Flight Mechanics Review

a) General Rigid Body Dynamics Equations in 3D (12 Equations)

A rigid body in 3D is fully described by 12 equations, split equally into 6 kinetic (dynamics) and 6 kinematic (geometric) equations.

Kinetic Equations (Dynamics)

These equations govern the forces and moments acting on the body.

Translational Dynamics (Newton's Second Law)

(1)

$$\dot{u} = (F_x/m) + r \cdot v - q \cdot w$$

(2)

$$\dot{v} = (F_Y/m) + p \cdot w - r \cdot u$$

(3)

$$\dot{w} = (Fz/m) + q \cdot u - p \cdot v$$

Where:

- u, v, w = Velocity components along the body x-, y-, and z-axes
- F_x, F_Y, Fz = External force components along the body axes
- m = Mass of the body
- p, q, r = Angular velocity components about the body x-, y-, and z-axes

Rotational Dynamics (Euler's Equations)

(4):

$$\dot{p} = (1/I_x) \cdot [M_x + (I_Y - Iz) \cdot q \cdot r]$$

(5):

$$\dot{q} = (1/I_Y) \cdot [M_Y + (Iz - I_x) \cdot r \cdot p]$$

(6):

$$\dot{r} = (1/Iz) \cdot [Mz + (I_x - I_Y) \cdot p \cdot q]$$

Where:

- I_x, I_Y, Iz = Moments of inertia about the body axes
- M_x, M_Y, Mz = External moment components about the body axes

Kinematic Equations (Geometry)

These equations relate the velocities and angular rates to the changes in position and orientation.

Translational Kinematics (Position Updates)

(7):

$$\dot{x} = u \cdot cos\theta \cdot cos\psi + v \cdot (sin\varphi \cdot sin\theta \cdot cos\psi - cos\varphi \cdot sin\psi) + w \cdot (cos\varphi \cdot sin\theta \cdot cos\psi + sin\varphi \cdot sin\psi)$$

(8):

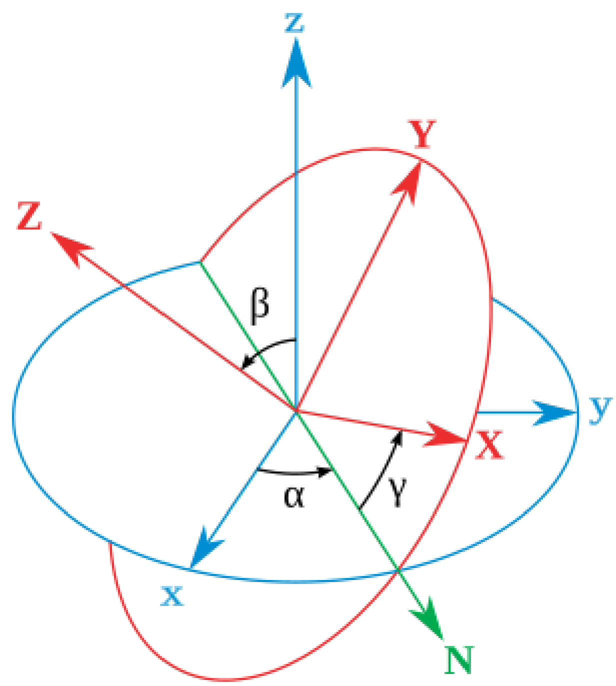
$$\dot{y} = u \cdot cos\theta \cdot sin\psi + v \cdot (sin\varphi \cdot sin\theta \cdot sin\psi + cos\varphi \cdot cos\psi) + w \cdot (cos\varphi \cdot sin\theta \cdot sin\psi - sin\varphi \cdot cos\psi)$$

(9):

$$\dot{z} = -u \cdot sin\theta + v \cdot sin\varphi \cdot cos\theta + w \cdot cos\varphi \cdot cos\theta$$

Where:

- x, y, z = Inertial (earth-fixed) coordinates
- φ, θ, ψ = Euler angles (roll, pitch, yaw)



Euler angles

Rotational Kinematics (Euler Angle Rates)

(10):

$$\dot{\varphi} = p + q \cdot sin\varphi \cdot tan\theta + r \cdot cos\varphi \cdot tan\theta$$

(11):

$$\theta = q \cdot cos\varphi - r \cdot sin\varphi$$

(12):

$$\dot{\psi} = (q \cdot sin\varphi + r \cdot cos\varphi) \,/\, cos\theta$$

These 12 equations together comprehensively describe the motion of a rigid body in 3D space.

