**Initial Condition:** A condition is on the solution at one point.

**Equation with variable Separable:** A first order equation is called variable separable if it can be written $\frac{dy}{dx} = \frac{f(x)}{g(y)}$.

## EXERCISE # 6.1

**Evaluate the following indefinite integrals.**

**(1)** $\int (x^2 + 4x + 13)\,dx.$

**Solution:** let $I = \int (x^2 + 4x + 13)\,dx.$

$I = \int x^2\,dx + \int 4x\,dx + \int 13dx \Rightarrow I = \int x^2\,dx + 4\int x\,dx + 13\int dx$

$I = \frac{x^3}{3} + \cancel{4}^{2}\frac{x^2}{\cancel{2}} + 13x + c \Rightarrow \boxed{I = \frac{x^3}{3} + 2x^2 + 13x + c}$ Ans

**(2)** $\int (3x^4 - 5x^3 - 4x^2 - 2)\,dx$

**Solution:** let $I = \int (3x^4 - 5x^3 - 4x^2 - 2)\,dx$

$I = \int 3x^4\,dx - \int 5x^3\,dx - \int 4x^2\,dx - \int 2x\,dx$

$I = 3\int x^4\,dx - 5\int x^3\,dx - 4\int x^2\,dx - 2\int dx$

$\boxed{I = \frac{3}{5}x^5 - \frac{5}{4}x^4 - \frac{4}{3}x^3 - 2x + c}$ Ans

**(3)** $\int (4x^3 - 12x^2 - 4x + 12)\,dx$

**Solution:** let $I = \int (4x^3 - 12x^2 - 4x + 12)\,dx$

$I = \int 4x^3\,dx - \int 12x^2\,dx - \int 4x\,dx + \int 12\,dx$

$I = 4\int x^3\,dx - 12\int x^2\,dx - 4\int x\,dx + 12\int dx$

$I = \cancel{4}\frac{x^4}{\cancel{4}} - {}^{4}\cancel{12}\frac{x^3}{\cancel{3}} - {}^{2}\cancel{4}\frac{x^2}{\cancel{2}} + 12x + c$

$\boxed{I = x^4 - 4x^3 - 2x^2 + 12x + c}$ Ans

**(4)** $\int x^2(x^2 - 4)\,dx$

**Solution:** let $I = \int x^2(x^2 - 4)\,dx$

$I = \int (x^4 - 4x^2)\,dx \Rightarrow I = \int x^4\,dx - \int 4x^2\,dx$

$I = \int x^4\,dx - 4\int x^2\,dx \Rightarrow \boxed{I = \frac{x^5}{5} - \frac{4}{3}x^3 + c}$ Ans

**(5)** $\int \sqrt{2x + 5}\,dx$

**Solution:** let $I = \int \sqrt{2x + 5}\,dx$

$I = \int (2x + 5)^{1/2}\,dx$

x & ÷ by 2



$I = \frac{1}{2}\int (2x + 5)^{1/2}.\,2\,dx \Rightarrow I = \frac{1}{2}\,\frac{(2x + 5)^{3/2}}{\frac{3}{2}} + c$

$I = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}(2x + 5)^{3/2} + c \Rightarrow \boxed{I = \frac{1}{3}(2x + 5)^{3/2} + c}$ Ans

**(6)** $\int (2x + 3)^{2/3}\,dx$

**Solution:** let $I = \int (2x + 3)^{2/3}\,dx$

x & ÷ by 2.

$I = \frac{1}{2}\int (2x + 3)^{2/3}.\,2\,dx \Rightarrow I = \frac{1}{2}\,\frac{(2x + 3)^{5/3}}{\frac{5}{3}} + c$

$\boxed{I = \frac{3}{10}(2x + 3)^{5/3} + c}$ Ans

**(7)** $\int (3x + 4)^{29}\,dx$

**Solution:** let $I = \int (3x + 4)^{29}\,dx$

x & ÷ by 3

$I = \frac{1}{3}\int (3x + 4)^{29}.3\,dx \Rightarrow I = \frac{1}{3}\,\frac{(3x + 4)^{30}}{30} + c$

$\boxed{I = \frac{(3x + 4)^{30}}{90} + c}$ Ans

**(8)** $\int \frac{du}{u^2}$

**Solution:** let $I = \int \frac{du}{u^2}$

$I = \int u^{-2}\,du \Rightarrow I = \frac{u^{-1}}{-1} + c \Rightarrow \boxed{I = \frac{-1}{u} + c}$ Ans

**(9)** $\int \frac{6}{v^5}\,dv$

**Solution:** let $I = \int \frac{6}{v^5}\,dv.$

$I = \int 6v^{-5}\,du \Rightarrow I = 6\int v^{-5}\,du \Rightarrow I = {}^{3}\cancel{6}\frac{v^{-4}}{-\cancel{4}_{2}} + c$

$\boxed{I = \frac{-3}{2v^4} + c}$ Ans

(10) $\int \frac{dy}{\sqrt{ay+b}}$

**Solution:** let $I = \int \frac{dy}{\sqrt{ay+b}}$

$I = \int (ay+b)^{-1/2}\, dy$

x & ÷ by a

$I = \frac{1}{a}\int (ay+b)^{-1/2}.\, a\, dy \Rightarrow I = \frac{1}{a}\frac{(ay+b)^{1/2}}{1/2} + c$

$\boxed{I = \frac{2}{a}\sqrt{ay+b} + c}$ Ans

(11) $\int x(x^3+1)^2\, dx$

**Solution:** let $I = \int x(x^3+1)^2\, du$

$\boxed{\therefore (a+b)^2 = a^2 + 2ab + b^2}$

$I = \int x(x^6 + 2x^3 + 1)\, dx$

$I = \int (x^7 + 2x^4 + x)\, dx$

$I = \int x^7\, dx + 2\int x^4\, dx + \int x\, dx$

$\boxed{I = \frac{x^8}{8} + \frac{2}{5}x^5 + \frac{x^2}{2} + c}$ Ans

(12) $\int (x-1)(x-2)(x-3)\, dx$

**Solution:** let $I = \int (x-1)(x-2)(x-3)\, dx$

$I = \int (x-1)(x^2 - 3x - 2x + 6)\, dx$

$I = \int (x-1)(x^2 - 5x + 6)\, dx$

$I = \int (x^3 - 5x^2 + 6x - x^2 + 5x - 6)\, dx$

$I = \int (x^3 - 6x^2 + 11x - 6)\, dx$

$I = \int x^3\, dx - \int 6x^2\, dx + \int 11x\, dx - \int 6\, dx$

$I = \int x^3\, dx - 6\int x^2\, dx + 11\int x\, dx - 6\int dx$

$I = \frac{x^4}{4} - \cancel{6}^{2}\frac{x^3}{\cancel{3}} + 11\frac{x^2}{2} - 6x + c$

$\boxed{I = \frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x + c}$ Ans

(13) $\int (\sqrt{x} - 1)^2\, dx$

**Solution:** let $= \int (\sqrt{x} - 1)^2\, dx$

$\boxed{\therefore (a-b)^2 = a^2 - 2ab + b^2}$

$I = \int (x - 2\sqrt{x} + 1)\, dx \Rightarrow I = \int x\, dx - \int 2\sqrt{x}\, dx + \int dx$

$I = \int x\, dx - 2\int x^{1/2}\, dx + \int dx$



$I = \frac{x^2}{2} - 2\frac{x^{3/2}}{\frac{3}{2}} + x + c \Rightarrow I = \frac{x^2}{2} - 2\times\frac{2}{3}x^{3/2} + x + c$

$\boxed{I = \frac{x^2}{2} - \frac{4}{3}x^{3/2} + x + c}$ Ans

(14) $\int_0^2 \frac{dx}{\sqrt{1+x} + \sqrt{x}}$

**Solution:** let $I = \int_0^2 \frac{dx}{\sqrt{1+x} + \sqrt{x}} \times \frac{\sqrt{1+x} - \sqrt{x}}{\sqrt{1+x} - \sqrt{x}}$

$I = \int_0^2 \frac{(\sqrt{1+x} - \sqrt{x})\, dx}{(\sqrt{1+x} + \sqrt{x})(\sqrt{1+x} - \sqrt{x})}$

$\boxed{\therefore a^2 - b^2 = (a+b)(a-b)}$

$I = \int_0^2 \frac{(\sqrt{1+x} - \sqrt{x})\, dx}{(\sqrt{1+x})^2 - (\sqrt{x})^2} \Rightarrow I = \int_0^2 \frac{(\sqrt{1+x} - \sqrt{x})\, dx}{1 + \cancel{x} - \cancel{x}}$

$I = \int_0^2 (\sqrt{1+x} - \sqrt{x})\, dx \Rightarrow I = \int_0^2 (1+x)^{1/2}\, dx - \int_0^2 x^{1/2}\, dx$

$I = \left[\frac{(1+x)^{3/2}}{3/2}\right]_0^2 - \left[\frac{x^{3/2}}{3/2}\right]_0^2$

$I = \frac{2}{3}\left[(1+x)^{3/2}\right]_0^2 - \frac{2}{3}\left[x^{3/2}\right]_0^2$

$I = \frac{2}{3}\{(1+2)^{3/2} - (1+0)^{3/2}\} - \frac{2}{3}\{2^{3/2} - 0^{3/2}\}$

$I = \frac{2}{3}\{3^{3/2} - 1\} - \frac{2}{3}\{2^{3/2}\} \Rightarrow I = \frac{2}{3}\{3\sqrt{3} - 1\} - \frac{2}{3}\{2\sqrt{2}\}$

$\boxed{I = \frac{2}{3}(3\sqrt{3} - 2\sqrt{2} - 1)}$ Ans

(15) $\int \frac{x^2 + 3\sqrt{x} + 4}{3x^{\frac{3}{4}}}\, dx$

**Solution:** let $I = \int \left(\frac{x^2 + 3x^{1/2} + 4}{3x^{\frac{3}{4}}}\right) dx$

$I = \int \left\{\frac{x^2}{3x^{\frac{3}{4}}} + \frac{\cancel{3}x^{\frac{1}{2}}}{\cancel{3}x^{\frac{3}{4}}} + \frac{4}{3x^{\frac{3}{4}}}\right\} dx$

$$I = \int \left\{ \frac{1}{3} x^2 \cdot x^{-3/4} + x^{\frac{1}{2}} \cdot x^{\frac{-3}{4}} + \frac{4}{3} x^{\frac{-3}{4}} \right\} dx$$

$$I = \int \left\{ \frac{1}{3} x^{5/4} + x^{-1/4} + \frac{4}{3} x^{-3/4} \right\} dx$$

$$I = \frac{1}{3} \int x^{5/4}\, dx + \int x^{-1/4}\, dx + \frac{4}{3} \int x^{-3/4}\, dx$$

$$I = \frac{1}{3} \frac{x^{9/4}}{\frac{9}{4}} + \frac{x^{3/4}}{3/4} + \frac{4}{3} \frac{x^{1/4}}{1/4} + c$$

$$I = \frac{1}{3} \times \frac{4}{9} x^{9/4} + \frac{4}{3} x^{3/4} + \frac{4}{3} \times 4\, x^{1/4} + c$$

$$\boxed{I = \frac{4}{27} x^{9/4} + \frac{4}{3} x^{3/4} + \frac{16}{3} x^{1/4} + c} \quad \text{Ans}$$

**(16)** $\int \frac{x+8}{\sqrt{x}}\, dx$

**Solution:** let $I = \int \frac{x+8}{\sqrt{x}}\, dx$

$$I = \int \left( \frac{x+8}{x^{1/2}} \right) dx \Rightarrow I = \int \left( \frac{x}{x^{1/2}} + \frac{8}{x^{1/2}} \right) dx$$

$$I = \int (x\, x^{-1/2} + 8\, x^{-1/2})\, dx \Rightarrow I = \int (x^{1/2} + 8x^{-1/2})\, dx$$

$$I = \int x^{1/2}\, dx + 8 \int x^{-1/2}\, dx \Rightarrow I = \frac{x^{3/2}}{3/2} + 8 \frac{x^{1/2}}{1/2} + c$$

$$\boxed{I = \frac{2}{3} x^{3/2} + 16\sqrt{x} + c} \quad \text{Ans}$$

**(17)** $\int \frac{(\sqrt{\theta} - 1)^3}{\sqrt{\theta}}\, d\theta$

**Solution:** let $I = \int \frac{(\sqrt{\theta} - 1)^3}{\sqrt{\theta}}\, d\theta$

$$\boxed{\therefore (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3}$$

$$I = \int \frac{(\theta^{3/2} - 3\theta + 3\theta^{1/2} - 1)}{\theta^{1/2}}\, d\theta$$

$$I = \int \left( \frac{\theta^{3/2}}{\theta^{1/2}} - \frac{3\theta}{\theta^{1/2}} + \frac{3\cancel{\theta^{1/2}}}{\cancel{\theta^{1/2}}} - \frac{1}{\theta^{1/2}} \right) d\theta$$

$$I = \int (\theta^{3/2}.\, \theta^{-1/2} - 3\theta.\theta^{-1/2} + 3 - \theta^{-1/2})\, d\theta$$

$$I = \int (\theta - 3\theta^{1/2} + 3 - \theta^{-1/2})\, d\theta$$

$$I = \int \theta\, d\theta - \int 3\theta^{1/2}\, d\theta + \int 3\, d\theta - \int \theta^{-1/2}\, d\theta$$

$$I = \int \theta\, d\theta - 3 \int \theta^{1/2}\, d\theta + 3 \int d\theta - \int \theta^{-1/2}\, d\theta$$



$$I = \frac{\theta^2}{2} - 3 \frac{\theta^{3/2}}{3/2} + 3\theta - \frac{\theta^{1/2}}{1/2} + c$$

$$I = \frac{\theta^2}{2} - \cancel{3} \times \frac{2}{\cancel{3}} \theta^{3/2} + 3\theta - 2\sqrt{\theta} + c$$

$$\boxed{I = \frac{\theta^2}{2} - 2\theta^{3/2} + 3\theta - 2\sqrt{\theta} + c} \quad \text{Ans}$$

**(18)** $\int \frac{dx}{(2x+3)^{2/3}}$

**Solution:** let $I = \int \frac{dx}{(2x+3)^{2/3}}$

$$I = \int (2x+3)^{-2/3}\, dx$$

$\times$ & $\div$ by 2

$$I = \frac{1}{2} \int (2x+3)^{-2/3}.\, 2dx \Rightarrow I = \frac{1}{2} \frac{(2x+3)^{1/3}}{1/3} + c$$

$$\boxed{I = \frac{3}{2} (2x+3)^{1/3} + c} \quad \text{Ans}$$

**(19)** $\int_{-1}^{1} (2x^2+4)^3 (4x)\, dx$

**Solution:** let $I = \int_{-1}^{1} (2x^2+4)^3 (4x)\, dx$

$$I = \left[ \frac{(2x^2+4)^4}{4} \right]_{-1}^{1} \Rightarrow I = \frac{1}{4} \left[ (2x^2+4)^4 \right]_{-1}^{1}$$

$$I = \frac{1}{4} \left[ \{2(1)^2 + 4\}^4 - \{2(-1)^2 + 4\}^4 \right]$$

$$I = \frac{1}{4} \left[ (6)^4 - (6)^4 \right] \Rightarrow I = [\cancel{1296} - \cancel{1296}]$$

$$I = \frac{1}{4} (0) \Rightarrow \boxed{I = 0} \quad \text{Ans}$$

**(20)** $\int_{0}^{2} (x^2 + bx + c)^{-2/3} \left(x + \frac{b}{2}\right) dx$

**Solution:** let $I = \int_{0}^{2} (x^2 + bx + c)^{-2/3} \left(x + \frac{b}{2}\right) dx$

$\times$ & $\div$ by 2

$$I = \frac{1}{2}\int_0^2 (x^2 + bx + c)^{-2/3}(2x + b)\,dx$$

$$I = \frac{1}{2}\left[\frac{(x^2 + bx + c)^{1/3}}{1/3}\right]_0^2 \Rightarrow I = \frac{3}{2}\left[(x^2 + bx + c)^{1/3}\right]_0^2$$

$$\boxed{I = \frac{3}{2}\left[(4 + 2b + c)^{1/3} - c^{1/3}\right]} \text{ Ans}$$

**(21)** $\int \frac{3x^2 + 2x + 1}{(x^3 + x^2 + x + 7)^{1/7}}\,dx$

**Solution:** let $I = \int \frac{3x^2 + 2x + 1}{(x^3 + x^2 + x + 7)^{1/7}}\,dx$

$$I = \int (x^3 + x^2 + x + 7)^{-1/7}(3x^2 + 2x + 1)\,dx$$

$$I = \frac{(x^3 + x^2 + x + 7)^{6/7}}{6/7} + c \Rightarrow \boxed{I = \frac{7}{6}(x^3 + x^2 + x + 7)^{6/7} + c} \text{ Ans}$$

**Integrate the following with respect to their independent variable.**

**(22)** $\int \left(\sqrt{\theta} - \frac{1}{\sqrt{\theta}}\right)d\theta$

**Solution:** let $I = \int \left(\sqrt{\theta} - \frac{1}{\sqrt{\theta}}\right)d\theta$

$$I = \int \sqrt{\theta}\,d\theta - \int \frac{1}{\sqrt{\theta}}\,d\theta \Rightarrow I = \int \theta^{1/2}\,d\theta - \int \theta^{-1/2}\,d\theta$$

$$I = \frac{\theta^{3/2}}{3/2} - \frac{\theta^{1/2}}{1/2} + c \Rightarrow \boxed{I = \frac{2}{3}\theta^{3/2} - 2\sqrt{\theta} + c} \text{ Ans}$$

**(23)** $\int (ax^2 + bx + c)^2\,dx$

**Solution:** let $I = \int (ax^2 + bx + c)^2\,dx$

$$\boxed{\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac}$$

$$I = \int (a^2x^4 + b^2x^2 + c^2 + 2abx^3 + 2bcx + 2acx^2)\,dx$$

$$I = \int \{a^2x^4 + 2abx^3 + (2ac + b^2)x^2 + 2bcx + c^2\}\,dx$$

$$I = a^2\int x^4\,dx + 2ab\int x^3\,dx + (2ac + b^2)\int x^2\,dx + 2bc\int x\,dx + c^2\int dx.$$

$$I = \frac{a^2x^5}{5} + \frac{\cancel{2}abx^4}{\cancel{4}^2} + (2ac + b^2)\frac{x^3}{3} + \cancel{2}bc\frac{x^2}{\cancel{2}} + c^2x + c$$

$$\boxed{I = \frac{1}{5}a^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}(b^2 + 2ac)x^3 + bcx^2 + c^2x + c} \text{ Ans}$$



**(24)** $\int \left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2 dt$

**Solution:** let $I = \int \left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2 dt$

$$\boxed{\therefore (a + b)^2 = a^2 + 2ab + b^2}$$

$$I = \int \left\{t + 2(\cancel{\sqrt{t}})\left(\frac{1}{\cancel{\sqrt{t}}}\right) + \frac{1}{t}\right\}dt$$

$$I = \int \left(t + 2 + \frac{1}{t}\right)dt \Rightarrow I = \int t\,dt + \int 2\,dt + \int \frac{1}{t}\,dt$$

$$I = \int t\,dt + 2\int dt + \int \frac{1}{t}\,dt \Rightarrow \boxed{I = \frac{t^2}{2} + 2t + \ell nt + c} \text{ Ans}$$

**(25)** $\int \left(\frac{1}{y^3} - y\right)dy$

**Solution:** let $I = \int \left(\frac{1}{y^3} - y\right)dy$

$$I = \int \frac{1}{y^3}\,dy - \int y\,dy \Rightarrow I = \int y^{-3}\,dy - \int y\,dy$$

$$I = \frac{y^{-2}}{-2} - \frac{y^2}{2} + c \Rightarrow \boxed{I = \frac{-1}{2y^2} - \frac{y^2}{2} + c} \text{ Ans}$$

**(26)** $\int x^3(x + 2)^3(x - 1)^2\,dx$

**Solution:** let $I = \int x^3(x + 2)^3(x - 1)^2\,dx$

$$\boxed{\begin{array}{l}\therefore (a - b)^2 = a^2 - 2ab + b^2 \\ \therefore (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\end{array}}$$

$$I = \int x^3(x^3 + 6x^2 + 12x + 8)(x^2 - 2x + 1)\,dx$$

$$I = \int (x^6 + 6x^5 + 12x^4 + 8x^3)(x^2 - 2x + 1)\,dx$$

$$I = \int (x^8 - 2x^7 + x^6 + 6x^7 - \cancel{12x^6} + 6x^5 + \cancel{12x^6} - 24x^5 + 12x^4 + 8x^5 - 16x^4 - 8x^3)\,dx$$

$$I = \int (x^8 + 4x^7 + x^6 - 10x^5 - 4x^4 + 8x^3)\,dx$$

$$I = \int x^8\,dx + 4\int x^7\,dx + \int x^6\,dx - 10\int x^5\,dx - 4\int x^4\,dx + 8\int x^3\,dx$$

$$I = \frac{x^9}{9} + {}^{1}\cancel{4}\frac{x^8}{\cancel{8}_2} + \frac{x^7}{7} - {}^{5}\cancel{10}\frac{x^6}{\cancel{6}_3} - 4\frac{x^5}{5} + {}^{2}\cancel{8}\frac{x^4}{\cancel{4}_1} + c$$

$$\boxed{I = \frac{x^9}{9} + \frac{x^8}{2} + \frac{x^7}{7} - \frac{5x^6}{3} - \frac{4x^5}{5} + 2x^4 + c} \text{ Ans}$$

**(27)** $\int_{-1}^{1} (x^4 + 2x^3 + 3x^2 + 4x + 5)^{2/5} (4 + 6x + 6x^2 + 4x^3)\, dx$

**Solution:** let $I = \int_{-1}^{1} (x^4 + 2x^3 + 3x^2 + 4x + 5)^{2/5} (4x^3 + 6x^2 + 6x + 4)\, dx$

$$I = \left[\frac{(x^4 + 2x^3 + 3x^2 + 4x + 5)^{7/5}}{7/5}\right]_{-1}^{1}$$

$$I = \frac{5}{7}\left[(x^4 + 2x^3 + 3x^2 + 4x + 5)^{7/5}\right]_{-1}^{1}$$

$$I = \frac{5}{7}\left[\{(1)^4 + 2(1)^3 + 3(1)^2 + 4(1) + 5\}^{7/5} - \{(-1)^4 + 2(-1)^3 + 3(-1)^2 + 4(-1) + 5\}^{7/5}\right]$$

$$I = \frac{5}{7}\{(1 + 2 + 3 + 4 + 5)^{7/5} - (1 - 2 + 3 - 4 + 5)^{7/5})\}$$

$$\boxed{I = \frac{5}{7}\{(15)^{7/5} - (3)^{7/5}\}} \text{ Ans}$$

**(28)** $\int_{-1}^{1} \frac{3x^2 + 1}{(x^3 + x + 6)^{1/2}}\, dx$

**Solution:** let $I = \int_{-1}^{1} \frac{3x^2 + 1}{(x^3 + x + 6)^{1/2}}\, dx$

$$I = \int_{-1}^{1} (x^3 + x + 6)^{-1/2} . (3x^2 + 1)\, dx$$

$$I = \left[\frac{(x^3 + x + 6)^{1/2}}{1/2}\right]_{-1}^{1} \Rightarrow I = 2\left[(x^3 + x + 6)^{1/2}\right]_{-1}^{1}$$

$$I = 2\left[\{(1)^3 + (1) + 6\}^{1/2} - \{(-1)^3 + (-1) + 6\}^{1/2}\right]$$

$$I = 2[8^{1/2} - 4^{1/2}] = 2\{2\sqrt{2} - 2\}$$

$$\boxed{I = 4(\sqrt{2} - 1)} \text{ Ans}$$

## EXERCISE # 6.2

**Evaluate the following integrals.**

**(1)** $\int \frac{dx}{x^2 - 4}$

**Solution:** let $I = \int \frac{dx}{x^2 - 4}\, dx$

$$I = \int \frac{1}{(x)^2 - (2)^2}\, dx$$



$$\boxed{\text{Using formula } \therefore \int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + c}$$

$$I = \frac{1}{2(2)} \ln\left(\frac{x - 2}{x + 2}\right) + c$$

$$\boxed{I = \frac{1}{4} \ln\left(\frac{x - 2}{x + 2}\right) + c} \text{ Ans}$$

**(2)** $\int \frac{dy}{1 - y^2}$

**Solution:** let $I = \int \frac{1}{1 - y^2}\, dy$

$$I = \int \frac{1}{(1)^2 - (y)^2}\, dy$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \ln\left(\frac{a + x}{a - x}\right) + c}$$

$$I = \frac{1}{2(1)} \ln\left(\frac{1 + y}{1 - y}\right) + c \Rightarrow \boxed{I = \frac{1}{2} \ln\left(\frac{1 + y}{1 - y}\right) + c} \text{ Ans}$$

**(3)** $\int \frac{dx}{4x^2 - 1}$

**Solution:** let $I = \int \frac{dx}{4x^2 - 1}\, dx$

$$I = \int \frac{1}{4(x^2 - \frac{1}{4})}\, dx \Rightarrow I = \frac{1}{4} \int \frac{1}{(x)^2 - \left(\frac{1}{2}\right)^2}\, dx$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \ln\left(\frac{x - a}{x + a}\right) + c}$$

$$I = \frac{1}{4}\left\{\frac{1}{\cancel{2}\left(\frac{1}{\cancel{2}}\right)} \ln\frac{1}{2}\left(\frac{x - \frac{1}{2}}{x + \frac{1}{2}}\right)\right\} + c$$

$$I = \frac{1}{4} \ln\left(\frac{\frac{2x - 1}{\cancel{2}}}{\frac{2x + 1}{\cancel{2}}}\right) + c \Rightarrow \boxed{I = \frac{1}{4} \ln\left(\frac{2x - 1}{2x + 1}\right) + c} \text{ Ans}$$

**(4)** $\int_0^2 \frac{dx}{9-x^2}$

**Solution:** let $I = \int_0^2 \frac{dx}{9-x^2}\,dx$

$$I = \int_0^2 \frac{1}{(3)^2-(x)^2}\,dx$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\,\ell n\left(\frac{a+x}{a-x}\right)+c}$$

$$I = \left[\frac{1}{2(3)}\,\ell n\left(\frac{3+x}{3-x}\right)\right]_0^2$$

$$I = \frac{1}{6}\left\{\ell n\left(\frac{3+2}{3-2}\right)-\ell n\left(\frac{3+0}{3-0}\right)\right\}$$

$$I = \frac{1}{6}\left\{\ell n\left(\frac{5}{1}\right)-\ell n\left(\frac{\cancel{3}}{\cancel{3}}\right)\right\} \Rightarrow I = \frac{1}{6}\{\ell n\,5-\ell n\,1\}$$

$$I = \frac{1}{6}\{\ell n\,5-0\}$$

Note: $\boxed{\therefore \ell n\,1 = 0}$

$$\boxed{I = \frac{1}{6}\ln 5} \text{ Ans}$$

**(5)** $\int \frac{du}{\sqrt{u^2+9}}$

**Solution:** let $I = \int \frac{du}{\sqrt{u^2+9}}\,du.$

$$I = \int \frac{1}{\sqrt{(u)^2+(3)^2}}\,du.$$

$$\boxed{\text{Using formula } \therefore \frac{1}{\sqrt{x^2+a^2}}\,dx = \ell n\left(x+\sqrt{x^2+a^2}\right)+c}$$

$$\boxed{I = \ell n\left(u+\sqrt{u^2+9}\right)+c} \text{ Ans}$$

**(6)** $\int \frac{dy}{\sqrt{y^2-1}}$

**Solution:** let $I = \int \frac{dy}{\sqrt{y^2-1}}\,dy$

$$I = \int \frac{1}{\sqrt{(y)^2-(1)^2}}\,dy$$



$$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ell n\left(x+\sqrt{x^2-a^2}\right)+c}$$

$$\boxed{I = \ell n\left(y+\sqrt{y^2-1}\right)+c} \text{ Ans}$$

**(7)** $\int \sqrt{25-x^2}\,dx$

**Solution:** let $I = \int \sqrt{25-x^2}\,dx$

$$I = \int \sqrt{(5)^2-(x)^2}\,dx$$

$$\boxed{\text{Using formula } \therefore \int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\,\text{Sin}^{-1}\frac{x}{a}+c}$$

$$I = \frac{1}{2}x\sqrt{25-x^2}+\frac{1}{2}(5)^2\,\text{Sin}^{-1}\frac{x}{5}+c$$

$$\boxed{I = \frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\,\text{Sin}^{-1}\frac{x}{5}+c} \text{ Ans}$$

**(8)** $\int \sqrt{v^2-36}\,dv.$

**Solution:** let $I = \int \sqrt{v^2-36}\,dv.$

$$I = \int \sqrt{(v)^2-(6)^2}\,dv$$

$$\boxed{\text{Using formula } \therefore \int \sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2}-\frac{1}{2}a^2\,\ell n\left(x+\sqrt{x^2-a^2}\right)+c}$$

$$I = \frac{1}{2}v\sqrt{v^2-36}-\frac{1}{2}(6)^2\,\ell n\left(v+\sqrt{v^2-36}\right)+c$$

$$I = \frac{v\sqrt{v^2-36}}{2}-\frac{\overset{18}{\cancel{36}}}{\cancel{2}}\,\ell n\left(v+\sqrt{v^2-36}\right)+c$$

$$\boxed{I = \frac{v\sqrt{v^2-36}}{2}-18\,\ell n\left(v+\sqrt{v^2-36}\right)+c} \text{ Ans}$$

**(9)** $\int \frac{dx}{x\sqrt{4x^2-9}}$

**Solution:** let $= \int \frac{dx}{x\sqrt{4x^2-9}}$

$$I = \int \frac{1}{x\sqrt{4\left(x^2-\frac{9}{4}\right)}}\,dx \Rightarrow I = \frac{1}{2}\int \frac{1}{x\sqrt{(x)^2-\left(\frac{3}{2}\right)^2}}\,dx$$

$$\boxed{\text{Using formula } \therefore \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\,\text{Sec}^{-1}\frac{x}{a}+c}$$

$$I = \frac{1}{2}\left\{\frac{1}{\frac{3}{2}}\ \text{Sec}^{-1}\frac{x}{\frac{3}{2}}\right\} + c \Rightarrow I = \frac{1}{\cancel{2}}\frac{\cancel{2}}{3}\ \text{Sec}^{-1}\frac{2x}{3} + c$$

$\boxed{I = \frac{1}{3}\text{Sec}^{-1}\left(\frac{2x}{3}\right) + c}$ Ans

(10) $\int \frac{dy}{25 - 16y^2}$

**Solution:** let $I = \int \frac{dy}{25 - 16y^2}\ dy$

$$I = \int \frac{1}{16\left(\frac{25}{16} - y^2\right)}\,dy. \Rightarrow I = \frac{1}{16}\int \frac{1}{\left(\frac{5}{4}\right)^2 - (y)^2}\,dy.$$

$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\ \ell n\left(\frac{a+x}{a-x}\right) + c}$

$$I = \frac{1}{16}\left\{\frac{1}{\cancel{2}\left(\frac{5}{\cancel{4}_2}\right)}\ \ell n\left(\frac{\frac{5}{4} + y}{\frac{5}{4} - y}\right)\right\} + c$$

$$I = \frac{1}{16}\left\{\frac{2}{5}\ell n\left(\frac{\frac{5+4y}{\cancel{4}}}{\frac{5-4y}{\cancel{4}}}\right)\right\} + c$$

$$I = \frac{1}{\cancel{16}_8}\times\frac{\cancel{2}}{5}\ \ell n\left(\frac{5+4y}{5-4y}\right) + c \Rightarrow \boxed{I = \frac{1}{40}\ \ell n\left(\frac{5+4y}{5-4y}\right) + c}\ \text{Ans}$$

(11) $\int \frac{dx}{\sqrt{25 - 16x^2}}$

**Solution:** let $I = \int \frac{dx}{\sqrt{25 - 16x^2}}$

$$I = \int \frac{1}{\sqrt{16\left(\frac{25}{16} - x^2\right)}}\,dx \Rightarrow I = \frac{1}{4}\int \frac{1}{\sqrt{\left(\frac{5}{4}\right)^2 - (x)^2}}\,dx$$

$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \text{Sin}^{-1}\frac{x}{a} + c}$

$$I = \frac{1}{4}\left(\text{Sin}^{-1}\frac{x}{\frac{5}{4}}\right) + c \Rightarrow \boxed{I = \frac{1}{4}\ \text{Sin}^{-1}\left(\frac{4x}{5}\right) + c}\ \text{Ans}$$



(12) $\int \frac{dx}{4x^2 + 9}$

**Solution:** let $I = \int \frac{dx}{4x^2 + 9}$

$$I = \int = \frac{1}{4\left(x^2 + \frac{9}{4}\right)}\,dx \Rightarrow I = \frac{1}{4}\int \frac{1}{(x)^2 + \left(\frac{3}{2}\right)^2}\,dx$$

$\boxed{\text{Using formula } \therefore \int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\ \tan^{-1}\frac{x}{a} + c}$

$$I = \frac{1}{4}\left\{\frac{1}{\frac{3}{2}}\ \tan^{-1}\frac{x}{\frac{3}{2}}\right\} + c \Rightarrow I = \frac{1}{4}\left\{\frac{2}{3}\tan^{-1}\frac{2x}{3}\right\} + c$$

$$I = \frac{1}{\cancel{4}_2}\times\frac{\cancel{2}}{3}\tan^{-1}\frac{2x}{3} + c \Rightarrow \boxed{I = \frac{1}{6}\ \tan^{-1}\left(\frac{2x}{3}\right) + c}\ \text{Ans}$$

(13) $\int \frac{dx}{\sqrt{4x^2 + 9}}$

**Solution:** let $I = \int \frac{dx}{\sqrt{4x^2 + 9}}$

$$I = \int \frac{1}{\sqrt{4\left(x^2 + \frac{9}{4}\right)}}\,dx \Rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{(x)^2 + \left(\frac{3}{2}\right)^2}}\,dx$$

$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + c}$

$$I = \frac{1}{2}\ell n\left(x + \sqrt{x^2 + \frac{9}{4}}\right) + c \Rightarrow I = \frac{1}{2}\ell n\left(x + \sqrt{\frac{4x^2 + 9}{4}}\right) + c$$

$$I = \frac{1}{2}\ell n\left(x + \frac{\sqrt{4x^2 + 9}}{2}\right) + c \Rightarrow I = \frac{1}{2}\ell n\left(\frac{2x + \sqrt{4x^2 + 9}}{2}\right) + c,$$

$$I = \frac{1}{2}\left\{\ell n\left(2x + \sqrt{4x^2 + 9}\right) - \ell n2\right\} + c$$

$$I = \frac{1}{2}\ell n\left(2x + \sqrt{4x^2 + 9}\right) + c - \frac{\ell n2}{2}$$

50-0.5=0

$\boxed{\text{Since } c - \frac{\ln 2}{2} = \text{constant} = c}$

$\boxed{I = \frac{1}{2}\ \ell n\left(2x + \sqrt{4x^2 + 9}\right) + c}$ Ans

**(14)** $\int \frac{dz}{\sqrt{9z^2-25}}$

**Solution:** let $I = \int \frac{dz}{\sqrt{9z^2-25}}\, dz = \int \frac{1}{\sqrt{9\left(z^2-\frac{25}{9}\right)}}\, dz$

$$I = \frac{1}{3}\int \frac{1}{\sqrt{(z)^2-\left(\frac{5}{3}\right)^2}}\, dz$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ell n\,(x+\sqrt{x^2-a^2})+c}$$

$$I = \frac{1}{3}\ell n\left(z+\sqrt{z^2-\frac{25}{9}}\right)+c$$

$$I = \frac{1}{3}\ell n\left(z+\sqrt{\frac{9z^2-25}{9}}\right)+c$$

$$I = \frac{1}{3}\ell n\left(z+\frac{\sqrt{9z^2-25}}{3}\right)+c$$

$$I = \frac{1}{3}\ell n\left(\frac{3z+\sqrt{9z^2-25}}{3}\right)+c$$

$$I = \frac{1}{3}\left\{\ell n\,\left(3z+\sqrt{9z^2-25}\right)-\ell n 3\right\}+c$$

$$I = \frac{1}{3}\ell n\,(3z+(3z+\sqrt{9z^2-25})+c-\frac{\ln 3}{3}$$

$$\boxed{\text{Since } c-\frac{\ell n 3}{3} = \text{constant} = c}$$

$$\boxed{I = \frac{1}{3}\ell n\,(3z+\sqrt{9z^2-25})+c} \quad \text{Ans}$$

**(15)** $\int \sqrt{3-4x^2}\, dx.$

**Solution:** let $I = \int \sqrt{3-4x^2}\, dx.$

$$I = \int \sqrt{4\left(\frac{3}{4}-x^2\right)}\, dx$$

$$I = 2\int \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2-(x)^2}\, dx$$

$$\boxed{\text{Using formula } \therefore \int \sqrt{a^2-x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\,\text{Sin}^{-1}\frac{x}{a}+c}$$



$$= 2\left\{\frac{1}{2}x\sqrt{\frac{3}{4}-x^2}+\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)^2\text{Sin}^{-1}\left(\frac{x}{\frac{\sqrt{3}}{2}}\right)\right\}+c$$

$$I = 2\left\{\frac{x}{2}\sqrt{\frac{3-4x^2}{4}}+\frac{3}{8}\,\text{Sin}^{-1}\left(\frac{2x}{\sqrt{3}}\right)\right\}+c.$$

$$I = 2\left\{\frac{x}{4}\sqrt{3-4x^2}+\frac{3}{8}\,\text{Sin}^{-1}\left(\frac{2x}{\sqrt{3}}\right)\right\}+c$$

$$I = \cancel{2}\times\frac{x}{\cancel{4}_2}\sqrt{3-4x^2}+\cancel{2}\times\frac{3}{\cancel{8}_2}\,\text{Sin}^{-1}\left(\frac{2x}{\sqrt{3}}\right)+c$$

$$\boxed{I = \frac{x}{2}\sqrt{3-4x^2}+\frac{3}{4}\,\text{Sin}^{-1}\left(\frac{2x}{\sqrt{3}}\right)+c} \quad \text{Ans.}$$

**(16)** $\int \sqrt{16-9x^2}\, dx$

**Solution:** let $I = \int \sqrt{16-9x^2}\, dx$

$$I = \int \sqrt{9\left(\frac{16}{9}-x^2\right)}\, dx \Rightarrow I = 3\int \sqrt{\left(\frac{4}{3}\right)^2-(x)^2}\, dx$$

$$\boxed{\text{Using formula } \therefore \int \sqrt{a^2-x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{1}{2}a^2\,\text{Sin}^{-1}\frac{x}{a}+c}$$

$$I = 3\left\{\frac{1}{2}x\sqrt{\frac{16}{9}-x^2}+\frac{1}{2}\left(\frac{4}{3}\right)^2\text{Sin}^{-1}\frac{x}{\frac{4}{3}}\right\}+c$$

$$I = 3\left\{\frac{x\sqrt{16-9x^2}}{6}+\frac{16}{18}\,\text{Sin}^{-1}\frac{3x}{4}\right\}+c$$

$$I = \cancel{3}\,\frac{x\sqrt{16-9x^2}}{\cancel{6}_2}+\cancel{3}\times\frac{\overset{8}{\cancel{16}}}{\underset{3}{\cancel{18}}}\,\text{Sin}^{-1}\frac{3x}{4}+c$$

$$\boxed{I = \frac{x\sqrt{16-9x^2}}{2}+\frac{8}{3}\,\text{Sin}^{-1}\left(\frac{3x}{4}\right)+c}$$

## EXERCISE # 6.3

**Evaluate**

**(i)** $\int 4x^3(x^4+1)^{3/2}\, dx$

**Solution:** let $I = \int 4x^3(x^4+1)^{3/2}\, dx$

$$I = \frac{(x^4+1)^{5/2}}{5/2}+c \Rightarrow \boxed{I = \frac{2}{5}(x^4+1)^{5/2}+c} \quad \text{Ans}$$

**(ii)** $\int \frac{8x^2}{(x^3+2)^3}\,dx$

**Solution:** let $I = \int \frac{8x^2}{(x^3+2)^3}\,dx$

$I = \int (x^3+2)^{-3}\, 8x^2\,dx \Rightarrow I = 8\int (x^3+2)^{-3} x^2\,dx.$

$\times$ & $\div$ by 3

$I = \frac{8}{3}\int (x^3+2)^{-3}\, 3x^2\,dx \Rightarrow I = \frac{\overset{4}{\cancel{8}}}{3}\frac{(x^3+2)^{-2}}{-\cancel{2}} + c$

$\boxed{I = \frac{-4}{3}(x^3+2)^{-2} + c}$ Ans

**(iii)** $\int 3x\sqrt{1-2x^2}\,dx$

**Solution:** let $I = \int (1-2x^2)^{1/2}\, 3x\,dx$

$I = 3\int (1-2x^2)^{1/2}\, x\,dx.$

$\times$ & $\div$ by $(-4)$

$I = \frac{-3}{4}\int (1-2x^2)^{1/2}(-4x)\,dx \Rightarrow I = \frac{-3}{4}\frac{(1-2x^2)^{3/2}}{3/2} + c$

$I = -\frac{\cancel{3}}{\cancel{4}_2}\times\frac{\cancel{2}}{\cancel{3}}(1-2x^2)^{3/2} + c \Rightarrow \boxed{I = \frac{-1}{2}(1-2x^2)^{3/2} + c}$

**(iv)** $\int \frac{y+3}{(y^2+6y)^{1/2}}\,dy.$

**Solution:** let $I = \int \frac{y+3}{(y^2+6y)^{1/2}}\,dy.$

$I = \int (y^2+6y)^{-1/2}.(y+3)\,dy.$

$\times$ & $\div$ by 2.

$I = \frac{1}{2}\int (y^2+6y)^{-1/2}(2y+6)\,dy. \Rightarrow I = \frac{1}{2}\frac{(y^2+6y)^{1/2}}{1/2} + c$

$I = \frac{1}{\cancel{2}}\times\cancel{2}\sqrt{y^2+6y} + c \Rightarrow \boxed{I = \sqrt{y^2+6y} + c}$ Ans

**(v)** $\int \sqrt[3]{1-x^2}\; x\,dx$

**Solution:** let $I = \int \sqrt[3]{1-x^2}\; x\,dx$

$I = \int (1-x^2)^{1/3}\, x\,dx$

$\times$ & $\div$ by $(-2)$

$I = \frac{-1}{2}\int (1-x^2)^{1/3}.(-2x)\,dx \Rightarrow I = \frac{-1}{2}\frac{(1-x^2)^{4/3}}{4/3} + c$

$I = \frac{-1}{2}\times\frac{3}{4}(1-x^2)^{4/3} + c \Rightarrow \boxed{I = \frac{-3}{8}(1-x^2)^{4/3} + c}$



**(vi)** $\int \sqrt{x^2-24x^4}\,dx$

**Solution:** let $I = \int \sqrt{x^2-24x^4}\,dx$

$I = \int \sqrt{x^2(1-24x^2)}\,dx \Rightarrow I = \int \sqrt{1-24x^2}\; x\,dx$

$I = \int (1-24x^2)^{1/2}.\, x\,dx$

$\times$ & $\div$ by $(-48)$

$I = \frac{-1}{48}\int (1-24x^2)^{1/2}.(-48x)\,dx \Rightarrow I = \frac{-1}{48}\frac{(1-24x^2)^{3/2}}{3/2} + c$

$I = \frac{-1}{\cancel{48}}\times\frac{\cancel{2}}{3}(1-24x^2)^{3/2} + c \Rightarrow \boxed{I = \frac{-1}{72}(1-24x^2)^{3/2} + c}$ Ans

**(vii)** $\int \frac{x\,dx}{3x^2-4}$

**Solution:** let $I = \int \frac{x\,dx}{3x^2-4}$

$\times$ & $\div$ by 6

$I = \frac{1}{6}\int \frac{6x}{3x^2-4}\,dx \Rightarrow \boxed{I = \frac{1}{6}\,\ell n\,(3x^2-4) + c}$ Ans

**(viii)** $\int \frac{x^2\,dx}{(1-2x^3)^{2/3}}$

**Solution:** let $I = \int \frac{x^2\,dx}{(1-2x^3)^{2/3}}$

$I = \int (1-2x^3)^{-2/3}\, x^2\,dx$

$\times$ & $\div$ by $(-6)$

$I = \frac{-1}{6}\int (1-2x^3)^{-2/3}.(-6x^2)\,dx.$

$I = \frac{-1}{6}\frac{(1-2x^3)^{1/3}}{1/3} + c \Rightarrow I = \frac{-1}{\cancel{6}}\times\cancel{3}(1-2x^3)^{1/3} + c$

$\boxed{I = \frac{-1}{2}(1-2x^3)^{1/3} + c}$ Ans

**(ix)** $\int \frac{x+2}{x-3}\,dx.$

**Solution:** let $I = \int \frac{x+2}{x-3}\,dx.$

This question belongs to special case # 01 make N (x) as D (x).

$I = \int \frac{x-3+5}{x-3}\,dx. \Rightarrow I = \left(\frac{\cancel{(x-3)}}{\cancel{(x-3)}} + \frac{5}{x-3}\right)dx.$

$I = \int dx + 5\int \frac{1}{x-3}\,dx \Rightarrow \boxed{I = x + 5\,\ell n\,(x-3) + c}$ Ans

(x) $\int \frac{(1+x)(1-2x)}{1-2x^2}\,dx$

**Solution:** let $I = \int \frac{(1+x)(1-2x)}{1-2x^2}\,dx$

$I = \int \frac{1-2x+x-2x^2}{1-2x^2}\,dx \Rightarrow I = \int \frac{1-2x^2-x}{1-2x^2}\,dx$

$I = \int \left( \frac{\cancel{(1-2x^2)}}{\cancel{(1-2x^2)}} + \frac{x}{1-2x^2} \right) dx \Rightarrow I = \int \left( 1 - \frac{x}{1-2x^2} \right) dx$

$I = \int dx - \int \frac{x}{1-2x^2}\,dx$

× & ÷ by (–4) on 2nd integral.

