Ex 3.1 (Abdul Rafeh _ 104)
Assignment # 02 Calculus

Q1: How is the number e defined?
Ans. e or euler's constant
we can defined e as
$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$
e is a irrational constant.

Q2: $f(x) = 2^{40}$
Solve $f(x) = 2^{40}$
Since the derivation of constant is zero.
So, $\boxed{\frac{d}{dx} 2^{40} = 0}$ Ans

Q3: $f(t) = 2 - \frac{2}{3}t$
Solve
Apply derivation
$f'(t) = 0 - \frac{2}{t}$
$\boxed{f'(t) = -\frac{2}{3}}$ Ans

Q4: $f(x) = x^3 - 4x + 6$
Solve $f(x) = x^3 - 4x + 6$
By Using Power rule of derivate.
$\boxed{f'(x) = 3x^2 - 4}$ Ans

Q5: $g(x) = x^2(1 - 2x)$
Solve $g(x) = x^2(1 - 2x)$
$\Rightarrow g(x) = x^2 - 2x^3$
$g'(x) = 2x - 6x^2$
OR $\boxed{g'(x) = 2x(1 - 3x)}$ Ans

Q6: $g(t) = 2t^{-1/4}$
Solve $g(t) = 2t^{-1/4}$
∵ Apply derivative
$g'(t) = -\frac{1}{4}(2) t^{-1/4 - 1}$
$\boxed{g'(t) = -\frac{1}{2} t^{-5/4}}$ Ans

Q7: $A(s) = \frac{12}{s^5}$
Solve $A(s) = \frac{12}{s^5}$
Simplify. $A(s) = -12 s^{-5}$
Apply derivation:
$A'(s) = +60 s^{-5-1}$
$A'(s) = 60 s^{-6}$
OR $\boxed{A'(s) = \frac{60}{s^6}}$ Ans

Row: Using Power rule

Q8: $R(a) = (3a + 1)^2$
Solve $R(a) = (3a + 1)^2$
$\Rightarrow R(a) = 9a^2 + 6a + 1$
Now apply derivative
$\boxed{R'(a) = 18a + 6}$ Ans

Q9: $S(P) = \sqrt{P} - P$
Solve
$S(P) = \sqrt{P} - P$
$S(P) = P^{1/2} - P$
Applying derivative:-
$S'(P) = \frac{1}{2} P^{1/2 - 1} - 1$
$S'(P) = \frac{1}{2} P^{1/2 - 1}$
OR $\boxed{S'(P) = \frac{1}{2\sqrt{P}} - 1}$ Ans

Row: By Using Power rule
$= \frac{1}{2} - 1$
$= -\frac{1}{2}$

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Q^{10}: $y = 3e^x + \frac{4}{\sqrt[3]{x}}$

Solve

Applying derivative

$$\frac{d}{dx} y = 3\frac{d}{dx} e^x + 4\frac{d}{dx} x^{-1/3}$$

$$y' = 3e^x + \left(-\frac{4}{3}\right) x^{-4/3}$$

$$\boxed{y' = 3e^x - \frac{4}{3x^{4/3}}} \text{ Ans}$$

R.w

$\frac{d}{dx} e^x = e^x$

&

$x^{-1/3-1}$

$x^{-4/3}$

Q^{11}: $h(u) = Au^3 + Bu^2 + Cu$

Solve $h(u) = Au^3 + Bu^2 + Cu$

Here A, B, C are Constants

Applying derivative Using Power rules:-

$$\Rightarrow \boxed{3Au^2 + Bu + C} \text{ Ans}$$

Q^{12}: $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$

Solve $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$

$$\Rightarrow y = x^{-1/2}(x^2 + 4x + 3)$$

$$\Rightarrow y = x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

Now Applying derivatives

$$y' = \frac{3}{2} x^{3/2-1} + \frac{4}{2} x^{1/2-1} + \left(\frac{-3}{2}\right) x^{-1/2-1}$$

$$\boxed{y' = \frac{3}{2} x^{1/2} + 2x^{-1/2} - \frac{3}{2} x^{-3/2}} \text{ Ans}$$

Q^{13}: $j(x) = x^{2.4} + e^{2.4}$

Solve $j(x) = x^{2.4} + e^{2.4}$

Here "$e^{2.4}$" is Constant.

Apply derivative on $x^{2.4}$,

$$j'(x) = 2.4x^{2.4-1} + 0$$

$$\boxed{j'(x) = 2.4x^{1.4}} \text{ Ans}$$

Q^{14}, $H(x) = (x + x^{-1})^3$

Solve

Simplifying. Using $(a+b)^3$ formula

$$H(x) = x^3 + 3x^2x^{-1} + 3x(x^{-1})^2 + (x^{-1})^3$$

∴ Now apply derivatives.

$$H'(x) = 3x^2 + 3 - 3 * x^{-2} - 3x^{-3-1}$$

$$\boxed{H'(x) = 3x^2 - \frac{3}{x^2} - \frac{3}{x^4} + 3} \text{ Ans}$$

Q^{15}: $u = \sqrt[5]{t} + 4\sqrt{t^5}$

Solve $u = \sqrt[5]{t} + 4\sqrt{t^5}$

Simplify: $u = t^{1/5} + 4t^{5/2}$

Now Applying derivative.

$$\frac{du}{dt} = \frac{1}{5} t^{1/5-1} + \frac{4}{2} \times 5\, t^{5/2-1}$$

$$\boxed{u' = \frac{1}{5} t^{-4/5} + 10t^{3/2}} \text{ Ans}$$

Q^{16}: $Z = \frac{A}{y^{10}} + Be^y$

Solve. $Z = \frac{A}{y^{10}} + Be^y$

Simplify $Z = Ay^{-10} + Be^y$

R.w

A & B are Constant.

After Applying derivate, we get

$$\boxed{Z' = -10Ay^{-11} + Be^y} \text{ Ans}$$

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Answer # 17

Solve $y = \sqrt[4]{x} \quad (1,1)$

$\Rightarrow y = x^{1/4}$

Apply derivative

$$\frac{d}{dx} y = \frac{1}{4} x^{1/4 - 1}$$

$$y' = \frac{1}{4} x^{-3/4}$$

OR

$$y' = \frac{1}{4x^{3/4}}$$

To find Slope with respect to "x" on it: Put $x = 1$

$$\boxed{m = \frac{1}{4(1)^{3/4}} = \frac{1}{4}}$$

Now to find tangent

$$m = \frac{y - y_1}{x - x_1}, \quad (x_1, y_1) = (1,1)$$

$$\because m = \frac{1}{4}$$

$$\frac{1}{4} = \frac{y-1}{x-1}$$

$$(x-1)\frac{1}{4} = y - 1$$

$$\Rightarrow y = \frac{1}{4}x - \frac{1}{4} + 1$$

$$\boxed{y = \frac{1}{4}x + \frac{3}{4}} \text{ Ans}$$

So the eqn. to tangent line is $\frac{1}{4}x + \frac{3}{4}$

Answer # 18

Solve $y = x^4 + 2x^2 - x \quad (1,2)$

Apply derivative, we get:

$$\frac{d}{dx} y = 4x^3 + 4x - 1$$

To find Slope with respect to "x"

$\because x = 1$

$$m = 4(1)^3 + 4(1) - 1$$

$$\boxed{m = 7}$$

Now find tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 7(x - 1)$$

$$y = 7x - 7 + 2$$

$$\boxed{y = 7x - 5} \text{ Ans}$$

eq. of tangent line

Answer # 19

Solve $y = x^4 + 2e^x \quad (0,2)$

Taking derivative

$$\frac{d}{dx} y = \frac{d}{dx} x^4 + 2e^x$$

$$y' = 4x^3 + 2e^x$$

To find Slope Put $x = 0$

$$m = 4(0) + 2e^0$$

$$\boxed{m_1 = 2}$$

Tangent Slope

R.W.

$e^0 = 1$

$1 \times 2 = 2$

And

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The slope of a normal line is a negative resiprocal of a tangent line.

So, $\boxed{m_2 = -\frac{1}{2}}$ normal line slope

To find eq. of tangent

$\Rightarrow y - y_1 = m_1(x - x_1)$

Here $m_1 = 2, (x_1, y_1) = (0, 2)$

So, $\Rightarrow y - 2 = 2(x - 0)$

$y - 2 = 2x$

$\boxed{y = 2x + 2}$

eq. of tangent line.

Now,

To find eq. of Normal line

$\Rightarrow y - y_1 = m_1(x - x_1)$

Here: $m_2 = -\frac{1}{2}, (x_1, y_1) = (0, 2)$

So, $y - 2 = -\frac{1}{2}(x - 0)$

$\boxed{y = -\frac{1}{2}x + 2}$ Ans

Answer # 20

Solve $y = x^2 - x^4 \quad (1, 0)$

First find derivative.

we get: $\frac{d}{dx} y = 2x - 4x^3$

Now to find slope, put $x = 1$

$2(1) - 4(1)^3$

$\boxed{m = -2}$

Now: To find eq. of tangent.

$y - y_1 = m(x - x_1)$

$y - 0 = -2(x - 1)$

$y = -2x + 1$

OR $\boxed{y = 1 - 2x}$

eq. of tangent line.

R.W

$m = -2$

$(x_1, y_1) = (1, 0)$

Ex # 3.2

Answer # 01

Solve $f(x) = (x^3+2x)e^x$

Using Product rule.

