PHY 1210

8/02/2017

MECHANICS

mechanics → Kinematic
mechanics → Dynamic
mechanics → Statics

① Kinematic ⇒ it described of all possible motions of materials ~~bodies~~ ~~of the~~ body

② Dynamic ⇒ is the study of laws which determine possible motion

③ Static ⇒ also is study of forces and system of forces which act on bodies at rest.

SPACE TIMES

Space time consist of three directions (x, y, z) and one ~~direction~~ (t) dymension (t) that combined into a single model. The frame of references is (x y z) coordinate.

Intial frame reference ⇒ This is references frame which numbros laws are valid or in which physics law are valid.

Measurement Units and Dimensions

Measurement is a method of comparation of a unknown physical quantity with unknown standard quantity.

A physical quatity it carries numerical value and unit when measured.

There are two types of physical quatities

1. fundemental or basic physical quantity
2. Derived physical quantity

The fundemental physical quantities of mechanics are mass, length and time with their corresponding units, with Kilo, metre, second,

The fundemental or basic physical quantities are basic below

Physics Quantity	Units	Dimensions
Length	metre (M)	L
mass	Kilogram (Kg)	M
Time	Second (S)	T
Temperature	°C or °K	Q
electric current	Ampere (A)	I
Luminecity	Candela (cd)	

- energy (J or Kgm^2

Amount of Substance { mole }

- all other physical quantities are derived quantities the unit of fundamental physical quantities are known as ~~derived unit as~~ basic or fundamental unit and unit of drive quantities are known as drive units

System units ⇒

1. ~~can~~ cgs (length, mass time)
2. FPS (foot, pound, & second)
3. MKS (metre, Kilogram & second)

The S. I system of unit abbraviation of "System - International" is Internationally accepted system of unit introduced by General Conference of weight and measures in 1960. It is modified MKS system.

Some metric prefixen

1	Giga	(G)	10^9
2	mega	(M)	10^6
3	micro	(μ)	10^{-6}
4	Nano	(N)	10^{-9}

Dimensions

The dimension of physical quantity is the relationship between the physical quantity and basic quantity

Length $\Rightarrow$ (L) mass $\Rightarrow$ (M) Time $\Rightarrow$ (T)

Examples

Dimension of area $= L \times b = L \times L = L^2$ (unit m^2)

Velocity $= \dfrac{\text{displacement}}{\text{Time}} = \dfrac{L}{L} = LT^{-1}$ (unit ms^{-1})

frequency $= 1/\text{period} = 1/T = T^{-1}$ (unit Hz or s^{-1})

Strain $= \dfrac{\text{extention}}{\text{original length}} = \dfrac{L}{L} =$ dimensions

Uses of Dimensions

With the help of dimensions equation

i — Convert one system of Unit to another

ii — check the Correctness mass of physical quantities

iii — Derive Important relations

Example

check homogeneity of equation

$S = ut^2 + \frac{1}{2}at$

$S = ut + \frac{1}{2} at^2$

$L \quad LT^{-1}(T^2) + LT^{-2}T$

$= LT^1 + LT^{-1}$

$= m_1 = ms + ms^{-1}$

$L = LT^{-1}\ T + LT^{-2}T^2$

$= (M) = (m) + (m)$

Example ⇒ Check pendlum equation

$T = 2\pi\sqrt{g/L}$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

Alternative unit for grantational feild strengh are NKg^{-1} and ms^{-2}

Show that the two are equavalent using method of dimensions.

$NKg^{-1} = \frac{\text{force}}{\text{mass}} = \frac{mLT^{-2}}{m} = LT^{-2}$

$ms^{-2} = \frac{L}{T^2} = LT^{-2}$

Quantity		Dimensions	SI
Length	(L)	L	m
Time	(T)	T	s
mass	(M)	M	Kg
Area	(A)	L^2	m^2
Velocity	(V)	LT^{-1}	~~m/s²~~ m^3
Acceleration		LT^{-2}	m/s^2
force	(F)	MLT^{-2}	N
pressure		$ML^{-1}T^{-2}$	pc
power		ML^2T^{-3}	W
Density		ML^{-3}	K/m^2
energy/work/heat		ML^2T^2	J

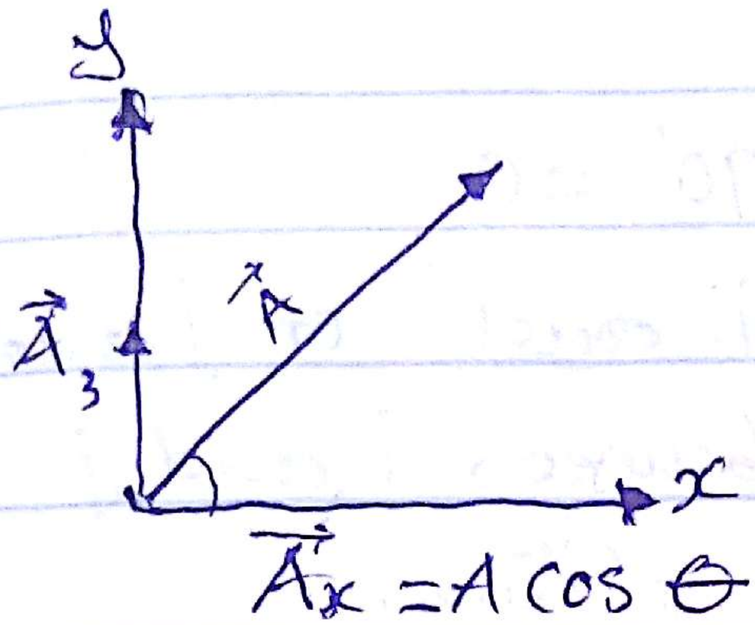
PHY 1210 mechanics 15/02/2017

SCALARS AND VECTORS

A vector quantity has both quantity and direction is normally repeated by straight line (magnitude) and arrow (direction)

magnitude of a vector $\vec{A}$, (the A here is boldface italic with arrow) $= A = |\vec{A}|$

Component of the vectors of $\vec{A}$ (in rectangular-coordinate) are: A_x A_y and A_3 so that the vector $\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$



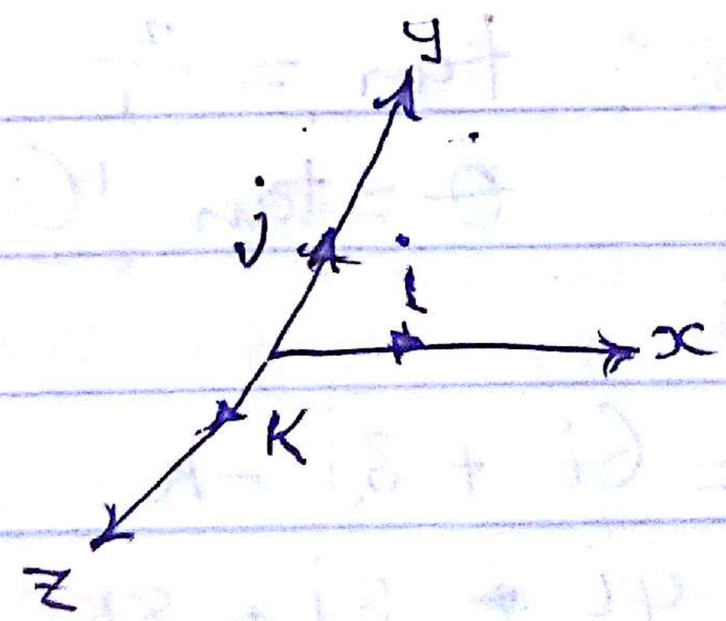
The magnitude of $\vec{A}$ is $A = \sqrt{A_x^2 + A_y^2}$ is 2D (dimension)
which is $A = \sqrt{A_x^2 + A_y^2 + A_3^2} = 3D$ (dimension)

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan\left(\frac{A_y}{1}\right)$$

UNIT VECTORS

A Unit Vector has a magnetude of 1 one, so that the Unit vector are: i j and K express the vector $\vec{A}$ in terms of it Compound