$I = \int dx + \frac{1}{4}\int \frac{-4x}{1-2x^2}\,dx \Rightarrow \boxed{I = x + \frac{1}{4}\ln(1-2x^2) + c}$ Ans

(xi) $\int \frac{x}{\sqrt{1+x}}\,dx$

**Solution:** let $I = \int \frac{x}{\sqrt{1+x}}\,dx$

This question belongs to special case # 01 make N (x) as D (x).

$I = \int \frac{1+x-1}{(1+x)^{1/2}}\,dx$

$I = \int \left( \frac{1+x}{(1+x)^{1/2}} - \frac{1}{(1+x)^{1/2}} \right) dx$

$I = \int \{(1+x)(1+x)^{-1/2} - (1+x)^{-1/2}\}\,dx$

$I = \int \{(1+x)^{1/2} - (1+x)^{-1/2}\}\,dx$

$I = \int (1+x)^{1/2}\,dx - \int (1+x)^{-1/2}\,dx$

$I = \frac{(1+x)^{3/2}}{3/2} - \frac{(1+x)^{1/2}}{1/2} + c$

$\boxed{I = \frac{2}{3}(1+x)^{3/2} - 2\sqrt{1+x} + c}$ Ans

(xii) $\int x^2\sqrt{4+x}\,dx$

**Solution:** let $I = \int \sqrt{4+x}\;x^2\,dx$ ——(1)

**This question belongs to special case # 02 break the derivative power.**

let $t = 4 + x \Rightarrow t - 4 = x \Rightarrow \boxed{x^2 = (t-4)^2}$

differentiate w.r to x



$\frac{dt}{dx} = 1 \Rightarrow \boxed{dt = dx}$

$(1) \Rightarrow I = \int t^{1/2}\,(t-4)^2\,dt$

$\boxed{\because (a-b)^2 = a^2 - 2ab + b^2}$

$I = \int t^{1/2}(t^2 - 8t + 16)\,dt \Rightarrow I = \int (t^{5/2} - 8t^{3/2} + 16t^{1/2})\,dt$

$I = \int t^{5/2}\,dt - 8\int t^{3/2}\,dt + 16\int t^{1/2}\,dt$

$I = \frac{t^{7/2}}{7/2} - 8\frac{t^{5/2}}{5/2} + 16\frac{t^{3/2}}{3/2} + c$

$I = \frac{2}{7}t^{7/2} - 8\times\frac{2}{5}t^{5/2} + 16\times\frac{2}{3}t^{3/2} + c$ but $t = 4 + x$

$\boxed{I = \frac{2}{7}(4+x)^{7/2} - \frac{16}{5}(4+x)^{5/2} + \frac{32}{3}(4+x)^{3/2} + c}$ Ans

(xiii) $\int \frac{3x+2}{\sqrt{x-1}}\,dx$

**Solution:** let $I = \int \frac{3x+2}{\sqrt{x-1}}\,dx$

**This question belongs to special case # 01 make N (x) as D (x).**

$I = \int \frac{3x-3+5}{(x-1)^{1/2}}\,dx.$

$I = \int \left\{ \frac{3\cancel{(x-1)}}{\cancel{(x-1)}^{1/2}} + \frac{5}{(x-1)^{1/2}} \right\} dx$

$I = \int \{3(x-1)^{1/2} + 5(x-1)^{-1/2}\}\,dx$

$I = 3\int (x-1)^{1/2}\,dx + 5\int (x-1)^{-1/2}\,dx$

$I = 3\frac{(x-1)^{3/2}}{3/2} + 5\frac{(x-1)^{1/2}}{1/2} + c$

$I = \cancel{3}\times\frac{2}{\cancel{3}}(x-1)^{3/2} + 5\times 2\sqrt{x-1} + c$

$\boxed{I = 2(x-1)^{3/2} + 10\sqrt{x-1} + c}$ Ans

(xiv) $\int (2x^2-3)^{4/3}\,x^3\,dx$

**Solution:** let $I = \int (2x^2-3)^{4/3}\,x^2 \cdot x\,dx$ ————(1)

**This question belongs to special case # 02 break derivative power.**

let $t = 2x^2 - 3 \Rightarrow t + 3 = 2x^2$

differentiate w.r to x. $\boxed{x^2 = \frac{t+3}{2}}$

$\frac{dt}{dx} = 4x. \Rightarrow \boxed{\frac{dt}{4} = x\,dx.}$

$(1) \Rightarrow I = \int t^{4/3}.\left(\frac{t+3}{2}\right)\frac{dt}{4} \Rightarrow I = \frac{1}{8}\int t^{4/3}(t+3)\,dt$

$I = \frac{1}{8}\int (t^{7/3} + 3t^{4/3})\,dt \Rightarrow I = \frac{1}{8}\int t^{7/3}\,dt + \frac{3}{8}\int t^{4/3}\,dx$

$I = \frac{1}{8}\,\frac{t^{10/3}}{10/3} + \frac{3}{8}\,\frac{t^{7/3}}{7/3} + c$

but $t = 2x^2 - 3$

$I = \frac{1}{8} \times \frac{3}{10}(2x^2-3)^{10/3} + \frac{3}{8} \times \frac{3}{7}(2x^2-3)^{7/3} + c$

$\boxed{I = \frac{3}{80}(2x^2-3)^{10/3} + \frac{9}{56}(2x^2-3)^{7/3} + c}$

**(xv)** $\int (x^3+1)^{7/5} \bullet x^5 \bullet dx$

**Solution:** let $I = \int (x^3+1)^{7/5} \bullet x^3 \bullet x^2\,dx$ ——— **(1)**

**This question belongs to special case # 02 break derivative power.**

let $t = x^3 + 1 \Rightarrow \boxed{x^3 = t - 1}$

differentiate w•r to x

$\frac{dt}{dx} = 3x^2 \Rightarrow \boxed{\frac{dt}{3} = x^2\,dx}$

$(1) \Rightarrow I = \int t^{7/5}(t-1)\frac{dt}{3} \Rightarrow I = \frac{1}{3}\int t^{7/5}(t-1)\,dt$

$I = \frac{1}{3}\int (t^{12/5} - t^{7/5})\,dt \Rightarrow I = \frac{1}{3}\int t^{12/5}\,dt - \frac{1}{3}\int t^{7/5}\,dt$

$I = \frac{1}{3}\,\frac{t^{17/5}}{17/5} - \frac{1}{3}\,\frac{t^{12/5}}{12/5} + c$

but $t = x^3 + 1$

$I = \frac{1}{3} \times \frac{5}{17}(x^3+1)^{17/5} - \frac{1}{3} \times \frac{5}{12}(x^3+1)^{12/5} + c$

$\boxed{I = \frac{5}{51}(x^3+1)^{17/5} - \frac{5}{36}(x^3+1)^{12/5} + c}$

**(xvi)** $\int (x^2 - 2x + 1)^{4/3}\,dx$

**Solution:** let $I = \int (x^2 - 2x + 1)^{4/3}\,dx$

$I = \int \{(x-1)^2\}^{4/3}\,dx \Rightarrow I = \int (x-1)^{8/3}\,dx$

$I = \frac{(x-1)^{11/3}}{11/3} + c \Rightarrow \boxed{I = \frac{3}{11}(x-1)^{11/3} + c}$ Ans



**Q2. Obtain**

**(i)** $\int e^{2-3x}\,dx$

**Solution:** let $I = \int e^{2-3x}\,dx$

$\times$ & $\div$ by $(-3)$

$I = \frac{-1}{3}\int e^{2-3x}(-3)\,dx \Rightarrow \boxed{I = \frac{-1}{3}e^{2-3x} + c}$ Ans.

**(ii)** $\int x\,e^{3x^2+2}\,dx$

**Solution:** let $I = \int\int x\,e^{3x^2+2}\,x\,dx$

$\times$ & $\div$ by 6

$I = \frac{1}{6}\int e^{3x^2+2} \bullet (6x)\,dx \Rightarrow \boxed{I = \frac{1}{6}\,e^{3x^2+2} + c}$ Ans

**(iii)** $\int a^{2y}\,dy.$

**Solution:** let $I = \int a^{2y}\,dy.$

$\boxed{\text{Using formula} \therefore \int a^x\,dx = \frac{a^x}{\ln a} + c} \Rightarrow \boxed{I = \frac{a^{2y}}{2\ln 2y} + c}$ Ans.

**(iv)** $\int \frac{e^{1/u}}{u^2}\,du.$

**Solution:** let $I = \int e^{1/u}\frac{du}{u^2}$ ——— (1)

$\times$ & $\div$ by $(-1)$

$I = -\int e^{1/u}\left(\frac{-1}{u^2}\right)du \Rightarrow \boxed{I = -e^{\frac{1}{u}} + c}$ Ans

**(v)** $\int \frac{x^2\,dx}{e^{2x^3+3}}$

**Solution:** let $I = \int \frac{x^2\,dx}{e^{2x^3+3}}$ ——— **(1)**

let $t = 2x^3 + 3$

differentiate w•r to x

$\frac{dt}{dx} = 6x^2 \Rightarrow \boxed{\frac{dt}{6} = x^2\,dx}$

$(1) \Rightarrow I = \int \frac{\frac{dt}{6}}{e^t} \Rightarrow I = \frac{1}{6}\int e^{-t}\,dt$

$I = \frac{1}{6}\left(\frac{e^{-t}}{-1}\right) + c \Rightarrow I = \frac{-1}{6\,e^t} + c$

but $t = 2x^3 + 3$

$$\boxed{I = \frac{-1}{6^{2x^3+3}} + c}$$

**(vi)** $\int e^x (2e^{3k} - 5)^{2/5}$ **dx.**

**Solution:** let $I = \int e^x (2e^{3k} - 5)^{2/5}\, dx$

$I = (2e^{3k} - 5)^{2/5} \int e^x\, dx \Rightarrow \boxed{I = (2e^{3k} - 5)^{2/5} e^x + c}$ Ans

**Q3. Determine:**

(i) $\int \text{Sin}\,(3x + 2)\, dx$.

**Solution:** let $I = \int \text{Sin}\,(3x + 2)\, dx$

$\times$ & $\div$ by (3)

$I = \frac{1}{3} \int \text{Sin}\,(3x + 2) \cdot 3dx \Rightarrow \boxed{I = \frac{-1}{3} \text{Cos}\,(3x + 2) + c}$ Ans

**(ii)** $\int$ **Cos2y dy**

**Solution:** let $I = \int \text{Cos}2y\, dy$

$\times$ & $\div$ by 2

$I = \frac{1}{2} \int \text{Cos}2y \cdot 2dy \Rightarrow \boxed{I = \frac{1}{2} \text{Sin}(2y) + c}$ Ans

**(iii)** $\int \text{Sin}^2x \,\text{Cos}x\, dx$.

**Solution:** let $I = \int \text{Sin}^2x \,\text{Cos}x$

$\boxed{I = \frac{1}{3} \text{Sin}^3x + c}$ Ans

**(iv)** $\int \frac{\text{Sec}\sqrt{x}}{\sqrt{x}}\, dx$.

**Solution:** let $I = \int \text{Sec}\sqrt{x}\, \frac{dx}{\sqrt{x}}$

$\times$ & $\div$ by 2

$I = 2 \int \text{Sec}\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\, dx \Rightarrow \boxed{I = 2\, \ell n \tan\left(\frac{\sqrt{x}}{2} + \frac{\pi}{4}\right) + c}$ Ans

**(v)** $\int x\, \text{Cot}x^2\, dx$

**Solution:** let $I = \int \text{Cot}x^2 . x dx$.

$\times$ & $\div$ by 2

$I = \frac{1}{2} \int \text{Cot}x^2 \cdot 2xdx \Rightarrow \boxed{I = \frac{1}{2}\, \ell n\, \text{Sin}x^2 + c}$ Ans



**(vi)** $\int$ **tanx dx.**

**Solution:** let $I = \int \tan x\, dx$

$\boxed{I = \ell n\, \text{Sec}x + c}$ Ans

**(vii)** $\int \text{Sec}^2 2ax$ **dx.**

**Solution:** let $I = \int \text{Sec}^2 2ax\, dx$

$\times$ & $\div$ by 2a

$I = \frac{1}{2a} \int \text{Sec}^2 2ax \cdot 2a\, dx \Rightarrow \boxed{I = \frac{1}{2a} \tan 2ax + c}$ Ans

**(viii)** $\int \frac{\text{Sin}x + \text{Cos}x}{\text{Cos}x}$ **dx.**

**Solution:** let $I = \int \frac{\text{Sin}x + \text{Cos}x}{\text{Cos}x}\, dx$.

$I = \int \left(\frac{\text{Sin}x}{\text{Cos}x} + \frac{\cancel{\text{Cos}x}}{\cancel{\text{Cos}x}}\right) dx \Rightarrow I = \int (\tan x + 1)\, dx$

$I = \int \tan x\, dx + \int dx \Rightarrow \boxed{I = \ell n\, \text{Sec}x + x + c}$ Ans

**(ix)** $\int (1 + \tan x)^2$ **dx.**

**Solution:** let $I = \int (1 + \tan x)^2\, dx$.

$\boxed{\therefore (a + b)^2 = a^2 + 2ab + b^2}$

$I = \int (1 + \tan^2x + 2\tan x)\, dx \Rightarrow \boxed{\therefore\ 1 + \tan^2x = \text{Sec}^2x.}$

$I = \int (\text{Sec}^2x + 2\tan x)\, dx \Rightarrow I = \int \text{Sec}^2x\, dx + 2 \int \tan x\, dx$

$\boxed{I = \tan x + 2\, \ell n\, \text{Sec}x + c}$ Ans

**(x)** $\int e^x \,\text{Cos}\, e^x$ **dx.**

**Solution:** let $I = \int \text{Cos}\, e^x .\, e^x\, dx$.

$\boxed{I = \text{Sin}e^x + c}$ Ans

**(xi)** $\int e^{3\cos 2x}$ **Sin2x dx.**

**Solution:** let $I = \int e^{3\cos 2x} .\ \text{Sin}2x\, dx$

$\times$ & $\div$ by $(-6)$

$I = \frac{-1}{6} \int e^{3\text{Cos}2x} (-6\, \text{Sin}2x)\, dx \Rightarrow \boxed{I = \frac{-1}{6} e^{3\text{Cos}2x} + c}$ Ans

**(xii)** $\int (\tan 2x + \text{Sec}2x)^2$ **dx.**

**Solution:** let $I = \int (\tan 2x + \text{Sec}2x)^2\, dx$

$\boxed{\therefore (a + b)^2 = a^2 + 2ab + b^2}$

$I = \int (\tan^2 2x + 2\tan 2x\, \text{Sec}2x + \text{Sec}^2 2x)\, dx$

$$\boxed{\therefore \tan^2 2x = \text{Sec}^2 2x - 1}$$

$$I = \int (\text{Sec}^2 2x - 1 + 2\tan 2x\,\text{Sec}2x + \text{Sec}^2 2x)\,dx$$

$$I = \int (2\text{Sec}^2 2x - 1 + 2\tan 2x\,\text{Sec}\,2x)\,dx$$

$$I = 2\int \text{Sec}^2 2x\,dx - \int dx + 2\int \tan 2x\,\text{Sec}2x\,dx$$

$$I = \cancel{2}\frac{\tan 2x}{\cancel{2}} - x + \cancel{2}\frac{\text{Sec}2x}{\cancel{2}} + c$$

$$\boxed{I = \tan 2x - x + \text{Sec}2x + c} \text{ Ans}$$

**(xiii)** $\int (\text{Sec}4x - 1)^2\,dx.$

**Solution:** let $I = \int (\text{Sec}4x - 1)^2\,dx$

$$\boxed{\therefore (a-b)^2 = a^2 - 2ab + b^2}$$

$$I = \int (\text{Sec}^2 4x - 2\text{Sec}4x + 1)\,dx$$

$$I = \int \text{Sec}^2 4x\,dx - 2\int \text{Sec}4x\,dx + \int dx.$$

$$I = \frac{\tan 4x}{4} - \frac{\cancel{2}^{1}}{\cancel{4}_{2}}\,\ell n\,(\text{Sec}4x + \tan 4x) + x + c$$

$$I = \frac{1}{4}\tan 4x - \frac{1}{2}\,\ell n\,(\text{Sec}4x + \tan 4x) + x + c$$

$$\boxed{I = \frac{1}{4}\tan 4x - \frac{1}{2}\ell n \tan\left(2x + \frac{\pi}{4}\right) + c} \text{ Ans}$$

**(xiv)** $\int \frac{\textbf{Secx tanx}}{\textbf{a + bSecx}}\,dx$

**Solution:** let $I = \int \frac{\text{Secx tanx}}{a + b\text{Secx}}\,dx$

$\times$ & $\div$ by b

$$I = \frac{1}{b}\int \frac{b\text{Secxtanxdx}}{a + b\text{Secx}} \Rightarrow \boxed{I = \frac{1}{b}\,\ell n\,(a + b\,\text{Secx}) + c}$$

**(xv)** $\int \frac{dx}{\textbf{Cos2x – Cot2x}}$

**Solution:** let $I = \int \frac{1}{\text{Cos}2x - \text{Cot}2x}\,dx.$

$$I = \int \frac{1}{\frac{1}{\text{Sin}2x} - \frac{\text{Cos}2x}{\text{Sin}2x}} \Rightarrow I = \int \frac{1}{\frac{1 - \text{Cos}2x}{\text{Sin}2x}}\,dx$$

$$I = \int \frac{\text{Sin}2x}{1 - \text{Cos}2x}\,dx$$

$\times$ & $\div$ by 2



$$I = \frac{1}{2}\int \frac{2\text{Sin}2x\,dx}{1 - \text{Cos}2x} \Rightarrow I = \frac{1}{2}\,\ell n\,(1 - \text{Cos}2x) + c$$

$$\boxed{\therefore 1 - \text{Cos}\theta = 2\text{Sin}^2\frac{\theta}{2}}$$

$$\boxed{\therefore 1 - \text{Cos}2x = 2\text{Sin}^2 x}$$

$$I = \frac{1}{2}\ell n\,\{\,2\text{Sin}^2 x\,\} + c \Rightarrow I = \frac{1}{2}\{\,\ell n2 + \ell n\text{Sin}^2 x\,\} + c$$

$$I = \frac{1}{2}\{\,\ell n2 + 2\ell n\,\text{Sinx}\,\} + c$$

$$I = \frac{\ell n2}{2} + \frac{\cancel{2}}{\cancel{2}}\ell n\,\text{Sinx} + c \Rightarrow I = \ell n\text{Sinx} + c + \frac{\ell n2}{2}$$

$$\boxed{\therefore c + \frac{\ell n2}{2} = \text{constant} = c} \Rightarrow \boxed{I = \ell n\text{Sinx} + c} \text{ Ans.}$$

**(xvi)** $\int \frac{\textbf{Sinlnx dx}}{\textbf{x (3 – Coslnx)}^{1/2}}$

**Solution:** let $I = \int \frac{\text{Sinlnx}\,dx}{x\,(3 - \text{Coslnxc})^{1/2}}$ ———— (1)

$$I = \int (3 - \text{Cos}\ell nx)^{-1/2} \cdot \frac{\text{Sin}\,\ell nx}{x}\,dx$$

$$\boxed{I = 2\,(3 - \text{Coslnx})^{1/2} + c} \quad \text{Ans}$$

**Q4. Find**

**(i)** $\int \frac{x^2\,dx}{\sqrt{1 - x^6}}$

**Solution:** let $I = \int \frac{x^2\,dx}{\sqrt{1 - x^6}}$

$$I = \int \frac{x^2\,dx}{\sqrt{1 - (x^3)^2}} \text{ ———— (1)}$$

let $t = x^3$

differentiate w.r to x

$$\frac{dt}{dx} = 3x^2 \Rightarrow \boxed{\frac{dt}{3} = x^2\,dx.}$$

$$(1) \Rightarrow I = \int \frac{\frac{dt}{3}}{\sqrt{1 - t^2}} \Rightarrow I = \frac{1}{3}\int \frac{1}{\sqrt{1 - t^2}}\,dt$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \text{Sin}^{-1}\frac{x}{a} + c}$$

$$I = \frac{1}{3}\,\text{Sin}^{-1}\frac{t}{1} + c$$

but $t = x^3$

$$\boxed{I = \frac{1}{3}\,\text{Sin}^{-1}x^3 + c} \text{ Ans}$$

**(ii)** $\int \frac{x\,dx}{x^4+3}$

**Solution:** let $I = \int \frac{x\,dx}{x^4+3}$

$$I = \int \frac{x\,dx}{(x^2)^2 + (\sqrt{3})^2} \text{ ———— (1)}$$

let $t = x^2$

differentiate w.r to x

$$\frac{dt}{dx} = 2x \Rightarrow \boxed{\frac{dt}{2} = x\,dx}$$

$$(1) \Rightarrow I = \int \frac{\frac{dt}{2}}{(t)^2 + (\sqrt{3})^2}$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c}$$

$$I = \frac{1}{2}\,\frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}} + c$$

but $t = x^2$

$$\boxed{I = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} + c} \quad \text{Ans}$$

**(iii)** $\int \frac{dx}{x\sqrt{x^4-1}}$

**Solution:** let $I = \int \frac{dx}{x\sqrt{x^4-1}}$

$$I = \int \frac{dx}{x\sqrt{(x^2)^2 - (1)^2}} \text{ ———— (1)}$$

let $t = x^2$

differentiate w.r to x

$$\frac{dt}{dx} = 2x \Rightarrow \boxed{\frac{dt}{2x} = dx.}$$



$$(1) \Rightarrow I = \int \frac{\frac{dt}{2x}}{x\sqrt{t^2-1}} \Rightarrow I = \int \frac{dt}{2x^2\sqrt{t^2-1}} \text{ but } t = x^2$$

$$I = \frac{1}{2}\int \frac{1}{t\sqrt{t^2-1}}\,dt$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\,\text{Sec}^{-1}\frac{x}{a} + c}$$

$$I = \frac{1}{2}\left(\frac{1}{1}\right)\text{Sec}^{-1}\left(\frac{t}{1}\right) + c$$

but $t = x^2$

$$\boxed{I = \frac{1}{2}\,\text{Sec}^{-1}(x^2) + c} \text{ Ans}$$

**(iv)** $\int \frac{dx}{\sqrt{4-(x+2)^2}}$

**Solution:** let $I = \int \frac{dx}{\sqrt{4-(x+2)^2}}$ ———— (1)

let $t = x + 2$

differentiate w.r to x

$$\frac{dt}{dx} = 1 \Rightarrow \boxed{dt = dx}$$

$$(1) \Rightarrow I = \int \frac{dt}{\sqrt{(2)^2 - (t)^2}}$$

$$\boxed{\text{using formula } \therefore \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \text{Sin}^{-1}\frac{x}{a} + c}$$

$$I = \text{Sin}^{-1}\frac{t}{2} + c \qquad \text{but } t = x + 2$$

$$\boxed{I = \text{Sin}^{-1}\left(\frac{x+2}{2}\right) + c}$$

**(v)** $\int \frac{\text{Sec}\theta \tan\theta\, d\theta}{9 + 4\,\text{Sec}^2\theta}$

**Solution:** let $I = \int \frac{\text{Sec}\theta \tan\theta\, d\theta}{9 + 4\,\text{Sec}^2\theta}$

$$I = \int \frac{\text{Sec}\theta \tan\theta\, d\theta}{(3)^2 + (2\text{Sec}\theta)^2} \text{ ———— (1)}$$

let $t = 2\,\text{Sec}\theta$

differentiate w.r to x

$$\frac{dt}{d\theta} = 2\,\text{Sec}\theta\,\tan\theta \Rightarrow \boxed{\frac{dt}{2} = \text{Sec}\theta\,\tan\theta\,d\theta}$$

$$(1) \Rightarrow I = \int \frac{\frac{dt}{2}}{(3)^2 + (t)^2}$$

$$I = \frac{1}{2}\int \frac{1}{(3)^2 + (t)^2}\,dt.$$

$$\boxed{\text{Using formula} \therefore \int \frac{1}{a^2 + x^2}\,dx = \tan^{-1}\frac{x}{a} + c}$$

$$I = \frac{1}{2}\,\frac{1}{3}\,\tan^{-1}\frac{t}{3} + c \qquad \text{but } t = 2\text{Sec}\theta$$

$$\boxed{I = \frac{1}{6}\tan^{-1}\frac{(2\,\text{Sec}\theta)}{3} + c} \text{ Ans}$$

**(vi)** $\int \frac{(x+3)\,dx}{\sqrt{1-x^2}}$

**Solution:** let $I = \int \frac{(x+3)\,dx}{\sqrt{1-x^2}}$ breaking L•C•M

$$I = \int \frac{x}{\sqrt{1-x^2}}\,dx + \int \frac{3}{\sqrt{1-x^2}}\,dx$$

$$I = \int (1-x^2)^{-1/2}\,x\,dx + 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$\times$ & $\div$ by $(-2)$ to 1st integral.

and using formula $\therefore \int \frac{1}{\sqrt{a^2-x^2}}\,dx = \text{Sin}^{-1}\frac{x}{a} + c$ on Second integral.

$$I = \frac{-1}{2}\int (1-x^2)^{-1/2}\cdot(-2x)\,dx + 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{-1}{2}\,\frac{(1-x^2)^{1/2}}{1/2} + 3\,\text{Sin}^{-1}\frac{x}{1} + c$$

**(iii)** $\int -$

$$= \frac{-1}{\cancel{2}} \times \cancel{2}\sqrt{1-x^2} + 3\,\text{Sin}^{-1}x + c$$

**Solutio**

$$\boxed{I = -\sqrt{1-x^2} + 3\,\text{Sin}^{-1}x + c} \text{ Ans}$$

$$\int \frac{2u-7}{u^2+9}\,du.$$

**on:** let $I = \int \frac{2u-7}{u^2+9}\,du.$

breaking L•C•M

$$I = \int \frac{2u}{u^2+9}\,du - 7\int \frac{1}{u^2+9}\,du.$$



$$\boxed{\text{Using formula} \therefore \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c}$$

on 2nd integral.

$$I = \ell n\,(u^2+9) - 7\left(\frac{1}{3}\tan^{-1}\frac{u}{3}\right) + c$$

$$\boxed{I = \ell n\,(u^2+9) - \frac{7}{3}\tan{-1}\frac{u}{3} + c} \text{ Ans}$$

**(viii)** $\int \frac{dt}{1+(3t-4)^2}$

**Solution:** let $I = \int \frac{dt}{1+(3t-4)^2}$ ———— (1)

let $u = 3t - 4$

differentiate w•r to x

$$\frac{du}{dt} = 3 \quad \Rightarrow \boxed{\frac{du}{3} = dt}$$

$$(1) \Rightarrow I = \int \frac{\frac{du}{3}}{(1)^2 + (u)^2}$$

$$I = \frac{1}{3}\int \frac{1}{(1)^2 + (u)^2}\,du.$$

$$\boxed{\textbf{Using formula} \therefore \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c}$$

$$I = \frac{1}{3}\,\frac{1}{1}\tan^{-1}\frac{u}{1} + c$$

but $u = 3t - 4$

$$\boxed{I = \frac{1}{3}\tan^{-1}(3t-4) + c} \qquad \text{Ans}$$

**Q5. by using Trigonometric identities or otherwise, evalu following integrals.**

**(i)** $\int \text{Sin}^2x\,dx.$

**Solution:** let $I = \int \text{Sin}^2x\,dx.$

$$\boxed{\textbf{Using formula} \therefore \text{Sin}^2x\,\frac{1-\text{Cos}2x}{2}}$$

$$I = \int\left(\frac{1-\cos2x}{2}\right)dx \Rightarrow I = \frac{1}{2}\int dx - \frac{1}{2}\int \text{Cos}2x\,dx$$

$\times$ & $\div$ by 2 on 2nd integral.

$$I = \frac{1}{2}\int dx - \frac{1}{2\times 2}\int Cos2x \cdot 2\,dx$$

$$\boxed{I = \frac{x}{2} - \frac{1}{4}\,Sin2x + c}\ \text{Ans}$$

**(ii)** $\int \mathbf{Cos^2 (3x + 2)\,dx}$

**Solution:** let $I = \int Cos^2 (3x + 2)\,dx$

$$\boxed{\textbf{Using formula } \therefore Cos^2x = \frac{1 + Cos2x}{2}}$$

$$I = \int \left\{\frac{1 + Cos2\,(3x + 2)}{2}\right\} dx$$

$$I = \int \left\{\frac{1 + Cos\,(6x + 4)}{2}\right\} dx$$

$$I = \frac{1}{2}\,dx + \frac{1}{2}\int Cos\,(6x + 4)\,dx$$

$\times$ & $\div$ by 6 on $2^{nd}$ integral.

$$I = \frac{1}{2}\int dx + \frac{1}{2\times 6}\int Cos\,(6x + 4)\cdot 6\,dx$$

$$\boxed{I = \frac{x}{2} + \frac{1}{12}\,Sin\,(6x + 4) + c}\ \text{Ans}$$

**(iii)** $\int \mathbf{Sin^3x\,dx}$

**Solution:** let $I = \int Sin^3x\,dx$

$$I = \int Sin^2x . Sinx\,dx$$

$$\boxed{\therefore Sin^2x = 1 - Cos^2x}$$

$$I = \int (1 - Cos^2x)\,Sinx\,dx \Rightarrow I = \int Sinx\,dx - \int Cos^2x\,Sinx\,dx.$$

$\times$ & $\div$ by $(-1)$ on $2^{nd}$ integral.

$$I = \int Sinx\,dx + \int Cos^2x.\,(-Sinx)\,dx$$

$$\boxed{I = -Cosx + \frac{Cos^3x}{3} + c}\ \text{Ans}$$

I = − Cos$^5$x dx.

n: let $I = \int Cos^5x\,dx$

$\frac{2u-7}{u^2+9}$ $= \int (cos^2x)^2.\,Cosx\,dx$

$$\boxed{Cos^2x = 1 - Sin^2x.}$$

on: let $I = \int\int (1 - Sin^2x)^2\,Cosx\,dx$

breaking L $\boxed{(a - b)^2 = a^2 - 2ab + b^2}$

$I = \int \frac{2u}{u^2+9}\int (1 - 2\,Sin^2x + Sin^4x)\,Cosx\,dx.$



$$I = \int Cosx\,dx - 2\int Sin^2x\,Cosx\,dx + \int Sin^4x\,Cosx\,dx$$

$$\boxed{I = Sinx - 2\,\frac{Sin^3x}{3} + \frac{sin^5x}{5} + c}\ \text{Ans}$$

**(v)** $\int \mathbf{Sin^2x\,Cos^3x\,dx}$

**Solution:** let $I = \int Sin^2x\,Cos^3x\,dx.$

$\downarrow$

**break**

$$I = \int Sin^2x.\,Cos^2x.Cosx\,dx$$

$$\boxed{\therefore Cos^2x = 1 - Sin^2x.}$$

$$I = \int Sin^2x\,Cosx\,(1 - Sin^2x)\,dx.$$

$$I = \int Sin^2x\,Cosx\,dx - \int Sin^4x\,Cosx\,dx.$$

$$\boxed{I = \frac{Sin^3x}{3} - \frac{Sin^5x}{5} + c}$$

**(vi)** $\int \mathbf{Cos^4 2x.\,Sin^3 2x\,dx}$

**Solution:** let $I = \int Cos^4 2x.sin^3 2x\,dx$

$\downarrow$

**break.**

$$I = \int Cos^4 2x.\,Sin2x.\,Sin^2 2x\,dx$$

$$\boxed{\therefore Sin^2 2x = 1 - Cos^2 2x}$$

$$I = \int Cos^4 2x.\,Sin2x\,(1 - Cos^2 2x)\,dx$$

$$I = \int Cos^4 2x\,Sin2x\,dx - \int Cos^6 2x\,Sin2x\,dx$$

$\times$ & $\div$ by $(-2)$ to both intergrals.

$$I = \frac{-1}{2}\int Cos^4 2x.(-2\,Sin2x)\,dx + \frac{1}{2}\int Cos^6 2x.\,(-2\,Sin2x)\,dx$$

$$I = \frac{-1}{2}\,\frac{Cos^5 2x}{5} + \frac{1}{2}\,\frac{Cos^7 2x}{7} + c$$

$$\boxed{I = -\frac{Cos^5 2x}{10} + \frac{Cos^7 2x}{14} + c}\ \text{Ans}$$

**(vii)** $\int \mathbf{Cos^3\frac{y}{3}\,dy.}$

**Solution:** let $I = \int Cos^3\frac{y}{3}\,dy.$

$$I = \int Cos^2\frac{y}{3}.\,Cos\frac{y}{3}\,dy. \quad \boxed{\therefore Cos^2\frac{y}{3} = 1 - Sin^2\frac{y}{3}}$$

$$I = \int \left(1 - Sin^2\frac{y}{3}\right) Cos\frac{y}{3}\,dy$$

$I = \int \cos\frac{y}{3}\,dy - \int \sin^2\frac{y}{3}\cos\frac{y}{3}\,dy$

× & ÷ by 3 on both integrals.

$I = 3\int \cos\frac{y}{3}\cdot\frac{1}{3}\,dy - 3\int \sin^2\frac{y}{3}\cdot\left(\frac{1}{3}\cos\frac{y}{3}\right)dy$

$I = 3\sin\frac{y}{3} - \frac{\cancel{3}\sin^3\frac{y}{3}}{\cancel{3}} + c \Rightarrow \boxed{I = 3\sin\frac{y}{3} - \sin^3\frac{y}{3} + c}$ Ans

**(viii) $\int \sin^4 x\,dx$.**

**<u>Solution:</u>** let $I = \int \sin^4 x\,dx$

$I = \int (\sin^2 x)^2\,dx$

$\boxed{\therefore \sin^2 x = \frac{1-\cos 2x}{2}}$

$I = \int\left(\frac{1-\cos 2x}{2}\right)^2 dx \Rightarrow I = \int\frac{(1-\cos 2x)^2}{4}\,dx.$

$\boxed{\therefore (a-b)^2 = a^2 - 2ab + b^2}$

$I = \int\frac{(1-2\cos 2x + \cos^2 2x)}{4}\,dx$

$I = \frac{1}{4}\int dx - \frac{\cancel{2}}{\cancel{4}_2}\int \cos 2x\,dx + \frac{1}{4}\int \cos^2 2x\,dx$

$I = \frac{1}{4}\int dx - \frac{1}{2}\int \cos 2x\,dx + \frac{1}{4}\int \cos^2 2x\,dx$

$\boxed{\therefore \cos^2 x = \frac{1+\cos 2x}{2}}$

$I = \frac{1}{4}\int dx - \frac{1}{2}\int \cos 2x\,dx + \frac{1}{4}\int\left(\frac{1+\cos 4x}{2}\right)dx$

$I = \frac{1}{4}\int dx - \frac{1}{2}\int \cos 2x\,dx + \frac{1}{8}\int dx + \frac{1}{8}\int \cos 4x\,dx.$

**× & ÷ by 2 on 2nd integral and**

**× & ÷ by 4 on 4th integral.**

$I = \frac{1}{4}\int dx - \frac{1}{2\times 2}\int \cos 2x\cdot 2dx + \frac{1}{8}\int dx + \frac{1}{8\times 4}\int \cos 4x\cdot 4dx$

$I = \frac{x}{4} - \frac{\sin 2x}{4} + \frac{x}{8} + \frac{\sin 4x}{32} + c$

$\boxed{I = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c}$ Ans



**(ix) $\int \sin^2 x \cos^2 x\,dx$**

**<u>Solution:</u>** let $I = \int \sin^2 x\cos^2 x\,dx$.

$\boxed{\therefore \sin^2 x = \frac{1-\cos 2x}{2}}$ $\boxed{\therefore \cos^2 x = \frac{1+\cos 2x}{2}}$

$I = \int\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx$

$I = \frac{1}{4}\int (1-\cos 2x)(1+\cos 2x)\,dx$

$\boxed{\therefore a^2 - b^2 = (a-b)(a+b)}$

$I = \frac{1}{4}(1-\cos^2 2x)\,dx \Rightarrow I = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2x\,dx$

$I = \frac{1}{4}\int dx - \frac{1}{4}\int\left(\frac{1+\cos 4x}{2}\right)dx$

$I = \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4x\,dx$

× & ÷ by 4 on 3rd integral.

$I = \frac{1}{4}\int dx - \frac{1}{8}\int dx - \frac{1}{8\times 4}\int \cos 4x\cdot 4dx$

$I = \frac{x}{4} - \frac{x}{8} - \frac{1}{32}\sin 4x + c \Rightarrow \boxed{I = \frac{x}{8} - \frac{\sin 4x}{32} + c}$ Ans

**(x) $\int \sin^4 3x\cos^3 3x\,dx$**

**<u>Solution:</u>** let $I = \int \sin^4 3x \underset{\text{break}}{\underset{\downarrow}{\cos^3 3x}}\,dx$

$I = \int \sin^4 3x.\cos 3x.\cos^2 3x\,dx$

$\boxed{\therefore \cos^2 3x = 1 - \sin^2 3x}$

$I = \int \sin^4 3x.\cos 3x.(1-\sin^2 3x)\,dx$

$I = \int \sin^4 3x\cos 3x\,dx - \int \sin^6 3x\cos 3x\,dx$

× & ÷ by 3 to both integrals.

$I = \frac{1}{3}\int \sin^4 3x.(3\cos 3x)\,dx - \frac{1}{3}\int \sin^6 3x.(3\cos 3x)\,dx.$

$I = \frac{1}{3}\frac{\sin^5 3x}{5} - \frac{1}{3}\frac{\sin^7 3x}{7} + c$

$\boxed{I = \frac{1}{15}\sin^5 3x - \frac{1}{7}\sin^7 3x + c}$ Ans

**(xi) $\int \sin 2z\sin 3z\,dz$.**

**<u>Solution:</u>** let $I = \int \sin 2z\sin 3z\,dz$.

$\boxed{\text{Using formula } \sin\alpha \sin\beta = \frac{-1}{2}\,[\cos(\alpha+\beta) - \cos(\alpha-\beta)]}$

$I = \int\left(\frac{-1}{2}\right)[\cos(2z+3z) - \cos(2z-3z)]\,dz$

$I = \frac{-1}{2}\int[\cos 5z - \cos(-z)]\,dz.$

$\boxed{\therefore \cos(-\theta) = \cos\theta}$

$I = \frac{-1}{2}\int[\cos 5z - \cos z]\,dz.$

$I = \frac{-1}{2}\int \cos 5z\,dz + \frac{1}{2}\int \cos z\,dz$

$I = \frac{-1}{2}\frac{\sin 5z}{5} + \frac{1}{2}\sin z + c$

$\boxed{I = -\frac{\sin 5z}{10} + \frac{\sin z}{2} + c}$ Ans

**(xii)** $\int \sin 3y \cos 5y\,dy$

**Solution:** let $I = \int \sin 3y \cos 5y\,dy$

$\boxed{\text{Using formula } \therefore \sin\alpha\cos\beta = \frac{1}{2}\,[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$

$I = \int\frac{1}{2}\,[\sin(3y+5y) + \sin(3y-5y)]\,dy.$

$I = \frac{1}{2}\int[\sin 8y + \sin(-2y)]\,dy.$

$\boxed{\therefore \sin(-\theta) = -\sin\theta}$

$I = \frac{1}{2}\int[\sin 8y - \sin 2y]\,dy. \Rightarrow I = \frac{1}{2}\int \sin 8y\,dy - \frac{1}{2}\int \sin 2y\,dy.$

$I = \frac{1}{2}\left(\frac{-\cos 8y}{8}\right) - \frac{1}{2}\left(\frac{-\cos 2y}{2}\right) + c$

$\boxed{I = \frac{-\cos 8y}{16} + \frac{\cos 2y}{4} + c}$ Ans

**(xiii)** $\int \cos 4x \cos 2x\,dx.$

**Solution:** let $I = \int \cos 4x \cos 2x\,dx$

$\boxed{\text{Using formula } \therefore \cos\alpha\cos\beta = \frac{1}{2}\,[\cos(\alpha+\beta) + \cos(\alpha-\beta)]}$

$I = \int\frac{1}{2}\,[\cos(4x+2x) + \cos(4x-2x)]\,dx$



$I = \int\frac{1}{2}\,[\cos 6x + \cos 2x]\,dx \Rightarrow I = \frac{1}{2}\int \cos 6x\,dx + \frac{1}{2}\int \cos 2x\,dx$

$I = \frac{1}{2}\frac{\sin 6x}{6} + \frac{1}{2}\frac{\sin 2x}{2} + c \Rightarrow \boxed{I = \frac{\sin 6x}{12} + \frac{\sin 2x}{4} + c}$ Ans

**(xiv)** $\int\sqrt{1-\cos x}\,dx.$

**Solution:** let $I = \int\sqrt{1-\cos x}\,dx$

$\boxed{\text{Using formula } \therefore 1 - \cos\theta = 2\sin^2\frac{\theta}{2}}$

$I = \int\sqrt{2\sin^2\frac{x}{2}}\,dx \Rightarrow I = \sqrt{2}\int \sin\frac{x}{2}\,dx.$

$I = \sqrt{2}\left(\frac{-\cos\frac{x}{2}}{\frac{1}{2}}\right) + c \Rightarrow \boxed{I = -2\sqrt{2}\cos\frac{x}{2} + c}$ Ans

**(xv)** $\int\sqrt{(1+\cos 3x)^3}\,dx$

**Solution:** let $I = \int\sqrt{(1+\cos 3x)^3}\,dx$

$\boxed{\therefore \text{ Using formula } 1 + \cos\theta = 2\cos^2\frac{\theta}{2}}$

$I = \int\sqrt{\left\{2\cos^2\frac{3x}{2}\right\}^3}\,dx \Rightarrow I = \int\sqrt{8\cos^6\frac{3x}{2}}\,dx$

$I = 2\sqrt{2}\int \cos^3\frac{3x}{2}\,dx \Rightarrow I = 2\sqrt{2}\int \cos^2\frac{3x}{2}.\cos\frac{3x}{2}\,dx$

$\boxed{\therefore \cos^2\frac{3x}{2} = 1 - \sin^2\frac{3x}{2}}$

$I = 2\sqrt{2}\int\left(1 - \sin^2\frac{3x}{2}\right)\cos\frac{3x}{2}\,dx$

$I = 2\sqrt{2}\int \cos\frac{3x}{2}\,dx - 2\sqrt{2}\int \sin^2\frac{3x}{2}\cos\frac{3x}{2}\,dx$

$I = 2\sqrt{2}\,\frac{\sin\frac{3x}{2}}{\frac{3}{2}} - 2\sqrt{2}\,\frac{\sin^3\frac{3x}{2}}{3\times\frac{3}{2}} + c$

$I = 2\sqrt{2}\times\frac{2}{3}\sin\frac{3x}{2} - 2\sqrt{2}\times\frac{2}{9}\sin^3\frac{3x}{2} + c$

$\boxed{I = \frac{4\sqrt{2}}{3}\sin\frac{3x}{2} - \frac{4\sqrt{2}}{9}\sin^3\frac{3x}{2} + c}$ Ans

**(xvi)** $\int \dfrac{dx}{\sqrt{1-\text{Sin}2x}}$

**Solution:** let $I = \int \dfrac{dx}{\sqrt{1-\text{Sin}2x}}$ ———— (1)

$\therefore \quad \text{Sin}\theta = \text{Cos}\left(\frac{\pi}{2}-\theta\right)$

$\therefore \quad \text{Sin}2x = \text{Cos}\left(\frac{\pi}{2}-2x\right)$

$1 - \text{Sin}2x = 1 - \text{Cos}\left(\frac{\pi}{2}-2x\right)$

$\boxed{\therefore\ 1-\text{Cos}\theta = 2\,\text{Sin}^2\frac{\theta}{2}}$

$1 - \text{Sin}2x = 2\text{Sin}^2\dfrac{\left(\frac{\pi-4x}{2}\right)}{2}$

$1 - \text{Sin}2x = 2\text{Sin}^2\left(\frac{\pi-4x}{4}\right)$

$1 - \text{Sin}2x = 2\,\text{Sin}^2\left(\frac{\pi}{4}-x\right)$

$(1) \Rightarrow I = \int \dfrac{dx}{\sqrt{2\,\text{Sin}^2\left(\frac{\pi}{4}-x\right)}} \Rightarrow I = \int \dfrac{dx}{\sqrt{2}\,\text{Sin}\left(\frac{\pi}{4}-x\right)}$

$I = \frac{1}{\sqrt{2}}\int \text{Cosec}\left(\frac{\pi}{4}-x\right)dx. \Rightarrow I = \frac{-1}{\sqrt{2}}\int \text{Cosec}\left(\frac{\pi}{4}-x\right)(-1)\,dx.$

$I = \left(\frac{-1}{\sqrt{2}}\right)\ell n \tan\left(\dfrac{\frac{\pi}{4}-x}{2}\right) + c$

$\boxed{I = \frac{-1}{\sqrt{2}}\ \ell n \tan\left(\frac{\pi}{8}-\frac{x}{2}\right) + c}$ Ans

**(xvii)** $\int \tan^4 x\,dx$

**Solution:** let $I = \int \tan^4 x\,dx$

$I = \int \tan^2 x.\tan^2 x\,dx$

$\boxed{\therefore \tan^2 x = \text{Sec}^2 x - 1}$

$I = \int (\text{Sec}^2 x - 1)\tan^2 x\,dx$

$I = \int \text{Sec}^2 x \tan^2 x\,dx - \int \tan^2 x\,dx$

$I = \int \text{Sec}^2 x \tan^2 x\,dx - \int (\text{Sec}^2 x - 1)\,dx$



$I = \int \text{Sec}^2 x \tan^2 x\,dx - \int \text{Sec}^2 x\,dx + \int dx$

$\boxed{I = \frac{\tan^3 x}{3} - \tan x + x + c}$ Ans

**(xviii)** $\int \tan^5 x\,dx$

**Solution:** let $I = \int \tan^5 x\,dx$

$I = \int \tan^3 x \cdot \tan^2 x\,dx.$

$\boxed{\therefore\ \tan^2 x = \text{Sec}^2 x - 1}$

$I = \int \tan^3 x.(\text{Sec}^2 x - 1)\,dx$

$I = \int \tan^3 x\,\text{Sec}^2 x\,dx - \int \tan^3 x\,dx$

$I = \int \tan^3 x\,\text{Sec}^2 x\,dx - \int \tan x \tan^2 x\,dx$

$I = \int \tan^3 x\,\text{Sec}^2 x\,dx - \int \tan x\,(\text{Sec}^2 x - 1)\,dx$

$I = \int \tan^3 x\,\text{Sec}^2 x\,dx - \int \tan x\,\text{Sec}^2 x\,dx + \int \tan x\,dx.$

$\boxed{I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ell n\,\text{Sec}x + c}$ Ans

**(xix)** $\int \text{Sec}^4 2u\,du.$

**Solution:** let $I = \int \text{Sec}^4 2u\,du.$

$I = \int \text{Sec}^2 2u.\ \text{Sec}^2 2u\,du.$

$\boxed{\therefore\ \text{Sec}^2 2u = 1 + \tan^2 2u}$

$I = \int (1+\tan^2 2u)\,\text{Sec}^2 2u\,du.$

$I = \int \text{Sec}^2 2u\,du + \int \tan^2 2u\,\text{Sec}^2 2u\,du$

$I = \frac{\tan 2u}{2} + \frac{\tan^3 2u}{3\,(2)} + c$

$\boxed{I = \frac{\tan 2u}{2} + \frac{\tan^3 2u}{6} + c}$ Ans

**(xx)** $\int \tan^3 3x\,\text{Sec}^4 3x\,dx$

**Solution:** let $I = \int \tan^3 3x\,\text{Sec}^4 3x\,dx$

$I = \int \tan^3 3x.\text{Sec}^2 3x.\text{Sec}^2 3x\,dx$

$\boxed{\therefore\ \tan^2 3x = \text{Sec}^2 3x - 1}$

$I = \int \tan^3 3x \cdot \text{Sec}^2 3x\,(\tan^2 3x + 1)\,dx$

$I = \int \tan^5 3x\,\text{Sec}^2 x\,dx + \int \tan^3 3x\,\text{Sec}^2 3x\,dx$

$I = \frac{1}{3}\int \tan^5 3x.\text{Sec}^2 3x \cdot 3dx + \frac{1}{3}\int \tan^3 3x.\ \text{Sec}^2 3x.3dx$

$I = \frac{1}{3}\frac{\tan^6 3x}{6} + \frac{1}{3}\frac{\tan^4 3x}{4} + c$

$\boxed{I = \frac{1}{18}\tan^6 3x + \frac{1}{12}\tan^4 3x + c}$ Ans

**(xxi)** $\int \tan^4\frac{x}{3}\,\text{Sec}^4\frac{x}{3}\,dx$

**Solution:** let $I = \int \tan^4\frac{x}{3}\,\text{Sec}^4\frac{x}{3}\,dx$

$= \int \tan^4\frac{x}{3}\cdot\text{Sec}^2\frac{x}{3}\cdot\text{Sec}^2\frac{x}{3}\,dx \Rightarrow = \int \tan^4\frac{x}{3}\left(1+\tan^2\frac{x}{3}\right)\text{Sec}^2\frac{x}{3}\,dx$

$= \int \tan^4\frac{x}{3}\,\text{Sec}^2\frac{x}{3}\,dx + \int \tan^6\frac{x}{3}\,\text{Sec}^2\frac{x}{3}\,dx$

$= 3\int \tan^4\frac{x}{3}\left(\text{Sec}^2\frac{x}{3}\cdot\frac{1}{3}\right)dx + 3\int \tan^6\frac{x}{3}\cdot\left(\text{Sec}^2\frac{x}{3}\cdot\frac{1}{3}\right)dx$

$$\boxed{I = \frac{3}{5}\tan^5\frac{x}{3} + \frac{3}{7}\tan^7\frac{x}{3} + c}$$

**(xxii)** $\int \tan^2 x\,\text{Sec}x\,dx$

**Solution:** let $I = \int \tan^2 x\cdot\text{Sec}x\,dx$

$I = \int \tan x\cdot\tan x\,\text{Sec}x\,dx$ ——— (1)

let $t = \text{Sec}x \quad \therefore\ 1+\tan^2 x = \text{Sec}^2 x \Rightarrow \tan x = \sqrt{t^2-1}$

$\frac{dt}{dx} = \text{Sec}x\tan x \Rightarrow \boxed{dt = \text{Sec}x\tan x\,dx}$

$(1) \Rightarrow I = \int \sqrt{t^2-1}\,dt$

$$\boxed{\therefore \int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\sqrt{x^2-a^2} - \frac{1}{2}a^2\,\ell n\,(x+\sqrt{x^2-a^2}) + c}$$

$= \frac{1}{2}\sqrt{t^2-1} - \frac{1}{2}(1)^2\,\ell n\,(t+\sqrt{t^2-1}) + c$

$= \frac{1}{2}\sqrt{\text{Sec}^2 x-1} - \frac{1}{2}\,\ell n\,\{\text{Sec}x + \sqrt{\text{Sec}^2 x - 1}\} + c$

$= \frac{1}{2}\sqrt{\tan^2 x} - \frac{1}{2}\,\ell n\,\{\text{Sec}x + \sqrt{\tan^2 x}\} + c$

$$\boxed{I = \frac{1}{2}\tan x - \frac{1}{2}\,\ell n\,\{\text{Sec}x + \tan x\} + c}$$ Ans.

