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Name :- Abdul Rafay

Reg No:- FA21-BEE-209

Assignment #2

(Question 1)

(a)

$$\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$$

$$x^2 = 4y^2$$

$$x = 2y$$

$$y = x/2$$

$$\frac{x^2-2(x)(x/2)}{x^2-4(x/2)^2}$$

$$\frac{x^2-x^2}{x^2-x^2} = \frac{0}{0}$$

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(b)

$$\lim_{(x,y)\to(0,0)} \frac{x-4y}{6y+7x}$$

let $6y+7x=0$

$$y=7x/6$$

$$\frac{x-4(7x/6)}{7x+7x}$$

$$\frac{6x-28x}{6(14x)} = \frac{-11}{52}$$

(c)

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{xy^3}$$

let $y=mx$

$$\frac{x^2-(mx)^2}{x(mx)^3}$$

$$= \frac{1-m^6x^3}{m^3x^2}$$

(d)

$$\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-4e^{2y}}{6x+2y-3z}$$

$$\frac{(-1)^3-4(0)e^2}{6(-1)+2(0)-3(4)} = \frac{1}{16}$$

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(Question # 2)

(a)

$f(x,y) = \cos(x/y)$ in $V = (3,-4)$

$$f\nabla = \frac{d}{dx}\left(\cos(x/y)\right)\hat{i} + \frac{d}{dy}\left(\cos(x/y)\right)\hat{j}$$

$$= -\frac{1}{y}\sin\left(\frac{x}{y}\right)\hat{i} + \frac{x}{y^2}\sin\left(\frac{x}{y}\right)\hat{j}$$

unit vector :-

$$\frac{3i-4j}{\sqrt{9+16}} \Rightarrow \frac{3}{\sqrt{25}}\hat{i} - \frac{4}{\sqrt{25}}\hat{j}$$

$$\text{Diff} = \frac{-3}{5y}\sin\left(\frac{x}{y}\right) + \frac{4x}{5y^2}\sin\left(\frac{x}{y}\right)$$

$$= \frac{1}{5y}\sin\left(\frac{x}{y}\right)\left(\frac{4x}{y} - 3\right)$$

(b)

$f(x,y,z) = x^2y^3 - 4xz$; $\vec{v} = (-1,2,0)$

$$\nabla f = (2y^3x\,i + (-4z)\hat{i}) + 3y^2x^2j - 4xk$$

$$\hat{v} = \frac{-i+2j+0k}{\sqrt{1+4}} = \frac{-1}{\sqrt{5}}i + \frac{2}{\sqrt{5}}j + 0k$$

$$\text{Diff} = \frac{-1}{\sqrt{5}}(2y^3x - 4z) + \frac{2}{\sqrt{5}}(3y^2x^2) + 0$$

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Question # 3

$$f(x,y,z) = 4x - y^2 e^{3xz}$$

$$\nabla f = 4 - y^2 e^{3xz}(3z)\hat{i} - 2ye^{3xz}\hat{j} + y^2 e^{3xz} \cdot 3x\,\hat{k}$$

$$= 4 - (-1)^2 e^{3(3)(0)}(3)\hat{i} - 2(-1)e^{3(3)(0)}\hat{j} - (-1)^2 e^{3(3)(0)} \cdot 3(3)$$

$$= 4\hat{i} - 2\hat{j} - 9\hat{k}$$

$$v = (-1, 4, 2)$$

$$\hat{v} = \frac{-1+4j+2k}{\sqrt{(1)+16+4}} = \frac{-1}{\sqrt{21}}\hat{i} + \frac{4}{\sqrt{21}}\hat{j} + \frac{2}{\sqrt{21}}\hat{k}$$

$$\frac{-1}{\sqrt{21}}(4) - \frac{2(4)}{\sqrt{21}} - \frac{4(2)}{\sqrt{21}}$$

$$\frac{-32}{\sqrt{21}}$$

(Question # 4)

(a)

$$f(x,y) = \sqrt{x^2+y^3} \quad \text{at } (-2,3)$$

$$\nabla f = \frac{1}{2}(x^2+y^3)^{-1/2}(2x)\hat{i} + \frac{1}{2}(x^2+y^3)^{-1/2}(3y^2)\hat{j}$$

$$\nabla f(-2,3) = \frac{-2}{\sqrt{13}}\hat{i} + \frac{27}{2\sqrt{13}}\hat{j}$$

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(6)

$f(x,y,z) = e^{2x}\cos(y-2z)$ at $(4,-2,0)$

$\nabla f = (e^{2x}\cdot 2\cos(y-2z))\,i + e^{2x}(-\sin(y-2z))(1)\,j + e^{2x}(-\sin(y-2z))(-2)\,k$

