UNIVERSITY OF DAR ES SALAAM
DEPARTMENT OF MATHEMATICS
MT 100: FOUNDATION OF ANALYSIS
TIMED TEST 2, 2019/2020.

Time: 50 minutes

Date: Wednesday, January 22, 2020

1. (a) Why a set of rational numbers ($\mathbb{Q}$) is an ordered field and not a complete ordered field?

   (b) Prove that $\forall x, y \in \mathbf{F}$, where $\mathbf{F}$ is a field, if $x \neq 0$ and $y \neq 0$, then $xy \neq 0$.

2. (a) Prove the transitive law that, $\forall x, y, z \in \mathbb{R}$, if $x < y$ and $y < z$, then $x < z$.

   (b) Prove that $\sqrt{3}$ is an irrational number.

3. (a) Prove by induction that $\left|\sum_{k=1}^{n} a_k\right| \leq \sum_{k=1}^{n} |a_k|$, $\forall n \in \mathbb{N}$.

   (b) Prove that between any two distinct rational numbers there is an irrational number.