Belirsiz integral

Trigonometrik

$\int \sin ax \sin bx\, dx$, $\int \sin ax \cdot \cos bx\, dx$, $\int \cos ax \cdot \cos bx\, dx$

$$\sin a + \sin b = 2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2 \cdot \sin\left(\frac{a-b}{2}\right) \cdot \cos\left(\frac{a+b}{2}\right)$$

$$\cos a + \cos b = 2 \cdot \cos\left(\frac{a+b}{2}\right) \cdot \cos\left(\frac{a-b}{2}\right)$$

$$\cos a - \cos b = -2 \cdot \sin\left(\frac{a+b}{2}\right) \cdot \sin\left(\frac{a-b}{2}\right)$$

4. Group $\quad a > b$

$$\sin a \cdot \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos a \cdot \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$$

$$\cos a \cdot \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin a \cdot \sin b = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

Örnek : $\int \sin 8x \cdot \cos 4x\, dx$

$$\frac{1}{2}[\sin(12x) + \sin(4x)]$$

$$\frac{1}{2}\int \sin 12x\, dx + \frac{1}{2}\int \sin 4x\, dx$$

$12x = t \qquad 4x = t_1$

$dx = \frac{dt}{12} \qquad dx = \frac{dt_1}{4}$

$$\Rightarrow \frac{1}{24}\int \sin t\, dt + \frac{1}{8}\int \sin t_1 \cdot dt_1$$

$$-\frac{1}{24}\cos 12x - \frac{1}{8}\cos 4x + C$$

Örnek: $\int \cos 4x \cdot \cos 3x\, dx$

$$\frac{1}{2}[\cos(7x) + \cos(x)]$$

$$\frac{1}{2}\int \cos 7x\, dx + \frac{1}{2}\int \cos x\, dx$$

$$\frac{1}{14}\cdot \sin 7x + \frac{1}{2}\sin x + C$$

Tipindeki integral

$\int \sin^m x \cdot \cos^n x \, dx$

1) m = Tek $\Rightarrow \cos x = t$, n = Tek $\Rightarrow \sin x = t$

örnek

$\int \sin^5 x \cdot \cos^2 x \, dx$

$\int \sin^4 x \cos^2 x \cdot \sin x \, dx$

$(\sin^2 x)^2$

$(1-\cos^2 x)^2$

$(1-t^2)^2$

$\cos x = t$

$\Rightarrow \sin x \, dx = dt$

$\int (1-t^2)^2 \cdot t^2 \cdot dt$

$\int (1+t^4-2t^2) t^2 \, dt$

$\int t^2 dt + \int t^6 dt - 2\int t^4 dt$

$\frac{t^3}{3} + \frac{t^7}{7} - \frac{2t^5}{5} + C$

$\frac{\cos^3 x}{3} + \frac{\cos^7 x}{7} - \frac{2\cos^5 x}{5}$

örnek

2C $\int \sin^3 x \cdot \cos^3 x \, dx$

$\int \sin^2 x \cdot \cos^3 x \cdot \sin x \, dx$

$\int (1-\cos^2 x) \cos^3 x \cdot \sin x \, dx$

$\cos x = t$

$\sin x \, dx = dt$

$\Rightarrow \int (1-t^2) t^3 \, dt$

$\int t^3 dt - \int t^5 dt$

$\frac{t^4}{4} - \frac{t^6}{6} + C$

$\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6} + C$

örnek

$\int \sin^7 x \, dx$

$\int (\sin^2 x)^3 \cdot \sin x \, dx$

$\int (1-\cos^2 x)^3 \cdot \sin x \, dx$

$\cos x = t$

$\sin x \, dx = dt$

örnek

$\int \sin^4 x \cdot \cos^2 x \, dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \left(\frac{1-\cos 2x}{4}\right)^2 \cdot \left(\frac{1+\cos 2x}{2}\right)$

$\frac{1}{8}\int (1+\cos^2 2x - 2\cos 2x)(1+\cos 2x) \, dx$

$\frac{1}{8}\int 1 - \cos^2 2x - \cos 2x + \cos^3 2x$

$C + \frac{x}{8} - \frac{\sin 2x}{16} - \frac{x}{16} - \frac{\sin 4x}{64} - \frac{\sin 2x}{16} + \frac{\sin^3 2x}{48}$

$-\frac{1}{8}\int \cos^2 2x = -\frac{1}{8}\int \frac{1+\cos 4x}{2} \, dx$

$-\frac{1}{16}\int 1 + \cos 4x \, dx$

$-\frac{x}{16} - \frac{\sin 4x}{64}$

$-\frac{1}{8}\int \cos^3 2x \, dx$

$-\frac{1}{8}\int (1-\sin^2 2x)\cos 2x \, dx$

$\sin 2x = t$

$dx \cdot \cos 2x = \frac{dt}{2}$

$-\frac{1}{16}\int (1-t^2) \, dt$

$-\frac{\sin 2x}{16} + \frac{\sin^3 2x}{48}$

$\int \tan^m x \cdot \sec^n x \, dx$

$\int \cot^m x \cdot \csc^n x \, dx$

1) n = çift $\Rightarrow$ $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$\cot x = t \quad -\csc^2 x \, dx = dt$

