

### Example 2.1

In a regular hexagon, each angle is 120 degrees. Therefore, the cosine law can be written as follows:

$$D^2 = R^2 + R^2 - 2R * R * \cos(120 \text{ degrees})$$

Simplifying the equation, we have:

$$D^2 = 2R^2 + R^2 - 2R^2 * \cos(120 \text{ degrees})$$

$$D^2 = 3R^2 - 2R^2 * (-0.5)$$

$$D^2 = 3R^2 + R^2$$

$$D^2 = 4R^2$$

Taking the square root of both sides, we get:

$$D = 2R$$

So, the distance between the center cell and the third neighboring cell is equal to  $2R$ .

ABDUL RAHEM CSC20S104 Assignment#02 WMC Sir Saddam Ahmed Shaikh  
thenumberofcellsthatcan be assigned the same channel.

Since we have established that the distance  $R$  is the same as the distance  $D$ , we can write:

$$Q^2 = 3R^2$$

Taking the square root of both sides, we have:

$$Q = \sqrt{3R^2}$$

$$\text{Therefore, } Q = \sqrt{3a^2} = \sqrt{3N}.$$

Hence, we have proved that for a hexagonal geometry, the co-channel reuse ratio is given by  $Q = \sqrt{3N}$ , where  $N = i^2 + ij + j^2$ .

## ABDUL RAHEM CSC20S104 Assignment#02 WMC Sir Saddam Ahmed Shaikh

### Example 2.3

(a) Omni-Directional Antennas:

For omni-directional antennas, the co-channel reuse ratio is given by  $Q = \sqrt{3N}$ . To find  $N$ , we divide both sides of the equation by  $\sqrt{3}$ :

$$N = Q / \sqrt{3}$$

Substituting the value of  $Q \approx 2378$ , we can calculate  $N$ :

$$N \approx 2378 / \sqrt{3} \approx 1374$$

(b) 120 Degree Sectoring:

For 120-degree sectoring, the co-channel reuse ratio is also given by  $Q = \sqrt{3N}$ .

Dividing both sides of the equation by  $\sqrt{3}$ , we get:

$$N = Q / \sqrt{3}$$

Substituting the value of  $Q \approx 2378$ , we can calculate  $N$ :

## ABDUL RAHEEM CSC20S104 Assignment#02 WMC Sir Saymad Ahmed Shaikh

calculated values of  $N$  are approximately:

- (a)  $N \approx 1.374$  for omni-directional antennas.
- (b)  $N \approx 1.374$  for 120-degree sectoring.
- (c)  $N \approx 0.793$  for 60-degree sectoring.

These values represent the optimal values of  $N$  for each antenna scheme.

### Example 2.4

(a) Omni-Directional Antennas:

Using the formula  $G = R * (SIR)^{(1/n)}$ , we can calculate the optimal value of  $N$  for omni-directional antennas.

Given:

$$SIR = 15 \text{ dB} = 10^{(15/10)}$$

$$n = 3$$

$$G = R * (SIR)^{(1/n)}$$

$$G = 1 * (10^{(15/10)})^{(1/3)}$$

$$G \approx 5.623$$



## ABDUL RAHEM CSC20S104 Assignment#02 WMC Sir Saddam Ahmed Shaikh

60-degree sectoring.

$$G = R * (SIR)^{(1/n)}$$

$$G = 1 * (10^{(15/10)})^{(1/3)}$$

$$G \approx 5.623$$

After recalculating the values of  $G$  for each case, we can see that the optimal value of  $n$  is approximately 5.623 for all antenna schemes when the path loss exponent is  $n = 3$ .

Therefore, with a path loss exponent of  $n = 3$ , the optimal value of  $n$  is approximately 5.623 for omni-directional antennas, 120-degree sectoring, and 60-degree sectoring.

## ABDUL RAHEEM CSC20S104 Assignment#02 WMC Sir Saad Ahmed Sheikh

### Example 2.5

1. Omni-directional antennas: For an  $N = 7$  system with  $P_H[\text{Blocking}] = 1\%$ , we can calculate the offered traffic ( $A$ ) using the Erlang B formula:

$A = N * \lambda * T$  where  $N$  is the number of channels,  $\lambda$  is the average per user call rate in calls per hour, and  $T$  is the average call duration in hours.

Given:

$N = 57$  channels  $P_H[\text{Blocking}] = 1\% (0.01)$

$\lambda = 1$  per hour (unknown, to be calculated)

$T = 2 \text{ minutes} = 2/60 \text{ hours} = 1/30 \text{ hours}$

From the Erlang B formula, we have:  $0.01 = (57 * \lambda * (1/30)) / (57 * \lambda * (1/30) + 1)$   
 $0.01 = \lambda / (\lambda + 1)$

Solving this equation, we can find the value of  $\lambda$ .

