30/June/2021 Professor: M. WAQAS Start

Discrete Structure ABDUL RAFEH CSC-20S-104

1 Ans

Whate Is Lattice

A lattice is on abstract structure studied in the mathmatical subdisciplines of order theory and abstract algebra. It consists of a partially ordered set in which every two elements have a unique supremum (also called a least upper bound or join) and a unique infimum (also called a greatest lower bound or meet.

Lattice partially ordered set in which every pair of elements has a unique glb and a unique lub.

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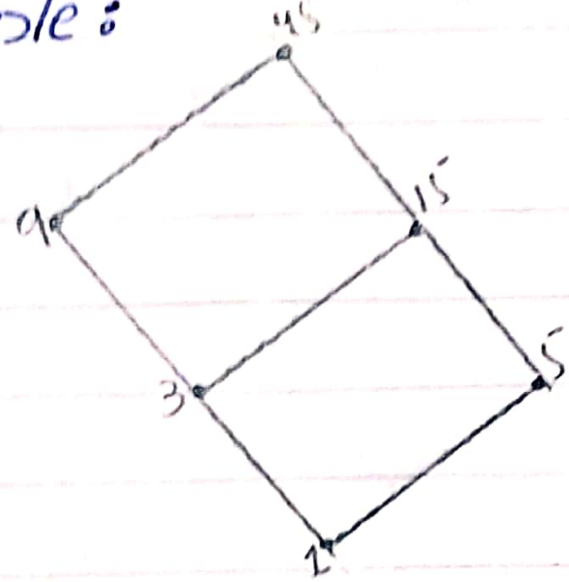
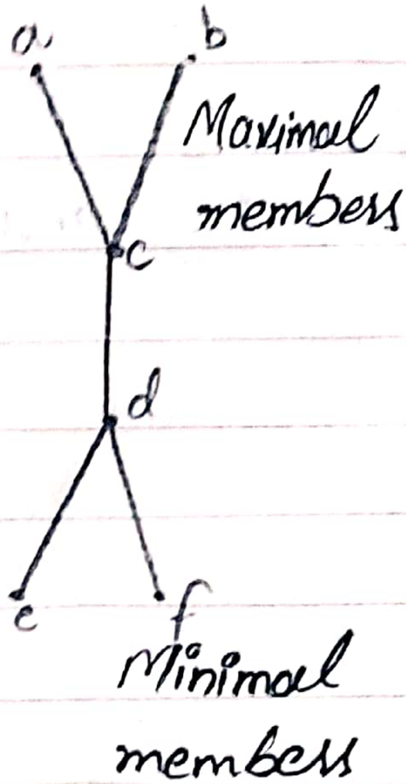
- least elements denoted as 0
- Greatest elements denoted as 1
- For each elements of a lattice $a \leq 1$ and $0 \leq a$

Example:



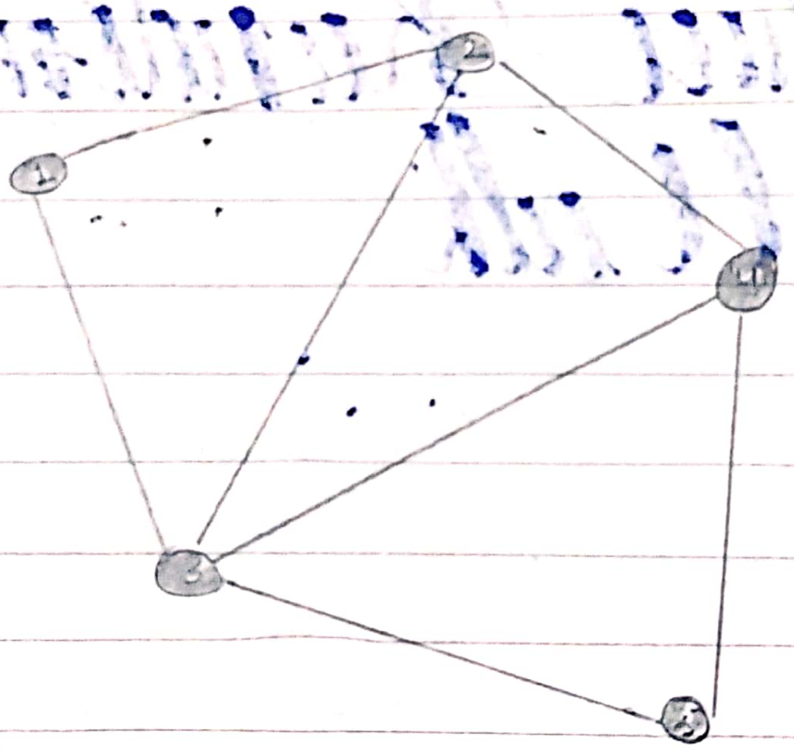


Let L be A Non-empty set closed under two binary operations called Meet and Join. Denoted by $\wedge$ and $\vee$. Then L is called A lattice if the following axioms hold where A, B, C are elements in L:

1) Commutative Law :- (A) $A \wedge B = B \wedge A$ (B) $A \vee B = B \vee A$
2) Associative Law :- (A) $(A \wedge B) \wedge C = A \wedge (B \wedge C)$
(B) $(A \vee B) \vee C =$
3) Absorption Law :- (A) $A \wedge (A \vee B) = A$ (B) $A \vee (A \wedge B) = A$

Lattice Random Walk

A popular random walk model is that of a random walk on a regular lattice where at each step the location jumps to another site according to some probability distribution. In a simple random walk, the location can only jumps to neighboring sits of the lattice, forming a lattice path. In simple symmetric random walk on a locally finite lattice, the probabilities of the location jumping to each one of its immediate neighbors are the same. The best-studied example is of random walk on the d-dimensional integer lattice (sometimes called the hypercubic lattice)



Lattice Walking Problem

On a lattice we want to looks at walks, that connect the vertices of the lattice. The basic component of a walk is a step, which essentially is nothing else than an edge.

Definition 1.2: let $\Lambda = (V, E)$. An-step lattice path or lattice walk or walk from $s \in V$ to $x \in V$ is a sequence $w = (w_0, w_1, \ldots, w_n)$ of elements in V. Such that 1. $w_0 = s$, $w_n = x$, 2. $(w_i, w_{i+1}) \in E$. The length $[w]$ of a lattice path is the number n of steps (edges) in the sequence w. In most cases of the work we are

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going to work on the Euclidean lattice which we define to consist of the vertices Z^d and to be periodic. The edges are mostly defined through a so called step set. On this lattice an alternative definition via the step set can be used.

Ans 2

Partially Ordered Sets (POSETS)

A partial order is a binary relation "$\leq$" over a set p which is reflexive, anti-symmetric, and transitive, i.e., which satisfies for all a, b and c in p.

- $a \leq a$ (reflexivity);
- if $a \leq b$ and $b \leq a$ then $a = b$ (anti-symmetry);
- if $a \leq b$ and $b \leq c$ then $a \leq c$ (transitivity).

A set with a partical order is called partially orderd set or poset.

In order theory, a Hasse diagram is a type of mathematical diagram used to represent partially ordered set, in the form of a drawing of its transitive reduction.

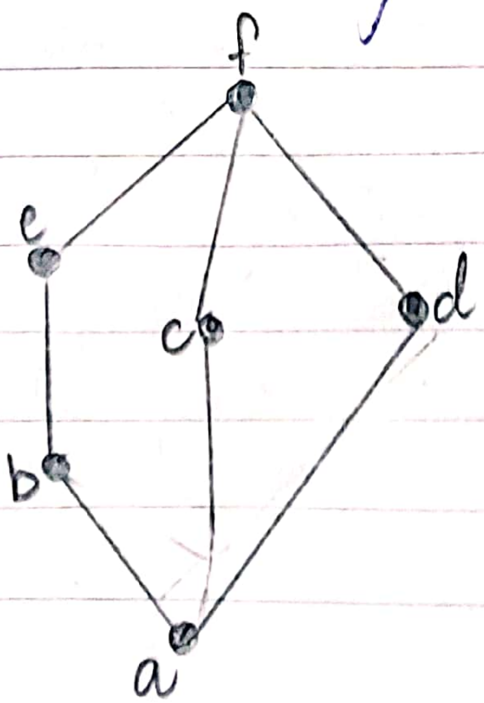
Named after helmet hasse (1898-1979);

The Hasse diagram of a finite poset P is the graph with vertices $x \in P$ and

- If $x < y$, then y is drawn above x in the diagram;
- If y covers x then there is an edge between x and y in the diagram.

