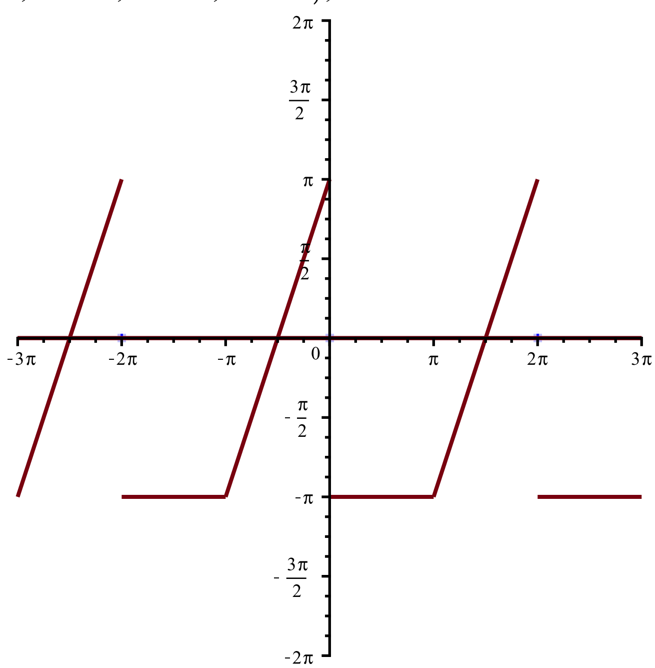
> $S7 := GetFourier(f, 7);$

$$S7 := -\frac{\pi}{2} + \frac{4\cos(x)}{\pi} - 2\sin(x) - \sin(2\,x) + \frac{4\cos(3\,x)}{9\,\pi} - \frac{2\sin(3\,x)}{3} - \frac{\sin(4\,x)}{2} + \frac{4\cos(5\,x)}{25\,\pi} - \frac{2\sin(5\,x)}{5} - \frac{\sin(6\,x)}{3} + \frac{4\cos(7\,x)}{49\,\pi} - \frac{2\sin(7\,x)}{7} \tag{8}$$

> $plot([S1, S3, S7, GetFourier(f, 100)], x=-3\cdot\text{Pi}..3\cdot\text{Pi}, discont=true);$

