UNIVERSITY OF DAR ES SALAAM

DEPARTMENT OF MATHEMATICS

MT 100: FOUNDATION OF ANALYSIS

TIMED TEST 1, 2016/2017.

Date: Friday, November 25, 2016 **Time: 50 minutes**

Instruction: Answer **all** questions.

1. Let P = You will pass, G = You goof off, A = You are attentive, R = It is raining, T = I play tennis and S = I play squash. Translate into symbols the statements:

   (a) You will pass unless you goof off, provided that you are attentive. $A \to (\sim G \to P)$

   (b) I will play tennis unless it is raining, in which case I will play squash. $(\sim R \to T) \wedge (R \to S)$

2. Fill in the blanks below to make the arguments valid. Support your response with a reason:

   reason: modus tollens

   (a) If Roza solved the first problem correctly, then the answer she obtained is 137. Roza's answer to the first problem is not 137. Therefore .... Roza solved the first problem incorrectly .

   (b) If integer 35244 is divisible by 396, then the integer 35244 is divisible by 66. .... Therefore, if the integer 35244 is divisible by 396, then the integer 35244 is divisible by 3. 2) (hypothetical syllogism)

3. (a) Prove that $\sqrt{3}$ is irrational number.

   (b) Fill in the blanks to complete the proof of the following theorem.

   THEOREM: Let A, B, and C be the subsets of universal set $\mathcal{U}$. Then $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$.
   Proof: We wish to prove that $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$. To this end, let $x \in A\backslash(B \cup C)$. Then $x \in A$ and ....(i). Since $x \notin B \cup C$, ...(ii) and $x \notin C$ (for if it were in either B or C then it would be in their union). Thus $x \in A$ and $x \notin B$ and $x \notin C$. Hence $x \in A\backslash B$ and $x \in A\backslash C$, which implies that ....(iii). We conclude that $A\backslash(B \cup C) \subseteq (A\backslash B) \cap (A\backslash C)$.
   Conversely, suppose that $x \in$ ....(iv). Then $x \in A\backslash B$ and $x \in A\backslash C$. But then $x \in$ ....(v) and $x \notin$ ....(vi) and $x \notin$ ....(vii). This implies that $x \notin (B \cup C)$, so $x \in$ ....(viii). Hence ....(ix) $\subseteq$ ....(x) as desired. ■

4. Determine whether the following propositions are true or false. Give a counterxample to False proposition and prove if the proposition is true.

   (a) $\forall x,\ x^2 \geq x$.

   (b) $\forall x \forall y,\ 2xy \leq x^2(x - 3y + 1) + y^2(3x - y + 1)$.

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DEPARTMENT OF MATHEMATICS

MT 100: FOUNDATION OF ANALYSIS

TIMED TEST 2, 2016/2017.

Date: Friday, January 13, 2017 **Time: 50 minutes**

Instruction: Answer **all** questions.

1. (a) Let $x, y, z \in \mathbb{R}$. Prove the transitive law that if $x < y$ and $y < z$, then $x < z$.

   (b) Find the sup $f(x)$ and inf $f(x)$ of the function $f(x) = \dfrac{x}{x-2}$ on $(-\infty, 2) \cup (2, \infty)$.

2. (a) Let $z_1, z_2 \in \mathbb{C}$. Without expressing z_1 and z_2 into real and imaginary parts, prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

   (b) Solve the inequality $|x + 3| + |x - 4| < 0$.

3. Let xRy defines the relation that $x - y$ is even. Show that R is equivalent relation and its equivalence classes partition $\mathbb{Z}$.

4. (a) When do we say set A is bounded for every $x \in A$?

   (b) Prove that $|z - w| \geq |z| - |w|, \forall z, w \in \mathbb{C}$.