**(xxiii)** $\int \tan^3 2x\,\text{Sec}^3 2x\,dx$

**Solution:** let $I = \int \tan^3 2x\cdot\text{Sec}^3 2x\,dx$

$I = \int \tan^2 2x\cdot\text{Sec}^3 2x\cdot\tan 2x\,dx$

$I = \int (\text{Sec}^2 2x - 1)\,\text{Sec}^3 2x\,\tan 2x\,dx$

$I = \int \text{Sec}^5 2x\cdot\tan 2x\,dx - \int \text{Sec}^3 2x\,\tan 2x\,dx$

$I = \frac{1}{2}\int \text{Sec}^4 2x\cdot(\text{Sec}2x\,\tan 2x\cdot 2)\,dx - \frac{1}{2}\int \text{Sec}^2 2x\cdot(\text{Sec}2x\cdot\tan 2x\cdot 2)\,dx$

$$\boxed{I = \frac{1}{10}\text{Sec}^5 2x - \frac{1}{6}\text{Sec}^3 2x + c}$$ Ans



**(xxiv)** $\int \text{Cot}^3 2x\,dx$

**Solution:** let $I = \int \text{Cot}^3 2x\,dx$

$I = \int \text{Cot}^2 2x\cdot\text{Cot}2x\,dx \Rightarrow I = \int (\text{Cosec}^2 2x - 1)\,\text{Cot}2x\,dx$

$I = \int \text{Cosec}^2 2x\,\text{Cot}2x\,dx - \int \text{Cot}2x\,dx$

$I = -\frac{1}{2}\int \text{Cot}2x\cdot(-2\,\text{Cosec}^2 2x)\,dx - \frac{1}{2}\int \text{Cot}2x\cdot 2dx$

$$\boxed{I = \frac{-1}{4}\text{Cot}^2 2x - \frac{1}{2}\,\ell n\,\text{Sin}2x + c}$$ Ans.

**(xxv)** $\int \text{Cot}^4 3z\,dx$

**Solution:** let $I = \int \text{Cot}^4 3z\,dx$

$I = \int \text{Cot}^2 3z\cdot\text{Cot}^2 3z\,dz$

$I = \int \text{Cot}^2 3z\,(\text{Cosec}^2 3z - 1)\,dz$

$I = \int \text{Cot}^2 3z\,\text{Cosec}^2 3z\,dx - \int \text{Cot}^2 3z\,dz$

$I = -\frac{1}{3}\int \text{Cot}^2 3z\,(-3\text{Cosec}^2 3z)\,dx - \frac{1}{3}\int \text{Cosec}^2 3z\cdot 3\,dz + \int dz$

$$\boxed{I = \frac{-1}{9}\text{Cot}^3 3z + \frac{1}{3}\text{Cot}3z + z + c}$$

**(xxvi)** $\int \text{Cosec}^6 x\,dx$

**Solution:** let $I = \int \text{Cosec}^6 x\,dx$

$I = \int \text{Cosec}^4 x\cdot\text{Cosec}^2 x\,dx = \int (\text{Cosec}^2 x)^2\,\text{Cosec}^2 x\,dx$

$I = \int (1+\text{Cot}^2 x)^2\,\text{Cosec}^2 x\,dx$

$I = \int (1+2\text{Cot}^2 x+\text{Cot}^4 x)\,\text{Cosec}^2 x\,dx$

$I = \int \text{Cosec}^2 x\,dx + 2\int \text{Cot}^2 x\cdot\text{Cosec}^2 x\,dx + \int \text{Cot}^4 x\,\text{Cosec}^2 x\,dx$

$I = \int \text{Cosec}^2 x\,dx - 2\int \text{Cot}^2 x\cdot(-\text{Cosec}^2 x\,dx) - \int \text{Cot}^4 x\cdot(-\text{Cosec}^2 x)\,dx$

$$\boxed{I = -\text{Cot}x - \frac{2}{3}\text{Cot}^3 x - \frac{1}{5}\text{Cot}^5 x + c}$$ Ans.

**(xxvii)** $\int \text{Cot}3x\,\text{Cosec}^4 3x\,dx$

**Solution:** let $I = \int \text{Cot}3x\,\text{Cosec}^4 3x\,dx$

$I = \int \text{Cot}3x\cdot\text{Cosec}^2 3x\cdot\text{Cosec}^2 3x\,dx$

$I = \int (1+\text{Cot}^2 3x)\,\text{Cot}3x\,\text{Cosec}^2 3x\,dx$

$I = \int \text{Cot}3x\cdot\text{Cosec}^2 3x\cdot dx + \int \text{Cot}^3 3x\,\text{Cosec}^2 3x\,dx$

$I = \frac{-1}{3}\int \text{Cot}3x\cdot(-3\text{Cosec}^2 3x)\,dx - \frac{1}{3}\int \text{Cot}^3 3x\,(-3\text{Cosec}^2 3x)\,dx$

$$\boxed{I = \frac{-1}{6}\text{Cot}^2 3x - \frac{1}{12}\text{Cot}^4 3x + c}$$ Ans.

**(xxviii)** $\int \text{Cot}^5\frac{x}{2}\,\text{Cosec}^3\frac{x}{2}\,dx$

**Solution:** let $I = \int \text{Cot}^5\frac{x}{2}\,\text{Cosec}^3\frac{x}{2}\,dx$

$$I = \int\left(\text{Cot}^2\frac{x}{2}\right)^2\cdot\text{Cot}\frac{x}{2}\cdot\text{Cosec}^3\frac{x}{2}\,dx$$

$$I = \int\left(\text{Cosec}^2\frac{x}{2}-1\right)^2\text{Cot}\frac{x}{2}\,\text{Cosec}^3\frac{x}{2}\,dx$$

$$I = \int\left(\text{Cosec}^4\frac{x}{2}-2\text{Cosec}^2\frac{x}{2}+1\right)\text{Cot}\frac{x}{2}\,\text{Cosec}^3\frac{x}{2}\,dx$$

$$I = \int\text{Cosec}^6\frac{x}{2}\cdot\text{Cosec}\frac{x}{2}\,\text{Cot}\frac{x}{2}\,dx - 2\int\text{Cosec}^4\frac{x}{2}\cdot\text{Cosec}\frac{x}{2}\cdot$$

$$\text{Cot}\frac{x}{2}\,dx + \int\text{Cosec}^2\frac{x}{2}\cdot\text{Cosec}\frac{x}{2}\,\text{Cot}\frac{x}{2}\,dx$$

$$I = -2\int\text{Cosec}^6\frac{x}{2}\cdot\left(\frac{-1}{2}\text{Cosec}\frac{x}{2}\,\text{Cot}\frac{x}{2}\right)dx$$

$$+\,4\int\text{Cosec}^4\frac{x}{2}\cdot\left(\frac{-1}{2}\text{Cosec}\frac{x}{2}\,\text{Cot}\frac{x}{2}\right)dx$$

$$-2\int\text{Cosec}^2\frac{x}{2}\left(\frac{-1}{2}\text{Cosec}\frac{x}{2}\,\text{Cot}\frac{x}{2}\right)dx$$

$$\boxed{I = \frac{-2}{7}\text{Cosec}^7\frac{x}{2}+\frac{4}{5}\text{Cosec}^5\frac{x}{2}-\frac{2}{3}\text{Cosec}^3\frac{x}{2}+c}$$ Ans.

**(xxix)** $\int \text{Cot}^2 2x\,\text{Cosec}^4 2x\,dx$

**Solution:** let $I = \int \text{Cot}^2 2x\cdot\text{Cosec}^4 2x\,dx$

$I = \int \text{Cot}^2 2x\cdot\text{Cos}^2 2x\cdot\text{Cosec}^2 2x\,dx$

$I = \int \text{Cot}^2 2x\cdot(1+\text{Cot}^2 2x)\,\text{Cosec}^2 2x\,dx$

$I = \int \text{Cot}^2 2x\cdot\text{Cosec}^2 2x\,dx + \int \text{Cot}^4 2x\,\text{Cosec}^2 2x\,dx$

$I = \frac{-1}{2}\int \text{Cot}^2 2x\cdot(-2\text{Cosec}^2 2x\,dx) - \frac{1}{2}\int \text{Cot}^4 2x\,(-2\text{Cosec}^2 2x)\,dx$

$$\boxed{I = \frac{-1}{6}\text{Cot}^3 2x - \frac{1}{10}\text{Cot}^5 3x + c}$$

**(xxx)** $\int \text{Cot}^2 x\,\text{Cosecx}\,dx$

**Solution:** let $I = \int \text{Cot}^2 x\cdot\text{Cosecx}\,dx$

$I = \int \text{Cotx}\cdot\text{Cotx Cosecx}\,dx$ ——— (1)

let $t = \text{Cosecx}$

$\frac{dt}{dx} = -\text{Cosecx Cotx} \Rightarrow \boxed{-dt = \text{CosecxCotx}\,dx}$



$\therefore\quad 1+\text{Cot}^2 x = \text{Cosec}^2 x \Rightarrow \boxed{\text{Cotx} = \sqrt{t^2-1}}$

$(1)\Rightarrow I = \int\sqrt{t^2-1}\,(-dt) = -\int\sqrt{t^2-1}\,dt$

$$\boxed{\therefore \int\sqrt{x^2-a^2}\,dx = \frac{1}{2}\sqrt{x^2-a^2}-\frac{1}{2}a^2\,\ell n\,(x+\sqrt{x^2-a^2})+c}$$

$$I = -\left\{\frac{1}{2}\sqrt{t^2-1}-\frac{1}{2}\ell n\,(t+\sqrt{t^2-1})\right\}+c$$

$$I = \frac{-1}{2}\sqrt{\text{Cosec}^2 x-1}+\frac{1}{2}\ell n\,\{\text{Cosecx}+\sqrt{\text{Cosec}^2 x-1}\}+c$$

$$\boxed{I = \frac{1}{2}\ell n\,\{\text{Cosecx}+\text{Cotx}\}-\frac{1}{2}\text{Cotx}+c}$$ Ans.

## EXERCISE # 6.4

**Evaluate the following integrals.**

**Q1.** $\int_0^3 \sqrt[3]{(3t-1)^2}\,dt$

**Solution:** let $I = \int_0^3 \sqrt[3]{(3t-1)^2}\,dt$

$I = \int_0^3 \{(3t-1)^2\}^{1/3}\,dt \Rightarrow I = \int_0^3 (3t-1)^{2/3}\,dt$

× & ÷ by 3.

$I = \frac{1}{3}\int_0^3 (3t-1)^{2/3}\cdot 3\,dt \Rightarrow I = \frac{1}{3}\left[\frac{(3t-1)^{5/3}}{5/3}\right]_0^3$

$I = \frac{1}{\cancel{3}}\times\frac{\cancel{3}}{5}\left[(3t-1)^{5/3}\right]_0^3$

$I = \frac{1}{5}\left[\{3(3)-1\}^{5/3}-\{3(0)-1\}^{5/3}\right]$

$I = \frac{1}{5}\{8^{5/3}-(-1)^{5/3}\} \Rightarrow I = \frac{1}{5}\{(2^{\cancel{3}})^{5/\cancel{3}}+1\}$

$I = \frac{1}{5}\{32+1\} \Rightarrow \boxed{I = \frac{33}{5}}$ Ans

**Q.2** $\int_{-2}^{1}\sqrt{2-x}\,dx$

**Solution:** let $I = \int_{-2}^{1}(2-x)^{1/2}\,dx$

× & ÷ by (−1)

$$I = -\int_{-2}^{1} (2-x)^{1/2}.(-1)\,dx \Rightarrow I = -\left[\frac{(2-x)^{3/2}}{3/2}\right]_{-2}^{1}$$

$$I = \frac{-2}{3}\left\{(2-1)^{3/2} - (2+2)^{3/2}\right\} \Rightarrow I = \frac{-2}{3}\left\{(1)^{3/2} - 4^{3/2}\right\}$$

$$I = \frac{-2}{3}\left\{1 - (2^{\cancel{2}})^{3/\cancel{2}}\right\} \Rightarrow I = \frac{-2}{3}\{1-8\} \Rightarrow I = \frac{-2}{3}(-7)$$

$\boxed{I = \frac{14}{3}}$ Ans

**Q3.** $\int_{-3}^{-1} \frac{dx}{(x-1)^3}$

**Solution:** let $I = \int_{-3}^{-1} \frac{dx}{(x-1)^3}$

$$I = \int_{-3}^{-1} (x-1)^{-3}\,dx \Rightarrow I = \left[\frac{(x-1)^{-2}}{-2}\right]_{-3}^{-1}$$

$$I = \frac{-1}{2}\left\{(-1-1)^{-2} - (-3-1)^{-2}\right\}$$

$$I = \frac{-1}{2}\left\{(-)^{-2} - (-4)^{-2}\right\} \Rightarrow I = \frac{-1}{2}\left\{\frac{1}{(-2)^2} - \frac{1}{(-4)^2}\right\}$$

$$I = \frac{-1}{2}\left\{\frac{1}{4} - \frac{1}{16}\right\} \Rightarrow I = \frac{-1}{2}\left\{\frac{4-1}{16}\right\} \Rightarrow I = \frac{-1}{2}\left(\frac{3}{16}\right)$$

$\boxed{I = \frac{-3}{32}}$ Ans

**Q4.** $\int_{-2}^{1} x\sqrt{2x^2+3}\,dx$

**Solution:** let $I = \int_{-2}^{1} (2x^2+3)^{1/2} \cdot x\,dx$

× & ÷ by 4

$$I = \frac{1}{4}\int_{-2}^{1} (2x^2+3)^{1/2}\,4x\,dx \Rightarrow I = \frac{1}{4}\left[\frac{(2x^2+3)^{3/2}}{3/2}\right]_{-2}^{1}$$

$$I = \frac{1}{\cancel{4}_2} \times \frac{\cancel{2}}{3}\left[\left\{2(1)^2+3\right\}^{3/2} - \left\{2(-2)^2+3\right\}^{3/2}\right]$$

$$I = \frac{1}{6}\left[5^{3/2} - 11^{3/2}\right] \Rightarrow \boxed{I = \frac{1}{6}\left\{\sqrt{125} - \sqrt{1331}\right\}}$$ Ans



**Q5.** $\int_{0}^{5} \frac{2x+3}{\sqrt{x^2+3x+1}}\,dx$

**Solution:** let $I = \int_{0}^{5} \frac{2x+3}{\sqrt{x^2+3x+1}}\,dx$

$$I = \int_{0}^{5} (x^2+3x+1)^{-1/2}.(2x+3)\,dx \Rightarrow I = \left[\frac{(x^2+3x+1)^{1/2}}{1/2}\right]_{0}^{5}$$

$$I = 2\left[\left\{(5)^2 + 3(5) + 1\right\}^{1/2} - \left\{(0)^2 + 3(0) + 1\right\}^{1/2}\right]$$

$$I = 2\left[(25+15+1)^{1/2} - (1)^{1/2}\right]$$

$\boxed{I = 2\left[\sqrt{41} - 1\right]}$ Ans

**Q6.** $\int_{1}^{2} (2x+1)\sqrt[3]{x^2+x+1}\,dx$

**Solution:** let $I = \int_{1}^{2} (x^2+x+1)^{1/3}.(2x+1)\,dx$

$$I = \left[\frac{(x^2+x+1)^{4/3}}{4/3}\right]_{1}^{2}$$

$$I = \frac{3}{4}\left[\{(2)^2+2+1\}^{4/3} - \{(1)^2+1+1\}^{4/3}\right]$$

$$I = \frac{3}{4}\left\{(7)^{4/3} - (3)^{4/3}\right\}$$

$\boxed{I = \frac{3}{4}\left[7\sqrt[3]{7} - 3\sqrt[3]{3}\right]}$ Ans

**Q7.** $\int_{-2}^{1} x\sqrt{x^2+1}\,dx$

**Solution:** let $I = \int_{-2}^{1} (x^2+1)^{1/2}.\,x\,dx$

× & ÷ by 2

$$I = \frac{1}{2}\int_{-2}^{1} (x^2+1)^{1/2}.2x\,dx \Rightarrow I = \frac{1}{2}\left[\frac{(x^2+1)^{3/2}}{3/2}\right]_{-2}^{1}$$

$$I = \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}\left[\{(1)^2+1)\}^{3/2} - \{(-2)^2+1\}^{3/2}\right]$$

$\boxed{I = \frac{1}{3}\left[2^{3/2} - 5^{3/2}\right]}$ Ans

**Q8.** $\int_{\pi/3}^{\pi/6} \text{Sin}^2x\ \text{Cosx}\ dx$

**Solution:** let $I = \int_{\pi/3}^{\pi/6} \text{Sin}^2x\ \text{Cosx}\ dx$

$$I = \left[\frac{\text{Sin}^3x}{3}\right]_{\pi/3}^{\pi/6} \Rightarrow I = \frac{1}{3}\left[\left(\text{Sin}\frac{\pi}{6}\right)^3 - \left(\text{Sin}\frac{\pi}{3}\right)^3\right]$$

$$I = \frac{1}{3}\left[\left(\frac{1}{2}\right)^3 - \left(\frac{\sqrt{3}}{2}\right)^3\right] \Rightarrow I = \frac{1}{3}\left[\frac{1}{8} - \frac{3\sqrt{3}}{8}\right]$$

$$I = \frac{1}{3}\left(\frac{1-3\sqrt{3}}{8}\right) \Rightarrow \boxed{I = \frac{1}{24}(1-3\sqrt{3})}\ \text{Ans}$$

**Q9.** $\int_0^{\pi/2} \text{Cos}^4x\ dx.$

**Solution:** let $I = \int_0^{\pi/2} \text{Cos}^4x\ dx.$

$$I = \int_0^{\pi/2} (\text{Cos}^2x)^2 dx \Rightarrow \boxed{\therefore \text{Cos}^2x = \frac{1+\text{Cos}2x}{2}}$$

$$I = \int_0^{\pi/2} \left(\frac{1+\text{Cos}2x}{2}\right)^2 dx \Rightarrow I = \frac{1}{4}\int_0^{\pi/2} (1+\text{Cos}2x)^2 dx.$$

$$\boxed{\therefore (a+b)^2 = a^2 + 2ab + b^2}$$

$$I = \frac{1}{4}\int_0^{\pi/2} (1 + 2\text{Cos}2x + \text{Cos}^2 2x)\ dx$$

$$I = \frac{1}{4}\int_0^{\pi/2} dx + \frac{\cancel{2}}{\cancel{4}_2}\int_0^{\pi/2} \text{Cos}2x\ dx + \frac{1}{4}\int_0^{\pi/2} \text{Cos}^2 2x\ dx$$

$$I = \frac{1}{4}\int_0^{\pi/2} dx + \frac{1}{2}\int_0^{\pi/2} \text{Cos}2x\ dx + \frac{1}{4}\int_0^{\pi/2}\left(\frac{1+\text{Cos}4x}{2}\right) dx$$

$$I = \frac{1}{4}\int_0^{\pi/2} dx + \frac{1}{2}\int_0^{\pi/2} \text{Cos}2x\ dx + \frac{1}{8}\int_0^{\pi/2} dx + \frac{1}{8}\int_0^{\pi/2} \text{Cos}4x\ dx$$

$$I = \frac{1}{4}\int_0^{\pi/2} dx + \frac{1}{2\times 2}\int_0^{\pi/2} \text{Cos}2x.2dx + \frac{1}{8}\int_0^{\pi/2} dx + \frac{1}{8\times 4}\int_0^{\pi/2} \text{Cos}4x.4dx$$



$$I = \frac{1}{4}[x]_0^{\pi/2} + \frac{1}{4}[\text{Sin}2x]_0^{\pi/2} + \frac{1}{8}[x]_0^{\pi/2} + \frac{1}{32}[\text{Sin}4x]_0^{\pi/2}$$

$$I = \frac{1}{4}\left[\frac{\pi}{2} - 0\right] + \frac{1}{4}\left[\text{Sin}\cancel{2}\left(\frac{\pi}{\cancel{2}}\right) - \text{Sin}(0)\right] + \frac{1}{8}$$

$$\left[\frac{\pi}{2} - 0\right] + \frac{1}{32}\left[\text{Sin}\cancel{4}^2\left(\frac{\pi}{\cancel{2}}\right) - \text{Sin}(0)\right]$$

$$I = \frac{1}{4}\left(\frac{\pi}{2}\right) + \frac{1}{4}[0-0] + \frac{1}{8}\left(\frac{\pi}{2}\right) + \frac{1}{32}(0-0)$$

$$I = \frac{\pi}{8} + \frac{\pi}{16} \Rightarrow I = \frac{2\pi+\pi}{16} \Rightarrow \boxed{I = \frac{3\pi}{16}}\ \text{Ans}$$

**Q10.** $\int_{\pi/6}^{\pi/2} \frac{\text{Cos}^3x\ dx}{\sqrt{\text{Sinx}}}$

**Solution:** let $I = \int_{\pi/6}^{\pi/2} \frac{\text{Cos}^3x\ dx}{\sqrt{\text{Sinx}}}$

$$I = \int_{\pi/6}^{\pi/2} \frac{\text{Cos}^2x.\text{Cosx}\ dx}{\sqrt{\text{Sinx}}}$$

$$\boxed{\therefore \text{Cos}^2x = 1 - \text{Sin}^2x}$$

$$I = \int_{\pi/6}^{\pi/2} \frac{(1-\text{Sin}^2x)\ \text{Cosx}\ dx}{\text{Sin}^{1/2}x} \qquad \text{———— (1)}$$

let t = Sinx.
diff w.r to x

$\frac{dt}{dx} = \text{Cosx}. \Rightarrow \boxed{dt = \text{Cosx}\ dx}$

| When $x = \frac{\pi}{2}$ | When $x = \frac{\pi}{6}$ |
|---|---|
| $t = \text{Sin}\frac{\pi}{2}$ | $t = \text{Sin}\frac{\pi}{6}$ |
| $\boxed{t = 1}$ | $\boxed{t = \frac{1}{2}}$ |

$$(1) \Rightarrow I = \int_{1/2}^{1} \frac{(1-t^2)\ dt}{t^{1/2}}$$

$$I = \int_{1/2}^{1}\left\{\frac{1}{t^{1/2}} - \frac{t^2}{t^{1/2}}\right\} dt \Rightarrow I = \int_{1/2}^{1}\{t^{-1/2} - t^{3/2}\}\ dt$$

$$I = \int_{1/2}^{1} t^{-1/2}\ dt - \int_{1/2}^{1} t^{3/2}\ dt$$

$$I = \left[\frac{t^{1/2}}{1/2}\right]_{1/2}^{1} - \left[\frac{t^{5/2}}{5/2}\right]_{1/2}^{1}$$

$$I = 2\,[\,t^{1/2}\,]_{1/2}^{1} - \frac{2}{5}\,[\,t^{5/2}\,]_{1/2}^{1}$$

$$I = 2\left\{(1)^{1/2} - \left(\frac{1}{2}\right)^{1/2}\right\} - \frac{2}{5}\left\{(1)^{5/2} - \left(\frac{1}{2}\right)^{5/2}\right\}$$

$$I = 2\left(1 - \frac{1}{\sqrt{2}}\right) - \frac{2}{5}\left(1 - \frac{1}{2^{5/2}}\right)$$

$$I = 2\left\{1 - \frac{1}{\sqrt{2}} - \frac{1}{5} + \frac{1}{5}\left(\frac{1}{2^{5/2}}\right)\right\}$$

$$I = 2\left\{\frac{5-1}{5} - \frac{1}{\sqrt{2}} + \frac{1}{5}\left(\frac{1}{4\sqrt{2}}\right)\right\}$$

$$I = 2\left\{\frac{4}{5} + \frac{1}{20\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right\} \Rightarrow I = 2\left\{\frac{4}{5} + \frac{1-20}{20\sqrt{2}}\right\}$$

$$I = 2\left\{\frac{4}{5} - \frac{19}{20\sqrt{2}}\right\} = \frac{8}{5} - \frac{19}{10\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$I = \frac{8}{5} - \frac{19\sqrt{2}}{20} \Rightarrow \boxed{I = \frac{32 - 19\sqrt{2}}{20}}$$

**Q11.** $\int_0^{\pi} \tan^3 x \,\text{Sec}x\, dx.$

**<u>Solution:</u>** let $I = \int_0^{\pi} \tan^3 x\, \text{Sec}x\, dx$

$$I = \int_0^{\pi} \tan x.\text{Sec}x.\tan^2 x\, dx \Rightarrow I = \int_0^{\pi} \tan x\, \text{Sec}x\, (\text{Sec}^2 x - 1)\, dx$$

$$I = \int_0^{\pi} \text{Sec}^3 x \tan x\, dx - \int_0^{\pi} \text{Sec}x \tan x\, dx$$

$$I = \int_0^{\pi} \text{Sec}^2 x.\ \sec x \tan x\, dx - \int_0^{\pi} \text{Sec}x \tan x\, dx$$

$$I = \left[\frac{\text{Sec}^3 x}{3}\right]_0^{\pi} - [\,\text{Sec}x\,]_0^{\pi}$$

$$I = \frac{1}{3}[\,(\text{Sec}\pi)^3 - (\text{Sec}0)^3] - [\,(\text{Sec}\pi) - (\text{Sec}0)]$$

$$I = \frac{1}{3}\,[-1-1\,] - [-1-1]$$



$$I = \frac{1}{3}(-2) - (-2) \Rightarrow I = \frac{-2}{3} + 2$$

$$I = \frac{-2+6}{3} \Rightarrow \boxed{I = \frac{4}{3}} \text{ Ans}$$

**Q12.** $\int_0^{\pi/4} \textbf{Sin}^2\textbf{2x. Cos}^2\textbf{2x dx}$

**<u>Solution:</u>** let $I = \int_0^{\pi/4} \text{Sin}^2 2x.\text{Cos}^2 2x\, dx$

$$I = \int_0^{\pi/4} \left(\frac{1 - \text{Cos}4x}{2}\right)\left(\frac{1 + \text{Cos}4x}{2}\right) dx$$

$$I = \frac{1}{4}\int_0^{\pi/4} (1 - \text{Cos}4x)\,(1 + \text{Cos}4x)\, dx$$

$$\boxed{\therefore\ a^2 - b^2 = (a-b)\,(a+b)}$$

$$I = \frac{1}{4}\int_0^{\pi/4} (1 - \text{Cos}^2 4x)\, dx$$

$$I = \frac{1}{4}\int_0^{\pi/4} \text{Sin}^2 4x\, dx. \Rightarrow I = \frac{1}{4}\int_0^{\pi/4} \left(\frac{1 - \text{Cos}8x}{2}\right) dx$$

$$I = \frac{1}{8}\int_0^{\pi/4} dx - \frac{1}{8}\int_0^{\pi/4} \text{Cos}8x\, dx$$

$$I = \frac{1}{8}[\,x\,]_0^{\pi/4} - \frac{1}{64}[\,\text{Sin}8x\,]_0^{\pi/4}$$

$$I = \frac{1}{8}\left(\frac{\pi}{4} - 0\right) - \frac{1}{64}\left\{\text{Sin}8\left(\frac{\pi}{4}\right) - \text{Sin}8\,(0)\right\}$$

$$I = \frac{1}{8}\left(\frac{\pi}{4}\right) - \frac{1}{64}(0-0) \Rightarrow \boxed{I = \frac{\pi}{32}} \text{ Ans}$$

## EXERCISE # 6.5

**Evaluate the following definite integrals.**

**(1)** $\int_0^1 \frac{dx}{\sqrt{4 - x^2}}$

**<u>Solution:</u>** let $I = \int_0^1 \frac{dx}{\sqrt{4-x^2}}$ ———— (1)

| | |
|---|---|
| let $x = a\,\text{Sin}\theta$<br>$x = 2\,\text{Sin}\theta$<br>differentiate w.r to $\theta$<br>$\frac{dx}{d\theta} = 2\,\text{Cos}\theta$<br>$\boxed{dx = 2\,\text{Cos}\theta d\theta}$ | when $\underline{x = 1}$<br>$x = 2\,\text{Sin}\theta$<br>$1 = 2\,\text{Sin}\theta$<br>$\text{Sin}\theta = \frac{1}{2}$<br>$\theta = \text{Sin}^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = \frac{\pi}{6}}$<br>when $\underline{x = 0}$<br>$0 = 2\,\text{Sin}\theta$<br>$\text{Sin}\theta = 0$<br>$\theta = \text{Sin}^{-1}(0) \Rightarrow \boxed{\theta = 0}$ |

$$(1) \Rightarrow I = \int_0^{\pi/6} \frac{2\,\text{Cos}\theta\, d\theta}{\sqrt{4 - 4\text{Sin}^2\theta}}$$

$$I = \int_0^{\pi/6} \frac{2\,\text{Cos}\theta\, d\theta}{\sqrt{4(1 - \text{Sin}^2\theta)}} \Rightarrow I = \int_0^{\pi/6} \frac{\cancel{2}\text{Cos}\theta\, d\theta}{\cancel{2}\sqrt{1 - \text{Sin}^2\theta}}$$

$$I = \int_0^{\pi/6} \frac{\text{Cos}\theta\, d\theta}{\sqrt{\text{Cos}^2\theta}} \Rightarrow I = \int_0^{\pi/6} \frac{\cancel{\text{Cos}\theta}\, d\theta}{\cancel{\text{Cos}\theta}} \Rightarrow$$

$$I = \int_0^{\pi/6} d\theta \Rightarrow I = [\theta]_0^{\pi/6} \Rightarrow I = \frac{\pi}{6} - 0 \Rightarrow \boxed{I = \frac{\pi}{6}} \text{ Ans}$$

**(2)** $\displaystyle\int_{-6}^{-2\sqrt{3}} \frac{dx}{x\sqrt{x^2 - 9}}$

**Solution:** let $I = \displaystyle\int_{-6}^{-2\sqrt{3}} \frac{dx}{x\sqrt{x^2 - 9}}$ ———— (1)

| | |
|---|---|
| let $x = a\,\text{Sec}\theta$<br>$x = 3\,\text{Sec}\theta$<br>differentiate w.r to $\theta$<br>$\frac{dx}{d\theta} = 3\,\text{Sec}\theta\tan\theta$<br>$\boxed{dx = 3\,\text{Sec}\theta\tan\theta\, d\theta}$ | when $\underline{x = -2\sqrt{3}}$<br>$x = 3\,\text{Sec}\theta$<br>$\frac{1}{x} = \frac{1}{3\,\text{Sec}\theta}$<br>$\text{Cos}\theta = \frac{3}{x} \Rightarrow \text{Cos}\theta = \frac{3}{-2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$<br>$\text{Cos}\theta = \frac{-\sqrt{3}}{2}$ |



$\theta = \text{Cos}^{-1}\left(\frac{-\sqrt{3}}{2}\right) \Rightarrow \boxed{\theta = \frac{5\pi}{6}}$

**when $\underline{x = 6}$**

$x = 3\,\text{Sec}\theta$

$\text{Cos}\theta = \frac{3}{x} \Rightarrow \text{Cos}\theta = \frac{\cancel{3}}{\cancel{-6}_2} = \frac{-1}{2}$

$\theta = \text{Cos}^{-1}\left(\frac{-1}{2}\right) \Rightarrow \boxed{\theta = \frac{2\pi}{3}}$

$$(1) \Rightarrow = \int_{2\pi/3}^{5\pi/6} \frac{\cancel{3\,\text{Sec}\theta}\tan\theta\, d\theta}{\cancel{3\,\text{Sec}\theta}\sqrt{9\,\text{Sec}^2\theta - 9}}$$

$$I = \int_{2\pi/3}^{5\pi/6} \frac{\tan\theta\, d\theta}{\sqrt{9(\text{Sec}^2\theta - 1)}} \Rightarrow I = \frac{1}{3}\int_{2\pi/3}^{5\pi/6} \frac{\tan\theta\, d\theta}{\sqrt{\text{Sec}^2\theta - 1}}$$

$$I = \frac{1}{3}\int_{2\pi/3}^{5\pi/6} \frac{\tan\theta\, d\theta}{\sqrt{\tan^2\theta}} \Rightarrow I = \frac{1}{3}\int_{2\pi/3}^{5\pi/6} \frac{\cancel{\tan\theta}\, d\theta}{\cancel{\tan\theta}}$$

$$I = \frac{1}{3}\int_{2\pi/3}^{5\pi/6} d\theta \Rightarrow I = \frac{1}{3}[\theta]_{2\pi/3}^{5\pi/6}$$

$$I = \frac{1}{3}\left\{\frac{5\pi}{6} - \frac{2\pi}{3}\right\} \Rightarrow I = \frac{1}{3}\left\{\frac{5\pi - 4\pi}{6}\right\} \Rightarrow \boxed{I = \frac{\pi}{18}} \text{ Ans}$$

**(3)** $\displaystyle\int_0^2 \frac{x^2\, dx}{\sqrt{x^2 + 4}}$

**Solution:** let $I = \displaystyle\int_0^2 \frac{x^2\, dx}{\sqrt{x^2 + 4}}$ ———— (1)

| | |
|---|---|
| let $x = a\tan\theta$<br>$x = 2\tan\theta$<br>differentiate w.r to $\theta$<br>$\frac{dx}{d\theta} = 2\text{Sec}\theta$<br>$\boxed{dx = 3\text{Sec}^2\theta\, d\theta}$ | **When $\underline{x = 2}$**<br>$x = 2\tan\theta$<br>$\cancel{2} = \cancel{2}\tan\theta$<br>$\tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1)$<br>$\boxed{\theta = \frac{\pi}{4}}$<br>**When $\underline{x = 0}$**<br>$x = 2\tan\theta$<br>$0 = 2\tan\theta$<br>$\tan\theta = 0$<br>$\theta = \tan^{-1}(0) \Rightarrow \boxed{\theta = 0}$ |

$$(1)\Rightarrow I=\int_0^{\pi/4}\frac{4\tan^2\theta\cdot 2\sec^2\theta\,d\theta}{\sqrt{4\tan^2\theta+4}} \Rightarrow I=\int_0^{\pi/4}\frac{4\tan^2\theta\cdot \cancel{2}\sec^2\theta\,d\theta}{\cancel{2}\sqrt{1+\tan^2\theta}}$$

$$I=4\int_0^{\pi/4}\frac{\tan^2\theta\cdot\sec^2\theta\,d\theta}{\sqrt{\sec^2\theta}} \Rightarrow I=4\int_0^{\pi/4}\frac{\tan^2\theta\cdot\sec^2\theta}{\sec\theta}\,d\theta$$

$$I=4\int_0^{\pi/4}\tan^2\theta\cdot\sec\theta\,d\theta$$

$$I=4\int_0^{\pi/4}\tan\theta\cdot\tan\theta\cdot\sec\theta\,d\theta \text{ ——— (2)}$$

let $x=\sec\theta$

diff w.r to $\theta$

$$\frac{dx}{d\theta}=\sec\theta\tan\theta \Rightarrow \boxed{dx=\sec\theta\tan\theta\,d\theta}$$

If $x=\sec\theta \Rightarrow x^2=\sec^2\theta \Rightarrow x^2=1+\tan^2\theta$

$\tan^2\theta=x^2-1 \Rightarrow \boxed{\tan\theta=\sqrt{x^2-1}}$

**When $\theta=0$**

$x=\sec\theta$

$x=\sec 0$

$\boxed{x=1}$

**When $\theta=\frac{\pi}{4}$**

$x=\sec\theta \Rightarrow x=\sec\frac{\pi}{4}$

$\boxed{x=\sqrt{2}}$

$$(2)\Rightarrow I=4\int_1^{\sqrt{2}}\sqrt{x^2-1}\,dx$$

$$\boxed{\therefore \int\sqrt{x^2-a^2}\,dx=\frac{1}{2}x\sqrt{x^2-a^2}-\frac{1}{2}a^2\ln\left(x+\sqrt{x^2-a^2}+c\right)}$$

$$I=4\left[\frac{1}{2}x\sqrt{x^2-1}-\frac{1}{2}\ln\left(x+\sqrt{x^2-1}\right)\right]_1^{\sqrt{2}}$$

$$I=4\left[\frac{\sqrt{2}}{2}\sqrt{2-1}-\frac{1}{2}\ln(\sqrt{2}+\sqrt{2-1})-\frac{1}{2}\sqrt{1-1}+\frac{1}{2}\ln(1+\sqrt{1-1})\right]$$

$$I=4\left[\frac{\sqrt{2}}{2}-\frac{1}{2}\ln(\sqrt{2}+1)\right] \Rightarrow \boxed{I=2\sqrt{2}-2\ln(1+\sqrt{2})} \text{ Ans.}$$

**(4)** $\displaystyle\int_0^2\frac{y^3\,dy}{\sqrt{16-y^2}}$

**Solution:** let $I=\displaystyle\int_0^2\frac{y^3\,dy}{\sqrt{16-y^2}}$ ——— (1)



$\therefore \sqrt{a^2-x^2} \Rightarrow y=a\sin\theta$

$y=4\sin\theta$

differentiate w.r to $\theta$

$$\frac{dy}{d\theta}=4\cos\theta$$

$\boxed{dy=4\cos\theta\,d\theta}$

when $\underline{y=2}$

$y=4\sin\theta$

$2=4\sin\theta \Rightarrow \frac{\cancel{2}}{\cancel{4}_2}=\sin\theta$

$\sin\theta=\frac{1}{2} \Rightarrow \theta=\sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\theta=\frac{\pi}{6}}$

when $\underline{y=0}$

$y=4\sin\theta$

$0=4\sin\theta$

$\sin\theta=0$

$\theta=\sin^{-1}(0)$

$\boxed{\theta=0}$

$$(1)\Rightarrow I=\int_0^{\pi/6}\frac{64\sin^3\theta.4\cos\theta\,d\theta}{\sqrt{16-16\sin^2\theta}} \Rightarrow I=\int_0^{\pi/6}\frac{64\sin^3\theta.4\cos\theta\,d\theta}{\sqrt{16(1-\sin^2\theta)}}$$

$$I=\int_0^{\pi/6}\frac{64\sin^3\theta.\cancel{4}\cos\theta\,d\theta}{\cancel{4}\sqrt{\cos^2\theta}} \Rightarrow I=\int_0^{\pi/6}\frac{64\sin^3\theta\,\cancel{\cos}\,\theta\,d\theta}{\cancel{\cos}\,\theta}$$

$$I=\int_0^{\pi/6}64\sin^3\theta\,d\theta \Rightarrow I=\int_0^{\pi/6}64.\sin\theta\sin^2\theta\,d\theta$$

$$I=\int_0^{\pi/6}64.\sin\theta.(1-\cos^2\theta)\,d\theta$$

$$I=64\int_0^{\pi/6}\sin\theta\,d\theta-64\int_0^{\pi/6}\cos^2\theta\sin\theta\,d\theta$$

$$I=64\left[-\cos\theta\right]_0^{\pi/6}+64\left[\frac{\cos^3\theta}{3}\right]_0^{\pi/6}$$

$$I=-64\left\{\cos\frac{\pi}{6}-\cos 0\right\}+\frac{64}{3}\left\{\left(\cos\frac{\pi}{6}\right)^3-(\cos 0)^3\right\}$$

$$I=-64\left\{\frac{\sqrt{3}}{2}-1\right\}+\frac{64}{3}\left\{\left(\frac{\sqrt{3}}{2}\right)^3-(1)^3\right\}$$

$$I=-32\sqrt{3}+64+\frac{64}{3}\left\{\frac{3\sqrt{3}}{8}-1\right\}$$

$I = -32\sqrt{3} + 64 + 8\sqrt{3} - \frac{64}{3} \Rightarrow I = \frac{192-94}{3} - 24\sqrt{3}$

$I = \frac{128}{3} - 24\sqrt{3} \Rightarrow \boxed{I = \frac{8}{3}(16 - 9\sqrt{3})}$ Ans