$\vec{A} = A_x i + A_y j + A_{3x}$

$\vec{B} = B_x i + B_y j + B_{3x}$

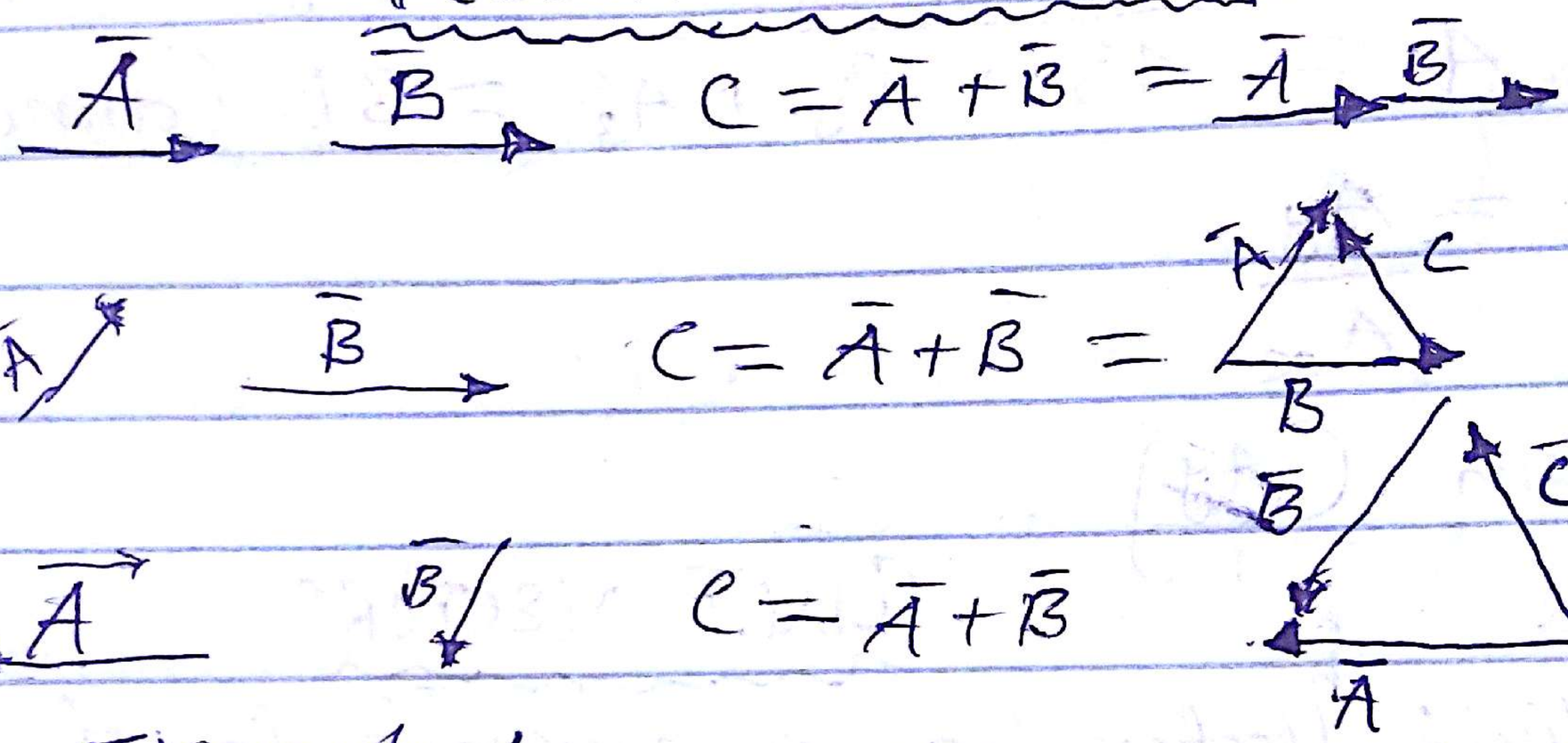
i j and K are perpendicular to each others

$i \cdot i = j \cdot j = K \cdot K = (1)(1)\cos 0^\circ = 1$

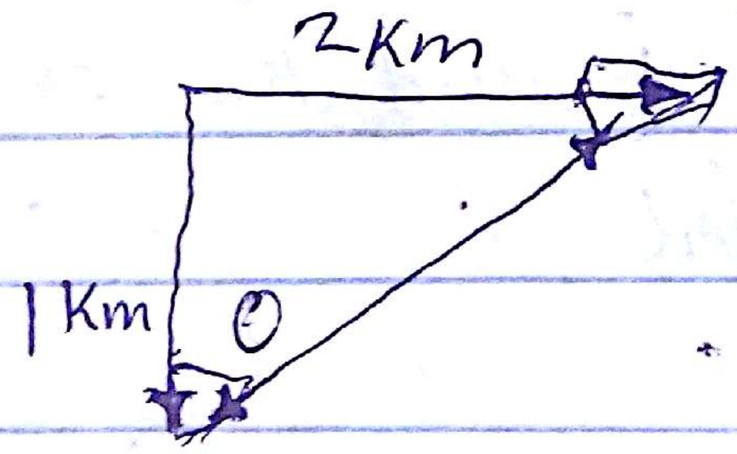
$i \cdot j = i \cdot k = j \cdot k = (1)(1) \cos 90^\circ = 0$

Whenever the angle between i and i or j and j is zero (0) while the angle between i and j is 90° degrees at any given condition.

VECTORS ADDITIONS



Example 1



$R = \sqrt{1^2 + 2^2}$

$\tan = \frac{2}{1}$

$\theta = \tan^{-1}(\cdot)$

Example 2.

Let Given $\bar{A} = 6i + 3j - k$

$\bar{B} = 4i - 5j + 8k$

(i) $\bar{A} + \bar{B} = (6i + 3j - k) + (4i - 5j + 8k)$

$\vec{A} + \vec{B} = (10i - 2j + 7k$ //

$2\vec{A} - B = (12i + 6$

(ii) $2\vec{A} - B = (12i + 6j - 2K) - (4i - 5j + 8K)$

$8i + 6j + 5j - 2 - 8$

$8i + 11j = 10K$ //

(iii) If Unit of $\bar{A}$ $\bar{B}$ and F are in metres, find the magnitude of F

$$|\bar{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$|\bar{F}| = \sqrt{(8i)^2 + (11j)^2 - (10)^2}$$

$$|F| = \sqrt{64 + 121 - 100}$$

A lady walk 3 Km due north and then 4Km due East - what is the resultant (or effected) of her journey)

SCALAR (OR DOT) PRODUCT

Given $\bar{A} = 2i + 3j + K$

$\bar{B} = -4i + 2j - K$

find the angle between $\bar{A}$ & $\bar{B}$

$\bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_3 B_z$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$B = \sqrt{B_x^2 + B_y^2 + B_z}$

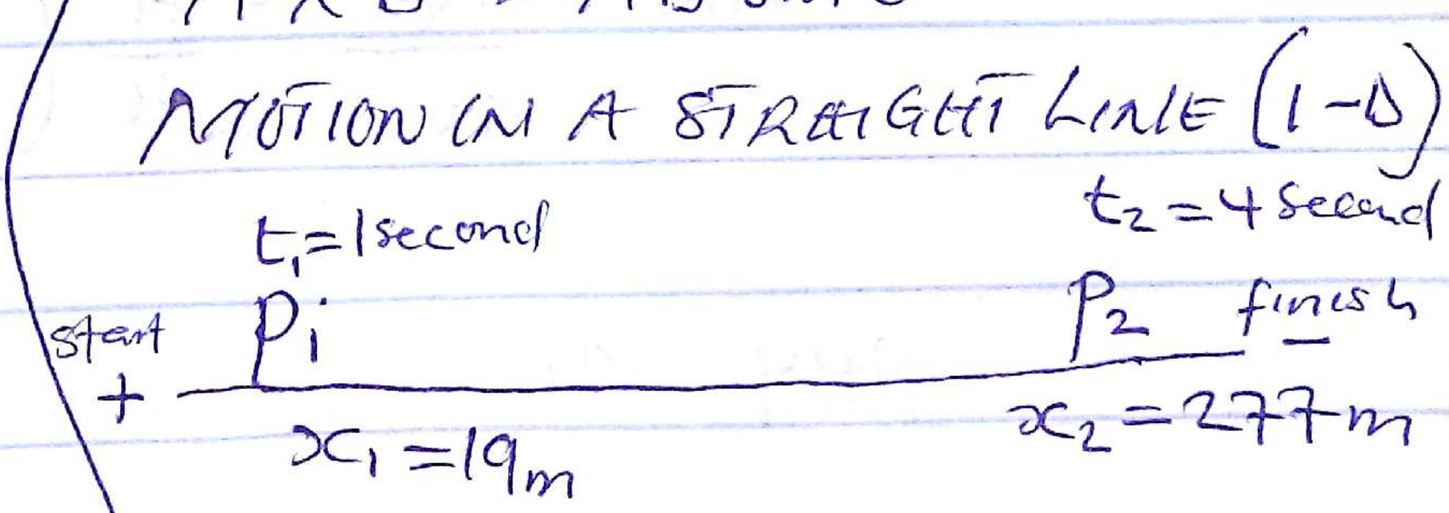
$\bar{A}\bar{B} = AB \cos\theta$

$$\cos\theta = \frac{\bar{A}\bar{B}}{AB}$$

Cross product

$\bar{A} \times B = AB \sin\theta$

MOTION IN A STRAIGHT LINE (1-D)



x_1 and x_2 are displacement at t_1 and t_2 respectively

Average Velocity $= \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$

The velocity at any time or specific instant if time or specific point along the path is known Instantaneous velocity.

To find Instantaneous velocity in (x direction) we take the derivative of (x) with respect to (t)

$$V_x = \frac{dx}{dt}$$

AVERAGE ACCELERATION

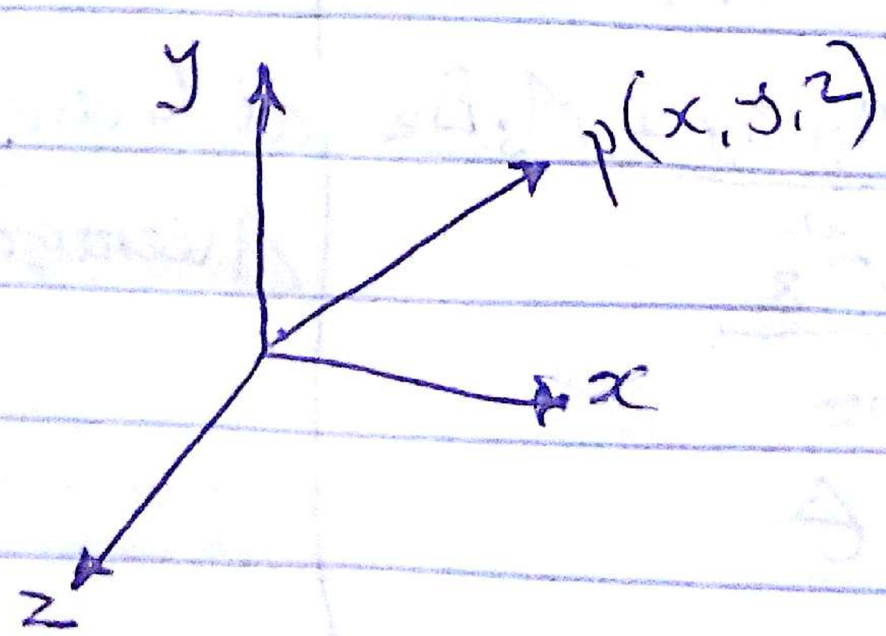
$$= \frac{\Delta V_x}{\Delta t} = \frac{V_{2x} - V_{1x}}{t_2 - t}$$

