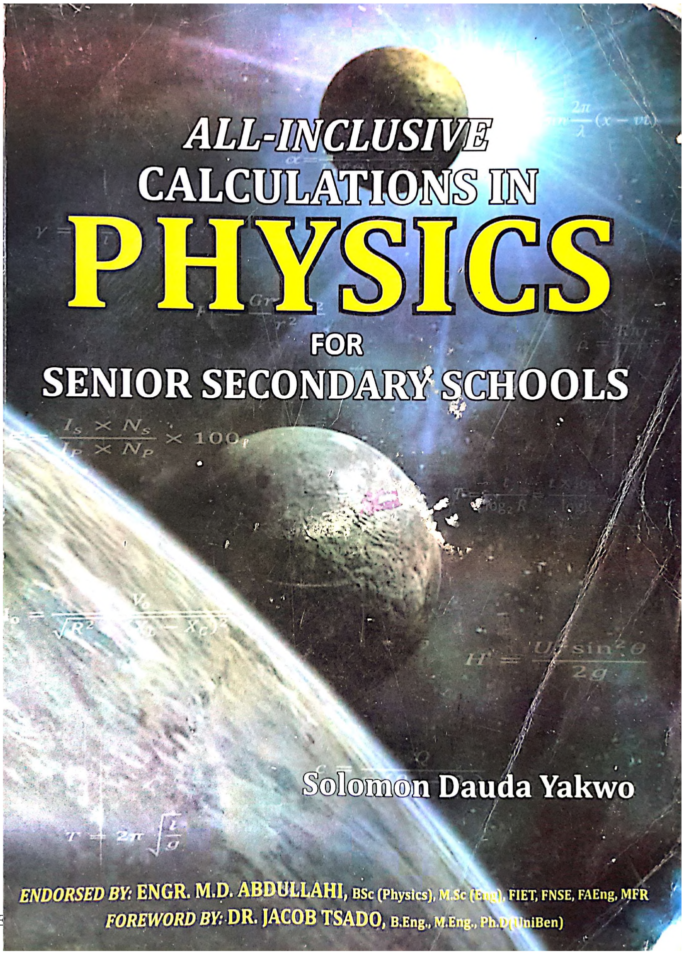
ALL-INCLUSIVE
CALCULATIONS IN
PHYSICS
FOR
SENIOR SECONDARY SCHOOLS
Solomon Dauda Yakwo
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FORE

mon Dauda Yakwo B.E

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FOREWORD

It has been ascertained that many students fail physics as a result of poor mathematical background. It is also true that 70% of physics is mathematics. If you are writing or preparing for physics in any of these examinations: WAEC, NECO, NABTEB, NDA, JAMB or Unified Tertiary Matriculation Examination, this book – ALL-INCLUSIVE CALCULATIONS IN PHYSICS – ***is your sure companion to success***.

This book has been greatly simplified to the most basic level and the examples arranged in proper ascending order. It has not only covered a lot of topics but treated them to the highest satisfactory level. The content is suitable for senior secondary school level, as well as students preparing for pre-degree and preliminary courses in universities, polytechnics and colleges of education. The exercises are numerous and widespread, covering examinations of many years.

This book seems to be the best of in its class. Thus, I strongly advise that you should have this book and at least one theory physics textbook. **However, if you cant afford many but one, let that one be THIS ONE**: *ALL-INCLUSIVE CALCULATIONS IN PHYSICS*.

The author is an injector and generator. I commend the author, Solomon Dauda Yakwo, for writing this book. It is a masterpiece. Above all, I congratulate him for the formulae he has produced in radioactivity. Through this book, the author has "valentined" mathematics and physics. Those who buy and study this book to the fullest can be sure of passing any physics calculation examination.

A tree cannot make a forest. The best of this book will include your constructive suggestions sent to the author.

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PREFACE

This book deals specifically with calculations in physics in an entirely different, highly simplified and very comprehensive manner. It is written to ensure that a student in Senior Secondary School acquires the basic principles and skills of physics calculations and prepares adequately in advance for WAEC, NECO, JAMB, NABTEB, and NDA examinations in such a way that nothing is left to chance.

Also, students undergoing pre-degree or preliminary programmes in universities, polytechnics and colleges of education will certainly find ***ALL-INCLUSIVE CALCULATION IN PHYSICS*** extremely useful to their understanding of basic calculations in physics as a prelude to the more complex calculations in Advanced Level Physics courses.

For every topic, concept or chapter, a brief theoretical explanation is given and the relevant equations are stated. Based on similarity, degree of difficulty and frequency of appearance, various typical examples are taken from a pool of ***ALL*** past JAMB (1978 - 2009), WAEC (1988 - 2009) and NECO (2000 – 2009) questions and solved in a detailed and simplified manner that will ensure that students are fully equipped to handle ***ANY*** calculation problems, no matter how tricky or difficult the question may be.

At the end of each chapter is a question and answer section that includes ALL other calculation questions (except those used as examples) in JAMB (1978 - 2009), WAEC (1988 - 2009) and NECO (2000 – 2009) related to the concepts or topics treated in that chapter. The author is bold and sincere to declare that if a student invests quality time and effort in going through all the solved examples and in solving all the exercises at the end of each chapter, then there is assurance that the student will confidently and correctly solve any calculation question in WAEC, NECO, JAMB, NABTEB, NDA examinations.

In fact, a renowned Nigerian physics author and prolific writer of a great many science textbooks, G.O. Ewelukwa, affirm that, "*...over 70% of the physics questions are mathematical problems...*" In other words, it is almost impossible for any student to pass any physics examination if he or she does not have a good grasp of calculations in physics. Therefore, ***ALL-INCLUSIVE CALCULATIONS IN PHYSICS*** is highly recommended to all physics students and teachers of physics in all senior secondary schools.

The author accepts responsibility for any error and encourages both students and teachers to write to the author if any mistake is detected. Suggestions for improvement of this book are welcomed and will be duly acknowledged in subsequent editions or reprints.

To all who contributed in making this book a success, I say, thank you and God bless you abundantly.

The author would like to duly acknowledge and profoundly thank the West African Examinations Council (WAEC), the National Examinations Council (NECO) and the Joint Admissions and Matriculation Board (JAMB) for using their past physics questions.

Solomon Dauda Yakwo

TO THE PHYSICS STUDENT

I wrote this book for you with the understanding that you may be facing one or more of the following challenges as a physics student.

1. While you accept the fact that physics is very relevant to your future career, you just don't like or understand the too many calculations involved in almost every concept in physics.
2. You are among the thousands of Nigerian physics student who have no physics teacher in their school.
3. Due to laziness or incompetence, your physics teacher only concentrates on the theoretical aspects of physics and partially or completely ignores the calculation problems.
4. After reading or been taught a topic or concept, you are always not sure if you have adequately prepare to solve any calculation question on that topic/concept that may come up in the future in any of these exams; WAEC, NECO, JAMB, NABTEB or NDA.
5. You are preparing for post-JAMB test or undergoing a preliminary or pre-degree physics course in a university, polytechnic or college of education, and know that you still do not have the competence and confidence in solving physics calculation problems.

Congratulations! The book you are holding will help you overcome one or more of the challenges you are facing. Read the preface if you have not done so. If you adhere strictly to the following suggestions it will go a long way in increasing your ability, confidence and speed when solving calculation problems in physics.

(i) Because this book deals with calculations, I strongly advise you to have and use a scientific calculator, preferably the ***Porpo®*** scientific calculator. Please read the manual again and again until you can efficiently and effectively use the calculator to solve problems without delay or mistakes. From my experience as a teacher, the ***Porpo®*** scientific calculator is the cheapest, most advanced and easy to use calculator best suited for senior secondary school students and even tertiary students.

(ii) Most questions and examples in this book are objective questions. The only difference between objective and essay question is that in objective questions, you are NOT required to show how you obtained your answers. It is absolutely unnecessary and will not fetch you any mark. In fact, you are allowed to use any shortcut method (and should devise various shortcut methods peculiar to you) because the examiner is ONLY interested in your answers. On the other hand, an essay question requires you to show in every detail how you arrived at your answers. This involves you stating the formula or equation to be used and showing how you carried out the substitution and calculations. It is very important for you to show ALL your workings or else you will lose a lot of marks.

(iii) This book deals mainly with calculations in physics and should be used in combination with other major textbooks that concentrate more on the theory of physics. Apart from M. Nelkon's *Principle of Physics* and A.F. Abbott's *Physics*, I recommend any of these textbooks written by great Nigerian authors: *Senior Secondary Physics SSS 1,2&3* by B.L.N. Ndupu, P.N. Okeke and O.A. Ladipo; *New School Physics for Senior Secondary Schools* by M.W. Anyakoha and *Senior Secondary Physics* by P.N. Okeke and M.W. Anyakoha.

(iv) If you find out any mistake in this book or have a suggestion on how any calculation method can be improved, please feel free and write to me. I will be extremely glad to read from you and will duly acknowledge you in subsequent edition or reprint.

OK ON

1

SPEED, VELOCITY AND ACCELERATION

SPEED

Speed is defined as the rate of change of distance with time.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{change in distance}}{\text{change in time}} = \frac{s}{t}$$

$$= \frac{\text{final distance} - \text{initial distance}}{\text{final time} - \text{initial time}} = \frac{s}{t}$$

The S.I. unit for speed is m/s.

Example 1

A student walks a distance of 3km in 20 minutes. Calculate his average speed.

Solution

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{3\text{km}}{20\text{min}} = \frac{3 \times 1000\text{m}}{20 \times 60\text{s}} = 2.5\text{m/s}$$

Example 2

A driver traveling at a speed of 115km/hr received a text message on his mobile phone. How far is he, in kilometers, 20s later from when he received the text?

Solution

speed = 115km/hr; time,t = 20s, $\left(20s = \frac{20}{3600}hr\right)$ distance,s = ?

$$\text{Distance} = \text{speed} \times \text{time} = \frac{115\text{km}}{\text{hr}} \times \frac{20\text{hr}}{3600} = \frac{2300}{3600} = 0.639\text{km}$$

VELOCITY

Velocity is defined as the rate of change of distance moved with time in a specified direction. It is also defined as the rate of change of displacement with time. Displacement means distance traveled in a specified direction.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{\text{change in displacement}}{\text{change in time}}$$

$$= \frac{\text{final displacement} - \text{initial displacement}}{\text{final time} - \text{initial time}}$$

However speed is used in place of velocity and vice versa. The S.I unit for velocity is m/s.

RECTILINEAR ACCELERATION

The term *rectilinear acceleration* means the rate of increase of velocity along a straight-line path in a unit time. When the velocity of an object changes it could be said to accelerate or decelerate. Acceleration is defined as the increasing rate of change of velocity with time. Deceleration on the other hand is defined as the decreasing rate of change of velocity with time.

Deceleration is also called retardation or negative acceleration.

$$\text{Acceleration (Deceleration)} = \frac{\text{Change in velocity}}{\text{Time taken for change}}$$

$$= \frac{\text{Final velocity - Initial velocity}}{\text{Final time - Initial time}}$$

Equations of Uniformly Accelerated Motion

Equations of motion for a body traveling along a straight line with uniform acceleration are derived as follows.

First equation of motion:-

If a body at rest or already moving with an initial velocity, u m/s, begins to accelerate at a m/s^2, after a particular time, t sec, it will obtain a final velocity v m/s. The acceleration, a, will be defined by

$$a = \frac{\text{Change in velocity}}{\text{Time interval}}$$

$$a = \frac{\text{Final velocity - Initial velocity}}{\text{Time interval}}$$

Substituting letters for words we have,

$$a = \frac{v - u}{t}$$

Cross-multiplying we have,

$$v - u = at$$

Rearranging,

$$v = u + at \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

This is the first equation of motion.

Second equation of motion:

The second equation of motion can be derived if we consider that a body moving with uniform or constant acceleration must have had an initial velocity u, before attaining a final velocity, v.

Therefore the average velocity is equal to the sum of the initial velocity, u and the final velocity, v divided by two.

