CSC 2323.

Course Outline.

→ propositional logic and predicate logic

→ Elementary Number theory and methods of proof.

→ Sequences, mathematical induction and recursion.

→ Relations

→ Matrices.

Distribution law $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Double negation law $\sim(\sim p) \equiv p$

De morgan's law $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

proposition logic

Proposition/statement.

Definition: Declarative sentence that is true or False b... not both.

Concept:

1) Abuja is the Capital of Nigeria. → True.

2) $2+2=4$ → True $1+1=3$ → False.

3) Do your homework (not proposition)

4) $x+1=6$ (Not a proposition)

Compound Statement

let p, q, r ... to denote propositions P_1 P_2 P_3 ----

Examples

It is hot and ~~but~~ not Sunny. → P and q

It is hot → P

It is not Sunny → q   P

Basic logical Operation.

1 Conjunction.

let P and q be propositions, Then Conjunction of P and q denoted by $(P \land q)$ is the proposition "P and q"

NOTE * The Conjunction $P \land q$ is true when both P and q is true and is false when both P and q is false.

6. Show...

P

T

F

7.

6. Show that $(p \wedge \sim p)$ is contradiction:

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

∴ Therefore $(p \wedge \sim p)$ is contradiction

7. If t is tautology and c is a contradiction, show that $p \wedge t \equiv p$ and $p \wedge c \equiv c$

p	t	$p \wedge t$
T	T	T
F	T	F

∴ therefore $p \wedge t \equiv p$.

p	c	$p \wedge c$
T	F	F
F	F	F

∴ $p \wedge c \equiv c$

Theorem 1.1 Given any statement variables p, q and r, a tautology t and a contradiction c, the following logical equivalences hold -

1) Commutative law $p \wedge q \equiv q \wedge p$ $\quad p \vee q \equiv q \vee p$
2) Associative law $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ $\quad p \vee (q \vee r) \equiv (p \vee q) \vee r$

8 Use theorem 1.1 to verify the logical equivalence of

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$$

De morgan's law $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$.

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$ by D.Lo

$\equiv (p \vee \sim q) \wedge (p \vee q)$ by Double negation la

$\equiv p \vee (\sim q \wedge q)$ by Distribute law

$\equiv p \vee (q \wedge \sim q)$ by Cummutative law

$\equiv p \vee c$ by Negation law

$\equiv p$ by Identity law

Therefore $\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$.

9. Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logica equivalent:

$\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are mutually equivalent.

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge (\sim(\sim p) \vee \sim q)$

* $\equiv \sim p \wedge (p \vee \sim q)$

$\equiv \sim p \wedge \sim q \vee p$

$\equiv \sim p \wedge \sim q \vee c$

$\equiv \sim p \wedge \sim q$

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$ by D.L

$\equiv \sim p \wedge (\sim(\sim p) \vee \sim q)$ by D.L

$\equiv \sim p \wedge (p \vee \sim q)$ by Double negation

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ by Distrible law

$\equiv (p \wedge \sim p) \vee (\sim p \wedge \sim q)$ cummutative law

$\equiv c \vee (\sim p \wedge \sim q)$ by negation law

$\equiv (\sim p \wedge \sim q) \vee c$ by Cummutative law

$\equiv \sim p \wedge \sim q$ by identity

CONDITIONAL:

Let p and q be propos

called CONDITIONAL STATE

$p \to q$ is false when

Otherwise. (when p is F

Truth Table of

p	q
T	F
T	F
F	T
F	F

Example: If you scor

Let p be you

q be I giv

Example: Construct a t

$\sim p$

p	q	$(p \vee \sim$
T	T	
T	F	
F	T	
F	F	

CONDITIONAL:

Let p and q be propositions. the statement "if p then q" is called CONDITIONAL STATEMENT and it is denoted by $p \to q$.

$p \to q$ is false when p is true and q is false, and true otherwise. (when p is false and T is true or when both are false

Truth Table of $(p \to q)$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$\to$

$\leftrightarrow$

Example: If you score p 100%, I will give q you A.

