UNIVERSITY OF DAR ES SALAAM

MATHEMATICS DEPARTMENT

SUPPLEMENTARY UNIVERSITY EXAMINATIONS 2013/2014

DATE: 15TH September 2014

DURATION: 2 HOURS

MT100: FOUNDATIONS OF ANALYSIS

instruction:

Attempt ALL Questions in Section A and ANY TWO questions in Section

SECTION A (30 Marks)

Q1. (a) Let $A = \{x : x^2 \le 9\}$, $B = \{x : |x| < 2\}$ and $C = \{x \in [x] : x^2 \le 3\}$. Find,

(i) (A n B)UC (ii) (A U C) n B

(b) Let $A = \{x : x \in N\}$ and $B = \{x : 0 < x^2 \le 4\}$. Sketch the set represented by (i) $B \times A$ (ii) $B \times B$?

Q2. (a) Find the solution set for

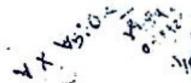
(b) Evaluate

ij.

ifi. lim (2n2-15n+9)

iv.

Q3. Let x, y and z be real numbers. For each of the



propositions below determine whether it is true or false. If the proposition is true prove it and if it is false give a counterexample.



(a)
$$\forall x \forall y$$
: $0 < \frac{1}{x^2 + y^2 + 2} < 0.5$

(2 marks)

(b)
$$\forall 33 \times \frac{x^2}{y^2+5} + \frac{3xy^2}{y^2+2} < 0$$

(2 marks)

(c)
$$\forall x \ \forall y \ \forall z : x^1 + y^2 + z^2 \le x^2 y^2 z^2$$

(2 marks)

(d)
$$\forall x : 0 < \frac{1}{x^2 + 1} \le 1$$

(2 marks)

Q4. (a) Let $A = \{x: x^3 > 0\}$, where x is a real number. Determine if set A (2 marks) is a number ring or number field or neither.

(b) Determine if Q the set of irrational numbers is a number ring or (2 marks) number field or neither.

SECTION B (30 Marks)

Q5. (a) Let $K = \left\{ x_n : x_n = \frac{(-1)^n n}{2n+1}, n \in \mathbb{N} \right\}$

Write the sub-sequences of even and odd term of the (i) sequence x.

max(K)

min(K) and

(8 marks)

(b) Use &-definition to prove that

$$\lim_{n \to \infty} \frac{3n+1}{2n+3} = \frac{3}{2}$$

(7 marks)

Q6. (a) Siven that x=1+i and x=1-2i are roots of the function $f(x) = x^6 - 4x^3 + 10x^4 - 10x^3 - x^2 + 14x - 10.$

Find other roots of the function f.

(6 marks)

(b) / Without writing z = x + iy, use the definition of modulus to simplify the inequality

$$\left|\frac{2iz+1}{z-i}\right| \leq \frac{1}{2}.$$