Q: 13

$(\mathbf{v})_S = ?$

$\mathbf{v} = c_1 v_1 + c_2 v_2 + c_3 v_3$

$(2,-1,3) = c_1(1,0,0) + c_2(2,2,0) + c_3(3,3,3)$

$$\begin{cases} 2 = 1c_1 + 2c_2 + 3c_3 \\ -1 = 0c_1 + 2c_2 + 3c_3 \\ 3 = 0c_1 + 0c_2 + 3c_3 \end{cases}$$

4.5 :-

Q 1, 3, 5, 7, 15, 17

Q 1

$$\left.\begin{aligned} x_1 + x_2 - x_3 &= 0 \\ -2x_1 - x_2 + 2x_3 &= 0 \\ -x_1 + x_3 &= 0 \end{aligned}\right]$$

(basis for solution space)

$$\begin{array}{cccc} & & & x \end{array}$$

$$\left[\begin{array}{cccc} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right]$$

$R_2 + (2)R_1$

$R_3 + (1)R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$R_3 + (-1)R_2 \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

non-trivial solution.

$$\left.\begin{array}{r} x_1 + x_2 - x_3 = 0 \\ x_2 = 0 \end{array}\right]$$

$$\text{No of Parameter} = \begin{pmatrix} \text{No of} \\ \text{variables} \end{pmatrix} - \begin{pmatrix} \text{No of} \\ \text{eqs} \end{pmatrix}$$

$$= 3 - 2 = 1$$

$\boxed{x_3 = t}$

$x_2 = 0$

$x_1 + 0 - t = 0$

$\boxed{x_1 = t}$

No of params = No of vectors $\longrightarrow \frac{t}{2}\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

basis for solution space is

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ or } \{(1, 0, 1)\}$$

Note:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t - 3s \\ t \\ s \end{bmatrix} = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix} + \begin{bmatrix} -3s \\ 0 \\ s \end{bmatrix}$$

$$= t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \cancel{s}(-s) \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

$$= \{(1,1,0), (3,0,-1)\}$$

basis for solution space.

Q: 7

$$3x \cancel{t} - 2y + 5z = 0$$

$$R\cancel{t} + y = s$$

$$z = t$$

$$3x - 2s + 5t = 0$$

$$x = \frac{2s - 5t}{3}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2s - 5t/3 \\ s \\ t \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2s}{3} - \frac{5t}{3} \\ s \\ t \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2s}{3} \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{-5t}{3} \\ 0 \\ t \end{bmatrix}$$

$$= \frac{s}{3} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} - \frac{t}{3} \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

SB 21/4/22 Linear Algebra

Standard Basis

For $R^2 = e_1 = (1,0), (0,1) = e_2$

For $R^3 = e_1 = (1,0,0)$

$e_2 = (0,1,0)$ independent & spanning

$e_3 = (0,0,1)$

For $R^4 = e_1 = (1,0,0,0)$ ..... $e_4 = (0,0,0,1)$

Ex: 4.5

Q: 15

$v_1 = (1,-2,3)$ } linearly independent

$v_2 = (0,5,-3)$

(enlarge $\{v_1, v_2\}$ to a basis for R^3)

↳ add a standard vector

For spanning R^3

$(x,y,z) \in R^3$

as a linear combination

$av_1 + bv_2 = (x,y,z)$

$a(1,-2,-3) + b(0,5,-3) = (x,y,z)$

$1a + 0b = x$

$-2a + 5b = y$

$-3a - 3b = z$

$\boxed{a = x}$

$-2x + 5b = y$

$\boxed{b = \frac{y+2x}{5}}$ how to check correct

→ not satisfy (3) So (1) is not possible.

add 1 vector $e_1 = (1,0,0)$

Now vector $\{v_1, v_2, e_1\}$ (independent ok)

for spanning

$a_1 v_1 + a_2 v_2 + a_3 e_1 = (x, y, z)$

$a_1(1,-2,3) + a_2(0,5,-3) + a_3(1,0,0) = (x,y,z)$

$$\left.\begin{aligned} a_1 + 0a_2 + a_3 &= x \\ -2a_1 + 5a_2 + 0a_3 &= y \\ 3a_1 - 3a_2 + 0a_3 &= z \end{aligned}\right]$$

if $|A| \neq 0$ solution exists

↳ So unique & spanning

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ -2 & 5 & 0 \\ 3 & -3 & 0 \end{vmatrix} = 1(0-0) - 0 + 1(6-15) = -9 \neq 0$$

↳ So solution is unique

(2) is true, spanning done

↳ Basis for R^3 is $\{v_1, v_2, e_1\}$

Q: 17:-

R^3 spanned by vector

↳ go to basis for R^3

$v_1 = (1,0,0)$

$v_2 = (1,0,1)$

$v_3 = (2,0,1)$

$v_4 = (0,0,-1)$

For

For independent

$av_1 + bv_2 + cv_3 + dv_4 = 0$

$$\left.\begin{aligned} a + b + c + 0d &= 0 \\ 0a + 0b + 0c + 0d &= 0 \\ 0a + 1b + 1c - 1d &= 0 \end{aligned}\right\}$$ now non-trivial solution is possible.