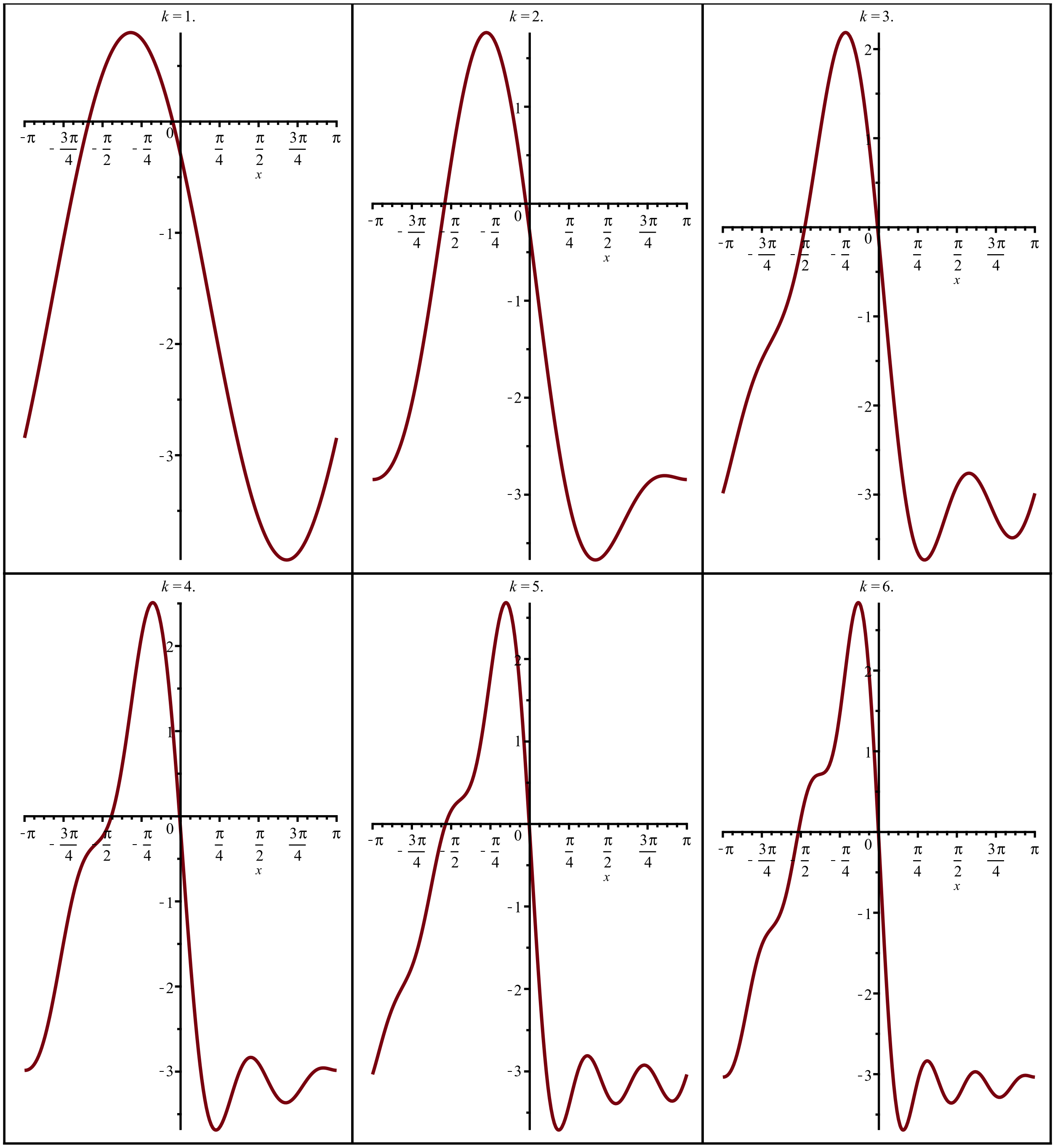


> $points := plot([[-2\cdot\pi, 0], [0, 0], [2\cdot\pi, 0]], style = point, color = blue)$:
$Func1 := plot(f(x), x = -3\cdot\pi..3\cdot\pi, y = -2\cdot\pi..2\cdot\pi, discont = true)$:
$Func2 := plot(f(x - 2\cdot\pi), x = -3\cdot\pi..3\cdot\pi, y = -2\cdot\pi..2\cdot\pi, discont = true)$:
$Func3 := plot(f(x + 2\cdot\pi), x = -3\cdot\pi..3\cdot\pi, y = -2\cdot\pi..2\cdot\pi, discont = true)$:
$plots[display](points, Func1, Func2, Func3)$;



>
$plots[animate](plot, [GetFourier(f, k), x = -\pi..\pi], k = [1, 2, 3, 4, 5])$;

>
$animation := plots[animate](plot, [GetFourier(f, k), x = -\mathrm{Pi}\ ..\ \mathrm{Pi}], k = [1, 2, 3, 4, 5, 6])$:
$plots[display](animation)$;



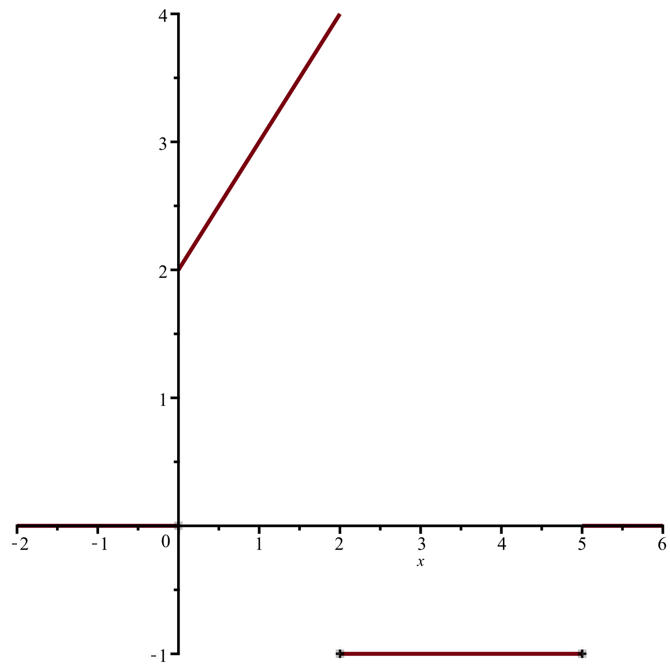
> *restart*

> *#Задание 2.*
#Получить разложение в тригонометрический ряд Фурье.
#Построить в одной системе координат графики частичных сумм $S_1(x)$, $S_3(x)$, $S_7(x)$ *ряда* и *его суммы* $S(x)$.