$= e^{4x}\cdot 2\cos(-2-0)\,i + e^{4x}(-\sin(-2))\,j + e^{4x}(-\sin(-2))(-2)$

$= e^{4x}(-2\cos\cdot 2\hat{i} - \sin(-2)\,j + 2\sin(-2)\,k$

(Question 5)

(a)

$\vec{F} = x^2y\,\hat{i} - (z^3-3x)\,\hat{j} + 4y^2\,\hat{k}$

$\nabla\cdot\vec{F} = \nabla f\cdot\hat{f}$

$= (2xy\,i)\cdot(x^2y\,i - (z^3-3x)\,j + 4y^3\,\hat{k})$

$\text{Div } f = 2x^3y^2$

$\text{Curve} = \nabla f \times f$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & -(z^3-x^3) & 4y^2 \end{vmatrix}$

$= (8y+3z^2)\,i - 0\,j + 7x^2\,\hat{k}$

$= (\cancel{8x} + 2z^2)\,i + \cancel{\frac{3}{2}y^2\,j + 7x^2\,\hat{k}}$

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(6)

$$F = (8x + 2z^2)\hat{i} + \frac{x^3y^2}{2}\hat{j} - (z - 7x)\hat{k}$$

$$div = \nabla f \cdot f$$

$$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right) \cdot \left(8x + 2z^2)\hat{i} + \frac{1}{2} x^3y^2 j - (z - 7x)\hat{k}\right)$$

$$= 2 + \frac{2x^3}{2} y - (1)$$

$$= 2 + \frac{2x^3}{2} y - 1$$

$$Curve = \nabla f \times f$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x + 2z^2 & \frac{x^3y^2}{2} & -(z - 7x) \end{vmatrix}$$

$$i\left(0 + \frac{x^3y^2}{2}\right) - j\,(7 - 4z) + k\left(\frac{3x^2y^2}{2} - 0\right)$$

$$\frac{x^3}{2}y^2 i - (7 - 4z)j + \frac{(3x^2y^2)}{2} k$$

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Sol (Question #6)

(a)

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}, \quad \frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$\frac{\partial N}{\partial z} = \frac{-2x^3}{z^3}, \quad \frac{\partial P}{\partial y} = \frac{-2x^3}{z^3}$$

$$\frac{\partial M}{\partial z} = \frac{-6x^2y}{z^3}, \quad \frac{\partial P}{\partial x} = -\frac{6x^2y}{z^3}$$

So vector is conservative.

Sol (b)

$$F = 6x\,i + (2x - y^2)\,j + (6z - x^3)\,k$$

$$\frac{\partial M}{\partial y} = 0 \; ; \quad \frac{\partial N}{\partial x} = 2$$

$$\frac{\partial N}{\partial z} = 0 \; , \quad \frac{\partial P}{\partial y} = 0$$

$$\frac{\partial M}{\partial z} = 0 \; , \quad \frac{\partial P}{\partial x} = -3x^3$$

Since $\partial P/\partial x$ and $\partial N/\partial x$ are not equal it is non-conservative

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Sol (Question #3)

(a)

$$z = \frac{x^2 - w}{y^4} \; ; \; x = t^3 + 2 \; ; \; y = \cos(2t) \; , \; w = 4t$$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt} + \frac{dz}{dw} \cdot \frac{dw}{dt}$$

$$= \left(\frac{2x}{y^4} \cdot 3t^2\right) + 4\left(\frac{1}{y^5}(x^2 - w)\right) \cdot (-\sin 2t)(2)$$

$$+ \frac{-1}{y^4} \cdot 4$$

$$= \frac{2}{y^4}\left(3x t^2 + \frac{4 \sin 2t \,(x^2 - w)}{y} - 2\right)$$

(b)

Sol

$$z = x^2 y^4 - 2y \; , \quad y = \sin(x^2)$$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

$$= (4y^3 x^2 - 2) \cdot \cos(x^2) \cdot 2x$$

$$= 8 y^3 x^3 \cos(x^2) - 4x \cos(x^2)$$

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(C)

$$x^2y^4 - 3 = \sin(xy)$$

$$\frac{d}{dx}(x^2y^4 - 3) = \frac{d}{dx}(\sin(xy))$$

$$2xy^4 - y\cos xy = x\frac{dy}{dx}\cos xy - 4y^3x^2\frac{dy}{dx}$$

$$2xy^4 - y\cos xy = \frac{dy}{dx}(x\cos xy - 4y^3x^2)$$

$$\frac{dy}{dx} = \frac{2xy^4 - y\cos xy}{x\cos xy - 4y^3x^2}$$