**(5)** $\int_0^{2\sqrt{3}} \frac{x^3\,dx}{\sqrt{x^2+4}}$

**Solution:** let $I = \int_0^{2\sqrt{3}} \frac{x^3\,dx}{\sqrt{x^2+4}}$ ———— (1)

$\therefore \sqrt{x^2+a^2} \qquad \Rightarrow x = a\tan\theta$

$x = 2\tan\theta$

differentiate w.r to $\theta$

$\frac{dx}{d\theta} = 2\,Sec^2\theta$

$\boxed{dx = 2\,Sec^2\theta\,d\theta}$

when $x = 0$

$x = 2\tan\theta$

$0 = 2\tan\theta$

$\tan\theta = 0$

$\theta = \tan^{-1}(0)$

$\boxed{\theta = 0}$

when $x = 2\sqrt{3}$

$x = 2\tan\theta$

$\cancel{2}\sqrt{3} = \cancel{2}\tan\theta$

$\theta = \tan^{-1}(\sqrt{3}) \Rightarrow \boxed{\theta = \frac{\pi}{3}}$

$(1) \Rightarrow I = \int_0^{\pi/3} \frac{8\tan^3\theta.\,2\,Sec^2\theta\,d\theta}{\sqrt{4\tan^2\theta + 4}} \Rightarrow I = \int_0^{\pi/3} \frac{8\tan^3\theta.\,2\,Sec^2\theta\,d\theta}{\sqrt{4(\tan^2\theta + 1)}}$

$I = \int_0^{\pi/3} \frac{8\tan^3\theta.\,\cancel{2}\,Sec^2\theta\,d\theta}{\cancel{2}\sqrt{1+\tan^2\theta}} \Rightarrow I = \int_0^{\pi/3} \frac{8\tan^3\theta.\,Sec^2\theta\,d\theta}{\sqrt{Sec^{\cancel{2}}\theta}}$

$I = \int_0^{\pi/3} \frac{8\tan^3\theta.\,\cancel{Sec^2\theta}\,d\theta}{\cancel{Sec\theta}} \Rightarrow I = \int_0^{\pi/3} 8.\tan^3\theta.\,Sec\theta d\theta$

$I = \int_0^{\pi/3} 8\tan^2\theta.\,\tan\theta\,Sec\theta\,d\theta$

$I = \int_0^{\pi/3} 8(Sec^2\theta - 1)\tan\theta\,Sec\theta\,d\theta$

$I = 8\int_0^{\pi/3} Sec^3\theta\tan\theta\,d\theta - 8\int_0^{\pi/3}\tan\theta\,Sec\theta\,d\theta$



$I = 8\int_0^{\pi/3} Sec^2\theta.Sec\theta\tan\theta\,d\theta - 8\int_0^{\pi/3}\tan\theta\,Sec\theta\,d\theta$

$I = 8\left[\frac{Sec^3\theta}{3}\right]_0^{\pi/3} - 8\,[Sec\theta]_0^{\pi/3}$

$I = \frac{8}{3}\left\{\left(Sec\frac{\pi}{3}\right)^3 - (Sec\,0)^3\right\} - 8\left\{\left(Sec\frac{\pi}{3}\right) - (Sec\,0)\right\}$

$I = \frac{8}{3}\{8-1\} - 8\{2-1\} \Rightarrow I = \frac{8}{3}(7) - 8(1)$

$I = \frac{56}{3} - 8 \Rightarrow I = \frac{56-24}{3} \Rightarrow \boxed{I = \frac{32}{3} = 10\frac{2}{3}}$ Ans

**(6)** $\int_0^1 \frac{x^3\,dx}{\sqrt{4-x^2}}$

**Solution:** let $I = \int_0^1 \frac{x^3\,dx}{\sqrt{4-x^2}}$ ———— (1)

$\therefore \sqrt{a^2-x^2} \qquad \Rightarrow x = a\,Sin\theta$

$x = 2\,Sin\theta$

differentiate w.r to $\theta$

$\frac{dx}{d\theta} = 2\,Cos\theta$

$\boxed{dx = 2\,Cos\theta\,d\theta}$

when $x = 0$

$x = 2\,Sin\theta$

$0 = 2\,Sin\theta$

$Sin\theta = 0$

$\theta = Sin^{-1}(0)$

$\boxed{\theta = 0}$

when $x = 1$

$x = 2\,Sin\theta$

$1 = 2\,Sin\theta \Rightarrow \frac{1}{2} = Sin\theta$

$\theta = Sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = \frac{\pi}{6}}$

$(1) \Rightarrow I = \int_0^{\pi/6} \frac{8\,Sin^3\theta.\,2\,Cos\theta\,d\theta}{\sqrt{4 - 4\,Sin^2\theta}} \Rightarrow I = \int_0^{\pi/6} \frac{8\,Sin^3\theta.\,2\,Cos\theta\,d\theta}{\sqrt{4(1 - Sin^2\theta)}}$

$I = \int_0^{\pi/6} \frac{8\,Sin^3\theta.\,\cancel{2}\,Cos\theta\,d\theta}{\cancel{2}\sqrt{1 - Sin^2\theta}} \Rightarrow I = \int_0^{\pi/6} \frac{8\,Sin^3\theta.\,Cos\theta\,d\theta}{\sqrt{Cos^{\cancel{2}}\theta}}$

$I = \int_0^{\pi/6} \frac{8\,Sin^3\theta.\,\cancel{Cos\theta}\,d\theta}{\cancel{Cos\theta}} \Rightarrow I = \int_0^{\pi/6} 8.\,Sin^3\theta.d\theta$

$$I = \int_0^{\pi/6} 8\ Sin^2\theta .\ Sin\theta\ d\theta$$

$$I = \int_0^{\pi/6} 8\ (1 - Cos^2\theta)\ Sin\theta\ d\theta$$

$$I = 8 \int_0^{\pi/6} Sin\theta\ d\theta - 8 \int_0^{\pi/6} Cos^2\theta\ Sin\theta\ d\theta$$

$$I = 8\left[-Cos\theta\right]_0^{\pi/6} + 8\left[\frac{Cos^3\theta}{3}\right]_0^{\pi/6}$$

$$I = -8\left\{Cos\frac{\pi}{6} - Cos\ 0\right\} + \frac{8}{3}\left\{\left(Cos\frac{\pi}{6}\right)^3 - (Cos\ 0)^3\right\}$$

$$I = -8\left\{\frac{\sqrt{3}}{2} - 1\right\} + \frac{8}{3}\left\{\left(\frac{\sqrt{3}}{2}\right)^3 - (1)^3\right\}$$

$$I = -8\left\{\frac{\sqrt{3}}{2} - 1\right\} + \frac{8}{3}\left\{\frac{3\sqrt{3}}{8} - 1\right\}$$

$$I = -4\sqrt{3} + 8 + \sqrt{3} - \frac{8}{3} \Rightarrow I = -3\sqrt{3} + \frac{24-8}{3}$$

$$I = -3\sqrt{3} + \frac{16}{3} \Rightarrow \boxed{I = \frac{1}{3}\left(16 - 9\sqrt{3}\right)} \text{ Ans}$$

**(7)** $\int_0^a \frac{dx}{(a^2 + x^2)^{3/2}}$

**Solution:** let $I = \int_0^a \frac{dx}{(a^2 + x^2)^{3/2}}$ ——— **(1)**

let $x = a\ tan\theta$

differentiate w.r to $\theta$

$\frac{dx}{d\theta} = a\ Sec^2\theta$

$\boxed{dx = a\ Sec^2\theta\ d\theta}$

| when $x = a$ | when $x = 0$ |
|---|---|
| $x = a\ tan\theta$ | $x = a\ tan\theta$ |
| $\cancel{a} = \cancel{a}\ tan\theta$ | $0 = a\ tan\theta$ |
| $tan\theta = 1$ | $tan\theta = 0$ |
| $\theta = tan^{-1}(1) \Rightarrow \boxed{\theta = \frac{\pi}{4}}$ | $\theta = tan^{-1}(0) \Rightarrow \boxed{\theta = 0}$ |



$$(1) \Rightarrow \int_0^{\pi/4} \frac{a\ Sec^2\theta\ d\theta}{(a^2 + a^2 tan^2\theta)^{3/2}} \Rightarrow I = \int_0^{\pi/4} \frac{a\ Sec^2\theta\ d\theta}{(a^2)^{3/2}(1 + tan^2\theta)^{3/2}}$$

$$I = \int_0^{\pi/4} \frac{\cancel{a}\ Sec^2\theta\ d\theta}{a^{\cancel{3}}(Sec^2\theta)^{3/2}} \Rightarrow I = \int_0^{\pi/4} \frac{Sec^2\theta\ d\theta}{a^2\ Sec^3\theta}$$

$$I = \frac{1}{a^2}\int_0^{\pi/4} \frac{1}{Sec\theta}\ d\theta \Rightarrow I = \frac{1}{a^2}\int_0^{\pi/4} Cos\theta\ d\theta$$

$$I = \frac{1}{a^2}\left[Sin\theta\right]_0^{\pi/4} \Rightarrow I = \frac{1}{a^2}\left\{Sin\frac{\pi}{4} - Sin\ 0\right\}$$

$$I = \frac{1}{a^2}\left\{\frac{1}{\sqrt{2}} - 0\right\} \Rightarrow \boxed{I = \frac{1}{a^2\sqrt{2}}} \text{ Ans}$$

**(8)** $\int_0^1 \frac{x^2\ dx}{(4 - x^2)^{3/2}}$

**Solution:** let $I = \int_0^1 \frac{x^2\ dx}{(4 - x^2)^{3/2}}$ ——— (1)

let $x = a\ Sin\theta$

$x = 2\ Sin\theta$

differentiate w.r to $\theta$

$\frac{dx}{d\theta} = 2\ Cos\theta$

$\boxed{dx = 2\ Cos\theta\ d\theta}$

| when $x = 0$ | when $x = 1$ |
|---|---|
| $x = 2\ Sin\theta$ | $x = 2\ Sin\theta \Rightarrow 1 = 2\ Sin\theta$ |
| $0 = 2\ Sin\theta$ | $Sin\theta = \frac{1}{2}$ |
| $Sin\theta = 0$ | |
| $\theta = Sin^{-1}(0) \Rightarrow \boxed{\theta = 0}$ | $\theta = Sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \boxed{\theta = \frac{\pi}{6}}$ |

$$(1) \Rightarrow I = \int_0^{\pi/6} \frac{4\ Sin^2\theta .\ 2\ Cos\theta\ d\theta}{(4 - 4\ Sin^2\theta)^{3/2}} \Rightarrow I = \int_0^{\pi/6} \frac{4\ Sin^2\theta .\ 2\ Cos\theta\ d\theta}{4^{3/2}(1 - Sin^2\theta)^{3/2}}$$

$$I = \int_0^{\pi/6} \frac{\cancel{4}\ Sin^2\theta .\ \cancel{2}Cos\theta\ d\theta}{\cancel{8}\ (Cos^2\theta)^{3/2}} \Rightarrow I = \int_0^{\pi/6} \frac{Sin^2\theta .\cancel{Cos\theta}\ d\theta}{Cos^3\theta}$$

$$I = \int_{0}^{\pi/6} \frac{Sin^2\theta}{Cos^2\theta}\, d\theta \Rightarrow I = \int_{0}^{\pi/6} tan^2\theta\, d\theta$$

$$I = \int_{0}^{\pi/6} (Sec^2\theta - 1)\, d\theta \Rightarrow I = \int_{0}^{\pi/6} Sec^2\theta\, d\theta - \int_{0}^{\pi/6} d\theta$$

$$I = [\tan\theta]_{0}^{\pi/6} - [\theta]_{0}^{\pi/6} \Rightarrow I = \left\{ tan\frac{\pi}{6} - tan\, 0 \right\} - \left\{ \frac{\pi}{6} - 0 \right\}$$

$$I = \frac{1}{\sqrt{3}} - 0 - \frac{\pi}{6} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{\pi}{6}$$

$$I = \frac{\sqrt{3}}{3} - \frac{\pi}{6} \Rightarrow \boxed{I = \frac{2\sqrt{3} - \pi}{6}} \text{ Ans}$$

**(9)** $\int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2\sqrt{x^2+9}}$

**Solution:** let $I = \int_{\sqrt{3}}^{3\sqrt{3}} \frac{dx}{x^2\sqrt{x^2+9}}$ ———— (1)

let $x = a\tan\theta$

$x = 3\tan\theta$

differentiate w • r to θ

$\frac{dx}{d\theta} = 3\, Sec^2\theta$

$\boxed{dx = 3\, Sec^2\theta\, d\theta}$

when $\underline{x = 3\sqrt{3}}$

$x = 3\tan\theta$

$\cancel{3}\sqrt{3} = \cancel{3}\tan\theta$

$\theta = tan^{-1}(\sqrt{3})$

$\boxed{\theta = \frac{\pi}{3}}$

when $\underline{x = \sqrt{3}}$

$x = 3\tan\theta \Rightarrow \sqrt{3} = 3\tan\theta$

$\tan\theta = \frac{\sqrt{3}}{3} \Rightarrow \tan\theta = \frac{\sqrt{\cancel{3}}}{\sqrt{\cancel{3}} \times \sqrt{3}}$

$\theta = tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \boxed{\theta = \frac{\pi}{6}}$

$$(1) \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\cancel{3}\, Sec^2\theta\, d\theta}{{}_3\cancel{9}\, tan^2\theta \sqrt{9\, tan^2\theta + 9}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{Sec^2\theta\, d\theta}{3\, tan^2\theta \sqrt{9\, (tan^2\theta + 1)}} \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{Sec^2\theta\, d\theta}{9\, tan^2\theta \sqrt{Sec^{\cancel{2}}\theta}}$$



$$I = \int_{\pi/6}^{\pi/3} \frac{Sec^{\cancel{2}}\theta\, d\theta}{9\, tan^2\theta . \cancel{Sec\theta}} \Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{Cot^2\theta}{9} . \frac{1}{Cos\theta}\, d\theta$$

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{9} \frac{Cos^{\cancel{2}}\theta}{Sin^2\theta} \times \frac{1}{\cancel{Cos\theta}}\, d\theta \Rightarrow I = \frac{1}{9} \int_{\pi/6}^{\pi/3} Cosec^2\theta . Cos\theta\, d\theta$$

$$I = \frac{1}{9} \int_{\pi/6}^{\pi/3} Cosec\theta .\, Cot\theta\, d\theta \Rightarrow I = \frac{1}{9} [-Cosec\theta]_{\pi/6}^{\pi/3}$$

$$I = \frac{1}{9} \left\{ Cosec\frac{\pi}{3} - Cosec\frac{\pi}{6} \right\} \Rightarrow I = \frac{-1}{9} \left\{ \frac{2}{\sqrt{3}} - 2 \right\}$$

$$\boxed{I = \frac{-2}{9}\left(\frac{1}{\sqrt{3}} - 1\right)} \text{ Ans}$$

**(10)** $\int_{0}^{\sqrt{5}} x^2\sqrt{5 - x^2}\, dx$

**Solution:** let $I = \int_{0}^{\sqrt{5}} x^2\sqrt{(\sqrt{5})^2 - (x)^2}\, dx$ ———— (1)

let $x = a\, Sin\theta$

$x = \sqrt{5}\, Sin\theta.$

differentiate w•r to θ

$\frac{dx}{d\theta} = \sqrt{5}\, Cos\theta$

$\boxed{dx = \sqrt{5}\, Cos\theta\, d\theta}$

when $\underline{x = \sqrt{5}}$

$x = \sqrt{5}\, Sin\theta \Rightarrow \cancel{\sqrt{5}} = \cancel{\sqrt{5}}\, Sin\theta$

$Sin\theta = 1$

$\theta = Sin^{-1}(1) \Rightarrow \boxed{\theta = \frac{\pi}{2}}$

when $\underline{x = 0}$

$x = \sqrt{5}\, Sin\theta \Rightarrow 0 = \sqrt{5}\, Sin\theta$

$Sin\theta = 0 \Rightarrow$

$\theta = Sin^{-1}(0)$

$\boxed{\theta = 0}$

$$(1) \Rightarrow I = \int_{0}^{\pi/2} 5\, Sin^2\theta \sqrt{5 - 5\, Sin^2\theta} . \sqrt{5}\, Cos\theta\, d\theta$$

$$I = \int_{0}^{\pi/2} 5\, Sin^2\theta \sqrt{5(1 - Sin^2\theta)} . \sqrt{5}\, Cos\theta\, d\theta$$

$$I = \int_{0}^{\pi/2} 5\, Sin^2\theta \sqrt{5} \sqrt{1 - Sin^2\theta} \sqrt{5}\, Cos\theta\, d\theta$$

$$I = 25 \int_{0}^{\pi/2} Sin^2\theta \sqrt{Cos^{\cancel{2}}\theta}\, Cos\theta\, d\theta$$

$$I = 25 \int_{0}^{\pi/2} Sin^2\theta\, Cos^2\theta\, d\theta$$

$$I = 25\int_0^{\pi/2}\left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right)d\theta$$

$$I = \frac{25}{4}\int_0^{\pi/2}(1-\cos^2 2\theta)\,d\theta$$

$$I = \frac{25}{4}\int_0^{\pi/2}d\theta - \frac{25}{4}\int_0^{\pi/2}\cos^2 2\theta\,d\theta$$

$$I = \frac{25}{4}\int_0^{\pi/2}d\theta - \frac{25}{8}\int_0^{\pi/2}\left(\frac{1+\cos 4\theta}{2}\right)d\theta$$

$$I = \frac{25}{4}\int_0^{\pi/2}d\theta - \frac{25}{8}\int_0^{\pi/2}d\theta - \frac{25}{8}\int_0^{\pi/2}\cos 4\theta\,d\theta$$

$$I = \frac{25}{4}\int_0^{\pi/2}d\theta - \frac{25}{8}\int_0^{\pi/2}d\theta - \frac{25}{8\times 4}\int_0^{\pi/2}\cos 4\theta . 4\,d\theta$$

$$I = \frac{25}{4}[\theta]_0^{\pi/2} - \frac{25}{8}[\theta]_0^{\pi/2} - \frac{25}{32}[\sin 4\theta]_0^{\pi/2}$$

$$I = \frac{25}{4}\left\{\frac{\pi}{2}-0\right\} - \frac{25}{8}\left\{\frac{\pi}{2}-0\right\} - \frac{25}{32}\left\{\sin 4\left(\frac{\pi}{2}\right) - \sin 4(0)\right\}$$

$$I = \frac{25}{4}\left(\frac{\pi}{2}\right) - \frac{25}{8}\left(\frac{\pi}{2}\right) - \frac{25}{32}\{0-0\} \Rightarrow I = \frac{25\pi}{8} - \frac{25\pi}{16}$$

$$I = \frac{50\pi - 25\pi}{16} \Rightarrow \boxed{I = \frac{25\pi}{16}} \text{ Ans}$$

**Determine the following integral.**

**Q11.** $\int \frac{x^3\,dx}{\sqrt{a^2-x^2}}$

**Solution:** let $I = \int \frac{x^3\,dx}{\sqrt{a^2-x^2}}$ ———— (1)

let $x = a\sin\theta$

differentiate w • r to θ

$$\frac{dx}{d\theta} = a\cos\theta$$

$$\boxed{dx = a\cos\theta\,d\theta}$$

$$(1) \Rightarrow I = \int \frac{a^3\sin^3\theta.\,a\cos\theta\,d\theta}{\sqrt{a^2 - a^2\sin^2\theta}} \Rightarrow I = \int \frac{a^4\sin^3\theta\cos\theta\,d\theta}{\sqrt{a^2(1-\sin^2\theta)}}$$

$$I = \int \frac{a^4\sin^3\theta\cos\theta\,d\theta}{a\sqrt{1-\sin^2\theta}} \Rightarrow I = \int \frac{a^3\sin^3\theta\cos\theta\,d\theta}{\sqrt{\cos^2\theta}}$$

$$I = \int \frac{a^3\sin^3\theta\,\cancel{\cos\theta}\,d\theta}{\cancel{\cos\theta}} \Rightarrow I = a^3\int \sin^3\theta\,d\theta$$



$$I = a^3\int \sin^2\theta.\,\sin\theta\,d\theta \Rightarrow I = a^3\int(1-\cos^2\theta)\,\sin\theta\,d\theta$$

$$I = a^3\int \sin\theta\,d\theta - a^3\int \cos^2\theta\sin\theta\,d\theta$$

$$I = a^3[-\cos\theta] + a^3\left[\frac{\cos^3\theta}{3}\right] + c$$

$$I = a^3\left\{-\cos\theta + \frac{\cos^3\theta}{3}\right\} + c \text{ ———— (1)}$$

let $x = a\sin\theta$

Squaring on b • s

$$x^2 = a^2\sin^2\theta \Rightarrow \frac{x^2}{a^2} = \sin^2\theta \Rightarrow \frac{x^2}{a^2} = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \frac{x^2}{a^2} \Rightarrow \cos^2\theta = \frac{a^2-x^2}{a^2} \Rightarrow \cos\theta = \frac{\sqrt{a^2-x^2}}{a}$$

$$(1) \Rightarrow I = a^3\left\{-\frac{\sqrt{a^2-x^2}}{a} + \frac{1}{3}\left(\frac{\sqrt{a^2-x^2}}{a}\right)^3\right\} + c$$

$$I = a^3\left\{-\frac{\sqrt{a^2-x^2}}{a} + \frac{1}{3a^3}(a^2-x^2)^{3/2}\right\} + c$$

$$\boxed{I = \frac{1}{3}(a^2-x^2)^{3/2} - a^2\sqrt{a^2-x^2} + c} \text{ Ans}$$

**Q12.** $\int \frac{x^3\,dx}{\sqrt{2x^2+7}}$

**Solution:** let $I = \int \frac{x^3\,dx}{\sqrt{2x^2+7}}$ ———— (1)

$$\therefore \sqrt{2x^2+7} = \sqrt{(\sqrt{2}\,x)^2 + (\sqrt{7})^2}$$

$$\therefore \sqrt{x^2+a^2} \Rightarrow x = a\tan\theta \Rightarrow \sqrt{2}\,x = \sqrt{7}\tan\theta \Rightarrow x = \frac{\sqrt{7}}{\sqrt{2}}\tan\theta$$

differentiate w • r to θ

$$\frac{dx}{d\theta} = \frac{\sqrt{7}}{\sqrt{2}}\sec^2\theta \Rightarrow \boxed{dx = \frac{\sqrt{7}}{\sqrt{2}}\sec^2\theta\,d\theta}$$

$$(1) \Rightarrow I = \frac{\int \frac{7\sqrt{7}}{2\sqrt{2}}\tan^3\theta \cdot \frac{\sqrt{7}}{\sqrt{2}}\sec^2\theta\,d\theta}{\sqrt{2\left(\frac{7}{2}\tan^2\theta\right) + 7}}$$

$$I = \frac{49}{4}\int \frac{\tan^3\theta\sec^2\theta\,d\theta}{\sqrt{7(1+\tan^2\theta)}} \Rightarrow I = \frac{49}{4}\int \frac{\tan^3\theta.\sec^2\theta\,d\theta}{\sqrt{7}\sqrt{1+\tan^2\theta}}$$

$$I = \frac{49}{4}\int \frac{\tan^3\theta.\sec^2\theta\, d\theta}{\sqrt{7}\sqrt{\sec^{\cancel{4}}\theta}} \Rightarrow I = \frac{49}{\frac{4}{\sqrt{7}}}\int \frac{\tan^3\theta.\sec^{\cancel{4}}\theta\, d\theta}{\cancel{\sec\theta}}$$

$$I = \frac{49}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \int \tan^2\theta \bullet \tan\theta.\sec\theta\, d\theta$$

$$I = \frac{49\sqrt{7}}{4\times 7}\int (\sec^2\theta - 1)\tan\theta \sec\theta\, d\theta$$

$$I = \frac{7\sqrt{7}}{4}\int \sec^2\theta.\sec\theta \tan\theta\, d\theta - \frac{7\sqrt{7}}{4}\int \sec\theta \tan\theta\, d\theta$$

$$I = \frac{7\sqrt{7}}{4}\int\left(\frac{\sec^3\theta}{3}\right) - \frac{49}{4\sqrt{7}}(\sec\theta) + c$$

$$I = \frac{7\sqrt{7}}{4}\left\{\frac{\sec^3\theta}{3} - \sec\theta\right\} + c \text{ ——— } (1)$$

$\therefore \sqrt{2}\,x = \sqrt{7}\tan\theta$

Squaring on b • S

$2x^2 = 7\tan^2\theta$

$$\frac{2x^2}{7} = \sec^2\theta - 1 \Rightarrow \sec^2\theta = \frac{2x^2}{7} + 1 \Rightarrow \sec^2\theta = \frac{2x^2+7}{7}$$

$$\sec\theta = \frac{\sqrt{2x^2+7}}{\sqrt{7}}$$

$$(1) \Rightarrow I = \frac{7\sqrt{7}}{4}\left\{\left[\frac{\sqrt{2x^2+7}}{\sqrt{7}}\right]^3 - \frac{\sqrt{2x^2+7}}{\sqrt{7}}\right\} + c$$

$$I = \frac{7\sqrt{7}}{4}\left\{\frac{1}{3}\left(\frac{2x^2+7}{7}\right)^{3/2} - \left(\frac{2x^2+7}{7}\right)^{1/2}\right\} + c \text{ Ans}$$

**Q13.** $\int u^3\sqrt{a^2u^2+b^2}\,du$.

**Solution:** let $I = \int u^3\sqrt{a^2u^2+b^2}\,du$.

$$I = \int u^3\sqrt{a^2\left(u^2+\frac{b^2}{a^2}\right)}\,du.$$

$$I = a\int u^3\sqrt{(u)^2+\left(\frac{b}{a}\right)^2}\,du. \text{ ——— } (1)$$

let $u = a\tan\theta$

$u = \frac{b}{a}\tan\theta$

differentiate w • r to θ

$$\frac{du}{d\theta} = \frac{b}{a}\sec^2\theta \Rightarrow \boxed{du = \frac{b}{a}\sec^2\theta\, d\theta}$$



$$(1) \Rightarrow I = \cancel{a}\int \frac{b^2}{a^{\cancel{3}}}\tan^3\theta\sqrt{\frac{b^2}{a^2}\tan^2\theta + \frac{b^2}{a^2}} \bullet \left(\frac{b}{a}\sec^2\theta\, d\theta\right)$$

$$I = \frac{b^3}{a^2}\int \tan^3\theta\sqrt{\frac{b^2}{a^2}(\tan^2\theta+1)}\;\frac{b}{a}\sec^2\theta\, d\theta$$

$$I = \frac{b^3}{a^2}\int \tan^3\theta\,\frac{b}{a}\sqrt{1+\tan^2\theta}\;\frac{b}{a}\sec^2\theta.d\theta$$

$$I = \frac{b^5}{a^4}\int \tan^3\theta\sqrt{\sec^{\cancel{4}}\theta}\;\sec^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int \tan^3\theta.\sec\theta.\sec^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int \tan^2\theta.\tan\theta.\sec^2\theta.\sec\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int (\sec^2\theta - 1)\sec^2\theta.\sec\theta\tan\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int (\sec^4\theta.\sec\theta\tan\theta - \sec^2\theta.\sec\theta\tan\theta)\, d\theta$$

$$I = \frac{b^5}{a^4}\int \sec^4\theta.\sec\theta\tan\theta\, d\theta - \frac{b^5}{a^4}\int \sec^2\theta.\sec\theta\tan\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int\left(\frac{\sec^5\theta}{5}\right) - \frac{b^5}{a^4}\left(\frac{\sec^3\theta}{3}\right) + c$$

$$I = \frac{b^5}{a^4}\left\{\frac{\sec^5\theta}{5} - \frac{\sec^3\theta}{3}\right\} + c \text{ ——— } (2)$$

$\therefore u = \frac{b}{a}\tan\theta$

Squaring on b • s

$$u^2 = \frac{b^2}{a^2}\tan^2\theta \Rightarrow \frac{a^2u^2}{b^2} = \sec^2\theta - 1 \Rightarrow \sec^2\theta = \frac{a^2u^2}{b^2} + 1$$

$$\sec^2\theta = \frac{a^2u^2+b^2}{b^2} \Rightarrow \boxed{\sec\theta\sqrt{\frac{a^2u^2}{b^2}+1}}$$

$$(1) \Rightarrow I = \frac{b^5}{a^4}\left\{\frac{1}{5}\left(\sqrt{\frac{a^2u^2}{b^2}+1}\right)^5 - \frac{1}{3}\left(\sqrt{\frac{a^2u^2}{b^2}+1}\right)^3\right\} + c$$

$$I = \frac{b^5}{a^4}\left\{\frac{1}{5}\left(\frac{a^2u^2+b^2}{b^2}\right)^{5/2} - \frac{1}{3}\left(\frac{a^2u^2+b^2}{b^2}\right)^{3/2}\right\} + c$$

$$I = \frac{b^5}{a^4}\left\{\frac{1}{5}\frac{(a^2u^2+b^2)^{5/2}}{b^5} - \frac{1}{3}\frac{(a^2u^2+b^2)^{3/2}}{b^3}\right\} + c$$

$$\boxed{I = \frac{1}{5a^4}\left\{(a^2u^2+b^2)^{5/2} - \frac{1}{3a^4}(a^2u^2+b^2)^{3/2}\right\} + c} \text{ Ans}$$

**(14)** $\int \frac{\sqrt{x^2-a^2}}{x}dx$

**Solution:** let $I = \int \frac{\sqrt{x^2-a^2}}{x} dx$ ———— (1)

$\therefore \sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

differentiate w • r to θ

$\frac{dx}{d\theta} = a\,\text{Sec}\theta\tan\theta$

$\boxed{dx = a\,\text{Sec}\theta\tan\theta\, d\theta}$

$(1)\Rightarrow I = \int \frac{\sqrt{a^2\text{Sec}^2\theta - a^2}}{\cancel{a}\,\cancel{\text{Sec}\theta}}\cancel{a}\,\cancel{\text{Sec}\theta}\tan\theta\, d\theta$

$I = \int\sqrt{a^2(\text{Sec}^2\theta-1)}\tan\theta\, d\theta \Rightarrow I = a\int\sqrt{\text{Sec}^2\theta-1}\tan\theta\, d\theta$

$I = a\int\sqrt{\tan^2\theta}\tan\theta\, d\theta \Rightarrow I = a\int\tan^2\theta\, d\theta$

$I = a\int(\text{Sec}^2\theta-1)d\theta \Rightarrow I = a\int\text{Sec}^2\theta d\theta - a\int d\theta$

$I = a\tan\theta - a\theta + c$ ———— (2)

$\therefore \sqrt{x^2-a^2} = a\tan\theta$

$\boxed{\tan\theta = \sqrt{\frac{x^2-a^2}{a^2}}}$

$1 + \tan^2\theta = \text{Sec}^2\theta \Rightarrow 1 + \left(\sqrt{\frac{x^2-a^2}{a^2}}\right)^2 = \text{Sec}^2\theta$

$1 + \frac{x^2-a^2}{a^2} = \text{Sec}^2\theta \Rightarrow \frac{\cancel{a^2}+x^2-\cancel{a^2}}{a^2} = \text{Sec}^2\theta$

$\text{Sec}^2\theta = \frac{x^2}{a^2} \Rightarrow \text{Sec}\theta = \frac{x}{a} \Rightarrow \boxed{\theta = \text{Sec}^{-1}\left(\frac{x}{a}\right)}$ Ans

$(1)\Rightarrow I = a\sqrt{\frac{x^2-a^2}{a^2}} - a\,\text{Sec}^{-1}\left(\frac{x}{a}\right) + c$

$I = \cancel{a}\frac{\sqrt{x^2-a^2}}{\cancel{a}} - a\,\text{Sec}^{-1}\left(\frac{x}{a}\right) + c$

$\boxed{I = (x^2-a^2)^{1/2} - a\,\text{Sec}^{-1}\left(\frac{x}{a}\right) + c}$ Ans

**15.** $\int\frac{du}{u^2\sqrt{a^2-u^2}}$

**Solution:** let $I = \int\frac{du}{u^2\sqrt{a^2-u^2}}$ ———— (1)



let $u = a\,\text{Sin}\theta$

differentiate w • r to θ

$\frac{du}{d\theta} = a\,\text{Cos}\theta \Rightarrow \boxed{du = a\,\text{Cos}\theta d\theta}$

$(1)\Rightarrow I = \int\frac{\cancel{a}\,\text{Cos}\theta\, d\theta}{a^{\cancel{2}}\,\text{Sin}^2\theta\sqrt{a^2-a^2\sin^2\theta}} \Rightarrow I = \frac{1}{a}\int\frac{\text{Cos}\theta\, d\theta}{\sqrt{a^2(1-\text{Sin}^2\theta)}}$

$I = \frac{1}{a}\int\frac{\text{Cos}\theta\, d\theta}{\text{Sin}^2\theta\sqrt{1-\text{Sin}^2\theta}} \Rightarrow I = \frac{1}{a^2}\int\frac{\text{Cos}\theta\, d\theta}{\text{Sin}^2\theta\sqrt{\text{Cos}^2\theta}}$

$I = \frac{1}{a^2}\int\frac{\cancel{\text{Cos}\theta}\, d\theta}{\text{Sin}^2\theta\,\cancel{\text{Cos}\theta}} \Rightarrow I = \frac{1}{a^2}\int\text{Cosec}^2\theta\, d\theta$

$I = \frac{1}{a^2}(-\text{Cot}\theta) + c$ ———— (2)

$\therefore u = a\,\text{Sin}\theta \Rightarrow \boxed{\text{Sin}\theta = \frac{u}{a}}$

$\therefore \text{Sin}^2\theta + \text{Cos}^2\theta = 1$

$\left(\frac{u}{a}\right)^2 + \text{Cos}^2\theta = 1$

$\text{Cos}^2\theta = 1 - \frac{u^2}{a^2} \Rightarrow \text{Cos}^2\theta = \frac{a^2-u^2}{a^2} \Rightarrow \boxed{\text{Cos}\theta = \frac{\sqrt{a^2-u^2}}{a}}$

$\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta} = \frac{\frac{\sqrt{a^2-u^2}}{\cancel{a}}}{\frac{u}{\cancel{a}}} = \frac{\sqrt{a^2-u^2}}{u}$

$(2)\Rightarrow \boxed{I = \frac{-1}{a^2}\frac{\sqrt{a^2-u^2}}{u} + c}$ Ans

**(16)** $\int\frac{dy}{y^2\sqrt{y^2-a^2}}$

**Solution:** let $I = \int\frac{dy}{y^2\sqrt{y^2-a^2}}$ ———— (1)

let $y = a\,\text{Sec}\theta$

differentiate w • r to θ

$\frac{dy}{d\theta} = a\,\text{Sec}\theta\tan\theta$

$\boxed{dy = a\,\text{Sec}\theta\tan\theta\, d\theta}$

$(1)\Rightarrow I = \int\frac{\cancel{a}\,\cancel{\text{Sec}\theta}\tan\theta\, d\theta}{a^{\cancel{2}}\,\text{Sec}^{\cancel{2}}\theta\sqrt{a^2\text{Sec}^2\theta - a^2}}$

$$I = \int \frac{\tan\theta\, d\theta}{a \operatorname{Sec}\theta \sqrt{a^2(\operatorname{Sec}^2\theta - 1)}} \Rightarrow I = \frac{1}{a^2}\int \frac{\tan\theta\, d\theta}{\operatorname{Sec}\theta\sqrt{\operatorname{Sec}^2\theta - 1}}$$

$$I = \frac{1}{a^2}\int \frac{\tan\theta\, d\theta}{\operatorname{Sec}\theta\sqrt{\tan^2\theta}} \Rightarrow I = \frac{1}{a^2}\int \frac{\cancel{\tan\theta}\, d\theta}{\operatorname{Sec}\theta\, \cancel{\tan\theta}}$$

$$I = \frac{1}{a^2}\int \operatorname{Cos}\theta\, d\theta \Rightarrow I = \frac{1}{a^2}\operatorname{Sin}\theta + c \text{ ———— (2)}$$

$\therefore$ $y = a \operatorname{Sec}\theta$

$$\frac{y}{a} = \operatorname{Sec}\theta \Rightarrow \frac{a}{y} = \frac{1}{\operatorname{Sec}\theta} \Rightarrow \boxed{\operatorname{Cos}\theta = \frac{a}{y}}$$

$\therefore$ $\operatorname{Cos}^2\theta + \operatorname{Sin}^2\theta = 1$

$$\frac{a^2}{y^2} + \operatorname{Sin}^2\theta = 1$$

$$\operatorname{Sin}^2\theta = 1 - \frac{a^2}{y^2} \Rightarrow \operatorname{Sin}^2\theta = \frac{y^2 - a^2}{y^2} \Rightarrow \boxed{\operatorname{Sin}\theta = \frac{\sqrt{y^2 - a^2}}{y}}$$

$$(2) \Rightarrow \boxed{I = \frac{1}{a^2}\left(\frac{\sqrt{y^2 - a^2}}{y}\right) + c} \text{ Ans}$$

**(17)** $\int \frac{dx}{(a^2 - x^2)^3}$

**Solution:** let $I = \int \frac{dx}{\sqrt{(a^2 - x^2)^3}}$ ———— (1)

let $x = a \operatorname{Sin}\theta$

differentiate w • r to $\theta$

$$\frac{dx}{d\theta} = a \operatorname{Cos}\theta$$

$\boxed{dx = a \operatorname{Cos}\theta\, d\theta}$

$$(1) \Rightarrow I = \int \frac{a \operatorname{Cos}\theta\, d\theta}{(a^2 - a^2 \operatorname{Sin}^2\theta)^{3/2}} \Rightarrow I = \int \frac{\cancel{a} \operatorname{Cos}\theta\, d\theta}{a^{\cancel{3}}(1 - \operatorname{Sin}^2\theta)^{3/2}}$$

$$I = \frac{1}{a^2}\int \frac{\operatorname{Cos}\theta\, d\theta}{(\operatorname{Cos}^2\theta)^{3/2}} \Rightarrow I = \frac{1}{a^2}\int \frac{\cancel{\operatorname{Cos}\theta}\, d\theta}{\operatorname{Cos}^3\theta}$$

$$I = \frac{1}{a^2}\int \frac{1}{\operatorname{Cos}^2\theta} d\theta \Rightarrow I = \frac{1}{a^2}\int \operatorname{Sec}^2\theta\, d\theta$$

$$I = \frac{1}{a^2}\tan\theta + c \text{ ———— (2)}$$

$$\therefore x = a \operatorname{Sin}\theta \Rightarrow \boxed{\operatorname{Sin}\theta = \frac{x}{a}}$$



$$\tan\theta = \frac{\operatorname{Sin}\theta}{\operatorname{Cos}\theta} \Rightarrow \tan\theta = \frac{\frac{x}{\cancel{a}}}{\frac{\sqrt{a^2 - x^2}}{\cancel{a}}} \Rightarrow \tan\theta = \frac{x}{\sqrt{a^2 - x^2}}$$

Consider $\operatorname{Sin}\theta = \frac{x}{a}$

Squaring on b • s

$$\operatorname{Sin}^2\theta = \frac{x^2}{a^2} \Rightarrow 1 - \operatorname{Sin}^2\theta = 1 - \frac{x^2}{a^2} \Rightarrow \operatorname{Cos}^2\theta = \frac{a^2 - x^2}{a^2}$$

$$\boxed{\operatorname{Cos}\theta = \frac{\sqrt{a^2 - x^2}}{a}}$$

$$(2) \Rightarrow \boxed{I = \frac{1}{a^2}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + c} \text{ Ans}$$

**(18)** $\int \frac{dx}{x^4\sqrt{a^2 - x^2}}$

**Solution:** let $I = \int \frac{dx}{x^4\sqrt{a^2 - x^2}}$ ———— (1)

let $x = a \operatorname{Sin}\theta$

differentiate w • r to $\theta$

$$\frac{dx}{d\theta} = a \operatorname{Cos}\theta$$

$\boxed{dx = a \operatorname{Cos}\theta\, d\theta}$

$$(1) \Rightarrow I = \int \frac{\cancel{a} \operatorname{Cos}\theta\, d\theta}{a^4 \operatorname{Sin}^4\theta\sqrt{a^2 - a^2 \operatorname{Sin}^2\theta}} \Rightarrow I = \frac{1}{a^3}\int \frac{\operatorname{Cos}\theta\, d\theta}{\operatorname{Sin}^4\theta\sqrt{a^2(1 - \operatorname{Sin}^2\theta)}}$$

$$I = \frac{1}{a^4}\int \frac{\operatorname{Cos}\theta\, d\theta}{\operatorname{Sin}^4\theta\sqrt{1 - \operatorname{Sin}^2\theta}} \Rightarrow I = \frac{1}{a^4}\int \frac{\operatorname{Cos}\theta\, d\theta}{\operatorname{Sin}^4\theta\sqrt{\operatorname{Cas}^2\theta}}$$

$$I = \frac{1}{a^4}\int \frac{\cancel{\operatorname{Cos}\theta}\, d\theta}{\operatorname{Sin}^4\theta . \cancel{\operatorname{Cos}\theta}} \Rightarrow I = \frac{1}{a^4}\int \operatorname{Cosec}^4\theta\, d\theta$$

$$I = \frac{1}{a^4}\int \operatorname{Cosec}^2\theta . \operatorname{Cosec}^2\theta\, d\theta$$

$$I = \frac{1}{a^4}\int (1 + \operatorname{Cot}^2\theta) \operatorname{Cosec}^2\theta\, d\theta$$

$$I = \frac{1}{a^4}\int \operatorname{Cosec}^2\theta\, d\theta + \frac{1}{a^4}\int \operatorname{Cot}^2\theta . \operatorname{Cosec}^2\theta\, d\theta$$

$$I = \frac{1}{a^4}\int \operatorname{Cosec}^2\theta\, d\theta - \frac{1}{a^4}\int \operatorname{Cot}^2\theta . (-\operatorname{Cosec}^2\theta)\, d\theta$$

$$I = \frac{1}{a^4}\int(-\text{Cot}\theta) - \frac{1}{a^4}\left(\frac{\text{Cot}^3\theta}{3}\right) + c \text{ ———— (2)}$$

$\therefore\ x = a\,\text{Sin}\theta \Rightarrow \text{Sin}\theta = \frac{x}{a}$

$$\text{Cot}\theta = \frac{\text{Cos}\theta}{\text{Sin}\theta} \Rightarrow \text{Cot}\theta = \frac{\frac{\sqrt{a^2-x^2}}{\cancel{a}}}{\frac{x}{\cancel{a}}} \Rightarrow \text{Cot}\theta = \frac{\sqrt{a^2-x^2}}{x}$$

Consider $\text{Sin}\theta = \frac{x}{a}$

Squaring on b • s

$\text{Sin}^2\theta = \frac{x^2}{a^2}$

$$1 - \text{Sin}^2\theta = 1 - \frac{x^2}{a^2} \Rightarrow \text{Cos}^2\theta = \frac{a^2-x^2}{a^2} \Rightarrow \boxed{\text{Cos}\theta = \frac{\sqrt{a^2-x^2}}{a}}$$

$$(2) \Rightarrow \boxed{I = \frac{-1}{a^4}\left\{\frac{1}{3}\,\frac{(a^2-x^2)^{3/2}}{x^3} + \frac{(a^2-x^2)^{1/2}}{x}\right\} + c} \text{ Ans}$$

**19.** $\int \frac{dx}{(a^2+x^2)^2}$

**Solution:** let $I = \int \frac{dx}{(a^2+x^2)^2}$ ———— (1)

let = $x = a\tan\theta$

differentiate w • r to $\theta$

$\frac{dx}{d\theta} = a\,\text{Sec}^2\theta$

$\boxed{dx = a\,\text{Sec}^2\theta\, d\theta}$

$$(1) \Rightarrow I = \int \frac{a\,\text{Sec}^2\theta\ d\theta}{(a^2 + a^2\tan^2\theta)^2} \Rightarrow I = \int \frac{\cancel{a}\,\text{Sec}^2\theta\ d\theta}{a^{\cancel{4}}(1+\tan^2\theta)^2}$$

$$I = \frac{1}{a^3}\int \frac{\text{Sec}^2\theta\, d\theta}{(\text{Sec}^2\theta)^2} \Rightarrow I = \frac{1}{a^3}\int \frac{\text{Sec}^{\cancel{2}}\theta\, d\theta}{\text{Sec}^{\cancel{4}}\theta} \Rightarrow I = \frac{1}{a^3}\int \frac{1}{\text{Sec}^2\theta}\, d\theta$$

$$I = \frac{1}{a^3}\int \text{Cos}^2\theta\ d\theta. \Rightarrow I = \frac{1}{a^3}\int\left(\frac{1+\text{Cos}2\theta}{2}\right)d\theta$$

$$I = \frac{1}{2a^3}\int d\theta + \frac{1}{2a^3}\int \text{Cos}2\theta.\, d\theta$$

$$I = \frac{1}{2a^3}\int d\theta + \frac{1}{4a^3}\int \text{Cos}2\theta.\, 2\, d\theta$$



$$I = \frac{1}{2a^3}(\theta) + \frac{1}{4a^3}(\text{Sin}2\theta) + c \text{ ———— (2)}$$

$\therefore\ x = a\tan\theta$

$$\tan\theta = \frac{x}{a} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{x}{a}\right)}$$

$$1 + \tan^2\theta = \text{Sec}^2\theta \Rightarrow 1 + \frac{x^2}{a^2} = \text{Sec}^2\theta$$

$$\frac{a^2+x^2}{a^2} = \text{Sec}^2\theta \Rightarrow \text{Sec}\theta = \frac{\sqrt{a^2+x^2}}{a} \Rightarrow \boxed{\text{Cos}\theta = \frac{a}{\sqrt{a^2+x^2}}}$$