Remove 1 $\longrightarrow v_4$

$aV_1 + bV_2 + cV_3 = 0$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$ (non-trivial solution)

↳ again dependent

Remove 1 (V_3)

$aV_1 + bV_2 = 0$

$a(1,0,0) + b(1,0,1) = (0,0,0)$

$$\left.\begin{aligned} a + b &= 0 \\ 0 + 0 &= 0 \\ 0 + b &= 0 \end{aligned}\right\}$$ no determinant

$$\left.\begin{aligned} a &= 0 \\ b &= 0 \end{aligned}\right\}$$ trivial solution independent

So, V_1 & V_2 are independent vectors and basis ~~for~~ (subspace of R^3)

↳ because less vectors.

Q: 19:-

$T_A(x) = 0$ $\quad\quad\quad\quad$ $T_A(x) = Ax.$

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$T_A(X) = 0$

$Ax = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\downarrow$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ t \end{bmatrix} = -t \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Ex : 4.6 (Change of Basis)
1, 3 (example (1)) (P-231)

$B = \{u_1, u_2, \ldots u_n\}$
$B' = \{u_1', u_2' \ldots u_n'\}$

(2)
$$[V]_B = P_{B' \to B} \cdot [V]_{B'}$$

$$[V]_{B'} = P_{B \to B'} [V]_B$$

(3) $(P_{B' \to B})(P_{B \to B'}) = I$

(~~view~~ new basis) (old basis)

$$\left[\begin{array}{cc|cc} 2 & 4 & 1 & -1 \\ 2 & -1 & 3 & -1 \end{array}\right] \text{———} (1)$$

$$\frac{1}{2}R_1 \left[\begin{array}{cc|cc} 1 & 2 & 1/2 & -1/2 \\ 2 & -1 & 3 & -1 \end{array}\right]$$

$$R_2 + (2)R_1 \left[\begin{array}{cc|cc} 1 & 2 & 1/2 & -1/2 \\ 0 & -5 & 2 & 0 \end{array}\right]$$

$$-\frac{1}{5}R_2 \left[\begin{array}{cc|cc} 1 & 2 & 1/2 & -1/2 \\ 0 & 1 & -2/5 & 0 \end{array}\right]$$ $R_1 + (-2)R_2$ —

$P_{B' \to B}$

26/4/2022 LINEAR ALGEBRA

Q:1 Bases:

$$B = \{u_1, u_2\}$$
$$B' = \{u_1', u_2'\}$$

$u_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, u_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \quad u_1' = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, u_2' = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

(a) $P_{B' \to B} = ?$ (transition matrix)

(b) $P_{B \to B'} = ?$

(c) $[w]_B = ?, \quad w = \begin{bmatrix} 3 \\ -5 \end{bmatrix} \quad \left([v]_{B'} = P_{B \to B'} [v]_B\right)$

also ~~the~~ complete $[w]_{B'}$

(d) Check your work by computing $[w]_{B'}$ directly

Solution:

(a) $P_{B' \to B} = \begin{bmatrix} 13/10 & -1/2 \\ -2/5 & 0 \end{bmatrix}$

(b) $P_{B \to B'} = ?$

old New

$\left[\begin{array}{c|c} \text{New} & \text{old} \\ \text{Basis} & \text{Basis} \end{array}\right]$

$\left[\begin{array}{cc|cc} 1 & -1 & 2 & 4 \\ 3 & -1 & 2 & -1 \end{array}\right]$ — (I)

$R_2 + (-3)R_1 \left[\begin{array}{cc|cc} 1 & -1 & 2 & 4 \\ 0 & 2 & -4 & 13 \end{array}\right]$

$\frac{1}{2}R_2 \left[\begin{array}{cc|cc} 1 & -1 & 2 & 4 \\ 0 & 1 & -2 & 13/2 \end{array}\right]$

$R_1 + (1)R_2 \left[\begin{array}{cc|cc} 1 & 0 & 0 & -5/2 \\ 0 & 1 & -2 & -13/2 \end{array}\right]$

$P_{B \to B'} = \begin{bmatrix} 0 & -5/2 \\ -2 & -13/2 \end{bmatrix}$

(c) $[W]_B = ?$

$W = c_1u_1 + c_2u_2$

$$\begin{bmatrix} 3 \\ -5 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$3 = 2c_1 + 4c_2$ — (i)

$-5 = 2c_1 - c_2$ — (ii)