Let $u = x^3+2x$, $v = e^x$

$f'(x) = (x^3+2x)\frac{d}{dx}e^x + \frac{d}{dx}(x^3+2x)$

$= (x^3+2x)(e^x) + e^x(3x^2+2)$

Taking "e^x" common

$\boxed{f'(x) = e^x(x^3+3x^2+2x+2)}$ Ans

Answer # 02

Solve $y = \frac{x}{e^x}$

Here, we use quotient rule:

$y' = \frac{e^x\frac{d}{dx}x - x\frac{d}{dx}e^x}{(e^x)^2}$

$y' = \frac{e^x - e^x x}{(e^x)^2}$

OR

$y' = \frac{\cancel{e^x}(1-x)}{\cancel{e^x}\cdot e^x}$

So,

$\boxed{y' = \frac{1-x}{e^x}}$ Ans

Answer # 03

Solve $g(x) = \frac{1+2x}{3-4x}$

Put values as quotient formula:

$u = 1+2x$, $v = 3-4x$

$g'(x) = \frac{(3-4x)\frac{d}{dx}(1+2x) - (1+2x)\frac{d}{dx}(3-4x)}{(3-4x)^2}$

$g'(x) = \frac{(3-4x)(2)-(1+2x)(-4)}{(3-4x)^2}$

$g'(x) = \frac{6-\cancel{8x}+4+\cancel{8x}}{(3-4x)^2}$

$\boxed{g'(x) = \frac{10}{(3-4x)^2}}$ Ans

Answer # 04

Solve: $H(u) = (u-\sqrt{u})(u+\sqrt{u})$

Close it in formula a^2-b^2:

$\Rightarrow H(u) = u^2 - (\sqrt{u})^2$

$H(u) = u^2 - u$

R.w Using Power rule

$\boxed{H'(u) = 2u-1}$ Ans

Answer # 05

Solve $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y+5y^3)$

Before take derivative, first Simplify it:

$f(y) = (y^{-2} - 3y^{-4})(y+5y^3)$

Now apply Product rule, Here:

$u = y^{-2} - 3y^{-4}$, $v = y+5y^3$

$\Rightarrow (y^{-2} - 3y^{-4})\frac{d}{dx}(y+5y^3) + (y+5y^3)\frac{d}{dx}(y^{-2}-3y^{-4})$

$= y^{-2} - 3y^{-4}(1+15y^2) + y+5y^3(-2y^{-3}+12y^{-5})$

$= (y^{-2} - 3y^{-4} + 15 - 45y^{-2}) + (-2y^{-2} - 10 + 12y^{-4} + 60y^{-2})$

$\Rightarrow (15 - 3y^{-4} - 44y^{-2}) + (-10 + 12y^{-4} + 58y^{-2})$

$\boxed{f'(y) = 5 + 14y^{-2} + 9y^{-4}}$ Ans

Answer # 06

Solve

$$y = \frac{x^3}{1-x^2}$$

Using quotient rule:

let: $u = x^3, V = 1-x^2$

$$y' = \frac{(1-x^2)\frac{d}{dx}(x^3) - x^3\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$y' = \frac{(1-x^2)(3x^2) - x^3(-2x)}{(1-x^2)^2}$$

$$y' = \frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2}$$

$$y' = \frac{3x^2 - x^4}{(1-x^2)^2}$$

OR

$$\boxed{y' = \frac{x^2(3-x^2)}{(1-x^2)^2}}$$

Answer # 07

Solve

$$y = \frac{t^2+2}{t^4-3t^2+1}$$

let, $u = t^2+2, \; V = t^4-3t^2+1$

Using Quotient rule

$$y' = \frac{(t^4-3t^2+1)\frac{d}{dx}(t^2+2) - (t^2+2)\frac{d}{dx}(t^4-3t^2+1)}{(t^4-3t^2+1)^2}$$

$$y' = \frac{(t^4+3t^2+1)(2t) - (t^2+2)(4t^3+6t)}{(t^4-3t^2+1)^2}$$

$$y' = \frac{2t^5 - 6t^3 + 2t - 4t^5 + 6t^3 - 8t^3 + 12t}{(t^4-3t^2+1)^2}$$

$$y' = \frac{-2t^5 - 8t^3 + 14t}{(t^4-3t^2+1)^2}$$

OR

$$\boxed{y' = \frac{t(-2t^4 - 8t^2 + 14)}{(t^4-3t^2+1)^2}}$$

Answer # 08

Solve

$$y = e^p(p + p\sqrt{p})$$

first simplify it:

$$y = e^p(p + p \cdot p^{1/2})$$

$$y = e^p(p + p^{3/2})$$

Now, Apply Product rule.

$$u = e^p, \; V = (p + p^{3/2})$$

$$y' = e^p\frac{d}{dx}(p + p^{3/2}) + (p + p^{3/2})\frac{d}{dx}e^p$$

$$y' = e^p\left(1 + \frac{3}{2}p^{1/2}\right) + (p + p^{3/2})(e^p)$$

$$= e^p\left(1 + \frac{3}{2}\sqrt{p}\right) + e^p(p + p^{3/2})$$

$$\boxed{y' = e^p\left[1 + \frac{3}{2}\sqrt{p} + p + p\sqrt{p}\right]}$$

Answer # 09

Solve

$$y = \frac{v^3 - 2v\sqrt{v}}{v}$$

Simplifying:

$$y = \frac{v^3}{v} - \frac{2v\sqrt{v}}{v}$$

$$y = v^2 - 2v^{1/2}$$

Simplify. Apply derivative here.

$$y' = 2v - \frac{\cancel{2}}{\cancel{2}}v^{-1/2}$$

$$\boxed{y' = 2v - \frac{1}{\sqrt{v}}}$$

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Answer # 10

Solu: $f(t) = \frac{2t}{2+\sqrt{t}}$

$u = 2t$, $V = 2+\sqrt{t}$

Using Quotient rule.

$$f'(t) = \frac{(2+\sqrt{t})\frac{d}{du}(2t) - (2t)\frac{d}{du}(2+\sqrt{t})}{(2+\sqrt{t})^2}$$

$$f'(t) = \frac{(2)(2+\sqrt{t}) - (2t)\left(\frac{1}{2\sqrt{t}}\right)}{(2+\sqrt{t})^2}$$

$$f'(t) = \frac{4+2\sqrt{t}-\sqrt{t}}{(2+\sqrt{t})^2}$$

$$\boxed{f'(t) = \frac{4+\sqrt{t}}{(2+\sqrt{t})^2}}$$ Ans

Answer # 11

Solu: $f(u) = \frac{A}{B+Ce^u}$

let $U = A$, $V = B+Ce^u$

Using Quotient rule.

$$f'(x) = \frac{(B+Ce^u)\frac{d}{du}A - A\frac{d}{du}(B+Ce^u)}{B+Ce^u}$$

$$= \frac{B+Ce^u(0) - A(0+Ce^u)}{B+Cex}$$

$$\boxed{f'(u) = \frac{-ACe^u}{(B+Ce^u)^2}}$$ Ans

Answer # 12

Solu: $f(u) = \frac{u}{u+\frac{c}{u}}$

To Simplify, Multiply " $\frac{u}{u}$ "

$$f(u) = \frac{u^2}{u^2+c}$$

Now apply Quotient rule.

$$f'(x) = \frac{(u^2+c)\frac{d}{du}u^2 - u^2\frac{d}{du}(u^2+c)}{(u^2+c)^2}$$

$$= \frac{(u^2+c)(2u) - (u^2)(2u)}{(u^2+c)^2}$$

$$f'(x) = \frac{(\cancel{u^2}+c-\cancel{u^2})\cdot 2u}{(u^2+c)^2}$$

$$\boxed{f'(u) = \frac{2cu}{(u^2+c)^2}}$$ Ans

Answer # 13

Solu: $f(u) = u^4 e^u$

Here we have to apply derivative

So, first take $f'(u)$.

By Using Product rule here.

$U = u^4$, $V = e^u$

$$f'(u) = u^4\frac{d}{du}e^u + e^u\frac{d}{du}u^4$$

$$= u^4e^u + e^u(4u^3)$$

$$f'(u) = e^u(u^4+4u^3)$$

Now apply $f'(u)$ on $f'(x)$:

Using Product rule.

$U = e^u$, $V = u^4+4u^3$

$$f''(x) = e^x \frac{d}{dx}(x^4 + 4x^3) + (x^4 + 4x^3)\frac{d}{dx}e^x$$

$$f''(x) = e^x(4x^3 + 12x^2) + e^x(x^4 + 4x^3)$$

Take e^x common.

$$f''(x) = e^x(x^4 + 4x^3 + 4x^3 + 12x^2)$$

$$f''(x) = e^x(x^4 + 8x^3 + 12x^2)$$

So, our answers of $f'(x)$ & $(f''(x))$ are.

$$\boxed{\begin{aligned} f'(x) &= e^x(x^4 + 4x^3) \\ f''(x) &= e^x(x^4 + 8x^3 + 12x^2) \end{aligned}}$$

Answer #14

Solu: $f(x) = \dfrac{x^2}{1+2x}$

$u = x^2$, $u' = 2x$

$v = 1+2x$, $v' = 2$

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$f'(x) = \frac{(2x)(1+2x) - 2(x^2)}{(1+2x)^2}$$

$$f'(x) = \frac{2x + \cancel{2x^2} - \cancel{2x^2}}{(1+2x)^2}$$

$$f'(x) = \frac{2x}{1+4x+4x^2}$$

Now;

$u = 2x$, $u' = 2$

$v = 1+4x+4x^2$, $v' = 4+8x$

$$f''(x) = \frac{2(1+4x+4x^2) - (2x)(4+8x)}{(1+4x+4x^2)^2}$$

$$f''(x) = \frac{2[1 + \cancel{4x} + 4x^2 - \cancel{4x} - 8x^2]}{((1+2x)^2)^2}$$

$$f''(x) = \frac{2[1-4x^2]}{((1+2x)^2)^2}$$

$$\boxed{f''(x) = \frac{2-8x^2}{(1+2x)^4}}$$

Answer #15

Solu:

$$y = \frac{x^2-1}{x^2+x+1} \quad (1, 0)$$

$$\because \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

Here.