$\therefore\ 1 + \text{Cot}^2\theta = \text{Cosec}^2\theta$

$1 + \frac{a^2}{x^2} = \text{Cosec}^2\theta$

$$\frac{x^2+a^2}{x^2} = \text{Cosec}^2\theta \Rightarrow \text{Cosec}\theta = \frac{\sqrt{x^2+a^2}}{a}$$

$$\boxed{\text{Sin}\theta = \frac{x}{\sqrt{x^2+a^2}}}$$

$$(2) \Rightarrow I = \frac{1}{2a^3}\theta + \frac{1}{_2\cancel{4}a^3}(\cancel{2}\text{Sin}\theta\,\text{Cos}\theta) + c$$

$$I = \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{2a^3}\left\{\frac{x}{\sqrt{x^2+a^2}} \times \frac{a}{\sqrt{x^2+a^2}}\right\} + c$$

$$\boxed{I = \frac{1}{2a^3}\left\{\frac{ax}{x^2+a^2} + \tan^{-1}\left(\frac{x}{a}\right)\right\} + c} \text{ Ans}$$

**(20)** $\int t^3\sqrt{a^2t^2 - b^2}\ dt$

**Solution:** let = $I\ \int t^3\sqrt{a^2t^2-b^2}\ dt$

$$I = \int t^3\sqrt{a^2\left(t^2 - \frac{b^2}{a^2}\right)}\,dt.$$

$$I = a\int\sqrt{(t)^2 - \left(\frac{b}{a}\right)^2}\ t^3\,dt. \text{ ———— (1)}$$

let $t = a\,\text{Sec}\theta$

$t = \frac{b}{a}\ \text{Sec}\theta$

differentiate w • r to $\theta$

$$\frac{dt}{d\theta} = \frac{b}{a}\ \text{Sec}\theta\tan\theta \Rightarrow \boxed{dt = \frac{b}{a}\ \text{Sec}\theta\tan\theta\, d\theta}$$

$$(1)\Rightarrow I = \cancel{a}\int \sqrt{\frac{b^2}{a^2}Sec^2\theta - \frac{b^2}{a^2}}\ \frac{b^3}{a^3}\ Sec^3\theta\ \frac{b}{\cancel{a}}\ Sec\theta \tan\theta\, d\theta$$

$$I = \sqrt{\frac{b^2}{a^2}(Sec^2\theta - 1)}\ \frac{b^4}{a^3} Sec^4\theta . \tan\theta\, d\theta$$

$$I = \int \frac{b}{a}\sqrt{Sec^2\theta - 1}\,\frac{b^4}{a^3}\ Sec^4\theta \tan\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int \sqrt{\tan^2\theta}\ Sec^4\theta \tan\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int Sec^4\theta \tan^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int Sec^2\theta . Sec^2\theta . \tan^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int (1 + \tan^2\theta)\, Sec^2\theta \tan^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int Sec^2\theta \tan^2\theta\, d\theta + \frac{b^5}{a^4}\int \tan^4\theta\, Sec^2\theta\, d\theta$$

$$I = \frac{b^5}{a^4}\int\left(\frac{\tan^3\theta}{3}\right) + \frac{b^5}{a^4}\left(\frac{\tan^5\theta}{5}\right) + c \text{ ——— } (2)$$

$$\therefore t = \frac{b}{a}\ Sec\theta$$

$$\frac{at}{b} = Sec\theta$$

Squaring on b • S

$$\frac{a^2t^2}{b^2} = Sec^2\theta$$

$$1 + \tan^2\theta = Sec^2\theta$$

$$\tan^2\theta = Sec^2\theta - 1$$

$$\tan^2\theta = \frac{a^2t^2}{b^2} - 1 \Rightarrow \tan^2\theta = \frac{a^2t^2 - b^2}{b^2}$$

$$\boxed{\tan\theta = \sqrt{\frac{a^2t^2}{b^2} - 1}}$$

$$(1)\Rightarrow I = \frac{b^5}{a^4}\left\{\frac{1}{3}\left(\sqrt{\frac{a^2t^2}{b^2} - 1}\right)^3\right\} + \frac{b^5}{a^4}\left\{\frac{1}{5}\left(\sqrt{\frac{a^2t^2}{b^2} - 1}\right)^5\right\} + c$$

$$I = \frac{\cancel{b^5}}{3a^4}\,\frac{(a^2t^2 - b^2)^{3/2}}{\cancel{b^3}} + \frac{\cancel{b^5}}{5a^4}\,\frac{(a^2t^2 - b^2)^{5/2}}{\cancel{b^5}} + c$$

$$\boxed{I = \frac{b^2}{3a^4}(a^2t^2 - b^2)^{3/2} + \frac{1}{5a^4}(a^2t^2 - b^2)^{5/2} + c}\ \text{Ans}$$



## EXERCISE # 6.6

**Calculate the following integrals.**

**(1)** $\int \frac{dx}{x^2 + 4x + 5}$

**Solution:** let $I = \int \frac{dx}{x^2 + 4x + 5}$ ——— (1)

$\therefore D(x) = x^2 + 4x + 5$

$D(x) = x^2 + 4x + 4 + 1 \Rightarrow D(x) = (x + 2)^2 + (1)^2$

$(1)\Rightarrow I = \int \frac{dx}{(x + 2)^2 (1)^2}$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c}$$

$I = \frac{1}{1}\cdot \tan^{-1}\frac{(x + 2)}{1} + c$

$\boxed{I = \tan^{-1}(x + 2) + c}$ Ans

**(2)** $\int \frac{dx}{\sqrt{5 + 4x - x^2}}$

**Solution:** let $I = \int \frac{dx}{\sqrt{5 + 4x - x^2}}$ ——— (1)

$\therefore D(x) = 5 + 4x - x^2$

$D(x) = -(x^2 - 4x - 5) \Rightarrow D(x) = -\{x^2 - 4x + 4 - 4 - 5\}$

$D(x) = -\{(x - 2)^2 - 9\} \Rightarrow D(x) = 9 - (x - 2)^2$

$(1)\Rightarrow I = \int \frac{dx}{\sqrt{(3)^2 - (x - 2)^2}}$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{a^2 - x^2}}dx = Sin^{-1}\frac{x}{a} + c}$$

$\boxed{I = Sin^{-1}\frac{(x - 2)}{3} + c}$ Ans

**(3)** $\int \frac{dx}{x^2 - x + 1}$

**Solution:** let $I = \int \frac{dx}{x^2 - x + 1}$ ——— (1)

$\therefore D(x) = x^2 - x + 1$

$D(x) = (x)^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + 1 - \frac{1}{4}$

$$D(x) = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$(1) \Rightarrow I = \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c}$$

$$I = \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1} \frac{\left(x - \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}} + c \Rightarrow \boxed{I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c} \text{ Ans}$$

**(4)** $\int \frac{ds}{\sqrt{4S - S^2}}$

**Solution:** let $I = \int \frac{ds}{\sqrt{4s - s^2}}$ ———— (1)

$\therefore D(S) = 4s - s^2$

$D(S) = -(s^2 - 4s)$

$D(S) = -\{(s)^2 - 2(s)(2) + (2)^2 - (2)^2\}$

$D(S) = -\{(s-2)^2 - (2)^2\}$

$D(S) = (2)^2 - (s-2)^2$

$$(1) \Rightarrow I = \int \frac{ds}{\sqrt{(2)^2 - (s-2)^2}}$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{\sqrt{a^2 - x^2}} dx = \text{Sin}^{-1} \frac{x}{a} + c}$$

$$\boxed{I = \text{Sin}^{-1}\left(\frac{s}{2} - 1\right) + c} \quad \text{Ans}$$

**(5)** $\int \frac{dx}{x^2 + 6x + 8}$

**Solution:** let $I = \int \frac{dx}{x^2 + 6x + 8}$ ———— (1)

$\therefore D(x) = x^2 + 6x + 8$

$D(x) = (x)^2 + 2(x)(3) + (3)^2 + 8 - 9$

$D(x) = (x+3)^2 - (1)^2$

$$(1) \Rightarrow I = \int \frac{dx}{(x+3)^2 - (1)^2}$$



$$\boxed{\text{Using formula} \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + c}$$

$$I = \frac{1}{2(1)} \ln\left(\frac{x+3-1}{x+3+1}\right) + c \Rightarrow \boxed{I = \frac{1}{2} \ln\left(\frac{x+2}{x+4}\right) + c} \text{ Ans}$$

**(6)** $\int \frac{dx}{4x - x^2}$

**Solution:** let $I = \int \frac{dx}{4x - x^2}$ ———— (1)

$\therefore D(x) = 4x - x^2$

$D(x) = -(x^2 - 4x)$

$D(x) = -\{(x)^2 - 2(x)(2) + (2)^2 - (2)^2\}$

$D(x) = -\{(x-2)^2 - (2)^2\}$

$D(x) = (2)^2 - (x-2)^2$

$$(1) \Rightarrow I = \int \frac{dx}{(2)^2 - (x-2)^2}$$

$$\boxed{\text{Using formula } \therefore \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + c}$$

$$I = \frac{1}{2(2)} \ln\left(\frac{\cancel{2} + x - \cancel{2}}{2 - x + 2}\right) + c \Rightarrow \boxed{I = \frac{1}{4} \ln\left(\frac{x}{4-x}\right) + c} \text{ Ans}$$

**(7)** $\int \frac{dx}{(x+2)\sqrt{x^2 + 4x + 3}}$

**Solution:** let $I = \int \frac{dx}{(x+2)\sqrt{x^2 + 4x + 3}}$ ———— (1)

$\therefore$ $D(x) = x^2 + 4x + 3$

$D(x) = (x)^2 + 2(x)(2) + (2)^2 + 3 - 4$

$D(x) = (x+2)^2 - (1)^2$

$$(1) \Rightarrow I = \frac{dx}{(x+2)\sqrt{(x+2)^2 - (1)^2}}$$

$$\boxed{\text{Using formula} \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \text{Sec}^{-1} \frac{x}{a} + c}$$

$$I = \frac{1}{1} \text{Sec}^{-1} \frac{(x+2)}{1} + c \Rightarrow \boxed{I = \text{Sec}^{-1}(x+2) + c} \text{ Ans}$$

**(8)** $\int \frac{dx}{(x-1)\sqrt{x^2 - 2x - 3}}$

**Solution:** let $I = \int \frac{dx}{(x-1)\sqrt{x^2 - 2x - 3}}$ ———— (1)

$\therefore D(x) = x^2 - 2x - 3$

$D(x) = (x)^2 - 2(x)(1) + (1)^2 - 3 - 1$

$D(x) = (x-1)^2 - (2)^2$

$$(1) \Rightarrow I = \int \frac{dx}{(x-1)\sqrt{(x-1)^2 - (2)^2}}$$

$$\boxed{\text{Using formula } \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\,\mathrm{Sec}^{-1}\frac{x}{a} + c}$$

$$\boxed{I = \frac{1}{2}\,\mathrm{Sec}^{-1}\left(\frac{x-1}{2}\right) + c} \text{ Ans}$$

**(9)** $\displaystyle\int \frac{dx}{\sqrt{5 - 2x + x^2}}$

**Solution:** let $I = \displaystyle\int \frac{dx}{\sqrt{5 - 2x + x^2}}$ ———— (1)

$\therefore D(x) = 5 - 2x + x^2$

$D(x) = x^2 - 2x + 5 \Rightarrow D(x) = (x)^2 - 2(x)(1) + (1)^2 + 5 - 1$

$D(x) = (x-1)^2 - (2)^2$

$$(1) \Rightarrow I = \int \frac{dx}{\sqrt{(x-1)^2 + (2)^2}}$$

$$\boxed{\text{Using formula } \int \frac{1}{\sqrt{x^2 + a^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right) + c}$$

$$I = \ln\left\{(x-1) + \sqrt{(x-1)^2 + (2)^2}\right\} + c$$

$$\boxed{I = \ln\left\{(x-1) + \sqrt{x^2 - 2x + 5}\right\} + c} \text{ Ans}$$

**(10)** $\displaystyle\int \frac{(2x-5)\,dx}{\sqrt{4x - x^2}}$

**Solution:** let $I = \displaystyle\int \frac{(2x-5)\,dx}{\sqrt{4x - x^2}}$ ———— (1)

$\therefore$ $D(x) = 4x - x^2 \Rightarrow D(x) = -(x^2 - 4x)$

$D(x) = -\{(x)^2 - 2(x)(2) + (2)^2 - (2)^2\}$

$D(x) = -\{(x-2)^2 - 4\}$

$D(x) = 4 - (2-x)^2 \Rightarrow D(x) = 4 - (2-x)^2$

$$(1) \Rightarrow I = \int \frac{(2x-5)\,dx}{\sqrt{4 - (2-x)^2}} \text{ ———— (2)}$$

let $t = 2 - x$. $\Rightarrow \boxed{x = 2 - t}$

differentiate w.r.t x

$\dfrac{dt}{dx} = -1 \quad \Rightarrow \boxed{-dt = dx}$



$$(1) \Rightarrow I = \int \frac{\{2(2-t) - 5\}(-dt)}{\sqrt{4 - t^2}} \Rightarrow I = \int \frac{(4 - 2t - 5)(-dt)}{\sqrt{4 - t^2}}$$

$$I = \int \frac{(-2t-1)(-dt)}{\sqrt{(2)^2 - (t)^2}} \Rightarrow I = \int \frac{(2t+1)\,dt}{\sqrt{(2)^2 - (t)^2}}$$

$$I = \int \frac{2t\,dt}{\sqrt{4 - t^2}} + \int \frac{dt}{\sqrt{(2)^2 - (t)^2}}$$

$$I = -\int (4 - t^2)^{-1/2}(-2t)\,dt + \int \frac{1}{\sqrt{(2)^2 - (t)^2}}\,dt.$$

$$\boxed{\therefore \int \frac{dx}{\sqrt{a^2 - x^2}} = \mathrm{Sin}^{-1}\left(\frac{x}{a}\right) + c}$$

$$I = \frac{(4 - t^2)^{1/2}}{1/2} + \mathrm{Sin}^{-1}\frac{t}{2} + c \qquad \text{but } t = 2 - x$$

$$I = 2\sqrt{4 - (2-x)^2} + \mathrm{Sin}^{-1}\left(\frac{2-x}{2}\right) + c$$

$$I = 2\sqrt{\cancel{4} - \cancel{4} + 4x - x^2} + \mathrm{Sin}^{-1}\left(1 - \frac{x}{2}\right) + c$$

$$\boxed{I = 2\sqrt{4x - x^2} + \mathrm{Sin}^{-1}\left(1 - \frac{x}{2}\right) + c} \text{ Ans}$$

**(11)** $\displaystyle\int \frac{(x+3)\,dx}{x^2 + 2x + 5}$

**Solution:** let $I = \displaystyle\int \frac{(x+3)\,dx}{x^2 + 2x + 5}$

$\times$ & $\div$ by 2

$$I = \frac{1}{2}\int \frac{2x + 6\,dx}{x^2 + 2x + 5} \text{ ———— (1)}$$

$\therefore D(x) = x^2 + 2x + 5$

$D(x) = (x)^2 + 2(x)(1) + (1)^2 + 5 - 1$

$D(x) = (x+1)^2 + (2)^2$

$$(1) \Rightarrow I = \frac{1}{2}\int \frac{2x + 6\,dx}{(x+1)^2 + (2)^2} \Rightarrow I = \frac{1}{2}\int \frac{(2x + 2 + 4)\,dx}{(x+1)^2 + (2)^2}$$

$$I = \frac{1}{2}\int \frac{2x+2}{x^2 + 2x + 5}\,dx + \frac{1}{\cancel{2}}\int \frac{\cancel{4}^{2}\,dx}{(x+1)^2 + (2)^2}$$

$$I = \frac{1}{2}\int \frac{2x + 2\,dx}{x^2 + 2x + 5} + 2\int \frac{1\,dx}{(x+1)^2 + (2)^2}$$

$$I = \frac{1}{2}\ln(x^2 + 2x + 5) + \cancel{2}\,\frac{1}{\cancel{2}}\tan^{-1}\frac{(x+1)}{2} + c$$

$$\boxed{I = \ln\sqrt{x^2 + 2x + 5} + \tan^{-1}\frac{(x+1)}{2} + c}$$

## EXERCISE # 6.7

**Q1. Integrate by parts the following,**

**(i)** $\int x \ln x \, dx.$

**Solution:** let $I = \int x \ln x \, dx.$

let $u = \ln x$ ; $v = x$

differentiate w • r to x $\int v\,dx = \int x\,dx$

$\boxed{\dfrac{du}{dx} = \dfrac{1}{x}}$ $\boxed{\int v dx = \dfrac{x^2}{2}}$

$\boxed{\text{Using Formula } \int u \bullet v dx = u \int v dx - \int \left\{ \dfrac{du}{dx} \int v\,dx \right\} dx}$

$I = \ln x \dfrac{x^2}{2} - \int \dfrac{1}{\cancel{x}} \bullet \dfrac{x^{\cancel{2}}}{2} dx \Rightarrow I = \ln x \dfrac{x^2}{2} - \dfrac{1}{2} \int x\,dx$

$I = \dfrac{x^2}{2} \ln x - \dfrac{1}{2}\dfrac{x^2}{2} + c \Rightarrow \boxed{I = \dfrac{x^2}{2} \ln x - \dfrac{x^2}{4} + c}$ Ans

**(ii)** $\int x \sin^2 x \cos x \, dx$

**Solution:** let $I = \int x \sin^2 x \cos x \, dx$

let $u = \ln x$ ; $v = \sin^2 x \cos x$

differentiate w • r to x $\int v\,dx = \int \sin^2 x \cos x\,dx$

$\boxed{\dfrac{du}{dx} = 1}$ $\boxed{\int v dx = \dfrac{\sin^3 x}{3}}$

$\boxed{\text{Using Formula } \int u \bullet v du = u \int v dx - \int \left\{ \dfrac{du}{dx} \int v\,dx \right\} dx}$

$I = \dfrac{\sin^3 x}{3} . x - \int (1) \dfrac{\sin^3 x}{3} .dx$

$I = \dfrac{\sin^3 x}{3} . x - \dfrac{1}{3} \int \sin^2 x . \sin x\,dx$

$I = \dfrac{\sin^3 x}{3} . x - \dfrac{1}{3} \int (1 - \cos^2 x) \sin x\,dx$

$I = x \dfrac{\sin^3 x}{3} - \dfrac{1}{3} \int \sin x\,dx + \dfrac{1}{3} \int \cos^2 x \sin x\,dx$

$I = x \dfrac{\sin^3 x}{3} + \dfrac{\cos x}{3} - \dfrac{1}{3} \dfrac{\cos^3 x}{3} + c$

$\boxed{I = \dfrac{1}{3}\left[ x \sin^3 x + \cos x - \dfrac{\cos^3 x}{3} \right] + c}$ Ans



**(iii)** $\int x^2 \tan^{-1} x \, dx.$

**Solution:** let $I = \int x^2 \tan^{-1} x \, dx$

let $u = \tan^{-1} x$ ; $v = x^2$

differentiate w • r to x $\int v\,dx = \int x^2\,dx$

$\boxed{\dfrac{du}{dx} = \dfrac{1}{1+x^2}}$ $\boxed{\int v dx = \dfrac{x^3}{3}}$

$\boxed{\text{Using Formula } \int u \bullet v dx = u \int v dx - \int \left\{ \dfrac{du}{dx} \int v\,dx \right\} dx}$

$I = \dfrac{x^3}{3} \tan^{-1} x - \int \dfrac{1}{1+x^2} \dfrac{x^3}{3} dx$

$I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{1}{3} \int \dfrac{x^3}{1+x^2} dx.$

$$\begin{array}{r|l} & x \\ \hline 1+x^2 & x^3 \\ & \underline{-x^3 \pm x} \\ & -x \end{array}$$

$I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{1}{3} \int \left( x - \dfrac{x}{1+x^2} \right) dx$

$I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{1}{3} \int x\,dx + \dfrac{1}{3} \int \dfrac{x\,dx}{1+x^2}$

$I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{1}{3} \int x\,dx + \dfrac{1}{6} \int \dfrac{2x\,dx}{1+x^2}$

$I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{1}{3} \dfrac{x^2}{3} + \dfrac{1}{6} \ln (1+x^2) + c$

$\boxed{I = \dfrac{x^3}{3} \tan^{-1} x - \dfrac{x^2}{6} + \dfrac{1}{6} \ln (1+x^2) + c}$ Ans

**(iv)** $\int x^3 \tan^{-1} x \, dx.$

**Solution:** let $I = \int x^3 \tan^{-1} x \, dx$

let $u = \tan^{-1} x$ ; $v = x^3$

differentiate w • r to x $\int v\,dx = \int x^3\,dx$

$\boxed{\dfrac{du}{dx} = \dfrac{1}{1+x^2}}$ $\boxed{\int v dx = \dfrac{x^4}{4}}$

$\boxed{\text{Using Formula } \int u \bullet v dx = u \int v dx - \int \left\{ \dfrac{du}{dx} \int v\,dx \right\} dx}$

$I = \dfrac{x^4}{4} \tan^{-1} x - \int \dfrac{1}{1+x^2} \dfrac{x^4}{4} dx$

$$I = \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int \frac{x^4}{1+x^2}\,dx$$

$$\begin{array}{r|c|l} 1+x^2 & x^4 & x^2-1 \\ & \cancel{x^4} \pm x^2 & \\ \hline & -\cancel{x^2} & \\ & -\cancel{x^2} - 1 & \\ & + \quad + & \\ \hline & 1 & \end{array}$$

$$I = \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int\left(x^2 - 1 + \frac{1}{1+x^2}\right)dx$$

$$I = \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int x^2\,dx + \frac{1}{4}\int dx - \frac{1}{4}\int\frac{1}{1+x^2}\,dx$$

$$I = \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\frac{x^3}{3} + \frac{1}{4}x - \frac{1}{4}\tan^{-1}x + c$$

$$\boxed{I = \frac{x^4}{4}\tan^{-1}x - \frac{x^3}{12} + \frac{x}{4} - \frac{1}{4}\tan^{-1}x + c} \text{ Ans}$$

**(v)** $\int x^2 \ln x\, dx.$

**Solution:** let $I = \int x^2 \ln x\, dx.$

let $u = \ln x$ ; $v = x^2$

differentiate w • r to x $\int v\,dx = \int x^2\,dx$

$\boxed{\frac{du}{dx} = \frac{1}{x}}$ $\boxed{\int v dx = \frac{x^3}{3}}$

$$\boxed{\text{Using Formula } \int u \bullet v dx = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx}$$

$$I = \frac{x^3}{3}\ln x - \int \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{3}}}{3}\,dx \Rightarrow I = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx$$

$$I = \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3} + c \Rightarrow \boxed{I = \frac{x^3}{3}\ln x - \frac{x^3}{9} + c} \text{ Ans}$$

**(vi)** $\int \tan^{-1}x\, dx.$

**Solution:** let $I = \int \tan^{-1}x\, dx.$

let $u = \tan^{-1}x$ ; $v = dx$

differentiate w • r to x $\int v\,dx = \int dx$

$\boxed{\frac{du}{dx} = \frac{1}{1+x^2}}$ $\boxed{\int v dx = x}$

$$\boxed{\text{Using Formula } \int u \bullet v dx = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx}$$



$$I = x\tan^{-1}x - \int\frac{1}{1+x^2}\cdot x\,dx \Rightarrow I = x\tan^{-1}x - \int\frac{x}{1+x^2}\,dx$$

$$I = x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}\cdot dx$$

$$\boxed{I = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c} \quad \text{Ans}$$

**(vii)** $\int x^4 \sin^2 x\, dx$

**Solution:** let $I = \int x^4 \sin^2 x\, dx$

$$I = \int x^4\left(\frac{1-\cos 2x}{2}\right)dx \Rightarrow I = \frac{1}{2}\int (x^4 - x^4\cos 2x)\,dx$$

$$I = \frac{1}{2}\int x^4\,dx - \frac{1}{2}\int x^4\cos 2x\,dx$$

$$I = \frac{x^5}{10} - \frac{1}{2}\int x^4\cos 2x\,dx \text{ ———— (1)}$$

Using by parts on 2nd integral.

let $u = x^4$ $v = \cos 2x.$

differentiate w • r to x $\int v dx = \frac{1}{2}\int \cos 2x \bullet 2dx$

$\boxed{\frac{du}{dx} = 4x^3}$ $\boxed{\int v dx = \frac{\sin 2x}{2}}$

$$\boxed{\text{Using Formula } \int u \bullet v dx = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx}$$

$$I = \frac{x^4 \sin 2x}{2} - \int {}^2\cancel{4}x^3 \cdot \frac{\sin 2x}{\cancel{2}}\,dx$$

$$I = \frac{x^4\sin 2x}{2} - 2\int x^3\sin 2x\,dx \text{ ———— (2)}$$

**again using by parts**

let $u = x^4$ $v = \sin 2x.$

$\boxed{\frac{du}{dx} = 3x^2}$ $\int v dx = \frac{1}{2}\int \sin 2x \bullet 2dx$

$\boxed{\int v dx = \frac{-\cos 2x}{2}}$

$$I = x^3\left(-\frac{\cos 2x}{2}\right) - \int 3x^2\left(\frac{-\cos 2x}{2}\right)dx$$

$$I = \frac{-x^3\cos 2x}{2} + \frac{3}{2}\int x^2\cos 2x\,dx \text{ ———— (3)}$$

again using by parts

 https://stbmathematics.blogspot.com/



let $u = x^2$ $\qquad$ $v = \text{Cos}2x.$

$\boxed{\dfrac{du}{dx} = 2x}$ $\qquad$ $\int v dx = \dfrac{1}{2}\int \text{Cos}2x \bullet 2dx$

$\boxed{\int v dx = \dfrac{\text{Sin}2x}{2}}$

$$I = \frac{x^2 \text{Sin}2x}{2} - \int \cancel{2}x.\left(\frac{\text{Sin}2x}{\cancel{2}}\right) dx$$

$$I = \frac{x^2 \text{Sin}2x}{2} - \int x \text{ Sin}2x \, dx \text{ ———— } (4)$$

Using again by parts

let $u = x$ $\qquad$ $v = \text{Sin}2x.$

$\boxed{\dfrac{du}{dx} = 1}$ $\qquad$ $\int v dx = \dfrac{1}{2}\int \text{Sin}2x \bullet 2dx$

$\boxed{\int v dx = \dfrac{-\text{Cos}2x}{2}}$

$$I = x\left(\frac{-\text{Cos}2x}{2}\right) - \int (1)\left(\frac{-\text{Cos}2x}{2}\right) dx$$

$$I = \frac{-x \text{ Cos}2x}{2} + \frac{1}{2.2}\int \text{Cos}2x \bullet 2dx$$

$$I = \frac{-x \text{ Cos}2x}{2} + \frac{\text{Sin}2x}{4}$$

put this result in (4)

$$(4) \Rightarrow I = \frac{x^2 \text{ Sin}2x}{2} - \left\{\frac{-x \text{ Cos}2x}{2} + \frac{\text{Sin}2x}{4}\right\}$$

$$I = \frac{x^2 \text{ Sin}2x}{2} + \frac{x \text{ Cos}2x}{2} - \frac{\text{Sin}2x}{4}$$

put this result in (3)

$$(3) \Rightarrow I = \frac{-x^3 \text{ Cos}2x}{2} + \frac{3}{2}\left\{\frac{x^2 \text{ Sin}2x}{2} + \frac{x \text{ Cos}2x}{2} - \frac{\text{Sin}2x}{4}\right\}$$

$$I = \frac{-x^3 \text{ Cos}2x}{2} + \frac{3}{4}x^2 \text{ Sin}2x + \frac{3}{4}x \text{ Cos}2x - \frac{3}{8}\text{Sin}2x$$

put this result in (2)

$$(2) \Rightarrow I = \frac{x^4 \text{ Sin}2x}{2} - 2\left\{\frac{-x^3 \text{ Cos}2x}{2} + \frac{3}{4}x^2 \text{ Sin}2x + \frac{3}{4}x \text{ Cos}2x - \frac{3}{8}\text{ Sin}2x\right\}$$

$$I = \frac{x^4 \text{ Sin}2x}{2} + x^3 \text{ Cos}2x - \frac{3}{2}x^2 \text{ Sin}2x - \frac{3}{2}x \text{ Cos}2x + \frac{3}{4}\text{Sin}2x.$$

put this result in (1)



$$I = \frac{x^5}{10} - \frac{1}{2}\left\{\frac{x^4 \text{ Sin}2x}{2} + x^3 \text{ Cos}2x - \frac{3}{2}x^2 \text{ Sin}2x - \frac{3}{2}x \text{ Cos}2x + \frac{3}{4}\text{Sin}2x\right\} + c$$

$$I = \frac{x^5}{10} - \frac{1}{4}x^4 \text{ Sin}2x - \frac{1}{2}x^3 \text{ Cos}2x + \frac{3}{4}x^2\text{Sin}2x + \frac{3}{4}x \text{ Cos}2x - \frac{3}{8}$$
$\text{Sin}2x + c$

$$I = \frac{x^5}{10} + \text{Sin}2x\left(\frac{3x^2}{4} - \frac{x^4}{4} - \frac{3}{8}\right) + \text{Cos}2x\left(\frac{3x}{4} - \frac{x^3}{2}\right) + c$$

$$I = \frac{x^5}{10} + \text{Sin}2x\left(\frac{6x^2 - 2x^4 - 3}{8}\right) + \text{Cos}2x\left(\frac{3x - 2x^3}{4}\right) + c$$

$$\boxed{I = \frac{x^5}{10} - \frac{1}{8}(2x^4 - 6x^2 + 3)\text{ Sin}2x - \frac{1}{4}(2x^3 - 3x)\text{ Cos}2x + c} \textbf{ Ans}$$

**(viii)** $\int x^2 \text{ Sin}^2 x \, dx.$

**Solution:** let $I = \int x^2 \text{ Sin}^2 x \, dx.$

let $u = x^2$ $\qquad$ ; $v = \text{Sin}^2 x.$

differentiate w – r to x $\qquad$ $\int v dx = \dfrac{1}{2}\int \text{Sin}^2 x \, dx$

$\boxed{\dfrac{du}{dx} = 2x}$ $\qquad$ $\int v dx = \int\left(\dfrac{1 - \text{Cos}2x}{2}\right) dx$

$\int v dx = \dfrac{1}{2}\int dx - \dfrac{1}{2 \times 2}\int \text{Cos}2x.2dx$

$\boxed{\int v dx = \dfrac{1}{2}x - \dfrac{\text{Sin}2x}{4}}$

$$I = u\int v dx - \int\left\{\frac{du}{dx}\int v \, dx\right\} dx$$

$$I = x^2\left(\frac{x}{2} - \frac{\text{Sin}2x}{4}\right) - \int 2x\left(\frac{x}{2} - \frac{\text{Sin}2x}{4}\right) dx$$

$$I = \frac{x^3}{2} - \frac{x^2 \text{ Sin}2x}{4} - \int x^2 dx + \frac{1}{2}\int x \text{ Sin}2x \, dx$$

$$I = \frac{x^3}{2} - \frac{x^2 \text{ Sin}2x}{4} - \frac{x^3}{3} + \frac{1}{2}\int x \text{ Sin}2x \, dx \text{ ———— } (1)$$

Consider $I = \int x \text{ Sin}2x \, dx.$

let $u = x$ $\qquad$ ; $v = \text{Sin}2x.$

differentiate w • r to x $\qquad$ $\int v dx = \int \text{Sin}2x \, dx$

$\boxed{\dfrac{du}{dx} = 1}$ $\qquad$ $\boxed{\int v dx = \dfrac{-\text{Cos}2x}{2}}$

$$I = u\int v dx - \int\left\{\frac{du}{dx}\int v dx\right\} dx$$

 https://stbmathematics.blogspot.com/

$$I = \frac{-x\,\mathrm{Cos}2x}{2} - \int (1)\left(\frac{-\mathrm{Cos}2x}{2}\right)dx$$

$$I = \frac{-x\,\mathrm{Cos}2x}{2} + \frac{1}{2\times 2}\int \mathrm{Cos}2x \bullet 2x\,dx$$

$$I = \frac{-x\,\mathrm{Cos}2x}{2} + \frac{\mathrm{Sin}2x}{4}$$

put this result in (1)

$$(1) \Rightarrow I = \frac{x^3}{2} - \frac{x^2\mathrm{Sin}2x}{4} - \frac{x^3}{2} + \frac{1}{2}\left\{\frac{-x\,\mathrm{Cos}2x}{2} + \frac{\mathrm{Sin}2x}{4}\right\} + c$$

$$\boxed{I = \frac{x^3}{6} - \frac{x^2\,\mathrm{Sin}2x}{4} - \frac{x\,\mathrm{Cos}2x}{4} + \frac{\mathrm{Sin}2x}{8} + c} \text{ Ans}$$

**(ix)** $\int (\ln x)^2\,dx.$

**Solution:** let $I = \int (\ln x)^2\,dx$

let $u = (\ln x)^2$ ; $v = dx$

differentiate w • r to x $\int v dx = \int dx$

$$\boxed{\frac{du}{dx} = 2(\ln x)\frac{1}{x}} \qquad \boxed{\int v dx = x}$$

$$I = uv - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx \Rightarrow I = x(\ln x)^2 - \int\left\{\frac{2(\ln x)}{\cancel{x}}\right\}\cancel{x}\,dx$$

$$I = x(\ln x)^2 - 2\int \ln x\,dx. \text{ ——— (1)}$$

Consider $I = \int \ln x\,dx$

let $u = \ln x$ ; $v = dx$

differentiate w • r to x $\int v dx = \int dx$

$$\boxed{\frac{du}{dx} = \frac{1}{x}} \qquad \boxed{\int v dx = x}$$

$$I = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx \Rightarrow I = x\ln x - \int \frac{1}{\cancel{x}}.\cancel{x}\,dx$$

$$I = x\ln x - \int dx \Rightarrow I = x\ln x - x$$

put this result in (1)

$$(1) \Rightarrow \boxed{I = x(\ln x)^2 - 2x\ln x + 2x + c} \text{ Ans}$$

**(x)** $\int \ln\left(x + \sqrt{x^2+1}\right)dx$

**Solution:** let $I = \int \ln\left(x+\sqrt{x^2+1}\right)dx.$

$u = \ln\left(x + \sqrt{x^2+1}\right)$ ; $v = 1$

differentiate w • r to x $\int v dx = \int dx$

$$\boxed{\int v dx = x}$$

$$\frac{du}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left\{1 + \frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cancel{2}x\right\}$$

$$\frac{du}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left\{1 + \frac{x}{\sqrt{x^2+1}}\right\}$$



$$\frac{du}{dx} = \frac{1}{\cancel{x+\sqrt{x^2+1}}}\left\{\frac{\cancel{\sqrt{x^2+1}+x}}{\sqrt{x^2+1}}\right\} \Rightarrow \boxed{\frac{du}{dx} = \frac{1}{\sqrt{x^2+1}}}$$

$$I = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = x\ln\left(x+\sqrt{x^2+1}\right) - \int \frac{1}{\sqrt{x^2+1}}\,x\,dx$$

$$I = x\ln\left(x+\sqrt{x^2+1}\right) - \int (x^2+1)^{-1/2}.\,x dx$$

$$I = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\int (x^2+1)^{-1/2}.2x\,dx$$

$$I = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{1}{2}\,\frac{\sqrt{x^2+1}}{1/2} + c$$

$$I = x\ln\left(x+\sqrt{x^2+1}\right) - \frac{\cancel{2}}{\cancel{2}}\sqrt{x^2+1} + c$$

$$\boxed{I = x\ln\left(x+\sqrt{x^2+1}\right) - \sqrt{x^2+1} + c} \text{ Ans}$$

**Q2. Use integration by parts to determine the following.**

**(i)** $\int \sqrt{4-x^2}\,dx$

**Solution:** let $I = \int \sqrt{4-x^2}\,dx$ ——— (1)

let $u = (4-x^2)^{1/2}$ ; $v = dx$

differentiate w • r to x $\int v dx = \int dx$

$$\frac{du}{dx} = \frac{1}{\cancel{2}}(4-x^2)^{-1/2}.(-\cancel{2}x) \qquad \boxed{\int v dx = x}$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{4-x^2}}$$

Using formula

$$I = u\int v dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = x\sqrt{4-x^2} - \int\left\{\frac{-x}{\sqrt{4-x^2}}\right\}x\,dx$$

$$I = x\sqrt{4-x^2} - \int \frac{-x^2\,dx}{\sqrt{4-x^2}}$$

$$I = x\sqrt{4-x^2} - \int\left\{\frac{4-x^2-4}{\sqrt{4-x^2}}\right\}dx.$$

$$I = x\sqrt{4-x^2} - \int \frac{(4-x^2)dx}{\sqrt{4-x^2}} + 4\int \frac{dx}{\sqrt{4-x^2}}$$

$$I = x\sqrt{4-x^2} - \int\sqrt{4-x^2}\,dx + 4\int\frac{1\,dx}{\sqrt{4-x^2}} \quad \text{by (1)}$$

$$I = x\sqrt{4-x^2} - I + 4\int\frac{1}{\sqrt{(2)^2-(x)^2}}\,dx$$

$$\boxed{\therefore \frac{1}{\sqrt{a^2-x^2}}\,dx = \text{Sin}^{-1}\frac{x}{a} + c}$$

$$I + I = x\sqrt{4-x^2} + 4\,\text{Sin}^{-1}\left(\frac{x}{2}\right) + c$$

$$2I = x\sqrt{4-x^2} + 4\,\text{Sin}^{-1}\left(\frac{x}{2}\right) + c.$$

$$\boxed{I = \frac{x\sqrt{4-x^2}}{2} + 2\,\text{Sin}^{-1}\left(\frac{x}{2}\right) + c} \quad \text{Ans}$$

**(ii)** $\int\sqrt{4-5x^2}\,dx.$

**Solution:** let $I = \int\sqrt{4-5x^2}\,dx.$ ———— (1)

let $u = (4-5x^2)^{1/2}$ ; $v = dx$

$\int v\,dx = \int dx$

$\frac{du}{dx} = \frac{1}{\cancel{2}}(4-5x^2)^{-1/2}(-\cancel{10}^{5}x)$ ; $\boxed{\int v\,dx = x}$

$$\boxed{\frac{du}{dx} = \frac{-5x}{\sqrt{4-5x^2}}}$$

$$I = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = x\sqrt{4-5x^2} - \int\left\{\frac{-5x}{\sqrt{4-5x^2}}\right\}x\,dx$$

$$I = x\sqrt{4-5x^2} - \int\frac{-5x^2\,dx}{\sqrt{4-5x^2}}$$

$$I = x\sqrt{4-5x^2} - \int\frac{(4-5x^2-4)}{\sqrt{4-5x^2}}\,dx$$

$$I = x\sqrt{4-5x^2} - \int\frac{4-5x^2}{\sqrt{4-5x^2}}\,dx + 4\int\frac{1}{\sqrt{(2)^2-(\sqrt{5}\,x)^2}}\,dx.$$

$$I = x\sqrt{4-5x^2} - \int\sqrt{4-5x^2}\,dx + 4\int\frac{1\,dx}{\sqrt{5\left(\frac{4}{5}-x^2\right)}} \quad \text{by (1)}$$

$$I = x\sqrt{4-5x^2} - I + 4\int\frac{1\quad dx}{\sqrt{5}\sqrt{\left(\frac{2}{\sqrt{5}}\right)^2-(x)^2}}$$



$$I + I = x\sqrt{4-5x^2} + \frac{4}{\sqrt{5}}\int\frac{1\,dx}{\sqrt{\left(\frac{2}{\sqrt{5}}\right)^2-(x)^2}}$$

$$2I = x\sqrt{4-5x^2} + \frac{4}{\sqrt{5}}\,\text{Sin}^{-1}\frac{x}{\frac{2}{\sqrt{5}}} + c$$

$$\boxed{I = x\sqrt{4-5x^2} + \frac{2}{\sqrt{5}}\,\text{Sin}^{-1}\left(\frac{\sqrt{5}\,x}{2}\right) + c} \quad \text{Ans}$$

**(iii)** $\int\sqrt{a^2-x^2}\,dx.$

**Solution:** let $I = \int\sqrt{a^2-x^2}\,dx.$ ———— (1)

let $u = (a^2-x^2)^{1/2}$ ; $v = 1$

differentiate w • r to x ; $\int v\,dx = \int dx$

$\frac{du}{dx} = \frac{1}{\cancel{2}}(a^2-x^2)^{-1/2}(-\cancel{2}x)$ ; $\boxed{\int v\,dx = x}$

$$\boxed{\frac{du}{dx} = \frac{-x}{\sqrt{a^2-x^2}}}$$

$$I = u\int v\,dx - \int\left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = x\sqrt{a^2-x^2} - \int\left(\frac{-x}{\sqrt{a^2-x^2}}\right)x\,dx.$$

$$I = x\sqrt{a^2-x^2} - \int\frac{a^2-x^2-a^2}{\sqrt{a^2-x^2}}\,dx$$

$$I = x\sqrt{a^2-x^2} - \int\frac{a^2-x^2}{\sqrt{a^2-x^2}}\,dx + a^2\int\frac{1}{\sqrt{a^2-x^2}}\,dx.$$

$$I = x\sqrt{a^2-x^2} - \int\sqrt{a^2-x^2}\,dx + a^2\int\frac{1}{\sqrt{a^2-x^2}}\,dx.$$

$$I = x\sqrt{a^2-x^2} - I + a^2\int\frac{1}{\sqrt{a^2-x^2}}\,dx.$$

$$I + I = x\sqrt{a^2-x^2} + a^2\int\frac{1}{\sqrt{a^2-x^2}}\,dx.$$

$$2I = x\sqrt{a^2-x^2} + a^2\,\text{Sin}^{-1}\frac{x}{a} + c$$

÷ by 2

$$\boxed{I = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\,\text{Sin}^{-1}\frac{x}{a} + c} \quad \text{Ans}$$

 https://stbmathematics.blogspot.com/

**Q3. Find the following indefinite integrals.**

**(i)** $\int x^3 e^{2x}\,dx$

**Solution:** let $I = \int x^3 e^{2x}\,dx$

$u = x^3$ ; $v = e^{2x}$

differentiate w.r to x ; $\int v dx = \int e^{2x} dx$

$\boxed{\frac{du}{dx} = 3x^2}$ ; $\boxed{\int v dx = \frac{e^{2x}}{2}}$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = \frac{x^3 e^{2x}}{2} - \int 3x^2 \frac{e^{2x}}{2}\,dx$$

$$I = \frac{x^3 e^{2x}}{2} - \frac{3}{2}\int x^2 e^{2x}\,dx \text{ ———— (1)}$$

Consider $I_1 = \int x^2 e^{2x}\,dx$

$u = x^2$ ; $v = e^{2x}$

differentiate w.r to x ; $\int v dx = \int e^{2x} dx$

$\boxed{\frac{du}{dx} = 2x}$ ; $\boxed{\int v dx = \frac{e^{2x}}{2}}$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = \frac{x^2 e^{2x}}{2} - \int \cancel{2}x.\frac{e^{2x}}{\cancel{2}}\,dx$$

$$I = \frac{x^2 e^{2x}}{2} - \int x e^{2x}\,dx \text{ ———— (2)}$$

Condiser $I_2 = \int x e^{2x}\,dx$

$u = x$ ; $v = e^{2x}$

differentiate w.r to x ; $\int v dx = \int e^{2x} dx$

$\boxed{\frac{du}{dx} = 1}$ ; $\boxed{\int v dx = \frac{e^{2x}}{2}}$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx \Rightarrow I = \frac{x e^{2x}}{2} - \int (1)\frac{e^{2x}}{2}\,dx$$

$$I = \frac{x e^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx \Rightarrow I = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4}$$

put this result in (2)

$$(2) \Rightarrow I = \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{e^{2x}}{4}$$

put this result in (1)



$$(1) \Rightarrow I = \frac{x^3 e^{2x}}{2} - \frac{3}{2}\left\{\frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{2} + \frac{e^{2x}}{4}\right\} + c$$

$$I = \frac{x^3 e^{2x}}{2} - \frac{3}{4}x^2 e^{2x} + \frac{3}{4}x e^{2x} - \frac{3}{8}e^{2x} + c$$

$$\boxed{I = \frac{1}{8}e^{2x}(4x^3 - 6x^2 + 6x - 3) + c} \text{ Ans}$$

**(ii)** $\int \mathbf{Sin^{-1}x\,dx}$

**Solution:** let $I = \int \text{Sin}^{-1}x\,dx$

$u = \text{Sin}^{-1}x$ ; $v = 1$

differentiate w.r to x ; $\int v dx = \int dx$

$\boxed{\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}}$ ; $\boxed{\int v dx = x}$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = x\,\text{Sin}^{-1}x - \int \frac{1}{\sqrt{1-x^2}}\,x\,dx$$

$$I = x\,\text{Sin}^{-1}x - \int (1-x^2)^{-1/2}\,x\,dx$$

$$I = x\,\text{Sin}^{-1}x + \frac{1}{2}\int (1-x^2)^{-1/2}(-2x)\,dx$$

$$I = x\,\text{Sin}^{-1}x + \frac{1}{2}\,\frac{(1-x^2)^{1/2}}{1/2} + c$$

$$I = x\,\text{Sin}^{-1}x + \frac{\cancel{2}}{\cancel{2}}\sqrt{1-x^2} + c$$

$$\boxed{I = x\,\text{Sin}^{-1}x + \sqrt{1-x^2} + c} \text{ Ans}$$

**(iii)** $\int \mathbf{x\tan^{-1}x\,dx}$.

**Solution:** let $I = \int x\tan^{-1}x\,dx$.

$u = \tan^{-1}x$ ; $v = x$

diff w.r to x ; $\int v dx = \int x\,dx$

$\boxed{\frac{du}{dx} = \frac{1}{1+x^2}}$ ; $\boxed{\int v dx = \frac{x^2}{2}}$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \int \frac{1}{1+x^2}\,\frac{x^2}{2}\,dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\,dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{\cancel{1+x^2}}{\cancel{1+x^2}}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$

$$I = \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}\frac{x}{1} + c$$

$$\boxed{I = \frac{x^2}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + c} \text{ Ans}$$