2.  $60^\circ$  sectored antennas: For  $60^\circ$  sectored antennas, we need to consider a different value of  $N$ , as the number of channels may change. Let's assume  $N$  channels for the  $60^\circ$  sectored case. Using the same Erlang B formula as above, but with

## ABDUL RAHEEM CSC20S104 Assignment #02 WMC Sir Saddam Ahmed Sheikh

the new value of  $n'$  channels, we can calculate the offered traffic ( $A'$ ).

Given:

$$n' = 57 \text{ channels (same as before)} \quad P_H[\text{blocking}] = 1\% (0.01)$$

$$\lambda' = 1 \text{ per hour (unknown, to be calculated)}$$

$$\tau = 2 \text{ minutes} = 2/60 \text{ hours} = 1/30 \text{ hours}$$

$$\text{From the Erlang B formula, we have: } 0.01 = (57 * \lambda' * (1/30)) / (57 * \lambda' * (1/30) + 1) \quad 0.01 = \lambda' / (\lambda' + 1)$$

Solving this equation, we can find the value of  $\lambda'$ .

$$\text{Traffic Capacity Loss} = A - A'$$

Example 2.7

(a) To calculate the maximum system capacity (total and per channel) in Erlangs with different numbers of channels and a 2% blocking probability, we can use the Erlang B formula.

Given: Blocking probability ( $P_H[\text{blocking}]$ )  
 $= 2\% = 0.02$  number of channels ( $n$ ) = 4,20,40 Using the Erlang B formula, we can solve for the offered traffic ( $A$ ) in Erlangs.



## ABDUL RAHEM CSC20S104 Assignment #02 WMC Sir Saddam Ahmed Sheikh

Offered traffic in Erlangs.

1. For 4 channels:  $N = 4$  channels  $0.02 = 4 * \lambda / (4 - \lambda)$

Solving this equation, we can find the offered traffic ( $A$ ) in Erlangs and the maximum system capacity per channel.

2. For 20 channels:  $N = 20$  channels  $0.02 = 20 * \lambda / (20 - \lambda)$

Solving this equation, we can find the offered traffic ( $A$ ) in Erlangs and the maximum system capacity per channel.

3. For 40 channels:  $N = 40$  channels  $0.02 = 40 * \lambda / (40 - \lambda)$

Solving this equation, we can find the offered traffic ( $A$ ) in Erlangs and the maximum system capacity per channel.

(b) To determine the number of users that can be supported with 40 channels at a 2% blocking probability, we need to calculate the offered traffic ( $A$ ) in Erlangs using the Erlang B formula.

Given: Blocking probability ( $P_B[\text{Blocking}]$ ) = 2% = 0.02 number of channels



## ABDUL RAHEEM CSC20S104 Assignment #02 WMC Sir Saddam Ahmed Sheikh

$$\begin{aligned} (N) &= 40 \text{ offered traffic } (A) = N * \lambda / (N - \lambda) \text{ average call duration } (H) \\ &= 105 \text{ seconds Call arrival rate } (\lambda) = 1 \text{ call/hour} \\ U &= A * H \end{aligned}$$

(c) To find the grade of service (GOS) in a lost call delayed system for the case of delays greater than 20 seconds, we need to use the Erlang B formula and account for the call holding time (H).

$$\begin{aligned} \text{Given: Blocking probability } (P_B[\text{Blocking}]) &= 2\% = 0.02 \text{ number of channels } (N) = \\ &4, 20, 40 \text{ Call holding time } (H) = 105 \text{ seconds} \\ \rho &= A / H \end{aligned}$$

If the maximum system capacity (per channel) calculated in part (a) is higher than the GOS for delays greater than

20 seconds (calculated in part (c)), it indicates that the system with a 20-second queue performs better. This means that a higher number of users can be supported with an acceptable GOS using a delayed system compared to a system that

## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Saad Ahmed Sheikh

clears blocked calls.

### Example 2.8

Given: Received power at the receiver  $(P) = 1 \text{ mW} = 0 \text{ dBm}$  Distance from the transmitter to the receiver  $(d) = d_0 = 1 \text{ meter}$  Required interference threshold  $(I_{\text{thresh}}) = -100 \text{ dBm}$  Path loss exponent  $(n) = 3$

The path loss formula can be expressed as:  $P = P_t - PL$  where  $P_t$  is the transmitted power and  $PL$  is the path loss.