$u = x^2 - 1$, $u' = 2x$

$v = x^2+x+1$, $V' = 2x+1$

$$\frac{dy}{dx} = \frac{2x(x^2+x+1) - (2x+1)(x^2-1)}{(x^2+x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2+x+1)(2x) - (x^2-1)(2x+1)}{(x^2+x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x^2+x+1)(2x)}{(x^2+x+1)^2} - \frac{(x^2-1)(2x+1)}{(x^2+x+1)^2}$$

for $x = 1$

$$\frac{dy}{dx} = \frac{2(1)}{((1)^2+(1)+1)} - \frac{(1^2-1)(2(1)+1)}{(1^2+1+1)^2}$$

$$y' = \frac{2}{3} - \frac{0 \times 3}{9}$$

$$y' = \frac{2}{3} \text{ or } m_1 = \frac{2}{3}$$

Now; $(y-b) = m(x-a)$

$$y - 0 = \frac{2}{3}(x-1)$$

$$3y = 2x - 2$$

$$\boxed{2x - 3y - 2 = 0}$$ eq. of Tangent

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Answer # 16

Solu: $y = 2xe^x$, $(0,0)$

$f(x) = 2xe^x$

Now differentiating w.r.t "x"

$\because (u.v)' = u'v - v'u$

$f'(x) = (2x)' \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(2x)$

$f'(x) = 2xe^x + 2e^x$

$f'(x) = e^x(2x+2)$

Slope of the tangent at $(0,0)$

$f'(0) = e^0(2(0)+2)$

$f'(0) = 2$

$m = 2$

eq. of tangent will be:

$(y-y_1) = m(x-x_1)$

$(y-0) = 2(x-0)$

$\boxed{y = 2x}$

$\because m_1 \times m_2 = -1$

$2 \times m_2 = -1$

$\boxed{m_2 = -1/2}$

$y - y_1 = m_2(x_2 - x_1)$

$y - 0 = -1/2(x^2 - 0)$

$\boxed{y = -\frac{x}{2}}$

Tangent line is $y = 2x$
Normal line is $y = -1/2x$

Aes

Answer # 17

Solve $y = \frac{2x}{x^2+1}$ $(1,1)$

differentiating w.r.t "x"

$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2x}{x^2+1}\right] \because \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v2}$

$y' = \frac{(2)(x^2+1) - (2x)(2x)}{(x^2+1)^2}$

$y' = \frac{2x^2+2-4x^2}{(x^2+1)^2}$

$y' = \frac{-2x^2+2}{(x^2+1)^2}$

Slope of tangent at $(1,1) = f'(1) = \frac{2-2}{(1^2+1)^2} = 0$

$m = 0$

Equation of the tangent line will be

$y - y_1 = m(x - x_1)$

$y - 1 = 0(x-1)$

$\frac{y-1}{x-1} = 0$

$\boxed{y = 1}$

Normal line is "⊥" to tangent Hence the normal line is $\boxed{x = 1}$

Tangent line is $y = 1$
Normal line is $x = 1$

Ae

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Answer # 18

Sol: $y = \frac{1}{(1+x^2)}$ $\left(-1, \frac{1}{2}\right)$

differentiating according to the formula. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

$$y' = \frac{(0)(1+x^2) - (2x)(1)}{(1+x^2)^2}$$

$$y' = \frac{-2x}{(1+x^2)^2}$$

when $x = -1$

$$y' = \frac{-2(-1)}{(1+(-1)^2)^2} = \frac{2}{(1+1)^2} = \frac{1}{2}$$

$$\boxed{m = \frac{1}{2}}$$

$\because y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = \frac{1}{2}(x+1)$

$2y - 1 = \frac{1}{\cancel{2}}(x+1)\cancel{2}$

$\boxed{y = \frac{x+2}{2}}$ eq. of Tangent

$\because m_1 \times m_2 = -1$

$\frac{1}{2} \times m_2 = -1$

$\boxed{m_2 = -2}$

$y - y_1 = m_2(x - x_1)$

$y - \frac{1}{2} = -2(x+1)$

$2y - 1 = -4x - 4$

$\boxed{y = \frac{-4x-3}{2}}$ eq. of Normal

Answer # 19

The Curve $y = \frac{x}{1+x^2}$ is called a Serpentine find an equation of the tangent line to this curve at the Point $(3, 0.3)$ illustrate Part (a) by graphing the curve & the tangent line on the Screen.

Solution: $y = \frac{x}{1+x^2}$ $\because \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

$$y' = \frac{(1)(1+x^2) - 2x(x)}{(1+x^2)^2}$$

$$y' = \frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$y' = \frac{1-x^2}{(1+x^2)^2}$$

at $x = 3$

$$y' = \frac{1-(3)^2}{(1+3^2)^2} = \frac{1-9}{(1+9)^2}$$

$$y' = \frac{-8}{100} \quad \boxed{= -0.08}$$

$m = -0.08$ or $-\frac{2}{25}$

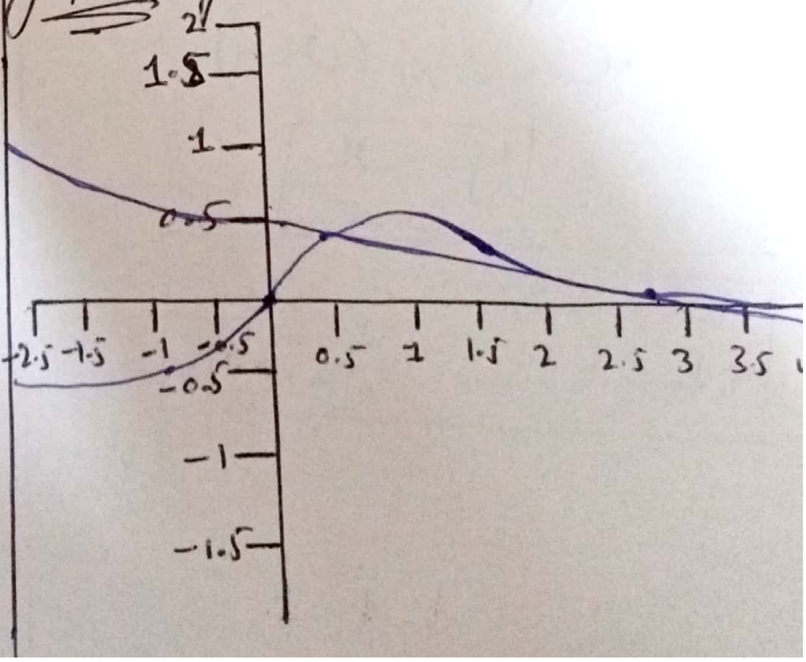
$y - y_1 = m(x - x_1)$

$y - 0.3 = -0.08(x-3)$

$\boxed{y = 0.54 - 0.08x}$

Required equation of the tangent.

Graphically:-



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EX 3.3

Answer # 01

Solve $f(x) = 3x^2 - 2\cos x$

Applying derivative we get

$$\boxed{f'(x) = 6x + 2\sin x}$$ Ans

R.W
$\frac{d}{dx}\cos x = -\sin x$

Answer # 02

Solve $f(x) = \sin x + \frac{1}{2}\cot x$

Applying derivative we get:

$$\boxed{f'(x) = \cos x - \frac{1}{2}\operatorname{cosec}^2 x}$$ Ans

R.W
$\frac{d}{dx}\sin x = \cos x$
$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

Answer # 03

Solve $y = \sec\theta\tan\theta$

Here we use Product rule.

$\because \frac{d}{d\theta}(u.v) = u\frac{d}{d\theta}v + v\frac{d}{d\theta}u$

So,
Here $u = \sec\theta$ & $v = \tan\theta$

$\Rightarrow y' = \sec\theta\frac{d}{du}\tan\theta + \tan\theta\frac{d}{d\theta}\sec\theta$

Applying derivative

$y' = \sec\theta(\sec^2\theta) + \tan\theta(\tan\theta, \sec\theta)$

$$\Rightarrow \boxed{y' = \sec^3\theta + \tan^2\theta\sec\theta}$$ Ans

Answer # 04

Solve $y = c\cos t + t^2\sin t$

Apply derivative

$\frac{d}{dt}y = \frac{d}{dt}c\cos t + \frac{d}{dt}t^2\sin t$

first function "c" is constant and in other function use Product rule (uv formula),

So, $\frac{dy}{dt} = c\frac{d}{dt}\cos t + t^2\frac{d}{dt}\sin t + \sin t\frac{d}{dt}t^2$

Now apply derivative we get:

$$\boxed{\frac{dy}{dt} = -c\sin t + t^2\cos t + 2t\sin t}$$ Ans

Answer # 05

Solve $y = \frac{x}{2-\tan x}$

$y' = \frac{d}{dx}\left[\frac{x}{2-\tan x}\right]$

Here we use quotient rule which is $\frac{d}{d\theta}\frac{u}{v}$ formula.