**(iv)** $\int x \,\mathrm{Sec}^{-1}x\,dx.$

**Solution:** let $I = \int x\,\mathrm{Sec}^{-1}\,dx.$

$u = \mathrm{Sec}^{-1}x$ ; $v = x$

differentiate w • r to x ; $\int v dx = \int x\,dx$

$$\boxed{\frac{du}{dx} = \frac{1}{x\sqrt{x^2-1}}} \qquad \boxed{\int v dx = \frac{x^2}{2}}$$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = \frac{x^2}{2}\mathrm{Sec}^{-1}x - \int \frac{1}{\cancel{x}\sqrt{x^2-1}}\,\frac{x^{\cancel{2}}}{2}\,dx$$

$$I = \frac{x^2}{2}\mathrm{Sec}^{-1}x - \frac{1}{2\times 2}\int (x^2-1)^{-1/2}\,2x\,dx$$

$$I = \frac{x^2}{2}\mathrm{Sec}^{-1}x - \frac{1}{4}\frac{(x^2-1)^{1/2}}{1/2} + c$$

$$I = \frac{x^2}{2}\mathrm{Sec}^{-1}x - \frac{\cancel{2}}{\cancel{4}_2}\sqrt{x^2-1} + c$$

$$\boxed{I = \frac{x^2}{2}\mathrm{Sec}^{-1}x - \frac{1}{2}\sqrt{x^2-1} + c} \text{ Ans}$$

**(v)** $\int x\,\mathrm{Cosec}^2\frac{x}{2}\,dx.$

**Solution:** let $I = \int x\,\mathrm{Cosec}^2\frac{x}{2}\,dx.$

$u = x$ ; $v = \mathrm{Cosec}^2\frac{x}{2}$

differentiate w • r to x ; $\int v dx = \int \mathrm{Cosec}^2\frac{x}{2}\,dx$



$$\boxed{\frac{du}{dx} = 1} \qquad \int v dx = \frac{-\mathrm{Cot}\frac{x}{2}}{\frac{1}{2}}$$

$$\boxed{\int v dx = -2\,\mathrm{Cot}\frac{x}{2}}$$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = -2x\,\mathrm{Cot}\frac{x}{2} - \int (1)\left(-2\,\mathrm{Cot}\frac{x}{2}\right)dx$$

$$I = -2x\,\mathrm{Cot}\frac{x}{2} + 2\int \mathrm{Cot}\frac{x}{2}\,dx$$

$$I = -2x\,\mathrm{Cot}\frac{x}{2} + 2\,\frac{\ln \mathrm{Sin}\frac{x}{2}}{\frac{1}{2}} + c$$

$$\boxed{I = -2x\,\mathrm{Cot}\frac{x}{2} + 4\ln \mathrm{Sin}\frac{x}{2} + c} \text{ Ans}$$

**(vi)** $\int \frac{2x\,dx}{\mathrm{Cos}^2 2x}$

**Solution:** let $I = \int \frac{2x\,dx}{\mathrm{Cos}^2 2x}$

$I = 2\int x\,\mathrm{Sec}^2 2x\,dx$ ——— (1)

$u = x$ ; $v = \mathrm{Sec}^2 2x$

differentiate w • r to x ; $\int v dx = \int \mathrm{Sec}^2 2x\,dx$

$$\boxed{\frac{du}{dx} = 1} \qquad \boxed{\int v dx = \frac{\tan 2x}{2}}$$

$$I = u\int v dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx \Rightarrow I = \frac{x\tan 2x}{2} - \int (1)\frac{\tan 2x}{2}\,dx$$

$$I = \frac{x\tan 2x}{2} - \frac{1}{2}\int \tan 2x\,dx \Rightarrow I = \frac{x\tan 2x}{2} - \frac{1}{2\times 2}\ln \mathrm{Sec}2x$$

put this result in (1)

$$I = 2\left\{\frac{x\tan 2x}{2} - \frac{1}{4}\ln \mathrm{Sec}2x\right\} + c$$

$$\boxed{I = x\tan 2x - \frac{1}{2}\ln \mathrm{Sec}2x + c} \text{ Ans}$$

**(vii)** $\int 9x\tan^2 3x\,dx.$

**Solution:** let $I = \int 9x\tan^2 3x\,dx.$

$I = \int 9x(\text{Sec}^2 3x - 1)\,dx$

$I = 9\int x\,\text{Sec}^2 3x - 9\int x\,dx$

$I = 9\int x\,\text{Sec}^2 3x - 9\dfrac{x^2}{2} + c$ ———— (1)

Consider $I = \int x\text{Sec}^2 3x\,dx$

$u = x$ ; $v = \text{Sec}^2 3x$

differentiate w.r to x $\quad \int v dx = \int \text{Sec}^2 3x\,dx$

$\boxed{\dfrac{du}{dx} = 1}$ $\quad \boxed{\int v dx = \dfrac{\tan 3x}{3}}$

$I = u\int v dx - \int\left\{\dfrac{du}{dx}\int v\,dx\right\}dx$

$I = \dfrac{x\tan 3x}{3} - \int(1)\dfrac{\tan 3x}{3}\,dx$

$I = \dfrac{x\tan 3x}{3} - \dfrac{1}{3}\int\tan 3x\,dx$

$I = \dfrac{x\tan 3x}{3} - \dfrac{1}{9}\ln\text{Sec}3x$

**put this result in (1)**

$(1) \Rightarrow I = 9\left\{\dfrac{x\tan 3x}{3} - \dfrac{1}{9}\ln\text{Sec}3x\right\} - \dfrac{9x^2}{2} + c$

$\boxed{I = 3x\tan 3x - \ln\text{Sec}3x - \dfrac{9x^2}{2} + c}$ Ans

**(viii)** $\int \text{Sin}\sqrt{2x}\,dx$

**Solution:** let $I = \int \text{Sin}\sqrt{2x}\,dx$ ———— (1)

let $t = \sqrt{2x}$

Squaringon b.s

$t^2 = 2x.$

differentiate w.r to x

$\cancel{2}t\dfrac{dt}{dx} = \cancel{2} \Rightarrow \boxed{dx = tdt}$

$(1) \Rightarrow I = \int \text{Sin}t \cdot tdt$

$I = \int t\,\text{Sin}\,tdt.$

$u = t$ ; $v = \text{Sin}\,t$

differentiate w.r to t $\quad \int vdt = \int \text{Sin}\,t\,dt$

$\boxed{\dfrac{du}{dx} = 1}$ $\quad \boxed{\int vdt = -\text{Cos}\,t}$



$I = u\int v\,dt - \int\left\{\dfrac{du}{dt}\int v\,dt\right\}dt$

$I = -t\,\text{Cos}\,t - \int(1)(-\text{Cos}\,t)\,dt \Rightarrow I = -t\,\text{Cost} + \int\text{Cos}\,t\,dt$

$I = -t\,\text{Cos}\,t + \text{Sin}\,t + c$

but $t = \sqrt{2x}$

$\boxed{I = -\sqrt{2x}\,\text{Cos}\sqrt{2x} + \text{Sin}\sqrt{2x} + c}$ Ans

**(ix)** $\int 6x^2\,\text{Sin}^{-1}2x\,dx.$

**Solution:** let $I = \int 6x^2\,\text{Sin}^{-1}2x\,dx.$

$I = 6\int x^2\,\text{Sin}^{-1}2x\,dx.$ ———— (1)

Using integration by parts.

let $u = \text{Sin}^{-1}2x$ ; $v = x^2\,dx$

$\quad \int vdx = \int x^2\,dt$

$\boxed{\dfrac{du}{dx} = \dfrac{2}{\sqrt{1-4x^2}}}$ $\quad \boxed{\int vdx = \dfrac{x^3}{3}}$

$I = u\int v\,dx - \int\left\{\dfrac{du}{dx}\int v\,dx\right\}dx$

$I = \dfrac{x^3}{3}\text{Sin}^{-1}2x - \int\dfrac{2}{\sqrt{1-4x^2}}\dfrac{x^3}{3}\,dx$

$I = \dfrac{x^3}{3}\text{Sin}^{-1}2x - \dfrac{2}{3}\int\dfrac{x^3}{\sqrt{1-4x^2}}\,dx$

put this result in (1)

$(1) \Rightarrow I = 2x^3\,\text{Sin}^{-1}2x - 4\int\dfrac{x^3}{\sqrt{1-4x^2}}\,dx$ ———— (2)

let $I = \int\dfrac{x^3\,dx}{\sqrt{1-4x^2}}$

$I = \int\dfrac{x^3\,dx}{\sqrt{4\left(\frac{1}{4} - x^2\right)}} = \dfrac{1}{2}\int\dfrac{x^3\,dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x)^2}}$

**Using Trigonometric Substitution method**

let $x = a\,\text{Sin}\theta \Rightarrow x = \dfrac{1}{2}\text{Sin}\theta$

differentiate w.r to θ

$\dfrac{dx}{d\theta} = \dfrac{\text{Cos}\theta}{2} \Rightarrow \boxed{dx = \dfrac{\text{Cos}\theta}{2}d\theta} \Rightarrow I = \dfrac{1}{2}\int\dfrac{\left(\frac{\text{Sin}\theta}{2}\right)^3\frac{\text{Cos}\theta}{2}\,d\theta}{\sqrt{\frac{1}{4} - \frac{1}{4}\text{Sin}^2\theta}}$

$$I = \frac{1}{2}\int \frac{\frac{Sin^3\theta}{8} \cdot \frac{Cos\theta}{2}\, d\theta}{\sqrt{\frac{1}{4}(1 - Sin^2\theta)}} \Rightarrow I = \frac{1}{32}\int \frac{Sin^3\theta\, Cos\theta\, d\theta}{\frac{1}{2}\sqrt{1 - Sin^2\theta}}$$

$$I = \frac{\cancel{2}}{\cancel{32}_{16}}\int \frac{Sin^3\theta\, Cos\theta\, d\theta}{\sqrt{Cos^2\theta}} \Rightarrow I = \frac{1}{16}\int \frac{Sin^3\theta\, \cancel{Cos\theta}\, d\theta}{\cancel{Cos\theta}}$$

$$I = \frac{1}{16}\int Sin^3\theta\, d\theta = \frac{1}{16}\int Sin^2\theta\, Sin\theta\, d\theta$$

$$I = \frac{1}{16}\int (1 - Cos^2\theta)\, Sin\theta\, d\theta$$

$$I = \frac{1}{16}\int Sin\theta\, d\theta - \frac{1}{16}\int Cos^2\theta\, Sin\theta\, d\theta$$

$$I = \frac{1}{16}\int Sin\theta\, d\theta + \frac{1}{16}\int Cos^2\theta.\, (-Sin\theta)\, d\theta$$

$$I = \frac{1}{16}(-Cos\theta) + \frac{1}{16}\frac{Cos^3\theta}{3} \Rightarrow I = \frac{-Cos\theta}{16} + \frac{Cos^3\theta}{48}$$

put this result (2)

$$(2) \Rightarrow I = 2x^3\, Sin^{-1}2x - 4\left\{\frac{-Cos\theta}{16} + \frac{Cos^3\theta}{48}\right\} + c$$

$$I = 2x^3\, Sin^{-1}2x + \frac{Cos\theta}{4} - \frac{Cos^3\theta}{12} + c$$

$\therefore Sin\theta = 2x$

$Sin^2\theta + Cos^2\theta = 1 \Rightarrow Cos\theta = \sqrt{1 - 4x^2}$

$$I = 2x^3\, Sin^{-1}2x + \frac{\sqrt{1-4x^2}}{4} - \frac{(1-4x^2)^{3/2}}{12} + c$$

$$\boxed{I = 2x^3\, Sin^{-1}2x - \frac{1}{4}\left\{\frac{1}{3}(1-4x^2)^{3/2} - (1-4x^2)^{1/2}\right\} + c} \text{ Ans}$$

**(x)** $\int x^m \ln x\, dx$

**Solution:** let $I = \int x^m \ln x\, dx$

Using by parts.

let $u = \ln x$ ; $v = x^m$

$\boxed{\frac{du}{dx} = \frac{1}{x}}$ $\int v dx = \int x^m dx$

$\boxed{\int v dx = \frac{x^{m+1}}{m+1}}$

$$I = u\int v\, dx - \int\left\{\frac{du}{dx}\int v\, dx\right\} dx$$



$$I = (\ln x)\left(\frac{x^{m+1}}{m+1}\right) - \int \frac{1}{x}\frac{x^{m+1}}{m+1}\, dx$$

$$I = (\ln x)\left(\frac{x^{m+1}}{m+1}\right) - \frac{1}{m+1}\int x^m\, dx$$

$$I = (\ln x)\left(\frac{x^{m+1}}{m+1}\right) - \frac{1}{m+1}\frac{x^{m+1}}{m+1} + c$$

$$\boxed{I = \frac{x^{m+1}}{m+1}\left\{\ln x - \frac{1}{m+1}\right\} + c} \text{ Ans}$$

**(xi)** $\int 2x^3 e^{x^2} dx$

**Solution:** let $I = \int 2x^3 e^{x^2} dx$ ———— (1)

let $t = x^2$

$\frac{dt}{dx} = 2x. \Rightarrow \boxed{dx = \frac{dt}{2x}}$

$(1) \Rightarrow I = \int \cancel{2}\cancel{x}^3 e^t \left(\frac{dt}{\cancel{2x}}\right)$

$I = \int x^2 e^t\, dt. \Rightarrow I = \int t\, e^t\, dt.$

using by parts.

let $u = t$ ; $v = e^t$

$\boxed{\frac{du}{dt} = 1}$ $\int v\, dt = \int e^t\, dt$

$\boxed{\int vdt = e^t}$

$$I = u\int v\, dt - \int\left\{\frac{du}{dt}\int v\, dt\right\} dt$$

$I = t\, e^t - \int (1)\, e^t\, dt. \Rightarrow I = t.e^t - \int e^t\, dt.$

$I = t\, e^t - e^t + c$ but $t = x^2$

$I = x^2 e^{x^2} - e^{x^2} + c \Rightarrow \boxed{I = e^{x^2}(x^2 - 1) + c}$ Ans

**(xii)** $\int e^{3x} Sin2x\, dx$

**Solution:** let $I = \int e^{3x} Sin2x\, dx$

Using integration by parts.

let $u = Sin2x$ ; $v = e^{3x} dx$

$\boxed{\frac{du}{dx} = 2\, Cos2x}$ $\int v\, dx = \int e^{3x}\, dx$

$\boxed{\int vdx = \frac{e^{3x}}{3}}$

$$I = u\int v\, dx - \int\left\{\frac{du}{dx}\int v\, dx\right\} dx$$

$$I = Sin2x.\frac{e^{3x}}{3} - \int 2\, Cos2x.\frac{e^{3x}}{3}\, dx$$

$I = \frac{1}{3}\text{Sin}2x\, e^{3x} - \frac{2}{3}\int e^{3x}.\text{Cos}2x\, dx$ ——— (1)

let $I = \int e^{3x}.\text{Cos}2x\, dx$

again using by parts.

$u = \text{Cos}2x$ ; $v = e^{3x}\, dx$

$\boxed{\frac{du}{dx} = -2\,\text{Sin}2x}$ $\int v\, dx = \int e^{3x}\, dx$

$\boxed{\int v dx = \frac{e^{3x}}{3}}$

$I = \text{Cos}2x\frac{e^{3x}}{3} + \frac{2}{3}\int e^{3x}\,\text{Sin}2x\, dx$

$I = \frac{\text{Cos}2x\, e^{3x}}{3} + \frac{2}{3}I.$

put this result in (1)

$(1) \Rightarrow I = \frac{1}{3}\,\text{Sin}2x\, e^{3x} - \frac{2}{3}\left\{\frac{\text{Cos}2x\, e^{3x}}{3} + \frac{2}{3}I\right\} + c$

$I = \frac{1}{3}\,\text{Sin}2x\, e^{3x} - \frac{2}{9}\,\text{Cos}2x\, e^{3x} - \frac{4}{9}I + c$

$I + \frac{4I}{9} = \frac{1}{3}\text{Sin}2x\, e^{3x} - \frac{2}{9}\,\text{Cos}2x\, e^{3x} + c$

$\frac{13I}{9} = \frac{1}{3}\text{Sin}2x\, e^{3x} - \frac{2}{9}\,\text{Cos}2x\, e^{3x} + c$

xplying throughout by $\frac{9}{13}$

$I = \frac{3}{13}\,\text{Sin}2x\, e^{3x} - \frac{2}{13}\,\text{Cos}2x\, e^{3x} + c$

$\boxed{I = \frac{e^{3x}}{13}(3\,\text{Sin}2x - 2\text{Cos}2x) + c}$ Ans

**(xiii)** $\int e^{-x}$ **Cos3x dx.**

**Solution:** let $I = \int e^{-x}\,\text{Cos}3x\, dx$.

Using integration by parts.

$u = \text{Cos}3x$ ; $v = e^{-x}$

$\boxed{\frac{du}{dx} = -3\text{Sin}3x}$ $\int v\, dx = \int e^{-x}\, dx$

$\boxed{\int v\, dx = -e^{-x}}$

$I = u\int v\, dx - \int\left\{\frac{du}{dx}\int v\, dx\right\}dx$

$I = -3\,\text{Cos}3x\, e^{-x} - \int\{-3\,\text{Sin}3x\}(-e^{-x})\, dx$

$I = -3\,\text{Cos}3x\, e^{-x} - 3\int e^{-x}\,\text{Sin}3x\, dx$ ——— (1)

let $I = \int e^{-x}\,\text{Sin}3x\, dx$



$u = \text{Sin}3x$ $v = e^{-x}$

$\boxed{\frac{du}{dx} = 3\,\text{Cos}3x}$ $\int v\, dx = \int e^{-x}\, dx$

$\boxed{\int v\, dx = -e^{-x}}$

$I = -\text{Sin}3x\, e^{-x} - \int(3\,\text{Cos}3x)(-e^{-x})\, dx$

$I = -\text{Sin}3x\, e^{-x} + 3\int \text{Cos}3x\, e^{-x} dx$

$I = -\text{Sin}3x\, e^{-x} + 3I$

put this result in (1)

$(1) \Rightarrow I = -3\text{Cos}3x\, e^{-x} - 3(-\text{Sin}3x\, e^{-x} + 3I) + c$

$I = -3\text{Cos}3x\, e^{-x} + 3\,\text{Sin}3x\, e^{-x} - 9I + c$

$I + 9I = e^{-x}\{3\,\text{Sin}3x - \text{Cos}3x\} + c$

$10I = e^{-x}\{3\,\text{Sin}3x - \text{Cos}3x\} + c$

$\boxed{I = \frac{e^{-x}}{10}\{3\,\text{Sin}3x - \text{Cos}3x\} + c}$ Ans

**(xiv)** $\int e^{ax}$ **Cosbx dx**

**Solution:** let $I = \int e^{ax}\,\text{Cosbx}\, dx$

let $u = \text{Cosbx}$ ; $v = e^{ax}$

$\boxed{\frac{du}{dx} = -b\,\text{Sinbx}}$ $\int v\, dx = \int e^{ax}\, dx$

$\boxed{\int v dx = \frac{e^{ax}}{3}}$

$I = u\int v\, dx - \int\left\{\frac{du}{dx}\int v\, dx\right\}dx$

$I = \text{Cosbx}\frac{e^{ax}}{a} - \int(-b\,\text{Sinbx})\frac{e^{ax}}{a}\, dx$

$I = \frac{\text{Cosbx}\, e^{ax}}{a} + \frac{b}{a}\int \text{Sinbx}\, e^{ax}\, dx$ ——— (2)

let $I = \int \text{Sinbx}\, e^{ax}\, dx$

again using by parts

$u = \text{Sinbx}$ $v = e^{ax}$

$\boxed{\frac{du}{dx} = b\,\text{Cosbx}}$ $\int v\, dx = \int e^{ax}\, dx$

$\boxed{\int v dx = \frac{e^{ax}}{3}}$

$I = \text{Sinbx}\frac{e^{ax}}{a} - \int b\,\text{Cosbx}.\frac{e^{ax}}{a}\, dx.$

$I = \text{Sinbx}\frac{e^{ax}}{a} - \frac{b}{a}\int \text{Cosbx}\, e^{ax}\, dx.$

$I = \text{Sinbx}\frac{e^{ax}}{a} - \frac{b}{a}I.$ by (1)

put this result in (2)

$$(2) \Rightarrow I = \frac{\text{Cosbx}}{a} e^{ax} + \frac{b}{a}\left\{ \text{Sinbx} \frac{e^{ax}}{a} - \frac{b}{a} I \right\} + c$$

$$I = \frac{\text{Cosbx}}{a} e^{ax} + \frac{b}{a^2} \text{Sinbx}\, e^{ax} - \frac{b^2}{a^2} I + c$$

$$I + \frac{b^2}{a^2} I = \frac{\text{Cosbx}}{a} e^{ax} + \frac{b}{a^2} \text{Sinbx}\, e^{ax} + c$$

$$I \left(\frac{a^2 + b^2}{a^2}\right) = \frac{\text{Cosbx}}{a} e^{ax} + \frac{b}{a^2} \text{Sinbx}\, e^{ax} + c$$

xplying throughout by $\left(\frac{a^2}{a^2 + b^2}\right)$

$$I = \frac{a}{a^2 + b^2} \text{Cosbx}\, e^{ax} + \frac{a}{a^2 + b^2} \text{Sinbx}\, e^{ax} + c$$

$$\boxed{I = \frac{e^{ax}}{a^2 + b^2} (a \text{ Cosbx} + b \text{ Sinbx}) + c} \text{ Ans}$$

**Q4. Calculate the following.**

**(i)** $\int e^x (\text{Sinx} + \text{Cosx})\, dx.$

**Solution:** let $I = \int e^x (\text{Sinx} + \text{Cosx})\, dx.$

$$\boxed{\text{Using formula} \int e^x \{ f(x) + f'(x) \}\, dx = e^x f(x) + c}$$

$$\boxed{I = e^x \text{ Sinx} + c} \text{ Ans}$$

**Second Method:**

let $I = \int e^x (\text{Sinx} + \text{Cosx})\, dx$

$I = \int e^x \text{ Sinx}\, dx + \int e^x \text{ Cosx}\, dx$ ———— (1)

Consider $I = \int e^x \text{ Sinx}\, dx$

$u = \text{Sinx}$ ; $v = e^{ax}$

$\boxed{\frac{du}{dx} = \text{Cosx}}$ ; $\int v\, dx = \int e^{ax}\, dx$ ; $\boxed{\int vdx = e^x}$

$$I = u \int v\, dx - \int \left\{ \frac{du}{dx} \int v\, dx \right\} dx$$

$$I = e^x \text{ Sinx} - \int \text{Cos } x\, e^x\, dx$$

Put this result in (1)

$$(1) \Rightarrow I = e^x \text{ Sinx} - \int \text{Cosx}\, \cancel{e^x}\, dx + \int \cancel{e^x \text{Cos}\theta}\, dx$$

$$\boxed{I = e^x \text{ Sinx} + c} \text{ Ans}$$

**(ii)** $\int e^x \left\{ \text{Sec}^{-1}x + \frac{1}{x\sqrt{x^2 - 1}} \right\} dx$

**Solution:** $I = \int e^x \left\{ \text{Sec}^{-1}x + \frac{1}{x\sqrt{x^2 - 1}} \right\} dx$



$$\boxed{\text{Using formula} \int e^x \{ f(x) + f'(x) \}\, dx = e^x f(x) + c}$$

$$\boxed{I = e^x \text{ Sec}^{-1}x + c} \text{ Ans}$$

**(iii)** $\int \frac{x\, ex}{(1 + x)^2}$

**Solution:** let $I = \int \frac{x\, ex}{(1 + x)^2}$

$$I = \int \frac{(x + 1 - 1)\, e^x\, dx}{(1 + x)^2} \Rightarrow I = \int \frac{\cancel{(x + 1)}\, e^x\, dx}{(1 + x)^{\cancel{2}}} - \int \frac{e^x\, dx}{(1 + x)^2}$$

$$I = \int \frac{e^x\, dx}{(1 + x)} - \int \frac{e^x\, dx}{(1 + x)^2} \text{ ———— (1)}$$

Consider $I = \int \frac{e^x\, dx}{(1 + x)}$

Using by parts.

$u = (1 + x)^{-1}$ ; $v = e^x$

differentiate w • r to x ; $\int v\, dx = \int e^x\, dx$

$\frac{du}{dx} = (-1)(1 + x)^{-2}$ ; $\boxed{\int vdx = e^x}$

$$\boxed{\frac{du}{dx} = \frac{-1}{(1 + x)^2}}$$

$$I = u \int v\, dx - \int \left\{ \frac{du}{dx} \int v\, dx \right\} dx$$

$$I = \frac{e^x}{(1 + x)} - \int \left\{ \frac{-1}{(1 + x)^2} \right\} e^x\, dx$$

$$I = \frac{e^x}{(1 + x)} - \int \frac{e^x}{(1 + x)^2}\, dx$$

put this result in (1)

$$(1) \Rightarrow I = \frac{e^x}{(1 + x)} - \cancel{\int \frac{e^x\, dx}{(1 + x)^2}} - \cancel{\int \frac{e^x\, dx}{(1 + x)^2}} + c$$

$$\boxed{I = \frac{e^x}{(1 + x)} + c} \text{ Ans}$$

**(iv)** $\int \frac{e^x (1 + x)\, dx}{(2 + x)^2}$

**Solution:** let $I = \int \frac{e^x (1 + x)\, dx}{(2 + x)^2}$

$$I = \int \frac{(2 + x - 1)\, e^x\, dx}{(2 + x)^2} \Rightarrow I = \int \frac{\cancel{(2 + x)}\, e^x\, dx}{(2 + x)^{\cancel{2}}} - \int \frac{e^x\, dx}{(2 + x)^2}$$

$$I = \int \frac{e^x\,dx}{(2+x)} - \int \frac{e^x\,dx}{(2+x)^2} \text{ ———— } (1)$$

Consider $I = \int \frac{e^x\,dx}{(2+x)}$

**Using by parts.**

$u = (2+x)^{-1}$ ; $v = e^x$

differentiate w • r to x $\int v\,dx = \int e^x\,dx$

$\frac{du}{dx} = (-1)(2+x)^{-2}$ $\boxed{\int v dx = e^x}$

$$\boxed{\frac{du}{dx} = \frac{-1}{(2+x)^2}}$$

$$I = u\int v\,dx - \int \left\{\frac{du}{dx}\int v\,dx\right\}dx$$

$$I = \frac{e^x}{(2+x)} - \int \left\{\frac{-1}{(2+x)^2}\right\} e^x\,dx$$

$$I = \frac{e^x}{(2+x)} - \int \frac{e^x}{(2+x)^2}\,dx$$

put this result in (1)

$$(1) \Rightarrow I = \frac{e^x}{(2+x)} - \cancel{\int \frac{e^x\,dx}{(2+x)^2}} - \cancel{\int \frac{e^x\,dx}{(2+x)^2}} + c$$

$$\boxed{I = \frac{e^x}{(2+x)} + c} \quad \text{Ans}$$

**(v)** $\int e^x \frac{1+\text{Sinx}}{1+\text{Cosx}}\,dx$

**<u>Solution:</u>** let $I = \int e^x \frac{1+\text{Sinx}}{1+\text{Cosx}}\,dx$

$$\boxed{\begin{array}{l} \therefore\ \text{Sinx} = 2\ \text{Sin}\frac{x}{2}\ \text{Cos}\frac{x}{2} \\ \therefore\ 1+\text{Cosx} = 2\,\text{Cos}^2\frac{x}{2} \end{array}}$$

$$I = \int e^x \left\{\frac{1 + 2\,\text{Sin}\frac{x}{2}\ \text{Cos}\frac{x}{2}}{2\,\text{Cos}^2\frac{x}{2}}\right\}dx$$

$$I = \int e^x \left\{\frac{1}{2\,\text{Cos}^2\frac{x}{2}} + \frac{\cancel{2}\text{Sin}\frac{x}{2}\ \cancel{\text{Cos}\frac{x}{2}}}{\cancel{2}\,\text{Cos}^{\cancel{2}}\frac{x}{2}}\right\}dx$$



$$I = \int e^x \left\{\frac{1}{2}\text{Sec}^2\frac{x}{2} + \tan\frac{x}{2}\right\}dx \Rightarrow I = \int e^x\left(\tan\frac{x}{2} + \frac{1}{2}\text{Sec}^2\frac{x}{2}\right)dx$$

Using formula

$\boxed{\text{Using formula} \int e^x \{ f(x) + f'(x) \}\,dx = e^x f(x) + c}$

$\boxed{I = e^x\,\text{Sec}^{-1}x + c}$ Ans

**(vi)** $\int \text{Cos}\left(b \ln\frac{x}{a}\right) + c$

**<u>Solution:</u>** let $I = \int \text{Cos}\left(b \ln\frac{x}{a}\right)dx$ ———— (1)

let $y = \ln\frac{x}{a} \Rightarrow e^y = \frac{x}{a}$

differentiate w • r to x $\boxed{x = ae^y}$

$$\frac{dy}{dx} = \frac{1}{\frac{x}{a}}\left(\frac{1}{a}\right) \Rightarrow \frac{dy}{dx} = \frac{\cancel{a}}{x}\cdot\frac{1}{\cancel{a}} \Rightarrow \boxed{xdy = dx}$$

$(1) \Rightarrow I = \int \text{Cos}\,(by)\,xdy. \Rightarrow I = \int \text{Cos}\,(by)\,ae^y)\,dy.$

$I = a\int e^y\,\text{Cos by dy}$ ———— (2)

let $I = \int e^y\,\text{Cos by dy}$

$u = \text{Cos by}$ $v = e^y$

$\boxed{\frac{du}{dy} = b\,\text{Sin by}}$ $\int v\,dy = \int e^y\,dx$

$\boxed{\int v\,dy = e^y}$

$$I = u\int v\,dy - \int \left\{\frac{du}{dy}\int v\,dy\right\}dy$$

$I = e^y\,\text{Cos by} - \int (-\,b\,\text{Sin by})\,e^y\,dy.$

$I = e^y\,\text{Cos by} + 6\int \text{Sin by}\,e^y\,dy.$ ———— (3)

let $I = \int \text{Sin by}\,e^y\,dy.$

$u = \text{Sin by}$ $v = e^y$

$\boxed{\frac{du}{dy} = b\,\text{Cos by}}$ $\int v\,dy = \int e^y\,dy$

$\boxed{\int v\,dy = e^y}$

$I = \text{Sin by}\,e^y - \int b\,\text{Cos by}\,e^y\,dy.$

$I = \text{Sin by}\,e^y - b\int \text{Cos by}\,e^y\,dy.$

$I = \text{Sin by}\,e^y - b\,I$

put this result in (3)

$(3) \Rightarrow I = e^y\,\text{Cos by} + b\,\{\ \text{Sin}\,b^y\,ey - b\,I\ \} + c$

$I = e^y\,\text{Cos by} + b\,\text{Sin by}\,e^y - b^2 I + c$

$I = b^2 I = e^y\,\text{Cos by} + b\,\text{Sin by}\,e^y + c$

$I = (1 + b^2) = e^y \text{ Cos by} + b \sin by\, e^y + c.$

$I = \dfrac{e^y \text{ Cos by}}{1 + b^2} + \dfrac{b \text{ Sin by } e^y}{1 + b^2}$

$I = \dfrac{a\, e^y \text{ Cos by}}{1 + b^2} + \dfrac{ab \text{ Sin by } e^y}{1 + b^2} + c$

but $y = \ln \dfrac{x}{a}$ $\quad ae^y = x$

$$I = \frac{x \text{ Cos}\left(b \ln \frac{x}{a}\right)}{1 + b^2} + \frac{b \text{ Sin}\left(b \ln \frac{x}{a}\right) x}{1 + b^2} + c$$

$$\boxed{I = \frac{x}{1 + b^2}\left\{\text{Cos}\left(b \ln \frac{x}{a}\right) + b \text{ Sin}\left(b \ln \frac{x}{a}\right)\right\} + c} \text{ Ans}$$

## EXERCISE # 6.8

**Resolve into partial fractions.**

**(1)** $\dfrac{7x - 25}{(x - 3)(x - 4)}$

**Solution:** let $\dfrac{7x - 25}{(x - 3)(x - 4)} = \dfrac{A}{x - 3} + \dfrac{B}{x - 4}$ ——— (1)

multiplying throughout eq$^n$ (1) by $(x - 3)(x - 4)$

$7x - 25 = A(x - 4) + B(x - 3)$ ——— (2)

put $x - 3 = 0 \Rightarrow \boxed{x = 3}$ in eq$^n$ (2) $\Rightarrow \boxed{A = 4}$

put $x - 4 = 0 \Rightarrow \boxed{x = 4}$ in eq$^n$ (2) $\Rightarrow \boxed{B = 3}$

put the values of A & B in (1)

$(1) \Rightarrow \boxed{\dfrac{7x - 25}{(x - 3)(x - 4)} = \dfrac{4}{x - 3} + \dfrac{3}{x - 4}}$ Ans.

**(2)** $\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

**Solution:** $\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$

we first divide the N (x) by D (x).

$$\begin{array}{r|l} & 2x + 3 \\ 3x^2 - 2x - 1 & 6x^3 + 5x^2 - 7 \\ & \underline{-6x^3 \mp 4x^2 \mp 2x} \\ & 9x^2 + 2x - 7 \\ & \underline{-9x^2 \mp 6x \mp 3} \\ & 8x - 4 \end{array}$$



$\therefore \dfrac{N(x)}{D(x)} = Q(x) + \dfrac{R(x)}{D(x)}$

$\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} = 2x + 3 + \dfrac{8x - 4}{3x^2 - 2x - 1}$

$\dfrac{8x - 4}{3x^2 - 2x - 1} = \dfrac{8x - 4}{(x - 1)(3x + 1)}$ (Now partial fraction)

$\dfrac{8x - 4}{(x - 1)(3x + 1)} = \dfrac{A}{x - 1} + \dfrac{B}{3x + 1}$ ——— (1)

Xplying throughout eq$^n$ (1) by $(x - 1)(3x - 1)$

$8x - 4 = A(3x + 1) + B(x - 1)$ ——— (2)

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (2) $\Rightarrow \boxed{A = 1}$

put $3x + 1 = 0 \Rightarrow \boxed{x = \dfrac{-1}{3}}$ in (2) $\Rightarrow \boxed{B = 5}$

Now $\boxed{\dfrac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} = 2x + 3 + \dfrac{1}{x - 1} + \dfrac{5}{3x + 1}}$ Ans.

**(3)** $\dfrac{2x^3 + x^2 - x - 3}{x(x - 1)(2x + 3)}$

**Solution:** $\dfrac{2x^3 + x^2 - x - 3}{x(x - 1)(2x + 3)} = \dfrac{2x^3 + x^2 - x - 3}{2x^3 + x^2 - 3x}$

first we divide N (x) by D (x)

$$\begin{array}{r|l} & 1 \\ 2x^3 + x^2 - 3x & 2x^3 + x^2 - x - 3 \\ & \underline{-2x^3 \pm x^2 \mp 3x} \\ & 2x - 3 \end{array}$$

$\dfrac{2x^3 + x^2 - x - 3}{x(x - 1)(2x + 3)} = 1 + \dfrac{2x - 3}{2x^3 + x^2 - 3x}$ —— (1)

Consider $\dfrac{2x - 3}{2x^3 + x^2 - 3x} = \dfrac{A}{x} + \dfrac{B}{x - 1} + \dfrac{C}{2x + 3}$ —— (2)

Xplying throughout eq$^n$ (2) by $x(x - 1)(2x + 3)$

$2x - 3 = A(x - 1)(2x + 3) + Bx(2x + 3) + cx(x - 1)$ — (3)

put $x = 0$ in (3) $\Rightarrow \boxed{A = 1}$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (3) $\Rightarrow \boxed{B = \dfrac{-1}{5}}$

put $2x + 3 = 0 \Rightarrow \boxed{x = \frac{-3}{2}}$ in (3) $\Rightarrow \boxed{c = \frac{-8}{5}}$

$$(1) \Rightarrow \boxed{\frac{2x^3 + x^2 - x - 3}{x(x-1)(2x+3)} = 1 + \frac{1}{x} - \frac{1}{5(x-1)} - \frac{-8}{5(2x+3)}} \quad \text{Ans.}$$

**(4)** $\dfrac{1}{(1-ax)(1-bx)(1-cx)}$

**Solution:** $\dfrac{1}{(1-ax)(1-bx)(1-cx)} = \dfrac{A}{1-ax} + \dfrac{B}{1-bx} + \dfrac{C}{1-cx}$ —(1)

Xplying throughout eq$^n$ (1) by $(1-ax)(1-bx)(1-cx)$

$1 = A(1-bx)(1-cx) + B(1-ax)(1-cx) + c(1-ax)(1-bx)$ —(2)

put $1 - ax = 0 \Rightarrow \boxed{x = \frac{1}{a}}$ in (2) $\Rightarrow \boxed{A = \frac{a^2}{(a-b)(a-c)}}$

put $1 - bx = 0 \Rightarrow \boxed{x = \frac{1}{b}}$ in (2) $\Rightarrow \boxed{B = \frac{b^2}{(b-a)(b-c)}}$

put $1 - cx = 0 \Rightarrow \boxed{x = \frac{1}{c}}$ in (2) $\Rightarrow \boxed{C = \frac{c^2}{(c-a)(c-b)}}$

$$(1) \Rightarrow \boxed{\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{a^2}{(a-b)(a-c)(1-ax)} + \frac{b^2}{(b-a)(b-c)(1-bx)} + \frac{c^2}{(c-a)(c-b)(1-cx)}}$$

**(5)** $\dfrac{x^2 + a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)}$ **(Hint : put $x^2 = y$)**

**Solution:** $\dfrac{x^2 + a^2}{(x^2+b^2)(x^2+c^2)(x^2+d^2)} = \dfrac{y + a^2}{(y+b^2)(y+c^2)(y+d^2)}$

$\dfrac{y + a^2}{(y+b^2)(y+c^2)(y+d^2)} = \dfrac{A}{y+b^2} + \dfrac{B}{y+c^2} + \dfrac{C}{y+d^2}$ —— (1)

Xplying throughout eq$^n$ (1) by $(y + b^2)(y + c^2)(y + d^2)$

$y + a^2 = A(y + c^2)(y + d^2) + B(y + b^2)(y + d^2) + c(y + c^2)(y + b^2)$ ——— (2)

put $y + b^2 = 0 \Rightarrow \boxed{y = -b^2}$ in (2)$\Rightarrow \boxed{A = \frac{a^2 - b^2}{(c^2-b^2)(d^2-b^2)}}$

put $y + c^2 = 0 \Rightarrow \boxed{y = -c^2}$ in (2)$\Rightarrow \boxed{B = \frac{a^2 - c^2}{(b^2-c^2)(d^2-c^2)}}$

put $y + d^2 = 0 \Rightarrow \boxed{y = -d^2}$ in (2)



$(2) \Rightarrow \boxed{C = \frac{d^2 - a^2}{(c^2-d^2)(b^2-d^2)}}$

put the values of A, B & C in (1)

$$\frac{y + a^2}{(y+b^2)(y+c^2)(y+d^2)} = \frac{a^2 - b^2}{(c^2-b^2)(d^2-b^2)(y+b^2)} + \frac{a^2 - c^2}{(b^2-c^2)(d^2-c^2)(y+c^2)} + \frac{d^2 - a^2}{(c^2-d^2)(b^2-d^2)(y+d^2)}$$

$$\boxed{\frac{x + a^2}{(x+b^2)(x+c^2)(x+d^2)} = \frac{a^2 - b^2}{(c^2-b^2)(d^2-b^2)(x+b^2)} + \frac{a^2 - c^2}{(b^2-c^2)(d^2-c^2)(x+c^2)} + \frac{d^2 - a^2}{(c^2-d^2)(b^2-d^2)(x+d^2)}}$$

Ans.

**(6)** $\dfrac{1}{x^4(x+1)}$

**Solution:** $\dfrac{1}{x^4(x+1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x^4} + \dfrac{E}{x+1}$ —— (1)

Xplying throughout eq$^n$ (1) by $x^4(x + 1)$

$1 = Ax^3(x + 1) + Bx^2(x + 1) + Cx(x + 1) + D(x + 1) + Ex^4$ ——— (2)

put $x = 0$ in eq$^n$ (2) $\Rightarrow \boxed{D = 1}$

eq$^n$ (2) can be written as

$1 = A(x^4 + x^3) + B(x^3 + x^2) + C(x^2 + x) + D(x + 1) + Ex^4$

equating the coefficients of $x^4, x^3, x^2, x$

$A + E = 0$ ——— (3)

$A + B = 0$ ——— (4)

$B + C = 0$ ——— (5)

$C + D = 0$ ——— (6)

put $D = 1$ in (6) $\Rightarrow \boxed{C = -1}$

put $C = -1$ in (5) $\Rightarrow \boxed{B = 1}$

put $B = 1$ in (4) $\Rightarrow \boxed{A = -1}$

put $A = -1$ in (3) $\Rightarrow \boxed{E = 1}$

put these values of A, B, C, D, E in (1)

$$(1) \Rightarrow \boxed{\frac{1}{x^4(x+1)} = \frac{-1}{x} + \frac{1}{x^2} - \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x+1}} \quad \text{Ans.}$$

**(7)** $\dfrac{4x^3}{(x+1)^2(x^2-1)}$

**Solution:** $\dfrac{4x^3}{(x+1)^2(x^2-1)} = \dfrac{4x^3}{(x+1)^2(x+1)(x-1)} = \dfrac{4x^3}{(x+1)^3(x-1)}$

$$\frac{4x^3}{(x+1)^3(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{D}{x-1} \text{ —— (1)}$$

Xplying throughout eq$^n$ (1) by $(x+1)^3(x-1)$

$4x^3 = A(x+1)^2(x-1) + B(x+1)(x-1) + C(x-1) + D(x+1)^3$ ——— (2)

put $x + 1 = 0 \Rightarrow \boxed{x=-1}$ in eq$^n$ (2) $\Rightarrow \boxed{c=2}$

eqn (2) can be written as

$4x^3 = A(x^3 + x^2 - x - 1) + B(x^2 - 1) + C(x - 1) + D(x^3 + 3x^2 + 3x + 1)$

equating the coefficient of $x^3$, $x^2$, x and constant terms.

$A + D = 4$ ——— (3)

$A + B + 3D = 0$ ——— (4)

$C - A + 3D = 0$ ——— (5)

$-A - B - C + D = 0$ ——— (6)

put $x - 1 = 0 \Rightarrow \boxed{x=1}$ in eq$^n$ (2) $\Rightarrow \boxed{D=\frac{1}{2}}$

put the value of D in (3) $\Rightarrow \boxed{A=\frac{7}{2}}$

put the values of A & D in eq$^n$ (4)

(4)$\Rightarrow \dfrac{7}{2} + B + \dfrac{3}{2} = 0 \Rightarrow \boxed{B=-5}$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \frac{4x^3}{(x+1)^3(x-1)} = \frac{\frac{7}{2}}{x+1} + \frac{-5}{(x+1)^2} + \frac{2}{(x+1)^3} + \frac{\frac{1}{2}}{x-1}$$

$$\boxed{\frac{4x^3}{(x+1)^3(x-1)} = \frac{7}{2(x+1)} - \frac{5}{(x+1)^2} + \frac{2}{(x+1)^3} + \frac{1}{2(x-1)}}$$

Ans.

**(8)** $\dfrac{x^4+1}{x^2(x-1)}$

**Solution:** $\dfrac{x^4+1}{x^2(x-1)} = \dfrac{x^4+1}{x^3-x^2}$

We first divide N (x) by D (x).



$$\begin{array}{r|l}
 & \quad x+1 \\ \hline
x^3-x^2 & \cancel{x^4}+1 \\
 & \underline{-\cancel{x^4} \mp x^3} \\
 & \quad \cancel{x^3}+1 \\
 & \underline{-\cancel{x^3} \mp x^2} \\
 & \quad x^2+1
\end{array}$$

$$\frac{x^4+1}{x^2(x-1)} = x + 1 + \frac{x^2+1}{x^2(x-1)} \qquad \boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$$

let $\dfrac{x^2+1}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$ ——— (1)

Xplying throughout eq$^n$ (1) by $x^2(x-1)$

$x^2 + 1 = Ax(x-1) + B(x-1) + Cx^2$ —— (2)

put x = 0 in eq$^n$ (2) $\Rightarrow \boxed{B=-1}$

put $x - 1 = 0 \Rightarrow \boxed{x=1}$ in eq$^n$ (2) $\Rightarrow \boxed{C=2}$

equating the coefficient of $x^2$

$A + C = 1 \Rightarrow A + 2 = 1 \Rightarrow \boxed{A=-1}$

$$(1) \Rightarrow \frac{x^2+1}{x^2(x-1)} = \frac{-1}{x} - \frac{1}{x^2} + \frac{2}{x-1}$$

$$\boxed{\therefore \frac{x^4+1}{x^2(x-1)} = x + 1 - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-1}}$$

Ans.

