Answer No# 1

Application of Function

- *ATM*
- *Money & Time Comparison*
- *Thermometer*

ATM

When the ATM card is inserted to the machine, the software inner is performing a characteristic to map the wide variety saved in the card to your current or saving account. This is essentially a one-to-one mapping, i.e., function. This is likely the most broadly and famous use of function.

Money & Time Comparison

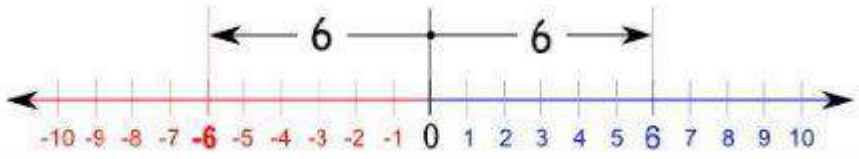
Money as a function of time. You in no way have extra than one quantity of cash at any time due to the fact you can usually add the entirety to supply one complete amount. By grasp how your cash modifications over time, you can layout to spend your cash sensibly. Businessmen discover it very beneficial to plot the layout of their cash over time so that they can see when they are spending too much.

Thermometer

Temperature as a feature of more than a few factors. Temperature is a very complicated feature due to the fact it has so many inputs, including: the time of day, the season, the quantity of clouds in the sky, the energy of the wind, the place you are and many more. But the vital component is that there is solely one temperature output when you measure it in a particular place. This is what thermometer offers with and is a very properly instance for many to one function.

Absolute Value Function

The absolute value function is often notion of as imparting the distance the range is from zero on a number line. Algebraically, for something the in-put value is, the output is the value barring regard to sign. Absolute value of -6 is 6, and absolute value of 6 is 6, each are 6 units from zero in the number line.



The actual absolute value function is non-stop everywhere. It is differentiable in all places barring for x = 0 It is monotonically lowering on the interval [−∞,0] and monotonically growing on the interval [0,+∞].Since a real number and its contrary have the equal absolute value, it is an even function, and is for this reason now not invertible. The real absolute value function is a piecewise linear, convex function.

Limits Function in Computer Sciences

In day to day computer programming, nothing is achieved which depends on the properties of real numbers and functions on real numbers. Computers can't honestly symbolize real numbers (they use various types of rational approximations, which consist of so-called "floating point" numbers).Most components of computer science do now not use the properties of actual numbers, and you won't find, in general, a lot point out of limits, continuity, or functions on real numbers when analyzing algorithms, data structures, automata theory, the lambda calculus, truing machines, and so on. The mathematics self-discipline of "Discrete" mathematics is a lot extra essential than the mathematics of continuity.

One component of computer science, algorithm analysis, on the different hand, can gain from cross-fertilization with non-stop mathematics. The well-known Computer science educator Donald Knuth, together with Oren Patashnick and Ron Graham, wrote a textbook known as "Concrete Mathematics", which covers the elements of non-stop and discrete math beneficial in computer science. There is a subfield of computer science, known as "numerical methods", which is committed to how to great calculate on computer systems numerical outcomes for issues in calculus and comparable disciplines, taking into account the boundaries of the rational approximations computer systems use.

Limits Function in Physics

There are many motives to use limits in physics. One beneficial limit is instantaneous velocity. If you understand the position of an object at two factors in time, you can calculate its common velocity. When you pick out smaller and smaller time increments, the common velocity approaches a certain value. Only limits can inform us what that is. The thought of the limiting value of the slope of a function is the derivative, as above. The derivative applies to any easy graph, which physics has in end-less scenarios.

This is usually achieved in order to approximate the conduct of a system when sure parameters come to be massive (i.e., limit as something goes to infinity) or when sure parameters become small (i.e., limit as something goes to zero).

Limits Function in Engineering

Much of engineering includes calculus. You won't get some distance in electrical engineering besides calculus, and nor will you in civil engineering. Calculus is used in electrical engineering for apparent motives in connection to alternating currents and transient outcomes in switching etc. In civil engineering it is essential to recognize vibrations of structure, and bending of beams. of route this is simply a couple of examples.

Limits and continuity are necessary for perception calculus.

Limits Function in Daily Life

Real-life limits are used any time you have some kind of real-world application method a steady-state solution. As an example, we should have a chemical reaction in a beaker begin with two chemical compounds that form a new compound over time. The quantity of the new compound is the limit of a function as time methods infinity. Similarly, if you drop an ice dice in a glass of warm water and measure the temperature with time, the temperature subsequently approaches the room temperature the place the glass is stored. Measuring the temperature is a limit once more as time approaches infinity.Limits are additionally used as real-life approximations to calculating derivatives. It is very hard to calculate a derivative of tricky motions in real-life situations. So, to make calculations, engineers will approximate a function the use of small variations in the a function and then attempt and calculate the derivative of the function by way of having smaller and smaller spacing in the function sample intervals. For example, when designing the engine of a new car, an engineer can also model the gasoline via the car's engine with small intervals referred to as a mesh, due to the fact the geometry of the engine is too complex to get exactly with genuinely functions such as polynomials. These approximations al-ways use limits.

Answer No# 2

Make a piece wise defined function with its graph for your electricity bill upto 700 units, as the cost of electricity is dependent upon number of units in different interval.

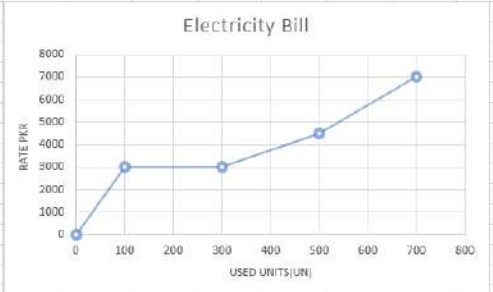
Units	Rates (PKR)
300	3000
500	4500
700	7000

X	Y
0	0
100	3000
300	3000
500	4500
700	7000

For units upto **300**, the rates is **3000 PKR**. For additional units upto **500**, the rates added **1500 PKR**. The units exceeded from **500** upto **700** units the rates charges is **2500 PKR** added in the electricity bill. Let **(Un)** represent the units you used.

The piecewise Function & Graph are:

$$f(Un) = \begin{cases} 3000, & if\ 0 < Un \le 300 \\ 4500, & if\ 300 < Un \le 500 \\ 7000, & if\ 500 < Un \le 700 \end{cases}$$



Answer No# 3

Abdul Rafeh Piecewise Defined Function Csc-205-104

Q:15:~ $f(x) = \begin{cases} 2x+1, & \text{if } x < 1 \\ -x+4, & \text{if } x \geq 1 \end{cases}$

Solution:

$f(0)$

$f(0) = 2(0)+1$

$f(0) = 0+1$

$f(0) = 1$

$f(1)$

$f(1) = -(1)+4$

$f(1) = 3$

Q:16:~ $f(x) = \begin{cases} 5x-1, & \text{if } x < -2 \\ x-9, & \text{if } x \geq -2 \end{cases}$

a) $f(-4)$ b) $f(-2)$ c) $f(0)$ d) $f(5)$

Solution:

a) $f(-4) = 5(-4)-1$

$f(-4) = -20-1$

$f(-4) = -21$

b) $f(-2) = 5(-2)-1$

$f(-2) = -10-1$

$f(-2) = -11$

c) $f(0) = 5(0)-1$

$f(0) = -1$

d) $f(5) = 5(5)-1$

$f(5) = 24$

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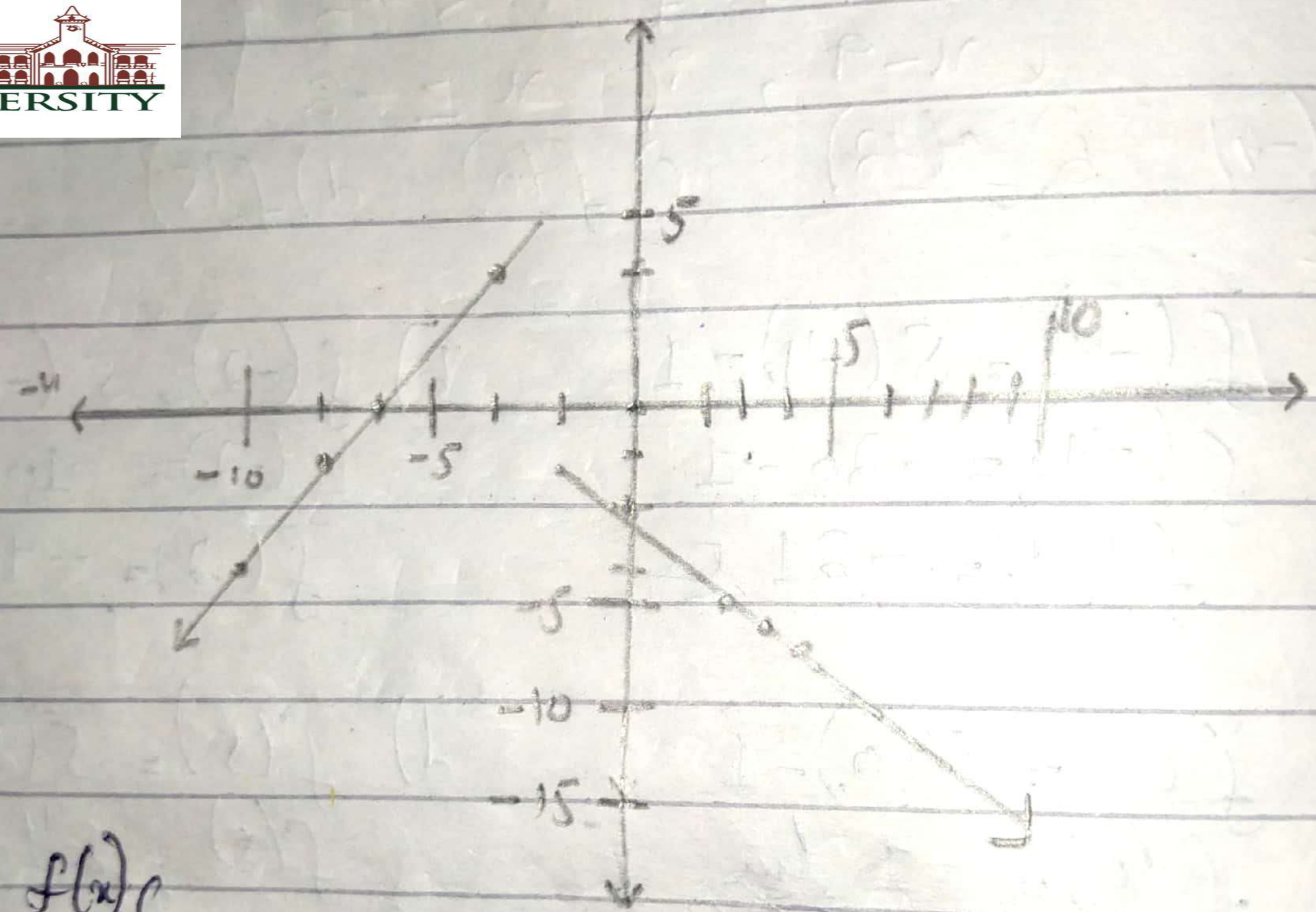
Q.17:- $f(x) = \begin{cases} x+6, & \text{if } x \le -3 \\ -\frac{2}{3}x - 3, & \text{if } x > -3 \end{cases}$

Solution:

x	-3	-6	-10	-1	0	3
$f(x)$	3	0	-4	-2.333	-3	-5

5	4	-7
-6.333	-5.667	-1

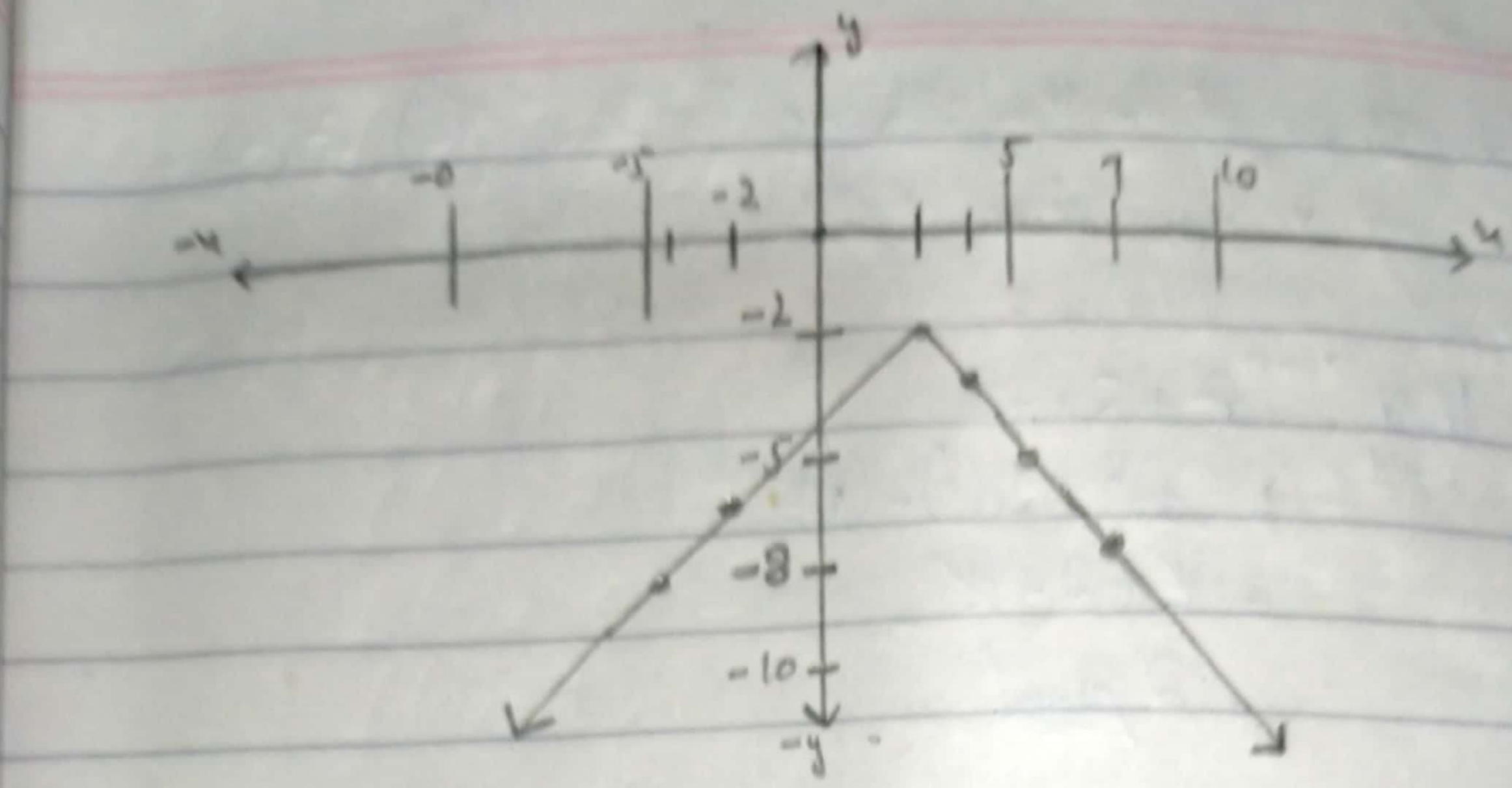


Q.18:- $f(x) = \begin{cases} -x, & \text{if } x > 2 \\ x-4, & \text{if } x \le 2 \end{cases}$

Solution:

x	2	0	-2	3	5	-4	7
$f(x)$	-2	-4	-6	-3	-5	-8	-7

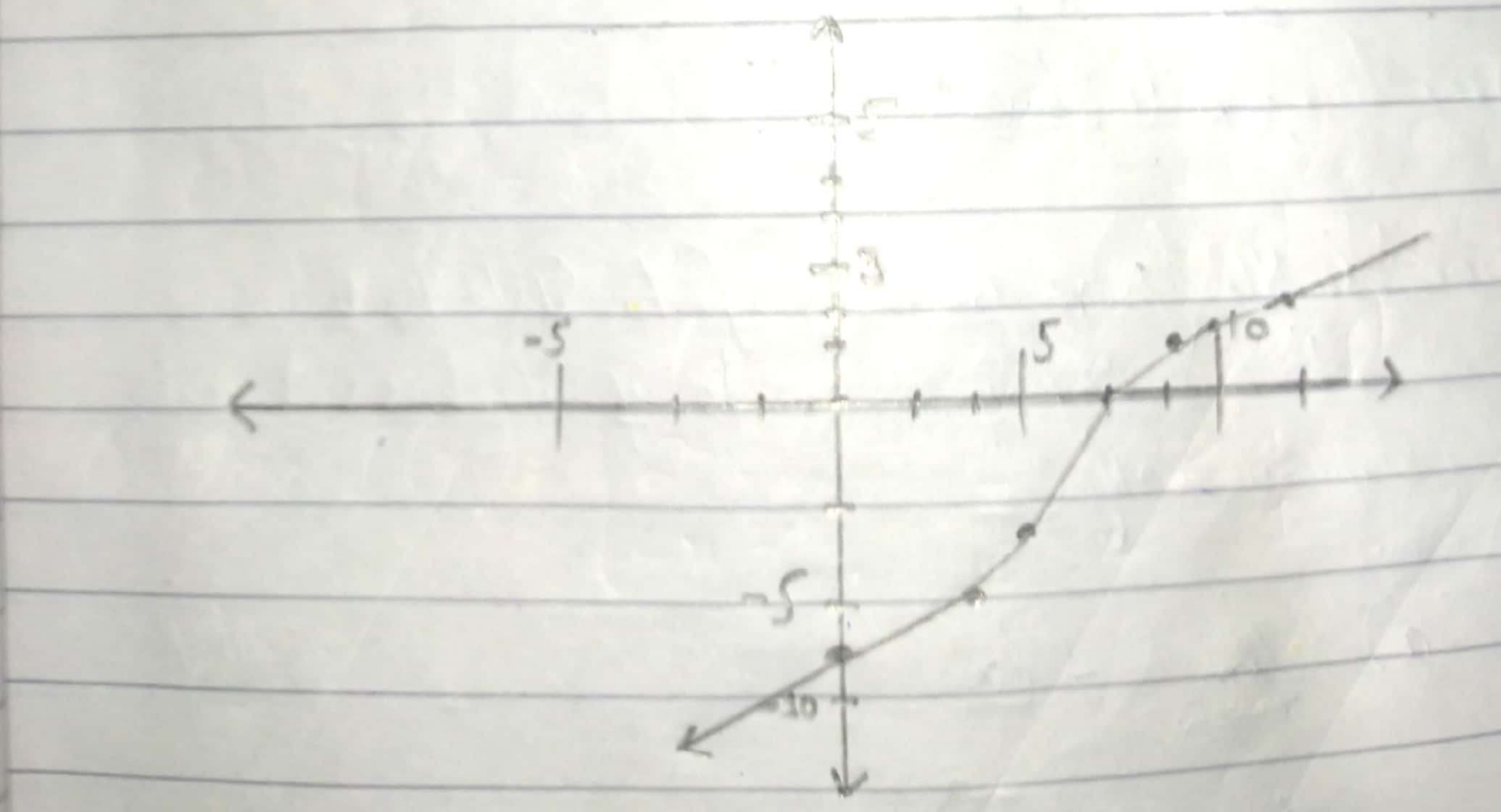
Graph on next Page.