Thus, $\text{Average velocity} = \frac{u + v}{2}$

Substituting the first equation of motion, $v = u + at$ into the above we obtain,

$$\text{Average velocity} = \frac{u + u + at}{2}$$

$$= \frac{2u + at}{2}$$

$$= \frac{2u}{2} + \frac{at}{2}$$

$$= u + \tfrac{1}{2}at$$

$$\text{Average velocity} = \frac{\text{displacement (distance)}}{\text{time}} = \frac{s}{t}$$

$$\frac{s}{t} = u + \tfrac{1}{2}at$$

Cross multiplying,

$$s = t(u + \tfrac{1}{2}at)$$

$$s = ut + \tfrac{1}{2}at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This is second equation of motion.

Third equation of motion:

A combination of first and second equation of motion yields the third equation of motion as follows:

Square both sides of first equation of motion, $v = u + at$

$$(v)^2 = (u + at)^2$$
$$v^2 = (u + at)(u + at)$$
$$v^2 = u^2 + uat + uat + (at)^2$$
$$v^2 = u^2 + 2uat + a^2t^2$$

Factor out *2a* from the last two terms of the right hand side of the above equation.

$$v^2 = u^2 + \frac{2a}{2a}\left(2uat + a^2t^2\right)$$
$$v^2 = u^2 + 2a\left(\frac{2uat}{2a} + \frac{a^2t^2}{2a}\right)$$
$$v^2 = u^2 + 2a(ut + \tfrac{1}{2}at^2)$$

Substituting the second equation of motion $s = ut + \frac{1}{2}at^2$ into the above, we obtain

$$v^2 = u^2 + 2as \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This is the third equation of motion.

The three equations of motion with uniform acceleration are:

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $v^2 = u^2 + 2as$

Where u = initial velocity in m/s or ms^{-1}
v = final velocity in m/s or ms^{-1}
t = time in sec
s = distance in m
a = uniform acceleration/deceleration in m/s^2 or ms^{-2}

ALWAYS REMEMBER:

1. When an object moves or accelerates ***from rest***, its initial velocity, u = 0.
2. When a body ***comes to rest or stops***, its final velocity, v = 0.
3. When a body's velocity is ***constant or not changing***, its acceleration, a = 0.

Problem Solving Using Equation Of Motion

Example 3

A particle accelerates uniformly from rest at $6.0m/s^2$ for 8s and then decelerates uniformly to rest in the next 5s. Determine the magnitude of the deceleration. *WAEC 2005*

Solution

You are to calculate the particle's deceleration. It decelerated to rest from a particular velocity which was attained during acceleration. So, you are to calculate the velocity attained as follows:

Initial velocity, u = 0; acceleration, a = $6m/s^2$; t = 8s; velocity attained, v = ?

Substituting into the most appropriate equation, $v = u + at$

$$v = 0 + 6 \times 8$$
$$.\ v = 48m/s$$

Next the particle decelerated to rest from a velocity of 48m/s, therefore, the deceleration is calculated thus:

Initial velocity, u = 48m/s; final velocity, v = 0; time, t = 5s; deceleration, a = ?

Substituting into the most appropriate equation,

$$v = u + at$$
$$0 = 48 + a \times 5$$
$$0 = 48 + 5a$$
$$-\ 5a = 48$$

$$\frac{-5a}{5} = \frac{48}{5} \quad \therefore \quad a = 9.6ms^{-2}$$

Example 4

A body accelerates uniformly from rest at the rate of $3ms^{-2}$ for 8s. Calculate the distance covered by the body during acceleration. *WAEC 1992*

Solution

Using appropriate symbols deduce, the known and unknown from the question.

Acceleration, a = $3m/s^2$; initial velocity, u = 0; time, t = 8s; distance, s = ?

Therefore the most appropriate equation that includes a, u, t and s is the second equation of motion:

$$s = ut + \tfrac{1}{2}at^2$$

Substituting:

$$s = 0\times8 + \tfrac{1}{2}\times3\times8^2$$
$$= 0 + \tfrac{1}{2}\times3\times64$$
$$= 96m$$

Example 5

Starting from rest, a *Formular One* car accelerates uniformly at $25m/s^2$ for 30s. What distance does it cover in the

(i) last one second of motion.

(ii) seventeenth second of motion.

Solution

The distance in the last one second of motion is the distance it covered between time interval of the 29th and 30th second. It is calculated by subtracting the distance in the 29th sec from that of the 30th sec.

Distance, s = $S_{30} - S_{29}$

But, $S_{30} = ut + \tfrac{1}{2}at_{30}^2$ and $S_{29} = ut + \tfrac{1}{2}at_{29}^2$

Starting from rest, initial velocity, u = 0. Therefore, ut = 0 in each case

$$\therefore \quad s = \tfrac{1}{2}at_{30}^2 - \tfrac{1}{2}at_{29}^2$$
$$= \tfrac{1}{2}a(t_{30}^2 - t_{29}^2)$$

From the question, acceleration, a = $25m/s^2$; t_{30} = 30s; t_{29} = 29s. So substituting into above equation:

$$s = \tfrac{1}{2} \times 25(30^2 - 29^2)$$
$$= 12.5\ (900 - 841)$$
$$= 12.5(59)$$
$$= 737.50m.$$

(ii) From question acceleration, a = $25m/s^2$; initial velocity, u = 0; time, t is not equal to 17s because the 17th second is actually the time interval between the 16th and 17th second, therefore time, t = $t_{17} - t_{16}$.

Substituting into $s = \tfrac{1}{2}a(t_{17}^2 - t_{16}^2)$

$$s = \tfrac{1}{2} \times 25(17^2 - 16^2)$$
$$= 12.5\ (289 - 256)$$
$$= 12.5 \times 33$$
$$= 412.5m.$$

Note: In calculating distance between time intervals, the second equation of motion, $s = ut + \tfrac{1}{2}at^2$ is modified to $s = \tfrac{1}{2}a(t_2^2 - t_1^2)$, where initial velocity u = 0, t_2 and t_1 are the time intervals.

Example 6

A body uniformly accelerates from rest at $8m/s^2$. In how much time will the body travel a distance of 2.5km?

Solution

Acceleration, $a = 8m/s^2$; initial velocity, $u = 0$;
distance, $s = 2.5km = 2.5\times1000m = 2500m$; time, $t = ?$
Substituting into the most appropriate equation, $s = ut + \frac{1}{2}at^2$

$$2500 = 0 + \tfrac{1}{2}\times8\times t^2$$
$$2500 = 4t^2$$
$$t^2 = \frac{2500}{4} = 625$$
$$t = \sqrt{625} = 25s$$

Example 7

A particle starts from rest and moves with a uniform acceleration of $4m/s^2$. What is its velocity after covering a distance of 8m.

Solution

Initial velocity, $u = 0$; acceleration, $a = 4m/s^2$; distance, $s = 8m$; final velocity, $v = ?$
The equation containing u, a, s, and v is the 3rd equation of motion, $v^2 = u^2 + 2as$.
Substituting in to most appropriate equation,

$$v^2 = 0 + 2\times4\times8$$
$$v^2 = 64$$
$$v = \sqrt{64} = 8m/s$$

Example 8

A motorist, travelling at 120km/hr sees a broken down truck at the middle of the road and immediately applies his brakes and comes to a stop with uniform retardation in 20s. What distance does the car travel after the brakes were applied?

Solution

Convert 120km/hr to m/s: $120km/hr = \frac{120\times1000\,m}{60\times60\,s} = 33.33m/s$

Initial velocity, $u = 33.33m/s$; final velocity, $v = 0$; time, $t = 20s$.

The retardation, a, must first be found using $v = u + at$

Reterdation, $a = \frac{v-u}{t} = \frac{0-33.33}{20} = 1.67m/s^2$

Then any of the other two equations of motion can be used to calculate distance

(i) $s = ut + \frac{1}{2}at^2$

$$= 33.33 \times 20 + \tfrac{1}{2}(-1.67) \times 20^2$$
$$= 666.6 - \tfrac{1}{2} \times 1.67 \times 400$$
$$= 666.6 - 334$$
$$= 332.60m$$

or

(ii) $v^2 = u^2 + 2as \quad \therefore \quad s = \frac{v^2 - u^2}{2a}$

$$s = \frac{0-(33.33)^2}{2\times(-1.67)} = \frac{-1110.89}{-3.34} = 332.60m$$

Example 9

A body which is uniformly retarded comes to rest in 5s after travelling a distance of 10m. What is its initial velocity? *NECO 2002*

Solution

Final velocity, v = 0; time, t = 5s; distance, s = 10m; initial velocity, u = ?

Did you notice that acceleration or deceleration is not given? Therefore, there is no single equation of motion that can be applied because all the 3 equations of motion has deceleration/acceleration. A combination of two equations will do.

The deceleration, a, is not given and must first be determined in order to calculate the initial velocity. Substituting into $v = u + at$

$$a = \frac{v - u}{t} = \frac{0 - u}{5}$$

$$\therefore \quad a = \frac{-u}{5}$$

Substituting this deceleration, $\frac{-u}{5}$ into the third equation, $v^2 = u^2 + 2as$

$$\therefore u^2 = v^2 - 2as$$

$$u^2 = 0 - 2 \times \left(\frac{-u}{5}\right) \times 10$$

$$u^2 = \frac{-2 \times -u \times 10}{5}$$

$$u^2 = \frac{20u}{5} = 4u$$

$$u^2 = 4u$$

$$\frac{u^2}{u} = \frac{4u}{u} \quad \text{dividing by u}$$

$$u = 4\text{m/s}$$

However, this type of question can be solved more easily using graphical method. See example 15 (Graphical solutions).

Instantaneous Speed/Velocity and Instantaneous Acceleration.

Apart from being average, uniform or non-uniform, the speed, velocity and acceleration of objects could also be instantaneous, that is, at particular point or instant in time.

Instantaneous speed/velocity. This is defined as the actual speed/velocity of an object at any particular point in time. The speedometer of a moving car gives the instantaneous speed/velocity of that car at any moment in time.

Instantaneous velocity at a given instant t is the final, limiting value approached by the average velocity when the average velocity is calculated for a smaller and smaller range of time intervals that includes the instant t. Mathematically, instantaneous speed or velocity is written as;

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

The instantaneous speed at a particular instant has the same value as the average speed provided two conditions are met:

1. The particular instant must be included in Δt.

2. The ratio $\frac{\Delta s}{\Delta t}$ must cover a very small part of the distance-time curve that is nearly a straight line segment.

Instantaneous acceleration at a given instant t is defined as the rate of change of velocity with time during an infinitesimally small interval of time that include the desired instant t. Mathematically, this is written as;

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Example 10

Calculate the instantaneous velocity at 7s of a rocket undergoing uniformly accelerated motion if the position is given by $s = 5t + 12t^2$, where s is in m and t is in seconds.

Solution

From differential calculus, we know that speed/velocity is obtained when distance/displacement is differentiated with respect to time.

If $\quad y = x^n$

Then $\quad \frac{dy}{dx} = nx^{n-1}$

Also, the derivative of a constant is zero.

Therefore, differentiating $s = 5t + 12t^2$

We obtain instantaneous velocity,

$$\frac{ds}{dt} = 5t^{1-1} + 2(12t^{2-1}) = 5t^0 + 24t^1 = 5 + 24t$$

$$\therefore \quad \text{for } t = 7s, \quad \frac{ds}{dt} = 5 + 24 \times 7$$

$$= 173ms^{-1}$$

Example 11

The velocity v of a space shuttle in a time t is given by $v = 25 + 3t^2$. Find the instantaneous acceleration at $t = 9s$.

Solution

When velocity is differentiated with respect to time, acceleration is obtained.

Differentiating $v = 25 + 3t^2$

We obtain instantaneous acceleration, $\frac{dv}{dt} = 0 + 2(3t^{2-1})$

$$= 6t$$

$$\therefore \quad \text{for } t = 9s, \quad \frac{dv}{dt} = 6 \times 9 = 54m/s$$

Graphical Solution To Problems In Rectilinear Acceleration

Velocity time (v – t) graph are commonly used to solve problems in Rectilinear acceleration and could be in different shapes depending on whether the velocity is increasing, decreasing or constant.