> $f := x \rightarrow piecewise(0 < x < 2, x + 2, 2 \le x \le 5, -1);$

$$f := x \mapsto \begin{cases} x+2 & 0 < x < 2 \\ -1 & 2 \le x \le 5 \end{cases} \tag{9}$$

> $plot(f(x), x = -2..6, discont = true);$



> $a0 := simplify\left(\frac{2}{5} \cdot int(f(x), x = 0..5)\right);$
$an := simplify\left(\frac{2}{5} \cdot int\left(f(x) \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint;$
$bn := simplify\left(\frac{2}{5} \cdot int\left(f(x) \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint;$

$$a0 := \frac{6}{5}$$

$$an := \frac{5\left(2\,\pi\,n \sin\left(\frac{4\,\pi\,n}{5}\right) + \cos\left(\frac{4\,\pi\,n}{5}\right) - 1\right)}{2\,n^2\,\pi^2}$$

$$bn := \frac{-10\,\pi\,n \cos\left(\frac{4\,\pi\,n}{5}\right) + 6\,\pi\,n + 5 \sin\left(\frac{4\,\pi\,n}{5}\right)}{2\,n^2\,\pi^2} \quad \textbf{(10)}$$

> $Fourier := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right), n = 1..k\right);$

$$Fourier := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{5}\right)\right) \quad \textbf{(11)}$$

>

> $S1 := Fourier(1);$

$$S1 := \frac{3}{5} + \frac{5\left(2\,\pi \sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} + \frac{\left(10\,\pi \cos\left(\frac{\pi}{5}\right) + 6\,\pi + 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} \tag{12}$$

> $S3 := Fourier(3);$

$$S3 := \frac{3}{5} + \frac{5\left(2\,\pi \sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} + \frac{\left(10\,\pi \cos\left(\frac{\pi}{5}\right) + 6\,\pi + 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} + \frac{5\left(-4\,\pi \sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2} + \frac{\left(-20\,\pi \cos\left(\frac{2\,\pi}{5}\right) + 12\,\pi - 5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2} + \frac{5\left(6\,\pi \sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2} + \frac{\left(-30\,\pi \cos\left(\frac{2\,\pi}{5}\right) + 18\,\pi + 5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2} \tag{13}$$

> $S7 := Fourier(7);$

$$S7 := \frac{3}{5} + \frac{5\left(2\,\pi \sin\left(\frac{\pi}{5}\right) - \cos\left(\frac{\pi}{5}\right) - 1\right)\cos\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} + \frac{\left(10\,\pi \cos\left(\frac{\pi}{5}\right) + 6\,\pi + 5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{2\,\pi\,x}{5}\right)}{2\,\pi^2} + \frac{5\left(-4\,\pi \sin\left(\frac{2\,\pi}{5}\right) + \cos\left(\frac{2\,\pi}{5}\right) - 1\right)\cos\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2} \tag{14}$$

$$+\frac{\left(-20\,\pi\cos\left(\frac{2\,\pi}{5}\right)+12\,\pi-5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{4\,\pi\,x}{5}\right)}{8\,\pi^2}$$

$$+\frac{5\left(6\,\pi\sin\left(\frac{2\,\pi}{5}\right)+\cos\left(\frac{2\,\pi}{5}\right)-1\right)\cos\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$

$$+\frac{\left(-30\,\pi\cos\left(\frac{2\,\pi}{5}\right)+18\,\pi+5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{6\,\pi\,x}{5}\right)}{18\,\pi^2}$$

$$+\frac{5\left(-8\,\pi\sin\left(\frac{\pi}{5}\right)-\cos\left(\frac{\pi}{5}\right)-1\right)\cos\left(\frac{8\,\pi\,x}{5}\right)}{32\,\pi^2}$$

$$+\frac{\left(40\,\pi\cos\left(\frac{\pi}{5}\right)+24\,\pi-5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{8\,\pi\,x}{5}\right)}{32\,\pi^2}-\frac{2\sin(2\,\pi\,x)}{5\,\pi}$$

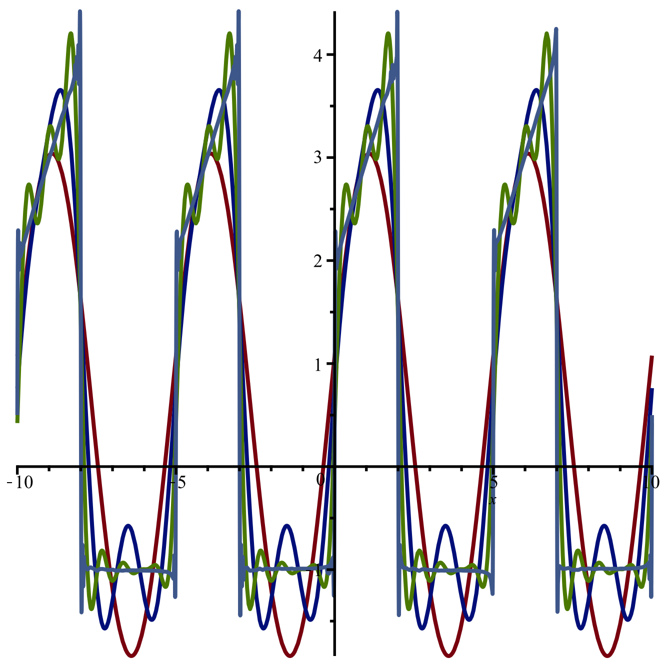
$$+\frac{5\left(12\,\pi\sin\left(\frac{\pi}{5}\right)-\cos\left(\frac{\pi}{5}\right)-1\right)\cos\left(\frac{12\,\pi\,x}{5}\right)}{72\,\pi^2}$$

