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$$n = \vec{AB} \times \vec{AC} = \begin{bmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{bmatrix}$$

$$= i(-4+15) - (24-0)\hat{j} + (-24)\hat{k}$$
$$= \hat{i} + 4\hat{j} + 4\hat{k}$$

$$d = o \cdot n = (4\hat{i} - 4\hat{j} + \hat{k}) \cdot (\hat{i} + 4\hat{j} + 4\hat{k})$$
$$= 4 - 16 + 4 = -8$$

Equation of plane.

$$r \cdot n = d$$
$$r(\hat{i} + 4\hat{j} + 4\hat{k}) = -8$$
$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + 4\hat{j} + 4\hat{k}) = -8$$

$$x + 4y + 4z + 8 = 0$$

b)

$$\text{prep distance} = \frac{d}{|n|} = \frac{+8}{\sqrt{(1)^2 + (4)^2 + (4)^2}}$$
$$= \frac{8}{33} = 1.39$$

c)

$$\text{Line } OD: r = a \times \lambda b$$

$$r\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}$$

as same value of λ

$$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$$

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$$2\lambda + 12\lambda - 12\lambda = -8$$

$$2\lambda = -8$$

$$\lambda = -4$$

$$r = \begin{bmatrix} 2(-4) \\ 2(-4) \\ -3(-4) \end{bmatrix} = (-8, -12, 12)$$

Question # 3

(a)

$$l_1 = t\hat{i} + \hat{j} \qquad -2\hat{i} - \hat{j}$$

$$l_2 = \hat{j} + t\hat{k} \qquad -2\hat{j} + \hat{k}$$

The shortest distance b/w l_1 and l_2 is $\sqrt{21}$.

$$r_1 = OA + \lambda AB$$

$$r_2 = OA + \lambda AB$$

$$r_1 = t\hat{i} + \hat{j} + \lambda 2\hat{i} - \hat{j}$$

$$l_1 = \vec{r_1} = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -10 \\ 0 \end{bmatrix}$$

$$l_2 = \vec{r_2} = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

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$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{21}$$

$$(a_2 - a_1) = -t\hat{i} + t\hat{k}$$

$$D = \frac{(-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})}{\sqrt{21}}$$

$$\sqrt{21} = t + 4t/\sqrt{21}$$

$$21 = 5t$$

$$t = 21/5$$

(b) $$\vec{r_1} = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$$

$$\vec{r_2} = \hat{j} - \frac{21}{5}\hat{k} + \mu(-2\hat{j} + \hat{k})$$

$$\pi_1 = r = \vec{OA} + \lambda\vec{AB} + \mu AC$$

$$\vec{r} = \frac{-21}{5}\hat{i} + \hat{j} + \pi(-2\hat{i} - \hat{j}) + \mu(-2\hat{j} + k)$$

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(C) $\lambda_2 = 5x - 6y + 5z = 0$

$$\lambda_2 = \frac{x-0}{0}, \quad \lambda_2 = \frac{y-1}{-2}$$

$$\lambda_2 = \frac{2-4.2}{1}$$

From l_2 direction vector is

$= \{0, -2, 1\}$

From π_2 normal vector is

$= (5, -6, 7)$

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|a|\,|b|}$$

$$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ -21/5 \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$= -6 + 29.4 = 23.4$

$$|a| = \sqrt{(1)^2 + \left(\frac{21}{5}\right)^2} = 4.3$$

$$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2} = 10.49$$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$\theta = 59.34°$

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(d)

$$\pi_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}, \quad \pi_2 \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$\pi_1 \cdot \bar{\pi}_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -21 - 6 = -27$$

$$|a| = \sqrt{\left(\frac{21}{5}\right)^2 + (1)^2} = 4.3$$

$$|b| = \sqrt{(5)^2 + (-6)^2 + 7^2} = 10.49$$

$$\theta = \cos^{-1} \frac{-27}{4.3 \times 6.49}, \quad \theta = 126.78$$

acute angle:-

$$\phi = 180 - 126.78$$
$$= 53.23^\circ$$

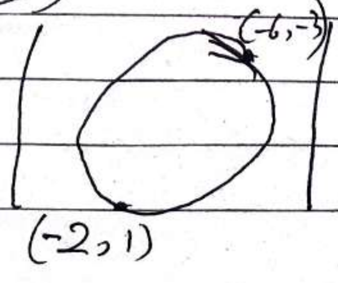
Question # 5

Sol,

Mid point.

$$\left(\frac{-2-6}{2}\right) \left(\frac{-1-3}{2}\right)$$

$$(-4, -2)$$



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so

$$r^2 = (4)^2 + (-3)^2$$

$$r^2 = \pm 5$$

neglat -ve.