The following are various v – t graphs:

1.

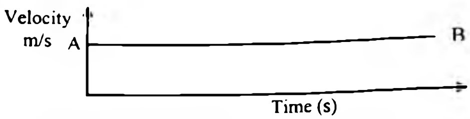


Fig.1.1: ***Velocity–Time Graph For An Object Moving With Constant Velocity.***

The graph AB is drawn parallel to the time axis.

2.

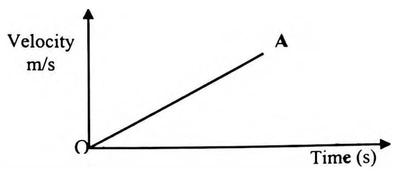


Fig.1.2 ***Velocity-Time Graph For An Object Accelerating From Rest.***

The graph OA begins from the origin. The slope of line OA gives the acceleration of the object.

3.

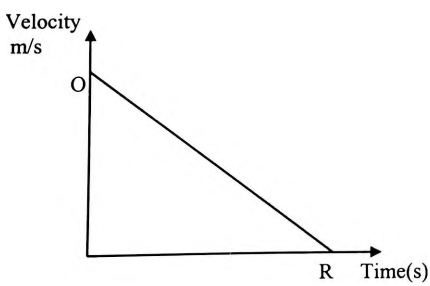


Fig 1.3 ***Velocity-Time*** **graph for an object decelerating to rest.**

The slope OR gives the deceleration or retardation of the object

Example 12

A body moving with uniform acceleration, a, has two points (5, 15) and (20, 60) on the velocity – time graph of its motion. Calculate a. *WAEC 1991*

Solution

On a velocity – time graph, the uniform acceleration, a, is equal to the gradient (slope) of the graph.

From question, first point is (5, 15), equivalent to (x_1, y_1)

Second point is (20, 60), equivalent to (x_2, y_2)

The two points are plotted below

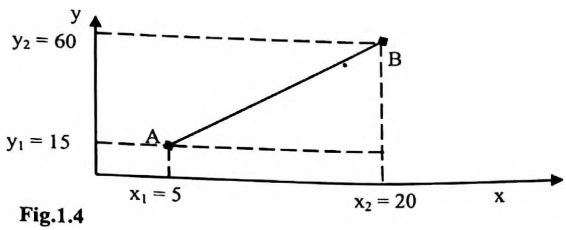


Fig.1.4

The slope of AB $= a = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{60-15}{20-5} = \frac{45}{15} = 3.00ms^{-2}$$

Example 13

A car runs at a constant speed of 15m/s for 300s and then accelerates uniformly to a speed of 25m/s over a period of 20s. This speed is maintained for 300s before the car is brought to rest with uniform deceleration in 30s. Draw a velocity – time graph to represent the journey described above. From the graph find

(i) The acceleration while the velocity changes from 15m/s to 25m/s.

(ii) The total distance travelled in the time described.

(iii) The average speed over the time described. *A.F. ABBOT (J.M.B.)*

Solution

"*...Constant speed of 15 m/s...*", is represented by AB drawn parallel to the time axis.

"*...for 300s...*", is represented by OH.

"*...accelerates uniformly to a speed of 25m/s...*" is shown by BC. "*... over a period of 20s...*", by HG.

"*...Speed (25m/s) is maintained...*", signifies constant or uniform speed/velocity and is therefore represented by CD drawn parallel to the time axis; "*...maintained for 300s...*" is presented by GF.

"*...brought to rest with uniform deceleration...*", is represented by downward sloping DE and FE represents the time (30s) for deceleration.

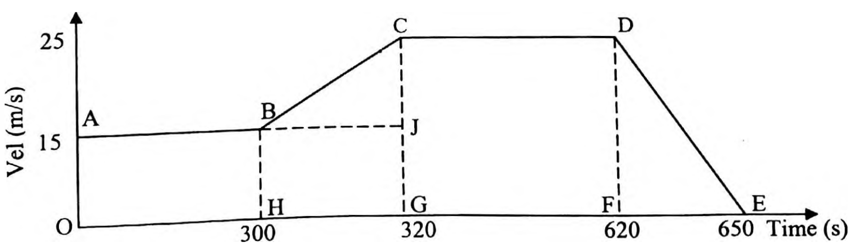


Fig. 1.5

(i) The acceleration, a is equal to the slope or gradient of BC.

$$\text{Acceleration}, a = \text{slope}\,BC = \frac{CJ}{BJ} = \frac{25-15}{320-300} = \frac{10}{20} = \tfrac{1}{2} = 0.5\text{m/s}^2$$

(ii) In a v-t graph, the total distance is equal to the area under the v-t graph.

Total distance travelled is equal to area OABCDE.

Area OABCDE = ABHO + BCGH + CDFG + DEF

Area ABHO = Distance covered during constant speed of 15m/s for 300s/

Area BCGH = Distance covered during acceleration or velocity change from 15m/s to 25m/s.

Area CDFG = Distance covered during when speed (25m/s) is maintained for 300s.

Area DEF = Distance covered during uniform deceleration in 30 seconds.

Total distance = OABCDE = ABHO + BCGH + CDFG + DEF

$= AB \times BH + \frac{1}{2}(BH + CG)HG + (CD \times DF) + \frac{1}{2}(FE \times DF)$

$= 300 \times 15 + \frac{1}{2}(15 + 25)20 + 300 \times 25 + \frac{1}{2}(30 \times 25)$

$= 4500 + 400 + 7500 + 375$

$= 12775\text{m}$

(iii) $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{12775}{650} = 19.65\text{m/s}$$

Note: Time taken is obtained from the graph, represented on the time axis by OE.

Example 14

A car starts from rest at a check point A and comes to rest at the next check point B, 6km away, in 3 minutes. It has first, a uniform acceleration for 40s, then a constant speed and is brought to rest with a uniform retardation after 20s. Sketch a velocity – time graph of the motion. Determine

(i) the maximum speed (ii) the retardation *NECO 2003*

Solution

Convert all units to S.I. units.

6km = 6 × 1000 = 6000m

3mins = 3 × 60s = 180s

The v – t graph is drawn as shown below.

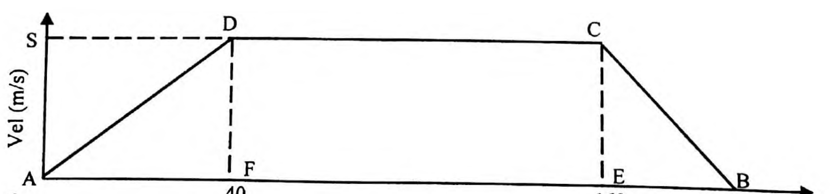


Fig. 1.6

(i) The maximum speed will be equal to S (on the graph) i.e. the distance between the parallel sides of the trapezium; which of course is equivalent to DF or CE.

Total distance, 6000m = Area of trapezium ADCB

$\therefore$ $6000 = \frac{1}{2}(AB + DC)\,S$

$6000 = \frac{1}{2}(180 + 120)\,S$

$6000 = \frac{1}{2}(300)\,S$

$$6000 = 150S$$

$$S = \frac{6000}{150} = 40ms^{-1}$$

(ii) Retardation, a, is equal to the slope of CB

$$\therefore a = \frac{CE}{EB} = \frac{40-0}{160-180} = \frac{40}{-20} = -2m/s^2$$

. Retardation = $2m/s^2$

Example 15

a) A body at rest is given an initial uniform acceleration of $8.0m/s^2$ for 30s after which the acceleration is reduced to $5.0m/s^2$ for the next 20s. The body maintains the speed attained for 60s after which it is brought to rest in 20s. Draw the velocity-time graph of the motion using the information given above.

b) Using the graph, calculate the:

i. maximum speed attained during the motion;
ii. average retardation as the body is being brought to rest;
iii. total distance travelled during the first 50s;
iv. average speed during the same interval as in (iii).

Solution

a) Graph

The velocity attained during acceleration ($8.0m/s^2$ for 30s) is calculated as follows: initial velocity, u = 0; acceleration, $a=8m/s^2$; time, t=30s

Sunbstituting into $v = u + at$

$$v = 0 + 30 \times 8 = 240m/s$$

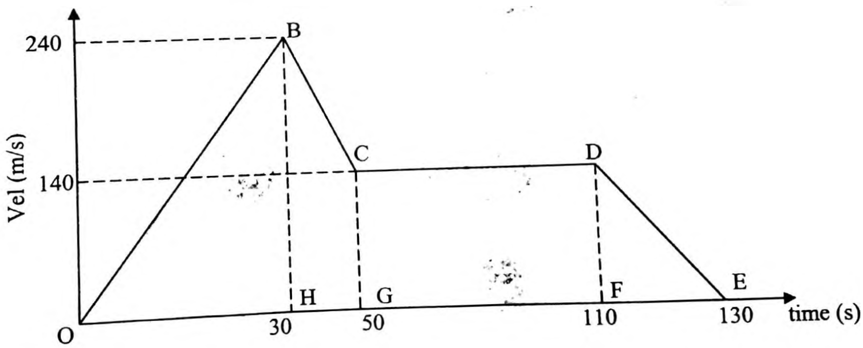


Fig. 1.7

With the value (240m/s) OB is drawn starting from the origin because the motion started from rest.

With this value (240m/s) OB is drawn starting from the origin because the motion started from rest.

"...*After which the acceleration is reduced to $5m/s^2$ for the next 20s...*". This signifies deceleration/retardation of $5m/s^2$, therefore the graph BC slopes downward.

The velocity attained after this retardation is found as follows:

The retardation begins from a speed of 240m/s $\therefore$ Initial velocity, u = 240m/s

Retardation is negative acceleration $\therefore$ Retardation, $a = 5m/s^2$

Time for deceleration is 20s $\therefore$ Time, t = 20s

So, final velocity (speed) is v = u + at

$$v = 240 + (-5 \times 20)$$
$$= 240 - 100 = 140\text{m/s}.$$

Next the graph (CD) is drawn parallel to the time axis to represent the period (60s) during which the body maintained the speed of 140m/s.

Finally, the graph (DE) slopes downward as the body retards to rest.

b) i. As shown in the graph the maximum speed attained is 240m/s.

ii. Average retardation = slope of DE $= \dfrac{140-0}{110-130} = \dfrac{140}{20} = 7\text{m/s}^2$

iii. Total distance during first 50s = Area of OBCG = Area of triangle OBH + Area of trapezium BCGH.

$$\text{Distance} = \tfrac{1}{2}\,OH \times BH + \tfrac{1}{2}(BH + CG)\,GH$$
$$= \tfrac{1}{2} \times 30 \times 240 + \tfrac{1}{2}[(240 + 140)]\,20$$
$$= 3600 + 3800$$
$$= 7400\text{m}$$

(iv) Average speed $= \dfrac{\text{total distance covered in 50s}}{\text{total time taken}} = \dfrac{7400}{50} = 148\text{m/s}$

Example 16

If a car starts from rest and moves with a uniform acceleration of 10m/s^2 for ten seconds, the distance it covers in the last one second of its motion is *JAMB 1985*

Solution

Initial velocity, u = 0; acceleration, a = 10m/s^2; time, t=10s

Final velocity, v = u + at

$$v = 0 + 10 \times 10 = 100\text{m/s}$$

The graph is drawn as shown below.

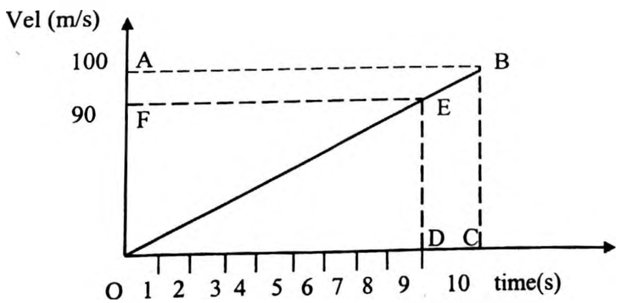


Fig. 1.8

The last one second of motion is the time between the 9^{th} and the 10^{th} second. Therefore the distance covered in the last one second of motion is:

=Distance in the 10^{th} sec – Distance in the 9^{th} sec.