Q:19:- $f(x) = \begin{cases} x-8, & \text{if } x < 9 \\ \frac{1}{3}x-2, & \text{if } x \geq 9 \end{cases}$

Solution:

x	0	3	5	8	9	10	12
$f(x)$	-8	-5	-3	0	1	1.33	2



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Absolute Value Function CSC-205-10

Abdul Rafeh

Q:10:~ $|2x+5|+4=3$

Solution:

$|2x+5|=-1$

No Solution.

Q:11:~ $\frac{1}{4}|2x-6|+1=2$

Solution:

$1/4\,|2x-6|=1$

$|2x-6|=4$

$2x-6=4$ $\qquad$ $2x-6=-4$

$2x=10$ $\qquad$ $2x=2$

$\boxed{x=5}$ $\qquad$ $\boxed{x=1}$

$\{1,5\}$ Ans

Q:12:~ $|x-7|+2=2$

Solution:

$|x-7|=0$

$\boxed{x=7}$ Ans

Q:13:~ $|x-4|=|4-x|$

Solution:- $|x-4|=4-x$

$x-4=-(4-x)$, $x-4=4-x$

$x-4=-4+x$, $x=4(4-x)$

$x=4(-4+x)$, $x=16-4x$

$x=-16+4x$, $5x=16$

$4x-x=16$, $x=16/5$

$3x=-16$

$x=-16/3$

Q:14:~ $|x+3|>4$

Solution:

$|x+3|>4$

$(x+3)=-4$

$x=-7$

$(-\infty,-7)\cup(-7,\infty)$

X ——— X Ans

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Abdul Rakh Continuity CSC-205-104

Q:-1 — $f(x) = |x-3|$ at $x = 3$

Solution:-

$f(3) = |3-3| = 0$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} |x-3| = \lim_{h \to 0} |3-h-3|$, putting $x = 3-h$

$= \lim_{h \to 0} |-h| = 0$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} |x-3| = \lim_{h \to 0} |3+h-3|$, putting $x = 3+h$

$= \lim_{h \to 0} |h| = 0$

Thus $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x) = f(3)$

Hence f is Continuous at $x = 3$

Q:2:- $f(x) \begin{cases} x-4 & \text{if } -1 < x \leq 2 \\ x^2-6 & \text{if } 2 < x < 5 \end{cases}$ at $x = 2$

Solution:-

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x-4) = -2$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x^2-6) = -2$

When $x = 2$, $f(x) = x-4$

Thus $f(2) = 2-4 = -2$

Therefore, $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$

Hence, the function is Continuous at $x = 2$.

Q:3:- $f(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ at $x = 0$

Solution.

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \sin \frac{1}{x} = \sin(-\infty)$ which is not a definite number. It can have any value in $[-1, 1]$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \sin \frac{1}{x} = \sin \infty$, which is also not a definite number.

Hence the limit of $f(x)$ does not exist as $x \to 0$, So the function is discontinuous at $x = 0$.

Q4:- $f(x) \begin{cases} \frac{x^2}{a} - a & \text{if } 0 < x < a \\ 0 & \text{if } x = a \\ a - \frac{a^2}{x} & \text{if } x > a \end{cases}$ at $x = a$

Solution:

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^-} \left(\frac{x^2}{a} - a \right) = 0$$

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^+} \left(a - \frac{a^2}{x} \right) = 0$$

Hence $f(x)$ is continuous at $x = a$.

Q:43:~ Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$f(x) = 3x^4 - 5x + \sqrt[3]{x^2+4}, \quad a = 2$$

Sol:

By Putting

$a = x = 2$

$$\lim_{x \to 2} \left(3x^4 - 5x + \sqrt[3]{x^2+4}\right)$$

$$= 3(2)^4 - 5(2) + \sqrt[3]{2^2+4}$$

$$= 40$$

$\therefore \Rightarrow f(x)$ is continuous at $a = 2$

Q:44:~ $f(x) = \dfrac{x^3-8}{x^2-4}$ → How would you remove the discontinuity of f? in other word. how would define f(2) in order to make f continuous at 2?

Sol:

By Simplify

$$f(x) = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

$$f(x) = \frac{x^2+2x+4}{x+2}$$

$$f(2) = \frac{2^2+2(2)+4}{2+2}$$

$$= 3$$

Sol: $f(2) = 3$

Evaluate, Using Theorem 4, 5, 7, and 9, why the function is continous at every number in its domain, state the domain

Q:45:~ $G(x) = \dfrac{x^2+1}{2x^2-x-1}$

Solu As:

Theorem "7" State all the rational function are continuous on this domain.

Q:46:~ $R(t) = \dfrac{e^{\sin t}}{2+\cos \pi t}$

Solu

Use quotient rule of trig function

So,

All the real numbers are domain.

Q:47:~ $B(x) = \dfrac{\tan x}{\sqrt{4-x^2}}$

Solu

$\tan x$ has a domain for all real numbers except $\dfrac{\pi}{2} + n\pi$

$\because$

$\sqrt{4-x^2}$ has domain $[-2, 2]$, outside this range will make negative.

Q:48:~ $N(r) = \tan^{-1}(1+e^{-r^2})$

Solu

Function is $1+e^{-r^2}$ and arc tan it is continuous on domain.

Domain are all real numbers.

Q: 49:~ Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

$$g(x) = -2\sqrt{3-x}, \quad (-\infty, 3)$$

Soln

According to theorem 9;

Since $\sqrt{3-x}$ and 2 both are continuous on their own.

$g(x)$ is continuous for given interval.

Q: 50:~ Explain why the function is discontinuous at the given number a.

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}, \quad a = -2$$

Soln

Function said to be continuous when it is continuous from both side.

from $\frac{1}{x+2}$, $x \neq -2$ is discontinuous from $x = -2$

So,

$f(x)$ $\lim_{x \to -2}$ does not exist.

Q.51:- $f(x) = \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$, $a=1$

Soln

Value $f(1)$ is defined to '1'

here,

$$= \frac{x^2-x}{x^2-1} \quad \text{a } x \neq 1$$

$$= \frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$

$$= \frac{x}{x+1}$$

It is discontinous at $a=1$ lim

Q.52:- Same question as (Q.43) only eq. change.

$$h(t) = \frac{2t-3t^2}{1+t^3}, \quad a=1$$

Soln

$$h(t) = \frac{2t-3t^2}{1+t^3}, \quad a=1$$

$a = t = 1$

$$\lim_{t \to 1} \left(\frac{2t-3t^2}{1+t^3} \right)$$

By putting $= \dfrac{2(1)-3(1)^2}{1+1^3}$

$= -1/2$ lim

$f(x)$ is continous at $a=1$ lim.