$$r = 5$$

(c)

$$y^2 = 100x$$

compare with

$$y^2 = 4ax$$

$$4a = 100$$

$$a = 25$$

equation of diretion = $x = -a$

$$x = -25$$

(d)

$$x^2 = 24y$$

compare with

$$x^2 = 4ay.$$

$$4a = 24.$$

$$a = 8$$

so facus is $F(a, 0) = F(8, 0)$

and equation of direction is.

$$x = -a.$$

$$x = -8$$

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equation of circle = $(x+h)^2+(y-k)^2=r^2$ — ①

$(x+4)^2+(y+2)^2=r^2$

$\times\ (x,y)=(-2,-1)$

$(-2+4)^2+(-1+2)^2=r^2$

$r^2=5$

put the value

$(x+4)^2+(y+2)^2=5$

(6) sol

equation of circle.

$(x-h)^2+(y-k)^2=r^2$

Let $x=0$, $y=b$

at point $(4,0)$

$(4)^2+(-b)^2=r^2$

$16+b^2=r^2$ — ①

at point $(0,2)$

$(0)^2+(2-b)^2=r^2$

$(2-b)^2=r^2$ — ②

comparing eqs.

$16+b^2=(2-b)^2$

$12+4b=0$

$b=-3$

put the values in ①

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(e)

$$\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$$

compare with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$$

$a = 5$

$b = 4$

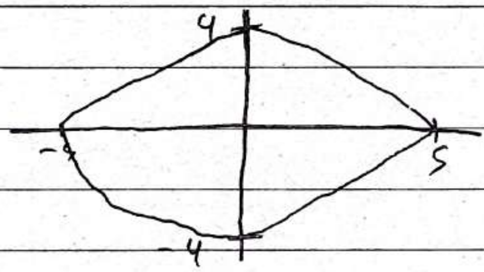
$c = \sqrt{a^2 - b^2} = \sqrt{25-16} = \pm 3$

$F_1 = (3, 0)$

$F_2 = (-3, 0)$

length of major axis $= 2a$

$= 2(5) = 10$



(f) ~~sol~~

major axis = 10

minor axis = 8

$2a = 10$

$a = 5$

$2b = 8$

$b = 4$

Equation of ellipse along x-axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

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(ii) $\pi_2 = \vec{r_2} = \vec{OA} + \lambda \vec{AB} + u \vec{AD}$

when $\lambda = 4$.

$\vec{AD} = \vec{OD} - \vec{OA}$

$$= \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$\pi_2 = \vec{r_2} = \vec{OA} + \lambda \vec{AB} + \mu \vec{AD}$

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 5 \end{bmatrix}$$

$x = 7 + 4\lambda - 5\mu$ — (I)
$y = 7 - \lambda + 3\mu$ — (II)
$z = -1 + \lambda + 5\mu$ — (III)

eq (II) + eq (III)

$y + z = 3 + 8\mu$ — (4)

Multiply eq (II) by 2

$y = 4 - \lambda + 3\mu$
$\times 4$

$4y = 16 - 4\lambda + 12u$ — (5)

eq (1) + eq (2)

$x = 7 + \cancel{4\lambda} - 5\mu$
~~$4y = 16 - 4\lambda + 12u$~~

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$x + 4y = 23 + 7u$

$\frac{x + 4y - 23}{7} = u$

put the values in eq

$y + z = 3 + 8\left(\frac{x + 4y - 23}{7}\right)$

multiply both side by 7

$7y + 7z = 21 + 8x + 32y - 184$

$8x + 25y - 7z = 163$

(Part C)

$\pi_1 = \vec{r_1} = \vec{OA} + \lambda \vec{AB} + \mu \vec{AD}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$x = 7 + 4\lambda - 5\mu$ —— 1

$y = 4 - \lambda + 3\mu$ —— 2

$z = -1 + \lambda + 2\mu$ —— 3

eq (ii) + eq (iii)

$y = 4 - \lambda + 3u$

$z = -1 + \lambda + 2u$

$y + z = 3 + 5u$ —— 4

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$|x_1| = \sqrt{5^2 + 13^2 + (-1)^2} = 4\sqrt{3}$

$|x_2| = \sqrt{8^2 + 25^2 + (-7)^2} = 3\sqrt{82}$

$\theta = \cos^{-1}\left(\frac{414}{4\sqrt{3} \times 3\sqrt{82}}\right.$

$\theta = 12.15^\circ$

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Question #2

$A = 7\hat{i} + 4\hat{j} - \hat{k}$, $B = 11\hat{i} + 3\hat{j}$

$C = 2\hat{i} + 6\hat{j} + 3\hat{k}$ $D = 2\hat{i} + 7j + \lambda\hat{k}$)