=Area of triangle OBC − Area of triangle OED

$=\frac{1}{2} \times OC \times BC - \frac{1}{2} \times OD \times DE$

$=\frac{1}{2} \times 10 \times 100 \quad \frac{1}{2} \times 9 \times 90$

$=500 - 405$

$= 95m.$

Example 17

A body which is uniformly retarded comes to rest in 5s after travelling a distance of 10m. What is its initial velocity? *NECO 2002*

Solution

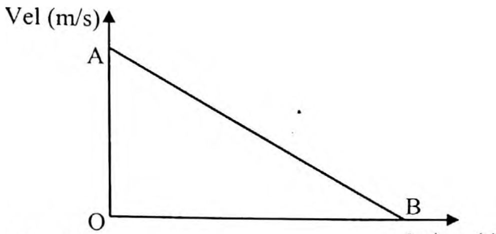


Fig. 1.9

The graph is drawn as shown above. The area of the graph is equal to the distance, 10m. Initial velocity is equal to the height of triangle OAB, which is length OA.

$\therefore$ Area of OAB = 10

$\frac{1}{2} \times OB \times OA = 10$

$\frac{1}{2} \times 5 \times OA = 10$

$$OA = \frac{20}{5} = 4ms^{-1}$$

Motion Under Gravity: Gravitational Acceleration

When an object is thrown upwards or released from a height, its motion is governed or affected by gravity. An object thrown upwards experiences a negative acceleration (-g) because its motion is in opposite direction to the gravitational pull of the earth. On the other hand, when an object falls downwards or is released from a height, it experiences positive acceleration (+g) because its motion is in the same direction as that of the gravitational attraction of the earth.

Equations of motion for gravitational acceleration are derived from the equations of motion for rectilinear acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

1. For a body thrown upwards, the following equations apply.

$v = u - gt$

$h = ut - \frac{1}{2}gt^2$

$v^2 = u^2 - 2gh$

2. For a body falling downwards or released from a height the following equations apply.

$v = u + gt$

$h = ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$

Where v = final velocity in m/s

u = initial velocity in m/s

h = height in m

t = time in s

g = acceleration due to gravity in m/s^2

Always remember:

1. When an object is thrown upwards, its final velocity, at its highest point, v = 0
2. When an object falls downward or is released from a height, its initial velocity u =0
3. The time it takes an object to travel upwards is the same time it takes to come down.

Example 18

At exactly 2hr:00min:00sec a pendulum bob is thrown vertically upwards from the ground with an initial velocity of 75m/s. At what time will the bob return to the ground? (Take $g=10m/s^2$)

Solution

Initial velocity, u = 75m/s; $g=10m/s^2$; final velocity, v = 0; time, t=?

The equation containing all the known and the unknown items is $v = u - gt$

Substituting:

$$0 = 75 - 10 \times t$$

$$10t = 75$$

$$t = \frac{75}{10} = 7.5s$$

The time, t = 7.5s is the time taken to go upwards which is the same time it takes to come down. Therefore time taken to return to the ground from the moment of throw is,

(7.5 + 7.5)s = 15s.

$\therefore$ Time of return will be 2hr:00min:15sec

Example 19

Calculate the maximum height a ball of mass 1.2kg will attain if projected vertically upward with an initial velocity of 17m/s.

Solution

Initial velocity, u = 17m/s; final velocity, v = 0; $g=10m/s^2$; h=?

The mass of the ball is not required.

Substitute into $v^2 = u^2 - 2gh$

$$0 = 17^2 - 2 \times 10 \times h$$

$$20h = 17^2$$

$$h = \frac{289}{20} = 14.45m$$

Example 20

An object falls freely from a height of 25m onto the roof of a building 5m high. Calculate the velocity with which the object strikes the roof. *WAEC 1997*

Solution

Initial velocity, u = 0; $g=10m/s^2$; h=20 i.e.(25 – 5), because the object does not reach the ground instead it fell on a 5m high roof.

Substituting into $v^2 = u^2 + 2gh$

$$v^2 = 0^2 + 2 \times 10 \times 20$$

$$v^2 = 400$$

$$v = \sqrt{400} = 20m/s$$

Example 21
A carpenter working at a construction site mistakenly drops his hammer from a height of 125m. How long does it take to reach the ground?
Solution
Initial velocity, u = 0; $g = 10m/s^2$; height, h=125m; time, t=?

Substituting into $h = ut + \frac{1}{2}gt^2$

$$125 = 0 + \frac{1}{2} \times 10 \times t^2$$
$$125 = 5t^2$$
$$t^2 = \frac{125}{5} = 25$$
$$t = \sqrt{25} = 5s$$

EXERCISE 1

1. A boy cycles continuously through a distance of 1.0km in 5 minutes. Calculate his average speed. *WAEC 1995* Ans: 3.33m/s
2. An air force jet flying with a speed of 335m/s went past an anti aircraft gun. How far is the aircraft 5s later when the gun was fired.
A. 838m B. 3350m C. 670m D. 1675m E. 67m *JAMB 1978* Ans: 1675m
3. A car travels with a constant velocity of 45 km/hr for 20s. What distance does it cover in this time? *NECO 2005* Ans: 250m
4. A body starts from rest and accelerates uniformly at $5m/s^2$ until it attains a velocity of 25m/s. Calculate the time taken to attain this velocity. *WAEC 2005* Ans: 5s
5. A train has an initial velocity of 44m/s and an acceleration of $-4m/s^2$. Its velocity after 10 sec is A. 2m/s B. 4m/s C. 8m/s D. 12m/s E. 16m/s
JAMB 1983 Ans: 4m/s
6. If a car starts from rest and moves with a uniform acceleration of $10m/s^2$ for ten seconds, the distance it covers in the last one second of motion is
A. 95m B. 100m C. 500m D. 905m E. 1000m *JAMB 1985* Ans: 95m
7. A body starts from rest and moves with uniform acceleration of $6m/s^2$. What distance does it cover in the third second?
A. 15m B. 18m C. 27m D. 30m *JAMB 1992* Ans: 15m
8. A body accelerates uniformly from rest at the rate of $3m/s^2$ for 8s. Calculate the distance covered by the body during acceleration. *WAEC 1992* Ans: 96m
9. A particle starts from rest and moves with a constant acceleration of $0.5m/s^2$. Calculate the time taken by the particle to cover a distance of 25m. *WAEC 1993* Ans: 10sec
10. How far will a body move in 4 seconds if uniformly accelerated from rest at the rate of $2ms^{-2}$. *WAEC 1993* Ans: 16m
11. A car takes off from rest and covers a distance of 80m on a straight road in 10s. Calculate the magnitude of its acceleration. *WAEC 2002* Ans: $1.6m/s^2$
12. A bus travelling at 15m/s accelerates uniformly at $4m/s^2$. What is the distance covered in 10s? *WAEC 2000* Ans: 350m
13. A body starts from rest and accelerates uniformly at $5ms^{-2}$. Calculate the time taken by the body to cover a distance of 1km. *WAEC 2005* Ans: 20 secs
14. A particle starts from rest and moves with a constant acceleration of $0.5m/s^2$. The distance covered by the particle in 10s is
A. 2.5m B. 5.0m C. 25.0m D. 50.0m *JAMB 1990* Ans: 25m

15. A body accelerates uniformly from rest at $2m/s^2$. Calculate its velocity after travelling 9m. *WAEC 1988* Ans: 6m/s

16. A boy moving with a velocity of 3m/s is brought to rest by a constant force after travelling 15m. Calculate the retardation. *NECO 2000* Ans: $0.3m/s^2$

17. A car moving with a speed of 90km/hr was brought to rest by the application of the brakes in 10s. How far did the car travel after the brakes were applied?
A. 125m B. 150m C. 250m D. 15km *JAMB 1990* Ans: 125m

18. An aeroplane lands on a runway at a speed of 180km/hr and is brought to a stop uniformly in 30 seconds. What distance does it cover on the runway before coming to rest? A. 360m B. 540m C. 750m D. 957m *JAMB 1993* Ans: 750m

19. Two points on a velocity time graph have coordinates ($5s, 10ms^{-1}$) and ($20s, 20ms^{-1}$). Calculate the mean acceleration between the two points.
A. $0.67ms^{-2}$ B. $0.83ms^{-2}$ C. $1.50ms^{-2}$ D. $2.00ms^{-2}$ *JAMB1989* Ans: $0.67ms^{-2}$

20. A motor vehicle is brought to rest from a speed of $15ms^{-1}$ in 20 seconds. Calculate the retardation.
A. $0.75ms^{-2}$ B. $1.33ms^{-2}$ C. $5.00ms^{-2}$ D. $7.50ms^{-2}$ *JAMB 1994* Ans: $0.75m/s^2$

21. A particle moving in a straight line with uniform deceleration has a velocity of 40m/s at a point P, 20m/s at a point Q and comes to rest at a point R where QR=50m. Calculate the: (i) distance PQ; (ii) time taken to cover PQ; (iii) time taken to cover PR.
WAEC 1990 Ans: (i) 150m; (ii) 5s (iii) 10s

22.

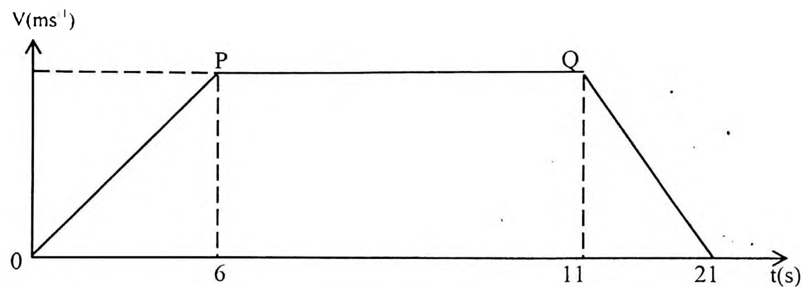


Fig.1.10

The diagram above (**Fig.1.10**) represents the velocity – time graph of a body in motion. The total distance travelled by the body is 195m. Calculate the acceleration of the body in section OP of the graph. *WAEC 1994* Ans: $2.5m/s^2$

23. The diagram below(**Fig. 1.11**) illustrates the velocity –time graph of the motion of a body. Calculate the total distance covered by the body. *WAEC 1996* Ans: 225m

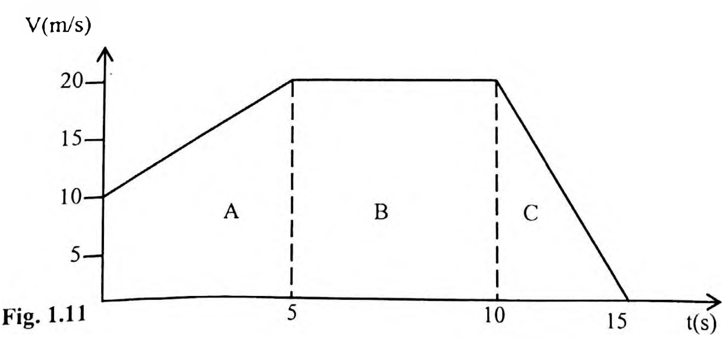


Fig. 1.11

24. The diagram below (**Fig. 1.12**) represents a velocity – time graph. Determine the distance covered in the first 20s. *WAEC 2005* Ans: 60m

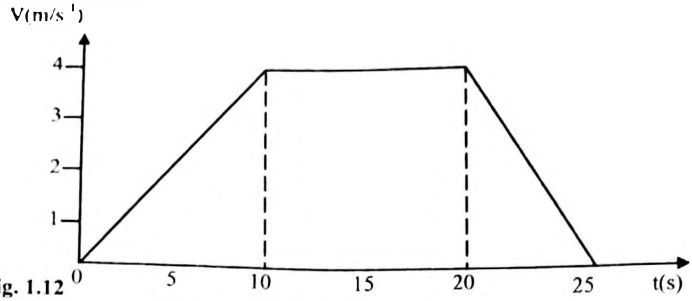


Fig. 1.12

25. The figure below (**Fig. 1.13**) shows the velocity – time graph of a car which starts from rest and is accelerated uniformly at the rate of $3m/s^2$ for 5 seconds. It attains a velocity which is maintained for 1 minute. The car is then brought to rest by a uniform retardation after another 3 seconds. Calculate the total distance covered.
A. 900meters B.1920meters C.1020meters D.960meters E.860meters
JAMB 1979 Ans: 960m

