SECTION A (30 marks).

1. State whether or not each of the following statement is True or False. Hence prove or give counterexample to each statement. **(7 marks)**

   (a) A set of irrational numbers is a number ring.

   (b) Every convergent sequence is Cauchy sequence.

   (c) If set A is bounded then $\inf(A) \leq \sup(A)$ for every $x \in A$.

   (d) For every $x, y, z \in \mathbb{R}$, then $\dfrac{4}{x^2 + y^2 + 4x - 2y + z^2 + 2z + 8} \leq 2$.

2. (a) Let $\wp(A)$ and $\wp(B)$ denote power sets of A and B. Prove that if $A \subseteq B$ then $\wp(A) \subseteq \wp(B)$. **(2 marks)**

   (b) Prove that between any two rational numbers there are infinitely many irrational numbers. **(3 marks)**

3. (a) Show that the sequence $\{a_n\}_{n=1}^{\infty}$ defined by $a_n = \dfrac{n^3 + 2^n}{n^2 + 3^n}$ is a null sequence. **(2 marks)**

   (b) Prove that $\sqrt{5}$ is not a rational number. **(3 marks)**

   (c) When is a function $f : A \longrightarrow B$ said to be bounded? **(1 mark)**

4. (a) Let R be an equivalence relation on set S and $[a]$ and $[b]$ be two distinct equivalence classes on set S. Prove the following results:

   i. $aRb \Rightarrow [a] = [b]$. **(2 marks)**

   ii. $[a] = [b] \Rightarrow aRb$. **(2 marks)**

   (b) Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$, $C = \{2, 3, 4\}$ with
   $R = \{(1, 4), (1, 5), (2, 6), (3, 4)\}$ a relation from A to B,
   $S = \{(4, 2), (4, 3), (6, 2)\}$ a relation from B to C. Show that
   $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$. **(2 marks)**

5. (a) Show that the mapping $f : \mathbb{R} \to [0, \infty)$ defined by $f(x) = x^2$ is surjective. Suggest the means of making it bijective. **(3 marks)**

   (b) Use induction method to prove that
   $1 \times 1! + 2 \times 2! + \cdots + n \times n! = (n + 1)! - 1$. **(3 marks)**

SECTION B (30 Marks).

6. (a) Use the special truth table to determine whether the following propositions are valid or not. If the proposition is valid use the principle of demonstration and Or indirect proof to prove it, and if not valid give a counterexample.

   i. $[(\neg r \to \neg p) \wedge (q \to p) \wedge (\neg q \to \neg r)] \to (r \leftrightarrow q)$. Valid **(7 marks)**

   ii. $[(p \to r) \wedge (q \vee s)] \to [(p \wedge q) \to (r \wedge s)]$. **(2 marks)**

   (b) Consider the function $f(x) = \dfrac{10}{x^2 - 2}$ which is defined on $(-\infty, -\sqrt{2})$.

   i. Determine $\mathrm{Im}(f)$. **(2 marks)**

   ii. Find $\sup f$, $\inf f$, $\max f$ and $\min f$ if they exist. **(2 marks)**

   iii. Determine whether f is monotonic or not. **(2 marks)**

7. (a) For $x, y \in \mathbb{Z}$, define xRy when $x^2 - y^2$ is even.

   i. Show that R is an equivalence relation. **(4 marks)**

   ii. Find distinct equivalence classes of R and show that they form partition on $\mathbb{Z}$. **(4 marks)**

   (b) Let $A = \{1, 2, 3, 4\}$ and let R and S be relations on set A defined as $(n, m) \in R$ if and only if $nm = 2k_1$ and $(x, y) \in S$ if and only if $xy = 2k_2 + 1$, $k_1, k_2 \in \mathbb{Z}^+$.

   i. List the elements of S and R. **(4 marks)**

   ii. Find $S \circ R^{-1}$. **(2 marks)**

   iii. Is $S \circ R^{-1}$ a function? Explain. **(1 mark)**

8. (a) Find all the points x which lie in the set A, where:

$$A = \left\{ x : \frac{5x+6}{x+2} > \frac{2x-3}{x-1} \right\}.$$ **(5 marks)**

   (b) Suppose $B = \left\{ z : \left| z^2 - \frac{5}{3} \right| < \frac{4}{3} \right\}$ be the set of complex numbers. Without expressing z into its real and imaginary parts, deduce that the inequality can be expressed as $3|z|^4 - 10\mathcal{R}e(z^2) < -3$. **(5 marks)**

   (c) Use $\epsilon - \mathrm{N}(\epsilon)$ definition to prove that if $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} b_n = L_2$, then $\lim_{n\to\infty} (a_n + b_n) = L_1 + L_2$. **(5 marks)**