Let p be you score 100%

q be I give you A

Example: Construct a truth table for the statement $(p \vee \sim q) \to \sim p$

p	q	$(p \vee \sim q)$	$(\sim p)$	$((p \vee \sim q) \to \sim p)$	$\sim q$
T	T	T	F	F	F
T	F	T	F	F	T
F	T	F	T	T	F
F	F	T	T	T	T

$p \to q \equiv \sim p \vee q \to$ Conditional disjuction equivalen[ce]

Example 2 Show that $p \to q \equiv \sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore p \to q \equiv \sim p \vee q$

3. Show that $\sim(p \to q)$ and $(p \wedge \sim q)$ are logically equivalent.

$\sim(p \to q) \equiv \sim(\sim p \vee q)$ by C.d.E.L

$\equiv \sim(\sim p) \wedge \sim q$ by D.L

$\equiv p \wedge \sim q$ by Double N.L

Therefore $\sim(p \to q) \equiv (p \wedge \sim q)$

4. Show that $(p \wedge q) \to (p \wedge q)$ is a tautology.

p	q	$p \wedge q$	$p \wedge q$	$(p \wedge q) \to (p \wedge q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	F	T

Contrapositive:

If p then q

$p \to q$

$\sim q \to \sim p$

Eg: Write each of the

form: a) If John c

can swim to the

b) If Today is Sunday

Solution:

Let p be today

q be tomm

Symbolically,

b) p

the contrapo

$\sim p$ tod

$\sim q$ to

$\therefore \sim q \to$

If tomorro

Conve

1) The conve

2) The inverse

Example: Converse:

Inverse: If

day.

Contrapositive:

If p then q

$p \to q$

$\sim q \to \sim p$

Eg. Write each of the following statements in contrapositive form: a) If John can swim across the lake, then John can swim to the island.

b) If Today is Sunday, then tomorrow is monday.

Solution:

Let p be today is sunday

q be tommorrow is monday.

Symbolically,

(b) $p \to q$

the contrapositive of $p \to q$ is $\sim q \to \sim p$

$\sim p$ today is not sunday

$\sim q$ tommorrow ~~today~~ is not monday.

$\therefore \sim q \to \sim p$

If tomorrow is not monday then today is not Sunday.

Converse and Inverse.

1) The converse of $p \to q$ is $q \to p$

2) The inverse of $p \to q$ is $\sim p \to \sim q$

Example; Converse: If tommorow is monday, then today is Sunday

Inverse: If tomorrow is not monday, then today is not Sunday.

$p \to q$

T F F

F T T

Biconditional:

Let p and q be propositions.

$p \longleftrightarrow q$ ie p iff q is reffered to as ~~biconditional~~ biconditional of p and q.

Truth table of $p \leftrightarrow q$.

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

~~Example: [illegible]~~

* Example 1.22

Arguments:

Try these.

1. $(p \vee q) \wedge r$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

② $\sim p \wedge (q \vee$

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	[illegible]
F	F	[illegible]
F	F	[illegible]

Try these.

1. $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

2. $\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim p$	$\sim r$	$q \vee \sim r$	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	F	T	F
T	T	F	F	T	T	F
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

Law of Algebra of logic

1. Impodent law

$P \vee P \equiv P$

$P \wedge P \equiv P$

ii. Associative law

$P \vee (q \vee r) \equiv (P \vee q) \vee r$

$P \wedge (q \wedge r) \equiv (P \wedge q) \wedge r$

iii. Cummutative law

$P \vee q \equiv q \vee P$

$P \wedge q \equiv q \wedge P$

iv) Distributive law

$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$

$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$.

v) Involution law / contradiction law.

$\sim(\sim P) \equiv P$

vi) Identity law.

$P \vee F \equiv P$ | $P \wedge F \equiv P$

$P \vee T \equiv P$ | $P \wedge T \equiv P$.

vii) Complement law.