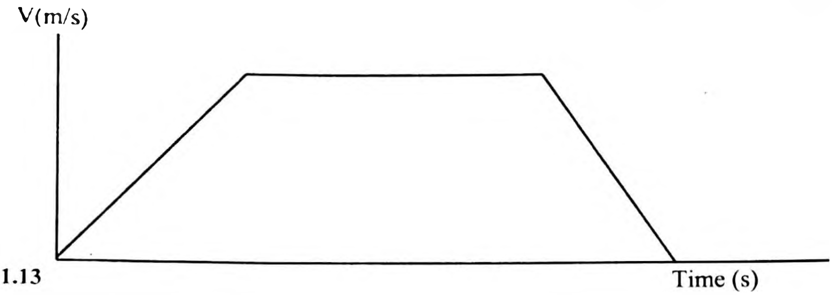


Fig. 1.13

26. A ball is dropped from a height of 45m above the ground. Calculate the velocity of the ball just before it strikes the ground. *WAEC 1994* Ans:$30ms^{-1}$
27. An orange is dropped from a height of 100m above the ground level. Calculate the velocity of the orange just before it strikes the ground. *WAEC 2005*Ans $44.7ms^{-1}$
28. The graph below(**Fig. 1.14**) describes the motion of a particle. The acceleration of the particle during the motion is A.$0.00ms^{-2}$ B.$0.25ms^{-2}$ C.$4.00ms^{-2}$ D.$8.00ms^{-2}$ E.$10.00ms^{-2}$
JAMB1985 Ans: $4.00ms^{-2}$

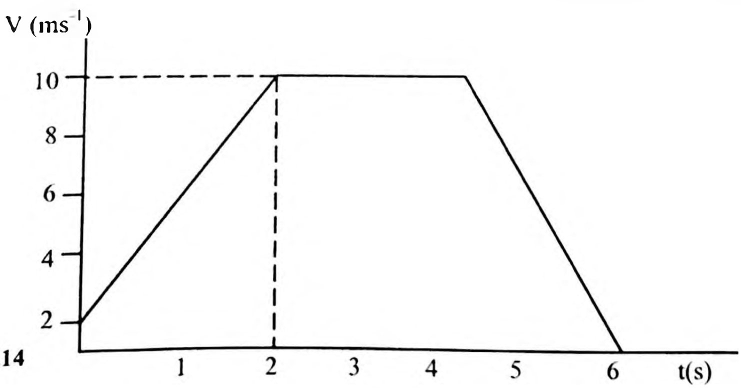


Fig. 1.14

30. A ball is thrown vertically upwards from the ground with an initial velocity of 50m/s. What is the total time spent by the ball in the air? *WAEC 1991* Ans: 10s
31. A stone is projected vertically upward with a velocity of $20ms^{-1}$. Two seconds later a second stone is similarly projected with the same velocity. When the two stones meet, the second one is rising at a velocity of $10ms^{-1}$. Neglecting air resistance, calculate the (i)

length of time the second stone is in motion before they meet. (ii) Velocity of the first stone when they meet. (Take $g=10m/s^2$) *WAEC 1998* Ans: (i) 1s; (ii) 10m/s

32. A ball bearing is projected vertically upwards from the ground with a velocity of $15ms^{-1}$. Calculate the time taken by the ball to return to the ground. [$g=10ms^{-2}$]
WAEC 2004 Ans: 3.0s

33. A stone of mass 0.7kg is projected vertically upwards with a speed of $5ms^{-1}$. Calculate the maximum height reached. [Take g as $10ms^{-2}$ and neglect air resistance)
NECO 2005 Ans: 1.25m

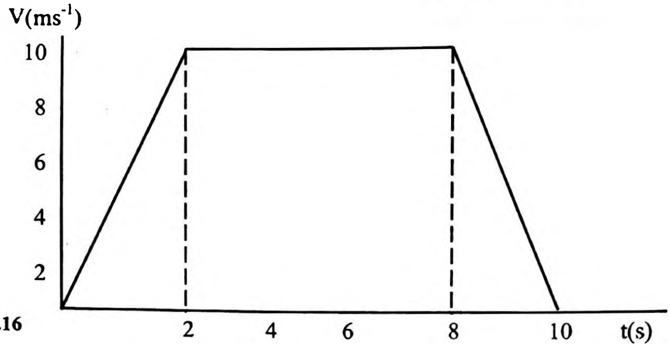


Fig. 1.16

34. What is the average velocity of the sprinter whose velocity – time graph is shown in the figure **Fig. 1.16** above?
A. $85.0ms^{-1}$ B. $17.0ms^{-1}$ C. $8.5ms^{-1}$ D. $1.7ms^{-1}$ *JAMB 1990* Ans: 8m/s

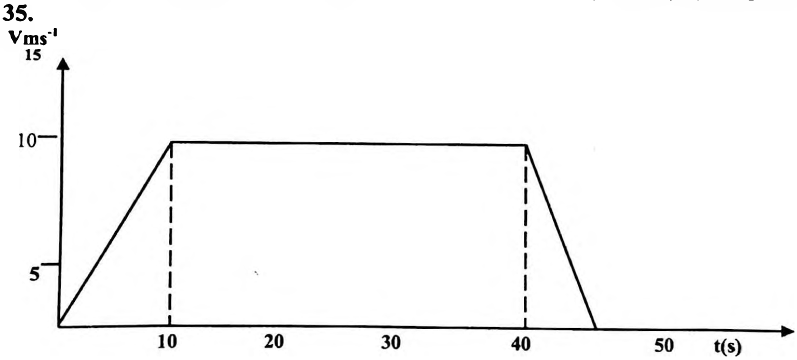
35.


Fig. 1.17
The diagram above (**Fig. 1.17**) shows a velocity- time graph representing the motion of a car. Find the total distance covered during the acceleration and retardation periods of the motion. A. 75m B. 150m C. 300m D. 375m *JAMB 1993* Ans: 75m

36. A stone is released from a height of 80m above the ground. Calculate its velocity just before it strikes the ground. *NECO 2003* Ans: 40m/s

37. A body falls freely under gravity ($g=9.8m/s^2$) from a height of 40m on to the top of a platform 0.8m above the ground. It's velocity on reaching the platform is
A. $784ms^{-1}$ B. $80ms^{-1}$ C. $78.4ms^{-1}$ D. $39.2ms^{-1}$ E. $27.7ms^{-1}$
JAMB 1981 Ans: $27.7ms^{-1}$

38. The diagram below (**Fig. 1.18**) shows the velocity-time graph of a vehicle. Its acceleration and retardation respectively are.
A. 8.0 ms^{-2}, 4.0 ms^{-2} B. 4.0 ms^{-2}, 8.0 ms^{-2} C. 4.0 ms^{-2}, 2.0 ms^{-2} D. 2.0 ms^{-2}, 4.0 ms^{-2}
JAMB 1999 Ans: $4ms^{-2}$, $2ms^{-2}$

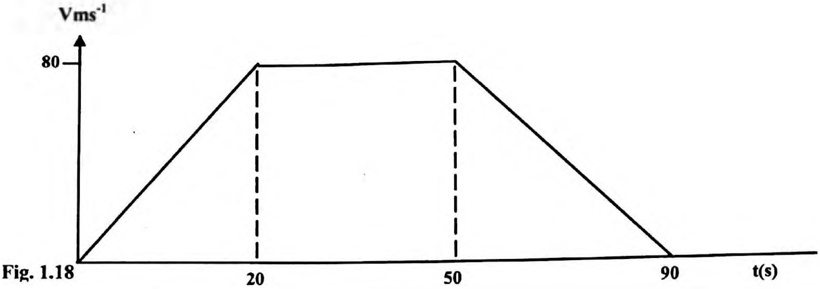


Fig. 1.18

39. A mango fruit drops from a branch 10m above the ground. Just before hitting the ground its velocity is A. $10\sqrt{2}ms^{-1}$ B. $\frac{10}{\sqrt{2}}ms^{-1}$ C. $100ms^{-1}$ D. $5\sqrt{2}ms^{-1}$ E. $200ms^{-1}$ *JAMB 1982* Ans: $10\sqrt{2}ms^{-1}$

40. An orange drops to the ground from the top of a tree 45m tall. How long does it take to reach the ground? *WAEC 1991* Ans: 3s

41. An object is released from rest at a height of 25m. Calculate the time it takes to fall to the ground. *WAEC 2002* Ans:2.24s

42. A palm fruit dropped to the ground from the top of a tree 45m tall. How long does it take to reach the ground? [$g=10ms^{-2}$]
A. 9s B. 4.5s C. 6s D. 7.5s E. 3s *JAMB 1978* Ans: 3s

43. A stone is dropped from thc top of a tower of height 11.25m. Calculate the time it will take to reach the ground. *NECO 2004* Ans: 1.50s

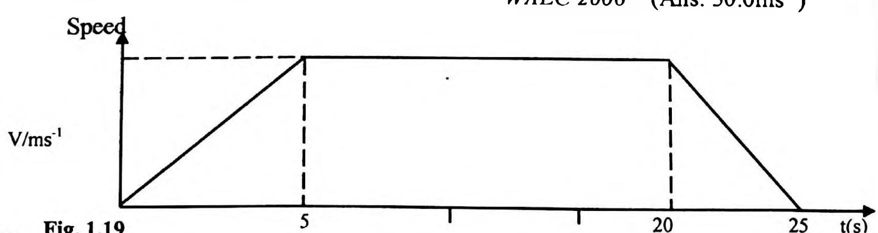
44. A small metal ball is thrown vertically upwards from the top of a tower with an initial velocity of $20ms^{-1}$. If the ball took a total of 6 seconds to reach the ground level, determine the height of the tower.($g=10ms^{-2}$
A. 60m B. 80m C. 100ms D. 120m *JAMB 1991* Ans: 20m

45. A piece of stone is projected vertically upwards with a speed of $25ms^{-1}$. Determine its speed at its highest point reached. *WAEC 2005* Ans:0.0m/s

46. In free fall, a body of mass 1kg drops from a height of 125m from rest in 5s. How long will it take another body of mass 2kg to fall from rest from the same height? [$g=10ms^{-2}$] A. 5s B. 10s C. 12s D. 15s *JAMB 1998* Ans:5s

47. A ball thrown vertically upward reaches a maximum height of 50m above the level of projection. Calculate the : (i) time taken to reach the maximum height.
(ii) speed of the throw. *WAEC 2001* Ans: (i) 3.16s (ii) 31.6m/s

48. A car accelerates uniformly from rest at $5ms^{-2}$. Determine its speed after 10s.
WAEC 2006 (Ans: $50.0ms^{-1}$)



49. Fig. 1.19

The diagram above (**Fig. 1.19**) shows the speed-time graph of a car. If the car covered a total distance of 600m in 25s, calculate its maximum speed. *WAEC 2006* Ans:$30ms^{-1}$

50. Two particles X and Y starting from rest cover the same distance. The acceleration of X is twice that of Y. The ratio of the time taken by X to that taken by Y is?
A.½ B. 2 C. $1/\sqrt{2}$ D. $\sqrt{2}$ E. 4 *JAMB 1981* Ans: $1/\sqrt{2}$ or $\sqrt{1/2}$

51. A bullet fired vertically upward from a gun held 2m above the ground, reaches its maximum height in 4s. Calculate the (i) magnitude of the initial velocity of the bullet; (ii) total distance the bullet travelled by the time it hits the ground. [$g = 10ms^{-2}$]

NECO 2006 Ans: (i) 40m/s (ii) 162m

52. What is the total time of flight of an object projected vertically upwards with a speed of $30ms^{-1}$? [$g = 10ms^{-2}$]

NECO 2007 Ans: 6.0s

53. A car starts from rest and accelerates uniformly at a rate of $5.0ms^{-2}$. Calculate the magnitude of its velocity after moving through 100m.

