Final

Ex 4.8

Q.1 a) $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$

For echeleon form

$R_2+(-2)R_1$
$R_3+(-3)R_1$
$R_4+(-4)R_1$
$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Basis for row space

$\{(1,2,-1,1)\}$

Basis for coloumn space

$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

Rank $A = 1$

Nullity of A

$Rank(A) + \cancel{Rank}\ N(A) = n$ $\quad \because n=4$ (no. of col)

$1 + N(A) = 4$

$N(A) = 4-1$

$N(A) = 3$

Eigon values & Eigon vector

A (matrix)

λ (eigon value)

x (eigon vector)

$Ax = \lambda x$

$\lambda = ?$

~~$|\lambda I - A|$~~

$|\lambda I - A| = 0$

(Characteristic eq)

Ex 5.1

Q. 1, 3, 5, 7, 9, (example 6, 7)

Q. 5 (a) . $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

Characteristic eq:?

4.9, 4.10, 4.11

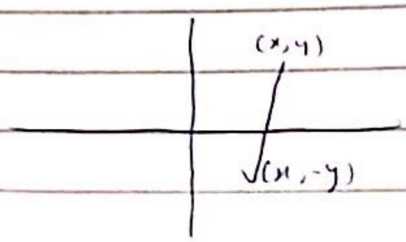
Note

1. Reflection about x-axis

$T(x,y) = (x,-y)$

Standard Matrix

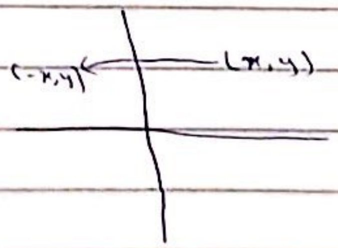
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Find reflection of (4,6) about x-axis by using standard matrices

$$(4,6) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \end{bmatrix} = (4,-6)$$

2. Reflection about y-axis



$T(x,y) = (-x,y)$

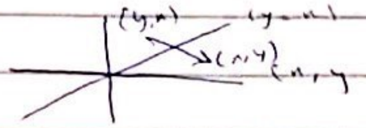
Standard Matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

→ Find reflection (6,-2) about y-axis by using standard vector

$$(6,-2) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} +6 \\ -2 \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \end{bmatrix} = (-6,-2)$$

3 Reflection about ~~y~~ the line $y = x$



$T(x,y) = (y,x)$

standard Matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Find reflection (6,1) about $y=x$ by using standard vector

$$(6,1) \to \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} = (1,6)$$

4. ~~[illegible]~~

Q Find inverse of $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ draw graph of each elementary matrices ~~with~~ by multiplying with matrix of ~~of~~ unit ~~to~~ ~~square~~ square matrix

Sol:-

$$\left[\begin{array}{cc|cc} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{array}\right], \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_{12}\left[\begin{array}{cc|cc} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{array}\right], \quad E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = E_1^{-1}$$

$$(-1)R_1\left[\begin{array}{cc|cc} 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{array}\right], \quad E_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = E_2^{-1}$$

$$(-1)R_2\left[\begin{array}{cc|cc} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{array}\right], \quad E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = E_3^{-1}$$

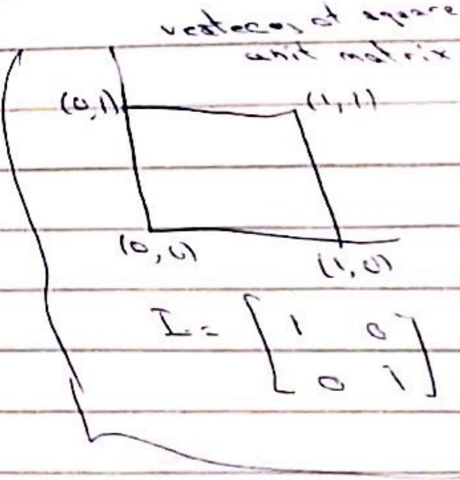
$$A^{-1} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$E_3 E_2 E_1 = A^{-1}$$

$$A^{-1} = E_3 E_2 E_1$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1}$$

$E_1 = ?$



$$E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

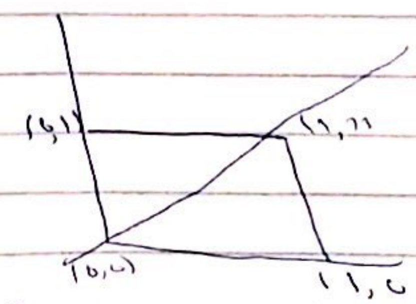
$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(0,0) \Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = (0,0)$$

$$(1,0) \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (0,1)$$

$$(1,1) \rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (1,1)$$

$$(0,1) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (1,0)$$



reflection about $y=x$

E_2 :

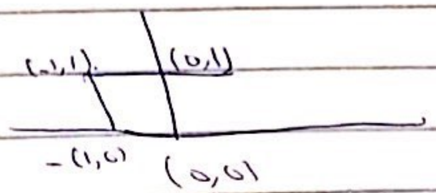
$E_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$(0,0) \rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = (0,0)$

$(1,0) \rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = (-1,0)$

$(1,1) \rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = (-1,1)$

$(0,1) \rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (0,1)$



reflection about y-axis

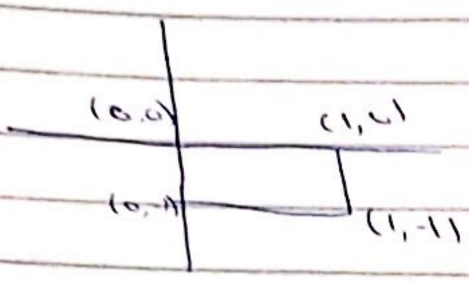
$E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$(0,0) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = (0,0)$

$(1,0) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = (1,0)$

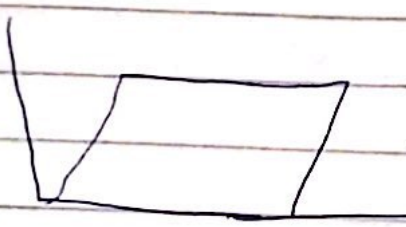
$(1,1) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (1,-1)$

$(0,1) \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (0,-1)$



reflection about x-axis

Note



→ sheer along x-axis

can be positive or -ve