Considering  $P_t = 1 \text{ mW}$  and using the path loss formula, we have:  $0 \text{ dBm} = 1 \text{ mW} - PL$   
 $PL = 1 \text{ mW} - 0 \text{ dBm} = 1 \text{ mW}$

Using the path loss formula and the path loss exponent, we have:  $PL = (d/d_0)^n$

Substituting the given values, we get:  $1 \text{ mW} = (1/1)^3$

$\text{mW} = 131 \text{ mW} = 1$  This equation is satisfied for any value of  $I$ . Therefore, there is no unique major radius for the 7-cell reuse pattern.

## ABDUL RAHEEM CSC20S104 Assignment #02 WMC Sir Saad Ahmed Sheikh

power at the receiver ( $P$ ) =  $1 \text{ mW} = 0 \text{ dBm}$  Distance from the transmitter to the receiver ( $d$ ) =  $d_0 = 1 \text{ meter}$  Required interference threshold ( $I_{\text{thresh}}$ ) =  $-100 \text{ dBm}$  Path loss exponent ( $n$ ) =  $3$

Using the path loss formula and the given values, we have:  $0 \text{ dBm} = 1 \text{ mW} - \text{PL}$   $\text{PL} = 1 \text{ mW} - 0 \text{ dBm} = 1 \text{ mW}$

Using the path loss formula and the path loss exponent, we have:  $\text{PL} = (d/d_0)^n$

Substituting the given values, we get:  $1 \text{ mW} = (1/1)^3 1 \text{ mW} = 1^3 1 \text{ mW} = 1$

This equation is also satisfied for any value of  $I$ . Therefore, there is no unique major radius for the 4-

cell reuse pattern. Example 2.9 Given: Cluster size ( $C$ ) =  $7$  Total channels ( $N$ ) =  $660$  Setup (control) channels ( $S$ ) =  $30$  Voice channels per cell ( $V$ ) =  $90$  Potential user density ( $D$ ) =  $9000 \text{ users/km}^2$  Average call rate per user ( $\lambda$ ) =  $1 \text{ call/hour}$  Call duration ( $H$ ) =  $1 \text{ minute}$



## ABDUL RAHEEM CSC20S104 Assignment #02 WMC Sir Saddam Ahmed Sheikh

First, let's calculate the traffic intensity per channel ( $\rho$ ) for a single cell:  $\rho = \lambda$

$$* H / V = (1 \text{ call/hour}) * (1/60 \text{ hour}) / 90 = 1/540$$

Now, let's calculate the offered traffic per channel ( $A$ ) for a single cell:  $A = \rho / (1 - \rho) = (1/540) / (1 - 1/540) \approx 1/539$

Since the system has a cluster size of 7, the offered traffic per cluster ( $A_{\text{cluster}}$ ) is given by:  $A_{\text{cluster}} = 7 * A$

Next, let's calculate the total offered traffic ( $A_{\text{total}}$ ) for the entire system:

$$A_{\text{total}} = A_{\text{cluster}} * (N - S) = (7 * A) * (660 - 30)$$

The traffic intensity ( $\rho_{\text{total}}$ ) for the entire system is given by:  $\rho_{\text{total}} = A_{\text{total}} / (N - S) = A_{\text{total}} / (660 - 30)$

Finally, using the Erlang C formula, we can calculate the probability that a user will experience a delay greater than 20 seconds in a queuing system:  $P_{\text{delay}} > 20s = ((\rho_{\text{total}})^C) /$



## ABDUL RAHEEM CSC20S104 Assignment #02 WMC Sir Saad Ahmed Sheikh

$$C! / (\sum (\rho_{\text{total}}^i / i!)) \text{ for } i = 0 \text{ to } C$$

Example 2.18

(a) To calculate the traffic intensity for each user, we need to determine the offered traffic ( $A$ ) per user. The traffic intensity ( $\rho$ ) is defined as the ratio of the offered traffic to the average service time.

Given: Average number of calls per user per hour ( $\lambda$ ) = 3 calls/hour  
Average call duration ( $H$ ) = 5 minutes =  $5/60$  hours

The offered traffic per user ( $A$ ) can be calculated as:  $A = \lambda * H$

Substituting the given values, we have:  $A = 3 * (5/60)$   $A = 1/4$

The traffic intensity ( $\rho$ ) for each user is given by:  $\rho = A / H$

Substituting the values, we get:  $\rho = (1/4) / (5/60)$   $\rho = 3/10$

Therefore, the traffic intensity for each user is  $3/10$ .

## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Saad Ahmed Sheikh

We can use the Erlang B formula.

Given: Blocking probability ( $P_B[\text{Blocking}]$ ) = 1% = 0.01 number of channels ( $N$ ) = 1

Using the Erlang B formula, we can calculate the offered traffic ( $A$ ) in Erlangs:

$$A = N * \rho / (N - \rho)$$

Substituting the values, we get:  $A = 1 * (3/10) / (1 - 3/10)$   $A = 3/7$

The number of users that could use the system with 1% blocking when only one channel is available is given by: number of users =  $A / \rho$

Substituting the values, we get: number of users =  $(3/7) / (3/10)$  number of users = 10 Therefore, 10 users could use the system with 1% blocking when only one channel is available.

(c) To find the number of users that could use the system with 1% blocking if five trunked channels are available, we can use the Erlang B formula with the new number of channels.

## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Sammad Ahmed Sheikh

probability ( $P_H[\text{Blocking}]$ ) = 10% = 0.01 number of channels ( $N$ ) = 5

Using the Erlang B formula, we can calculate the offered traffic ( $A$ ) in Erlangs:

$$A = N * \rho / (N - \rho)$$

Substituting the values, we get:  $A = 5 * (3/10) / (5 - 3/10)$   $A = 15/23$

The number of users that could use the system with 10% blocking when five trunked channels are available is given by: number of users =  $A / \rho$

Substituting the values, we get: number of users =  $(15/23) / (3/10)$  number of users =  $50/23$

Therefore, approximately 2.17 (or  $50/23$ ) users could use the system with 10% blocking when five trunked channels are available.

### Example 2.20

(a) To determine the number of base stations (cell sites) that can be installed for \$6 million, we need to consider the cost of each base station. Given:

Cost of each base station = \$500,000



## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Saïmad Ahmed Sheikh

Total budget available = \$6,000,000

Number of base stations = Total budget available / Cost of each base station

Number of base stations = \$6,000,000 / \$500,000 Number of base stations = 12

Therefore, you will be able to install 12 base stations for \$6 million.

(b) Assuming the earth is flat and subscribers are uniformly distributed on the ground, the coverage area of each cell site can be approximated as a hexagon. In a hexagonal cell layout, the coverage area of each cell is equilateral triangular. The major radius of each cell in a hexagonal mosaic can be calculated using the formula:  $R = (3 * A) / (2 * \sqrt{3})$  where  $R$  is the major radius and  $A$  is the area of each cell.

Given:

Coverage area of the license = 140 square km

The area of each cell in a hexagonal mosaic can be calculated as:  $A =$



## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Saïmad Ahmed Sheikh

Coverage area / number of cells  $A = 140 \text{ square km} / 12 \Rightarrow A \approx 11.67 \text{ square km}$

Using the formula, we can calculate the major radius of each cell:  $R = (3 * 11.67) / (2 * \sqrt{3}) \Rightarrow R \approx 6.74 \text{ km}$

Therefore, assuming a hexagonal mosaic, the major radius of each cell will be approximately 6.74 km.

(c) To determine the minimum number of customers needed on the first day of service in order to earn \$10 million in gross billing revenues by the end of the 4th year, we need to consider the revenue generated by each customer over the 4-year period.

Given:

Average customer payment per month = \$50  
Revenue earned per year = Average customer payment per month \* 12 months

To calculate the minimum number of customers, we can work backward from the desired gross billing revenue.

Total revenue earned over 4 years = Desired

## ABDUL RAHEH CSC20S104 Assignment #02 WMC Sir Saïmad Ahmed Sheikh

gross billing revenue = \$10 million

Simplifying the equation, we have:  $\$10 \text{ million} = (\text{Revenue earned per year} * \text{number of customers on the first day}) * (1 + 2 + 2 + 2)$   
 $\text{number of customers on the first day} = \$10 \text{ million} / (\$50 * 12 * 7)$

number of customers on the first day  $\approx 2381$

(d) To calculate the number of users per square km needed on the first day of service to reach the \$10 million mark after the 4th year, we can divide the total number of customers on the first day by the coverage area. Given: Number of customers on the first day = 2381 Coverage area = 140 square km

number of users per square km = number of customers on the first day / Coverage area

number of users per square km =  $2381 / 140$

number of users per square km  $\approx 17$  Therefore, approximately 17 users per square km are needed on

ABDUL RAFEH CSC20S104 Assignment#02 WMC Sir Sammad Ahmed Sheikh

the first day of service in order to reach the \$10 million mark after the 4th year.

-----x-----x-----