The Intantaneous of acceleration (in x-axis) can be gotten by taking the derivative of V_x with respect to t :

$$a_x = \frac{dV_x}{at} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

PHY 1210 22/02/2017

motion in 2 or 3-D (Dimension)



The provision vector is y a particle contain insistant by
$ix + jy$ (for 2-D) or 2 Dimension
$ix + jy + xz$ (for 3-D) or 3 Dimension

Average velocity vector $V_{ox} = \frac{r_2 - r_1}{t_2 - t_1} = \frac{\Delta r}{\Delta t}$

Instentaneous velocity vector is $\frac{d\vec{r}}{dt}$

Component of a $\vec{V}$ are

$$\frac{u}{x} = \frac{dx}{dt} \quad \frac{u}{y} = \frac{dy}{dt} \quad \frac{u}{z} = \frac{dz}{dt}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} i + \frac{dy}{dt} j + \frac{dz}{dt} K$$

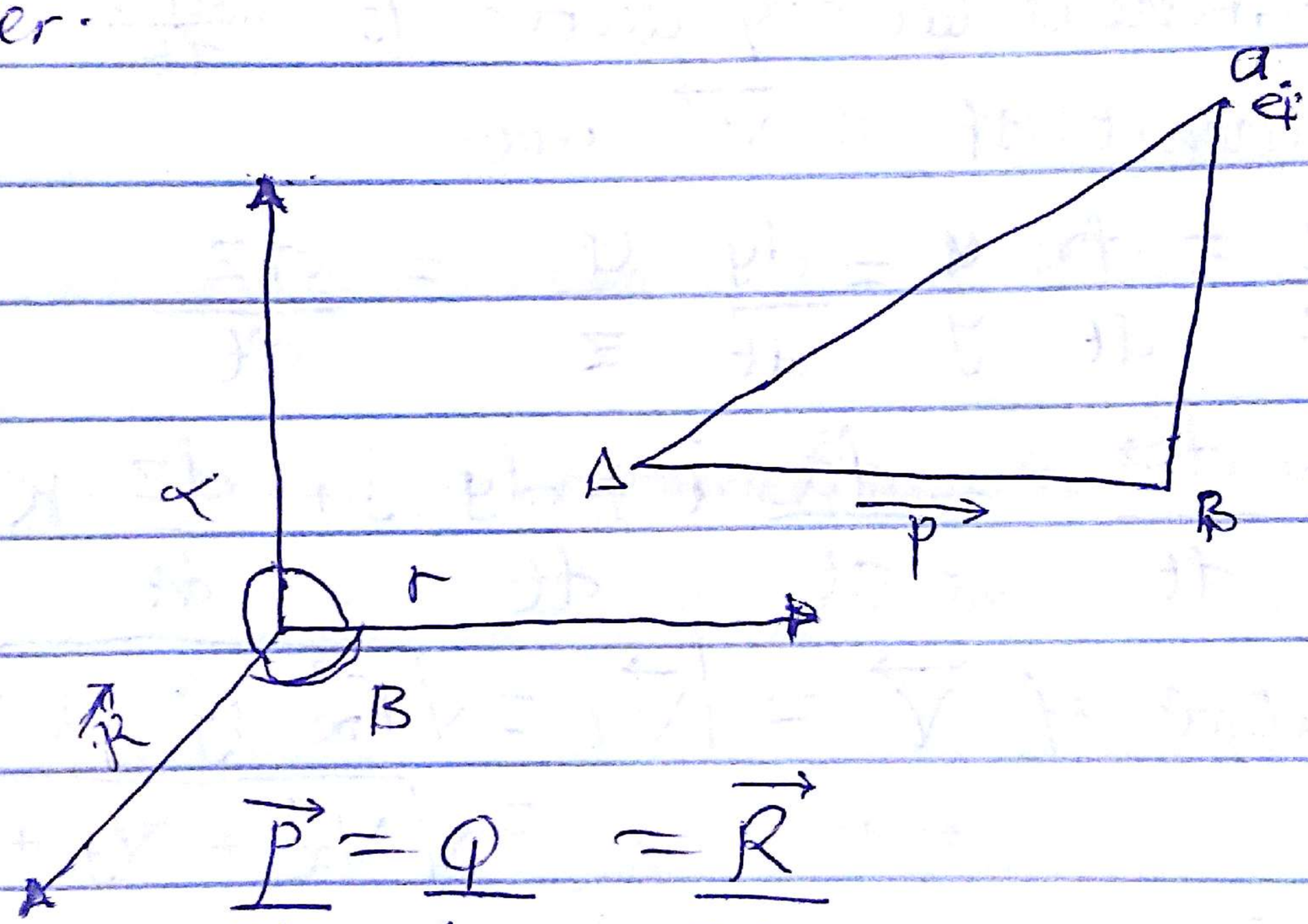
Magnitude of $\vec{V} = |\vec{V}| = \sqrt{V_x^2 \; V_y^2}$ (2-D)
$= \sqrt{V_x^2 + V_y^2 + V_z}$ (3-D)

The Component of acceleration
$\vec{a} \; \frac{dv}{dt} = \frac{dr^2 \rightarrow}{dt^2}$

The Component are

TRIANGLE LAW AND VECTOR:

If three force acting as a point a body are in equilibrium and are join head to tail to form a cross triangle, then each of the force can represented with magnitude and direction by the size side of the triangle taken in order.



$$\frac{\vec{P}}{ab} = \frac{Q}{bc} = \frac{\vec{R}}{ac}$$

LAMI'S THEOREM:-

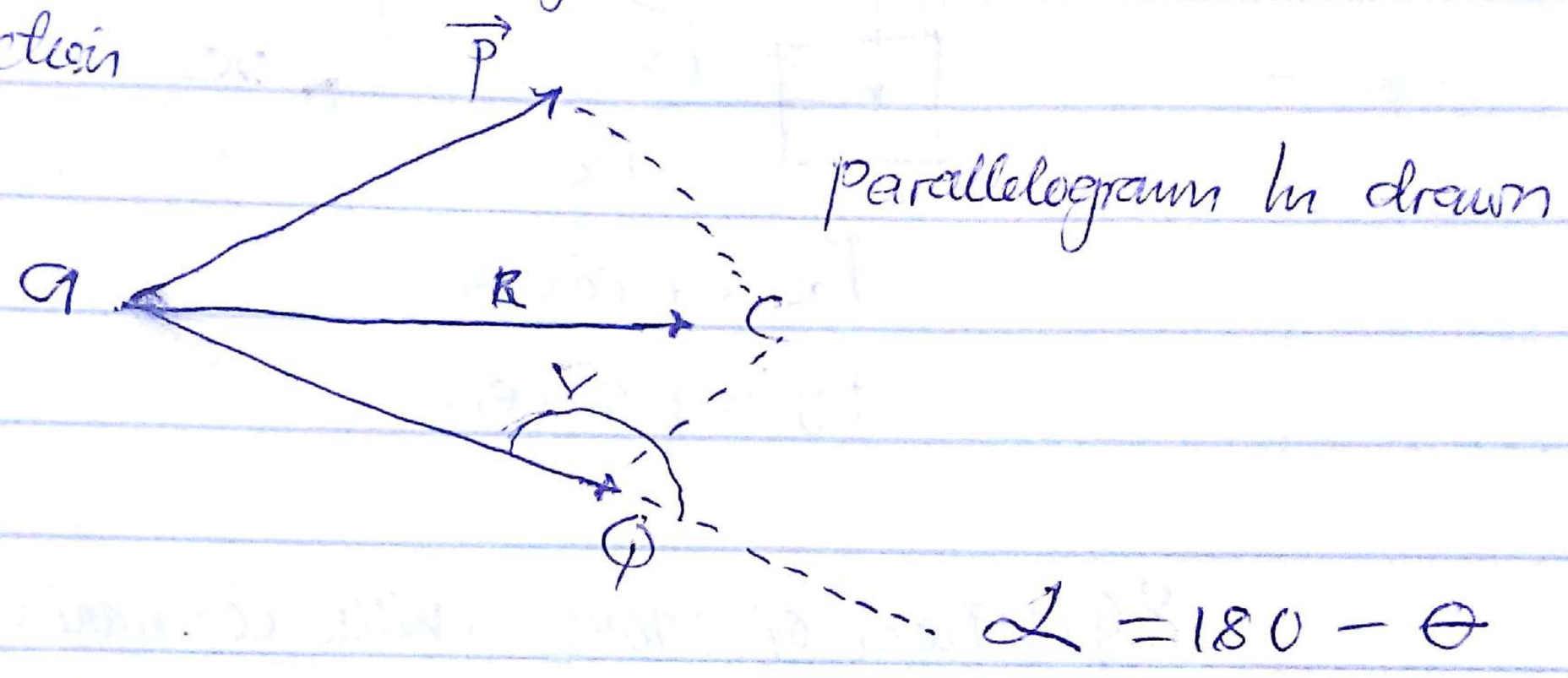
If three force acting at a point are in equilibrium of the force is proportional to the sin of the angle between the two vector

Mathematically

$$\frac{\vec{P}}{\sin\theta} = \frac{\vec{\theta}}{\sin B} = \frac{\vec{R}}{\sin\gamma}$$