$$+\frac{\left(60\,\pi\cos\left(\frac{\pi}{5}\right)+36\,\pi+5\sin\left(\frac{\pi}{5}\right)\right)\sin\left(\frac{12\,\pi\,x}{5}\right)}{72\,\pi^2}$$

$$+\frac{5\left(-14\,\pi\sin\left(\frac{2\,\pi}{5}\right)+\cos\left(\frac{2\,\pi}{5}\right)-1\right)\cos\left(\frac{14\,\pi\,x}{5}\right)}{98\,\pi^2}$$

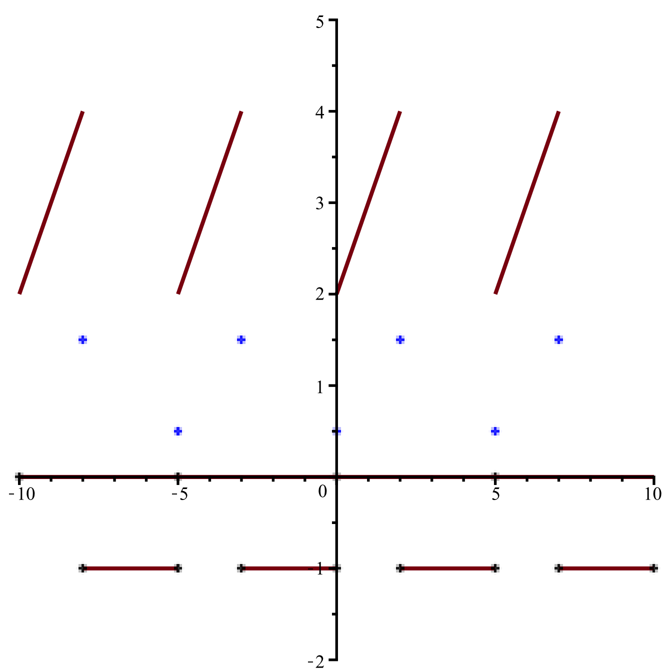
$$+\frac{\left(-70\,\pi\cos\left(\frac{2\,\pi}{5}\right)+42\,\pi-5\sin\left(\frac{2\,\pi}{5}\right)\right)\sin\left(\frac{14\,\pi\,x}{5}\right)}{98\,\pi^2}$$

```
> plot([S1, S3, S7, Fourier(100)], x = -10..10, discont = true);
```