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LIMITS

Csc-205-104

Abdul Rafeh

Q:5:- $\lim_{x \to \pm\infty} \left[\frac{x^2}{x+1} - \frac{x^2}{x+3} \right]$

Solution:

$$= \lim_{x \to \pm\infty} \left[\frac{x^2(x+3) - x^2(x+1)}{(x+1)(x+3)} \right]$$

$$= \lim_{x \to \pm\infty} \left[\frac{\cancel{x^3} + 3x^2 - \cancel{x^3} - x^2}{x^2 + 3x + x + 3} \right]$$

$$= \lim_{x \to \pm\infty} \left[\frac{2x^2}{x^2 + 4x + 3} \right]$$

$$= \lim_{x \to \pm\infty} \left[\frac{2\cancel{x^2}}{\cancel{x^2}\left(1 + \frac{4}{x} + \frac{3}{x^2}\right)} \right]$$

$$= \frac{2}{1+0+0}$$

$$= 2 \text{ Ans}$$

Q:6:- $\lim_{x \to \infty} \frac{5x^3 + 2x^2 - 1}{x - 4x^4}$

Solution:

$$= \lim_{x \to \infty} \frac{x^3(5 + 2/x - 1/x^3)}{x^4(1/x^3 - 4)}$$

$$= \lim_{x \to \infty} \frac{5 + 2/x - 1/x^3}{x(1/x^3 - 4)}$$

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$$= \frac{5 + 2/\infty - 1/\infty}{\infty\,(1/\infty - 4)}$$

$$= \frac{5 + 0 - 0}{\infty\,(1/\infty - 4)}$$

$$= \frac{5}{\infty}$$

$$= 0 \text{ Ans.}$$

Q.7:- $\lim_{x \to -1} \frac{x^{1/3} + 1}{x + 1}$

Solution:-

$$= \lim_{x \to -1} \frac{x^{1/3} + 1}{(x^{1/3})^3 + (1)^{1/3\,3}}$$

$$= \lim_{x \to -1} \frac{\cancel{x^{1/3} + 1}}{\cancel{(x^{1/3} + 1)}\,(x^{2/3} + 1 - x^{1/3})}$$

$$= \lim_{x \to -1} \frac{1}{x^{2/3} - x^{1/3} + 1}$$

$$= \frac{1}{(-1)^{2/3} - (-1)^{1/3} + 1} = \frac{1}{+1 - (-1) + 1}$$

$$= \frac{1}{+1 + 1 + 1}$$

$$= \frac{1}{3} \text{ Ans.}$$

Q:8:- $\lim_{x \to \infty} \frac{x + \sin x}{x}$

Solution:-

$$= \lim_{x \to \infty} \left(\frac{\cancel{x}}{\cancel{x}} + \frac{\sin x}{x} \right)$$

$$= \lim_{x \to \infty} \left(1 + \frac{\sin x}{x} \right)$$

$$= \lim_{x \to \infty} (1) + \lim_{x \to \infty} \frac{\sin x}{x}$$

$$= 1 + 0$$

$$= 1 \text{ Ans}$$

Q:9:- $\lim_{x \to 2^-} \frac{\sqrt{4 - x^2}}{\sqrt{6 - 5x + x^2}}$

Solution:-

$$= \lim_{x \to 2^-} \frac{\sqrt{(2-x)(2+x)}}{\sqrt{6 - 3x - 2x + x^2}}$$

$$= \lim_{x \to 2^-} \frac{\sqrt{2-x}\sqrt{2+x}}{\sqrt{3(2-x) - x(2-x)}}$$

$$= \lim_{x \to 2^-} \frac{\sqrt{2-x}\sqrt{2+x}}{\sqrt{(2-x)(3-x)}}$$

$$= \lim_{x \to 2^-} \frac{\cancel{\sqrt{2-x}}\sqrt{2+x}}{\cancel{\sqrt{2-x}}\sqrt{3-x}}$$

$$= \lim_{x \to 2^-} \frac{\sqrt{2+x}}{\sqrt{3-x}}$$

$$= \frac{\sqrt{4}}{1}$$

$$= 2 \text{ Ans}$$

Q:20:~ Show that the limits of function $f(x) = x^2 + 2x - 1$ as x approaches 1 is equal to 2.

Solution:

$$\lim_{x \to 1} x^2 + 2x - 1$$
$$= (1)^2 + 2(1) - (1)$$
$$= \cancel{1} + 2 - \cancel{1}$$
$$= 2 \text{ proved}$$

Q:21:~ Find the limit of function $f(x) = 2x^2 - 3x + 5$ as x approaches to -1.

Solution:

$$\lim_{x \to -1} 2x^2 - 3x + 5$$
$$\Rightarrow 2(1)^2 - 3(1) + 5$$
$$\Rightarrow 2 - 3 + 5$$
$$\Rightarrow -1 + 5$$
$$\Rightarrow 4 \text{ Ans.}$$

Q:22:~ Use the given graph of f to state the value of each quantity if it exists. if it does not exist. Explain why?

Solution:

a) $\lim_{x \to 2^-} f(x)$

$$\lim_{x \to 2^-} f(x) = 3$$

b) $\lim_{x \to 2^+} f(x)$

$$f(x) = 1$$

c) $\lim_{x \to 2} f(x)$

$f(x)$ is not exist because the value of $f(x)$ from L.H. & R.H are different.

d) $f(2)$

$f(2) = 3$

e) $\lim_{x \to 4} f(x)$

$f(x) = 4$

f) $f(4)$

$\Rightarrow f(4) = 4$

Q:23: For the function f whose graph is shown. State following?

a) $\lim_{x \to -7} f(x)$ b) $\lim_{x \to -3} f(x)$ c) $\lim_{x \to 0} f(x)$

d) $\lim_{x \to 6^-} f(x)$ e) $\lim_{x \to 6^+} f(x)$

(f) The equations of the vertical asymptotes.

Solution

a) $-\infty$, b) $+\infty$, c) not exist , d) $-\infty$

e) $+\infty$, f) $-7, -3, +6$

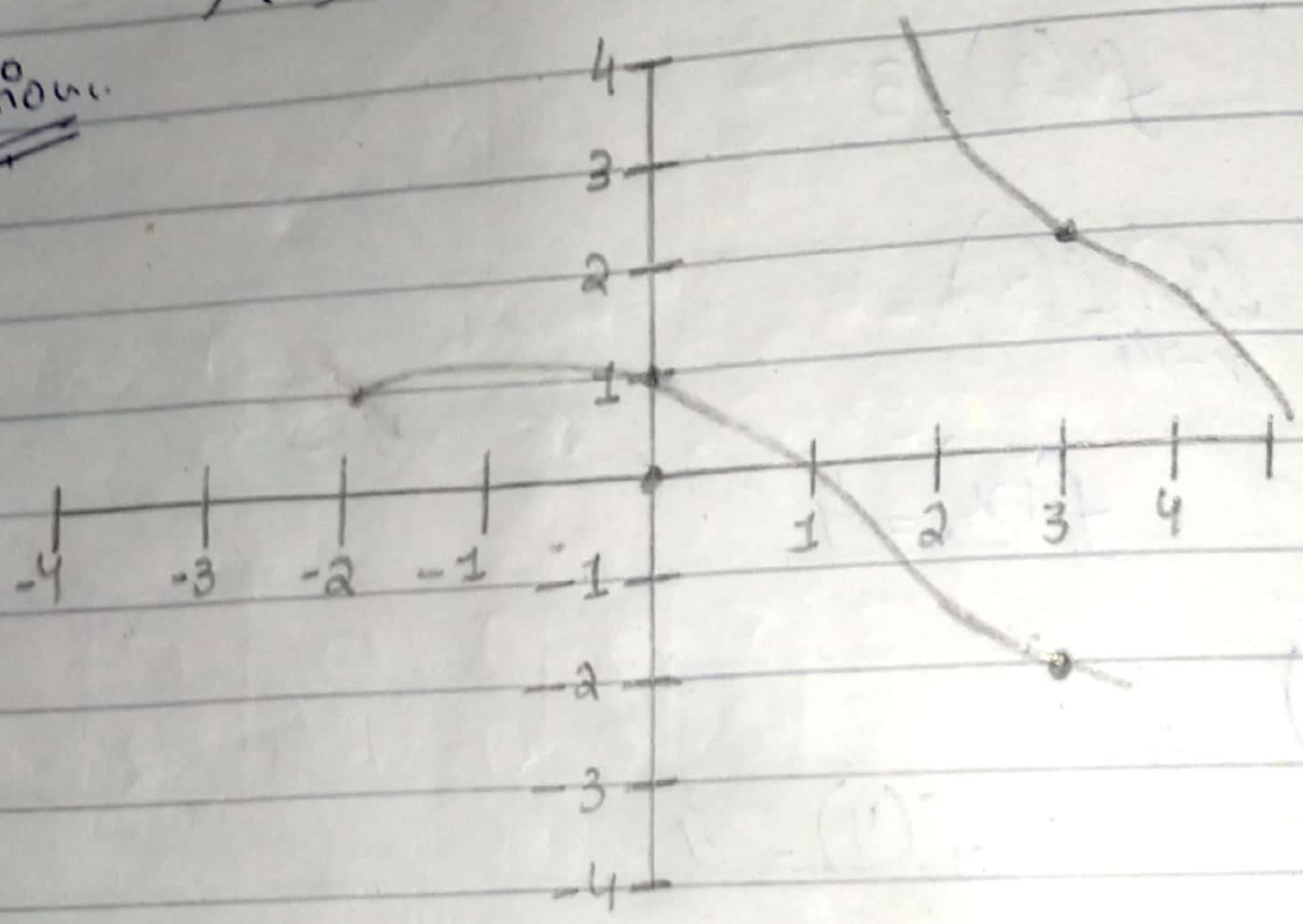
are asymptotes

Sketch the graph of an example of a function f that satisfies all of the given condition.