PARALLELOGRAM LAW

If two forces are represented in magnitude and direction by adjecent sides of a constructed parallelogram their resultant by the diagonal drawn from the common point of action



parallelogram in drawn

Applying cosine rules abc

$R^2 = p^2 + Q^2 + 2pQ \cos\theta$

$= p^2 + Q^2 - 2pQ \cos\alpha$

When $\theta = 0°$, $12 = p + Q$

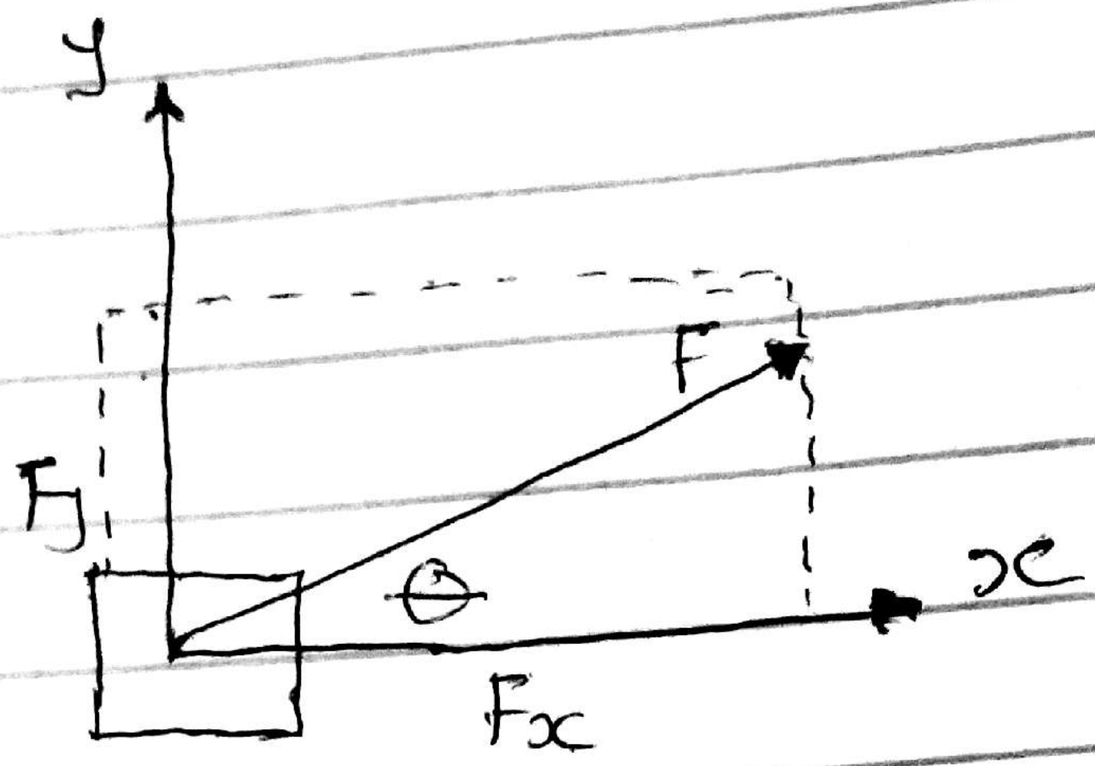
$\theta = 90°$, $R^2 = p^2 + Q^2$

$\theta = 180°$, $R = p - Q$

VECTOR RESOLUTION

This is the method of obtaining two vectors from a single vector,

if two vector can be represented by their resultant, conversely one vector can be represent by two vectors e.g. a vector (force) acting as a body at angle θ



$F_x = F\cos\theta$

$F_y = F\sin\theta$

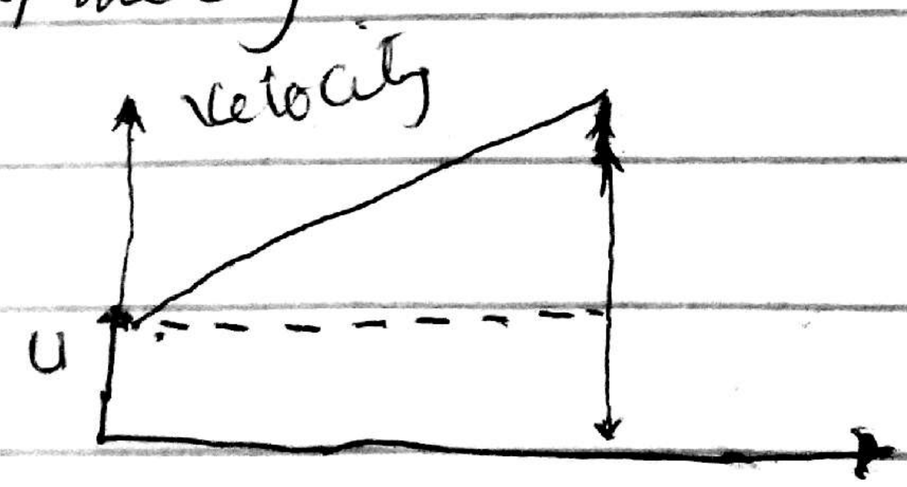
EQUATION OF MOTION (WHEN CONSTANTS ACCELERATION)

A body moving with initial velocity u and then final velocity v in time t

The rate of change of velocity, $a = \frac{v-u}{t}$

$$v = u + at \quad \text{--- (1)}$$

Graphically



The distance travelled S is given by the area under the velocity time graph.

$$S = ut + \frac{1}{2}(v-u)t$$

$$= \frac{1}{2}ut + \frac{1}{2}vt$$

$$\boxed{S = \frac{1}{2}(u+v)t} \quad \text{--- (2)}$$

that $v = u + at$

$$S = \frac{1}{2}(u + u + at)t$$

$$\boxed{S = ut + \frac{1}{2}at^2} \quad \text{——— (3)}$$

Let eliminating t using equation (1) $t = \frac{v-u}{a}$

$$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)$$

$$V^2 = u^2 + 2as \text{ - - - - - - - (4)}$$

Example

A driver of a car driving at 50 miles per hours apply the breaks and the car accelerate at constant rate. what is the velocity of a car 0.8 miles, Further along the road, if the car reach at 1.2 minutes after breaks where applied.

$$t = \frac{1.2}{60} = 1.2\,hr = 0.02\,hr$$

$u = 50$

$u = 50\,mil/hrs$

$S = 0.8$ miles

$V = ?$

$$S = \frac{(u+v)}{2}t$$

final velocity $V = \frac{25}{t} - u = 30$ meter/hours

A car start rest with an acceleration of $2m^{s-2}$
What is the velocity after

$u = 0$ $\quad a = 2 m/s^2$

$V = ?$ $\quad s = 200m$

PHY 1210 01/03/2017

MECHANICS

First law (1)

NEWTONS LAWS OF MOTION

Everybody, a particle persist in its state of rest or uniform motion in a straight line (constant velocity) unless its compelled with change.

This properties of an objects to resist change in its state of rest or motion is known as Inertia

2nd law

Law, the rate of change of linea momentum is proportional to applied force. It is a direct rein in which the force act

Rate of change of momentum $\propto$ force F

i.e. $\frac{mv - mu}{t} \propto F$

$$\frac{mv - mu}{t} = KF = \boxed{FKma}$$

in SI unit if $F = 1N$ and $m = 1Kg$

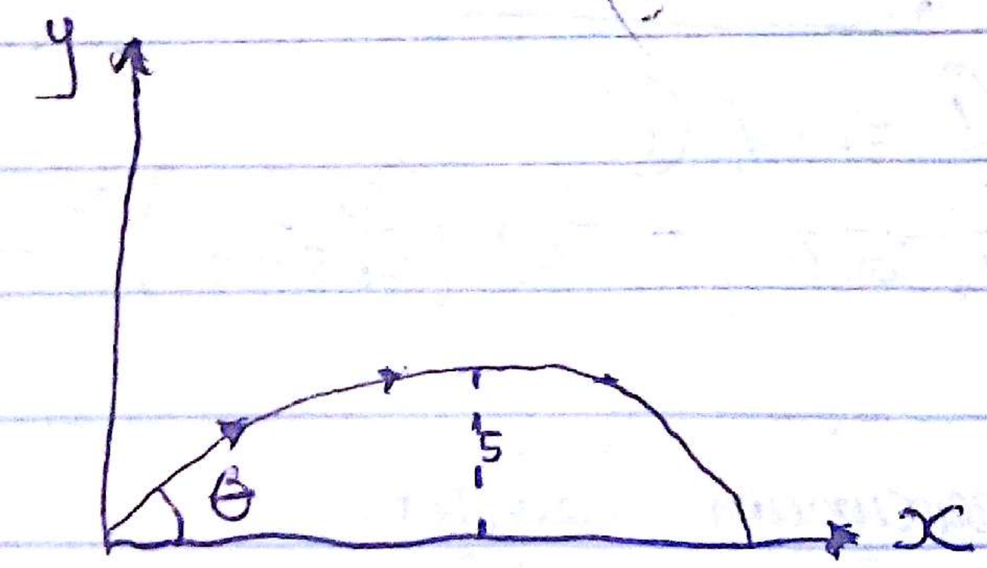
then $K = 1$ $\boxed{F = ma}$

{third law} ⟶ if two particle exert forces of one another, these forces are equal in magnitude and opposite in direction an act along the line joining F_1.