$\Rightarrow y' = \frac{\left(2-\tan x\frac{d}{dx}x\right) - \left(x\frac{d}{dx}(2-\tan x)\right)}{(2-\tan x)^2}$

$y' = \frac{2-\tan x(1) - (x)(-\sec^2 x)}{(2-\tan x)^2}$

$$\boxed{y' = \frac{2-\tan x + x\sec^2 x}{(2-\tan)^2}}$$ Ans

Answer # 06

Solve $f(\theta) = \frac{\sec\theta}{1+\sec\theta}$

By using Quotient rule

$u = \sec\theta$, $v = 1+\sec\theta$

If we take derivative of u & v. we get.

$u' = \sec\theta\tan\theta$, $v' = \sec\theta\tan\theta$

So,

$\Rightarrow f'(\theta) = \frac{\sec\theta\tan\theta + \sec^2\theta\tan\theta - \sec^2\theta\tan\theta}{(1+\sec\theta)^2}$

After cancel out:

$$\boxed{f'(\theta) = \frac{\sec\theta\tan\theta}{(1+\sec\theta)^2}}$$ Ans

Answer # 07

Solve. $y = \frac{t \sin t}{1+t}$

By Using Quotient rule.

let: $u = t \sin t$, $v = 1+t$,

$\Rightarrow$ for $u = t \sin t$ use Product rule

as $u = t$, $v = \sin t$ here:

So, $u' = \sin t' + \cos t$ eq $\rightarrow$ (1)

& $V = 1+t = 1$ eq $\rightarrow$ (2)

Now Put both eq (1) & (2) in quotient formula.

$$y' = \frac{(\sin t + t \cos t)(1+t) - t \sin t (1)}{(1+t)^2}$$

$$= \frac{\sin t + t \cos t + t^2 \cos t}{(1+t)^2}$$

$$\boxed{y' = \frac{(t^2+t)\cos t + \sin t}{(1+t)^2}}$$ Ans

Answer # 08

Solve. $f(x) = xe^x \csc x$

let. $u = xe^x$ & $V = \csc x$

Apply Product rule:

$$f'(x) = xe^x \frac{d}{dx} \csc x + \csc x \frac{d}{dx} \frac{xe^x}{uv}$$

After apply derivative, we get.

$$= -xe^x \sin x + \csc x \left[x \frac{d}{dx} e^x + e^x \frac{dx}{dx}\right]$$

$$= -xe^x \sin x + xe^x \operatorname{cosec} x + e^x (1)$$

$$= f'(x) = xe^x \csc x - xe^x \sin x + e^x$$

taking common of " e^x ".

$$\boxed{f'(x) = e^x (x \csc x - x \sin x + 1)}$$ Ans

Answer # 09

Solve

Prove $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$

Using quotient rule here.

$$\Rightarrow \frac{d}{dx} \csc x - \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

So, Using $\frac{1}{\sin x}$ here

$$\frac{d}{dx} \csc x = \frac{\sin x \frac{d}{dx}(1) - (1) \frac{d}{dx} \sin x}{(\sin x)^2}$$

$$= \frac{\sin x (0) - (1) \cos x}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin^2 x}$$

R.W $\frac{\cos x}{\sin x} = \cot x$

OR

$$= (-1)\frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

$$\boxed{\frac{d}{dx} \csc x = -\operatorname{cosec} x \cot x}$$ Proved

Ans

Answer # 10

Solve

Prove $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$

use Quotient rule here.

$$\Rightarrow \frac{d}{dx} \cot x = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

Apply $\frac{\cos x}{\sin x}$ here.

$$\frac{d}{dx} \cot x = \frac{\sin x \frac{d}{dx} \cos x - \cos x \frac{d}{dx} \sin x}{\sin^2 x}$$

$$= \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

R.w $\sin^2\theta + \cos^2\theta = 1$

OR

$$\boxed{\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x \text{ Proved}}$$

Answer # 11

Solu: $y = \sec x \left(\frac{\pi}{3}, 2\right)$

first take derivative:

$y' = \sec x \tan x$

Now To find Slope of tangent line Put $x = \frac{\pi}{3}$ in it.

We get:

$m = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$

$$\boxed{m = 2\sqrt{3}}$$

Now Put values $(x_1, y_1) = \left(\frac{\pi}{3}, 2\right)$ in a tangent formula.

$\Rightarrow y - y_1 = m(x - x_1)$

$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$

OR $\boxed{y = 2\sqrt{3}\left(x - \frac{\pi}{3}\right) + 2}$ tx

Answer # 12

Solu $y = \cos x - \sin x \; (\pi, -1)$

Take derivative we get: $y' = -\sin x - \cos x$

Put $x = \pi$ to find Slope (m).

$\Rightarrow m = -\sin\pi - \cos\pi$

$$\boxed{m = 0 - (-1) = 1}$$

Now, Put value in tangent formula.

$y - y_1 = m(x - x_1)$

$\Rightarrow y - (-1) = 1(x - \pi)$

$$\boxed{y = x - \pi - 1}$$ tx

Ex 3.4 Answer # 01

Solu $y = \sqrt[3]{1 + 4x}$

$\Rightarrow y = (1 + 4x)^{1/3}$

Apply derivative Using Chain rule.

Let $u = (1 + 4x)$

$\frac{d}{dx}y = \frac{d}{dx}u^{1/3}$

$\Rightarrow y' = \frac{1}{3}u^{-2/3} \times \frac{d}{dx}u$

Put $u = (1 + 4x)$ again,

$y' = \frac{1}{3}(1 + 4x)^{-2/3} \times \frac{d}{dx}(1 + 4x)$

$y' = \frac{1}{3}(1 + 4x)^{-2/3}(4)$

OR

$y' = \frac{4}{3}(1 + 4x)^{-2/3}$ Ans

Answer # 02

Solve: $y = \tan \pi x$

Using chain rule Put $u = \pi$

Solve by inner and outer values

$\Rightarrow \frac{d}{dx} y = \tan u$

$y' = \sec^2 x u (u)$

Put $u = \pi$:

$y' = \sec^2 x \pi (\pi)$

$y' = \pi \sec^2 (\pi x)$ Ans

Answer # 03

Solve: $y = e\sqrt{x}$ $\quad e^{\sqrt{x}}$

Differentiate Using chain rule:

let $u = \sqrt{x}$

$\frac{dy}{dx} = \frac{de^u}{dx}$

$\Rightarrow \frac{dy}{dx} = e^u \times \frac{d}{dx} u$

Row: $\frac{d}{dx} e^u = e^u$

Put $u = \sqrt{x}$

$y' = e\sqrt{x} \frac{d}{dx} x^{1/2}$

$y' = \frac{1}{2} e^x x^{-1/2}$

$y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ Ans

Answer # 04

Solve: $f(x) = (x^4 + 3x^2 - 2)^5$

let $z = x^4 + 3x^2 - 2$

So, Using chain rule.

Apply derivative. $\frac{d}{dz}(f(x)) = z^5$

$f'(x) = 5z^4 \frac{d}{dx} z$

Put $z = x^4 + 3x^2 - 2$

$f'(x) = 5(x^4 + 3x - 2)^4 \frac{d}{dx}(x^4 + 3x^2 - 2)$

$f'(x) = 5(x^4 + 3x - 2)^4 (4x^3 + 6x)$ Ans

Answer # 05

Solve: $F(x) = \sqrt{1-2x}$

To use chain rule let: $u = (1-2x)$

and apply derivative on it.

$f'(x) = \frac{d}{dx} \sqrt{u}$

$f'(u) = \frac{1}{2} u^{-1/2} \frac{d}{dx} u$

Put $u = 1-2x$ again

$f'(x) = \frac{1}{2}(1-2x)^{1/2} \times \frac{d}{dx}(1-2x)$

$f'(x) = \frac{1}{\cancel{2}\sqrt{1-2x}} \times (-\cancel{2})$

$f'(x) = \frac{-1}{\sqrt{1-2x}}$ Ans

Answer # 06

Solve: $F(z) = \frac{1}{z^2+1}$

OR

$f(z) = (z^2+1)^{-1}$

Now, apply derivative Using chain rule. $u = z^2 + 1$

$f'(z) = \frac{d}{dz} u^{-1} \frac{d}{dz} u$

$f'(z) = -u^{-2} \frac{d}{dz} u$

Put $u = (z^2+1)$

$f'(z) = -(z^2+1)^{-2} \frac{d}{dx}(z^2+1)$

$F'(z) = -\frac{2z}{(z^2+1)^2}$ Ans

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Answer # 07

Solu: $y = \cos(a^3 + x^3)$

Apply derivative Using chain rule:

$$\frac{d}{dx} y = \frac{d}{dx} \cos(a^3 + x^3)$$

let; $u = a^3 + x^3$

$$y' = \frac{d}{du} \cos(u)$$

$$y' = -\sin(u) \frac{d}{dx} u$$

Put $u = x^3 + a^3$

$$y' = -\sin(x^3 + a^3)(3x^2)$$

$$y' = -3x^3 \sin(x^3 + a^3)$$

Ans

Answer # 08

Solx $y = xe^{-kx}$

Differentiating

$$\frac{dy}{dx} = \frac{d}{dx}(xe^{-kx})$$

$\because (u \cdot v)' = u'v + v'u$

$$\frac{dy}{dx} = x \cdot \frac{d}{dx}(e^{-kx}) + e^{-kx} \cdot \frac{dx}{dx}$$

$$\frac{dy}{dx} = \frac{x\, d(e^{-kx})}{d(-kx)} \times \frac{d(-kx)}{dx} + e^{-kx}$$