Q:24:~ $\lim_{x\to 0} f(x) = 1$, $\lim_{x\to 3^-} f(x) = 2$, $\lim_{x\to 3^+} f(x) = 2$,

$f(0) = -1$, $f(3) = 1$.

Solution:

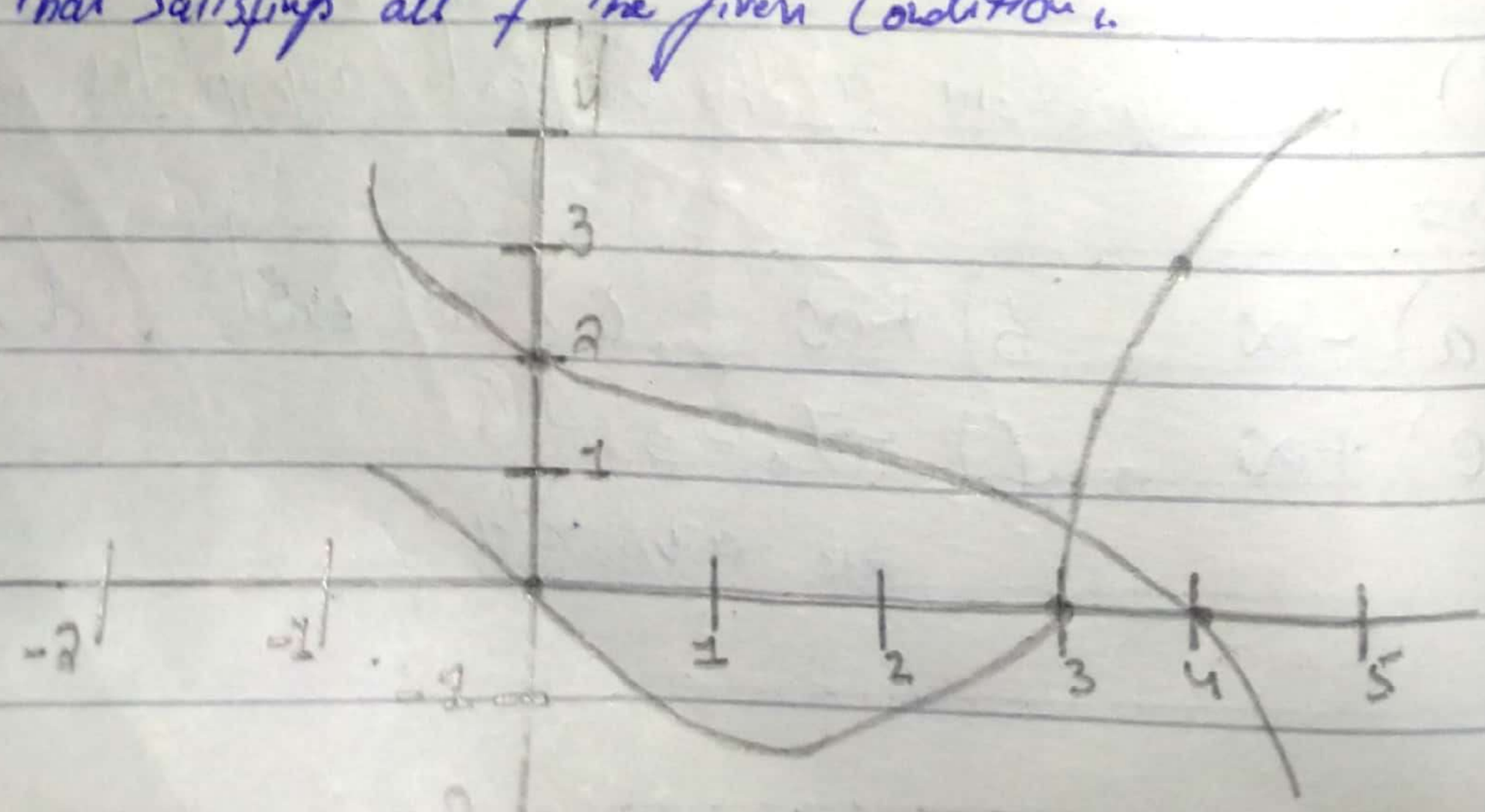


Q:25:~ $\lim_{x\to 0^-} f(x) = 2$, $\lim_{x\to 0^+} f(x) = 0$, $\lim_{x\to 4^-} f(x) = 3$,

$\lim_{x\to 4^+} f(x) = 0$, $f(0) = 2$, $f(4) = 1$

Solution:

That satisfies all of the given condition.



Q: 26 $\lim_{x \to 0} \frac{\tan 3x}{\tan 5x}$

Solution

$$\lim_{x \to 0} \frac{\sin 3x}{\cos 3x} \div \frac{\sin 5x}{\cos 5x}$$

$\lim_{x \to 0}$ Multiply & Divide by $3x$ & $5x$

$$\lim_{x \to 0} \frac{\frac{3x \sin 3x}{3x}}{\cos 3x} \times \frac{\cos 5x}{\frac{5x \sin x}{5x}}$$

$$\lim_{x \to 0} \frac{3x}{\cos 3x} \times \frac{\cos 5x}{5x}$$

$\therefore \lim_{x \to 0} \left[\sin \frac{x}{x} \right] \cdot 1$

$$\frac{3 \times \cos(5(0))}{5 \times \cos(3(0))}$$

$\therefore \cos(0) = 1$

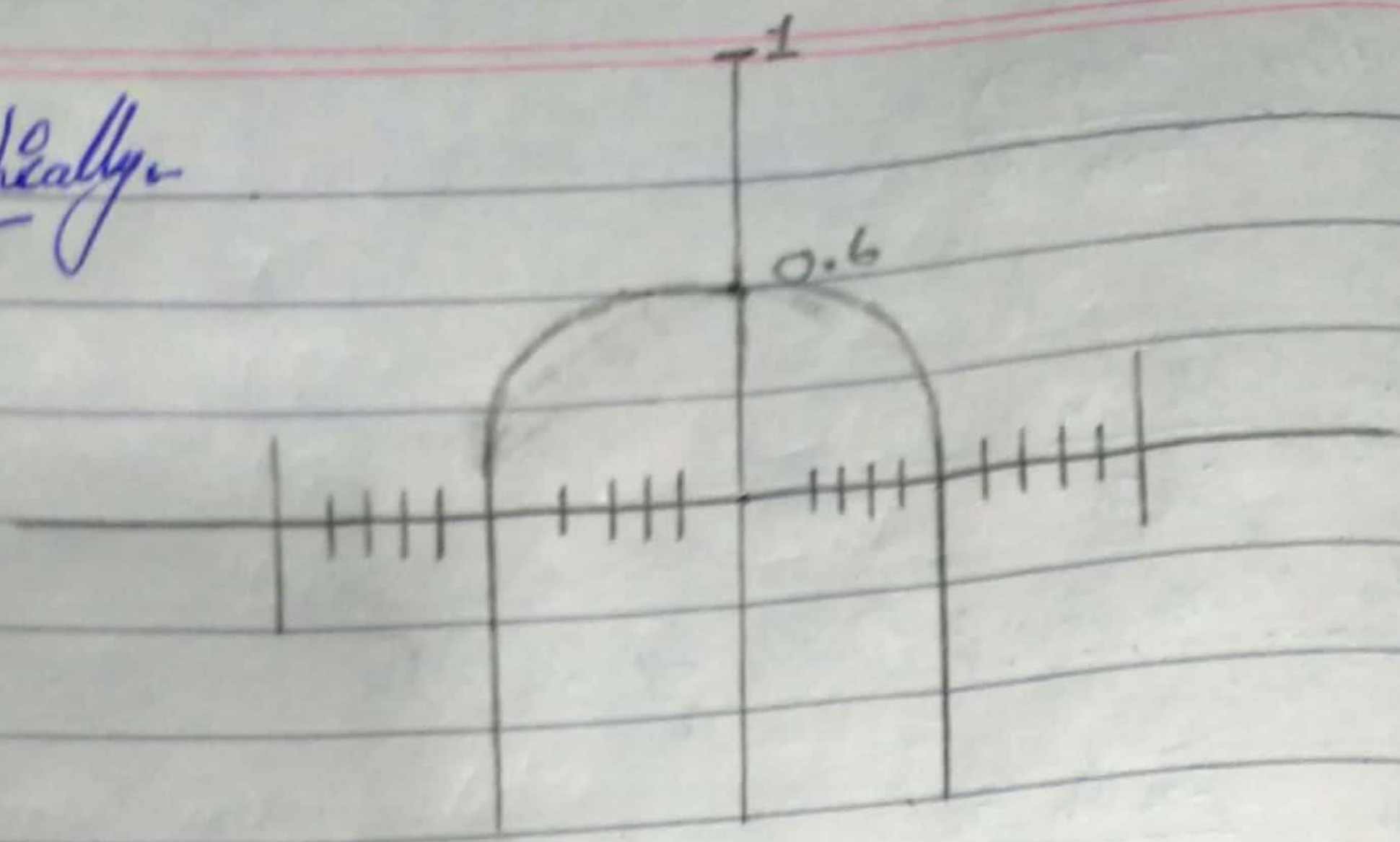
$$\Rightarrow \frac{3}{5}$$

Numerically $\tan 3x / \tan 5x$

x^+	$f(x)$
0.1	0.5666 23621
0.01	0.599983
0.001	0.5999968
0.0001	0.599999997

x^-	$f(x)$
−0.1	0.5662361
−0.001	0.5999968
−0.0001	0.59999997

Graphically



Q.27:- $\lim_{x \to 0} \frac{9^x - 5^x}{x}$

Solution:- apply derivative

$$\Rightarrow \lim_{x \to 0} \frac{\frac{d}{dx}(9^x) - \frac{d}{dx}(5^x)}{\frac{d}{dx}(x)}$$