(I) - (II)

$$\begin{aligned} 3 &= 2c_1 + 4c_2 \\ -5 &= 2c_1 - c_2 \\ \hline 8 &= 5c_1 \end{aligned}$$

$c_2 = 8/5$ Put in (II)

$-5 = 2c_1 - 8/5$

$\frac{8}{5} - 5 = 2c_1$

$-17/5 = 2c_1$

$\boxed{-17/10 = c_1}$

$[W]_B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

$[W]_B = \begin{bmatrix} -17/10 \\ 8/5 \end{bmatrix}$

use

$[W]_{B'} = P_{B \to B'} [W]_B$

$$= \begin{bmatrix} 0 & -5/2 \\ -2 & -13/2 \end{bmatrix} \begin{bmatrix} -17/10 \\ 8/5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 - 4 \\ 17/5 - 4/10 \end{bmatrix} = \begin{bmatrix} -4 \\ \frac{-34 - 104}{10} \end{bmatrix}$$

$$[W]_{B'} = \begin{bmatrix} -4 \\ -7 \end{bmatrix}$$

(d) $[W]_{B'} = ?$

$$W = C_1 U_1' + C_2 U_2'$$

$$\begin{bmatrix} 3 \\ -5 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + C_2 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$3 = C_1 - C_2$ — (I)

$-5 = 3C_1 - C_2$ — (II)

(I) - (II)

$$\begin{aligned} 3 &= C_1 - C_2 \\ -5 &= 3C_1 - C_2 \\ \hline 8 &= 2C_1 \end{aligned}$$

$-4 = C_1 \qquad C_2 = -7$

$$[W]_{B'} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -7 \end{bmatrix} \text{. true}$$

Bases for row space
column space
null space → basis of solution space of $Ax = 0$
nullity

(rank = 1)

Q: 9

(a) $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

echelon form.

$R_2 + (-5)R_1$
$R_3 + (-7)R_1$ $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 1 & -19 \end{bmatrix}$

$R_3 + (-1)R_2$ $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -19 \\ 0 & 0 & 0 \end{bmatrix}$ — (1)

Basis for row space.

$\{(1,-1,3), (0,1,-19)\}$

Basis for column space

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$ (columns contains leading elements 1)

4.7:

$$Ax = 0$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x - y + 3z = 0$$
$$5x - 4y - 4z = 0$$
$$2x - 6y + 2z = 0$$

Basis for null space

$$1x - 1y + 3z = 0$$
$$y - 19z = 0$$

Nullity is number of vectors in the basis for null space.

Let $z = t$

$$x - y + 3t = 0$$
$$y - 19t = 0$$
$$y = 19t$$

$$x - 19t + 3t = 0$$
$$x - 16t = 0$$
$$x = 16t$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16t \\ 19t \\ t \end{bmatrix}$$

$$t = \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix}$$

Basis for null space is

$$\left\{ \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \right\}$$

or $\{(16, 19, 1)\}$

Nullity $(A) = 1$

Note

$$\underbrace{\text{Rank}(A)}_{\text{Rank}(A) = 2} + \text{nullity}(A) = \underbrace{n}_{\text{no of column of vector } A}$$

$2 + N(A) = 3$

$N(A) = 1$

---

example (16)

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 6 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

$\equiv$

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ --- } (1)$$

$\{ (1, -3, 4, -2, 5, 4),$
$(0, 0, 1, 3, -2, -6),$
$(0, 0, 0, 0, 1, 5) \}$

Basis:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Basis for null space.

from (1)

$$x_1 - 3x_2 + 4x_3 - 2x_4 + 5x_5 + 4x_6 = 0$$

$$x_3 + 3x_4 - 2x_5 - 6x_6 = 0$$

$$x_5 + 5x_6 = 0$$

Put $\boxed{\begin{aligned} x_6 &= s \\ x_4 &= t \\ x_2 &= u \end{aligned}}$

$$x_1 - 3u + 4x_3 - 2t + 5x_5 + 4s = 0$$

$$x_3 + 3t - 2x_5 - 6s = 0$$

$$x_5 + 5s = 0$$

$$x_5 = -5s$$

$$x_3 + 3t - 2(-5s) - 6s = 0$$

$$x_3 + 3 + 4s = 0$$

$$x_3 = -3t - 4s$$

$$x_1 - 3u + 4(-3t - 4s) - 2t$$

$$x_1 - 3u - 12t - 37s = 0$$

$$x_1 = 3u + 12t + 37s$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \begin{bmatrix} 3u + 12t + 37s \\ u \\ -3t - 4s \\ t \\ -5s \\ s \end{bmatrix}$$

$$= \begin{bmatrix} 3u \\ u \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 12t \\ 0 \\ -3t \\ t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 37s \\ 0 \\ -4s \\ 0 \\ -5s \\ s \end{bmatrix}$$

$$u \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 12 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 37 \\ 0 \\ -4 \\ 0 \\ -5 \\ 1 \end{bmatrix}$$