**(9)** $\dfrac{2x+1}{(x+3)(x-1)(x+2)^2}$

**Solution:**

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2} \text{ — (1)}$$

Xplyng throughout eq$^n$ (1) by $(x+3)(x-1)(x+2)^2$

$2x + 1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x-1)(x+3)(x+2) + D(x+3)(x-1)$ —— (2)

put $x + 3 = 0 \Rightarrow \boxed{x=-3}$ in eq$^n$ (2) $\Rightarrow \boxed{A=\frac{5}{4}}$

put $x - 1 = 0 \Rightarrow \boxed{x=1}$ in eq$^n$ (2) $\Rightarrow \boxed{B=\frac{1}{12}}$

put $x + 2 = 0 \Rightarrow \boxed{x=-2}$ in eq$^n$ (2) $\Rightarrow \boxed{D=1}$

eq$^n$ (2) can be written as

$2x + 1 = A(x^3 + 3x^2 - 4) + B(x^3 + 7x^2 + 16x + 12) + C(x^3 + 4x^2 + x - 6) + D(x^2 + 2x - 3)$

equating the coefficient of $x^3$

$$A + B + C = 0 \Rightarrow \frac{5}{4} + \frac{1}{12} + C = 0 \Rightarrow \boxed{C = \frac{-4}{3}}$$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{\frac{5}{4}}{x+3} + \frac{\frac{1}{12}}{x-1} - \frac{\frac{4}{3}}{x+2} + \frac{1}{(x+2)^2}$$

$$\boxed{\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}}$$

Ans

**(10)** $\dfrac{1}{(x+1)(x^2+1)}$

**Solution:** let $\dfrac{1}{(x+1)(x^2+1)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x+1)(x^2+1)$

$1 = A(x^2+1) + Bx(x+1) + C(x+1)$ —— (2)

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in eq$^n$ (2) $\Rightarrow \boxed{A = \frac{1}{2}}$

eq$^n$ (2) can be written as

$1 = (A+B)x^2 + (B+C)x + (A+C)$

equating the coefficients of $x^2$ & x.

$$A + B = 0 \Rightarrow \frac{1}{2} + B = 0 \Rightarrow \boxed{B = \frac{-1}{2}}$$

$$B + C = 0 \Rightarrow \frac{-1}{2} + C = 0 \Rightarrow \boxed{C = \frac{1}{2}}$$

put the values of A, B & C in (1)

$$(1) \Rightarrow \frac{1}{(x+1)(x^2+1)} = \frac{\frac{1}{2}}{x+1} + \frac{\frac{-1}{2}x + \frac{1}{2}}{x^2+1}$$

$$\boxed{\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{1-x}{2(x^2+1)}}$$

Ans.

**(11)** $\dfrac{3x+7}{(x^2+x+1)(x^2-4)}$

**Solution:** $\dfrac{3x+7}{(x^2+x+1)(x^2-4)} = \dfrac{3x+7}{(x^2+x+1)(x-2)(x+2)}$



let $\dfrac{3x+7}{(x^2+x+1)(x^2-4)} = \dfrac{Ax+B}{x^2+x+1} + \dfrac{C}{x-2} + \dfrac{D}{x+2}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x^2+x+1)(x^2-4)$

$3x + 7 = Ax(x^2-4) + B(x^2-4) + C(x+2)(x^2+x+1) + D(x-2)(x^2+x+1)$ —— (2)

put $x - 2 = 0 \Rightarrow \boxed{x = 2}$ in eq$^n$ (2) $\Rightarrow \boxed{C = \frac{13}{28}}$

put $x + 2 = 0 \Rightarrow \boxed{x = -2}$ in eq$^n$ (2) $\Rightarrow \boxed{D = \frac{-1}{12}}$

eq$^n$ (2) can be written as

$3x + 7 = (A + C + D)x^3 + (B + 3C - D)x^2 + (3C - 4A - D)x - 4B + 2C - 2D$

equating the coefficients of $x^3$, $x^2$ we have

$$A + C + D = 0 \Rightarrow A + \frac{13}{28} - \frac{1}{12} = 0 \Rightarrow A + \frac{39-7}{84} = 0$$

$$A + \frac{\overset{8}{\cancel{32}}}{\underset{21}{\cancel{84}}} = 0 \Rightarrow \boxed{A = \frac{-8}{21}}$$

$$B + 3C - D = 0 \Rightarrow B + \frac{39}{28} + \frac{1}{12} = 0$$

$$B + \frac{117+7}{84} = 0 \Rightarrow B + \frac{\overset{31}{\cancel{124}}}{\underset{21}{\cancel{84}}} = 0 \Rightarrow \boxed{B = \frac{-31}{21}}$$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \frac{3x+7}{(x^2+x+1)(x^2-4)} = \frac{\frac{-8}{21}x - \frac{31}{21}}{x^2+x+1} + \frac{\frac{13}{28}}{x-2} + \frac{\frac{-1}{12}}{x+2}$$

$$\boxed{\frac{3x+7}{(x^2+x+1)(x^2-4)} = \frac{-1}{21}\left(\frac{8x+31}{x^2+x+1}\right) + \frac{13}{28(x-2)} - \frac{1}{12(x+2)}}$$

Ans.

**(12)** $\dfrac{x^5}{x^4-1}$

**Solution:** first we ÷ N (x) by D (x)

$$\frac{x^5}{x^4-1} = x + \frac{x}{x^4-1} \text{ —— (1)}$$

$$\boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$$

$$\begin{array}{r|l} & x \\ x^4 - 1 & x^5 \\ & -x^5 \mp x \\ \hline & x \end{array}$$

let $\frac{x}{x^4 - 1} = \frac{x}{(x-1)(x+1)(x^2+1)}$

$$\frac{x}{x^4 - 1} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx + D}{x^2 + 1} \text{———— (2)}$$

Xplying throughout eq$^n$ (2) by $(x-1)(x+1)(x^2+1)$

$x = A(x+1)(x^2+1) + B(x-1)(x^2+1) + Cx(x^2-1) + D(x^2 - 1)$ ——— (3)

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in eq$^n$ (3) $\Rightarrow \boxed{A = \frac{1}{4}}$

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in eq$^n$ (3) $\Rightarrow \boxed{B = \frac{1}{4}}$

eq$^n$ (2) can be written as

$x = (A + B + C)x^3 + (A - B + D)x^2 + (A + B - C)x + (A - B - D)$

equating the coefficient of $x^3$ & $x^2$

$A + B + C = 0 \Rightarrow \frac{1}{4} + \frac{1}{4} + C = 0 \Rightarrow \boxed{C = \frac{-1}{2}}$

$A - B + D = 0 \Rightarrow \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + D = 0 \Rightarrow \boxed{D = 0}$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \boxed{\frac{x^5}{x^4 - 1} = x + \frac{1}{4(x-1)} + \frac{1}{4(x+1)} - \frac{x}{2(x^2+1)}} \text{ Ans.}$$

**(13)** $\dfrac{\mathbf{x + a}}{\mathbf{x^2 (x - a)(x^2 + a^2)}}$

**Solution:** $\frac{x + a}{x^2(x-a)(x^2+a^2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-a} + \frac{Dx + E}{x^2 + a^2}$ —— (1)

Xplying throughout eq$^n$ (1) by $x^2(x-a)(x^2+a^2)$

$x + a = Ax(x - a)(x^2 + a^2) + B(x-a)(x^2+a^2) + C(x^2+a^2)x^2 + Dx^3(x-a) + Ex^2(x-a)$ —— (2)

put $x = 0$ in (2) $\Rightarrow \boxed{B = \frac{-1}{a^2}}$

put $x - a = 0 \Rightarrow \boxed{x = a}$ in (2) $\Rightarrow \boxed{C = \frac{1}{a^3}}$

eq$^n$ (2) can be written as

$x + a = (A + C + D)x^4 + (B - aA - aD + E)x^3 + (a^2A - aB + a^2C - aE)x^2 + (a^2B - a^3A)x - a^3B$

equating the coefficient of $x^4$, $x^3$ & x



$a^2B - a^3A = 1 \Rightarrow \cancel{a^2}\left(\frac{-1}{\cancel{a^2}}\right) - a^3A = 1 \Rightarrow -1 - 1 = a^3A$

$\boxed{A = \frac{-2}{a^3}}$

$A + C + D = 0 \Rightarrow \frac{-2}{a^3} + \frac{1}{a^3} + D = 0 \Rightarrow \boxed{D = \frac{1}{a^3}}$

$B - aA - aD + E = 0 \Rightarrow \frac{-1}{a^2} - a\left(\frac{-2}{a^3}\right) - a\left(\frac{1}{a^3}\right) + E = 0$

$\frac{-1}{a^2} + \frac{2a}{a^3} - \frac{1}{a^2} + E = 0 \Rightarrow E + \frac{2 - 2}{a^2} = 0 \Rightarrow \boxed{E = 0}$

put the values of A, B, C, D & E in (1)

$$(1) \Rightarrow \frac{x + a}{x^2(x-a)(x^2+a^2)} = \frac{\frac{-2}{a^3}}{x} + \frac{\frac{-1}{a^2}}{x^2} + \frac{\frac{1}{a^3}}{x - a} + \frac{\frac{1}{a^3}x}{x^2 + a^2}$$

$$\boxed{\frac{x + a}{x^2(x-a)(x^2+a^2)} = \frac{-2}{a^3x} - \frac{1}{a^2x^2} + \frac{1}{a^3(x-a)} + \frac{x}{a^3(x^2+a^2)}} \text{ Ans.}$$

**(14)** $\dfrac{\mathbf{4x^4 + 3x^3 + 6x^2 + 5x}}{\mathbf{(x - 1)(x^2 + x + 1)^2}}$

**Solution:**

$$\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2} = \frac{A}{x-1} + \frac{Bx + c}{x^2 + x + 1} + \frac{Dx + E}{(x^2+x+1)^2} \text{—— (1)}$$

Xplying throughout eq$^n$ (1) by $(x-1)(x^2+x+1)$

$4x^4 + 3x^3 + 6x^2 + 5x = A(x^2+x+1)^2 + Bx(x-1)(x^2+x+1) + C(x-1)(x^2+x+1) + Dx(x-1) + E(x-1)$ —— (2)

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (2) $\boxed{A = 2}$

eq$^n$ (2) can be written as

$4x^3 + 3x^3 + 6x^2 + 5x = (A + B)x^4 + (2A + C)x^3 + (3A + D)x^2 + (2A - B - D + E)x + (A - C - E)$

equating the Coefficient of $x^4$, $x^3$, $x^2$ and constant term

$A + B = 4 \Rightarrow 2 + B = 4 \Rightarrow \boxed{B = 2}$

$2A + C = 3 \Rightarrow 2(2) + C = 3 \Rightarrow \boxed{C = -1}$

$3A + D = 6 \Rightarrow 3(2) + D = 6 \Rightarrow \boxed{D = 0}$

$A - C - E = 0 \Rightarrow 2 + 1 - E = 0 \Rightarrow \boxed{E = 3}$

put the values of A, B, C, D & E in (1)

$$(1) \Rightarrow \boxed{\frac{4x^4 + 3x^3 + 6x^2 + 5x}{(x-1)(x^2+x+1)^2} = \frac{2}{x-1} + \frac{2x - 1}{x^2 + x + 1} + \frac{3}{(x^2+x+1)^2}} \text{ Ans.}$$

**(15)** $\dfrac{x^3-15x^2-8x-7}{(1+x^2)^2(2x-5)}$

**Solution:** $\dfrac{x^3-15x^2-8x-7}{(1+x^2)^2(2x-5)}=\dfrac{Ax+B}{1+x^2}+\dfrac{Cx+D}{(1+x^2)^2}+\dfrac{E}{2x-5}$ —(1)

Xplying throughout eq$^n$ (1) by $(1+x^2)^2(2x-5)$

$x^3-15x^2-8x-7=Ax(2x-5)(1+x^2)+B(2x-5)(1+x^2)+Cx(2x-5)+D(2x-5)+E(1+x^2)^2$ —— (2)

put $2x-5=0 \Rightarrow \boxed{x=\frac{5}{2}}$ in (2)

$(2)\Rightarrow \dfrac{125}{8}-\dfrac{375}{4}-\dfrac{40}{2}-7=E\left(\dfrac{29}{4}\right)^2$

$\dfrac{125-750-160-56}{8}=E\left(\dfrac{841}{16}\right)$

$-\dfrac{841}{8}=E\left(\dfrac{841}{16}\right)\Rightarrow \boxed{E=-2}$

eq$^n$ (2) can be written as

$x^3-15x^2-8x-7=(2A+E)x^4+(2B-5A)x^3+(2A-5B+2C+2E)x^2+(2B-5A-5C+2D)x-(5B+5D-E)$

equating the coefficients of $x^4$, $x^3$, $x^2$ and the constant terms.

$2A+E=0\Rightarrow 2A-2=0\Rightarrow \boxed{A=1}$

$2B-5A=1\Rightarrow 2B-5(1)=1\Rightarrow \boxed{B=3}$

$2A-5B+2C+2E=-15\Rightarrow 2(1)-5(3)+2C+2(-2)=-15$

$\Rightarrow \boxed{C=1}$ $\cancel{-}(5B+5D-E)=\cancel{-}7\Rightarrow 5(3)+5D+2=7$

$\Rightarrow \boxed{D=-2}$

put the values of A, B, C, D & E in (1)

$$\boxed{\dfrac{x^3-15x^2-8x-7}{(1+x^2)^2(2x-5)}=\dfrac{x+3}{1+x^2}+\dfrac{x-2}{(1+x^2)^2}-\dfrac{2}{2x-5}}\quad \text{Ans.}$$

**(16)** $\dfrac{8x^2}{(1-x^4)(1+x^2)}$

**Solution:** $\dfrac{8x^2}{(1-x^4)(1+x^2)}=\dfrac{8x^2}{(1-x)(1+x)(1+x^2)(1+x^2)}$

(or) $\dfrac{8x^2}{(1-x^4)(1+x^2)}=\dfrac{8x^2}{(1-x)(1+x)(1+x^2)^2}$

let $\dfrac{8x^2}{(1-x^4)(1+x^2)}=\dfrac{A}{1-x}+\dfrac{B}{1+x}+\dfrac{Cx+D}{1+x^2}+\dfrac{Ex+F}{(1+x^2)^2}$ —(1)



Xplying throughout eq$^n$ (1) by $(1-x)(1+x)(1+x^2)^2$

$8x^2=A(1+x)(1+x^2)^2+B(1-x)(1+x^2)^2+Cx(1-x^2)(1+x^2)+D(1-x^2)(1+x^2)+Ex(1-x^2)+F(1-x^2)$ —(2)

put $1-x=0\Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{A=1}$

put $1+x=0\Rightarrow \boxed{x=-1}$ in (2) $\Rightarrow \boxed{B=1}$

eq$^n$ (2) can be written as

$8x^2=(A-B-C)x^2+(A+B-D)x^4+(2A-2B-E)x^3+(2A+2B-F)x^2+(A-B+C+E)x+A+B+D+F$

equating the coefficient of $x^5$, $x^4$, $x^3$ & $x^2$

$A-B-C=0\Rightarrow \cancel{1}-\cancel{1}-C=0\Rightarrow \boxed{C=0}$

$A+B-D=0\Rightarrow 1+1-D=0\Rightarrow \boxed{D=2}$

$2A-2B-E=0\Rightarrow 2(1)-2(1)-E=0\Rightarrow \boxed{E=0}$

$2A+2B-F=8\Rightarrow 2(1)+2(1)-F=8\Rightarrow \boxed{F=-4}$

put the values of A, B, C, D, E & F in (1)

$$(1)\Rightarrow \boxed{\dfrac{8x^2}{(1-x^4)(1+x^2)}=\dfrac{1}{1-x}+\dfrac{1}{1+x}+\dfrac{2}{1+x^2}-\dfrac{4}{(1+x^2)^2}}\quad \text{Ans.}$$

**(17)** $\dfrac{x^3+1}{(x^2-1)^2}$

**Solution:** $\dfrac{x^3+1}{(x^2-1)^2}=\dfrac{\cancel{(x+1)}(x^2-x+1)}{(x-1)^2(x+1)^2}=\dfrac{x^2-x+1}{(x-1)^2(x+1)}$

let $\dfrac{x^2-x+1}{(x+1)(x-1)^2}=\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x+1)(x-1)^2$

$x^2-x+1=A(x-1)^2+B(x^2-1)+C(x+1)$ —— (2)

put $x+1=0\Rightarrow \boxed{x=-1}$ in (2) $\Rightarrow \boxed{A=\frac{3}{4}}$

put $x-1=0\Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{C=\frac{1}{2}}$

eq$^n$ (2) can be written as

$x^2-x+1=(A+B)x^2+(C-2A)x+A-B+C$

equating the Coefficient of $x^2$

$A+B=1\Rightarrow \frac{3}{4}+B=1\Rightarrow \boxed{B=\frac{1}{4}}$

put the values of A, B & C in (1)

$$\boxed{\frac{x^3+1}{(x^2-1)^2}=\frac{3}{4(x+1)}+\frac{1}{4(x-1)}+\frac{1}{2(x-1)^2}}\quad \text{Ans.}$$

**(18)** $\dfrac{x^2}{x^4-5x^2+4}$

**Solution:** $\dfrac{x^2}{x^4-5x^2+4}=\dfrac{x^2}{(x^2-4)(x^2-1)}=\dfrac{x^2}{(x-2)(x+2)(x-1)(x+1)}$

let $\dfrac{x^2}{(x-2)(x+2)(x-1)(x+1)}=\dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{C}{x-1}+\dfrac{D}{x+1}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x-2)(x+2)(x-1)(x+1)$

$x^2=A(x+2)(x-1)(x+1)+B(x-2)(x-1)(x+1)+C(x-2)(x+2)(x+1)+D(x-2)(x+2)(x-1)$ —— (2)

put $x-2=0\Rightarrow \boxed{x=2}$ in (2) $\Rightarrow \boxed{A=\frac{1}{3}}$

put $x+2=0\Rightarrow \boxed{x=-2}$ in (2) $\Rightarrow \boxed{B=\frac{-1}{3}}$

put $x-1=0\Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{C=\frac{-1}{6}}$

put $x+1=0\Rightarrow \boxed{x=-1}$ in (2) $\Rightarrow \boxed{D=\frac{1}{6}}$

put the values of A, B, C & D in (1)

$$(1)\Rightarrow \boxed{\frac{x^2}{x^4-5x^2+4}=\frac{1}{3(x-2)}-\frac{1}{3(x+2)}-\frac{1}{6(x-1)}+\frac{1}{6(x+1)}}$$

Ans.

**(19)** $\dfrac{x^2+2x-1}{x^3-27}$

**Solution:** $\dfrac{x^2+2x-1}{x^3-27}=\dfrac{x^2+2x-1}{(x-3)(x^2+3x+9)}$

let $\dfrac{x^2+2x-1}{(x-3)(x^2+3x+9)}=\dfrac{A}{x-3}+\dfrac{Bx+C}{x^2+3x+9}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x-3)(x^2+3x+9)$

$x^2+2x-1=A(x^2+3x+9)+Bx(x-3)+C(x-3)$ —— (2)

put $x-3=0\Rightarrow \boxed{x=3}$ in (2) $\Rightarrow \boxed{A=\frac{14}{27}}$

eq$^n$ (2) can be written as



$x^2+2x-1=(A+B)x^2+(3A-3B+C)x+9A-3C$

equating the coefficient of $x^2$ & constant terms

$A+B=1\Rightarrow \frac{14}{27}+B=1\Rightarrow \boxed{B=\frac{13}{27}}$

$9A-3C=-1\Rightarrow 9\left(\frac{14}{27}\right)-3C=1$

$\frac{126}{27}+1=3C\Rightarrow \frac{\overset{51}{\cancel{153}}}{27\times\cancel{3}}=C\Rightarrow \boxed{C=\frac{51}{27}}$

put the values of A, B & C in (1)

$$(1)\Rightarrow \frac{x^2+2x-1}{x^3-27}=\frac{14}{27(x-3)}+\frac{\frac{13}{27}x+\frac{51}{27}}{x^2+3x+9}$$

$$\boxed{\frac{x^2+2x-1}{x^3-27}=\frac{14}{27(x-3)}+\frac{13x+51}{27(x^2+3x+9)}}\quad \text{Ans.}$$

**(20)** $\dfrac{x^2-3x+5}{x^4-8x^2+16}$

**Solution:** $\dfrac{x^2-3x+5}{x^4-8x^2+16}=\dfrac{x^2-3x+5}{(x^2-4)^2}=\dfrac{x^2-3x+5}{(x-2)^2(x+2)^2}$

let $\dfrac{x^2-3x+5}{(x-2)^2(x+2)^2}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+2}+\dfrac{D}{(x+2)^2}$ —— (1)

Xplying throughout eq$^n$ (1) by $(x-2)^2(x+2)^2$

$x^2-3x+5=A(x-2)(x+2)^2+B(x+2)^2+C(x+2)(x-2)^2+D(x-2)^2$ —— (2)

put $x-2=0\Rightarrow \boxed{x=2}$ in (2) $\Rightarrow \boxed{B=\frac{3}{16}}$

put $x+2=0\Rightarrow \boxed{x=-2}$ in (2) $\Rightarrow \boxed{D=\frac{15}{16}}$

eq$^n$ (2) can be written as

$x^2-3x+5=(A+C)x^3+(2A+B-2C+D)x^2+4(B-A-C-D)x-8A+4B+8C+4D$

equating the coefficient of $x^3$ & $x^2$

$A+C=0\Rightarrow A=-C$ —— (3)

$2A+B-2C+D=1\Rightarrow 2(-c)+\frac{3}{16}-2C+\frac{15}{16}=1$

$-4C+\frac{18}{16}=1\Rightarrow \frac{18}{16}-1=4c\Rightarrow \frac{\cancel{2}}{16\times\cancel{4}_2}=C$

$\boxed{C = \frac{1}{32}}$ put in (3) $\Rightarrow$ $\boxed{A = \frac{-1}{32}}$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \boxed{\frac{x^2 - 3x + 5}{x^4 - 8x^2 + 16} = \frac{-1}{32(x-2)} + \frac{3}{16(x-2)^2} + \frac{1}{32(x+2)} + \frac{15}{16(x+2)^2}}$$

**(21)** $\dfrac{8x^2}{(1 - x^4)(1 + x^2)}$

**Solution:** $\dfrac{8x^2}{(1-x^4)(1+x^2)} = \dfrac{8x^2}{(1-x^2)(1+x^2)(1+x^2)}$

$= \dfrac{8x^2}{(1-x)(1+x)(1+x^2)^2}$

let $\dfrac{8x^2}{(1-x)(1+x)(1+x^2)^2} = \dfrac{A}{1-x} + \dfrac{B}{1+x} + \dfrac{Cx+D}{1+x^2} + \dfrac{Ex+F}{(1+x^2)^2}$ —(1)

Xplying throughout eq$^n$ (1) by $(1-x)(1+x)(1+x^2)^2$

$8x^2 = A(1+x)(1+x^2)^2 + B(1-x)(1+x^2)^2 + Cx(1-x^2)(1+x^2) + D(1-x^2)(1+x^2) + Ex(1-x^2) + F(1-x^2)$ —(2)

put $1 - x = 0 \Rightarrow \boxed{x = 1}$ in (2) $\Rightarrow \boxed{A = 1}$

put $1 + x = 0 \Rightarrow \boxed{x = -1}$ in (2) $\Rightarrow \boxed{B = 1}$

eq$^n$ (2) can be written as

$8x^2 = (A - B - C)x^5 + (-D + 2A - B - E)x^3 + (2A + 2B - F)x^2 + (A - B + C + E)x + (A + B + D + F)$

equating the coefficients of $x^5, x^3, x^2, x$

$A - B - C = 0 \Rightarrow \cancel{1} - \cancel{1} - C = 0 \Rightarrow \boxed{C = 0}$

$2A + 2B - F = 8 \Rightarrow 2(1) + 2(1) - F = 8 \Rightarrow \boxed{F = -4}$

$A - B + C + E = O \Rightarrow \cancel{1} - \cancel{1} + O + E = O \Rightarrow \boxed{E = O}$

$A + B + D + F = O \Rightarrow 1 + 1 + D - 4 = 0 \Rightarrow \boxed{D = 2}$

put the values of A, B, C, D, E & F in (1)

$$(1) \Rightarrow \boxed{\frac{8x^2}{(1-x^4)(1+x^2)} = \frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} - \frac{4}{(1+x^2)^2}}$$

**(22)** $\dfrac{x^2}{(x-1)^3(x+1)}$

**Solution:** $\dfrac{x^2}{(x+1)(x-1)^3} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2} + \dfrac{D}{(x-1)^3}$ —(1)

Xplying throughout eq$^n$ (1) by $(x+1)(x-1)^3$

$x^2 = A(x-1)^3 + B(x+1)(x-1)^2 + C(x+1)(x-1)^2 + D(x+1)$ —(2)



put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in (2) $\Rightarrow \boxed{A = \frac{-1}{8}}$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (2) $\Rightarrow \boxed{D = \frac{1}{2}}$

eq$^n$ (2) can be written as

$x^2 = (A + B)x^3 + (C - 3A - B)x^2 + (3A - B + D)x - A + B - C + D$

equating the coefficient of $x^3$ & $x^2$

$A + B = O \Rightarrow \frac{-1}{8} + B = O \Rightarrow \boxed{B = \frac{1}{8}}$

$C - 3A - B = 1 \Rightarrow C + \frac{3}{8} - \frac{1}{8} = 1 \Rightarrow C = 1 - \frac{2}{8} \Rightarrow \boxed{C = \frac{3}{4}}$

put the values of A, B, C & D in (1)

$$(1) \Rightarrow \boxed{\frac{x^2}{(x+1)(x-1)^3} = \frac{-1}{8(x+1)} + \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3}}$$

**(23)** $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$

**Solution:** $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = \dfrac{x^3 - 6x^2 + 11x - 6}{x^3 - 15x^2 + 74x - 120}$

first we $\div$ N (x) by D (x)

$$\begin{array}{r|l}
 & 1 \\
x^3 - 15x^2 + 74x - 120 & x^3 - 6x^2 + 11x - 6 \\
 & \underline{-x^3 \mp 15x^2 \pm 74x \mp 120} \\
 & 9x^2 - 63x + 114
\end{array}$$

(or) $3(3x^2 - 21x + 38)$

$\dfrac{x^3 - 6x^2 + 11x - 6}{x^3 - 15x^2 + 74x - 120} = 1 + 3\left\{\dfrac{3x^2 - 21x + 38}{(x-4)(x-5)(x-6)}\right\}$

Suppose $\dfrac{3x^2 - 21x + 38}{(x-4)(x-5)(x-6)} = \dfrac{A}{x-4} + \dfrac{B}{x-5} + \dfrac{C}{x-6}$ —(1)

Xplying throughout (1) by $(x-4)(x-5)(x-6)$

$3x^2 - 21x + 38 = A(x-5)(x-6) + B(x-4)(x-6) + C(x-4)(x-5)$ —(2)

put $x - 4 = 0 \Rightarrow \boxed{x = 4}$ in (2) $\Rightarrow \boxed{A = 1}$

put $x - 5 = 0 \Rightarrow \boxed{x = 5}$ in (2) $\Rightarrow \boxed{B = -8}$

put $x - 6 = 0 \Rightarrow \boxed{x = 6}$ in (2) $\Rightarrow \boxed{C = 10}$

put the values of A, B & C in (1)

$$\boxed{\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + 3\left\{\frac{1}{x-4} - \frac{8}{x-5} + \frac{10}{x-6}\right\}} \text{ Ans.}$$

**(24)** $\dfrac{x^3+3x^2-2x+1}{x^4+5x^2+4}$

**Solution:** $\dfrac{x^3+3x^2-2x+1}{x^4+5x^2+4} = \dfrac{x^3+3x^2-2x+1}{(x^2+4)(x^2+1)}$

$$\frac{x^3+3x^2-2x+1}{x^4+5x^2+4} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{x^2+1} \text{ ——— (1)}$$

Xplying throughout (1) by $(x^2+4)(x^2+1)$

$x^3+3x^2-2x+1 = Ax(x^2+1) + B(x^2+1) + Cx(x^2+4) + D(x^2+4)$ ——— (2)

$x^3+3x^2-2x+1 = A(x^3+x) + B(x^2+1) + C(x^3+4x) + D(x^2+4)$

$x^3+3x^2-2x+1 = (A+C)x^3 + (B+D)x^2 + (A+4C)x + B + 4D$

equating the Coefficient of $x^3$, $x^2$, x and constant terms

$A + C = 1$ ——— (3)

$B + D = 3$ ——— (4)

$A + 4C = -2$ ——— (5)

$B + 4D = 1$ ——— (6)

(3) – (5)

$$\begin{array}{r} \cancel{A} + C = 1 \\ -\cancel{A} \pm 4C = \mp 2 \\ \hline -3C = 3 \Rightarrow \boxed{C = -1} \end{array}$$

put the value of C in (3)

(3)⇒ $A - 1 = 1 \Rightarrow \boxed{A = 2}$

(4) – (6)

$$\begin{array}{r} \cancel{B} + D = 3 \\ -\cancel{B} \pm 4D = -1 \\ \hline -3D = 2 \Rightarrow \boxed{D = \frac{-2}{3}} \end{array}$$

put the value of D in (4)

(4)⇒ $B - \dfrac{2}{3} = 3 \Rightarrow \boxed{B = \dfrac{11}{3}}$

put the values of A, B, C & D in (1)

$$\frac{x^3+3x^2-2x+1}{x^4+5x^2+4} = \frac{2x + \frac{11}{3}}{x^2+4} + \frac{(-1)x - \frac{2}{3}}{x^2+1}$$



$$\frac{x^3+3x^2-2x+1}{x^4+5x^2+4} = \frac{6x+11}{3(x^2+4)} - \frac{(3x+2)}{3(x^2+1)}$$

$$\boxed{\frac{x^3+3x^2-2x+1}{x^4+5x^2+4} = \frac{1}{3}\left[\frac{6x+11}{x^2+4} - \frac{(3x+2)}{x^2+1}\right]} \text{ Ans.}$$

## EXERCISE # 6.9

**Calculate the following integrals.**

**(1)** $\displaystyle\int \frac{dx}{x^2-25}$

**Solution:** let $I = \displaystyle\int \frac{dx}{x^2-25}$ let partial fraction

$$\frac{1}{x^2-25} = \frac{1}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5} \text{ ——— (1)}$$

$$\frac{1}{\cancel{(x-5)(x+5)}} = \frac{A(x+5) + B(x-5)}{\cancel{(x-5)(x+5)}}$$

$1 = A(x+5) + B(x-5)$ ——— (2)

put $x - 5 = 0 \Rightarrow \boxed{x = 5}$ in (2)

(2)⇒ $1 = 10A \Rightarrow \boxed{A = \dfrac{1}{10}}$

put $x + 5 = 0 \Rightarrow \boxed{x = -5}$ in (2)

(2)⇒ $1 = -10B \Rightarrow \boxed{B = \dfrac{-1}{10}}$

put the values of A & B in (1)

(1)⇒ $\dfrac{1}{(x-5)(x+5)} = \dfrac{\frac{1}{10}}{x-5} - \dfrac{\frac{1}{10}}{x+5}$ on integration

$$\int \frac{1}{(x-5)(x+5)}dx = \frac{1}{10}\int \frac{1}{x-5}dx - \frac{1}{10}\int \frac{1}{x+5}dx$$

$$I = \frac{1}{10}\ell n(x-5) - \frac{1}{10}\ell n(x+5) + c$$

$$\boxed{I = \frac{1}{10}\ell n\frac{x-5}{x+5} + c} \text{ Ans.}$$

**(2)** $\displaystyle\int \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}dx$

**Solution:** let $I = \displaystyle\int \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}dx$

let partial fraction

$$\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3} \text{ ——— (1)}$$

Xplying throughout (1) by $(x-1)(x-2)(x-3)$

$3x^2-12x+11 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$ ——— (2)

put $x-1=0 \Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{A=1}$

put $x-2=0 \Rightarrow \boxed{x=2}$ in (2) $\Rightarrow \boxed{B=1}$

put $x-3=0 \Rightarrow \boxed{x=3}$ in (2) $\Rightarrow \boxed{C=1}$

put the values of A, B & C in (1)

$$(1)\Rightarrow \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}=\frac{1}{x-1}+\frac{1}{x-2}+\frac{1}{x-3} \text{ on integration}$$

$$\int\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}dx=\int\frac{1}{x-1}dx+\int\frac{1}{x-2}dx+\int\frac{1}{x-3}dx$$

$$I=\ell n(x-1)+\ell n(x-2)+\ell n(x-3)+\ell nc$$

$$\boxed{I=\ell n[c(x-1)(x-2)(x-3)]} \quad \text{Ans.}$$

**(3)** $\displaystyle\int\frac{(2x-1)\,dx}{x(x-1)(x-3)}$

**Solution:** let $I=\displaystyle\int\frac{(2x-1)\,dx}{x(x-1)(x-3)}$

We first resolve into partial fraction

$$\text{let } \frac{2x-1}{x(x-1)(x-3)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x-3} \text{ ——— (1)}$$

Xplying throughout eq$^n$ (1) by $x(x-1)(x-3)$

$2x-1=A(x-1)(x-3)+Bx(x-3)+Cx(x-1)$ —— (2)

put $x=0$ in (2) $\Rightarrow -1=3A \Rightarrow \boxed{A=\frac{-1}{3}}$

put $x-1=0 \Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{B=\frac{-1}{2}}$

put $x-3=0 \Rightarrow \boxed{x=3}$ in (2) $\Rightarrow \boxed{C=\frac{5}{6}}$

put the values of A, B & C in (1)

$$(1)\Rightarrow \frac{2x-1}{x(x-1)(x-3)}=\frac{-\frac{1}{3}}{x}-\frac{\frac{1}{2}}{x-1}+\frac{\frac{5}{6}}{x-3} \text{ on integration}$$



$$\int\frac{(2x-1)dx}{x(x-1)(x-3)}=\frac{-1}{3}\int\frac{1}{x}dx-\frac{1}{2}\int\frac{1}{x-1}dx+\frac{5}{6}\int\frac{1}{x-3}dx$$

$$I=\frac{-1}{3}\ell nx-\frac{1}{2}\ell n(x-1)+\frac{5}{6}\ell n(x-3)+\ell nc$$

$$I=-\ell nc^{1/3}-\ell n(x-1)^{1/2}+\ell n(x-3)^{5/6}+\ell nc$$

$$I=\ell nc(x-3)^{5/6}-\{\ell nx^{1/3}+\ell n(x-1)^{1/2}\}$$

$$I=\ell nc(x-3)^{5/6}-\ell nx^{1/3}\cdot(x-1)^{1/2}$$

$$\boxed{I=\ell n\left[\frac{c(x-3)^{5/6}}{x^{1/3}.(x-1)^{1/2}}\right]}$$

**(4)** $\displaystyle\int\frac{dx}{2x^2-5x+2}$

**Solution:** let $I=\displaystyle\int\frac{dx}{2x^2-5x+2}$

$$I=\int\frac{1}{2x^2-4x-x+2}dx=\int\frac{1}{2x(x-2)-(x-2)}dx$$

$$I=\int\frac{1}{(x-2)(2x-1)}dx \text{ ——— (1)}$$

We first resolve into partial fraction

$$\text{let } \frac{1}{(x-2)(2x-1)}=\frac{A}{x-2}+\frac{B}{2x-1} \text{ ——— (2)}$$

Xplying throughout eq$^n$ (2) by $(x-2)(2x-1)$

$1=A(2x-1)+B(x-2)$ ——— (3)

put $x-2=0 \Rightarrow \boxed{x=2}$ in (3) $\Rightarrow \boxed{A=\frac{1}{3}}$

put $2x-1=0 \Rightarrow \boxed{x=\frac{1}{2}}$ in (3) $\Rightarrow \boxed{B=\frac{-2}{3}}$

put the values of A & B in (2)

$$(2)\Rightarrow \frac{1}{(x-2)(2x-1)}=\frac{\frac{1}{3}}{x-2}+\frac{\frac{-2}{3}}{2x-1} \qquad \text{On integration}$$

$$\int\frac{1}{(x-2)(2x-1)}dx=\frac{1}{3}\int\frac{1}{x-2}dx-\frac{1}{3}\int\frac{2}{2x-1}dx$$

$$I=\frac{1}{3}\ell n(x-2)-\frac{1}{3}\ell n(2x-1)+\ell nc$$

$$I=\frac{1}{3}\ell n\frac{(x-2)}{(2x-1)}+\ell nc \Rightarrow \boxed{I=\frac{1}{3}\ell n\left\{\frac{c(x-2)}{(2x-1)}\right\}} \quad \text{Ans.}$$

**(5)** $\int \dfrac{\text{Cosx dx}}{\text{Sinx}(2+\text{Sinx})}$

**Solution:** let $I = \int \dfrac{\text{Cosx dx}}{\text{Sinx}(2+\text{Sinx})}$ ———— (1)

let y = Sinx diff w.r to x

$\dfrac{dy}{dx} = \text{Cosx} \Rightarrow \boxed{dy = \text{Cosx dx}}$

$(1) \Rightarrow I = \int \dfrac{dy}{y(2+y)}$ ———— (2)

We first resolve into partial fraction

$\dfrac{1}{y(y+2)} = \dfrac{A}{y} + \dfrac{B}{y+2}$ ———— (3)

Xplying throughout eq$^n$ (3) by y (y + 2)

$1 = A(y+2) + By$ ———— (4)

put y = 0 in (4) $\Rightarrow \boxed{A = \dfrac{1}{2}}$

put $y + 2 = 0 \Rightarrow \boxed{y = -2}$ in (4) $\Rightarrow \boxed{B = \dfrac{-1}{2}}$

put the values of A & B in (3)

$(3) \Rightarrow \dfrac{1}{y(y+2)} = \dfrac{\frac{1}{2}}{y} + \dfrac{\frac{-1}{2}}{y+2}$ on integration

$\int \dfrac{1}{y(y+2)} dy = \dfrac{1}{2}\int \dfrac{1}{y} dy - \dfrac{1}{2}\int \dfrac{1}{y+2} dy$

$I = \dfrac{1}{2}\ell ny - \dfrac{1}{2}\ell n(y+2) + c$

$I = \dfrac{1}{2}\ell n\left\{\dfrac{y}{y+2}\right\} + c$ $\therefore$ y = Sinx

$\boxed{I = \dfrac{1}{2}\ell n\left\{\dfrac{\text{Sinx}}{2+\text{Sinx}}\right\} + c}$ Ans.

**(6)** $\int \dfrac{\text{Sinx dx}}{(1+\text{Cosx})(2+\text{Cosx})}$

**Solution:** let $I = \int \dfrac{\text{Sinx dx}}{(1+\text{Cosx})(2+\text{Cosx})}$ ———— (1)

let y = Cosx diff w.r to x

$\dfrac{dy}{dx} = -\text{Sinx} \Rightarrow \boxed{-dy = \text{Sinx dx}}$

$(1) \Rightarrow I = \int \dfrac{-dy}{(1+y)(2+y)}$ ———— (2)



we first resolve into partial fraction

$\dfrac{-1}{(1+y)(2+y)} = \dfrac{A}{1+y} + \dfrac{B}{2+y}$ ———— (3)

Xplying throughout eq$^n$ (3) by (1 + y) (2 + y)

$-1 = A(2+y) + B(1+y)$ ———— (4)

put $1 + y = 0 \Rightarrow \boxed{y = -1}$ in (4) $\Rightarrow \boxed{A = -1}$

put $2 + y = 0 \Rightarrow \boxed{y = -2}$ in (4) $\Rightarrow \boxed{B = 1}$

put the values of A & B in (3)

$(3) \Rightarrow \dfrac{-1}{(1+y)(2+y)} = \dfrac{-1}{1+y} + \dfrac{1}{2+y}$ on integration

$\int \dfrac{-1\ dy}{(1+y)(2+y)} = -\int \dfrac{1}{1+y} dy + \int \dfrac{1}{y+2} dy$

$I = -\ell n(1+y) + \ell n(y+2) + c$

$I = \ell n\left\{\dfrac{2+y}{1+y}\right\} + c$ $\therefore$ y = Cosx

$\boxed{I = \ell n\left\{\dfrac{2+\text{Cosx}}{1+\text{Cosx}}\right\} + c}$ Ans.

**(7)** $\int \dfrac{x^2+3x+4}{x-2} dx$

**Solution:** let $I = \int \dfrac{x^2+3x+4}{x-2} dx$

We first divide the N (x) by D (x)

$I = \int \left\{x + 5 + \dfrac{14}{x-2}\right\} dx$

$$\begin{array}{r|l} & x+5 \\ \hline x-2 & x^2+3x+4 \\ & -x^2 \mp 2x \\ \hline & 5x+4 \\ & -5x \mp 10 \\ \hline & 14 \end{array}$$

$\boxed{\therefore \dfrac{N(x)}{D(x)} = Q(x) + \dfrac{R(x)}{D(x)}}$

$I = \int x dx + 5\int dx + 14\int \dfrac{1}{x-2} dx$

$\boxed{I = \dfrac{x^2}{2} + 5x + 14\ell n(x-2) + c}$ Ans.

**(8)** $\int \dfrac{x^3 - x^2 + 2x + 3}{x^2 + 3x + 2} dx$

**Solution:** let $I = \int \dfrac{x^3 - x^2 + 2x + 3}{x^2 + 3x + 2} dx$

We first divide the N (x) by D (x)

$$\begin{array}{r|l} & \quad x - 4 \\ x^2 + 3x + 2 & x^3 - x^2 + 2x + 3 \\ & \underline{-x^3 \pm 3x^2 \pm 2x} \\ & \quad -4x^2 + 3 \\ & \quad \underline{\mp 4x^2 \mp 12x \mp 8} \\ & \quad 12x + 11 \end{array}$$

$$\boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$$

$$I = \int \left\{x - 4 + \frac{12x + 11}{x^2 + 3x + 2}\right\} dx$$

$$I = \int x\,dx - 4\int dx + \int \frac{12x + 11}{x^2 + 3x + 2}\,dx \text{ ——— (1)}$$

$$\text{let } I = \int \frac{12x + 11\,dx}{x^2 + 3x + 2}$$

resolve into partial fraction

$$\frac{12x + 11}{x^2 + 3x + 2} = \frac{12x + 11}{(x + 2)(x + 1)} = \frac{A}{x + 1} + \frac{B}{x + 2} \text{ ——— (2)}$$

Xplying throughout eq$^n$ (2) by $(x + 1)(x + 2)$

$$12x + 11 = A(x + 2) + B(x + 1) \text{ ——— (3)}$$

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in (3) $\Rightarrow \boxed{A = -1}$

put $x + 2 = 0 \Rightarrow \boxed{x = -2}$ in (3) $\Rightarrow \boxed{B = 13}$

put the values of A & B in (2)

$$(2) \Rightarrow \frac{12x + 11}{(x + 2)(x + 1)} = \frac{-1}{x + 1} + \frac{13}{x + 2} \text{ on integration}$$

$$\int \frac{12x + 11}{(x + 2)(x + 1)}\,dx = -\int \frac{1}{x + 1}\,dx + 13\int \frac{1}{x + 2}\,dx$$

put this in (1)

$$I = \int x\,dx - 4\int dx - \int \frac{1}{x + 1}\,dx + 13\int \frac{1}{x + 2}\,dx$$

$$\boxed{I = \frac{x^2}{2} - 4x - \ell n(x + 1) + 13\ell n(x + 2) + c}$$

**(9)** $\displaystyle\int \frac{2x\,dx}{(1 + x^2)(3 + x^2)}$

**Solution:** let $I = \displaystyle\int \frac{2x\,dx}{(1 + x^2)(3 + x^2)}$ ——— (1)

let $y = x^2$ diff w.r to x

$$\frac{dy}{dx} = 2x \Rightarrow \boxed{2x\,dx = dy}$$



$$(1) \Rightarrow I = \int \frac{dy}{(1 + y)(3 + y)}$$

Now resolve into partial fraction

$$\frac{1}{(1 + y)(3 + y)} = \frac{A}{1 + y} + \frac{B}{3 + y} \text{ ——— (2)}$$

Xplying throught eq$^n$ (2) by $(1 + y)(3 + y)$

$$1 = A(3 + y) + B(1 + y) \text{ ——— (3)}$$

put $1 + y = 0 \Rightarrow \boxed{y = -1}$ in (3) $\Rightarrow \boxed{A = \frac{1}{2}}$

put $3 + y = 0 \Rightarrow \boxed{y = -3}$ in (3) $\Rightarrow \boxed{B = \frac{-1}{2}}$

put the values of A & B in (2)

$$(2) \Rightarrow \frac{1}{(1 + y)(3 + y)} = \frac{\frac{1}{2}}{1 + y} - \frac{\frac{1}{2}}{3 + y} \text{ on integration}$$

$$\int \frac{1}{(1 + y)(3 + y)}\,dy = \frac{1}{2}\int \frac{1}{1 + y}\,dy - \frac{1}{2}\int \frac{1}{3 + y}\,dy$$

$$I = \frac{1}{2}\ell n(1 + y) - \frac{1}{2}\ell n(3 + y) + c$$

$$I = \frac{1}{2}\ell n\left\{\frac{1 + y}{3 + y}\right\} + c \qquad \therefore y = x^2$$

$$\boxed{I = \frac{1}{2}\ell n\left\{\frac{1 + x^2}{3 + x^2}\right\} + c} \quad \text{Ans.}$$

**(10)** $\displaystyle\int \frac{x^2 + 2x + 3}{x^2 - 3x + 2}\,dx$

**Solution:** let $I = \displaystyle\int \frac{x^2 + 2x + 3}{x^2 - 3x + 2}\,dx$

first we divide the N (x) by D (x)

$$\begin{array}{r|l} & \quad 1 \\ x^2 - 3x + 2 & x^2 + 2x + 3 \\ & \underline{-x^2 \pm 3x^2 \pm 2} \\ & \quad 5x + 1 \end{array}$$

$$\boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$$

$$I = \int \left\{1 + \frac{5x + 1}{x^2 - 3x + 2}\right\} dx \Rightarrow I = \int \left\{1 + \frac{5x + 1}{x^2 - 2x - x + 2}\right\} dx$$

$I = \int dx + \int \frac{(5x+1)}{(x-2)(x-1)} dx$ ——— (1)

let $I = \int \frac{5x+1}{(x-2)(x-1)} dx$

resolve into partial fraction

$\frac{5x+1}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$ ——— (2)

Xplying throught eq$^n$ (2) by $(x-2)(x-1)$

$5x + 1 = A(x-1) + B(x-2)$ ——— (3)

put $x - 2 = 0 \Rightarrow \boxed{x = 2}$ in (3) $\Rightarrow \boxed{A = 11}$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (3) $\Rightarrow \boxed{B = -6}$

put the values of A & B in (2)

(2)$\Rightarrow \frac{5x+1}{(x-2)(x-1)} = \frac{11}{x-2} - \frac{6}{x-1}$ on integration

$\int \frac{(5x+1)\,dx}{(x-2)(x-1)} = 11\int \frac{1}{x-2} dx - 6\int \frac{1}{x-1} dx$

put this result in (1)

(1)$\Rightarrow I = \int dx + 11\int \frac{1}{x-2} dx - 6\int \frac{1}{x-1} dx$

$\boxed{I = x + 11\ell n(x-2) - 6\,\ell n(x-1) + c}$ Ans

**(11)** $\int \frac{x^4+1}{x^3-x} dx$

**Solution:** let $I = \int \frac{x^4+1}{x^3-x} dx$

first we divide N (x) by D (x)

$$\begin{array}{r|l} & x \\ x^3 - x & x^4 + 1 \\ & -x^4 \mp x^2 \\ \hline & x^2 + 1 \end{array}$$

$\boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$

$I = \int \left\{ x + \frac{x^2+1}{x^3-x} \right\} dx$

$I = \int x dx + \int \frac{x^2+1\ dx}{x(x-1)(x+1)}$ ——— (1)

let $I = \int \frac{x^2+1\ dx}{x(x-1)(x+1)}$



resolve into partial fraction

$\frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$ ——— (2)

Xplying throught eq$^n$ (2) by $x(x-1)(x+1)$

$x^2 + 1 = A(x^2-1) + Bx(x+1) + Cx(x-1)$ ——— (3)

put $x - 0$ in (3) $\Rightarrow \boxed{A = -1}$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (3) $\Rightarrow \boxed{B = 1}$

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in (3) $\Rightarrow \boxed{C = 1}$

put the values of A, B and C in (2)

(2)$\Rightarrow \frac{x^2+1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{x-1} + \frac{1}{x+1}$ on integration

$\int \frac{x^2+1\ dx}{x(x-1)(x+1)} = -\int \frac{1}{x} dx + \int \frac{1}{x-1} dx + \int \frac{1}{x+1} dx$

put this result in (1)

(1)$\Rightarrow I = \int x dx - \int \frac{1}{x} dx + \int \frac{1}{x-1} dx + \int \frac{1}{x+1} dx$

$I = \frac{x^2}{2} - \ell nx + \ell n(x-1) + \ell n(x+1) + c$

$I = \frac{x^2}{2} + \ell n(x^2-1) - \ell nx + c$

$\boxed{I = \frac{x^2}{2} + \ell n\left\{\frac{x^2-1}{x}\right\} + c}$ Ans.

