TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Logic Programming and Argumentation Group

ADVANCED PROBLEM SOLVING AND SEARCH

Lecture 1

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Dresden

DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

What to expect

- The course has 12 lectures, 10 tutorials
- Lectures will take place on Tuesday in DS2, 9:20-10:50, in room GÖR/0226/H.
- There are 2 tutorial groups,
  1. Thursday DS4, 13:00-14:30, in room APB E005,
  2. Monday DS4, 13:00-14:30, in room TOE/317/H.
- Schedule will be available at course web-page `https://iccl.inf.tu-dresden.de/web/Advanced_Problem_Solving_and_Search_(WS2024)`
- Additionally, the lecture material will be available in OPAL `https://bildungsportal.sachsen.de/opal/auth/RepositoryEntry/46238597126` and the **forum** will be used for discussions related to lecture and tutorial.
- Any questions related to the course must be asked in the forum, not via email. Also, you are strongly encouraged to answer your peer's questions if you know the solution.

Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Evolutionary Algorithms/ Genetic Algorithms
6. Answer-set Programming (ASP)
7. Constraint Satisfaction Problems (CSP)
8. Structural Decomposition Techniques (Tree/Hypertree Decompositions)

What are the Ages of my Three Sons?

Two men meet on the street. One gives the other a puzzle

A: "All three of my sons celebrate their birthday this very day! So, can you tell me how old each of them is?"

B: "Sure, but you'll have to tell me something about them."

A: "The product of the ages of my sons is 36."

B: "That's fine but I need more than just this."

A: "The sum of their ages is equal to the number of windows in that building."

B: "Still, I need an additional hint to solve your puzzle."

A: "My oldest son has blue eyes."

B: "Oh, this is sufficient!"

What are the Ages of my Three Sons? ctd.

"The product of the ages of my sons is 36."

son 1	son 2	son 3
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

son 1	son 2	son 3
36	1	1
18	2	1
12	3	1
9	4	1
9	2	2
6	6	1
6	3	2
4	3	3

What are the Ages of my Three Sons? ctd.

"The sum of their ages is equal to the number of windows in that building."

36	+	1	+	1	=	38
18	+	2	+	1	=	21
12	+	3	+	1	=	16
9	+	4	+	1	=	14
9	+	2	+	2	=	13
6	+	6	+	1	=	13
6	+	3	+	2	=	11
4	+	3	+	3	=	10

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36	+	1	+	1	=	38
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"My oldest son has blue eyes."

$$
\begin{array}{ccccccc}
36 & + & 1 & + & 1 & = & 38 \\
18 & + & 2 & + & 1 & = & 21 \\
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\hline
\multicolumn{1}{|c}{9} & + & 2 & + & 2 & = & \multicolumn{1}{c|}{13} \\
\multicolumn{1}{|c}{6} & + & 6 & + & 1 & = & \multicolumn{1}{c|}{13} \\
\hline
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\end{array}
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What was difficult on this problem?

Problem Solving

- Where to begin?
- You have to create the plan for generating a solution.
- Always consider all of the available data.
- Can you make connections between the goal and what is given?

Why are Some Problems Difficult to Solve?

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- The number of possible solutions in the search space is too large for an exhaustive search.
- The problem is too complicated, and simplified models of the problem are useless.
- The evaluation function of the quality of a solution is noisy or varies with time, which requires an entire series of solutions.
- There are so many constraints that finding even one feasible answer is difficult, let alone searching for an optimal solution.
- The person solving the problem is inadequately prepared.

The Size of the Search Space

Boolean Satisfiability Problem (SAT)

Make a compound statement of Boolean variables evaluate to TRUE.

- For example, consider the following problem of 100 variables given in conjunctive normal form (CNF):

$$F(x) = (x_{17} \vee \neg x_{37} \vee x_{73}) \wedge (\neg x_{11} \vee \neg x_{56}) \wedge \cdots \wedge (x_2 \vee x_{43} \vee \neg x_{77} \vee \neg x_{89} \vee \neg x_{97}).$$

- Challenge: find the truth assignment for each variable x_i, for all $i = 1, \ldots 100$ s.t. $F(x) = \text{TRUE}$.

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Space of possible solutions.

- Any binary string of length 100 is a possible solution.
- Two choices for each variable, and taken over 100 variables, generates 2^{100} possibilities.

The Size of the Search Space ctd.

- Size of the search space $\mathcal{S}$ is
  $|\mathcal{S}| = 2^{100} \approx 10^{30} = 1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000.$
- The number of bacterial cells on Earth is estimated at around 5×10^{30}.
- If we had a computer that could test 1000 strings per second and could have started at the beginning of time itself, 15 billion years ago (Big Bang!) we would have examined fewer than 1% of all the possibilities by now!
- Trying out all alternatives is out of the question.
- Choice of which evaluation function to use.
- Solutions closer to the right answer should yield better evaluations than those who are far away.
- If we try a string x and $F(x)$ returns TRUE, we are done. But what if $F(x)$ returns FALSE?
- How to find a function which gives more than just "right" or "wrong"?