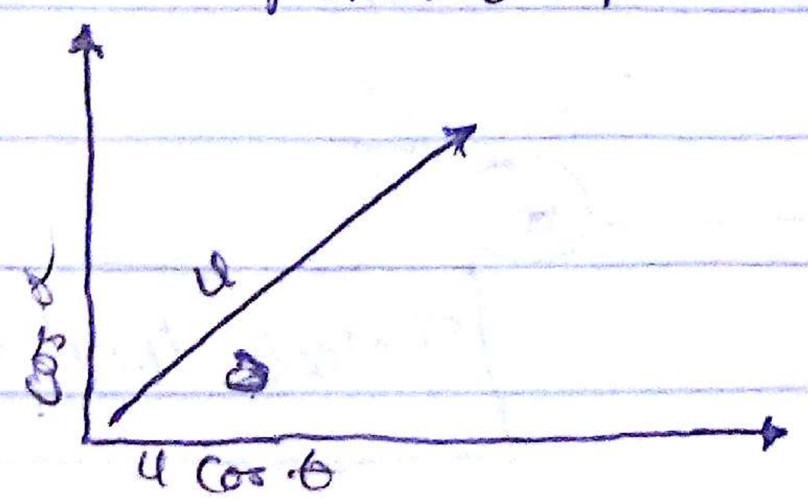
$\boxed{\text{mathematical language } F_{12} = -F_{21}}$

PROJECTILE MOTION (In a non resistative medium)

this example of 2D (two dimension)



it has two Component of velocity



u_0 = Initial velocity

$u_x = u \cos\theta$

$u_y = u \sin\alpha$

(i) the maximum vertical distance S_y

$S_y = ut + \frac{1}{2}at^2 = ut - \frac{1}{2}gt^2$ (against gravity $A - a = g$)

$= u_y t - \frac{1}{2}gt^2$ —— (1)

or from $v^2 - u^2 = 2as = -2gs$

($u = 0$ at highest point)

i.e. $(u \sin\theta)^2 = 2gs_y$

$$\boxed{S_y = \frac{(u\sin\theta)^2}{2g}} \quad \text{—— (2)}$$

(ii) Horizontal distance S_A

$S_A \quad U_x t$ —— (3)

$t = \frac{S_x}{u_x}$, i.e. $\boxed{t = \frac{S_x}{u\cos\theta}}$ —— (4)

from equation (1) and (4)

$$S_y = V_y\left(\frac{S_x}{u_0\cos\theta}\right) - \frac{1}{2g}\left(\frac{S_x}{u_0\sin\theta}\right)^2 \quad \text{—— (5)}$$

(iii) Time to reach maximum height

$V = u - gt$ ($v = u + at$)

$0 = u_0 \sin\theta - gt$

$$\boxed{t = \frac{u\sin\theta}{g}} \quad \text{—— (6)}$$

$$\boxed{\text{Total time} = \frac{2u\sin\theta}{g}}$$

(iv) Range of projectile

$$S_x = U_x t = (U_0 \cos\theta)(2 U_0 \sin\theta)$$

$$= \frac{U_0^2 \sin^2\theta}{g} = \text{horizontal range}$$

$$\boxed{R = \frac{U_0^2 \sin 2\theta}{g}}$$

(v) Ratio between $S_x = R$ and S_y

$$\frac{S_y}{S_x} = \frac{(U_0 \sin\theta)^2}{2g} \times \frac{g}{4\cos\theta} = \frac{\sin\theta}{4\cos\theta}$$

$$\boxed{\frac{S_y}{S_x} = \frac{\sin\theta}{4\cos\theta}}$$

Example 1

prove that the maximum range of projectile is achieved when its thrown at an angles 45°

Soln

$$R = \frac{U_0^2 \sin 2\theta}{g}$$

R is maximum when $2\theta = 90^\circ$

ie $\theta = \frac{90^\circ}{2} = 45^\circ$

Example 2.

A marble travelling at 100 cm/s rolles off the edge of a level table, if its hit the floor 30 cm away from the spot directly below the edges of the table, how high is the table?

Solen

$U_{ox} = 100$ cm/

For horizontal motion, S_{ox} $U_{ox} t$

$\Rightarrow 30 = 100t$

$\Rightarrow t = 0.3s$

For vertical motion $= 44.1$ cm

A projectile is lunch with muzzle with velocity of 2900 Km/h with an angle of 60^0 and land on the same place find

a- maximum highet

b- Total time of flight

c- find the range

d- the time to reach maximum height

e- Speed after 1 minute of flight

f- the Speed at the height of 10,000 Km

a- $S_y = \frac{(U_{ox} \sin\theta)^2}{2g} = 24.8$ Km

b- Total time $= \frac{2 u_0 \sin\theta}{g} = 71.11$ second

~~c- Range $R = \frac{u_0^2}{g}$~~

c- Range $R = \frac{u_0^2 \sin 2\theta}{g}$

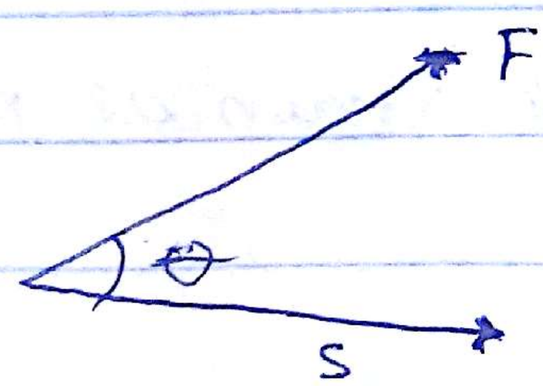
08/03/2017

⇐ Work energy and power ⇒

work involves (i) a force and (ii)

Consider a body that under goes in displacement by in a straight line when constant force F act on its

work $W = F \cdot S$ — $(\cos\theta)$



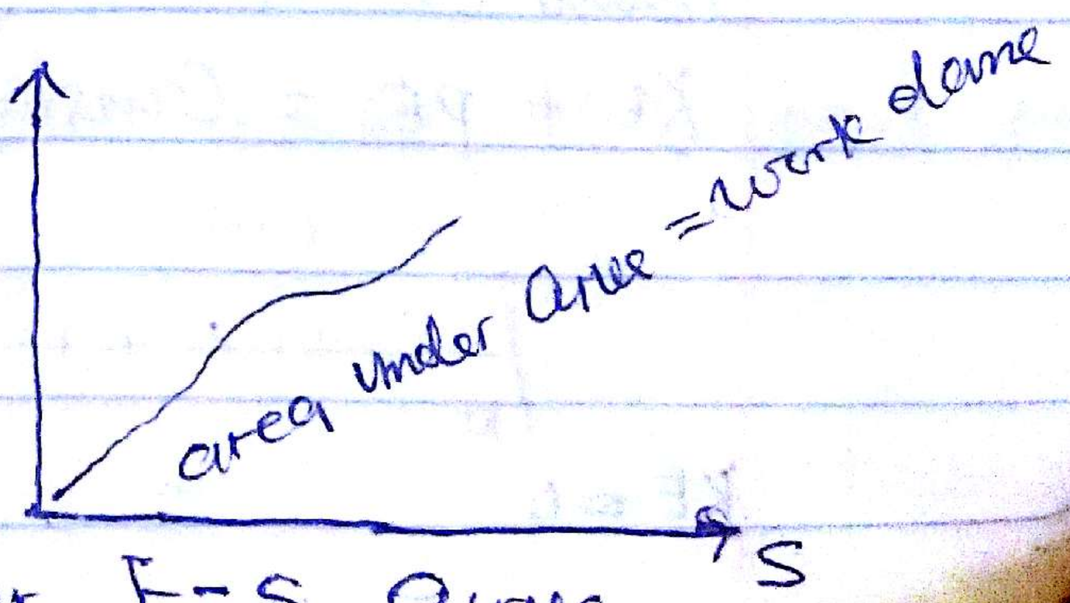
$W = F \cdot S \cos\theta$

the unit of W is Joul (J)

$1J = 1N \cdot 1m$

If force is variable

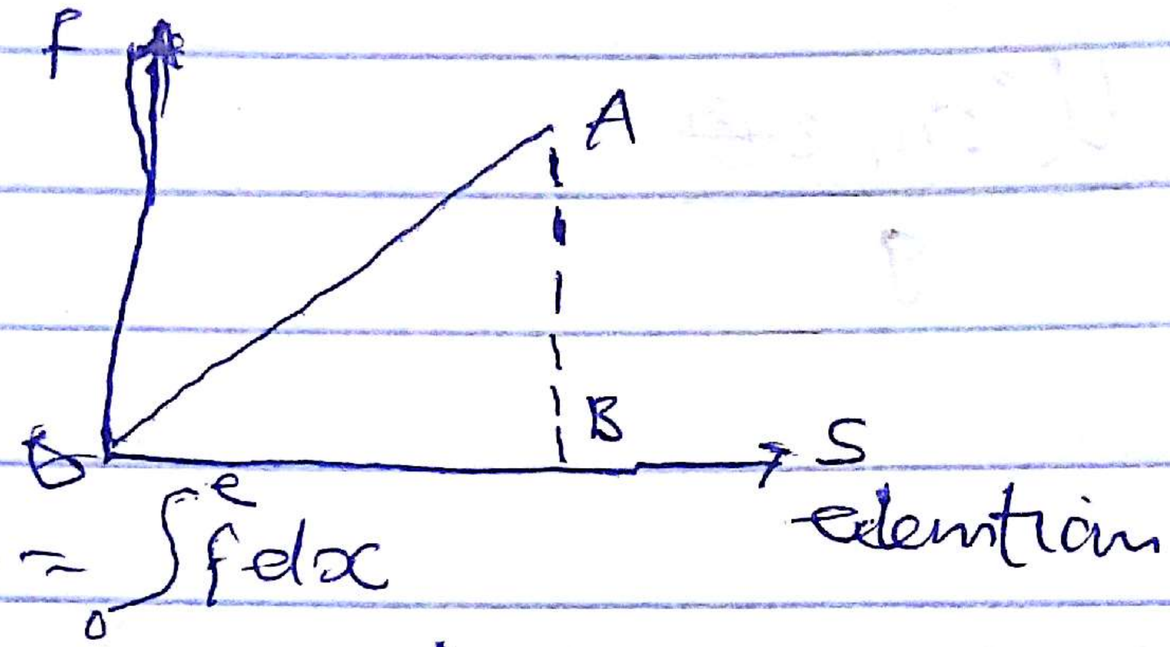
$W = \int_0^x F\,ds$



Work = area under F-S curve.

