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DEPARTMENT OF MATHEMATICS
MT 100: FOUNDATION OF ANALYSIS
TIMED TEST 1, 2019/2020.

Date: Thursday, December 19, 2019 **Time: 50 minutes**

Instruction: Answer **all** questions.

1. Write down the negation of each of the following statements.
   (a) Some logic students understand quantifiers.
   (b) All logic students understand quantifiers.
   (c) $\forall x \leq 1, f(x) \neq 3$.
   (d) $\exists x$ in $A \ni \forall y$ in B, $(x \geq y) \vee (y \geq 1)$.
   (e) $\exists x \ni \forall y \exists z \ni x + y + z > xyz$.

2. Fill in the blanks below to make the arguments valid. Support your response with reasons:
   (a) He studies very hard. *Therefore,* either .... or he is a very bad student.
   (b) If it rains, I will take a leave. If it is hot outside, I will go for a shower. Either I will not take a leave or I will not go for a shower. *Therefore,*................ .

3. (a) Prove that if m^2 is an even number then, m is an even number.
   (b) Fill in the blanks to complete the proof of the following theorem. Write only the response of each item in a separate sheet of paper provided.

   Theorem 1. *Let A be a subset of a universal set U. Then $A \cup (U \backslash A) = U$.*

   Proof. **By direct proof.** If $x \in A \cup (U \backslash A)$, then $x \in$(i) .... or $x \in$(ii) .... Since both A and $U \backslash A$ are subsets of U, in either the case we have (iii)........ . Thus (iv) ....$\subseteq$(v)........ .
   On the other hand, suppose that $x \in$(vi)........ . Now either $x \in A$ or $x \notin A$. If $x \notin A$, then $x \in$(vii)........ . In either case $x \in$(viii)........ .
   Hence (ix)........$\subseteq$(x)........ . □

4. (a) Determine whether the following propositions are true or false. Give a counterexample to False proposition and prove if the proposition is true for $x, y, z \in \mathbb{R}$.
   i. $\exists x \ni \forall y \exists z \ni x^2 + y^2 = z$. True.
   ii. $\forall x,\ 2x^2 \geq x$. — ~~False~~ True
   (b) Prove by contrapositive that if $A \cap B = A$, then $A \subseteq B$.