$r_1 = \vec{OA} - \vec{AB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$$= \begin{bmatrix} 11 \\ 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$\vec{r_1} = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$r_2 = \vec{OC} - \vec{CD}$

$\vec{CD} = \vec{OD} - \vec{OC}$

$$= \begin{bmatrix} 2 \\ 7 \\ \lambda \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \lambda-3 \end{bmatrix}$$

$$b_1 \times b_2 = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ 0 & 1 & \lambda-3 \end{bmatrix}$$

$$= i \begin{vmatrix} -1 & 1 \\ 1 & \lambda-3 \end{vmatrix} - \hat{j} \begin{vmatrix} 4 & 1 \\ 0 & \lambda-3 \end{vmatrix} + \hat{k} \begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix}$$

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$$= (2-\lambda)\hat{i} - \hat{j}(4\lambda - 12) + 4\hat{k}$$

$$a_2 - a_2 = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$$

$$(b_1 \times b_2) = \sqrt{(2-\lambda)^2 + (4\lambda - 12)^2 + (4)^2}$$

$$= \sqrt{17\lambda^2 - 100\lambda + 164}$$

$$(b_1 \times b_2) \cdot (a_1 - a_2) = [(2-\lambda)\hat{i} - \hat{j}(4\lambda - 12) + 4\hat{k}] \cdot (-5\hat{i} + 2\hat{j} + 4\hat{k})$$

$$= -5(2-\lambda) - 2(4\lambda - 12) + 16$$

$$= 30 - 3\lambda$$

$$d = \frac{(b_1 \times b_2) \cdot (a_1 - a_2)}{|b_1 \times b_2|}$$

$$3 = \frac{30 - 3\lambda}{\sqrt{17\lambda^2 - 100\lambda + 164}}$$

Taking square on b.s.

$$a = \frac{(30 - 3\lambda)^2}{(\sqrt{17\lambda^2 - 100\lambda + 164})^2}$$

$$a = \frac{900 + 9\lambda^2 - 180\lambda}{17\lambda^2 - 100\lambda + 164}$$

$$144\lambda^2 - 720\lambda + 576 = 0$$
$$4(36\lambda^2 - 180\lambda + 144) = 0$$
$$4(9\lambda^2 - 45\lambda + 36) = 0$$
$$9(\lambda^2 - 5\lambda + 4) = 0$$

$$\lambda^2 - 4\lambda - \lambda + 4 = 0$$
$$\lambda - 4 = 0 \quad , \quad \lambda - 1 = 0$$
$$\lambda = 4 \quad , \quad \lambda = 1$$

put $\lambda = 4$ in eq ①

$$(4)^2 - 5(4) + 4 = 0$$
$$0 = 0$$

put $\lambda = 1$ in eq ①

$$(1)^2 - 5(1) + 4 = 0$$
$$0 = 0$$

sol (Part 2)

i) $\vec{r}_1 = \vec{r}_1 = \vec{OA} + s\vec{AB} + t\vec{AD}$

when $\lambda = 1$

$$\vec{AD} = \vec{OD} - \vec{OA}$$
$$= \begin{bmatrix} 2 \\ 7 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 2 \end{bmatrix}$$

$$\vec{r}_1 = 7\hat{i} + 4\hat{j} - \hat{k} + s(4\hat{i} - \hat{j} + \hat{k})$$
$$+ t(-5\hat{i} + 3\hat{j} + 2\hat{k})$$

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multiply eq (11) by 4

$4y = 16 - 4\lambda + 12$ —— (5)

Add eq (1) and (5)

$x = 7 + 4\lambda - 5u$

$4y = 16 - 4\lambda + 2$

$x + 4y = 23 + 7u$

$$u = \frac{x + 4y - 23}{7}$$

$$y + z = 3 + 5\left(\frac{x + 4y - 23}{7}\right)$$

multiply b.s by 7

$7y + 7z = 21 + 5x + 20y - 115$

$-5x - 13y + 7z + 94 = 0$

$5x + 13y - 7z = 94$

$$\theta = \cos^{-1} \frac{\pi_1 \cdot \pi_2}{|\pi_1| |\pi_2|}$$

$$\pi_1 \cdot \pi_2 = \begin{bmatrix} 5 \\ 13 \\ -7 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 25 \\ -7 \end{bmatrix}$$

$= \cancel{49} = 414$