Eg

force required to extended a spring is directly proportional to the extention



$$W = \int_0^e f\,dx$$

= area of $\triangle$ OBA

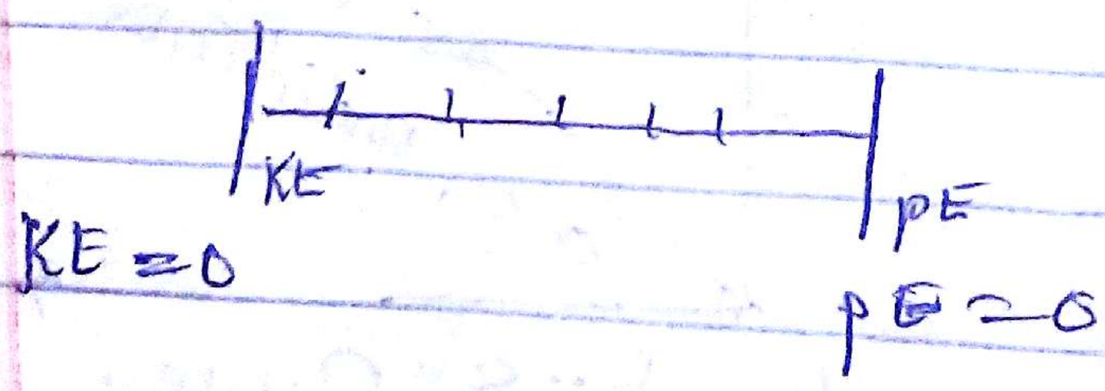
$$\boxed{W = \frac{1}{2} fe}$$

⇐ ENERGY ⇒

Is the ability to do work is known as energy.
The energy of an objects due to its motion position or physical condition is known as mechanical energy

* Mechanical energy can either be KE or PE
means (Kinetic energy) or (potential energy)

KE + PE = Constant



KINATIC ENERGY AND WORK ENERGY THEOREM ⇒

$$W = F \cdot S$$
$$= (ma)$$
$$V^2 = V^2 + 2as$$

If $u = 0$, $S = \frac{V^2}{2a}$

$$W = ma\left(\frac{V^2}{2a}\right) = \frac{1}{2}mv^2 = \text{Kinetic energy}$$

$$V_2^2 = V_1^2 + 2a_x s \Rightarrow a_x = \frac{V_2^2 - V_1^2}{2s}$$

$$F = ma_x = m\frac{V_2^2 - V_1^2}{2s}$$

$$W = F \cdot S = \frac{1}{2}mV_2^2 - \frac{1}{2}V_1^2 = KE_2 - K \cdot E_1$$

the work done by the net force from partele it equal change in a partele Kinetic energy

$$\boxed{W = K \cdot E_2 - KE_1}$$

The energy stored in a body because of its position is Known as potential energy.

A mass (m) raised through a hight (h) in a equal work against gravity.

Gain in gravitational potential = work done against gravity

$$= F h$$
$$= mgh$$

PRINCIPLE OF CONSERVATION OF ENERGY

The total energy of the closed system is always conserved, is a system where no external system forces act on bodies in the system.

Total energy $= K.E + PE =$ Constant

⇐ POWER ⇒

power is rate of which work is done or the rate at which energy is delivered.

A quantity of work Δw done during a time interval Δt,

The average work done per unit time or average power P_{av} is $P_{av} = \frac{\Delta w}{\Delta t}$ (average power)

When rate at which work is done is not constant ~~then~~ we power p as the quotient in ~~equation~~ the equation above as Δt approaches zero

$$p = \lim_{t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt}$$

$$\boxed{p = \frac{dw}{dt}}$$

$$\boxed{p = \frac{f.s}{t} = fv}$$

Unit of power is watt (w)

$1W = 1J/s$

A large Unit called horse power (hp) is also Used

$$\boxed{hp = 746w}$$

A fifty (50Kg) marathon runner runs up stairs to the top of 443m tall Sears tower, to left herself to the top in 15 minutes, what must be her average power output,

(i) watt

(ii) Kilowatt

(iii) horse power

Work she must do is

$W = (mg)h = (50kg)(9.8m/s)(443)$

$t = 15 \text{ minute} = 15 \times 60 = 900$

$$\text{Average power} = \frac{\text{work}}{\text{time}}$$

A Manufectural claims that the maximum power delivered by the engine of car of mass 1200Kg is 90Kw, find the maximum time in which the car will accelerate from rest to 30m/s

Sol

$m = 1200Kg, \; u = 0, \; v = 30m/s, \; p = 90Kw$

$w = f \cdot s = (ma)s$

$$V^2 = \cancel{V^2}^{\to 0} + 2as$$

$$s = \frac{1}{2}\frac{v^2}{a}$$

$$p = \frac{\text{work done}}{\text{time interval}} = \frac{mv^2}{2t}.$$

IMPULSE OF A AND CHANGE IN MOMENTUM

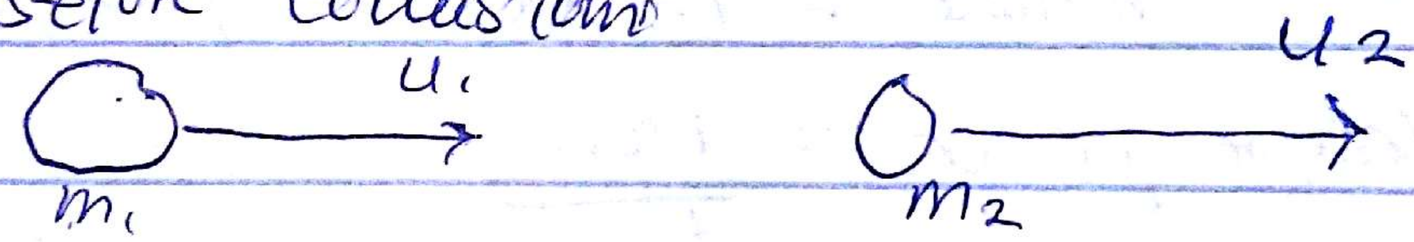
$$F = ma = m\left(\frac{V-u}{t}\right)$$

$$Ft = mv - mu = \text{Impulse of the force} = \Delta p$$

22 = 03 = 2017

Conversation of Linear momentum

Before Collision



After Collision

* Total momentum before Collision :- $m_1u_1 + m_2u_2$
* Total momentum after Collision :- $m_1v_1 + m_2v_2$

$\Longrightarrow$ When total momentum before Collision equals total momentum after Collision, momentum is said

to be conserved mathematically

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Newton law Restitution

When two bodies colides, relative velocity after collision = e x relative velocity before collision,

where e = Coefficient of restitution relative velocity is

$$V_1 - V_2 = -e(u_1 - u_2)$$

i ⇒ $e = 1$ for completly elastic collision

ii ⇒ $e = 0$ for completly elastic collision

iii ⇒ $0 < e < 1$ for partially Inelastic collision

Elastic collision :— is the one which total Kinetic energy after collision is equal that before collision.

Therefore Kinetic energy (K.E) and linear momentum and energy are conserved

for an Inelastic collision ($e=0$), the body coalines after collision (in the case Kinetic energy is not conserved but linear momentum and energy are conserved)

for elastic collision $e = 1$

$$V_1 - V_2 = U_1 + U_2 \longrightarrow \text{Newton law In restitution}$$

and $\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 + v_2}$

Example :- A body of mass 4.2 kg moving with a velocity of 10 m/s due east, hit a stationary body of mass 2.8 kg. If they stick together after collision and move with the velocity V due east calculate

$m_1 = 4.2 kg$ $u_1 = 10 m/s$

$m_2 = 2.8$ $u_2 = 0$

$V = ?$

equation

$m_1 u_1 = (m_1 + m_2) V$

Question 1

Net force $F = (3x^2 N) I$ with x in meters act on a particle changing only Kinetic energy (K.E) of the particle, how much work is done on the particle as its moves from coordinate (2m, 3m) to (3m, 0m)

Question 2

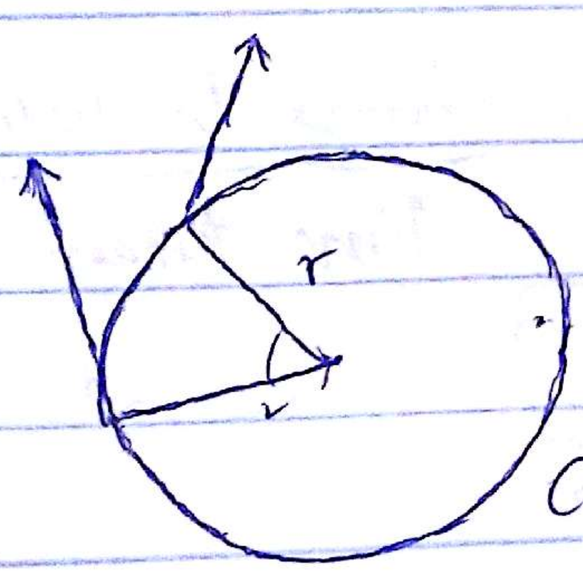
A girls whose mass is 55 kg stand on a spring weighting machines inside a acceleration is 2 m/s. What will be the reading on the machine ?

(take $g = 10 m/s$)

Question 3:

A Uniform bar is 15m long is balanced on a pivot placed its midpoint. A boy of mass 55Kg sit on one arm of the bar at point 5m away from the pivot. what mass can be placed 2m away from the other ends of the bar to keep the bar horizontal

CIRCULAR MOTION



When a particle move along a ~~circle~~ circular path, the velocity continuously change direction but the magnitude remain constant.

When a particles moves in a circle with a constant speed or velocity the motion is called Uniform circular motion

arc $\underline{\quad s \quad} = r\theta$ angular distance

$$\frac{s}{t} = \frac{r\theta}{t}$$

$\boxed{V = r\omega}$ → is velocity in any instant and its always tangential to the circle and its known as the cartesian velocity of the particle

$$\frac{du}{dt} = r\frac{dw}{dt}$$

Where $a = \frac{dv}{dt}$ = tangentical acceleration

$\alpha = \frac{dw}{dt}$ = angular acceleration

Centripetal acceleration : An Object In Uniform - Circular motion experiences centripetal acceleration which is the acceleration directed toward the centre of circle.