$\tan^2 x + 1 = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

Örnek: $\int \tan^6 x \cdot \sec^4 x \, dx$

$\int \tan^6 x \cdot \sec^2 x \cdot \sec^2 x \, dx$

$\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$\int t^6 (1+t^2) \, dt$

$\int t^6 + t^8 \, dt$

$\frac{t^7}{7} + \frac{t^9}{9} + C$

$(\sec x)' = \tan x \cdot \sec x$

$(\csc x)' = -\csc x \cdot \cot x$

2) n = tek , m = Tek

$\sec x = t$

$\csc x = t$

$\int \tan^5 x \cdot \sec^3 x \, dx$

$\int \tan^4 x \cdot \sec^2 x \cdot \sec x \cdot \tan x \, dx$

$(\tan^2 x)^2$

$(\sec^2 x - 1)^2$

$\sec x = t$

$\sec x \cdot \tan x \, dx = dt$

$\int (t^2 - 1)^2 \cdot t^2 \cdot dt$

$\int \sec x \, dx$

$\sec x - \tan x \quad \sec x + \tan x$

$\csc x - \cot \quad \cot x + \csc x$

$\int \sec x \frac{(\sec x + \tan x)}{\sec x + \tan x} \, dx$

$\int \frac{\sec^2 x + \sec x \cdot \tan x}{\sec x + \tan x} \, dx$

$\sec x + \tan x = t \Rightarrow (\sec x \cdot \tan x + \sec^2 x) \, dx = dt$

$\int \frac{dt}{t}$

$\ln|t| + C$

$\ln|\sec x + \tan x| + C$

$\underbrace{\int \sec^3 x}_{I}$

$\int \underbrace{\sec x}_{u} \cdot \underbrace{\sec^2 x\, dx}_{dv}$

$\sec x = u \qquad \sec^2 x\, dx = dv$

$\tan x \cdot \sec x = du \qquad \tan x = v$

$\int u\,dv = u \cdot v - \int v\,du$

$I = \tan x \cdot \sec x - \int \tan^2 x \cdot \sec x\, dx$

$- \int (\sec^2 x - 1) \cdot \sec x\, dx$

$I = \tan x \cdot \sec x - \underbrace{\int \sec^3 x\, dx}_{I} + \int \sec x\, dx$

$2I = \tan x \cdot \sec x + \ln|\tan x + \sec x| + C$

$\int \sec^3 x = \frac{1}{2}\left[\tan x \cdot \sec x + \ln|\tan x + \sec x| + C\right]$

$\int \csc^3 x\, dx$

$\int \underbrace{\csc x}_{u} \cdot \underbrace{\csc^2 x\, dx}_{dv}$

$\int \frac{\sin^n x}{\cos^m x}\, dx$

$\int \frac{\sin^3 x}{\cos^2 x}\, dx = \int \frac{\sin^2 x}{\cos^2 x} \cdot \sin x\, dx$ ($\sin^2 x = 1 - \cos^2 x$)

$\cos x = t$

$-\sin x\, dx = dt$

$-\int \frac{1 - t^2}{t^2}\, dt$

$-\int \frac{1}{t^2}\, dt + \int dt$

$\frac{1}{t} + t + C$

$\csc x + \cos x + C$

$\int \cot^5 x\, dx$

$\int \cot^4 x \cdot \cot x\, dx$

$-\int \underbrace{\cot^4 x \cdot (-\csc x \cdot \cot x)\, dx}_{\csc x}$

$\csc x = t \Rightarrow -\csc x \cdot \cot x\, dx = dt$

$(\cot^2 x)^2$

$(\csc^2 x - 1)^2$

$-\int \frac{(t^2 - 1)^2\, dt}{t} = -\int \frac{t^4}{t} - \frac{2t^2}{t} + \frac{1}{t}\, dt$

$\int t^3 - 2t + \frac{1}{t}$

$\frac{t^4}{4} - t^2 + \ln|t| + C$

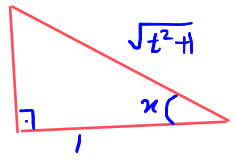
$\frac{\csc^4 x}{4} - \csc^2 x + \ln|\csc x| + C$

$$\int \frac{dx}{\sin^2 x \cdot \cos^4 x}$$

$\tan x = t$

$\arctan t = x$

$\frac{dt}{1+t^2} = dx$



$\sin x = \frac{t}{\sqrt{t^2+1}}$

$\cos x = \frac{1}{\sqrt{t^2+1}}$

$$\int \frac{\frac{dt}{1+t^2}}{\frac{t^2}{1+t^2} \cdot \frac{1}{(t^2+1)^2}} \Rightarrow \int \frac{(t^2+1)^2}{t^2} dt$$

$$\int \frac{t^4}{t^2} + \frac{2t^2}{t^2} + \frac{1}{t^2} dt$$

$$\frac{t^3}{3} + 2t - \frac{1}{t} + C$$

$$\frac{\tan^3 x}{3} + 2\tan x - \cot x + C$$

$-\frac{1}{t}$