Chapter – 3.1

1) Extreme values of functions:

a. The Extreme value theorem states that if a function is continuous on a closed interval [a, b], then the function must have a maximum and a minimum on the interval. This makes sense: when a function is continuous you can draw its graph without lifting the pencil, so you must hit a high point and a low point on that interval.

b. An **extreme value**, or **extremum** (plural *extrema*), is the smallest (minimum) or largest (maximum) value of a function, either in an arbitrarily small neighbourhood of a point in the function's domain — in which case it is called a *relative* or *local* extremum — or on a given set contained in the domain (perhaps all of it) — in which case it is called an absolute or global extremum (the latter term is common when the set is all of the domain).

c. Note that in the case of relative/local extrema, it is common to concentrate on *where* the extrema occur (i.e., the "x-values") rather than what the extreme values actually are (the "y-values"), whereas in the case of absolute/global extrema it is common to concentrate on the extreme value itself (the "y-value"). However, in either case both values may be given — e.g., $f(2) = 5$ if the extreme value 5 occurs at $x = 2$.

d. Extrema can be found by taking the derivative of a function and setting it to equal zero. If the second derivative at this point is positive, it is a minimum, and vice-versa.

2) Function Analysis:

a. Function analysis is a method for analysing and developing a function structure. A function structure is an abstract model of the new product, without material features such as shape, dimensions and materials of the parts. It describes the functions of the product and its parts and indicates the mutual relations. The underlying idea is that a function structure may be built up from a limited number of elementary (or general) functions on a high level of abstraction. Functions are abstractions of what a product should do. Being forced to think about the product in an abstract way stimulates creativity, and prevents you from 'jumping to solutions', i.e. immediately elaborating on the first idea that comes to mind, which may not be the best.

b. In function analysis, the product is considered as a technical-physical system. The product functions, because it consists of a number of parts and components which fulfil subfunctions and the overall function. By choosing the appropriate form and materials, a designer can influence the subfunctions and the overall function. The principle of function analysis is first to specify what the product should do, and then to infer from there what the parts - which are yet to be developed - should do. Function analysis forces designers to distance themselves from known products and components in considering the question: what is the new product intended to do and how could it do that? The method is useful to accomplish a breakthrough in thinking in conventional solutions.

c. A function analysis often precedes the [morphological method](). The functions and subfunctions that are identified in the function analysis serve as the parameters in the morphological chart.

When Can You Use a Function Analysis?

d. A function analysis is typically carried out at the beginning of idea generation.

e. http://wikid.io.tudelft.nl/WikID/index.php/Function_analysis

3) Least Square Method:

a.

Least squares approximations

We have the observations:

$$f_1, f_2, \ldots, f_m \in \mathbb{R}$$

at time moments:

$$t_1, t_2, \ldots, t_m \in \mathbb{R}$$

We assume that the observed phenomenon can be described by a `model`:

$$F(t) = \sum_{j=1}^{n} x_j \varphi_j(t)$$

- the φ_j's are `given` functions, the building blocks of our model.
- the x_j's are real numbers, the `unknown` coefficients of the model.

b.

There are infinitely many functions of this type. We are going to choose the one, for which

$$J(x) = \sum_{i=1}^{m} (F(t_i) - f_i)^2$$

is `minimal`.
The model F determined in this way is called a `least-squares` approximation of the data.

c. The least squares method is a statistical procedure to find the best fit for a set of data points by minimizing the sum of the offsets or residuals of points from the plotted curve.

d. Least squares regression is used to predict the behaviour of dependent variables.

Example of the Least Squares Method

An example of the least squares method is an analyst who wishes to test the relationship between a company's stock returns, and the returns of the index for which the stock is a component. In this example, the analyst seeks to test the dependence of the stock returns on the index returns. To achieve this, all of the returns are plotted on a chart. The index returns are then designated as the independent variable, and the stock returns are the dependent variable. The line of best fit provides the analyst with coefficients explaining the level of dependence.

The Line of Best Fit Equation

The line of best fit determined from the least squares method has an equation that tells the story of the relationship between the data points. Line of best fit equations may be determined by computer software models, which include a summary of outputs for analysis, where the coefficients and summary outputs explain the dependence of the variables being tested.

Least Squares Regression Line

If the data shows a leaner relationship between two variables, the line that best fits this linear relationship is known as a least squares regression line, which minimizes the vertical distance from the data points to the regression line. The term “least squares” is used because it is the smallest sum of squares of errors, which is also called the "variance".