A/c to the formula;

$$\frac{d(e^{y})}{dx} = e^{y}$$

$$\frac{dy}{dx} = x \cdot e^{-kx} \times (-k) + e^{-kx}$$

$$\frac{dy}{dx} = e^{-kx}[-kx + 1]$$

$$\boxed{\frac{dy}{dx} = e^{-kx}[1 - kx]}$$

Answer # 09

Solu $y = \left(\frac{x^2+1}{x^2-1}\right)^3$

$$\because \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

Now differentiating the above eq.

$$y' = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \left(\frac{(2x)(x^2-1) - (2x)(x^2+1)}{((x^2-1))^2}\right)$$

$$y' = \frac{3(x^2+1)^2}{(x^2-1)^2}\left(\frac{\cancel{2x^3} - 2x - \cancel{2x^3} - 2x}{(x^2-1)^2}\right)$$

$$y' = \frac{3(x^2+1)^2(-4x)}{(x^2-1)^4}$$

$$\boxed{y' = \frac{-12x(x^2+1)^2}{(x^2-1)^4}}$$

Ans

Answer # 10

Sol: $y = \sqrt{1 + 2e^{3x}}$

$$y = \sqrt{1+2e^{3x}} = (1 + 2e^{3x})^{1/2}$$

Now differentiating

$$y' = \frac{1}{2}(1+2e^{3x})^{-1/2} \times \frac{d}{dx}(1+2e^{3x})$$

$$y' = \frac{1}{2\sqrt{1+2e^{3x}}} \times 6e^{3x}$$

$$y' = \frac{\overset{3}{\cancel{6}}e^{3x}}{\cancel{2}\sqrt{1+2e^{3x}}}$$

$$\boxed{y' = \frac{3e^{3x}}{\sqrt{1+2e^{3x}}}}$$

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Answer # 11

Solve: $y = 5^{-1/x}$

Now differentiating:

$$y' = \frac{d}{dx}\left(5^{-1/x}\right)$$

$$y' = \frac{d(5^{-1/x})}{d(-1/x)} \times \frac{d}{dx}\left(-1/x\right)$$

$$y' = 5^{-1/x} \ln 5 \times 1/x^2$$

$$\boxed{y' = \frac{5^{-1/x} \ln 5}{x^2}}$$

Answer # 12

Solu: $y = \frac{r}{\sqrt{r^2+1}}$ $\because \left[\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}\right]$

$$\frac{dy}{dx} = \frac{(r^2+1)^{1/2}(1) - (r)\left[\frac{1}{2}(r^2+1)^{-1/2}(2r)\right]}{(r^2+1^{1/2})^2}$$

$$\frac{dy}{dx} = \frac{(r^2+1) - \frac{1}{\cancel{2}}(r^2+1)^{-1/2}\,\cancel{2}r}{r^2+1}$$

$$\frac{dy}{dx} = \frac{(r^2+1)^{1/2} - r^2(r^2+1)^{-1/2}}{(r^2+1)}$$

$$\frac{dy}{dx} = \frac{(r^2+1)^{1/2} - \frac{r^2}{(r^2+1)^{1/2}}}{r^2+1}$$

$$y' = \frac{(r^2+1)^{1/2} - \frac{r^2}{(r^2+1)^{1/2}}}{r^2+1} \times \frac{(r^2+1)^{1/2}}{(r^2+1)^{1/2}}$$

$$y' = \frac{r^2+1-r^2}{(r^2+1)^{3/2}} = \frac{1}{(r^2+1)^{3/2}}$$

$$\boxed{\frac{dy}{dx} = (r^2+1)^{-3/2}}$$

Answer # 13

Solu

$F(t) = e^{t\sin 2t}$

$F(t) = y = e^{t\sin 2t}$

let, $y = e^u$

$u = vw$

$v = t$

$w = \sin x$

$x = 2t$

$$\frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt}$$

$$= \frac{dy}{du}\frac{d}{dt}(vw)$$

$$= \frac{dy}{du}\left[v\frac{dw}{dt} + w\frac{dv}{dt}\right)$$

$$= \frac{dy}{du}\left(v\frac{dw}{dx}\frac{dx}{dt} + w\frac{dv}{dt}\right)$$

$$= e^u\left(t \times \cos x * 2 + \sin x \times 1\right)$$

$$= e^{t\sin 2t}\left(2t \times \cos 2t x + \sin 2t\right)$$

$$= e^{t\sin 2t}\left(2t \times \cos 2t + \sin 2t\right)$$

$$\boxed{= e^{t\sin 2t}\left(2t\cos 2t + \sin 2t\right)}$$

Answer # 14

Solu

$y = \sin(\tan 2x)$

Differentiating

$$\frac{dy}{dx} = \frac{d}{dx}\sin(\tan 2x)$$

$$\frac{dy}{dx} = \frac{d\sin(\tan 2x)}{dx(\tan 2x)} \times \frac{d(\tan 2x)}{d(2x)} \times \frac{d(2x)}{dx}$$

$$y' = \cos(\tan 2x) \times \sec^2 2x \times 2$$

$$\boxed{y' = 2\sec^2 2x \cos x(\tan 2x)}$$

Answer # 15

Solu $y = 2^{\sin \pi x}$

Differentiating

$$\frac{dy}{dx} = \frac{d}{dx}\left(2^{\sin \pi x}\right)$$

Now Using Chain rule.

$$\frac{dy}{dx} = \frac{d\left(2^{\sin \pi x}\right)}{d(\sin \pi x)} \times \frac{d(\sin \pi x)}{d\,\pi x} \times \frac{d(\pi x)}{dx}$$

$$\frac{dy}{dx} = \left(2^{\sin \pi x} \times \ln 2\right) \times \cos \pi x \times \pi$$

$$\boxed{y' = \pi \ln 2 \cdot 2^{\sin \pi x} \cos \pi x}$$

Answer # 16

Solu $y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$

$$y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}} \times \frac{e^{-x}}{e^{-x}}\right)$$

$$y = \cos\left(\frac{e^{-x}-e^{2x-x}}{e^{-x}+e^{2x-x}}\right)$$

$$y = \cos\left(\frac{e^{-x}-e^{x}}{e^{-x}+e^{x}}\right)$$

$\because \frac{e^{-x}-e^{x}}{e^{-x}+e^{x}} = \tanh x$

$$y = \cos(\tanh(x))$$

$$\frac{dy}{dx} = \frac{d\cos(\tanh(x))}{dx}$$

$$\frac{dy}{dx} = \frac{d(\cos(\tanh(x)))}{d\tanh x} \times \frac{d\tanh x}{dx}$$

$$y' = -\sin(\tanh(x)) \times (-\operatorname{sech}^2 x)$$

$$y' = \operatorname{sech}^2 x \sin(\tanh(x))$$

$$\operatorname{sech} x = \frac{2}{e^{x}+e^{-x}}$$

$$\boxed{\frac{dy}{dx} = \frac{4}{(e^{x}+e^{-x})^2} \cdot \sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)}$$

Answer # 17

Solve

$$y = \cot^2(\sin\theta)$$

diff. w.r.t θ.

$$y' = 2\cot(\sin\theta) \cdot (\cot(\sin\theta))'$$

$$y' = 2\cot(\sin)\left(-\operatorname{cosec}^2(\sin\theta)\cos\theta\right)$$

$$\boxed{y' = -2\cot(\sin\theta)\operatorname{cosec}^2(\sin\theta)\cos\theta}$$

Answer # 18

Sol:- $f(t) = \tan(e^t) + e^{\tan t}$

Differentiating;

$$f'(t) = \frac{d}{dt}(\tan(e^t))$$

$$f'(t) = \frac{d}{dt}(\tan(e^t)) + \frac{d}{dt}\left(e^{\tan t}\right)$$

$$f'(t) = \sec^2(e^t)\frac{d}{dt}(e^t) + e^{\tan t} \times \frac{d(\tan t)}{dt}$$

$$f'(t) = \sec^2(e^t)e^t + e^{\tan t}\sec^2 t$$

$$\boxed{f'(t) = e^t(\sec^2(e^t)) + e^{\tan t}\sec^2 t}$$

Answer # 19

Solve $f(t) = \sin^2\left(e^{\sin^2 t}\right)$

Differentiating,

$$f'(t) = \frac{d}{dt}\left(\sin^2\left(e^{\sin^2 t}\right)\right)$$

$$f'(t) = \frac{d\left[\sin^2\left(e^{\sin^2 t}\right)\right]}{d\left[\sin\left(e^{\sin^2 t}\right)\right]} \times \frac{d\left[\sin\left(e^{\sin^2 t}\right)\right]}{d\left[e^{\sin^2 t}\right]}$$

