UDSM

Dept of maths

MT 100: Foundations of Analysis

Timed Test 1 - Marking Scheme 2020/2021

1.(a) All maths student study computer science. (½ mark)

(b) Some maths students do not study computer science. (½ mark)

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(c) $\forall x \geq 1,\ f(x) < 3$ (1 mark)

(d) $\exists x$ in $A \ni \forall y$ in B, $1 \geq y$ or $y > x$ (1 mark)

(e) $\exists x \forall y \exists z \ni x+y+z \neq xyz$. (½ mark)

2.(a) not a novelist. (1 mark)

4

Reason: Addition (1 mark)

(b) Juma is chatting (1 mark)

Reason: Disjunctive syllogism (1 mark)

3(b) (i) B (ii) A∪B (iii) B

(iv) B (v) B (vi) A∪B

5

(vii) A∪B (viii) B

(ix) A (x) B @ ½ [5 marks]

3.(a) Proof: By contradiction

Assume $3n+2$ is odd and n is even. It follows that (½ mark)

$\exists k \in \mathbb{Z} \ni n = 2k$.

2

$$\Rightarrow 3n+2 = 3(2k)+2 = 2(3k+1)$$

$\Rightarrow 3n+2$ is even (½ mark)

This contradicts our hypothesis that $3n+2$ is odd (½ mark)

i.e. the number can not be both even and odd at the same time. Thus the assumption made that n is even WAS FALSE. Therefore if $3n+2$ is an odd then n is odd. # (½ mark)

4.(a)(i) $\forall x \forall y \exists z \ni x^2+y^2 = z$ is TRUE (½ mark)

Proof: Let us express the existence of z in terms of x and y:

Suppose $z = y^2 + x^2$ (½ mark)

$\forall x \forall y,\ x^2+y^2 = y^2+x^2$ is TRUE

(i.e. Commutative property of addition in real numbers) (½ mark)

(ii) $\forall x \forall y,\ x^2+y^2$ is non-negative is TRUE (½ mark)

3

Proof: Required to show that $\forall x, y \in \mathbb{R},\ x^2+y^2 \geq 0$

Let $x-y \in \mathbb{R} \Rightarrow (x-y)^2 \geq 0$

Let $x+y \in \mathbb{R} \Rightarrow (x+y)^2 \geq 0$

$\Rightarrow (x-y)^2 + (x+y)^2 \geq 0$ (½ mark)

$\Rightarrow x^2-2xy+y^2+x^2+2xy+y^2 \geq 0$

$\Rightarrow 2x^2+2y^2 \geq 0$

$\Rightarrow \frac{2x^2}{2} + \frac{2y^2}{2} \geq \frac{0}{2}$.

$\Rightarrow x^2+y^2 \geq 0$ # (½ mark)

4(b) Proof: By contradiction

Assume $\sqrt{7}$ is rational

i.e $\sqrt{7} = \frac{m}{n}$, ----(1), where $m, n \in \mathbb{Z}, n \neq 0$

and $\frac{m}{n}$ is expressed in its lowest terms

(i.e. no common factor between m and n) -- ($\frac{1}{2}$ mark)

$\Rightarrow \quad 7 = \frac{m^2}{n^2}$

$\Rightarrow m^2 = 7n^2$ ----- (2)

$\Rightarrow m^2$ is divisible by 7

$\Rightarrow m$ is also divisible by 7 ----- (3) -- ($\frac{1}{2}$ mark)

$\Rightarrow \exists k \in \mathbb{Z} \ni m = 7k$ ------ (4) -- ($\frac{1}{2}$ mark)

3

Substituting (4) into (2) we have:

$(7k)^2 = 7n^2$

$49k^2 = 7n^2$

$\Rightarrow n^2 = 7k^2$

$\Rightarrow n^2$ is divisible by 7

$\Rightarrow n$ is divisible by 7 ----- (5) -- ($\frac{1}{2}$ mark)

We observe that from (3) and (5) both m and n have common factor 7. This contradicts the assumption that no common factor between m and n.

The assumption made that $\sqrt{7}$ is RATIONAL WAS WRONG. ($\frac{1}{2}$ mark)

Therefore $\sqrt{7}$ is IRRATIONAL. --- ($\frac{1}{2}$ mark)

UNIVERSITY OF DAR ES SALAAM

DEPARTMENT OF MATHEMATICS

MT 100: FOUNDATION OF ANALYSIS

TIMED TEST 1, 2018/2019.

Date: Friday, January 04, 2019 **Time: 50 minutes**

Instruction: Answer **all** questions.

1. Write down the negation of each of the following statements.

   (a) Some pencils are blue.

   (b) All chairs have four legs.

   (c) $\exists x > 1 \ni f(x) = 3$.

   (d) $\forall x$ in $A, \exists y$ in $B \ni x < y < 1$.

   (e) $\forall x \exists y \ni \forall z, x + y + z \leq xyz$.

2. Fill in the blanks below to make the arguments valid. Support your response with a reason:

   (a) If anyone is born of God, then he loves his brothers. Martin does not love his brothers. *Therefore*..............

   (b) If A implies B. .... *Therefore* A implies C.

3. (a) Prove that $\sqrt{3}$ is irrational number.

   (b) Fill in the blanks to complete the proof of the following theorem. Write only the response of each item in a separate sheet of paper provided.

   Theorem 1. $A \subseteq B$ *iff* $A \cup B = B$.

   Proof. **By direct proof**. Suppose that $A \subseteq B$. If $x \in A \cup B$, then $x \in A$ or $x \in$(i)..B... . Since $A \subseteq B$, in either case we have $x \in B$. Thus (ii)A∪B $\subseteq$(iii)..B.. . On the other hand, if $x \in$(iv)..B.., then $x \in A \cup B$, so (v)..B..$\subseteq$(vi)A∪B. Hence $A \cup B = B$. Conversely, suppose that $A \cup B = B$. If $x \in A$, then $x \in$(vii)..A∪B. But $A \cup B = B$, so $x \in$(viii)..B... . Thus (ix)..A..$\subseteq$(x)..B.. . □

4. Determine whether the following propositions are true or false. Give a counterexample to False proposition and prove if the proposition is true for $x, y, z \in \mathbb{R}$.

   (a) $\exists x \ni \forall y \exists z \ni x + y = z$.

   (b) $\forall x,\ x^2 \geq x$.