b) Classification into Kinetics and Kinematics

Kinetic Equations (6 in total):

- Equations (1) – (3): Translational dynamics
- Equations (4) – (6): Rotational dynamics

Kinematic Equations (6 in total):

- Equations (7) – (9): Relate body velocities (expressed in body frame) to inertial position changes
- Equations (10) – (12): Relate body angular rates to Euler angle rates

c) Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)

To specialize the general RBD equations for a fixed-wing airplane, the following are added: •

Aerodynamic Force and Moment Models:

Aerodynamic forces are expressed as functions of dynamic pressure, wing area, and non-dimensional coefficients (e.g., Cx, Cy, Cz). Similarly, aerodynamic moments (roll L, pitch M, yaw N) are modeled using coefficients that depend on angle of attack, sideslip, and control surface deflections.

- ## Gravitational Force Projection:

The weight of the airplane is resolved along the body axes via the transformation from the inertial (or earth-fixed) frame.

- ## Velocity-to-Angle Relationships:

Equations that relate the components of velocity in the body frame to the airspeed, angle of attack (α), and sideslip angle (β).

- ## Control Input Effects:

Additional relations that quantify how control surface deflections affect the aerodynamic forces and moments.

d) Assumptions in Deriving Airplane Equations of Motion.

Common assumptions include:

- **Rigid Body Approximation:** The aircraft is assumed not to deform.
- **Constant Mass and Inertia:** Effects of fuel burn or payload shifts are neglected.
- **Quasi-Steady Aerodynamics:** Aerodynamic forces and moments are assumed to respond instantaneously to changes in flight conditions.
- **Small Perturbations:** Linearization is often performed around a trim condition.
- **Neglection of High-Order Effects:** Compressibility, viscous effects, and higher-order nonlinearities are typically ignored in preliminary analyses.
- **Uniform Atmospheric Conditions:** The air is assumed to have steady and uniform properties.

e) Mathematical Classification of the Airplane EOM.

Order	The equations are represented as first-order differential equations in state-space form.
Type	They are Ordinary Differential Equations (ODEs), with time being the independent variable.
Linearity	The full equations are nonlinear, although linearization is common near steady flight conditions.

Coupling	The equations are generally coupled, as translational and rotational dynamics interact. Under certain assumptions (such as decoupling of longitudinal and lateral-directional dynamics), the equations can be simplified to an uncoupled form.

f) Difference Between Body Axes and Earth (Inertial) Axes.

- ### Body Axes:

A coordinate system fixed to the airplane, typically defined as:

- $x - axis$: along the fuselage (forward),
- $y - axis$: towards the right wing,
- $z - axis$: downward.

The body axes rotate with the aircraft.

- ### Earth (Inertial) Axes:

A fixed or quasi-fixed coordinate system relative to the Earth, often defined as North-East-Down (NED) or a similar scheme, serving as an absolute reference frame.

g) Difference Between Pitch Angle (θ) vs. Angle of Attack (α) and Sideslip Angle (β) vs. Heading Angle (ψ).

- ### Pitch Angle (θ) vs. Angle of Attack (α):

The pitch angle θ is the orientation of the aircraft's longitudinal axis relative to the horizon, while the angle of attack α is the angle between the chord line (or x-axis) and the oncoming airflow. In steady, coordinated flight these angles are related, but during maneuvers or in the presence of wind, they can differ significantly.

- ### Sideslip Angle (β) vs. Heading Angle (ψ):

The sideslip angle β is the angle between the aircraft's longitudinal axis and the relative wind, whereas the heading angle ψ is the navigational direction of the aircraft relative to a fixed reference (such as geographic North).

h) Attitude Representations: Advantages and Disadvantages

- ### Euler Angles:

- **Advantages**: Intuitive (roll, pitch, yaw) and straightforward.
- **Disadvantages**: Suffer from singularities (gimbal lock) when, for example, the pitch angle approaches $\pm 90^{\circ}$.

- **Direction Cosine Matrix (DCM):**
  - Advantages: No singularity issues and provides a full transformation matrix.
  - Disadvantages: Requires 9 elements along with orthogonality constraints, which can complicate numerical implementation.

- **Quaternions:**
  - Advantages: Compact (4 parameters), computationally efficient, and free from singularities.
  - Disadvantages: Less intuitive and involve a double-cover issue (i.e., q and −q represent the same orientation).

- **Axis-Angle Representation:**
  - Advantages: Provides a clear geometric interpretation (rotation by a specific angle about a fixed axis).
  - Disadvantages: Less practical for sequential rotations or for numerical integration when rotations are small.