① $$\times \frac{d\left(e^{\sin^2 t}\right)}{d\,\sin^2 t}$$

② $$\times \frac{d(\sin^2 t)}{d(\sin t)} \times \frac{d}{dt}(\sin t)$$

$$f'(t) = \left(2\sin e^{\sin^2 t} \times \cos e^{\sin^2 t}\right) \times$$

$$e^{\sin^2 t} \times 2\sin t \times \cos t$$

$$f'(t) = \sin\left(2e^{\sin^2 t}\right) \times e^{\sin^2 t} \times \sin 2t$$

$$\because 2\sin\theta\cos\theta = \sin 2\theta$$

$$\boxed{f'(t) = \sin 2t \cdot e^{\sin^2 t} \sin\left(2e^{\sin^2 t}\right)}$$

Ans

~~Answer #20~~

Solve

$$g(x) = \left(2ra^{rx} + x\right)^p$$

Differentiating:

$$g'(x) = p\left(2ra^{rx}+x\right)^{p-1} \times \left(2ra^{rx}+x\right)'$$

$$g'(x) = p\left(2ra^{rx}+x\right)^{p-1} \times \left(2r(\ln a)a^{rx}(rx)' + 0\right)$$

$$g'(x) = p\left(2ra^{rx}+x\right)^{p-1}\left(2r(\ln a)a^{rx}(r)\right)$$

$$\boxed{g'(x) = 2r^2 p(\ln a)\left(2ra^{rx}+x\right)^{p-1} a^{rx}}$$

Ans

~~Answer #21~~

Solve.

$$y = \cos\sqrt{\sin(\tan \pi x)}$$

Differentiating w.r.t "x"

$$y' = \frac{d\left(\cos\sqrt{\sin(\tan \pi x)}\right)}{dx}$$

$$y' = \frac{d}{d}\left[\frac{\cos\sqrt{\sin(\tan\pi x)}}{\sqrt{\sin(\tan\pi x)}}\right] \times \frac{d\left(\sqrt{\sin(\tan\pi x)}\right)}{dx}$$

$$y' = -\sin\sqrt{\sin(\tan\pi x)} \times \frac{d\left[\sqrt{\sin(\tan\pi x)}\right]}{d[\sin(\tan\pi x)]} \times \frac{d}{dx}\sin(\tan\pi x)$$

$$y' = -\sin\sqrt{\sin(\tan\pi x)} \times \frac{1}{2\sqrt{\sin(\tan(\pi x))}} \times \frac{d}{d}$$

$$\frac{\sin(\tan\pi x)}{\tan\pi x} \times \frac{d}{dx}\tan\pi x$$

$$y' = \frac{-\sin\sqrt{\sin(\tan\pi x)}}{2\sqrt{\sin(\tan\pi x)}} \cdot \cos(\tan(\pi x)) \times \frac{d}{d}\frac{(\tan\pi x)}{(\pi x)} \times \frac{d\pi x}{dx}$$

$$y' = \frac{-\sin\sqrt{\sin(\tan\pi x)} \times \cos(\tan(\pi x)) \times \sec^2\pi x \times \pi}{2\sqrt{\sin(\tan\pi x)}}$$

$$\boxed{y' = \frac{-\pi\sec^2\pi x\cos(\tan\pi x) \cdot \sin\sqrt{\sin(\tan\pi x)}}{2\sqrt{\sin(\tan\pi x)}}}$$

Ans.

~~Answer #22~~

Solve

$$y = \cos(x^2)$$

Differentiating w.r.t "x"

$$y' = \frac{d(\cos(x^2))}{dx}$$

$$y' = -\sin x^2 \times \frac{d}{dx}(x^2)$$

$$y' = -\sin x^2 \times 2x$$

$$y' = -2x\sin x \quad \because u.v = u'v + v'u$$

$$y' = (-2x)'\sin x^2 + (-2x)(\sin(x^2))'$$

$$y' = -2\sin x^2 - 2x\cos x^2 \times (x^2)'$$

$$y' = -2\sin x^2 - 2x\cos x^2 \times 2x$$

$$\boxed{y' = -2\sin x^2 - 4x^2\cos x^2}$$

Ans

Answer # 23

Solve: $y = e^{\alpha x}\sin\beta x$.

$\because (uv)' = u'v + v'u$

$y' = (e^{\alpha x})'\sin\beta x + e^{\alpha x}(\sin\beta x)'$

$y' = \alpha e^{\alpha x}\sin\beta x + e^{\alpha x}\cos\beta x \times \beta$

$y' = e^{\alpha x}(\alpha\sin\beta x + \beta\cos\beta x)$

Now,

$y' = (e^{\alpha x})'(\alpha\sin\beta x + \beta\cos\beta x) + e^{\alpha x}(\alpha\sin\beta x + \beta\cos\beta x)'$

$y' = \alpha e^{\alpha x}(\alpha\sin\beta x + \beta\cos\beta x) + e^{\alpha x}(\alpha\beta\cos\beta x - \beta^2\sin\beta x)$

$y' = e^{\alpha x}(\alpha^2\sin\beta x + \alpha\beta\cos\beta x) + e^{\alpha x}(\alpha\beta\cos\beta x - \beta^2\sin\beta x)$

$y' = e^{\alpha x}(\alpha^2\sin\beta x + \alpha\beta\cos\beta x + \alpha\beta\cos\beta x - \beta^2\sin\beta x)$

$\boxed{y' = e^{\alpha x}\left[(\alpha^2 - \beta^2)\sin\beta x + 2\alpha\beta\cos\beta x\right]}$ Ans

Answer # 24

Solve: $y = (1+2x)^{10}, \ (0, 1)$

Differentiating w.r.t. "x"

$y' = 10(1+2x)^9 \cdot 2$.

$y' = 20(1+2x)^9$

Now for finding the value of the slope of the tangent put the value of the tangent in the differentiated equation.

and we get:

$y' = 20(1+(2(0)))^9$

$y' = 20(1+0)^9$

$y' = 20$ or $m = 20$

and we know that;

$y - y_1 = m(x - x_1)$

$y - 1 = 20(x - 0)$

$\boxed{\begin{aligned} y - 1 &= 20x \\ y &= 20x + 1 \end{aligned}}$ OR Ans

Answer # 25

Solve: $y = \sin(\sin x)\ (\pi, 0)$

Differentiating w.r.t. "x"

$y' = \cos(\sin x)\cos x \times 1$

$y' = \cos(\sin x)\cos x$

Now substituting the value of "x" in order to find the slope of tangent in the differentiated equation.

$y' = \cos(\sin\pi)\cos\pi$

$y' = \cos(0)(-1)$

$y' = -1$ or $\boxed{m = -1}$

Here we know that;

$y - y_1 = m(x - x_1)$

So the equation of the tangent line to the curve will be

$y - 0 = (-1)(x - \pi)$

$\boxed{\begin{aligned} y &= -x + \pi \text{ OR} \\ y &= \pi - x \end{aligned}}$ Ans

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Ex 3.5 Answer # 1

Solve:

$x^3 + y^3 = 1$

Now differentiating w.r.t "x"

$= \frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) \frac{d}{dx}(1)$

$= 3x^2 + 3y^2 \frac{dy}{dx} = 0$

$= \frac{dy}{dx} = -\frac{3x^2}{3y^2}$

$= \frac{dy}{dx} = \frac{-x^2}{y^2}$

Ans.

Answere no 2.

Solve:

$\because uv = u'v + v'u$

$x^2 + xy - y^2 = 4$

differentiating w.r.t. "x"

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(xy) - \frac{d}{dx}(y^2)$

$= \frac{d}{dx}(4)$

$= 2x + (x'y + y'x) - 2y\frac{dy}{dx} = 0$

$= 2x + y + x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 2y) = -2x - y$

$\frac{dy}{dx} = -\frac{(2x+y)}{(x-2y)}$

$\frac{dy}{dx} = \frac{2x+y}{2y-x}$

Ans:

Answere no 3

Solve:

$x^4(x+y) = y^2(3x-y)$

$\because (u.v)' = u'.v + v'u$

Now differentiating w.r.t "x"

$x^4 \frac{d(x+y)}{dx} + (x+y)\frac{d(x^4)}{dx}$

$= (3x-y).\frac{d(y^2)}{dx} + y^2\frac{d(3x-y)}{dx}$

$x^4 \frac{d}{dx}(x+y) + (x+y)\frac{d(x^4)}{dx} = (3x-y)$

$\frac{d}{dy}y^2 . \frac{dy}{dx} + y^2\left[3 - \frac{dy}{dx}\right.$

$\left[x^4 + x^4\frac{dy}{dx}\right] + (4x^4 + 4x^3y)$

$= (6x - 2y^2)\frac{dy}{dx} + \left[3y^2 - y^2\frac{d}{d}\right.$

$x^4\frac{dy}{dx} + 5x^4 + 4x^3y = (6xy - 3y^2)$

$\frac{dy}{dx} + 3y^2$

$(x^4 - 6xy + 3y^2)\frac{dy}{dx} = 3y^2 - 5x^4 - 4x^3y$

$\frac{dy}{dx} = \frac{3y^2 - 5x^4 - 4x^3y}{x^2 - 6xy + 3y^2}$

Ans:

Answere no 4.