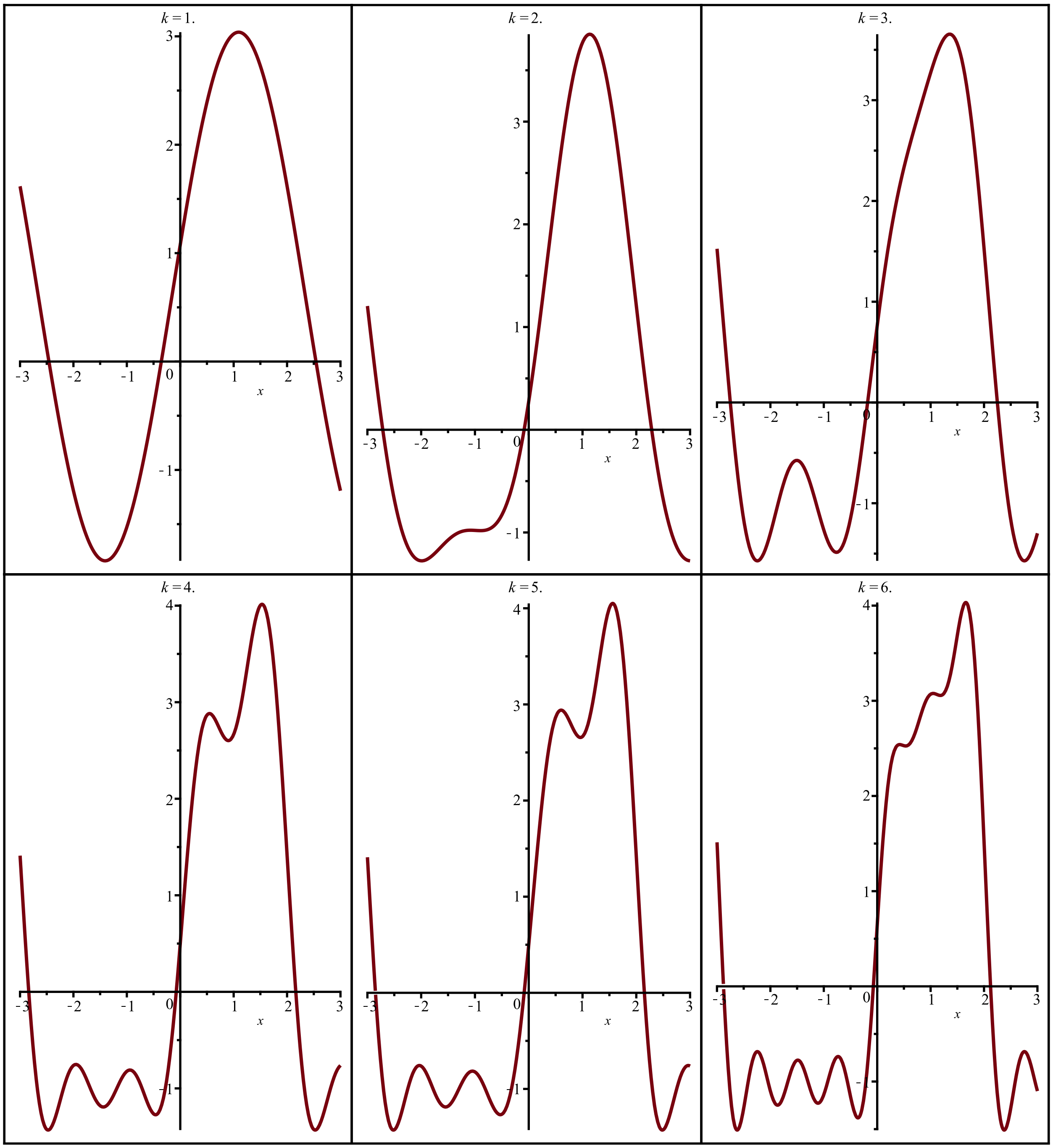

```
> points := plot([[-8, 1.5], [-5, 0.5], [-3, 1.5], [0, 0.5], [2, 1.5], [5, 0.5], [7, 1.5]], style
      = point, color = blue) :
  Func1 := plot(f(x), x = -10..10, y = -2..5, discont = true) :
  Func2 := plot(f(x - 5), x = -10..10,  y = -2..5, discont = true) :
  Func3 := plot(f(x + 5), x = -10..10, y = -2..5, discont = true) :
  Func4 := plot(f(x + 10), x = -10..10, y = -2..5, discont = true) :

  plots[display](points, Func1, Func2, Func3, Func4);
```




```
> animation := plots[animate](plot, [Fourier(k), x = -3 .. 3], k = [1 , 2, 3, 4, 5 , 6]) :
```

plots[*display*](*animation*);

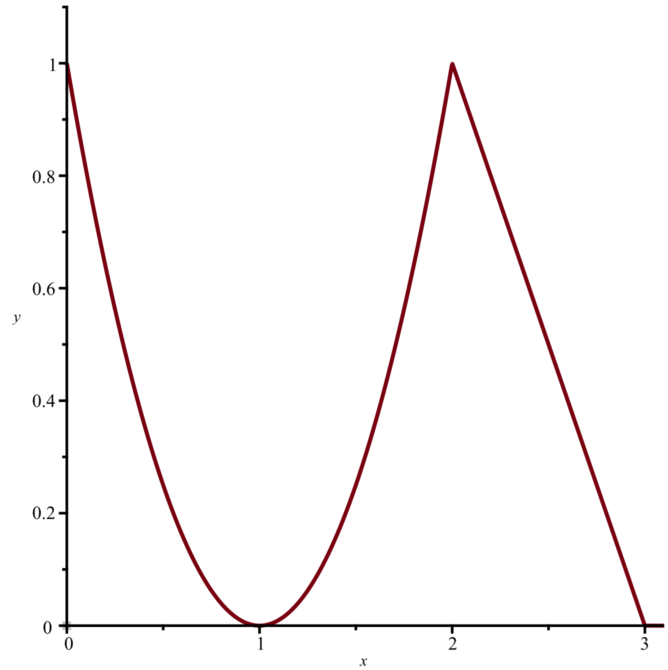


> *restart*;
#Задание 3
#полный период

$f := x \rightarrow piecewise\left(0 < x < 2, (x-1)^2, 2 < x < 3, -(x-2)+1\right);$

$$f := x \mapsto \begin{cases} (x-1)^2 & 0 < x < 2 \\ -x+3 & 2 < x < 3 \end{cases} \tag{15}$$