Example: If $P = \{a, b, c, d, e, f\}$ and $a < b$, $a < c$, $a < d$, $b < e$, $e < f$, $c < f$, $d < f$

Then the hasse diagram will be ....

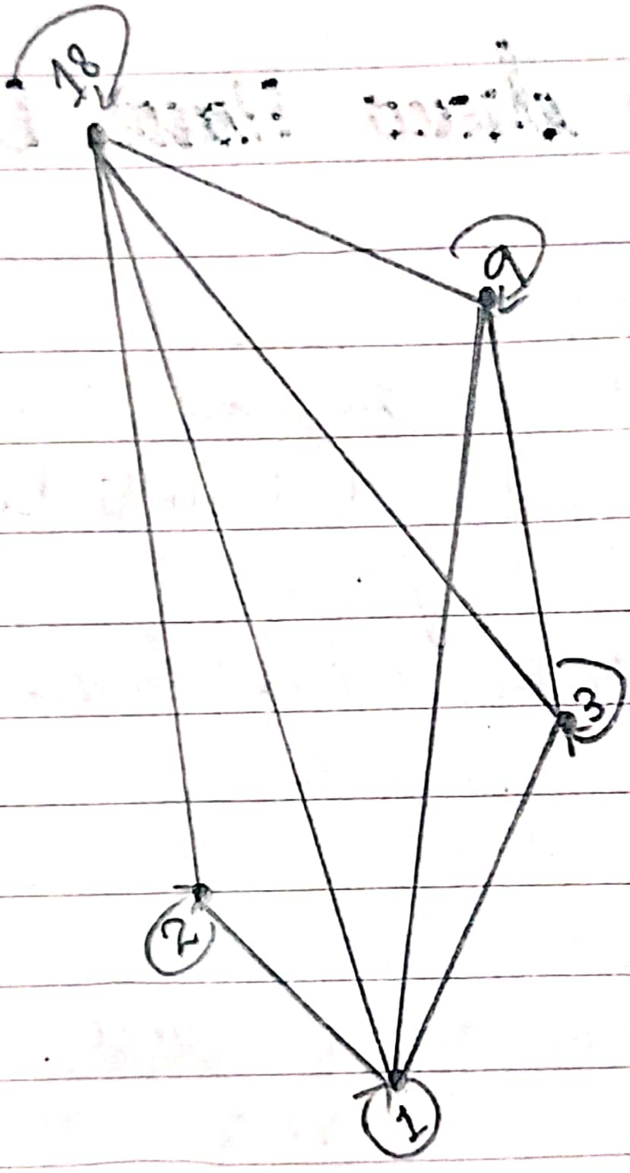


Procedure to draw Hasse Diagram

1 Start with diagraph of partical order.
2 Remove the loop at the each vertex (reflexivity)
3 Remove all the edges that must be present because of transitivity.
4 Arrange each edges so that all arrows point upwards.
5 Remove all arrowheads.

Q. Let $A = \{1, 2, 3, 9, 18\}$ and consider the 'devides' relation A: for all $a, b \in A$, $a/b \Leftrightarrow b = Ka$ for some integer K.

The directed graph for the given relation is $\rightarrow \rightarrow \rightarrow$


18
9
3
2
1

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The Growth Of Function

The growth of functions is usally describe using the big-O notation.

Definition: Let f and g be functions from the integers or the real numbers to the real numbers. We say that $f(X)$ is $O(g(X))$ if there are constants C and K such that

$$|f(X)| \leq C|g(X)|$$

whenever $X > K$.

When we analyze the growth of complexity functions, $f(X)$ and $g(X)$ are always positive.

Therefore, we can simplify the big-O requirment to

$f(x) \leq C \cdot g(x)$ whenever $x > K$.

If we want to show that $f(x)$ is $O(g(x))$, we only need to find one pair (C, K) (which is never unique).