$$\Rightarrow \lim_{x \to 0} \frac{9^x \ln 9 - 5^x \ln 5}{1}$$

$$\Rightarrow \frac{9^0 \ln 9 - 5^0 \ln 5}{1}$$

$$\Rightarrow \ln 9 - \ln 5$$

$$\Rightarrow \ln \frac{9}{5} = 0.5878 \text{ ans.}$$

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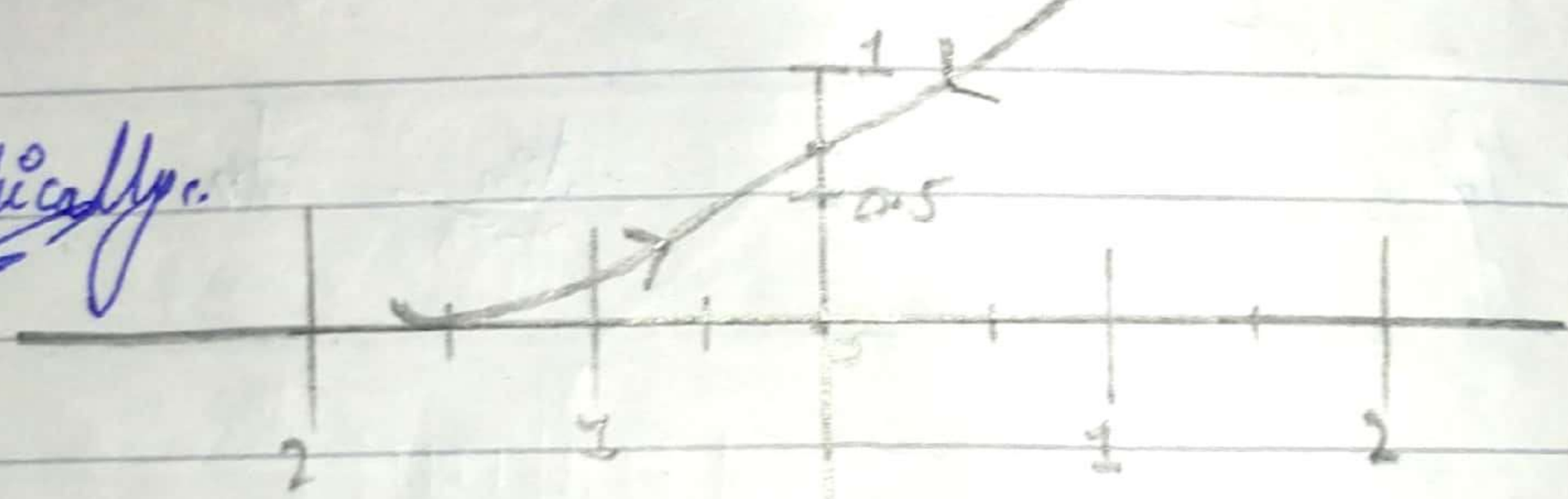
Numerically $\lim \frac{9^x - 5^{-x}}{x}$

x^+	$f(x)$
0.001	0.588890
0.0001	0.58789855
0.000001	0.587787778

x^-	$f(x)$
−0.001	0.58666
−0.0001	0.58789
−0.000001	0.587785

Graphically:



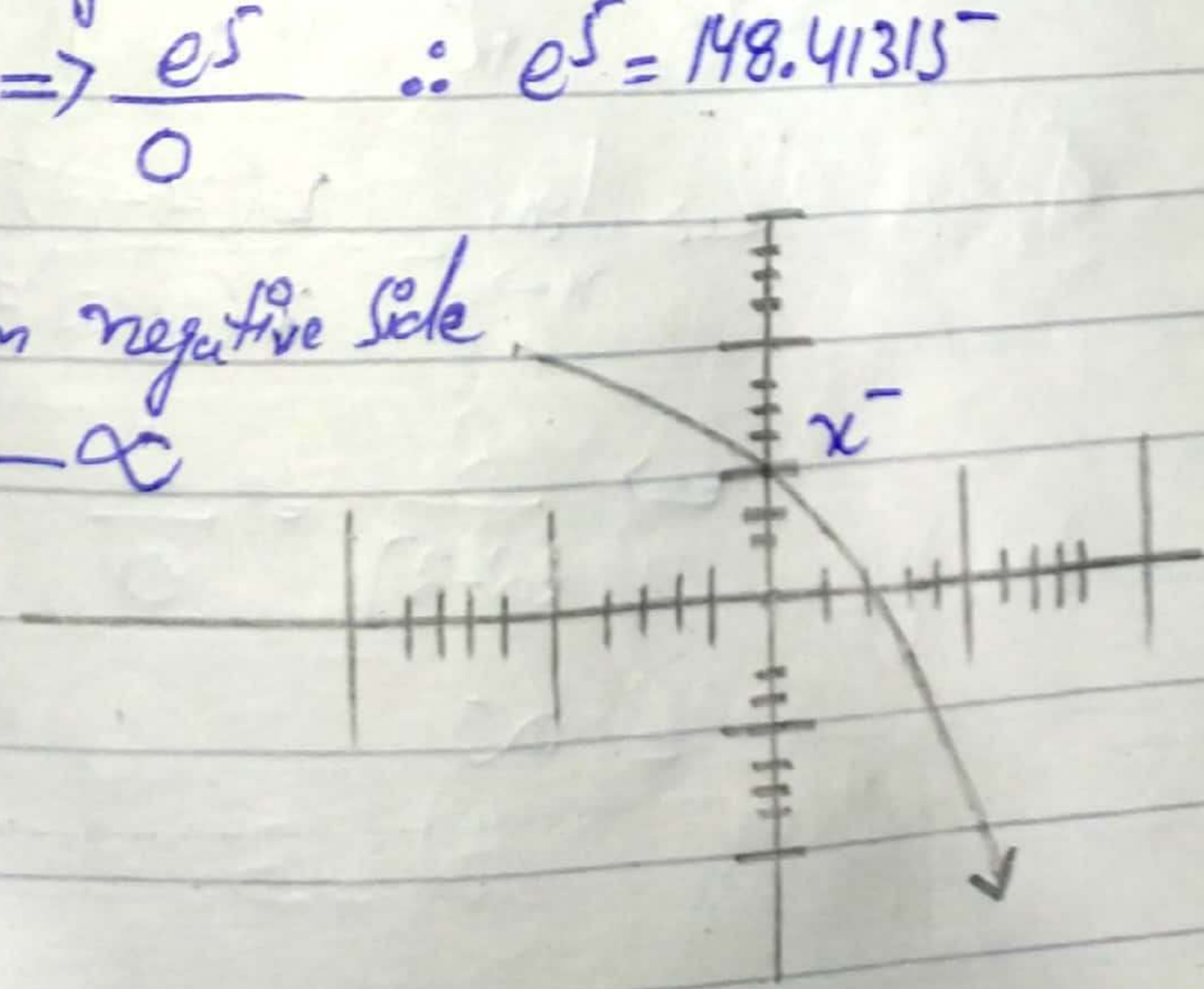
Q:28: $\lim_{x \to 5^-} \frac{e^x}{(x-5^-)^3}$

Solution: By Substituting Value

$\Rightarrow \frac{e^5}{(5-5)^3} \Rightarrow \frac{e^5}{0}$ $\therefore e^5 = 148.41315^-$

we get higher values from negative side.

$\lim_{x \to 5^-} \frac{e^x}{(x-5)^3} = -\infty$



Numerically.