Solve:

$$y'\cos x - y\sin x = 2x + 2yy'$$

$$y'\cos x - 2yy' = 2x + y\sin x$$

$$y'(\cos x - 2y) = 2x + y\sin x$$

$$y' = \frac{2x + y\sin x}{\cos x - 2y}$$

OR

$$\frac{dy}{dx} = \frac{2x + y\sin x}{\cos x - 2y}$$

Ans:

Answere no 5

Solve:

$$4\cos x \sin y = 1$$

differentiating w.r.t "x"

$$4\frac{d}{dx}\left[\cos x \sin y\right] = \frac{d}{dx}(1)$$

$$\boxed{4\cos x\, y'\cos y - 4\sin x \sin y = 0}$$

$$y' \cdot 4\cos x \cos y = 4\sin x \sin y$$

$$y' = \tan x \tan y \quad \left[\begin{array}{l} \tan x = \frac{\sin x}{\cos x} \\ \tan y = \frac{\sin y}{\cos y} \end{array}\right]$$

$$\frac{dy}{dx} = \tan x \tan y$$

Ans:

Answere no 6

Solve:

$$e^{x/y} = x - y$$

Differentiating w.r.t "x"

$$\frac{d}{dx}(e^{x/y}) = \frac{d}{dx}(x-y)$$

$$\frac{d(e^{x/y})}{d(x/y)} \times \frac{d(x/y)}{dx}$$

$$= \frac{d(x)}{dx} - \frac{dy}{dx}$$

$$e^{x/y} \times \frac{d}{dx}(x/y) = 1 - \frac{dy}{dx}$$

$$e^{x/y} \times \frac{y\frac{dx}{dx} - x\frac{dy}{dx}}{y^2} = 1 - \frac{dy}{dx}$$

$$(e^{x/y}) \times \frac{y - x\frac{dy}{dx}}{y^2} = 1 - \frac{dy}{dx}$$

$$-x\frac{dy}{dx} + \frac{y^2}{e^{x/y}} \cdot \frac{dy}{dx}$$

$$= \frac{y^2}{e^{x/y}} - y$$

Now: $$\frac{dy}{dx} = \frac{\frac{y^2}{e^{x/y}} - y}{\frac{y^2}{e^{x/y}} - x}$$

$$\frac{dy}{dx} = \frac{y^2 - ye^{x/y}}{y^2 - xe^{x/y}}$$

Ans:

Answere no 7.

Solve:

$\tan^{-1}(x^2y) = x + xy^2$

Differentiating w.r.t "x"

$$\frac{d}{dx}(\tan^{-1} x^2y) = \frac{d}{dx}(x + xy^2)$$

$$\frac{d}{d}\left[\frac{\tan^{-1}(x^2y)}{x^2y}\right] \times \frac{d(x^2y)}{dx}$$

chain rule

$$= \underbrace{\frac{d}{dx} + x\frac{d(y^2)}{dx}}_{\text{Chain Rule}} \frac{dy + y^2}{dx} \quad \frac{dx}{dx}$$

$$\frac{d}{d}\left[\frac{\tan^{-1}(x^2y)}{(x^2y)}\right] \times \left[\underbrace{x^2\frac{dy}{dx} + y\frac{d(x^2)}{dx}}_{\text{Chain Rule}}\right]$$

$$= \frac{dx}{dx} + x\underbrace{\frac{d(y^2)}{dy}\frac{dy}{dx}}_{\text{chain Rule}} + y^2\frac{dy}{dx}$$

$$\therefore \left[\frac{d(\tan^{-1}u)}{dx} = \frac{1}{1+x^2}\right]$$

Reverse function of tan

$$\frac{1}{1+(x^2+y)^2} \times \left[x^2\frac{dy}{dx} + y(2x)\right] = 1 + x(2y)\frac{dy}{dx} + y^2 + 1$$

$$\frac{1}{1+x^4y^2} \times \left[x^2\frac{dy}{dx} + 2xy\right] = 1 + 2xy\frac{dy}{dx} + y^2$$

$$\frac{x^2}{1+x^4y^2} \times \frac{dy}{dx} + \frac{2xy}{1+x^2y^2} = (1+y^2) + 2xy\frac{dy}{dx}$$

$$\left[\frac{x^2}{1+x^4y^2} - 2xy\right]\frac{dy}{dx} = (1+y^2) - \frac{2xy}{1+x^4y^2}$$

$$\left[\frac{x^2 - 2xy(1+x^4y^2)}{1+x^4y^2}\right]\frac{dy}{dx} =$$

$$\frac{(1+y^2)(1+x^4y^2) - 2xy}{1+x^4y^2}$$

$$\frac{dy}{dx} = \frac{(1+y^2)(1+x^4+y^2) - 2xy}{x^2 - 2xy(1+x^4y^2)}$$

$$\frac{dy}{dx} = \frac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$$

Answere no 8:

Solve:

$e^y \cos x = 1 + \sin xy$

differentiating w.R.t "x"

$$\frac{d}{dx}(e^y \cos x) = \frac{d(1+\sin xy)}{dx}$$

$$\frac{d}{dx}(\sqrt{e^u}) = e^u\frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

Continue next page

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$-e^y \sin x + \cos x \cdot e^y \frac{dy}{dx} = \left(x\frac{dy}{dx} + y\right)\cos yx$$

$$-e^y \sin x + \cos x\, e^y \frac{dy}{dx} = x\cos xy \frac{dy}{dx} + y\cos xy$$

$$e^y \cos x \frac{dy}{dx} - x\cos xy \frac{dy}{dx} = y\cos xy - e^y \sin x$$

$$\frac{dy}{dx} = \frac{y\cos xy - e^y \sin x}{e^y \cos x - x\cos xy}$$

Answere no 9

Solve:-

$$y \sin 2x = x\cos 2y \quad \boxed{\left(\frac{\pi}{2}, \frac{\pi}{4}\right)}$$

Now differentiating w.r.t $\boxed{\text{"}x\text{"}}$

$$\frac{d}{dx}(y\sin 2x) = \frac{d}{dx}(\cos 2y\, x)$$

: Now Using chain Rule:

$$\sin 2x \frac{d}{dx}(y) + y\frac{d}{dx}(\sin 2x)$$

$$= \cos 2y \frac{dy}{dx}(x) + x\frac{d}{dx}\boxed{(\cos 2y)}$$

$$\sin 2x \frac{dy}{dx} + 2y\cos 2x =$$

$$\cos 2y - 2x\sin 2y \frac{dy}{dx}$$

$$\sin 2x \frac{dy}{dx} + 2x\sin 2y \frac{dy}{dx} = \cos 2y - 2y\cos 2x$$

$$(\sin 2x + 2x\sin 2y)\frac{dy}{dx} = \cos 2y - 2y\cos 2x$$

$$\frac{dy}{dx} = \frac{\cos 2y - 2y\cos 2x}{\sin 2x + 2x\sin 2y}$$

Now substituting the coordinates

$$\frac{dy}{dx} = \frac{\cos 2\left(\frac{\pi}{4}\right) - 2\left(\frac{\pi}{4}\right)\cos 2\left(\frac{\pi}{2}\right)}{\sin 2\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{4}\right)}$$

$$\frac{dy}{dx} = \frac{\cos\left(\frac{\pi}{2}\right) - \frac{\pi}{2}\cos\pi}{\sin\pi + \pi\sin\left(\frac{\pi}{2}\right)}$$

Ans: $$\frac{dy}{dx} = \frac{0 - \frac{\pi}{2}(-1)}{0 + \pi(1)}$$

$$m = \frac{1}{2}$$

$$\because (y - y_1) = m(x - x_1)$$

$$\left(y - \frac{\pi}{4}\right) = \frac{1}{2}\left(x - \frac{\pi}{2}\right)$$

$$y = \frac{1}{2}x$$

Ans:

Answer # 10

Solve:

$x^2+xy+y^2=3$ (1, 1) ellipse

Differtiating w.r.t "x"

$$\frac{d}{dx}(x^2)+\underbrace{\frac{d}{dx}(xy)}_{\text{chain rule}}+\frac{d}{dx}(y^2)=\frac{d}{dx}(3)$$

$$2x+y\cdot\frac{dx}{dx}+x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$2x+y+x\frac{dy}{dx}+2y\frac{dy}{dx}=0$$

$$(2x+y)+(x+2y)\cdot\frac{dy}{dx}=0$$

$$(x+2y)\frac{dy}{dx}=-2x-y$$

$$\frac{dy}{dx}=-\frac{(2x+y)}{(x+2y)}$$

In Order to find the slope substituting the co-ordinates.

$$\frac{dy}{dx}=-\frac{(2(1)+1)}{(1+2(1))}$$

$$\frac{dy}{dx}=\frac{-\cancel{3}}{\cancel{3}}=-1$$

$$m=-1$$

$$\because\ (y-y_1)=m(x-x_1)$$

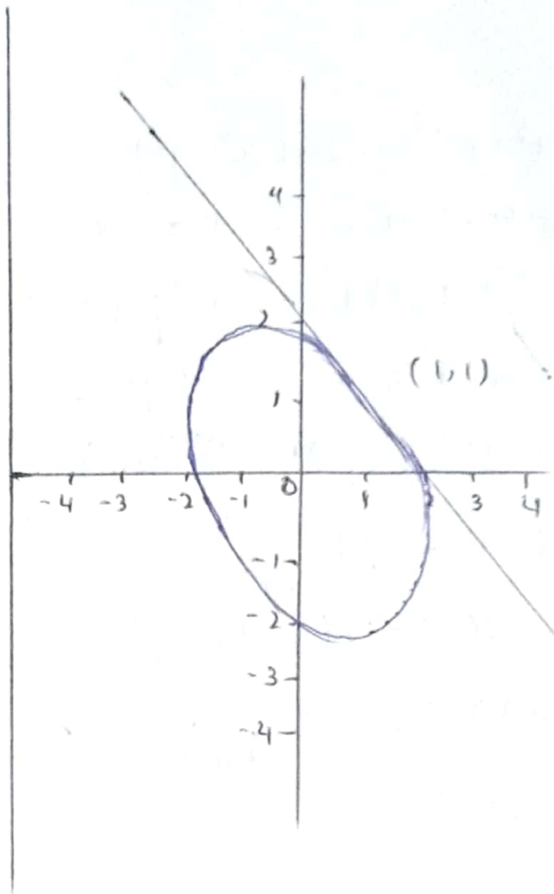
$$(y-1)=(-1)(x-1)$$

$$\frac{y-1}{x-1}=-1$$

$$y-1=1-x$$

$\boxed{y+x=2}$ or $\boxed{y=2-x}$ Answer.