NECO 2007 Ans: $31.62ms^{-1}$

54. An object of mass 2kg moves with uniform speed of $10ms^{-1}$ for 5s along a straight path. Determine the magnitude of its acceleration.

WAEC 2007 Ans: $0\ ms^{-2}$

55. A body moving with an initial velocity u accelerates until it attains a velocity of v within time t. The distance, s covered by the body is given by the expression.

A. $s = \left(\frac{v-u}{2}\right)t$ B. $s = \left(\frac{v+2u}{2}\right)t$ C. $s = \left(\frac{v+u}{2}\right)t$ D. $s = \left(\frac{2v-u}{2}\right)t$

WAEC 2007 Ans:C

56. A body accelerates uniformly from rest at $2ms^{-2}$. Calculate the magnitude of its velocity after traveling 9m.

WAEC 2007 Ans: $6.0ms^{-1}$

57. A body moving with a velocity of $50ms^{-1}$ is brought to rest in 30s by a constant retarding force. Calculate the distance covered by the body. *NECO 2008 Ans: 750m*

58. A ball is thrown vertically upwards with a speed of $20ms^{-1}$. Calculate the maximum height reached. ($g = 10ms^{-2}$)

NECO 2008 Ans: 20.0m

59. A bullet fired vertically upward from a gun held 2.0m above the ground reaches its maximum height in 4.0s. Calculate its initial velocity. ($g = 10\ ms^{-2}$)

A. $10\ ms^{-1}$ B. $8\ ms^{-1}$ C. $40\ ms^{-1}$ D. $20\ ms^{-1}$ JAMB 2009[8] Ans: C

60.

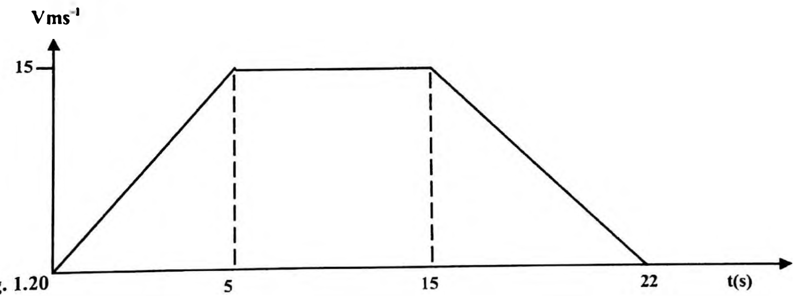


Fig. 1.20

The velocity-time graph above represents the motion of a car. Calculate the total distance travelled by the car.

NECO 2008 Ans: 240

61. What is the acceleration between two points on a velocity-time graph which has coordinates (10 s, 15 ms^{-1}) and (20 s, 35 ms^{-1})?

A. 1.75 ms^{-2} B. 3.50 ms^{-2} C. 1.00 ms^{-2} D. 2.00 ms^{-2} JAMB 2009[4] Ans: D

62. A car accelerates uniformly from rest at 4 ms^{-2}. How far will it travel in the fifth complete second?

A. 100m B. 50m C. 32m D. 18m JAMB 2009[5] Ans: D

63. A car moves with a speed of $30ms^{-1}$. Calculate the distance travelled in 30s.

WAEC 2009[7] Ans: 900m

64. A body at rest is given an initial acceleration of $6.0ms^{-2}$ for 20s after which the acceleration is reduced to $4.0ms^{-2}$ for the next 10s. The body maintains the speed attained for 30s.

Draw the velocity-time graph of the motion using the information given above. From the graph, calculate:

(i) Maximum speed attained during the motion;

(ii) Total distance travelled during the first 30s;

(iii) Average speed during the same time interval as in (ii) above

WAEC 2009[E11] Ans: (i) $120ms^{-1}$ (ii) 2200m (iii) $73.33ms^{-1}$

65. An object of mass 2kg moves for 5s with a uniform velocity of $10ms^{-1}$. What is the magnitude of its acceleration in ms^{-2}? NECO 2009[4] Ans: 0.0

67. A car starts from rest and accelerates uniformly for 5s until it attains a velocity of $30ms^{-1}$. It then travels with uniform velocity for 15s before decelerating uniformly to rest in 10s.

(i) Sketch a graph of the motion.

(ii) Using the graph above, calculate the

I. acceleration during the first 5s,

II. deceleration during the last 10s,

III. total distance covered throughout the motion.

NECO 2009[E11] Ans: I. $6ms^{-2}$ II. $3ms^{-2}$ III. 675m

2

FRICTION

Friction is force which opposes relative motion between two surfaces in contact with each other. Coefficient of static friction, μ is defined as the ratio of the limiting frictional force (F) to the normal reaction (R).

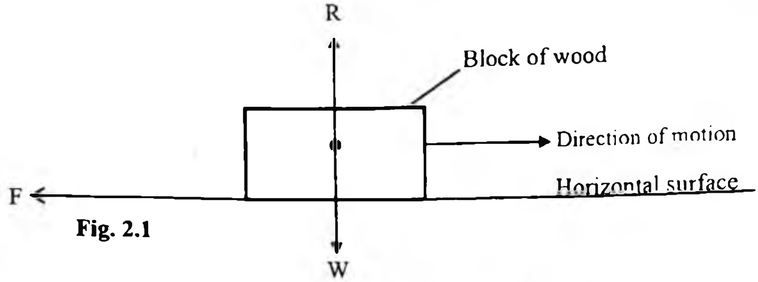


Fig. 2.1

$$\text{Coefficient of static friction} = \mu = \frac{F}{R} = \frac{\textit{Frictional force}}{\textit{Normal reaction}}$$

The normal reaction, R is equal to the weight, W of the block i.e. R = W = mg, where m is the mass of the block in kg and g is the acceleration due to gravity.

However, for a block of wood (or any object) sliding down an inclined plane, AB as shown below the coefficient of friction is obtained as follows:

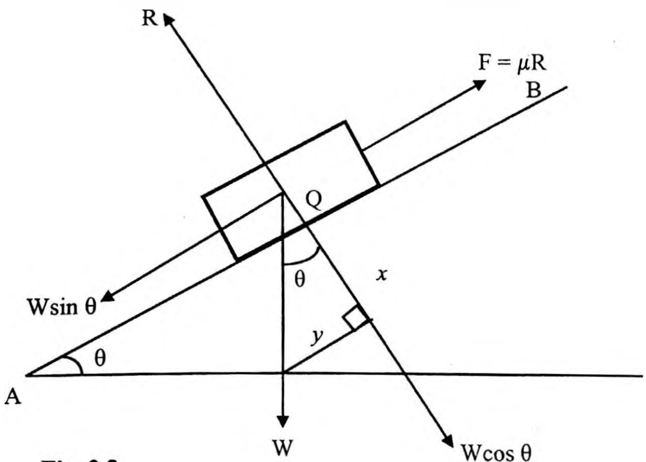


Fig. 2.2

The weight, W, of the block Q acts vertically downwards and can be resolved into two components:

(i) Component y, parallel to the plane: $\sin\theta = \dfrac{y}{W}$ $\quad\therefore\quad$ $y = W\sin\theta$

ii) Component x, perpendicular to the plane: $Cos\,\theta = \dfrac{x}{W}$ $\quad\therefore\quad$ $x = W\cos\theta$

To obtain the coefficient of static friction, μ, the angle of inclination of the plane, θ, is slowly increased until the block just begins to slide down the plane. At this point, the parallel component of the weight, W$\sin\theta$ acting downwards along the plane is equal to the limiting frictional force, F(=μR) acting upwards along the plane.

That is, $F = W\sin\theta$ .... (1)

or $\mu R = W\sin\theta$

Also, on an inclined plane, the normal reaction R is not equal to the weight, W, but to the perpendicular component of the weight, W$\cos\theta$.

That is $R = W\cos\theta$ .... (2)

Combining equations (1) and (2),

The coefficient of friction, $\mu = \frac{F}{R} = \frac{W\sin 6}{W\cos 6}$

Alternatively, $\mu = \operatorname{Tan}\theta = \frac{Opp}{Adj} = \frac{y}{x} = \frac{W\sin 6}{W\cos\theta}$

The angle of inclination, θ, is called the angle of friction.

Example 1

A wooden block of mass 1.6kg rests on a rough horizontal surface. If the limiting frictional force between the block and the surface is 8N, calculate the coefficient of friction ($g=10m/s^2$). *WAEC 2000*

Solution

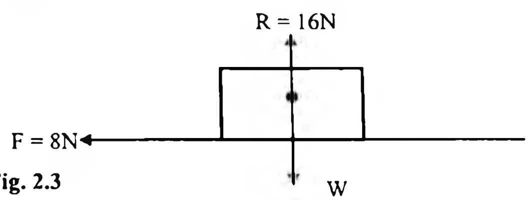


Fig. 2.3

Frictional force, F = 8N; normal reaction, $R = W = mg = 1.6 \times 10 = 16N$

Coefficient of friction, $\mu = \frac{F}{R} = \frac{8N}{16N} = 0.5$

Example 2

Calculate the magnitude of the force required to just move a 20kg object along a horizontal surface if the coefficient of friction is 0.2.

A.400.0N B.40.0N C.4.0N D. 0.4N ($g=10ms^{-2}$) *JAMB 1994*

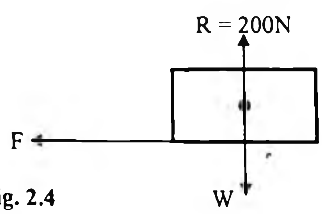


Fig. 2.4

Solution

Coefficient of friction, $\mu = 0.2$; normal reaction, $R = W = mg = 20 \times 10 = 200N$

From $\mu = \frac{F}{R}$, frictional force, $F = \mu R = 0.2 \times 200 = 40N$

Example 3

If an object just begins to slide on a surface inclined at 30° to the horizontal, the coefficient of friction is A. $\sqrt{3}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{1}{\sqrt{2}}$ D. $\frac{1}{\sqrt{3}}$ *JAMB 2003*

Solution

The coefficient of friction for an inclined plane is the tangent of the angle of the plane when an object on the plane *just begins to slide*.

Therefore, coefficient of friction = Tan 30

From trigonometry, Tan 30 = $\frac{1}{\sqrt{3}}$

Example 4

A horizontal force of 45N applied to a crate of mass 9kg is just sufficient to move it. If the crate is now pulled at an angle of 50° to the horizontal, find the force required to move the crate over the horizontal surface. ($g=10m/s^2$)

Solution

The coefficient of friction, μ is found from the first sentence of the question as follows:

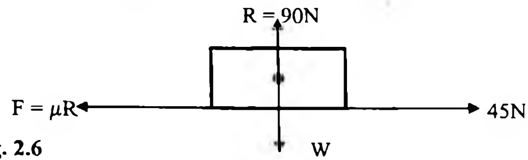


Fig. 2.6

Normal reaction, R = W = mg = 9×10 = 90N

Frictional force, F = 45N; $\theta = 50^0$

Coefficient of friction, $\mu = \frac{F}{R} = \frac{45N}{90N} = 0.5$

The second part of the question is represented below. The pulling force, P has been resolved into its vertical component (Psin θ) and horizontal component (Pcos θ).