Centripetal acceleration : $\frac{\text{change In velocity}}{\text{time taken}} = \frac{\Delta t}{\Delta t}$

is $\boxed{a = vw}$

but $v = rw$

$$a = vw = (rw)w = rw^2 = \frac{v^2}{r}$$

$$\boxed{a = \frac{v^2}{r}}$$

Centripetal acceleration $F = ma = m(vw) = mrw^2 = m\frac{v^2}{r}$

$$\boxed{F = m\frac{v^2}{r}}$$

Suppose particles goes round once In the time T

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T}$$

The time (T) taken for the object to move a complete rotation is known as its period.

$$r\omega = \frac{2\pi r}{T} \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$$

angular Frequency

frequency of rotation $F = \frac{1}{T} = \frac{1}{2\pi\omega}$

$$\boxed{\omega = 2\pi f}$$

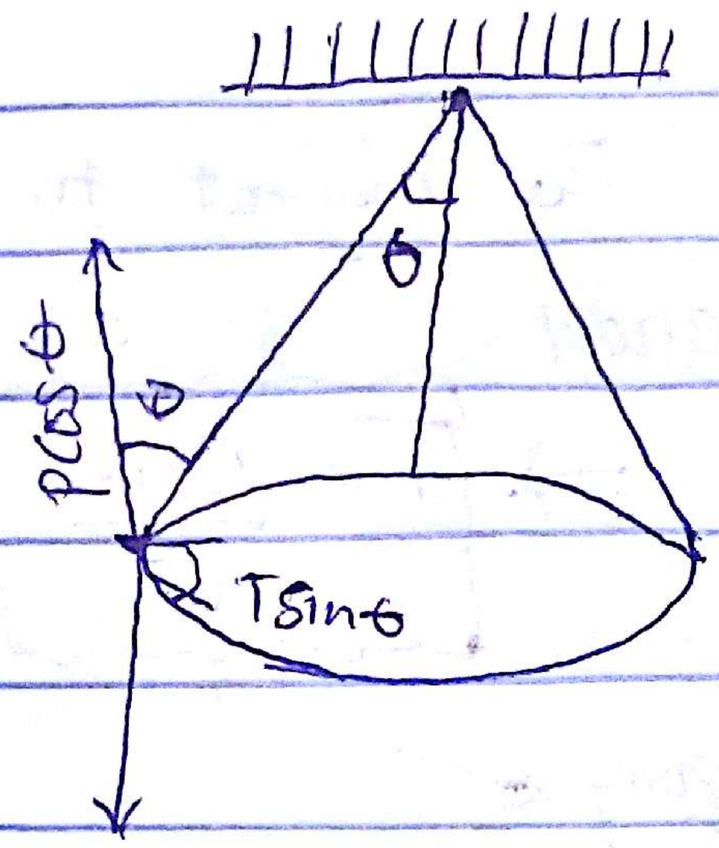
⟹ equation of motion with constant angular acceleration compared with linear

Linear	Angular
$a = \frac{v-u}{t}$	$\alpha = \frac{\omega_2 - \omega_1}{t}$
$v = u + at$	$\omega = \omega_2 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_2 t + \frac{1}{2}\alpha t^2$
$u^2 = v^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

29/03/2017

[THE CONICAL PENDULUM]

A pendulum both attach to string l and made to revolve in a horizontal circle of radius (r)



$\Sigma f_y = 0$

$\Sigma f_x = ma = mr^2 = T\sin\theta$

$= \frac{mr^2}{r}$

$w = mg$

$\Sigma f_y = 0 \Rightarrow T\cos\theta - mg = 0 \quad (1) \div (2)$

$\frac{T\sin\theta}{T\cos\theta} = \frac{mr^2}{r\,mg}$

$\boxed{\tan\theta = \frac{r^2}{rg}}$

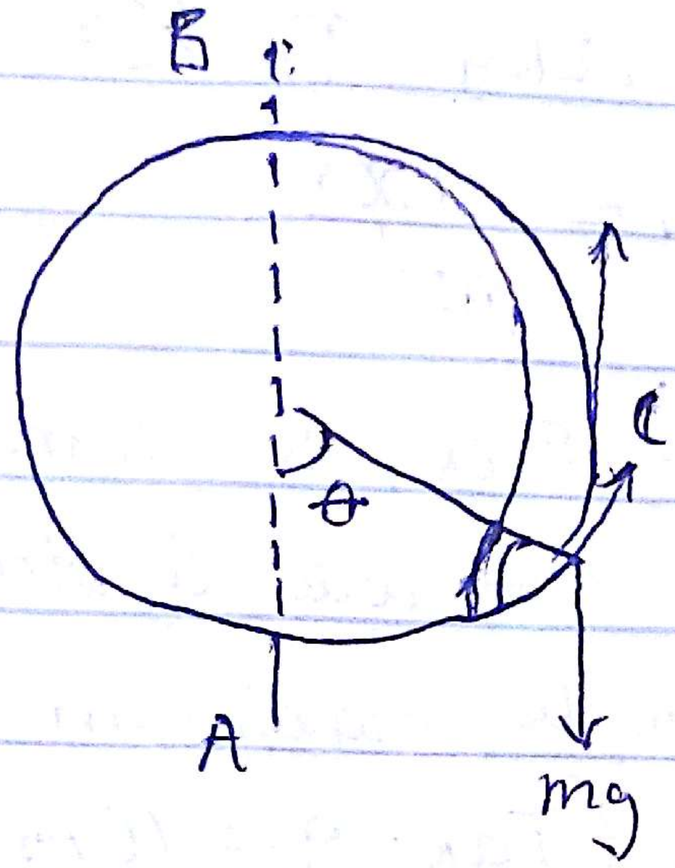
$r = l\sin\theta$

MOTION IN HORIZONTAL CIRCLE

A can is whirled on the end of a string round the horizontal circle (neglecting gravity) If the speed V is constant and r is the radius of the circle, then the tension T is $\boxed{T = \frac{mr^2}{r}}$

If the spring break, the tension T disapear and there is no more centrapetal force then, therefore fly off at a tangent to the circle with velocity = Instantenous Velocity

MOTION IN A VERTICAL CIRCLE



A pail of water whirled at the end of a ~~circle~~ rope in a vertical circle, the tension T in the rope radius with the position

$$F = ma = \frac{mr^2}{r}$$

Net force toward centre of Circle O

$T - mg\cos\theta$

$T - mg\cos\theta = \frac{mr^2}{r}$ $\quad$ v = speed of pail

r = radius of the circle

$$\boxed{T = \frac{mr^2}{r} + mg\cos\theta}$$

(i) At point A when $\theta = 0$ $\quad T = m\left(g + \frac{r^2}{r}\right)$

(ii) At point B when $\theta = 180°$ $\Rightarrow T = \left(m\left(-g + \frac{r^2}{r}\right)\right) = m\left(\frac{r^2}{r} - g\right)$

(iii) At C when $\theta = 90°$ $\quad T = \frac{mr^2}{r}$

Example 1

A box of mass 0.22kg revolve uniformally in a circle of radius 0.2m. If the box makes 2 revolution per second, what is the centrapetal force exerted on it by the cord

Soln

$F = ma = \frac{mr^2}{r} = mrw^2 = mrw$

$r = 0.2m$, $m = 0.22Kg$ $T = \frac{1}{2}s = 0.5s$

$T = \frac{m4\pi^2 r}{T^2}$ $V = \frac{4\pi r}{2T}$

Example 2. An Object of mass 8.0Kg is whirled in a Vertical Circle of radius $r = 2m$ with Constant speed of $V = 6m/s$. Calculate the maximum and the minimum tension in the Spring (Take $g = 10m/s^2$

Soln

$T = m\left(\frac{v^2}{r} - g\right)$

$T = m\left(\frac{v^2}{r} + g\right)$

Example 3. A ball is dropped on the flew from the height of 1.5m bounce back 0.85m

Soln

$V_1 - V_2 = e(V_1 - V_2)$

$V_1^2 = V^2 + 2gh$ ——— (1)

$V_2^2 = V_2^2 - 2gh$ ——— (2)

flow does not move so $V_2 = V_2 = 0$

Consider up and does

$u_1 = -\sqrt{2gh}$ $e = \frac{V_1 - V_2}{u_1 - u_2} = \frac{0 - V_1}{u - 0}$

$V_1 = +\sqrt{2gh}$

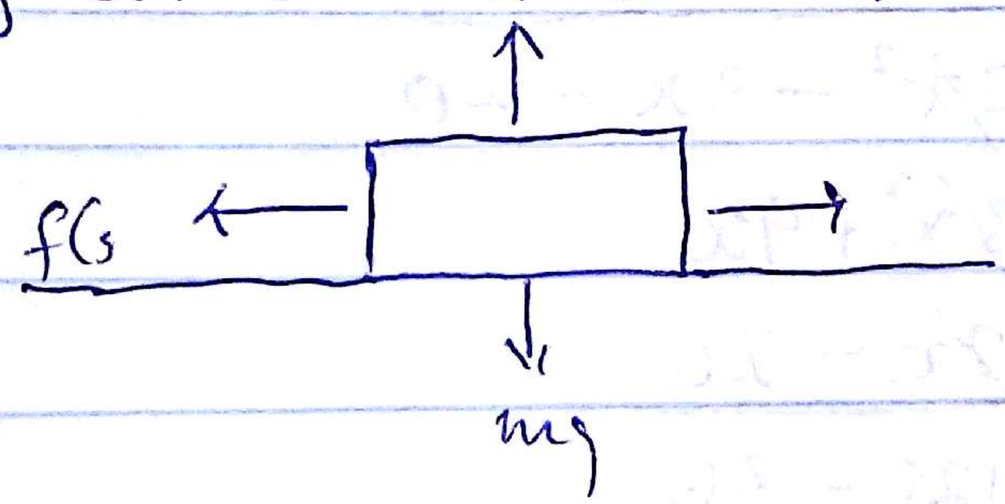
A passenger travel at Constant Speed with a circle of radius of 5m, if they make one Complete Circle in of Second, what is the acceleration