Graphically.



Answer # 11

Solve:

$$x^2+y^2=(2x^2+2y^2-x)^2$$

$$x^2+y^2=(2x^2+2y^2-x)^2$$

Now differentiating w.r.t "x"

$$2x+2y\,y'=2(2x^2+2y^2-x)(4x+4y\,y'-1)$$

at a point when $x=0$, $y=\frac{1}{2}$

$$2(0)+y'=2\left(\tfrac{1}{2}(2y'-1)\right)$$

$$\Rightarrow y'=2y'-1$$

$y'=1$ or $\boxed{m=1.1}$

Now the equation of the tangent line will be,

$y-\frac{1}{2}=1(x-0)$ or

$\boxed{y=x+\frac{1}{2}}$ Answer.

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Answer # 12

Solve:

$$2(x^2+y^2)^2 = 25(x^2-y^2)$$

Differentiating w.r.t "x"

$$4(x^2+y^2)(2x+2y\frac{dy}{dx}) = 25(2x-2y\frac{dy}{dx})$$

Now substitute $x = 3$ and $y = 1$

$$4((3)^2+(1)^2)(2(3)+2(1)dy/dx)$$

$$25(2(3)-2(1)\frac{dy}{dx}) =$$

$$25(2(3)-2(1)dy/dx)$$

$$4(9+1)(6+2dy/dx) = 25(6-2\frac{dy}{dx})$$

$$40(6+2\,dy/dx) = 25(6-2dy/dx)$$

$$240+80\frac{dy}{dx} = 150-50\frac{dy}{dx}$$

$$130\frac{dy}{dx} = 150-240$$

$$130\frac{dy}{dx} = -9\cancel{0}$$

$$\frac{dy}{dx} = -9/13 \text{ or } m = -9/13$$

As we know,

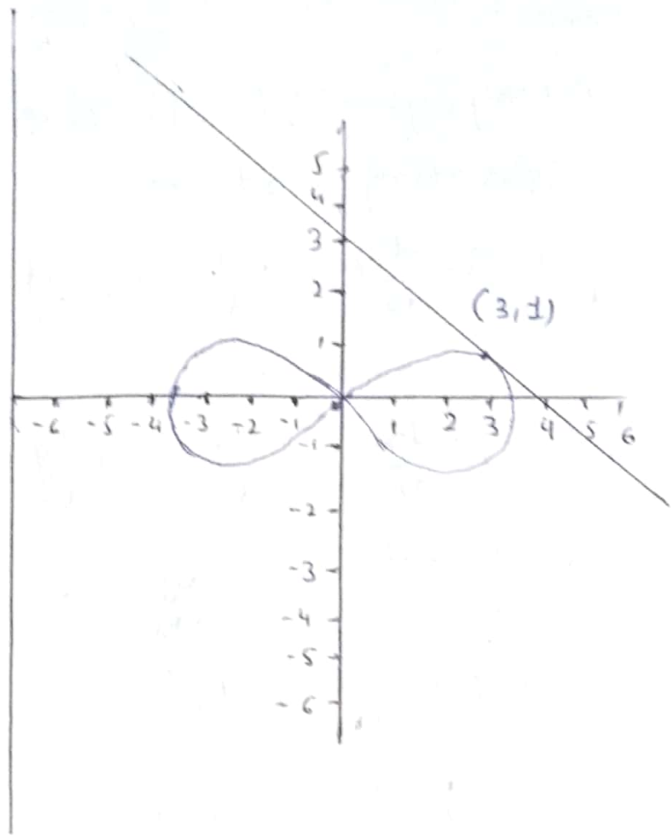
$$y-y_1 = m(x-x_1)$$

$$y-1 = (-9/13)(x-3)$$

$$\boxed{9x+13y = 40}$$

Answer.

Graphically:



Integration Answer # 01

Solve $\int_{-2}^{3} (x^2-3)\,dx$

$\int_{-2}^{3} x^2\,dx - 3\int_{-2}^{3} dx$

$\left[\frac{x^3}{3}\right]_{-2}^{3} - 3\left[x\right]_{-2}^{3}$

$\left[\frac{3^3}{3} - \frac{(-2)^3}{3}\right] - 3\left[3-(-2)\right]$

$\left[9+\frac{8}{3}\right] - 3[5]$

$\left[\frac{27+8}{3}\right] - 15$

$\frac{35}{3} - 15 = \frac{35-45}{3}$

$\boxed{= \frac{-10}{3}}$

$\boxed{\Rightarrow \int_{-2}^{3} (x^2-3)\,dx = \frac{-10}{3}}$

Answer # 02

Solve.

$\Rightarrow \int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x})\,dx$

$\left[\frac{3}{7}x^{1/3} + \frac{4}{9}x^{9/4}\right]_0^1$

$\left[\frac{3}{7}+\frac{4}{9}\right] - [0+0]^0$

$\boxed{= \frac{55}{63}}$

Answer # 03

Solve $\int_1^2 (x/2 - 2/x)\,dx$

$\Rightarrow \int_1^2 = \left[\frac{x^2}{4} - 2\ln x\right]_1^2$

$= \int_1^2 = (1-2\ln 2) - \frac{1}{9}$

$\boxed{\Rightarrow \int_1^2 = \left(\frac{x}{2}-\frac{2}{x}\right) = \frac{3}{2} - 2\ln 2}$

Answer # 04

Solve $\int_{-2}^{0}\left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t\right)dt$

$\left[\frac{t^5}{5\times2} + \frac{t^4}{4\times4} - \frac{t^2}{2}\right]_{-2}^{0}$

$\left[\frac{t^5}{10} + \frac{t^4}{16} - \frac{t^2}{2}\right]_{-2}^{0}$

$\left[\frac{0}{10}+\frac{0}{16}-\frac{0}{2}\right] - \left[\frac{(-2)^5}{\cancel{10}\,5} + \frac{(-2)^4}{16} - \frac{(-2)^2}{2}\right]$

$0 - \left[-\frac{21}{5}\right] = \frac{21}{5}$ or 4.2

Answer # 05

Solve $\int_0^2 (2x-3)(4x^2+1)\,dx$

$\Rightarrow \left[\frac{\cancel{8}\,2}{\cancel{4}}x^4 - \frac{16}{3}x^3 + \frac{\cancel{2}}{\cancel{2}}x^2 - 3x\right]_0^2$

$\Rightarrow \left[2(2)^4 - \frac{16}{3}(2)^3 + (2)^2 - 3(2)\right]$

$\left[2(0)^4 - \frac{16}{3}(0)^3 + (0)^2 - 3(0)\right]$

$\Rightarrow \boxed{-2}$

Answer # 06

Soln $\int_0^{\pi} (5e^x + 3\sin x)\,dx$

$\int_0^{\pi} (5e^x+3) - (5e^0 - 3)$

$\int_0^{\pi} 5e^{\pi} + 3 - 2$

$\Rightarrow \boxed{5e^{\pi}+1}$ Ans

Answer # 07

Soln $\int_1^4 \left(\frac{4+6u}{\sqrt{u}}\right)du$

$\left[4t^{-1/2} + 6u^{1/2}\right]_1^4 du$

$\left[(4u^{1/2})(2) + (6u^{3/2})(2/3)\right]_1^4$

$\left[(8u^{1/2}) + (4u^{3/2})\right]_1^4$

$\Rightarrow 48 - 12$

$\Rightarrow \boxed{36}$ Ans

Answer # 08

Soln: $\int_0^1 (x^{10} + 10^x)\,dx$

$\int_0^1 = \left[\frac{x^{11}}{11} + \frac{10^x}{\ln 10}\right]_0^1$

$= \boxed{\frac{1}{11} + \frac{9}{\ln 10}}$ Ans

Answer # 09

Solve $\int_0^1 \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

$\left[\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right]_0^1 d\theta$

$\left[\frac{1}{\cos^2\theta} + 1\right]_0^1 d\theta$

$\left[\sec^2\theta + 1\right]_0^1 d\theta$

$\because f'(x) = F(g) - f(\theta)$

So;

$I = (\tan\theta + \theta)_0^{\pi/2}$

$= \frac{\pi}{4} + \tan\frac{\pi}{4} - 0 + \tan 0$

$= \frac{\pi}{4} + 1$

$\boxed{\Rightarrow \int_0^{\pi/4} \frac{1+\cos^2\theta\,d\theta}{\cos^2\theta} = \frac{\pi}{4}+1}$

Ans