$P \vee \sim P \equiv T$ || $P \wedge \sim P \equiv F$.

viii) De-morgan's law

$\sim(P \vee q) \equiv \sim P \wedge \sim q$

$\sim(P \wedge q) \equiv \sim P \vee \sim q$

Proving by l

1. Impodent lau

$P \vee P \equiv P$

$P \wedge P \equiv P$.

p	p	p∨
T	T	T
T	T	T
F	F	F
F	F	F

$\therefore P \vee P \equiv$

ii. Associative

$P \vee (q \vee r)$

p	q
T	T
T	T
T	F
T	F
F	T
F	T
F	F
F	F

$\therefore P \vee (q$

Proving by law of Algebra logic :

1. Impodent law

$p \vee p \equiv p$

$p \wedge p \equiv p$.

p	p	p ∨ p	p	p	p ∧ p
T	T	T	T	T	T
T	T	T	T	T	T
F	F	F	F	F	F
F	F	F	F	F	F

$\therefore p \vee p \equiv p$ // $p \wedge p \equiv p$.

2. Associative law

$p \vee (q \vee r) \equiv (p \vee q) \vee r$.

p	q	r	q ∨ r	p ∨ (q ∨ r)	p ∨ q	(p ∨ q) ∨ r
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	F

$\therefore p \vee (q \vee r) \equiv (p \vee q) \vee r$

3) Commutative law :

$p \wedge q \equiv q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

4. Distributive law

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$p \vee (q \wedge r)$	$q \wedge r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F
T	F	T	T	T	T	T	F
T	F	F	T	T	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F

Therefore, $p \vee (q \wedge r) \equiv \underline{(p \vee q)} \wedge (p \vee r)$

$p \to q \not\equiv q \to p$

* Construct a truth ta

$(p \vee q) \to (p \wedge q$

p	q	$p \vee$
T	T	T
T	F	-
F	T	-
F	F	

ii) $(p \to q) \vee ($

p	q
T	T
T	T
T	F
T	F
F	
F	
F	
F	

$p \Rightarrow q \neq q \Rightarrow p$

* Construct a truth table of the following compound statement.

$(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

ii) $(p \rightarrow q) \vee (\sim p \rightarrow r)$

p	q	r	$\sim p$	$p \rightarrow q$	$\sim p \rightarrow r$	$(p \rightarrow q) \vee (\sim p \rightarrow r)$
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	F	T	T
T	F	F	F	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Therefore, $(p \rightarrow q) \vee (\sim p \rightarrow r)$ is a Tautology.

Order of Operations

1. $\sim$ 2. $\wedge$ 3. $\vee$ 4. $\rightarrow$ 5. $\leftrightarrow$

Arguments

Let $P_1, P_2, P_3 - - - P_n$, Q be proposition.

$\underbrace{P_1, P_2, P_3 - - - P_n}_{\text{Premise}} \vdash \underbrace{Q}_{\text{Conclusion}}$

* Arguments of the form is said to be valid if Q is true whenever P_i's are true.
* If argument is not valid, we say it is FALLACY.

Example: Determine the validity of the following arguments.

If Socrates is a man, then Socrates is mortal.
Socrates is a man.
$\therefore$ Socrates is mortal. – Q (Conclusion).

Let p be socrates is man
q be socrates is mortal

In symbols:

If p then q ($p \rightarrow q$)

$p \rightarrow q$
p
$\therefore q \rightarrow Q$(conclusion)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example 2:

P_1 $p \rightarrow q \vee$
P_2 $q \rightarrow p \wedge$
Q $\therefore p \rightarrow r$
~~p q~~

p	q	r	$\sim r$	$q \vee \sim r$
T	T	T	F	T
T	T	F	T	T
T	F	T	F	F
T	F	F	T	T
F	T	T	F	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

The argument is FAL
when is Conclusi

Example 3: Generali

(a) p
$\therefore p \vee q$

Sol.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example 2: Determine the validity of

P_1 $p \to q \vee \sim r$

P_2 $q \to p \wedge r$

Q $\therefore p \to r$ → conclusion

p q r ~r q∨~r p∧r

p	q	r	~r	q∨~r	p∧r	p→q∨~r	q→p∧r	p→r	
T	T	T	F	T	T	T	T ✓	T ✓	✓
T	T	F	T	T	F	T	F	F	
T	F	T	F	F	T	F	T	T	
T	F	F	T	T	F	T	T ✓	F *	X
F	T	T	F	T	F	T	F	T	
F	T	F	T	T	F	T	F	T	
F	F	T	F	F	F	T	T ✓	T ✓	✓
F	F	F	T	T	F	T	T ✓	T ✓	✓

The argument is FALLACY since all the argument in $p \to r$ which is conclusion are not all true.