> $plot(f(x), x = 0 ..3.1, y = 0 ..1.1, discont = true);$



> $a0 := simplify\left(\frac{2}{3} int(f(x), x = -3 ..3)\right);$

$$a0 := \frac{7}{9} \tag{16}$$

> $an := simplify\left(\frac{2}{3} int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x \cdot 2}{3}\right), x = 0 ..3\right)\right)$ assuming $n :: posint;$

$$an := \frac{9\,\pi\,n\cos\left(\frac{4\,\pi\,n}{3}\right) + 3\,\pi\,n - 9\sin\left(\frac{4\,\pi\,n}{3}\right)}{2\,n^3\,\pi^3} \tag{17}$$

> $bn := simplify\left(\frac{2}{3} int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x \cdot 2}{3}\right), x = 0 ..3\right)\right)$ assuming $n :: posint;$

$$bn := \frac{2\,\pi^2\,n^2 + 9\,\pi\,n\sin\left(\frac{4\,\pi\,n}{3}\right) + 9\cos\left(\frac{4\,\pi\,n}{3}\right) - 9}{2\,n^3\,\pi^3} \tag{18}$$

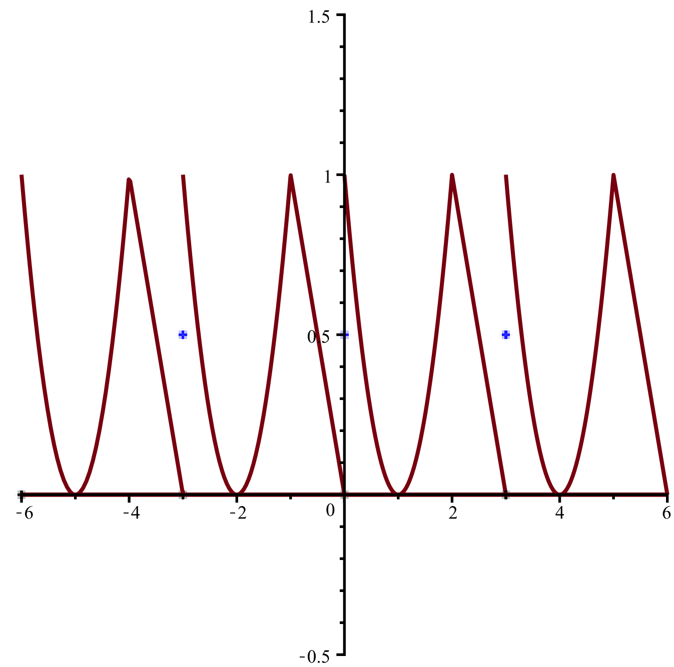
> $Fourier := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{3}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{3}\right), n = 1 ..k\right);$

$$Fourier := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k} \left(an \cdot \cos\left(\frac{2 \cdot \pi \cdot n \cdot x}{3}\right) + bn \cdot \sin\left(\frac{2 \cdot \pi \cdot n \cdot x}{3}\right)\right) \tag{19}$$

> $points := plot([[-3, 0.5], [0, 0.5], [3, 0.5]], style = point, color = blue) :$
$Func1 := plot(f(x), x = -6 ..6, y = -0.5 ..1.5, discont = true) :$

$Func2 := plot(f(x-3), x=-6..6, y=-0.5..1.5, discont=true):$
$Func3 := plot(f(x+3), x=-6..6, y=-0.5..1.5, discont=true):$
$Func4 := plot(f(x+6), x=-6..6, y=-0.5..1.5, discont=true):$

$plots[display](points, Func1, Func2, Func3, Func4);$

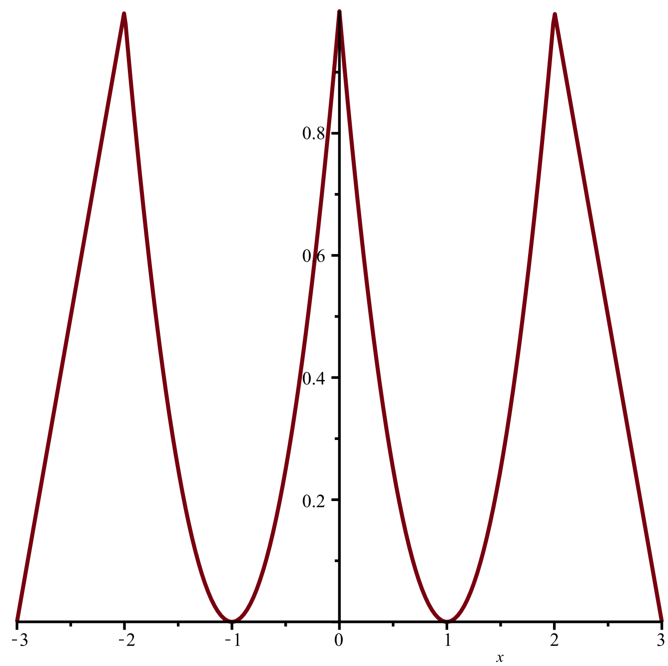


\> #четная

$f := x \to piecewise\left(0 < x < 2, (x-1)^2, 2 < x < 3, -(x-2)+1, -2 < x < 0, (x+1)^2, -3 < x < -2, (x+2)+1\right);$