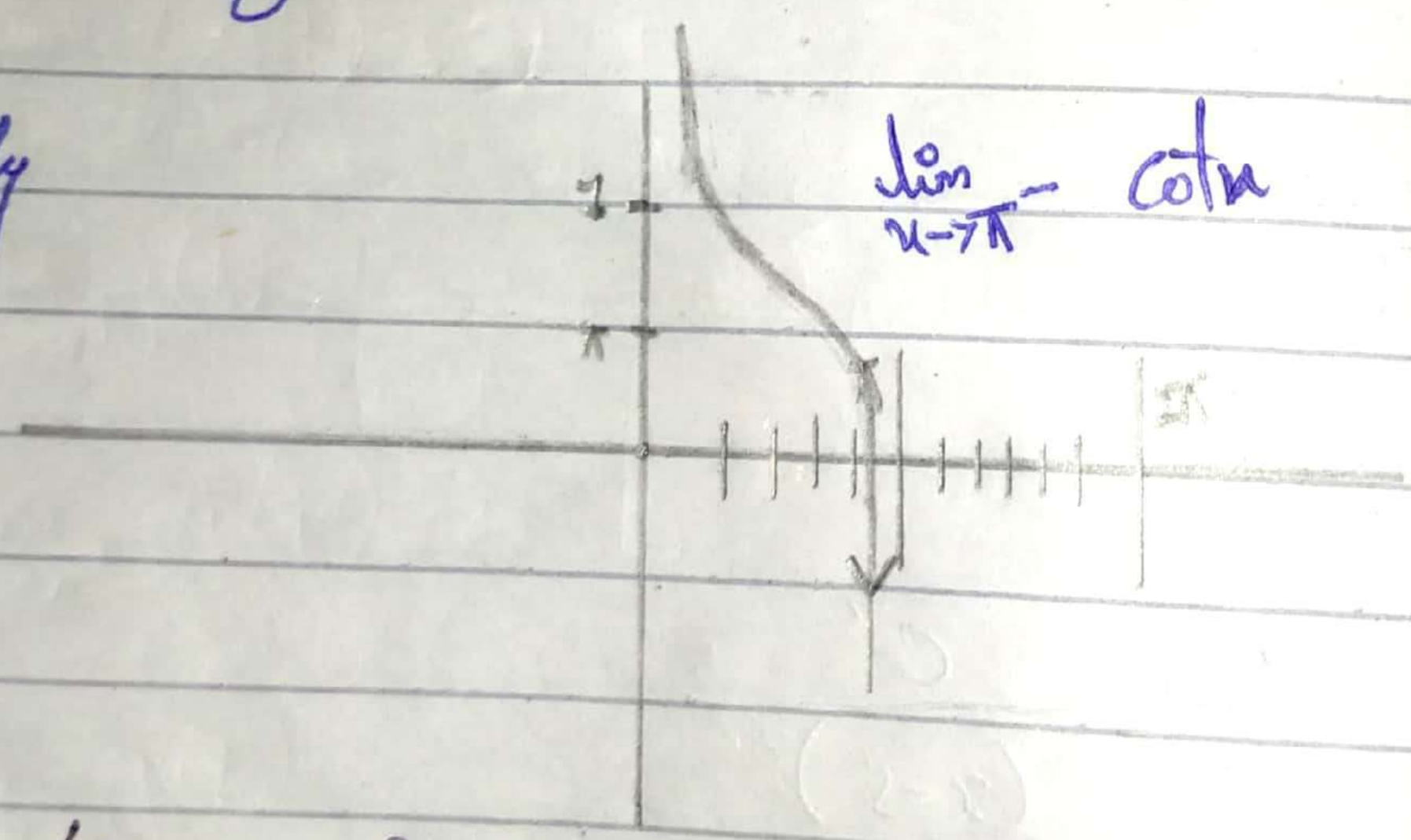
x^-	$f(x)$
4.5	-720.1370
4.8	-15188.80
4.999	$-1483983185 2.840$

Q:29:- $\lim_{x \to \pi^-} \cot x$

Solution $\Rightarrow \lim_{x \to \pi^-} \frac{\cos x}{\sin}$

$\Rightarrow \frac{-1}{0} = \infty$

Graphically



Q:30:- $\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$

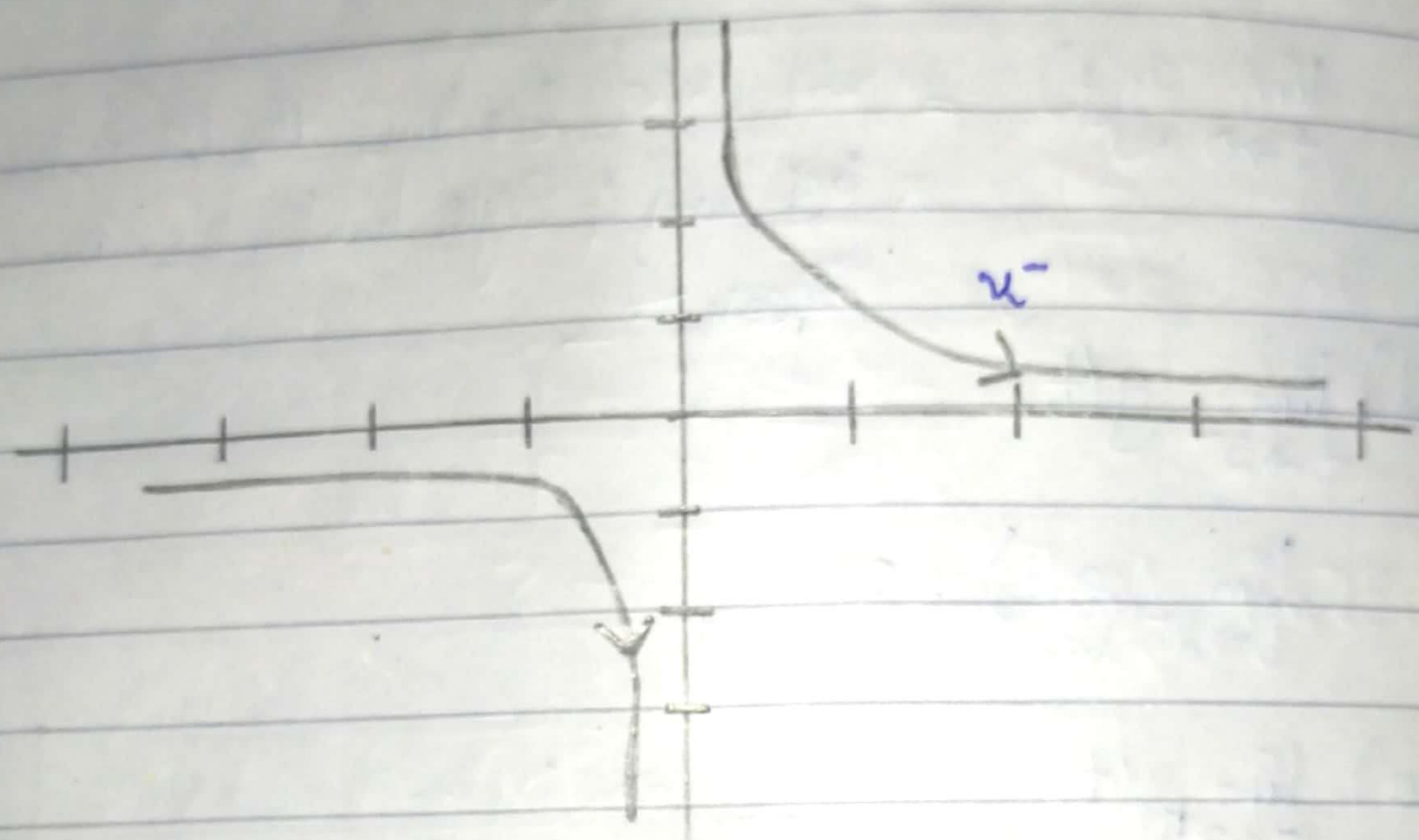
Solution $\Rightarrow \lim_{x \to 2^-} \frac{x\cancel{(x-2)}}{x(x-2)^{\cancel{2}}}$

$\Rightarrow \frac{2}{(2-2)} = \frac{2}{0}$

$= \infty$

Graphically:-

x^-	$f(x)$
1.9	-19
1.99	-1.99
1.999	-1999
1.9999	-19999



Q:31:~ For the function f whose graph is given. State the value of each quantity if it exists. if it does not exist. Explain why?

a) $\lim_{t\to 0^-} g(t)$ b) $\lim_{t\to 0^+} g(t)$ c) $\lim_{t\to 0} g(t)$

d) $\lim_{t\to 2^-} g(t)$ e) $\lim_{t\to 2^+} g(t)$ f) $\lim_{t\to 2} g(t)$

g) $g(2)$ h) $\lim_{t\to 4} g(t)$

Solution:

a) $\lim_{t\to 0^-} g(t)$

$\lim_{t\to 0^-} g(t) = -1$

b) $\lim_{t \to 0^+} g(t)$

$\lim_{t \to 0^+} g(t) = -2$

c) $\lim_{t \to 0} g(t)$

$\lim_{t \to 0} g(t)$ = not exist because the limit from left hand & right hand side are different.

d) $\lim_{t \to 2^-} g(t)$

$\lim_{t \to 2^-} g(t) = 2$

e) $\lim_{t \to 2^+} g(t)$

$\lim_{t \to 2^+} g(t) = 0$

f) $\lim_{t \to 2} g(t)$

= limit does not exist because both sides limit are different.