Example 3: Generalization: Valid

(a) $\dfrac{p}{\therefore p \vee q}$ (b) $\dfrac{q}{\therefore p \vee q}$

Sol.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

q	p	p∨q
T	T	T
F	T	T
T	F	T
F	F	F

4) Specialization : Valid

a) $\dfrac{p \wedge q}{\therefore p}$ (b) $\dfrac{p \wedge q}{\therefore q}$

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

5) Elimination : Valid-

(a) $p \vee q$ P_1
$\sim q$ P_2
$\therefore p$ Q

(b) $p \vee q$ P_3
$\sim p$ P_4
$\therefore q$ Q

p	q	~q	~p	p ∨ q	
T	T	F	F	T	P_2
T	F	T	F	T	
F	T	F	T	T	P_4
F	F	T	T	F	

6) Transitivity: Valid.

P_1 $p \to q$
P_2 $q \to r$
Q $\therefore p \to r$

p	q	p → q	q → r	p → r
T	T	T	T	
T	F	F	T	
F	T	T	F	
F	F	T	T	

p
T
T
T
T
F
F
F
F

∴ Th

7) Dete

soln

p	q
T	T
T	F
T	T
T	F
F	T
F	F
F	T
F	F

			P_1	P_2	Q	
p	q	r	$p \to q$	$q \to r$	$p \to r$	
T	T	T	T	T	T	
T	T	F	T	F	F	∴TH
T	F	T	F	T	T	
T	F	F	F	T	F	
F	T	T	T	T	T	
F	T	F	T	F	T	
F	F	T	T	T	T	
F	F	F	T	T	T	

∴ The argument is valid since $P_1, P_2 \vdash Q$.

7) Determine the validity of the following.

x is positive or x is negative. $\quad p \vee q \quad P_1$

If x is positive then $x^2 > 0 \quad p \to r \quad P_2$

If x is negative then $x^2 > 0 \quad q \to r \quad P_3$

$\therefore x^2 > 0 \quad \therefore r \quad Q$.

Soln: Let p be x is positive, q be x is negative, r be $x^2 > 0$

p	p	q	r	$p \vee q$	$p \to r$	$q \to r$	r
		T	T				
	T	F	F				
	T						
	T	F	T				
	T	F	F				
	F	T	T				
	F	F	F				
	F	F	T				
	F	F	F				

p	q	r	$p \vee q$ (P_1)	$p \to r$ (P_2)	$q \to r$ (P_3)	r (Q)
T	T	T	T	T	T	T ✓
T	T	F	T	F	F	F
T	F	T	T	T	T	T ✓
T	F	F	T	F	T	F
F	T	T	T	T	T	T ✓
F	T	F	F	T	F	F
F	F	T	T	T	T	T ✓
F	F	F	F	T	T	F

∴ The argument
$p \vee q$ P_1
$p \to r$ P_2 . $P_1, P_2, P_3 \vdash Q$.
$q \to r$ P_3
$\therefore r$ Q is VALID.

Theorem:
$P_1, P_2, P_3 \ldots P_n \vdash Q$ is valid.
iff $P_1 \wedge P_2 \wedge \ldots \wedge P_n \to Q$ i.e. it is a tautology.

Example: Transitivity
P_1 $p \to q$
P_2 $q \to r$
$\therefore p \to r$

Soln
$(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

p	q	r	$p \to$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	-
F	F	T	T
F	F	F	T

$\therefore (p \to q) \wedge (q \to r) \to (p$

QUANTIFIE

Universal Q

A Proposition

Example: x

$A = \{x \mid x$

$A = \{x \mid x$

$\Rightarrow \forall$ "for

Exam Find Tp o

Let
a) $P(x)$ be, $x +$

b) let $P(x)$ be

c) let $P(x)$ be

$\forall x \in$

p	q	r	$p \to q$	$q \to r$	$(p\to q)\wedge(q\to r)$ Y	$p \to r$ Z	$Y \to Z$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore (p\to q)\wedge(q\to r) \to (p \to r)$ is a tautology.