$plot(f1(x), x=-3..3);$

$$f := x \mapsto \begin{cases} (x-1)^2 & 0 < x < 2 \\ -x+3 & 2 < x < 3 \\ (x+1)^2 & -2 < x < 0 \\ 3+x & -3 < x < -2 \end{cases}$$



> $a0 := simplify\left(\frac{1}{3} int(f(x), x=-3..3)\right);$

$$a0 := \frac{7}{9} \tag{20}$$

> $an := simplify\left(\frac{1}{3} int\left(f(x) \cdot \cos\left(\frac{\pi \cdot n \cdot x}{3}\right), x=-3..3\right)\right)$ assuming $n :: posint;$

$$an := \frac{18\,\pi\,n\cos\left(\frac{2\,\pi\,n}{3}\right) - 6\,\pi\,(-1)^n\,n + 12\,\pi\,n - 36\sin\left(\frac{2\,\pi\,n}{3}\right)}{n^3\,\pi^3} \tag{21}$$

> $bn := simplify\left(\frac{1}{3} int\left(f(x) \cdot \sin\left(\frac{\pi \cdot n \cdot x}{3}\right), x=-3..3\right)\right)$ assuming $n :: posint;$

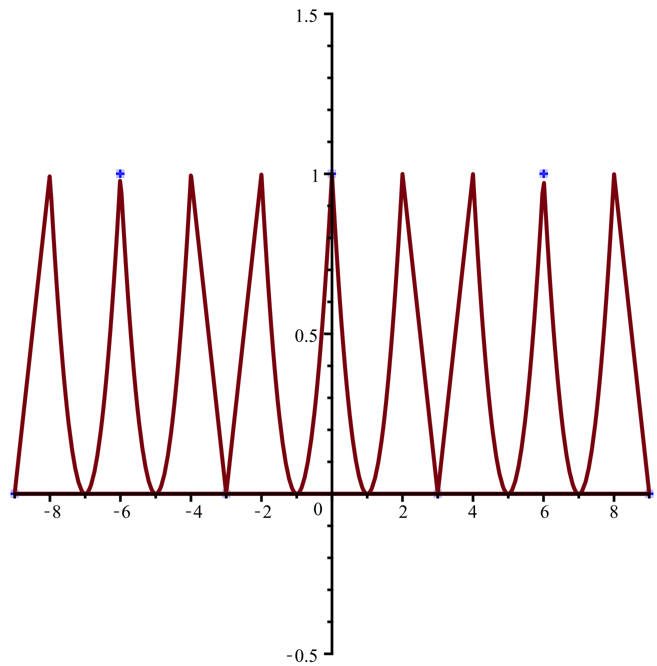
$$bn := 0 \tag{22}$$

> $Fourier := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{3}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{3}\right), n=1..k\right);$

$$Fourier := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{3}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{3}\right)\right) \tag{23}$$

> $points := plot([[-9, 0], [-3, 0], [3, 0], [9, 0], [-6, 1], [0, 1], [6, 1]], style=point, color=blue):$
$Func1 := plot(f(x), x=-9..9, y=-0.5..1.5, discont=true):$
$Func2 := plot(f(x-6), x=-9..9, y=-0.5..1.5, discont=true):$
$Func3 := plot(f(x+6), x=-9..9, y=-0.5..1.5, discont=true):$