The idea behind the big-O notation is to establised an upper boundary for the growth of a function $f(x)$ for large x.

This boundary is specified by a function $g(x)$ that is usally much simpler than $f(x)$.

We accept the constant C in the requirment $f(x) \leq C \cdot g(x)$ whenever $x > K$,

because C does not grow with x.

We are only interested in large x, so it is ok if $f(x) > C \cdot g(x)$ for $x \leq K$.

Example:

Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$.

for $x > 1$ we have:

$x^2 + 2x + 1 \leq x^2 + 2x^2 + x^2$

$\Rightarrow x^2 + 2x + 1 \leq 4x^2$

Therefore, for $C = 4$ and $K = 1$;

$f(x) \leq Cx^2$ whenever $x > K$.

$\Rightarrow f(x)$ is $O(x^2)$.

Question: If $f(x)$ is $O(x^2)$, is it also $O(x^3)$?

Yes: x^3 grows faster than x^2, so x^3 grows also faster than $f(x)$

Therefore, we also have to find the smallest simple function $g(x)$ for which $f(x)$ is $O(g(x))$.

"popular" function g(n) are n log n, 1, 2^n, n^2, n!, n, n^3, log n

List from slowest to fasted growth:

- 1
- log n
- n
- n log n
- n^2
- n^3
- 2n
- n!

A problem that can be solved with polynomial worst-case complexity is called tractable:

problems of higher complexity are called intractable.

problems that no algorithm can solved are called unsolvable.

for any polynomial $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers, $f(x)$ is $O(x^n)$.

If $f_1(x)$ is $O(g(x))$ and $f_2(x)$ is $O(g(x))$, then $(f_1 + f_2)(x)$ is $O(g(x))$.

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then $(f_1 + f_2)(x)$ is $O(\max(g_1(x), g_2(x)))$

If $f_1(x)$ is $O(g_1(x))$ and $f_2(x)$ is $O(g_2(x))$, then $(f_1 f_2)(x)$ is $O(g_1(x) g_2(x))$.

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Complexity - Examples

What does the following algorithm computes?

Procedure who-Knows ($a_1, a_2 \ldots, a_n$: integers)
who-Knows := 0

for i := 1 to n-1
  for j := i+1 to n
    if $|a_i - a_j|$ > who-Knows then who-Knows := $|a_i - a_j|$ {who-Knows is the maximum difference between any two numbers in the input sequence}

Comparisons : $n-1 + n-2 + n-3 \ldots + 1$
$= (n-1)n/2 = 0.5n^2 - 0.5n$
The complexity is $O(n^2)$.

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Ch 4

Pascal Triangle :-

Introduction

- What is the pascal's triangle and how does it apply?

- We will be showing you have the pascal's triangle works and where it come pirm. We will also be shawing you have to use it.

What is it?

- The pascal's triangle is one of the most interesting number patterns in mathematics. This is something that the chinese and the percians used in the elevent century and mathematics today still use it.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

French mathematician;
Blaise pascal is the founder of the pascal's triangle, but the persians and Chinese also used it before the birth of pascal-1623. It is said that mathematics used this method even in the eleventh century

by the persians and chinese. But it 1654, Blaise pascal completed the traite du triangle arithmetique, which had properties and applications of the triangle. pascal had made lots of other contributions to mathematics but the writings of his triangles are very famous.

Patterns In the Pascal Triangle

- We used pascal's triangles for many things. for example we used it a lot in algebra We also us it to find probabilites and combinatorics. We will be telling you about some patterns in the pascal's tringle.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

1 10 45 120 210 252 210 120 45 10 1

Abracadabra

Many words and phrases relating to rituales, talismans and pentacles have a symbolic meaning, either in them-selves or in the way they are used, which is expressed either phoneti-cally or, more frequently, graphically. This word was in frequent use during the middle Ages as a magic formula. It is derived from the hebrew pharase abreq and habra, meaning 'hurl your thunderbolt even unto death'. It was usally inscribed inside an inverted triangle, or was set out so that it formed a triangle (39) thus:

A B R A C A D A B A
A B R A C A D A B
A B R A C A D A B
A B R A C A D A
A B R A C A D
A B R A C A
A B R A C
A B R A
A B R
A B
A