QUANTIFIERS -

Universal Quantifier -

A Propositional function is defined as $P(x)$.

Example: $x + 2 > 10$

$A = \{x \mid x > 8,\ x \in \mathbb{Z}\} = T_p$.

$A = \{x \mid x > 8,\ x \in \mathbb{R}\} = T_p$.

$\Rightarrow \forall$ "for all" or "for every" $\forall \Rightarrow$ (universal quan)

Exa: find T_p of $P(x)$ ~~between~~ defined on the set of $\mathbb{N}$

(a) let $P(x)$ be, $x + 2 > 7$. $T_p = \{x \mid x > 5,\ x \in \mathbb{N}\}$

b) let $P(x)$ be $x + 5 < 3$ $T_p = \{\}$ or $\emptyset$

c) let $P(x)$ be $x + 5 > 1$ $T_p = \mathbb{N}$

$\forall x \in \mathbb{N}$, $P(x)$ is true.

Example

a) $\forall x \in \mathbb{N}\ (x+4 > 3)$ is true since $T_p = \mathbb{N}$.

b) $\forall x \in \mathbb{N}\ (x+2 \leq 8)$ is false since $T_p = \{x \mid x \in \mathbb{N} / x \leq 6\}$

Existential Quantifier:

$\exists$ to denote.

(there exists) or (for some)

$\exists x \in A,\ p(x)$

Example: a) The proposition $(\exists x \in \mathbb{N})(x+4 < 7)$

b) The proposition $(\exists x \in \mathbb{N})(x+6 < 4)$

Soln

a) The proposition $(\exists x \in \mathbb{N})(x+4) <$) is true since $\{x \mid x \in \mathbb{N}, x < 3\}$.

b) The proposition is $(\exists x \in \mathbb{N})(x+6 < 4)$ is false cos $\{x \mid x \in \mathbb{N},\ x+6 < 4\} = \emptyset$

$x > 5$ $T_p = \{x / x > 5, x \in \mathbb{N}\}$

1 (w 20 $\in \mathbb{N}$ $\mathbb{Z}$ R

Number Theory

Definition: n is e

n is

Direct proof:

use definition

a) is 0 even?

b) is -301 odd

c) if a and b a

d) if a and b ar

e) if every intege

Solution.

a) is 0 even?

$0 = 2(0)$,

$\therefore$ 0 is eve

b) $-301 = 2(-151)$

$\therefore -301$ is o

c) ~~$6a^2b$ = for~~

Example

$7a^5$ ~~term~~

e since $T_P = \mathbb{N}$.

e since $T_P = \{x \mid x \in N / x\ 8\}$

e)

$\in \mathbb{N})(x+4<7)$

$\in N)(x+6<4)$

$N)(x+4) <\ )$ is true since

$N)(x+6<4)$ is false cos

$\varnothing$

$\{x \in N\}$

$N \notin R$

Number Theory and Methods of proof.

Definition: n is even iff $n = 2k$, for some integer k.

n is odd iff $n = 2k+1$, for some integer k.

Direct proof:

use definition of even and odd to justify your answer

(a) is 0 even?

(b) is -301 odd

(c) if a and b are integer, is $6a^2b$ odd?

(d) if a and b are integers, is $10a + 8b + 1$ odd?

(e) is every integer either even or odd?

Solution.

(a) is 0 even?

$0 = 2(0)$, $0 \in \mathbb{Z}$

$\therefore$ 0 is even

(b) $-301 = 2(-151) + 1$

$\therefore -301$ is odd

(c) ~~$6a^2b = (3a^2b)$~~ $6a^2b = 2(3a^2b)$

$\therefore$ NO $6a^2b$ is even.

Example

7 ~~and 5~~ ~~7 and 5~~ -2 and 8

Theorem: the sum of any two even integers is even.