g) $g(2)$

$g(2) = 1$

h) $\lim_{t \to 4} g(t)$

$\lim_{t \to 4} g(t) = 3$

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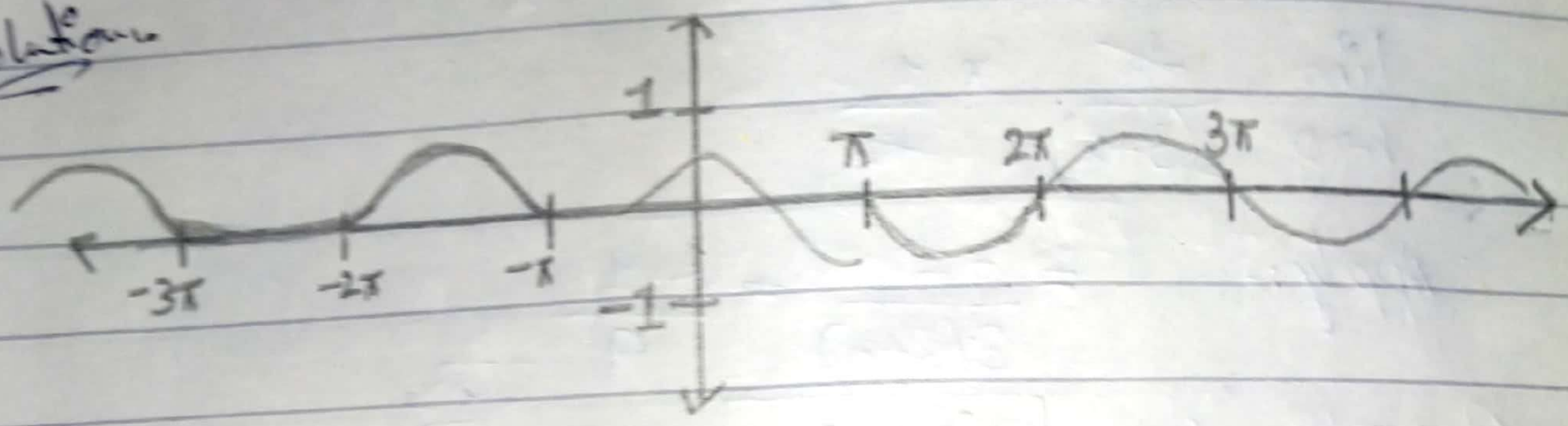
Q:32:~ Sketch the graph of the function and use it to determine the values of a for which $\lim_{x \to a} f(x)$ exists.

$$f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x \leq \pi \\ \sin x & \text{if } x > \pi \end{cases}$$

Solution:



limit exist, except $x = \pi$

Q:33:~ $\lim_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

Sol: Simplifying nominator & denominator

$$\lim_{x \to 4} \frac{x\,\cancel{(x-4)}}{(x+1)\,\cancel{(x-4)}}$$

$$\lim_{x \to 4} \frac{x}{x+1}$$

After Applying limit:
we get:

$$\Rightarrow \frac{4}{4+1} \Rightarrow \frac{4}{5}$$

Q:34:- $\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h}$

Sol:-

$$\lim_{h\to 0} \left(\frac{1}{(3+h)} - \frac{1}{3}\right) \times \frac{1}{h}$$

Taking L.C.M

$$\lim_{h\to 0} \left(\frac{3-3-h}{3(3+h)}\right) \times \frac{1}{h}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{3(3+h)} \times \frac{1}{\cancel{h}}$$

Apply:- $= \frac{-1}{3(3+0)} = \frac{-1}{9}$ Ans

Q:35:- $\lim_{x\to 4} \frac{\sqrt{x^2+9} - 5}{x+4}$

Sol:- $\lim_{x\to 4} \frac{\sqrt{x^2+9} - 5}{x-4} \times \frac{\sqrt{x^2-9} + 5}{\sqrt{x^2+9} + 5}$

$$\lim_{x\to 4} \frac{(x^2+9) - 25}{(x+4)(\sqrt{x^2+9} + 5)}$$

$$\lim_{x\to 4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9} + 5)}$$

$$\lim_{x\to 4} \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(\sqrt{x^2+9} + 5)}$$

Apply:-

$$= \frac{-4-4}{\sqrt{(-4)^2+9} + 5} \Rightarrow \frac{\cancel{-8}\,4}{\cancel{10}\,5} \Rightarrow \frac{-4}{5}$$ Ans

Q:36:~ $\lim_{h\to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

Soln Taking L.C.M on Nominator.

$$\lim_{h\to 0} \frac{x^2-(x+h)^2}{x^2(x+h)^2} \div h$$

R.w h in divide changes into $\frac{1}{h}$

In nominator Apply $(a+b)^2$ formula and $\div h$ becomes $\frac{1}{h}$.

$$\lim_{h\to 0} \frac{x^2-[(x^2)+2(x)(h)+(h)^2]}{x^2(x+h)^2} \times \frac{1}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{x^2(x+h)^2} \times \frac{1}{h}$$

$$\lim_{h\to 0} \frac{-(2xh+h^2)}{x^2(x+h)^2} \times \frac{1}{h}$$

$$\lim_{h\to 0} \frac{-(2x+h)\cancel{h}}{x^2(x+h)^2} \times \frac{1}{\cancel{h}}$$

Apply..

$$= \frac{-(2x+0)}{x^2(x+0)^2} = \frac{-2\cancel{x}}{x^{\cancel{4}}}$$

$$= \frac{-2}{x^3} \text{ Ans}$$

Q:37:- $\lim_{x \to -1} \frac{x^2+2x+1}{x^4-1}$

Soln $\lim_{x \to -1} \left(\frac{x^2+2x+1}{x^4-1} \right)$

$\lim_{x \to -1} \frac{(x+1)^2}{(x^2+1)(x^2-1)}$

$\lim_{x \to -1} \frac{(x+1)^{\cancel{2}}}{(x^2+1)\cancel{(x+1)}(x+1)}$

$\lim_{x \to -1} \frac{x+1}{(x^2+1)(x-1)}$

Apply

$= \frac{\cancel{1}+\cancel{1}}{[(-1)^2+1](-1-1)} = \frac{0}{(2)(-2)}$

$= 0$

Q:38:- $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right)$

Soln $\lim_{t \to 0} \frac{1}{t} - \frac{1}{t(t+1)}$

$\lim_{t \to 0} \frac{t^2+\cancel{t}-\cancel{t}}{t(t+1)}$: taking L.C.M

$\lim_{t \to 0} \frac{t}{(t+1)}$

Apply

$$= \frac{0}{0+1} = 0 \text{ Ans}$$

Q39:- $\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$

Sol:- $\lim_{h \to 0} \frac{h^3 + 6h^2 + 12h + \cancel{8} - \cancel{8}}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(h^2 + 6h + 12)}{\cancel{h}}$

$\lim_{h \to 0} h^2 + 6h + 12$

Apply

$\Rightarrow 0^2 + 6(0) + 12$

$= 12$ Ans

Q:40:- $\lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u-2}$

Sol:- $\lim_{u \to 2} \frac{\sqrt{4u+1} - 3}{u-2} \times \frac{\sqrt{4u+1} + 3}{\sqrt{4u+1} + 3}$

$\lim_{u \to 2} \frac{(\sqrt{4u+1})^2 - (3)^2}{(u-2)(\sqrt{4u+1} + 3)}$

$\lim_{u \to 2} \frac{4u + 1 - 9}{(u-2)(\sqrt{4u+1} + 3)}$

$\lim_{u \to 2} \frac{4u - 8}{(u-2)(\sqrt{4u+1} + 3)}$

$$\lim_{x \to 2} \frac{4\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{4x+1}+3)} = \frac{4}{\sqrt{4x+1}+3}$$

Apply $x = 2$

$$= \frac{4}{\sqrt{4(2)+1}+3}$$

$$= \frac{4}{\sqrt{9}+3} = \frac{\cancel{4}\,2}{\cancel{6}\,3}$$

$$= \frac{2}{3} \text{ Ans.}$$

Q:41:- $\lim_{x \to -1} \frac{2x^2+3x+1}{x^2-2x-3}$

Sol:- $\lim_{x \to -1} \frac{(2x+1)\cancel{(x+1)}}{(x-3)\cancel{(x+1)}}$

$$\lim_{x \to -1} \frac{2x+1}{x-3}$$

Apply

$$= \frac{2(-1)+1}{(-1)-3}$$

$$= \frac{1}{4} \text{ Ans.}$$

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Q:42:~ $\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1}$

Solu

$$\lim_{t \to 1} \frac{(t^2+1)(t+1)\cancel{(t-1)}}{\cancel{(t-1)}(t^2+t+1)}$$

$$\lim_{t \to 1} \frac{(t^2+1)(t+1)}{t^2+t+1}$$

Apply

$$= \frac{(1^2+1)(1+1)}{1^2+1+1}$$

$$= \frac{4}{3} \text{ Ans}$$

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