Basis of null space:

$$\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 37 \\ 0 \\ -4 \\ 0 \\ -5 \\ 1 \end{bmatrix} \right\}$$

nullity (A) = 3

28/4/2022. LINEAR ALGEBRA

Ch: 3 — 4.8 (Mid II)

MATRIX TRANSFORMATION (Linear transformation)

(1.8, 4.9, 4.10, 4.11)

Let $T: R^n \longrightarrow R^m$

T is called Linear transformation if it satisfy the following

i) $T(u+v) = T(u) + T(v)$ $u, v \in U$ α scalar.

ii) $T(\alpha u) = \alpha T(u) \longrightarrow \alpha = 0$, $T(0) = 0$

must for linear, but for Linear we prove above two conditions

1.8

Q: 21, 22.

Q: 21

$T(0) \neq 0$ gives decision of T is not linear

(a) $T(x, y) = (3x+y, x-y)$

is T Linear = ?

$T: R^2 \longrightarrow R^2$ components.

$T(0,0) = (0,0)$ so not not-linear

$u = (u_1, u_2)$

$v = (v_1, v_2) \in R$

i) $T(u+v) = T(u) + T(v)$

$u+v = (u_1, u_2) + (v_1, v_2) = (u_1+v_1, u_2+v_2)$

$T(u+v) = T(u_1+v_1, u_2+v_2)$

$= \left(3(u_1+v_1) + u_2+v_2, (u_1+v_1) - (u_2+v_2)\right)$

$= (3u_1 + 3v_1 + u_2 + v_2, u_1 + v_1 - u_2 - v_2)$

$= \left((3u_1+u_2) + (3v_1+v_2), (u_1-u_2) + (v_1-v_2)\right)$

$= (3u_1+u_2, u_1-u_2) + (3v_1+v_2, v_1-v_2)$

$= T(u_1, u_2) + T(v_1, v_2) = T(u) + T(v)$ (T)

ii) $T(\alpha u) = T(\alpha(u_1, u_2))$

$= T(\alpha u_1, \alpha u_2)$

$= (3\alpha u_1 + \alpha u_2, \alpha u_1 - \alpha u_2)$

$= \alpha(3u_1 + u_2, u_1 - u_2)$

$= \alpha T(u_1, u_2)$

$= \alpha T(u)$

$= (T)$

(b) $T(x_1, x_2, x_3) = (x_1, x_3, x_1 + x_2)$

$T : R^3 \longrightarrow R^3$

domain   codomain

i) $T(u + v) = T(u) + T(v)$

let $u = (u_1, u_2, u_3) \in R^3$

$v = (v_1, v_2, v_3)$

Q : 22

Q : 24

a) $T(x, y) = (x, y+1)$

$T(0,0) = (0, 0+1)$

$0 = (0,1) \neq 0$

Q : 23.

(a) $T(x, y) = (x^2, y)$ T is not linear.

$T(x,y) = (x^2, y)$

i) $T(u+v) = T(u) + T(v)$

$u = (u_1, u_2)$

$v = (v_1, v_2)$

$(u+v) = (u_1+v_1, u_2+v_2)$

$T(u+v) = T(u_1+v_1, u_2+v_2)$

$T(u+v) = ((u_1+v_1)^2, u_2+v_2)$

$T(u) + T(v) = T(u_1, u_2) + T(v_1, v_2)$

$= (u_1^2, u_2) + (v_1^2 + v_2)$

$T(u) + T(v) = (u_1^2 + v_1^2, u_2 + v_2)$ — II

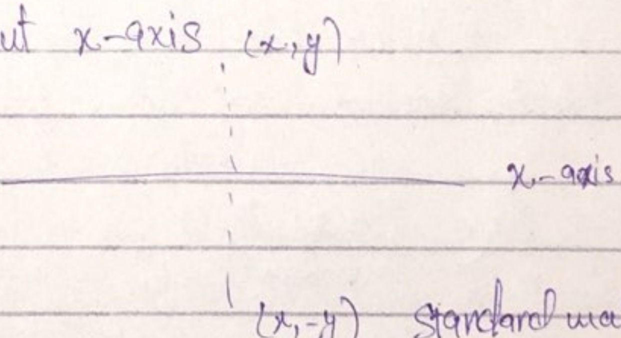
from I, II. $T(u+v) \neq T(u) + T(v)$ (F)

So, T is not Linear.

P-259) For R^2

(i) Reflection about x-axis

$T(x,y) = (x,-y)$



standard matrix

$T(e_1) = T(1,0) = (1,0)$

$T(e_2) = T(0,1) = (0,-1)$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$