Solution

Suppose a and b are integers, By definition

$a = 2p$ for some integer p

$b = 2q$ for some integer q

Goal: $(a+b)$ is even

Now $a+b = 2p+2q$

$= 2(p+q)$

$= 2t$, for some integer $t = (p+q)$

2) Show that the difference of any odd integer and any even integer is odd.

soln.

3) Show that for every odd integer n, n^2 is odd.

If n is odd, then n^2 is odd.

soln

Goal: n^2 is odd

Suppose n to be odd.

By definition: $n = 2k+1$ for $k \in \mathbb{Z}$

$n^2 = (2k+1)^2$

$= (2k+1)(2k+1)$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$= 2t + 1$ for some $t = 2k^2 + 2k \in \mathbb{Z}$

proof by

Example

is odd,

Soln 1,

Assume

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proof by Contraposition

Example: prove that if n is an integer and $3n+2$ is odd, then n is odd.

Soln "If $3n+2$ is odd, the n is odd"
Premise Conclusion.

Assume $3n+2$ is odd.

By definition $3n+2 = 2k+1$

$$3 = \frac{2k-1}{3} = \frac{2k}{3} - \frac{1}{3}$$ (it is not possible to use direct proof)

Suppose the statement "if $3n+2$ is odd, then n is odd" is false. Assume n is even, and $3n+2$ is odd.

By definition: $n = 2k$, for $k \in \mathbb{Z}$

$$3n+2 = 3(2k)+2$$
$$= 6k+2$$
$$= 2(3k+1), \quad w = 3k+1$$
$$= 2w \text{ for } w \in \mathbb{Z}.$$

this last step implies that $3n+2$ is even.

Therefore $3n+2$ must be odd.

* Prove that if $n = a \times b$, where a and b are positive integers then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

Soln: "If $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$"

suppose ($a \leq \sqrt{n}$ or $b \leq \sqrt{n}$) is false.

Goal: To show $n = ab$, $\sim(a \leq \sqrt{n} \vee b$

$\sim(a \leq \sqrt{n} \vee b \leq \sqrt{n}) \equiv \sim(a \leq \sqrt{n}) \wedge \sim(b \leq \sqrt{n})$

$\equiv a > \sqrt{n} \wedge b > \sqrt{n}$

$a > \sqrt{n}$ and $b > \sqrt{n}$

$a \cdot b > \sqrt{n} \cdot \sqrt{n} = n$

$ab > n$

$ab \neq n$

therefore

Exercise ~~ex~~: Prove that if n is an integer and n^2 is odd then n is odd.

2. Show that the proposition $P(0)$ is true, when $P(n)$ is "if $n > 1$, then $n^2 > n$", and domain consist of all integers.

Solution

"if $n > 1$, then $n^2 > n$" $= P(n)$

$P(0)$ is the statement.

"if $0 > 1$, then $0^2 > 0$"

But we know $0 > 1$ is false.

$\therefore$ the whole statement $P(0)$ is true.

3. Prove that if n is an integer with $10 \leq n \leq 15$ which is a perfect square, then n is also a perfect cube.

Soln: If $10 \leq n \leq 15$ ~~is then~~ n is P.S, then n is P.C"

Observe that $3^2 = 9$, $4^2 = 16$

therefore there is no such integer n with $(10 \leq n \leq 15)$ which is a perfect square

i.e. p is false.

Adebiyi ~~Suga~~
Ruqayah
~~Atheyem~~
~~Uknown~~
Abdur-Rahman

Number Theory

Use definition

a) is 0 even

c) if a & b are int

d) if a & b are in

e) is every integ

Solution

Recall that: n is e

n is

a) is 0 even

$\therefore n = 2k$, $2(0)$

b) is -301 odd.

$n = 2k+1 =$

c) is $6a^2b$ odd?

$n = 2k+1$

$n = 2(3a^2b) +$

$\therefore 6a^2b$ is an

d) $10a + 8b + 1$

$n = 2k+1 =$

$\therefore 10a + 8b + 1$

* Theorem: The

Suppose a & b

$a = 2p$ $\therefore$ recal

$b = 2q$ $n =$

$\therefore$ Sum of two