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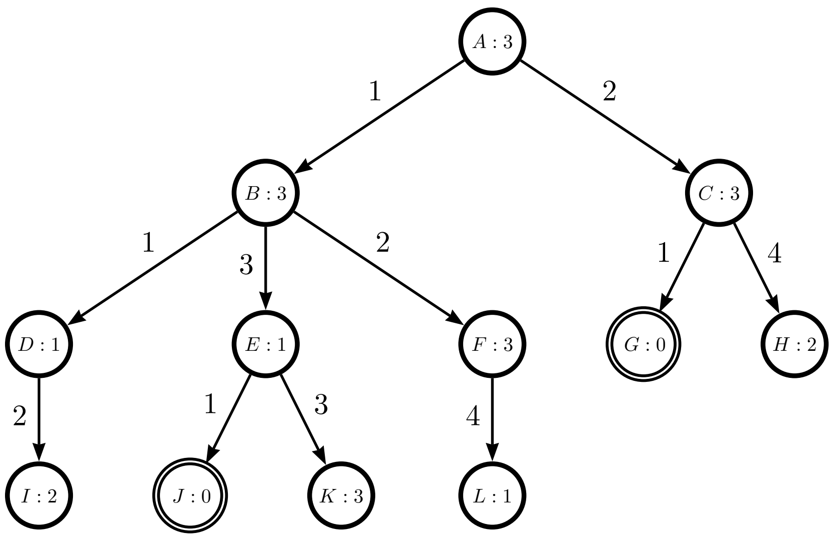
Introduction to Artificial Intelligence, Winter Term 2020
Problem Set 3

Exercise 3-1

The following tree is a full search tree for some state space. Arc labels denote branch costs, double circles indicate goal nodes. The numbers following the ":" indicate the value of the heuristic function h for the corresponding node.



For each of the following search strategies, indicate the order in which nodes will be chosen for expansion:

a) Greedy search.

Solution:

Expansion Sequence: A B D E J
Path to goal: A B E J

b) A* search.

Solution:

Expansion Sequence: A B D C G
Path to goal: A C G

Exercise 3-2

In exercise 2-1, what is a suitable admissible heuristic function?

Solution:

$h(\langle T, V\rangle) = \min(\{\textbf{Length}(v)|\textbf{Topics}(v) \cap T \neq \varnothing\})$.

Exercise 3-3 (From R&N[1])

Consider the problem of constructing crossword puzzles: fitting words into a grid of horizontal and vertical squares. Assume that a list of words (i.e., a dictionary) is provided, and that the task is to fill in the squares using any subset of this list. Go through a complete goal and problem formulation for this domain, and choose a search strategy to solve it. Specify a heuristic function, if you think one is needed.

Exercise 3-4 (From R&N)

Prove that if the heuristic function h obeys the triangle inequality, then the f-cost along any path in the search tree is nondecreasing. (The triangle inequality says that the sum of the costs from A to B and B to C must not be less than the cost from A to C directly.)

Solution:

For a search tree having node n as a parent of node p, with the actual cost from n to p equivalent to d, the heuristic obeys the triangular inequality if it satisfies the following: $h(n) \leq d + h(p)$

```
      ..
      n
    / d
  p
 ..
```

$\because h$ obeys the triangular inequality:
$\therefore h(n) \leq d + h(p)$
$\therefore f(n) - g(n) \leq g(p) - g(n) + f(p) - g(p)$
$\therefore f(n) \leq f(p)$
$\therefore f$ is a monotonically non-decreasing function

Exercise 3-5 (From R&N)

It was shown in Chapter 4 that an admissible heuristic h (when combined with monotonicity) leads to monotonically nondecreasing f values along any path (i.e., $f(\mathit{successor}(n)) \geq f(n)$). Does the implication go the other way? That is, does monotonicity of f imply admissibility of the associated h?

Solution:

Proof by induction:
Basis:

[1] Stuart Rusell and Peter Norvig (2003) *Artificial Intelligence: A Modern Approach*. Prentice Hall, Upper Saddle River, N.J.

$\because f$ is a monotonically non-decreasing function
$\therefore f(n) \leq f(n+1) \therefore g(n) + h(n) \leq g(n+1) + h(n+1)$
$\therefore h(n) \leq g(n+1) - g(n) + h(n+1)$
$\therefore$ If $n+1$ is a goal node, then $h(n+1) = 0$
$\therefore h(n) \leq g(n+1) - g(n) \therefore h(n)$ is less than the actual cost to the goal.

Induction Step:

```
              ..
              n
            / d1
           n+1
          / d2
         n+2
        /
       ..
       p (goal node)
```

Assume that $h(n+2) \leq g(p) - g(n+2)$ −> (1)
We need to prove this property for node $n+1$ (We will prove it by contradiction)
If we assume $h(n+1) > g(p) - g(n+1)$
$\therefore h(n+1) + g(n+1) > g(p)$
$\therefore f(n+1) > g(p)$ −> (2)
$\because f(n+2) \leq g(p)$ (from (1))
Therefore, (2) will lead to:
$f(n+2) \leq g(p) < f(n+1)$
Since f is a monotonically non-decreasing function, we have a contradiction.
Therefore, $h(n+1) \leq g(p) - g(n+1)$
Therefore, h is an admissible function.