**(12)** $\int \frac{x^3+3}{x^2+4} dx$

**Solution:** let $I = \int \frac{x^3+3}{x^2+4} dx$

first we divide N (x) by D (x)

$$\begin{array}{r|l} & x \\ x^2 - 4 & x^3 + 3 \\ & -x^3 \pm 4x \\ \hline & -4x + 3 \end{array}$$

$\boxed{\therefore \frac{N(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}}$

$I = \int \left\{ x + \frac{-4x+3}{x^2+4} \right\} dx \Rightarrow I = \int x dx - \int \frac{4x dx}{x^2+4} + 3\int \frac{1}{x^2+4} dx$

$I = \int x dx - 2\int \frac{2x\,dx}{x^2+4} + 3\int \frac{1}{(x)^2+(2)^2} dx$

$$\boxed{\therefore \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c}$$

$$\boxed{I = \frac{x^2}{2} - 2\,\ell n\,(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2} + c}$$ Ans.

**(13)** $\int \frac{(x^2-2)\,dx}{(x+1)(x-1)^2}$

**Solution:** let $I = \int \frac{(x^2-2)\,dx}{(x+1)(x-1)^2}$

first we resolve into partial fraction

$$\frac{x^2-2}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \text{ —— (1)}$$

Xplying through out eq$^n$ (1) by $(x+1)(x-1)^2$

$$x^2 - 2 = A(x-1)^2 + B(x^2-1) + C(x+1) \text{ —— (2)}$$

put $x + 1 = 0 \Rightarrow \boxed{x=-1}$ in (3) $\Rightarrow \boxed{A = \frac{-1}{4}}$

put $x - 1 = 0 \Rightarrow \boxed{x=1}$ in (3) $\Rightarrow \boxed{C = \frac{-1}{2}}$

eq$^n$ (2) can be written as

$x^2 - 2 = A(x^2 - 2x + 1) + B(x^2 - 1) + C(x+1)$

$x^2 - 2 = (A + B)x^2 + (C - 2A)x + (A - B + C)$

equating the coefficient of $x^2$

$A + B = 1 \Rightarrow \frac{-1}{4} + B = 1 \Rightarrow \boxed{B = \frac{5}{4}}$

put the values of A, B & C in (1)

$$(1) \Rightarrow \frac{x^2-2}{(x+1)(x-1)^2} = \frac{\frac{-1}{4}}{x+1} + \frac{\frac{5}{4}}{x-1} + \frac{\frac{-1}{2}}{(x-1)^2}$$

on integration

$$\int \frac{(x^2-2)}{(x+1)(x-1)^2}dx = \frac{1}{4}\int\frac{1}{x+1}dx + \frac{5}{4}\int\frac{1}{x-1}dx$$

$$-\frac{1}{2}\int (x-1)^{-2}dx$$

$$I = \frac{1}{4}\ell n\,(x+1) + \frac{5}{4}\ell n\,(x-1) + \frac{1}{2(x-1)} + c$$

$$\boxed{I = \frac{1}{4}\ell n\left\{\frac{(x-1)^5}{x+1}\right\} + \frac{1}{2(x-1)} + c}$$ Ans.



**(14)** $\int \frac{(x^2+3x+3)\,dx}{(x+1)(x^2+1)}$

**Solution:** let $I = \int \frac{(x^2+3x+3)\,dx}{(x+1)(x^2+1)}$

first resolve into partial fraction

$$\frac{x^2+3x+3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+c}{x^2+1} \text{ ——— (1)}$$

Xplying throughout eq$^n$ (1) by $(x+1)(x^2+1)$

$$x^2 + 3x + 3 = A(x^2+1) + B(x^2+x) + C(x+1) \text{ —— (2)}$$

put $x + 1 = 0 \Rightarrow \boxed{x=-1}$ in (2) $\Rightarrow \boxed{A = \frac{1}{2}}$

eqn (2) can be written as

$x^2 + 3x + 3 = (A + B)x^2 + (B + C)x + (A + C)$

equating the coefficient of $x^2$ & $x$

$1 = A + B \Rightarrow 1 = \frac{1}{2} + B \Rightarrow \boxed{B = \frac{1}{2}}$

$3 = B + C \Rightarrow 3 = \frac{1}{2} + C \Rightarrow \boxed{C = \frac{5}{2}}$

put the values of A, B and C in (1)

$$(1) \Rightarrow \frac{x^2+3x+3}{(x+1)(x^2+1)} = \frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{2}x + \frac{5}{2}}{x^2+1} \quad \text{on integration}$$

$$\int \frac{(x^2+3x+3)\,dx}{(x+1)(x^2+1)} = \frac{1}{2}\int\frac{1}{x+1}dx + \frac{1}{2\times 2}\int\frac{2x\,dx}{x^2+1}$$

$$+\frac{5}{2}\int\frac{1}{(x)^2+(1)^2}dx$$

$$\boxed{\therefore \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c}$$

$$\boxed{I = \frac{1}{2}\ell n\,(x+1) + \frac{1}{4}\ell n\,(x^2+1) + \frac{5}{2}\tan^{-1}x + c}$$

**(15)** $\int \frac{x^2-2x+3}{(x-1)(x^2+2x+2)}dx$

**Solution:** let $I = \int \frac{x^2-2x+3}{(x-1)(x^2+2x+2)}dx$

first we resolve into partial fraction

$$\frac{x^2-2x+3}{(x-1)(x^2+2x+2)} = \frac{A}{x-1} + \frac{Bx+c}{x^2+2x+2}$$

Xplying throughout eq$^n$ (1) by $(x-1)(x^2+2x+2)$

$$x^2-2x+3 = A(x^2+2x+2)+B(x^2-x)+C(x-1) \text{—— (2)}$$

put $x-1=0 \Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{A=\frac{2}{5}}$

eq$^n$ (2) can be written as

$$x^2-2x+3=(A+B)x^2+(2A-B+C)x+(2A-C)$$

equating the Coefficient of $x^2$ & constant terms

$$1=A+B \Rightarrow 1=\frac{2}{5}+B \Rightarrow \boxed{B=\frac{3}{5}}$$

$$3=2A-C \Rightarrow 3=2\left(\frac{2}{5}\right)-C \Rightarrow \boxed{C=\frac{-11}{5}}$$

Put the values of A, B & C in (1)

$$(1) \Rightarrow \frac{x^2-2x+3}{(x-1)(x^2+2x+2)}=\frac{\frac{2}{5}}{x-1}+\frac{\frac{3}{5}x-\frac{11}{5}}{x^2+2x+2} \quad \text{on integration}$$

$$\int\frac{(x^2-2x+3)\,dx}{(x-1)(x^2+2x+2)}=\frac{2}{5}\int\frac{1}{x-1}dx+\frac{1}{5}\frac{(3x-11)}{x^2+2x+2}dx$$

$$I=\frac{2}{5}\int\frac{1}{x-1}dx+\frac{1}{5\times 2}\int\frac{(6x-22)}{x^2+2x+2}dx$$

$$I=\frac{2}{5}\int\frac{1}{x-1}dx+\frac{1}{10}\int\frac{(6x+6)\,dx}{x^2+2x+2}-\frac{1}{10}\int\frac{28}{(x^2+2x+1)+(1)^2}dx$$

$$I=\frac{2}{5}\int\frac{1}{x-1}dx+\frac{3}{10}\int\frac{(2x+2)\,dx}{x^2+2x+2}-\frac{28}{10}\int\frac{1}{(x+1)^2+(1)^2}dx$$

$$\boxed{\therefore \int\frac{1}{x^2+a^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+c}$$

$$I=\frac{2}{5}\ln(x-1)+\frac{3}{10}\ln(x^2+2x+2)-\frac{\overset{14}{\cancel{28}}}{\underset{5}{\cancel{10}}}\tan^{-1}(x+1)+c$$

$$\boxed{I=\frac{2}{5}\ln(x-1)+\frac{3}{10}\ln(x^2+2x+2)-\frac{14}{5}\tan^{-1}(x+1)+c}$$

Ans

**(16)** $\displaystyle\int\frac{(x-3)\,dx}{(x+1)^2(x-2)}$

**Solution:** let $I=\displaystyle\int\frac{(x-3)\,dx}{(x+1)^2(x-2)}$

We first resolve into partial fraction

$$\frac{(x-3)}{(x+1)^2(x-2)}=\frac{A}{x-2}+\frac{B}{x+1}+\frac{C}{(x+1)^2} \text{—— (1)}$$



Xplying through out eq$^n$ (1) by $(x+1)^2(x-2)$

$$x-3=A(x+1)^2+B(x-2)(x+1)+C(x-2) \text{—— (2)}$$

put $x-2=0 \Rightarrow \boxed{x=2}$ in (3) $\Rightarrow \boxed{A=\frac{-1}{9}}$

put $x+1=0 \Rightarrow \boxed{x=-1}$ in (3) $\Rightarrow \boxed{C=\frac{4}{3}}$

The eq$^n$ (3) can be written as

$$x-3=A(x^2+2x+1)+B(x^2+x-2x-2)+C(x-2)$$

$$x-3=(A+B)x^2+(2A-B+C)x+(A-2B-2C)$$

equating the coefficient of $x^2$

$$A+B=0 \Rightarrow \frac{-1}{9}+B=0 \Rightarrow \boxed{B=\frac{1}{9}}$$

put the values of A, B & C in (1)

$$(1) \Rightarrow \frac{x-3}{(x+1)^2(x-2)}=\frac{\frac{-1}{9}}{x-2}+\frac{\frac{1}{9}}{x+1}+\frac{\frac{4}{3}}{(x+1)^2} \quad \text{on integration}$$

$$\int\frac{(x-3)\,dx}{(x+1)^2(x-2)}=\frac{-1}{9}\int\frac{1}{x-2}dx+\frac{1}{9}\int\frac{1}{x+1}dx+\frac{4}{3}\int(x+1)^{-2}dx$$

$$I=\frac{-1}{9}\ln(x-2)+\frac{1}{9}\ln(x+1)-\frac{4}{3(x+1)}+C$$

$$\boxed{I=\frac{1}{9}\ln\left\{\frac{x+1}{x-2}\right\}-\frac{4}{3(x+1)}+C} \quad \text{Ans.}$$

**(17)** $\displaystyle\int\frac{(x^2+1)}{(x-1)^3}dx$

**Solution:** let $I=\displaystyle\int\frac{(x^2+1)\,dx}{(x-1)^3}$

We first resolve into partial fraction

$$\frac{(x^2+1)}{(x-1)^3}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3} \text{—— (1)}$$

Xplying throughout eq$^n$ (1) by $(x-1)^3$

$$x^2+1=A(x-1)^2+B(x-1)+C \text{—— (2)}$$

put $x-1=0 \Rightarrow \boxed{x=1}$ in (2) $\Rightarrow \boxed{C=2}$

eq$^n$ (2) can be written as

$$x^2+1=A(x^2-2x+1)+Bx-B+C$$

$$x^2+1=Ax^2+(B-2A)x+(A-B+C)$$

equating the coefficient of $x^2$ & x.

$\boxed{A=1}$ $B-2A=0 \Rightarrow B-2(1)=0 \Rightarrow \boxed{B=2}$

put the values of A , B & C in (1)

$$(1) \Rightarrow \frac{(x^2+1)}{(x-1)^3} = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{2}{(x-1)^3} \quad \text{on integration}$$

$$\int \frac{(x^2+1)\,dx}{(x-1)^3} = \int \frac{1}{x-1}\,dx + 2\int (x-1)^{-2}\,dx + 2\int (x-1)^{-3}\,dx$$

$$\boxed{I = \ell n\,(x-1) - \frac{2}{(x-1)} - \frac{1}{(x-1)^2} + C} \quad \text{Ans}$$

**(18)** $\int \frac{2x^2-1}{(x+1)^2(x-3)}\,dx$

**Solution:** let $I = \int \frac{2x^2-1}{(x+1)^2(x-3)}\,dx$

resolve into partial fraction

$$\frac{2x^2-1}{(x+1)^2(x-3)} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \text{——} (1)$$

Xplying throughout eq$^n$ (1) by $(x-3)(x+1)^2$

$$2x^2 - 1 = A(x+1)^2 + B(x+1)(x-3) + C(x-3) \text{——} (2)$$

put $x - 3 = 0 \Rightarrow \boxed{x = 3}$ in (2) $\Rightarrow \boxed{A = \frac{17}{16}}$

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in (2) $\Rightarrow \boxed{C = \frac{-1}{4}}$

eq$^n$ (2) can be written as

$$2x^2 - 1 = A(x^2 + 2x + 1) + B(x^2 - 2x - 3) + C(x-3)$$

$$2x^2 - 1 = (A+B)x^2 + (2A - 2B + C)x + (A - 3B - 3C)$$

equating the coefficient of $x^2$

$$2 = A + B \Rightarrow 2 = \frac{17}{16} + B \Rightarrow \boxed{B = \frac{15}{16}}$$

put the values of A, B and C in (1)

$$(1) \Rightarrow \frac{2x^2-1}{(x+1)^2(x-3)} = \frac{\frac{17}{16}}{x-3} + \frac{\frac{15}{16}}{x+1} + \frac{\frac{-1}{4}}{(x+1)^2} \quad \text{on integration}$$

$$\int \frac{(2x^2-1)\,dx}{(x+1)^2(x-3)} = \frac{17}{16}\int \frac{1}{x-3}\,dx + \frac{15}{16}\int \frac{1}{x+1}\,dx - \frac{1}{4}\int (x+1)^{-2}\,dx$$

$$\boxed{I = \frac{17}{16}\ell n\,(x-3) + \frac{15}{16}\ell n\,(x+1) + \frac{1}{4(x+1)} + C}$$

**(19)** $\int \frac{(x^3+1)\,dx}{(x^2-1)^2}$

$(x^3+1)\,dx$



$$I = \int \frac{\cancel{(x+1)}(x^2-x+1)\,dx}{(x-1)^2\cancel{(x+1)^2}} \Rightarrow I = \int \frac{(x^2-x+1)\,dx}{(x+1)(x-1)^2}$$

resolve into partial fraction

$$\frac{x^2-x+1}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \text{——} (1)$$

Xplying throughout eq$^n$ (1) by $(x+1)(x-1)^2$

$$x^2 - x + 1 = A(x-1)^2 + B(x^2-1) + C(x+1) \text{——} (2)$$

put $x + 1 = 0 \Rightarrow \boxed{x = -1}$ in (2) $\Rightarrow \boxed{A = \frac{3}{4}}$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in (2) $\Rightarrow \boxed{C = \frac{1}{2}}$

eq$^n$ (2) can be written as

$$x^2 - x + 1 = A(x^2 - 2x + 1) + B(x^2 - 1) + C(x+1)$$

$$x^2 - x + 1 = (A+B)x^2 + (C - 2A)x + (A - B + C)$$

equating the coefficient of $x^2$

$$1 = A + B \Rightarrow 1 = \frac{3}{4} + B \Rightarrow \boxed{B = \frac{1}{4}}$$

put the values of A, B & C in (1)

$$(1) \Rightarrow \frac{x^2-x+1}{(x+1)(x-1)^2} = \frac{\frac{3}{4}}{x+1} + \frac{\frac{1}{4}}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} \quad \text{on integration}$$

$$\int \frac{(x^2-x+1)\,dx}{(x+1)(x-1)^2} = \frac{3}{4}\int \frac{1}{x+1}\,dx + \frac{1}{4}\int \frac{1}{x-1}\,dx + \frac{1}{2}\int (x-2)^{-2}\,dx$$

$$I = \frac{3}{4}\ell n\,(x+1) + \frac{1}{4}\ell n\,(x-1) - \frac{1}{2(x-1)} + c$$

$$\boxed{I = \frac{1}{4}\ell n\,\{(x+1)^3(x-1)\} - \frac{1}{2(x-1)} + C}$$

**(20)** $\int \frac{(x^3 - 2x^2 + 3x - 4)\,dx}{(x-1)^2(x^2+2x+2)}$

**Solution:** let $I = \int \frac{(x^3 - 2x^2 + 3x - 4)\,dx}{(x-1)^2(x^2+2x+2)}$

Resolve into partial fraction

$$\frac{x^3 - 2x^2 + 3x - 4}{(x-1)^2(x^2+2x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx + D}{x^2+2x+2} \text{——} (1)$$

Xplying throughout eq$^n$ (1) by $(x-1)^2(x^2+2x+2)$

$$x^3 - 2x^2 + 3x - 4 = A(x-1)(x^2+2x+2)$$
$$+ B(x^2+2x+2) + Cx(x-1)^2 + D(x-1)^2 \text{——} (2)$$

put $x - 1 = 0 \Rightarrow \boxed{x = 1}$ in eq$^n$ (2) $\Rightarrow \boxed{B = \frac{-2}{5}}$

$x^3 - 2x^2 + 3x - 4 = A(x^3 + x^2 - 2) + B(x^2 + 2x + 2) + C(x^3 - 2x^2 + x) + D(x^2 - 2x + 1)$

$x^3 - 2x^2 + 3x - 4 = (A + C)x^3 + (A + B - 2C + D)x^2 + (2B + C - 2D)x + (2B - 2A + D)$

equating the coefficient of $x^3$, $x^2$, x & constants

$A + C = 1$ ——— (3)

$A + B - 2C + D = -2$ ——— (4)

$2B + C - 2D = 3$ ——— (5)

$2B - 2A + D = -4$ ——— (6)

put $B = \frac{-2}{5}$ & from (3) $A = 1 - C$ in (4)

(4)$\Rightarrow 1 - C - \frac{2}{5} - 2C + D = -2$

$-3C + D = -2 - \frac{3}{5} \Rightarrow -3C + D = \frac{-13}{5}$ ——— (7)

put $B = \frac{-2}{5}$ in (5)

(5)$\Rightarrow 2\left(\frac{-2}{5}\right) + C - 2D = 3 \Rightarrow C - 2D = 3 + \frac{4}{5}$

$C - 2D = \frac{19}{5}$ ——— (8)

Xplying throught by 3

$3C - 6D = \frac{57}{5}$ ——— (9)

eqn (7) + eqn (9) $\quad -3C + D = \frac{-13}{5}$

$3C - 6D = \frac{57}{5}$

$-5D = \frac{44}{5} \Rightarrow \boxed{D = \frac{-44}{25}}$

put the value of D in (8)

(8)$\Rightarrow C - 2\left(\frac{-44}{25}\right) = \frac{19}{5}$

$C = \frac{19}{5} - \frac{88}{25} \Rightarrow C = \frac{95 - 88}{25} \Rightarrow \boxed{C = \frac{7}{25}}$

put $\boxed{C = \frac{7}{25}}$ in (3)



(3)$\Rightarrow A = \frac{7}{25} = 1 \Rightarrow \boxed{A = \frac{18}{25}}$

put the values of A, B, C & D in (1)

(1)$\Rightarrow \frac{x^3 - 2x^2 + 3x - 4}{(x - 1)^2(x^2 + 2x + 2)} = \frac{18}{25}\int \frac{1}{x - 1}dx$

$-\frac{2}{5}\int (x - 1)^{-2}dx + \frac{1}{25 \times 2}\int \frac{14x - 88}{x^2 + 2x + 2}dx$

$I = \frac{18}{25}\int \frac{1}{x - 1}dx - \frac{2}{5}\int (x - 1)^{-2}dx + \frac{1}{50}\int \frac{14x + 14 - 102}{x^2 + 2x + 2}dx$

$I = \frac{18}{25}\int \frac{1}{x - 1}dx - \frac{2}{5}\int (x - 1)^{-2}dx + \frac{1}{50}\int \frac{14x + 14}{x^2 + 2x + 2}dx$

$-\frac{1}{50}\int \frac{102}{x^2 + 2x + 2}dx$

$I = \frac{18}{25}\int \frac{1}{x - 1}dx - \frac{2}{5}\int (x - 1)^{-2}dx + \frac{7}{50}\int \frac{2x + 2}{x^2 + 2x + 2}dx$

$-\frac{1}{50}\int \frac{102\,dx}{(x + 1)^2 + (1)^2}$

$I = \frac{18}{25}\ell n(x - 1) + \frac{2}{5(x - 1)} + \frac{7}{50}\ell n(x^2 + 2x + 2)$

$-\frac{\cancel{102}^{51}}{\cancel{50}_{25}}\tan^{-1}(x + 1) + C$

$$\boxed{I = \frac{18}{25}\ell n(x - 1) + \frac{2}{5(x - 1)} + \frac{7}{50}\ell n(x^2 + 2x + 2) \; \frac{-51}{25}\tan^{-1}(x + 1) + c}$$

(21) $\int \frac{x^2 - 3x + 5}{x^4 - 8x^2 + 16}dx$

ation

**Solution:** let $I = \int \frac{x^2 - 3x + 5}{x^4 - 8x^2 + 16}dx$

$I = \int \frac{x^2 - 3x + 5}{(x^2 - 4)^2}dx = \int \frac{x^2 - 3x + 5}{(x - 2)^2(x + 2)^2}dx$

resolve into partial fraction

$\frac{x^2 - 3x + 5}{(x - 2)^2(x + 2)^2} = \frac{A}{x - 2} + \frac{B}{(x - 2)^2} + \frac{C}{x + 2} + \frac{D}{(x + 2)^2}$ — (1)

Xplying throughout eq$^n$ (1) by $(x - 2)^2(x + 2)^2$

$x^2 - 3x + 5 = A(x - 2)(x + 2)^2 + B(x + 2)^2 + C(x + 2)(x - 2)^2 + D(x - 2)^2$ ——— (2)

 https://stbmathematics.blogspot.com/



put $x - 2 = 0 \Rightarrow \boxed{x = 2}$ in (2) $\Rightarrow \boxed{B = \frac{3}{16}}$

put $x + 2 = 0 \Rightarrow \boxed{x = -2}$ in (2) $\Rightarrow \boxed{D = \frac{15}{16}}$

eq$^n$ (2) can be written as

$x^2 - 3x + 5 = A(x^3 + 2x^2 - 4x - 8) + B(x^2 + 4x + 4) + C(x^3 - 2x^2 - 4x + 8) + D(x^2 - 4x + 4)$

$x^2 - 3x + 5 = (A + C)x^3 + (2A + B - 2C + D)x + (4B - 4A - 4C - 4D)x + (4B - 8A + 8C + 4D)$

equating the coefficient of $x^3$, $x^2$ and constant term

$A + C = 0$ ——— (3)

$2A + B - 2C + D = 1$ ——— (4)

$4B - 8A + 8C + 4D = 5$ ——— (5)

put the values of B & D in (4)

(4)$\Rightarrow$ $2A + \frac{3}{16} - 2C + \frac{15}{16} = 1$

Xplying throughout by 16.

$32A + 3 - 32C + 15 = 16$

$32A - 32C = -2$ ——— (6)

Xply eq$^n$ (3) by 32 + eq$^n$ (6)

$32A + \cancel{32C} = 0$

$\underline{32A - \cancel{32C} = -2}$

$\underset{32}{\cancel{64}}A = -\cancel{2} \Rightarrow \boxed{A = \frac{-1}{32}}$

put the value of A in (3)

$\frac{-1}{32} + C = 0 \Rightarrow \boxed{C = \frac{1}{32}}$

put the values of A, B, C & D in (1)

p(1)$\Rightarrow$ $\frac{x^2 - 3x + 5}{(x-2)^2(x+2)^2} = \frac{\frac{-1}{32}}{x-2} + \frac{\frac{3}{16}}{(x-2)^2} + \frac{\frac{1}{32}}{x+2} + \frac{\frac{15}{16}}{(x+2)^2}$

(8)$\Rightarrow$ ( on integration

$\int \frac{(x^2 - 3x + 5)\,dx}{(x-2)^2(x+2)^2} = \frac{-1}{32}\int \frac{1}{x-2}dx + \frac{3}{16}\int (x-2)^{-2}dx$

$+ \frac{1}{32}\int \frac{1}{x+2}dx + \frac{15}{16}\int (x+2)^{-2}dx$

$I = \frac{-1}{32}\ln(x-2) - \frac{3}{16(x-2)} + \frac{1}{32}\ln(x+2) - \frac{15}{16(x+2)} + C$



$\boxed{I = \frac{-3}{16(x-2)} - \frac{1}{32}\ln\left\{\frac{x-2}{x+2}\right\} - \frac{15}{16(x+2)} + C}$

**(22)** $\int \frac{x^2 + 3x + 5}{x^3 + 8}dx$

**Solution:** let $I = \int \frac{x^2 + 3x + 5}{x^3 + 8}dx$

$I = \int \frac{x^2 + 3x + 5}{(x+2)(x^2 - 2x + 4)}dx$

resolve into partial fraction

$\frac{x^2 + 3x + 5}{(x+2)(x^2 - 2x + 4)} = \frac{A}{x+2} + \frac{Bx + c}{x^2 - 2x + 4}$ ——— (1)

Xplying throughout eq$^n$ (1) by $(x + 2)(x^2 - 2x + 4)$

$x^2 + 3x + 5 = A(x^2 - 2x + 4) + 13x(x + 2) + C(x + 2)$ — (2)

put $x + 2 = 0 \Rightarrow \boxed{x = -2}$ in (2) $\Rightarrow \boxed{A = \frac{1}{4}}$

eq$^n$ (2) can be written as.

$x^2 + 3x + 5 = (A + B)x^2 + (2B - 2A + C)x + 4A + 2C$

equating the coefficient of $x^2$ & x

$A + B = 1 \Rightarrow \frac{1}{4} + B = 1 \Rightarrow \boxed{B = \frac{3}{4}}$

$2B - 2A + C = 3$

$2\left(\frac{3}{4}\right) - 2\left(\frac{1}{4}\right) + C = 3 \Rightarrow \frac{6}{4} - \frac{2}{4} + C = 3$

$\frac{\cancel{4}}{\cancel{4}} + C = 3 \Rightarrow C + 1 = 3 \Rightarrow \boxed{C = 2}$

put the values of A, B & C in (1) $x^2$

(1)$\Rightarrow$ $\frac{x^2 + 3x + 5}{(x+2)(x^2 - 2x + 4)} = \frac{\frac{1}{4}}{x+2} + \frac{\frac{3}{4}x + 2}{x^2 - 2x + 4}$ on integration

$\int \frac{(x^2 + 3x + 5)\,dx}{(x+2)(x^2 - 2x + 4)} = \frac{1}{4}\int \frac{1}{x+2}dx + \frac{1}{4}\int \frac{3x + 8}{x^2 - 2x + 4}dx$

$I = \frac{1}{4}\int \frac{1}{x+2}dx + \frac{1}{8}\int \frac{(6x + 18)\,dx}{x^2 - 2x + 4}$

$I = \frac{1}{4}\int \frac{1}{x+2}dx + \frac{1}{8}\int \frac{(6x - 6 + 22)\,dx}{x^2 - 2x + 4}$

$I = \frac{1}{4}\int \frac{1}{x+2}dx + \frac{3}{8}\int \frac{(2x - 2)\,dx}{x^2 - 2x + 4} + \frac{22}{8}\int \frac{1}{(x - 1)^2 + (\sqrt{3})^2}dx$

https://stbmathematics.blogspot.com/ https://stbmathematics.blogspot.com/

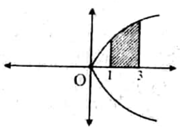
$$I = \frac{1}{4}\ell n(x+2) + \frac{3}{8}\ell n(x^2-2x+4) + \frac{\cancel{22}^{11}}{\cancel{8}_{4}} \times \frac{1}{\sqrt{3}}\tan^{-1}\frac{(x-1)}{\sqrt{3}} + C$$

$$\boxed{I = \frac{1}{4}\ell n(x+2) + \frac{3}{8}\ell n(x^2-2x+4) + \frac{11\sqrt{3}}{12}\tan^{-1}\frac{(x-1)}{\sqrt{3}} + C}$$

## EXERCISE # 6.10

**Find the area, above the x–axis , under the following curves, between the given ordinates.**

**(1) $y^2 = x$ , $x = 1$ , $x = 3$**




**Solution:** $y^2 = x$ $a = 1, b = 3$

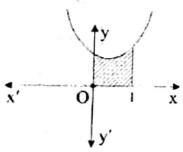
$\Rightarrow$ $y = \sqrt{x} \Rightarrow y = x^{1/2}$

$$A = \int_a^b y\,dx$$

$$A = \int_1^3 x^{1/2}\,dx = \left[\frac{x^{3/2}}{3/2}\right]_1^3 = \frac{2}{3}[x^{3/2}]_1^3$$

$$A = \frac{2}{3}\{3^{3/2} - 1^{3/2}\} = \frac{2}{3}\{3\sqrt{3} - 1\}\text{ Unit}^2$$

**(2) $y = x^2 - 2x + 5$ , $x' = 0$ , $x = 1$**




**Solution:** $y = x^2 - 2x + 5$ , $a = 0$ , $b = 1$

$$A = \int_a^b y\,dx$$

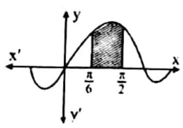
$$A = \int_0^1 (x^2 - 2x + 5)\,dx$$

$$A = \left[\frac{x^3}{3} - \frac{\cancel{2}x^2}{\cancel{2}} + 5x\right]_0^1 = \left[\frac{x^3}{3} - x^2 + 5x\right]_0^1$$

$$A = \left[\frac{(1)^3}{3} - (1)^2 + 5(1)\right] - \left[\frac{(0)^3}{3} - (0)^2 + 5(0)\right]$$

$$A = \frac{1}{3} - 1 + 5 = \frac{1}{3} + 4 = \frac{13}{3} \Rightarrow \boxed{A = 4\frac{1}{3}\text{ Unit}^2}$$

**(3) $y = \text{Sinx}$ , $x = \frac{\pi}{6}, x = \frac{\pi}{2}$**




**Solution:** $y = \text{Sinx}$ , $a = \frac{\pi}{6}$ , $b = \frac{\pi}{2}$

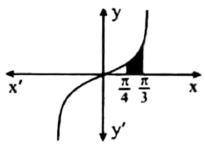


$$A = \int_a^b y\,dx \Rightarrow A = \int_{\pi/6}^{\pi/2} \text{Sinx}\,dx = [-\text{Cosx}]_{\pi/6}^{\pi/2}$$

$$A = -\left[\text{Cos}\frac{\pi}{2} - \text{Cos}\frac{\pi}{6}\right] = -\left[0 - \frac{\sqrt{3}}{2}\right].$$

$\boxed{A = \frac{\sqrt{3}}{2}\text{ Unit}^2}$ Ans.

**(4) $y = \tan x$ , $x = \frac{\pi}{4}$ , $x = \frac{\pi}{3}$**




**Solution:** $y = \tan x$ , $a = \frac{\pi}{4}$ , $b = \frac{\pi}{3}$

$$A = \int_a^b y\,dx \Rightarrow A = \int_{\pi/4}^{\pi/3} \tan x\,dx$$

$$A = [\ell n\,\text{Secx}]_{\pi/4}^{\pi/3} = \ell n\,\text{Sec}\frac{\pi}{3} - \ell n\,\text{Sec}\frac{\pi}{4}$$

$$A = \ell n\left[\frac{\text{Sec}\frac{\pi}{3}}{\text{Sec}\frac{\pi}{4}}\right] = \ell n\left[\frac{\text{Cos}\frac{\pi}{4}}{\text{Cos}\frac{\pi}{3}}\right] = \ell n\left[\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right]$$

$$A = \ell n\left(\frac{2}{\sqrt{2}}\right) \Rightarrow A = \ell n(1.4142) \Rightarrow \boxed{A = 0.3465\text{ Unit}^2}$$

**(5) $y = 3x^4 - 2x^3 + 1$ , $x = 1$ , $x = 2$**

**Solution:** $y = 3x^4 - 2x^3 + 1$ , $a = 1$ , $b = 2$

$$A = \int_a^b y\,dx = \int_1^2 (3x^4 - 2x^3 + 1)\,dx \Rightarrow A = \left[\frac{3x^5}{5} - \frac{2x^4}{4} + x\right]_1^2$$

$$A = \left[\frac{3}{5}(2)^5 - \frac{2}{5}(2)^4 + 2\right] - \left[\frac{3}{5}(1)^5 - \frac{2}{5}(1)^4 + 1\right]$$

$$A = \frac{96}{5} - 6 - \frac{11}{10} = \frac{960 - 300 - 55}{50}$$

$A = \frac{605}{50} \Rightarrow \boxed{A = 12\frac{1}{10}\text{ unit}^2}$ Ans.

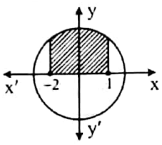
**(6) $x^2 + y^2 = 9$ , $x = -2$, $x = 1$**

**Solution:** $x^2 + y^2 = 9$ , $a = -2$ , $b = 1$

$$y = \sqrt{9 - x^2} \Rightarrow A = \int_a^b y\,dx \Rightarrow A\int_{-2}^{1}\sqrt{9 - x^2}\,dx$$

$$\boxed{\therefore \int\sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2\,\text{Sin}^{-1}\left(\frac{x}{a}\right) + c}$$

$$A = \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{x}{3}\right)\right]_{-2}^{1}$$




$$A = \left[\frac{(1)}{2}\sqrt{9-(1)^2} + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{1}{3}\right)\right] - \left[\frac{(-2)}{\cancel{2}}\sqrt{9-(-2)^2} + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{-2}{3}\right)\right]$$

$$A = \left[\frac{\sqrt{8}}{2} + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{1}{3}\right)\right] - \left[-\sqrt{5} - \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{2}{3}\right)\right]$$

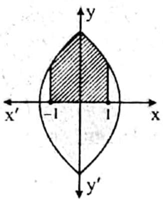
$$A = \frac{\sqrt{8}}{2} + \sqrt{5} + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{1}{3}\right) + \frac{9}{2}\,\mathrm{Sin}^{-1}\left(\frac{2}{3}\right)$$

$$A = 1.4142 + 2.2360 + \frac{9}{2}\left\{\mathrm{Sin}^{-1}\left(\frac{1}{3}\right) + \mathrm{Sin}^{-1}\left(\frac{2}{3}\right)\right\}$$

$$A = 3.6502 + \frac{9}{2}\{\,0.3398 + 0.7297\,\}$$

$$A = 3.6502 + 4.5\,(1.0695) \Rightarrow \boxed{A = 8.46295\ \text{Unit}^2} \qquad \text{Ans.}$$

**(7)** $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **,** $x = -1$ **,** $x = 1$




**Solution:** $\frac{x^2}{4} + \frac{y^2}{9} = 1$ , $a = -1$ , $b = 1$

$$\frac{y^2}{9} = 1 - \frac{x^2}{4} \Rightarrow \frac{y^2}{9} = \frac{4-x^2}{4}$$

$$y^2 = \frac{9\,(4-x^2)}{4} \Rightarrow y = \frac{3}{2}\sqrt{4-x^2}$$

$$A = \int_a^b y\,dx \Rightarrow A = \frac{3}{2}\int_{-1}^{1}\sqrt{4-x^2}\,dx$$

$$\boxed{\therefore \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\,\mathrm{Sin}^{-1}\left(\frac{x}{a}\right) + c}$$

$$A = \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\,\mathrm{Sin}^{-1}\left(\frac{x}{2}\right)\right]_{-1}^{1}$$

$$A = \frac{3}{2}\left[\left\{\frac{1}{2}\sqrt{4-(1)^2} + 2\,\mathrm{Sin}^{-1}\left(\frac{1}{2}\right)\right\} - \left\{\frac{(-1)}{2}\sqrt{4-(-1)^2} + 2\,\mathrm{Sin}^{-1}\left(\frac{-1}{2}\right)\right\}\right]$$

$$A = \frac{3}{2}\left[\frac{\sqrt{3}}{2} + 2\mathrm{Sin}^{-1}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2} + 2\mathrm{Sin}^{-1}\left(\frac{1}{2}\right)\right]$$

$$A = \frac{3}{2}\left[\frac{\cancel{2}(\sqrt{3})}{\cancel{2}} + 2\left\{\mathrm{Sin}^{-1}\left(\frac{1}{2}\right) + \mathrm{Sin}^{-1}\left(\frac{1}{2}\right)\right\}\right]$$



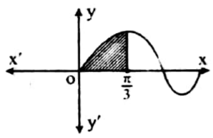
$$A = \frac{3}{2}[\,\sqrt{3} + 2\,(0.5235 + 0.5235)\,]$$

$\boxed{A = 5.7390\ \text{Unit}^2}$ Ans.

**Find the area under the curve given between the ordinates x = a , x = b.**

**(8)** $y = 3\mathrm{Sin}x$ **,** $a = 0$ **,** $b = \frac{\pi}{3}$

**Solution:** $y = 3\mathrm{Sin}x$ , $a = 0$ , $b = \frac{\pi}{3}$




$$A = \int_a^b y\,dx = \int_0^{\pi/3} 3\mathrm{Sin}x\,dx$$

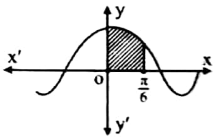
$$A = 3\int_0^{\pi/3}\mathrm{Sin}x\,dx = 3\,[\,-\mathrm{Cos}x\,]_0^{\pi/3}$$

$$A = -3\left[\mathrm{Cos}\frac{\pi}{3} - \mathrm{Cos}0\right] = -3\left[\frac{1}{2} - 1\right]$$

$\boxed{A = \frac{3}{2}\ \text{Sq. Units}}$ Ans.

**(9)** $y = 2\mathrm{Cos}3x$ **,** $a = 0$ **,** $b = \frac{\pi}{6}$

**Solution:** $y = 2\mathrm{Cos}3x$ , $a = 0$ , $b = \frac{\pi}{6}$




$$A = \int_a^b y\,dx = \int_0^{\pi/6} 2\mathrm{Cos}3x\,dx$$

$$A = 2\int_0^{\pi/6}\mathrm{Cos}3x\,dx = 2\left[\frac{\mathrm{Sin}3x}{3}\right]_0^{\pi/6}$$

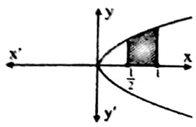
$$A = \frac{2}{3}\left[\,\mathrm{Sin}3\left(\frac{\pi}{6}\right) - \mathrm{Sin}3\,(0)\,\right]$$

$$A = \frac{2}{3}[\,1 - 0\,] \Rightarrow \boxed{A = \frac{2}{3}\ \text{Sq. Units}} \quad \text{Ans.}$$

**(10)** $y^2 = 6x$ **,** $a = \frac{1}{2}$ **,** $b = 1$

**Solution:** $y^2 = 6x$ , $a = \frac{1}{2}$ , $b = 1$

$$y = \sqrt{6}\,x^{1/2}$$



$A = \int_a^b y\,dx = \int_{1/2}^{1} \sqrt{6}\, x^{1/2} dx$

$A = \sqrt{6}\left[\frac{x^{3/2}}{3/2}\right]_{\frac{1}{2}}^{1} = \frac{2\sqrt{6}}{3}\left[(1)^{3/2} - \left(\frac{1}{2}\right)^{3/2}\right]$

$A = \frac{2\sqrt{6}}{3}\left[1 - \frac{1}{2\sqrt{2}}\right] = \frac{\cancel{2}\sqrt{6}}{3}\left[\frac{2\sqrt{2}-1}{\cancel{2}\sqrt{2}}\right]$

$A = \frac{\sqrt{3}}{3}(2\sqrt{2} - 1) \Rightarrow \boxed{A = 1.005 \text{ Sq. Units}}$ Ans.

**(portion of the curve in the first quadrant)**

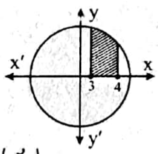
**(11)** $x^2 + y^2 = 25$ , $a = 3$ , $b = 4$

**Solution:** $x^2 + y^2 = 25, a = 3, b = 4, y = \sqrt{25 - x^2}$

$A = \int_a^b y\,dx = \int_3^4 \sqrt{25 - x^2}\, dx$

$\boxed{\therefore \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\text{Sin}^{-1}\left(\frac{x}{a}\right) + c}$

$A = \left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\text{Sin}^{-1}\left(\frac{x}{5}\right)\right]_3^4$




$A = \left[\frac{4}{2}\sqrt{25 - 16} + \frac{25}{2}\text{Sin}^{-1}\left(\frac{4}{5}\right)\right] - \left[\frac{3}{2}\sqrt{25 - 9} + \frac{25}{2}\text{Sin}^{-1}\left(\frac{3}{5}\right)\right]$

$A = 2\sqrt{9} + \frac{25}{2}\text{Sin}^{-1}\left(\frac{4}{5}\right) - \frac{3}{2}\sqrt{16} - \frac{25}{2}\text{Sin}^{-1}\left(\frac{3}{5}\right)$

$A = \cancel{6} - \cancel{6} + \frac{25}{2}\left[\text{Sin}^{-1}\left(\frac{4}{5}\right) - \text{Sin}^{-1}\left(\frac{3}{5}\right)\right]$

$A = \frac{25}{2}[\,0.9272 - 0.6435\,]$

$\boxed{A = 3.54625 \text{ Sq. Units}}$ Ans.

**(12)** $y = \tan^2 x$ , $a = \frac{\pi}{6}$ , $b = \frac{\pi}{4}$

**Solution:** $y = \tan^2 x$ , $a = \frac{\pi}{6}$ , $b = \frac{\pi}{4}$

$y = \text{Sec}^2 x - 1$

$A = \int_a^b y\,dx = \int_{\pi/6}^{\pi/4} (\text{Sec}^2 x - 1)\, dx = [\,\tan x - x\,]_{\pi/6}^{\pi/4}$

$A = \left\{\tan\frac{\pi}{4} - \frac{\pi}{4}\right\} - \left\{\tan\frac{\pi}{6} - \frac{\pi}{6}\right\}$



$A = \left(\tan\frac{\pi}{4} - \tan\frac{\pi}{6}\right) - \left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$A = \left(1 - \frac{1}{\sqrt{3}}\right) - \left(\frac{3\pi - 2\pi}{12}\right)$

$\boxed{A = 1 - \frac{1}{\sqrt{3}} - \frac{\pi}{12} \text{ Sq. Units}}$ Ans.

**(13)** $y = 2e^{3x}$ , $a = 2$ , $b = 5$

**Solution:** $y = 2e^{3x}$ , $a = 2$ , $b = 5$

$A = \int_a^b y\,dx = \int_2^5 2e^{3x}\, dx = \left[\frac{2e^{3x}}{3}\right]_2^5$

$A = \frac{2}{3}[\,e^{3(5)} - e^{3(2)}\,] = \frac{2}{3}\{\,e^{15} - e^{6}\,\}$ Sq. Units

**(14)** $y = x - \frac{5}{x^2}$ , $a = 2$ , $b = 4$

**Solution:** $y = x - \frac{5}{x^2}$ , $a = 2$ , $b = 4$

$A = \int_a^b y\,dx = \int_2^4 (x - 5x^{-2})\, dx$

$A = \left[\frac{x^2}{2} - \frac{5x^{-1}}{-1}\right]_2^4 = \left[\frac{x^2}{2} + \frac{5}{x}\right]_2^4$

$A = \left[\frac{(4)^2}{2} + \frac{5}{4}\right] - \left[\frac{(2)^2}{2} + \frac{5}{2}\right] = \left[8 + \frac{5}{4} - 2 - \frac{5}{2}\right]$

$A = \frac{32 + 5 - 8 - 10}{4} \Rightarrow A = \frac{19}{4} \Rightarrow \boxed{A = 4\frac{3}{4} \text{ Sq Units}}$ Ans.