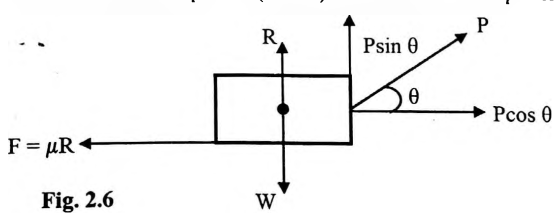


Fig. 2.6

In such a case as this, the normal reaction, R is not equal to the weight, W. The vertical component must also be considered. Therefore,

$$W = R + P\sin\theta \quad \ldots\ldots\ldots\ldots (1)$$

At the point in which the crate moves forward, the horizontal component of the pulling force, Pcos θ is equal to the limiting frictional force, F. Therefore,

$$F = P\cos\theta$$

or $$\mu R = P\cos\theta \quad \ldots\ldots\ldots\ldots (2)$$

Equation 1 and 2 are solved simultaneously to obtain the value of the pulling force, P. Substitute the given and calculated values ($\theta = 50^0$, W = 90N, F = 45N, $\mu = 0.5$) into equation 1 and 2.

$$90 = R + P\sin 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$0.5R = P\cos 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$90 = R + 0.766P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1b)$$

$$0.5R = 0.643P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2b)$$

From (2b), $0.5R = 0.643P$

$$R = \frac{0.643P}{0.5} = 1.286P$$

Substitute $R = 1.286P$ into (1b)

$$90 = R + 0.766P$$

$$90 = 1.286P + 0.766P$$

$$90 = 2.052P$$

$$\therefore P = \frac{90}{2.052} = 43.86\text{N}$$

Example 5

An object of mass 15kg is at the point of sliding down a plane inclined at 36^0 to the horizontal. Find the least force parallel to the plane required to make the object to begin to move up the plane. ($g=10\text{m/s}^2$)

Solution

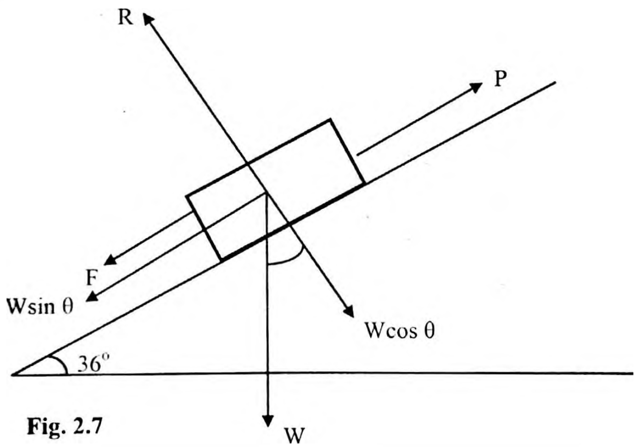


Fig. 2.7

From the question; $\theta = 36^0$; $m = 15\text{kg}$; $g = 10\text{m/s}^2$; $W = mg = 15 \times 10 = 150\text{N}$.

For the object to move up the plane, it must overcome the opposing limiting frictional force, F and the parallel component of the weight, Wsinθ. That is, the upward pulling force, P is

P = Parallel component of weight + Limiting frictional force

$$P = W\sin\theta + F$$

$$= W\sin\theta + \mu R \qquad [F = \mu R]$$

$$P = W\sin\theta + \mu W\cos\theta \qquad [R = W\cos\theta]$$

Coefficient of friction, μ is required to find P

$$\therefore \quad \mu = \frac{W\sin\theta}{W\cos\theta} = \frac{150 \times \sin 36}{150 \times \cos 36} = \frac{150 \times 0.588}{150 \times 0.809} = 0.73$$

Alternatively, the coefficient of friction, μ is calculated on the fact that the angle of inclination, θ at the point of sliding is called the angle of friction. The tangent of this angle of friction gives the coefficient of friction.

$$\mu = \tan\theta = \tan 36 = 0.73$$

Substitute into $P = W\sin\theta + \mu W\cos\theta$

$= 150\sin 36 + 0.73 \times 150 \times \cos 36$

$= 150 \times 0.588 + 0.73 \times 150 \times 0.809$

$= 88.20 + 88.59$

$= 176.79N$

Example 6

A concrete block of mass 25 kg is placed on a wooden plank inclined at an angle of 32° to the horizontal. Calculate the force parallel to the inclined plane that will keep the block at rest if the coefficient of friction between the block and the plank is 0.45.

Solution

$R = W = mg = 25 \times 10 = 250N$; $\theta = 32°$; $\mu = 0.45$

As shown in the figure below, the block is under the influence of 2 parallel and opposite forces:

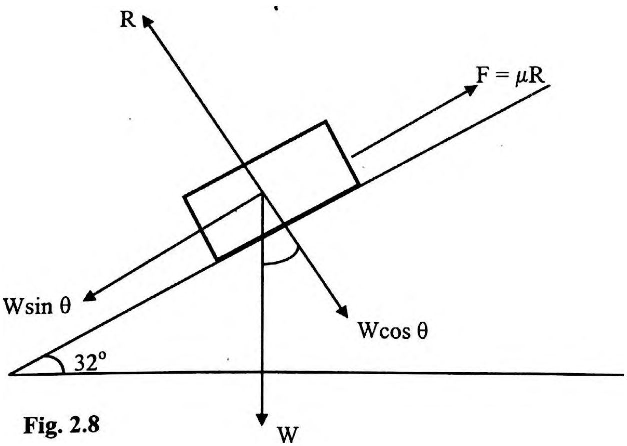


Fig. 2.8

The diagram above can be redrawn as follows:

(i) Wsin θ, the parallel component of the weight of the block acting downwards along the plane.

$$W\sin\theta = 250 \times \sin 32 = 250 \times 0.53 = 132.48N$$

(ii) F ($=\mu R$), the limiting frictional force acting upwards along the plane;

$F = \mu R = \mu W\cos\theta$

$= 0.45 \times 250 \times \cos 32 = 112.5 \times 0.848$

$= 95.41N$

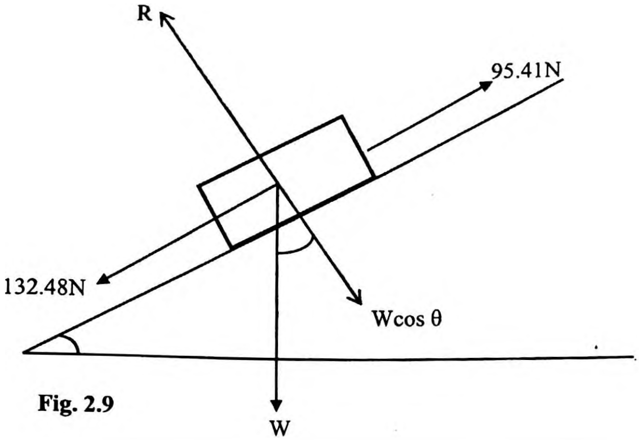


Fig. 2.9

It becomes obvious that the force required to keep the block at rest is the difference between the downward sloping force, Wsinθ (132.48N) and the upward sloping frictional force, $\mu W\cos\theta$ (95.41N);

$$132.48 - 95.41 = 37.07\text{N}$$

Example 7

A body of mass 25kg, moving at $3ms^{-1}$ on a rough horizontal floor is brought to rest after sliding through a distance of 2.50m on the floor. Calculate the coefficient of sliding friction. *WAEC 2005*

Solution

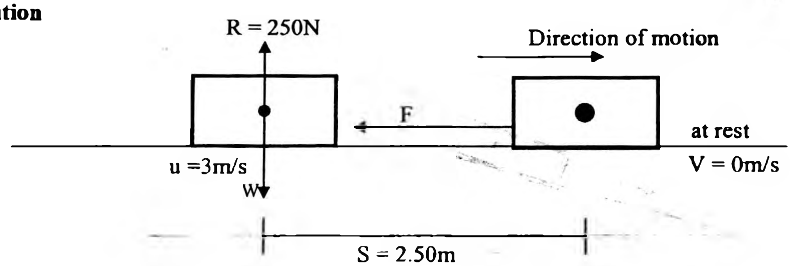


Fig. 2.10

The acceleration (in actual sense, deceleration) is calculated first.
Initial velocity, u = 3m/s; final velocity, v = 0; distance, s = 2.50m.

Substitute into $v^2 = u^2 + 2as$

$$0^2 = 3^2 + 2 \times a \times 2.50$$
$$0 = 9 + 5a$$
$$5a = -9$$
$$a = \frac{-9}{5} = -1.8ms^{-2} \quad \therefore \text{ Deceleration} = 1.8m/s^2$$

Normal reaction, R = W = mg = 25 × 10 = 250N.

Frictional force, F = ma; where m is mass in kg and a is acceleration (deceleration) in m/s^2

$\therefore$ F = ma = 25 × 1.8 = 45N

Coefficient of sliding friction, $\mu = \frac{F}{R} = \frac{45}{250} = 0.18$

Example 8

The diagram above shows a body resting on an inclined plane. If the body slides down the plane, what will be its acceleration? ($g = 10 m/s^2$)

WAEC 1988

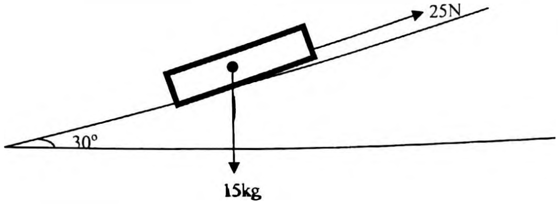


Fig. 2.11

Solution

As shown in the diagram below, $mg\sin\theta$ is the force that makes the object to slide downward the plane, though it is opposed by the 25N frictional force.

$\therefore$ The resultant force, $F = mg\sin\theta - 25N$

where mass, $m = 15kg$; $\theta = 30$; $g = 10m/s^2$

$\therefore$ $F = 15 \times 10 \times \sin 30 - 25$

$= 75 - 25 = 50N$

From $F = ma$, the acceleration $a = \frac{F}{m} = \frac{50}{15} = 3.33 m/s^2$

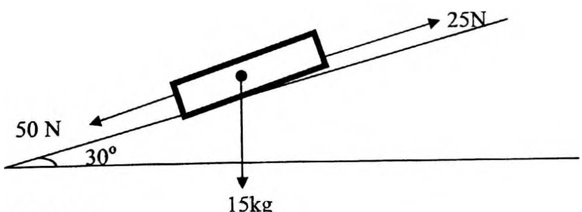


Fig. 2.12

Example 9

Two bodies X and Y of masses 5.0kg and 7.5kg, respectively, are connected by a light inextensible string as illustrated in the diagram above. If X is placed on a rough surface of coefficient of friction 0.5, calculate the magnitude of the:

i. Normal reaction on X
ii. Frictional force between X and the surface,
iii. Tension in the string ($g = 10ms^{-2}$)

NECO 2007[E11]

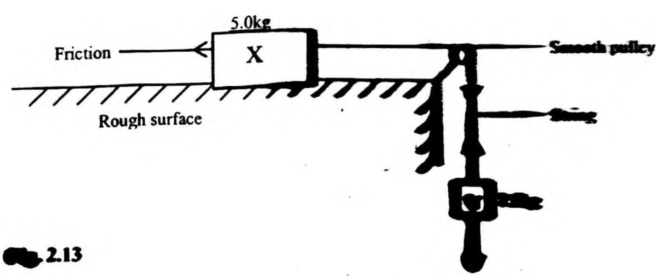


Fig. 2.13

Solution

i. Normal reaction on X is, R = mg

$= 5 \times 10 = 50N$

ii. Frictional Force, $F = \mu R$

$= 0.5 \times 50 = 25N$

iii. For 5kg mass, $T - \mu R = ma$

Substituting, $T - 0.5 \times 50 = 5a$

$T - 25 = 5a$ .... (1)

For 7.5kg mass, $mg - T = ma$

Substituting, (m = 7.5kg), $7.5 \times 10 - T = 7.5a$

$75 - T = 7.5a$

Rearranging $-T + 75 = 7.5a$ .... (2)

Equation (1) and (2) are simultaneous equations and can be solved as follows:

Add equation (1) and (2)

$$\begin{array}{r} T - 25 = 5a \\ -T + 75 = 7.5a \\ \hline 0 + 50 = 12.5a \end{array}$$

$\therefore 50 = 12.5a$

$$acceleration,\ a = \frac{50}{12.5} = 4ms^{-2}$$

The tension T is found by substituting $a = 4ms^{-2}$ into any of equations (1) and (2). Substituting into (1), $T - 25 = 5a$, we obtain

$T - 25 = 5 \times 4$

$\therefore T = 20 + 25 = 45N$

Example 10

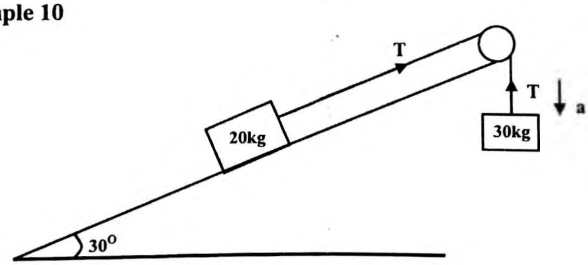


Fig. 2.14

The acceleration of the system shown above is

a. $2ms^{-2}$ b. $4ms^{-2}$ c. $6ms^{-2}$ d. $8ms^{-2}$ *JAMB 1995*

Solution

The equation for the mass on the inclined plane takes into consideration the angle of inclination. The parallel component of the mass is mg sin θ.