The Size of the Search Space ctd.

Traveling Salesperson Problem (TSP)

- Given n cities and the distances between each pair of cities;
- Traveling salesperson must visit every city exactly once and return home covering the shortest distance.

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Seach Space

- Set of permutations of n cities.
- $2n$ different ways (for symmetrical TSP) to represent one tour.
- There are $n!$ ways to permute n numbers.
- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$

The Size of the Search Space ctd.

- $|\mathcal{S}| = n!/(2n) = (n-1)!/2$
- For any $n > 6$, number of possible solutions to the TSP with n cities is larger than the number of possible solutions to the SAT problem with n variables.
- For $n = 6$: $5!/2 = 60$ solutions to the TSP and $2^6 = 64$ solutions to a SAT.
- For $n = 7$: 360 solutions to the TSP and 128 to the SAT.
- Search space increases very quickly with increasing n.
- A 50-city TSP has more solutions than existing liters of water on the planet.
- However, the evaluation function for the TSP is more straightforward than for SAT.
- Table with distances between each pair of cities.
- After n addition operations we could calculate the distance of any candidate tour and use this to evaluate its merit.
- $cost = dist(15,3) + dist(3,11) + \cdots + dist(6,15)$

Modeling the problem

- We only find the solution to a model of the problem.
- All models are simplifications of the real world.
- Problem $\rightarrow$ Model $\rightarrow$ Solution
  1. Use an approximate model of a problem and find the precise solution: Problem $\rightarrow$ Model$_a$ $\rightarrow$ Solution$_p$(Model$_a$)
  2. Use a precise model of the problem and find an approximate solution: Problem $\rightarrow$ Model$_p$ $\rightarrow$ Solution$_a$(Model$_p$)
- Which one is better?

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- Which one is better?
- Solution$_a$(Model$_p$) is better than Solution$_p$(Model$_a$).

Change over time

Problems my change

- before you model them,
- while you derive a solution, and
- after you execute the solution.

TSP - Travel time between two cities depends on many factors:

- traffic lights
- slow-moving trucks
- flat tire
- weather
- many more...

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Constraints

- Almost all practical problems pose constraints
- Two types of constraints:
  - Hard constraints, and
  - Soft constraints.
- Constraints make the search space smaller, but
  - It is hard to create operators that will act on feasible solution and generate in turn new feasible solutions that are an improvement of previous solution.
  - The geometry of search space gets tricky.

Constraints ctd.

Timetable of the classes at a college in one semester

We are given

- list of courses that are offered;
- list of students assigned to each class;
- professors assigned to each class;
- list of available classrooms, and information for size and other facilities that each offer.

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Construct timetables that fulfill hard constraints:

- Each class must be assigned to an available room that has enough seats and requisite facilities.
- Students who are enrolled in more than one class can not have their classes held at the same time on the same day.
- Professors can not be assigned to teach courses that overlap in time.

Constraints ctd.

Timetable - Soft Constraints:

- Courses that meets twice a week should preferably be assigned to Mondays and Wednesdays or Tuesdays and Thursdays.
- Courses that meets three times per week should preferably be assigned to Mondays, Wednesdays, and Fridays.
- Course time should be assigned so that students do not have to take final exams for multiple courses without any break in between.
- If more than one room satisfies the requirements for a course and is available at the designated time, the course should be assigned to the room with the capacity that is closest to the class size.

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- Any timetable that meets the hard constraints is feasible.
- The timetable has to be optimized in the light of soft constraints.
- Each soft constraint has to be quantified.
- We can evaluate two candidate assignments and decide that one is better than other.

Solve the Problem!

- Mr. Smith and his wife invited four other couples for a party.
- When everyone arrived, some of the people in the room shook hands with some of the others.
- Nobody shook hands with their spouse and nobody shook hands with the same person twice.
- After that, Mr. Smith asked everyone how many times they shook someone's hand.
- He received different answers from everybody.
- How many times did Mrs. Smith shake someone's hand?

Summary

Problem solving is difficult for several reasons:

- Complex problems often pose an enormous number of possible solutions.
- To get any sort of solution at all, we often have to introduce simplifications that make the problem tractable. As a result, the solutions that we generate may not be very valuable.
- The conditions of the problem change over time and might even involve other people who want to fail you.
- Real-world problems often have constraints that require special operations to generate feasible solutions.

References

Zbigniew Michalewicz and David B. Fogel.
How to Solve It: Modern Heuristics, volume 2. Springer, 2004.