$$v = \frac{2\pi r}{T}, \quad a = \frac{r^2}{r} = \frac{4\pi^2 r}{T^2}$$

friction

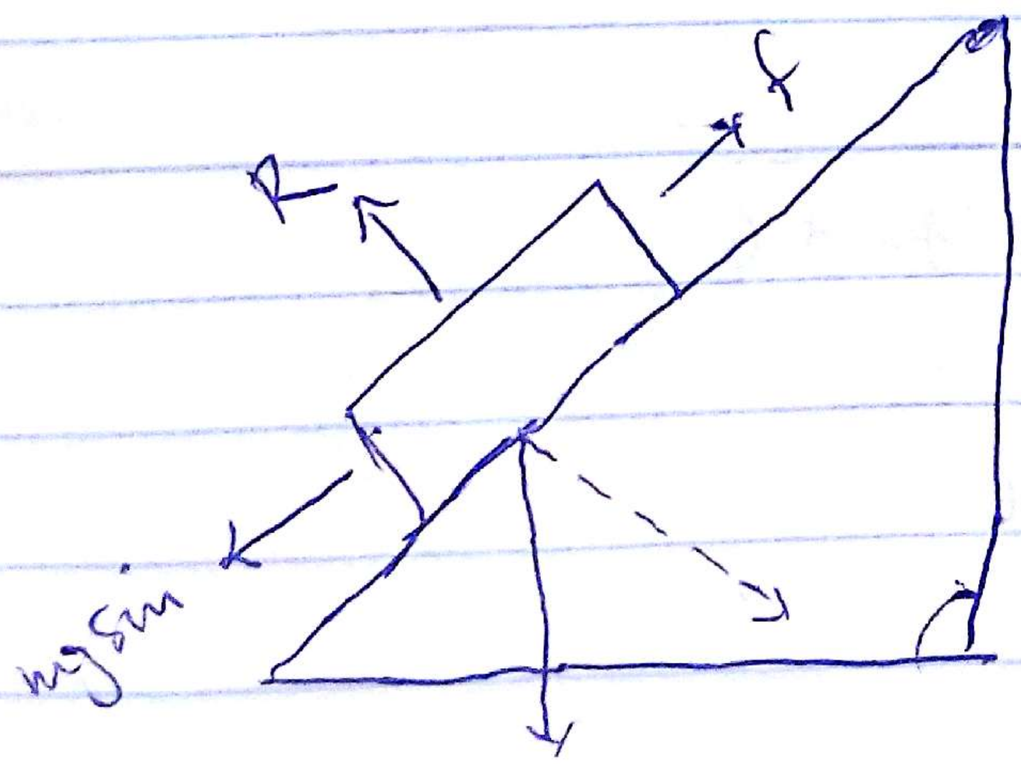
There are two types → Static, → Dynamic

The force of resistance coming to play as a result of motion of one body on another is called



The ratio

$\frac{F_s}{R}$ = Coefficient of Limiting friction $= \mu = \frac{f_s}{R}$



$$\mu = \frac{f_s}{R} = \frac{mg\sin\theta}{mg\cos\theta}$$

θ = Limiting angle = angle of friction

= NO angle

GRAVITATIONAL

Newtons law of gravitation :

$f \propto m_1 m_2$ $\qquad$ $f \propto \frac{m_1 m_2}{r^2}$

$f \propto \frac{1}{r^2}$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

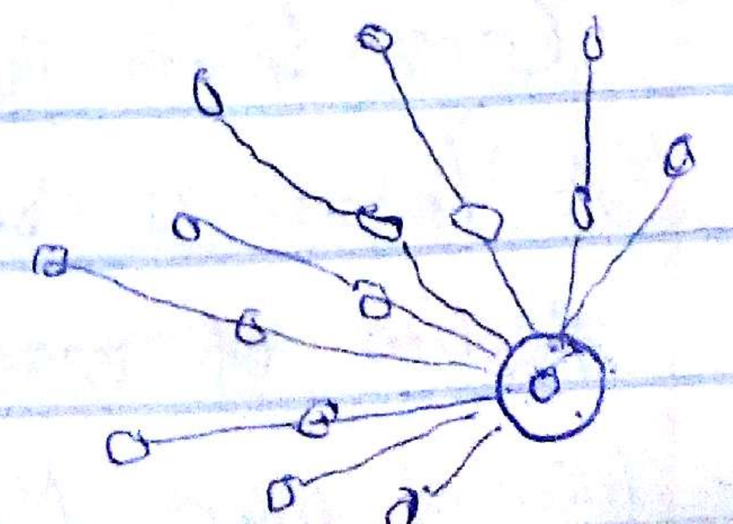
$$G = \frac{F r^2}{m_1 m_2} = \frac{N m^2}{kg^2} = N m^2 kg^{-2}$$

$$\text{Dimension of } G = \frac{(MLT^{-2})L^2}{M^2} = M^{-1} L^3 T^{-2}$$

SUPERPOSITION OF GRAVITATIONAL FORCE

Many star in the sky are system of two or more bodies held togather by gravitational attraction force.

When two or more body exotent gravitational force on a particular body, the total gravitational force on that body is a vector sum of all the forces as exerted by the other



Symetric ⇒ due

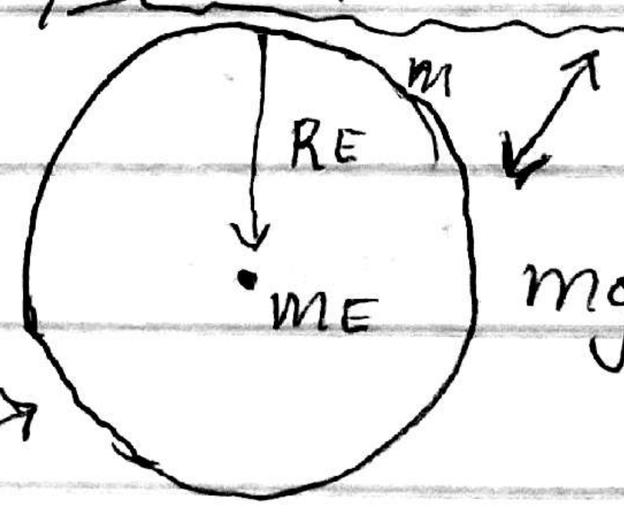
The weight of the body is the total gravitational force exerted on the body by all other bodies in the Universe, Consider the earth spherically symetric with radius R_E and mass M_E, mass of the small body m at the earth surface (the distance R_E from the earth Centre.

mass acceleration due to gravity

$$\boxed{W = mg} = \frac{G M_E m}{R_E^2}$$

$$\boxed{g = \frac{G M_E}{R_E^2}}$$

mass of earth



$$mg = \frac{G M_E}{R_E^2}$$

weight of body at earth surface

$$\boxed{M_E = \frac{g R_E^2}{G}} \quad g \simeq \frac{9.8 (m/s^2) \times (6.38 \times 10^6)^2 m^2}{6.67 \times 10^{-11} Nm^2 kg^{-2}}$$

$$\simeq 5.97 \times 10^{24} Kg$$

Density of earth

volume of earth $\boxed{V_E = \frac{4\pi}{3} R_E^3}$

Density of earth $\rho_E = \frac{m_E}{V_E} = \frac{g R_E^2 / G}{\frac{4}{3}\pi R_E^2} = \frac{3g}{4\pi G R_E} = \boxed{5500 Kg/m^3}$

Gravity on planet mars?

A lander is sent to mars

Radius of mars $R_m = 3.4 \times 10^6 m$

Mass of mars $M_m = 6.42 \times 10^{23} kg$

If the earth weight of lander is 3920 N, Calculate the lander weight on mars

mass of lander $m = \frac{W}{g} = \frac{3920N}{9.8 m/s^2}$

mass is the same weather lander is on earth or mars In between $F_g = \frac{GMmM}{r^2} = \frac{GMmM}{R_m + h} \simeq 194N$

The acceleration due gravity of mars at this point

$$\boxed{g_m = \frac{F_g}{m}} \simeq \frac{194}{400} \simeq 0.4 m s^{-2}$$

Variation of acceleration due to gravity g

At earth surface $g = \frac{GM_E}{R^2_E}$

At altitude h above earth surface $g_h = \frac{GM_E}{(R_E+h)^2}$

$$\frac{g_h}{g} = \frac{R_E^2}{(R_E+h)^2} \quad \text{or } g_h = \frac{gR^2_E}{(R_E+h)^2} \simeq g\frac{1}{\left(1+\frac{h}{R_E}\right)} = g\left(1-\frac{2h}{R_E}\right)$$

g decreases as we go above earth surface

At a depth of d below the earth surface if we consider the earth as a homogeneous sphere of Uniform density ρ, it can be shown that the depth of d, their acceleration due to gravity g is $g = g\left(1-\frac{h}{R_E}\right)$

g decreases with depth from earth surface and consider zero at centre.

GRAVITATIONAL POTENTIAL

The work done by the gravitational attraction to ~~bring~~ bring a unit mass from Infinity to 0 Zero point of Reference is the gravitational potential at that point V (or U)

$U =$ Gravitational potential $= -\frac{Gm}{r} = \boxed{U = -\frac{Gm}{R_E}}$

U is never positive, and it is zero when two bodies parallel

$$U = \frac{Gm}{R_E} = -\frac{gR_E^2}{R_E} = -gR_E \quad \boxed{U = gR_E}$$

ESCAPE VELOCITY

To take an object from the surface of the earth to Infinity, the same amount of energy is required.

The lunch a rocket of a mass (m) from the earth Surfeace, So that it escapes Completely from the earth gravitational attraction, the rocket must take off from earth with $K.E \geq \frac{GM_E m}{R_E}$ $\quad \frac{1}{2}mv^2 \geq \frac{GM_E m}{R_E}$

$$\boxed{U \geq \sqrt{\frac{2GM_E}{R_E}}}$$

The least speed required for an Object to escape Completely from the surface of the earth (and never to Come back) is Known as escape Velocity

$$\boxed{U = \sqrt{\frac{2GM_E}{R_E}}} \qquad GM_E = gR_E^2 = \boxed{U = \sqrt{2gR_E}}$$