التكامل بالتجزئة والكسور البسيطة

التكامل بالتجزئة

$$\int (u \cdot v)' = \int du \cdot v + \int dv \cdot u$$

$$u \cdot v = \int v \cdot du + \int u \cdot dv$$

$$\boxed{\int u \cdot dv = u \cdot v - \int v \cdot du}$$

L = Logaritim {log, ln}
A = Arc
P = polinom
T = Trigonometri
Ü = üstel $\Rightarrow \{a^x, e^x\}$

örnek:

$$\int \underbrace{\ln x}_{u} \underbrace{dx}_{dv}$$

$u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x}$ $\quad v = x$

$$\int \ln x \, dx = x \cdot \ln x - \int \frac{x}{x} dx$$
$$= x \cdot \ln x - \int dx$$
$$\boxed{\int \ln x \, dx = x \cdot \ln x - x + C}$$

örnek: $\int \underbrace{\arctan x}_{u} \cdot \underbrace{dx}_{dv}$

$dv = dx$

$\arctan x = u$

$\frac{1}{1+x^2} dx = du$ $\quad v = x$

$$\int \arctan x \, dx = x \cdot \arctan x - \int \underbrace{\frac{x}{1+x^2} dx}$$

$1 + x^2 = t$

$2x \, dx = dt$

$x \, dx = \frac{dt}{2}$

$\frac{1}{2} \int \frac{dt}{t}$

$\ln |t|$

$\frac{1}{2} \ln |1 + x^2| + C$

$$\int \arctan x \, dx = x \cdot \arctan x - \frac{1}{2} \ln |1 + x^2| + C$$

örnek:

$$\int e^x \cdot \cos x \, dx = I$$

$\cos x = u$ $\quad dv = e^x \cdot dx$

$-\sin x \, dx = du$ $\quad v = e^x$

$$I = e^x \cdot \cos x + \int e^x \sin x \, dx$$

$\sin x = u$ $\quad dv = e^x \cdot dx$

$\cos x \, dx = du$ $\quad v = e^x$

$$\Rightarrow \quad I = e^x \cdot \cos x + \left[e^x \cdot \sin x - \underbrace{\int e^x \cdot \cos x \, dx}_{I} \right]$$

$$2I = e^x \cdot \cos x + e^x \cdot \sin x$$

$$I = \frac{e^x}{2} [\cos x + \sin x] + C$$

$\int e^x \cdot x^3 \, dx$

D	I
$+ x^3$	e^x
$- 3x^2$	e^x
$+ 6x$	e^x
$- 6$	e^x
$+ 0$	

$= e^x [x^3 - 3x^2 + 6x - 6] + C$

$\int \cos x \cdot x^2 \, dx$

D	I
$+ x^2$	$\cos x$
$- 2x$	$\sin x$
$+ 2$	$-\cos x$
$- 0$	$-\sin x$

$x^2 \sin x + 2x \cos x - 2 \sin x + C$

$\int \sec^3 x \, dx$

$\int \overset{I}{\sec^2 x} \cdot \sec x \, dx$

$\sec^2 x \, dx = dv$

$\sec x = u$

$\sec x \tan x \, dx = du$ $\quad \tan x = v$

$I = \tan x \sec x - \int \tan^2 x \sec x \, dx$

$\tan^2 x = (\sec^2 x - 1)$

$\int (\sec^2 x - 1) \sec x \, dx$

$\int \sec^3 x - \sec x \, dx$

$\int \sec^3 x \, dx - \int \sec x \, dx$

$I = \tan x \sec x - I + \ln|\sec x + \tan x|$

$2I = \text{——}$

$$\int \sec^3 x = \frac{1}{2}\left[\tan x \sec x + \ln|\sec x + \tan x|\right] + C$$

الكسور البسيطة

$$\frac{P(x)}{Q(x)} = R(x) + \frac{k(x)}{Q(x)}$$

$$\int \frac{x+1}{x-1} \, dx = \int \frac{x+1-1+1}{x-1} \, dx = \int \frac{x-1}{x-1} \, dx + 2\int \frac{1}{x-1} \, dx$$

$$= x + 2\ln|x-1| + C$$

$$\int \frac{ax+b}{(x+m)(x+n)} = \frac{A}{(x+m)} + \frac{B}{(x+n)}$$

$$\int \frac{ax+b}{(mx^2+t)(nx+k)} = \frac{A}{(nx+k)} + \frac{Bx+C}{(mx^2+t)}$$

$\Delta < 0$ $\qquad$ $\Delta < 0$

$n > 1$

$\Delta < 0$

$\frac{M}{px+q}$, $\frac{N}{(px+q)^n}$, $\frac{Ax+B}{ax^2+bx+c}$, $\frac{cx+D}{(ax^2+bx+c)^n}$ $\qquad$ $\Delta = b^2 - 4ac < 0$

$$\int \frac{x^3-8x^2+2}{x+2}\,dx$$

$$\begin{array}{r|l} x^3-8x^2+2 & x+2 \\ x^3+2x^2 & x^2-10x+20 \\ \hline -10x^2+2 & \\ -10x^2-20x & \\ \hline 20x+2 & \\ 20x+40 & \\ \hline -38 & \end{array}$$

$$\int x^2-10x+20-\frac{38}{x+2}\,dx$$

$$\frac{x^3}{3}-\frac{10x^2}{2}+20x-38\int\frac{1}{x+2}\,dx$$

$$\frac{x^3}{3}-5x^2+20x-38\ln|x+2|+C$$

örnek: $\int \frac{x+1}{x^2-4}\,dx = \frac{A}{x-2}+\frac{B}{x+2}$

$$\frac{x+1}{x^2-4}=\frac{A(x+2)+B(x-2)}{x^2-4}$$

$$x+1 = Ax+2A+Bx-2B$$

$$x(A+B) \qquad 2(A-B)=1$$

$$\begin{cases} A+B=1 \\ A-B=\frac{1}{2} \end{cases} \quad 2A=\frac{3}{2}$$

$$A=\frac{3}{4}$$

$$B=\frac{1}{4}$$

$$\int \frac{\frac{3}{4}}{x-2}+\frac{\frac{1}{4}}{x+2}\,dx \qquad \frac{3}{4}\int\frac{1}{x-2}\,dx+\frac{1}{4}\int\frac{1}{x+2}\,dx$$

$$\frac{3}{4}\ln|x-2|+\frac{1}{4}\ln|x+2|+C$$

örnek:

$$\int \frac{2x^4-6x^3+7x^2-2x-2}{x^3-3x^2+3x-1}\,dx$$

$$\int 2x+\frac{x^2-2}{(x-1)^3}\,dx$$

$$x^2+\int\frac{x^2-2}{(x-1)^3}\,dx$$

$$\frac{x^2-2}{(x-1)^3}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$

$$x^2-2 = A(x-1)^2+B(x-1)+C$$

$$= Ax^2-2Ax+A+Bx-B+C$$

$$x^2-2 = Ax^2+x(B-2A)+A+C-B$$

$$A=1$$

$$B-2A=0$$

$$B=2$$

$$A+C-B=-2$$

$$-1+C=-2$$

$$C=-1$$

$$I = x^2+\int\frac{1}{x-1}+\frac{2}{(x-1)^2}-\frac{1}{(x-1)^3}\,dx$$

$$x^2+\ln|x-1|-\frac{2}{(x-1)}+\frac{1}{2(x-1)^2}+C$$