$plots[display](points, Func1, Func2, Func3);$

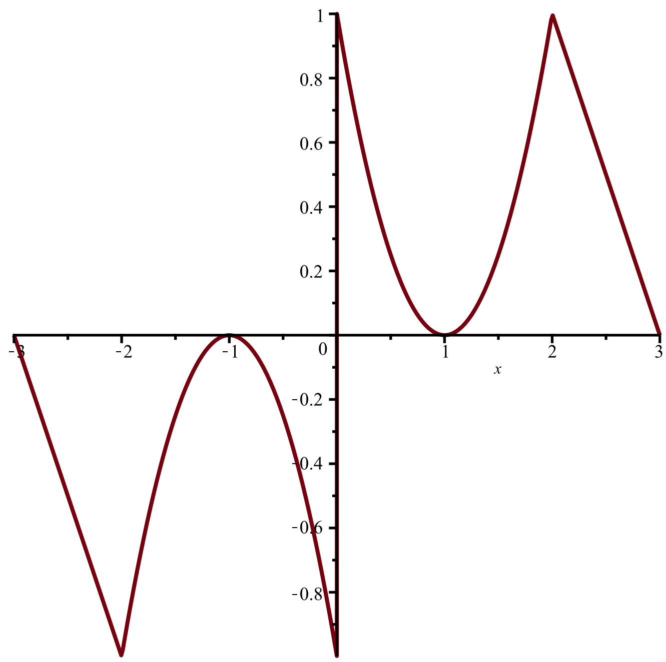


> *#нечетная*

$f := x \rightarrow piecewise\left(0 < x < 2, \left(x-1\right)^2, 2 < x < 3, -(x-2)+1, -2 < x < 0, -\left(x+1\right)^2, -3 < x < -2, -(x+2)-1\right);$

$plot(f(x), x=-3..3);$

$$f := x \mapsto \begin{cases} (x-1)^2 & 0 < x < 2 \\ -x+3 & 2 < x < 3 \\ -(x+1)^2 & -2 < x < 0 \\ -x-3 & -3 < x < -2 \end{cases}$$



> $a0 := simplify\left(\frac{1}{3} int(f(x), x=-3..3)\right);$

$$a0 := 0 \tag{24}$$

> $an := simplify\left(\frac{1}{3} int\left(f(x)\cdot\cos\left(\frac{\pi\cdot n\cdot x}{3}\right), x=-3..3\right)\right)$ assuming $n :: posint;$

$$an := 0 \tag{25}$$

> $bn := simplify\left(\frac{1}{3} int\left(f(x)\cdot\sin\left(\frac{\pi\cdot n\cdot x}{3}\right), x=-3..3\right)\right)$ assuming $n :: posint;$

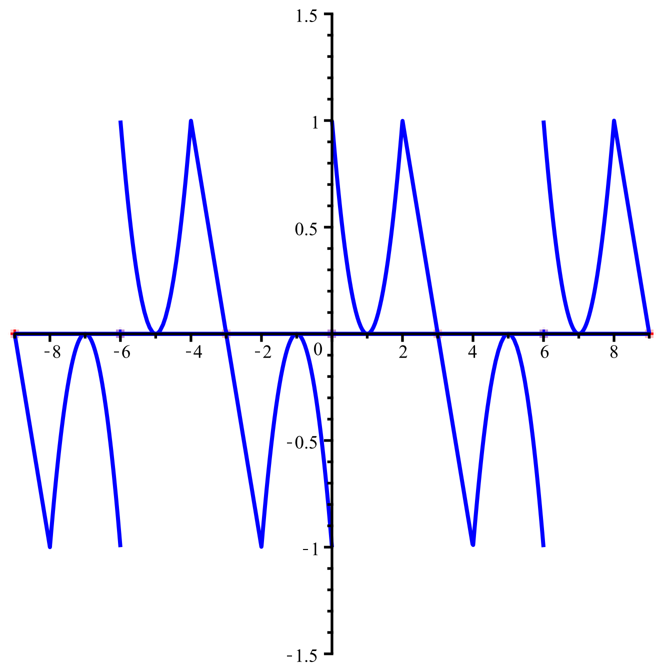
$$bn := \frac{2\pi^2 n^2 + 18\pi n \sin\left(\frac{2\pi n}{3}\right) + 36\cos\left(\frac{2\pi n}{3}\right) - 36}{n^3\pi^3} \tag{26}$$

> $Fourier := k \rightarrow \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{\pi\cdot n\cdot x}{3}\right) + bn\cdot\sin\left(\frac{\pi\cdot n\cdot x}{3}\right), n=1..k\right);$

$$Fourier := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an\cdot\cos\left(\frac{\pi\cdot n\cdot x}{3}\right) + bn\cdot\sin\left(\frac{\pi\cdot n\cdot x}{3}\right)\right) \tag{27}$$

> $points := plot([[-9, 0], [-3, 0], [3, 0], [9, 0], [-6, 0], [0, 0], [6, 0]], style=point, color=red) :$
$Func1 := plot(f(x), x=-9..9, y=-1.5..1.5, discont=true, color=blue) :$
$Func2 := plot(f(x-6), x=-9..9, y=-1.5..1.5, discont=true, color=blue) :$
$Func3 := plot(f(x+6), x=-9..9, y=-1.5..1.5, discont=true, color=blue) :$

$plots[display](points, Func1, Func2, Func3);$


1.5
1
0.5
-0.5
-1
-1.5
-8
-6
-4
-2
0
2
4
6
8

>