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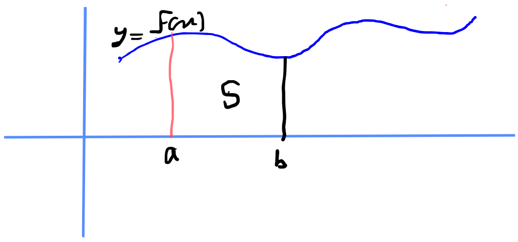
(f, P) $\sum_{k=1}^{n} |f(x)\,\Delta x_k|$

$\lim_{P\to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k = I$

$I = \int_a^b f(x)\,dx$ $\qquad \int_a^b f(x)\,dx$



$$\int_a^b f(x)\,dx = F(b) - F(a)$$

özellikler:

1) $\int_a^a f(x)\,dx = 0$ $\quad = \int_2^2 x\,dx = 0 \quad \frac{x^2}{2}\Big|_2^2 = \frac{4}{2} - \frac{4}{2} = 0$

2) $\frac{d}{dx}\left[\int_a^b f(x)\,dx\right] = 0$ $\quad \frac{d}{dx}\left[\int_0^1 x\,dx\right] = \frac{d}{dx}\left[\frac{x^2}{2}\Big|_0^1\right] = \frac{d}{dx}\left[\frac{1}{2}\right] = 0$

3) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ $\quad = \int_1^2 x\,dx = -\int_2^1 x\,dx$

4) $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$ $\quad \int_1^3 3x\,dx = 3\int_1^3 x\,dx$

5) $\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$ $\quad \int_0^{\pi} \sin x + \cos x\,dx = \int_0^{\pi} \sin x\,dx + \int_0^{\pi} \cos x\,dx$

6) $a < c < b \Rightarrow \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ $\quad \int_1^3 x\,dx = \int_1^2 x\,dx + \int_2^3 x\,dx$

7) $\int_a^b u\,dv = u.v\Big|_a^b - \int_a^b v.du$ $\quad \int_1^e \ln x\,dx = x.\ln x\Big|_1^e - \int_1^e \frac{x}{x}\,dx$

8) $\frac{d}{dx}\int_{h(x)}^{g(x)} f(x)\,dx = g'(x).f(g(x)) - h'(x).f(h(x))$

$$\frac{d}{dx}\int_{x^2}^{x^3+1} x^2+2x\,dx = 3x^2.\left[(x^3+1)^2 + 2(x^3+1)\right] - 2x\left[x^4 + 2x^2\right]$$

örnekler:

1) $\int_0^{\pi/3} \frac{\tan x}{\sqrt{\sec x}}\,dx = \int_0^{\pi/3} \frac{\frac{\sin x}{\cancel{\cos x}}}{\frac{1}{\cancel{(\cos x)^{\frac{1}{2}}}}}\,dx = \int_0^{\frac{\pi}{3}} \frac{\sin x}{(\cos x)^{\frac{1}{2}}}\,dx$

$\Rightarrow \int_0^{\frac{\pi}{3}} (\cos x)^{-\frac{1}{2}}.\sin x\,dx$

$\cos x = t \quad \Rightarrow \quad x = \frac{\pi}{3} \Rightarrow \cos\frac{\pi}{3} = t = b$

$\frac{1}{2} = t$

$-\sin x\,dx = dt \quad x = 0 \Rightarrow 1 = t = a$

$\int_1^{\frac{1}{2}} t^{-\frac{1}{2}}\,dt \Rightarrow -\int_{\frac{1}{2}}^{1} t^{-\frac{1}{2}}\,dt = -2\,t^{\frac{1}{2}}\Big|_{\frac{1}{2}}^{1}$

$-2 + 2\sqrt{\frac{1}{2}}$

2) $\int_{-1}^{1} |x| [\![x]\!] \, dx = \int_{-1}^{0} -x(-1)\, dx + \int_{0}^{1} x(0)\, dx = \int_{-1}^{0} x \, dx = \frac{x^2}{2}\Big|_{-1}^{0} = 0 - \frac{1}{2} = -\frac{1}{2}$

3) $\int_{-1}^{1} x \cdot sgn(x) \, dx = \int_{-1}^{0} x(-1)\, dx + \int_{0}^{1} x(1)\, dx = -\int_{-1}^{0} x \, dx + \int_{0}^{1} x \, dx$

4) $\int_{-1}^{3} \frac{|x^2-1| \cdot [\![x]\!]}{sgn(x+3)} \, dx = \int_{-1}^{0} \frac{-(x^2-1)(-1)}{1} dx + \int_{0}^{1} 0 + \int_{1}^{2} \frac{(x^2-1)}{1} dx + \int_{2}^{3} \frac{(x^2-1)\,2}{1} dx$

$\int_{-1}^{0} x^2-1 \, dx + \int_{1}^{2} x^2-1 \, dx + 2\int_{2}^{3} (x^2-1)\, dx$

$\frac{x^3}{3} - x \Big|_{-1}^{0} + \frac{x^3}{3} - x \Big|_{1}^{2} + 2\left[\frac{x^3}{3} - x \Big|_{2}^{3}\right]$

$0 \cancel{+\frac{1}{3}} \cancel{-1} + \cancel{\frac{8}{3}} \cancel{-2} \cancel{-\frac{1}{3}} \cancel{+1} + \cancel{18} \cancel{-6} - \cancel{\frac{16}{3}} \cancel{+4} = 14 - \frac{8}{3} = \frac{34}{3}$

5) $\int_{1}^{4} |x^2-4| \, dx$

$-2 \quad 1 \quad 2 \quad 4$

x^2-4	$+$	$-$	$+$
$\lvert x^2-4\rvert$	x^2-4	$-(x^2-4)$	x^2-4

$-\int_{1}^{2} (x^2-4)\, dx + \int_{2}^{4} (x^2-4)\, dx$

$\frac{-x^3}{3} + 4x \Big|_{1}^{2} + \frac{x^3}{3} - 4x \Big|_{2}^{4} = -\frac{19}{3}$

6) $\int_{-2}^{3} |x^2 - x - 2| \, dx$

$(x-2)(x+1)$

$-2 \quad -1 \quad 2 \quad 3$

x^2-x-2	$+$	$-$	$+$
$\lvert x^2-x-2\rvert$	x^2-x-2	$-x^2+x+2$	x^2-x-2

$\int_{-2}^{-1} x^2 - x - 2 \, dx + \int_{-1}^{2} -x^2 + x + 2 \, dx$

$+ \int_{2}^{3} x^2 - x - 2 \, dx$

$\frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-2}^{-1} - \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-1}^{2} + \frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{2}^{3}\right.$

7) $\int_{-2}^{3} sgn(x+1) \, dx$

$-2 \quad -1 \quad 3$

$x+1$	$-$	$+$
$sgn(x+1)$	-1	1

$-\int_{-2}^{-1} dx + \int_{-1}^{3} dx$

$-x \Big|_{-2}^{-1} + x \Big|_{-1}^{3}$

8) $\int_{-1}^{2} |x| \cdot sgn(x-1) \, dx = \int_{-1}^{0} -x(-1)\, dx + \int_{0}^{1} x \cdot (-1) + \int_{1}^{2} x(1)$

$\int_{-1}^{0} x \, dx - \int_{0}^{1} x \, dx + \int_{1}^{2} x$

$\frac{x^2}{2}\Big|_{-1}^{0} - \frac{x^2}{2}\Big|_{0}^{1} + \frac{x^2}{2}\Big|_{1}^{2}$