UNIVERSITY OF DAR ES SALAAM

MATHEMATICS DEPARTMENT

SUPPLEMENTARY UNIVERSITY EXAMINATIONS 2013/2014

DATE: 15TH September 2014 **DURATION: 2 HOURS**

MT100: FOUNDATIONS OF ANALYSIS

Instruction:

Attempt ALL Questions in Section A and ANY TWO questions in Section B.

SECTION A (30 Marks)

Q1. (a) Let $A=\{x : x^2 \le 9\}$, $B=\{x : |x| < 2\}$ and $C=\{x : 1 \le x \le 3\}$. Find,

(i) $(A \cap B) \cup C$ (ii) $(A \cup C) \cap B$ (4 marks)

(b) Let $A=\{x : x \in \mathbb{N}\}$ and $B=\{x : 0 < x^2 \le 4\}$. Sketch the set represented by (i) $B \times A$ (ii) $B \times B$ (4 marks)

Q2. (a) Find the solution set for $\frac{x}{x+2} \ge \frac{1}{3}$. (3 marks)

(b) Evaluate

i. $\lim\limits_{n\to\infty} \frac{3n^2-3n+7}{2n^3+5n^2-2}$

ii. $\lim\limits_{n\to\infty} \frac{5^6+5^n}{5^n-7^n}$

iii. $\lim\limits_{n\to\infty} (2n^2-15n+9)$

iv. $\lim\limits_{n\to\infty} \sqrt{n^2+n}-\sqrt{n^2-1}$

Q3. Let x, y and z be real numbers. For each of the propositions below determine whether it is true or false. If the proposition is true prove it and if it is false give a counterexample.

(a) $\forall x \; \forall y: \; 0 < \frac{1}{x^2+y^2+2} < 0.5$ (2 marks)

(b) $\forall y \; \exists x: \; \frac{x^2}{y^4+5} + \frac{3xy^2}{y^2+2} < 0$ (2 marks)

(c) $\forall x \; \forall y \; \forall z: \; x^2+y^2+z^2 \le x^2y^2z^2$ (2 marks)

(d) $\forall x \; : 0 < \frac{1}{x^2+1} \le 1$ (2 marks)

Q4. (a) Let $A = \{x : x^2 > 0\}$, where x is a real number. Determine if set A is a number ring or number field or neither. (2 marks)

(b) Determine if $\mathbb{Q}'$ the set of irrational numbers is a number ring or number field or neither. (2 marks)

SECTION B (30 Marks)

Q5. (a) Let $K = \left\{ x_n : x_n = \frac{(-1)^n n}{2n+1} \quad , \quad n \in \mathbb{N} \right\}$

(i) Write the sub-sequences of even and odd term of the sequence x_n.

(ii) Find $\sup(K)$, $\inf(K)$, $\max(K)$ and $\min(K)$

(iii) Determine if (x_n) converges or diverges (8 marks)

(b) Use ε-definition to prove that

$$\lim_{n \to \infty} \frac{3n+1}{2n+3} = \frac{3}{2}$$ (7 marks)

Q6. (a) Given that $x = 1+i$ and $x = 1-2i$ are roots of the function

$$f(x) = x^6 - 4x^5 + 10x^4 - 10x^3 - x^2 + 14x - 10.$$

Find other roots of the function f. (6 marks)

(b) Without writing $z = x+iy$, use the definition of modulus to simplify the inequality

$$\left| \frac{2iz+1}{z-i} \right| \le \frac{1}{2}.$$