For 20kg mass, $T - mg \sin\theta = ma$

Substituting, $T - 20 \times 10 \times \sin 30 = 20a$

$T - 100 = 20a$ .... (1)

For 30kg mass, $mg - T = ma$

$$30 \times 10 - T = 30a$$

$$300 - T = 30a$$

Rearranging $\quad -T + 300 = 30a$

Add equations (1) and (2)

$$\begin{array}{r} T - 100 = 20a \\ -T + 300 = 30a \\ \hline 0 + 200 = 50a \end{array}$$

$$\therefore \quad 200 = 50a$$

$$acceleration, \ a = \frac{200}{50} = 4ms^{-2}$$

Example 11

A motorcycle of mass 100kg moves round in a circle of radius 10m with a velocity of $5ms^{-1}$. Find the coefficient of friction between the road and the tyres.

A. 25.00 B. 2.50 C. 0.50 D. 0.25 ($g = 10m/s^2$) *JAMB 2008'13*

Solution

Mass m = 100kg; radius r = 10m;

velocity v = $5ms^{-1}$, coefficient of friction μ = ?

When an object of mass m moves with a velocity v, the force at work is the inward acting centripetal force (F) given by $F = \frac{mv^2}{r}$

$$\therefore \quad F = \frac{100 \times 5^2}{10} = \frac{100 \times 25}{10} = 250N$$

R = mg = 100 × 10 = 1000N

Coefficient of friction, $\mu = \frac{F}{R} = \frac{250}{1000} = 0.25$

EXERCISE 2.

1. A force of 20N applied parallel to the surface of a horizontal table is just sufficient to make a block of mass 4kg move on the table. Calculate the coefficient of friction between the block and the table ($g=10m/s^2$). Ans: 0.5 *WAEC 1992*

2. A metal block of mass 5kg lies on a rough horizontal platform. If a horizontal force of 8N applied to the block through its center of mass just slides the block on the platform, then the coefficient of limiting friction between the block and the platform is
A. 0.16 B. 0.63 C. 0.80 D. 1.60 E. 2.00 Ans:0.16 *JAMB 1985*

3. A 100Kg box is pushed along a road with a force of 500N. If the box moves with a uniform velocity, the coefficient of friction between the box and the road is

A. 0.5 B. 0.4 C. 1.0 D. 0.8 Ans: 0.5 *JAMB 2004*

4. What is the coefficient of static friction between a load of mass 2kg and a horizontal surface, if the limiting frictional force is 10N. Ans: 0.5 *NECO 2000*

5. A block weighing 15N rests on a flat surface and a horizontal force of 3N is exerted on it. Determine the frictional force on the block. Ans:3N *WAEC 2001*

6. A concrete block of mass 35kg is pulled along a horizontal floor with the aid of a rope inclined at an angle of 30° to the horizontal. If the coefficient of friction is 0.75, calculate the force required to move the block over the floor. Ans: 211.52N

7. A body of mass 10kg rests on a rough inclined plane whose angle of tilt θ is variable. θ is gradually increased until the body starts to slide down the plane at 30°. The coefficient of limiting friction between the body and the plane is?
A. 0.30 B. 0.50 C. 0.58 D. 0.87 Ans :0.58: *JAMB 1990*

8. A force, 10N drags a mass 10kg on a horizontal table with an acceleration of $0.2m/s^2$. If the acceleration due to gravity is $10m/s^2$, the coefficient of friction between the moving mass and the table is? A. 0.02 B. 0.08 C. 0.20 D. 0.80
Ans:0.08 *JAMB 1988*

9. The coefficient of static friction between a 40kg crate and a concrete surface is 0.25. Find the magnitude of the minimum force needed to keep the crate stationary on the concrete base inclined at 45° to the horizontal.
A. 400N B. 300N C. 283N D. 212N ($g = 10ms^{-1}$) Ans :212 N *JAMB 1987*

10. When a box of mass 45kg is given an initial speed of 5m/s, it slides along a horizontal floor a distance of 3m before coming to rest.
What is the coefficient of the kinetic friction between the box and the floor?
A. $\frac{5}{6}$ B. $\frac{5}{12}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$ Ans: 0.417 or $\frac{5}{12}$ *JAMB 1986*

11.

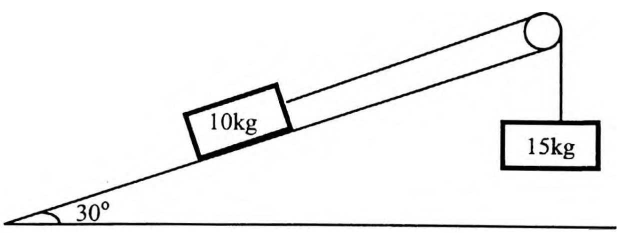


Fig. 2.15

A body of mass 10kg on a smooth inclined plane is connected over a smooth pulley to a mass of 15kg as shown (Fig. 1.13). The acceleration of the system is
A. $\frac{1}{4}g$ B. $\frac{8}{25}g$ C. $\frac{1}{2}g$ D. g E. $\frac{3}{4}g$ Ans :$^{2}/_{5}g$ *JAMB 1982*

12.

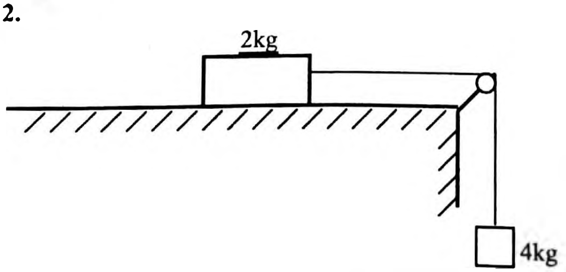


Fig. 2.16

A mass of 2kg on a surface ($\mu = \frac{1}{2}$) is connected to a second mass of 4kg over a frictionless pulley as shown above. If the acceleration due to gravity is $9.8ms^{-2}$- the masses will

A. Accelerate at $4.9 ms^{-2}$
B. Remain stationary
C. Accelerate at $9.8 ms^{-2}$
D. Accelerate at $19.6 ms^{-2}$
E. Accelerate at $39.2 ms^{-2}$ Ans: A *JAMB 1981*

13. An object of mass 80kg is pulled on a horizontal rough ground by a force of 500N. Find the coefficient of static friction. ($g = 10 ms^{-2}$) JAMB 2009[11] Ans: D

A. 0.8 B. 0.4 C. 1.0 D. 0.6

3

ELASTIC PROPERTIES OF SOLIDS

Hooke's law states that the change in length or extension, e, of an elastic body is directly proportional to the applied force or load, F, provided that the elastic limit of the body is not exceeded.

Force α Extension

$F \propto e$

$\therefore F = Ke$

where K is the force constant or stiffness constant of the material in Nm^{-1}.
F is the force or weight in N. F = mg (m is mass in kg and g is acceleration due to gravity in m/s^2)
e is extension or compression of the elastic material in m.

Young Modulus

Young Modulus, γ is defined as the ratio of tensile stress to tensile strain.

$$\gamma = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

Tensile strain is the ratio of the extension of a material to its original length.

$$\text{Strain} = \frac{\text{Change in lenght}}{\text{Original lenght}} = \frac{\text{Extension/Compression}}{\text{Original lenght}} = \frac{e}{l}$$

Tensile strain has no unit.
Tensile stress is the ratio of the force acting on a material to the cross sectional area of the material.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{F}{\pi r^2}$$

Therefore Young Modulus, $\gamma = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{e/l} = \dfrac{Fl}{Ae}$

The S.I unit for γ is N/m^2

Work Done In Springs and Elastic Strings

If an applied force, F caused an elastic spring of original length, l, to undergo an extension or compression, e, then the average force is;

$$= \frac{0+F}{2} = \tfrac{1}{2}F$$

Work done = force × Distance

= average force × Extension

$= {}^{F}/_{2} \times e$

$W = \frac{1}{2}Fe$ or $\frac{1}{2}Ke^2$ $[F = Ke]$

This is the work done by a spring when compressed or extended by a force.
The energy stored in a spring or string is also given by $W = \frac{1}{2}Fe$ or $\frac{1}{2}Ke^2$
The unit of work or energy is Joules, J.

Example 1 An object weighing 0.6N is hung on a spiral spring and causes it to extend by 6.0cm. The object is removed and a block of wood replacing it causes the spring to extend by 10.0cm. What is the weight of the block of wood? *NECO 2002*

Solution

Applied force, F = 0.60N; Extension, e = 6.0cm = 0.06m

$$\therefore \text{ Force constant, } K = \frac{F}{e} = \frac{0.60}{0.06} = 10\,\text{N/m}$$

For e = 10.0cm = 0.1m; and K = 10N/m

Weight of wood, F = Ke = 10 × 0.1 = 1.0N

Example 2

A load of 5N gives an extension of 0.56cm in a wire which obeys Hooke's law. What is the extension caused by a load of 20N?

A. 1.12cm B. 2.14cm C. 2.24cm D. 2.52cm *JAMB 1987*

Solution

Applied load, F = 5N; Extension, e = 0.56cm

$$\text{Force constant, } K = \frac{F}{e} = \frac{5\text{N}}{0.56\text{cm}} = 8.93\text{N/cm}$$

For F = 20N and K = 8.93N/cm

$$\text{Extension } e = \frac{F}{K} = \frac{20\text{N}}{8.93\text{N/cm}} = 2.24\text{cm}$$

Example 3

Use the following data to determine the length, L of a wire when a force of 30N is applied, assuming Hooke's law is obeyed.

Force applied(N)	0	5	10	30
Length of wire (mm)	500.0	500.5	501.0	L

Solution

Change in force applied, F = 5 – 0 = 5N

Change in length, e = 500.5 – 500.0 = 0.5mm

$$\text{From, } F = Ke\text{, force constant, } K = \frac{F}{e} = \frac{5\text{N}}{0.5\text{mm}} = 10\text{N/mm}$$

For F = 30N;

$$\text{Extension, } e = \frac{F}{K} = \frac{30\text{N}}{10\text{N/mm}} = 3\text{mm}$$

L = Original length + Extension

= 500.0mm + 3mm = 503.0mm

Example 4

A spiral spring balance is 25.0cm long when 5N hangs on it and 30.0cm when the weight is 10N. What is the length of the spring if the weight is 2N, assuming Hooke's law is obeyed? *NECO 2000*

Solution

Let the original length of the spiral spring be L

Extension, e = New length resulting from applied force – Original length

First case: extension, e = 25.0cm – L

applied force, F = 5N

2[nd] case: extension, e = 30.0cm – L

applied force, F = 10N

Substitute into F = Ke for 1[st] and 2[nd] cases.

1[st] $5 = (25 - L)K$ ....(1)

2[nd] $10 = (30 - L)K$ ....(2)

From (1), $K = \frac{5}{25 - L}$ .... (3)

From (2), $K = \frac{10}{30 - L}$ ....(4)

Equate (3) and (4): $\frac{5}{25 - L} = \frac{10}{30 - L}$

Cross multiplying,
$$5(30-L) = 10(25-L)$$
$$150 - 5L = 250 - 10L$$
$$10L - 5L = 250 - 150$$
$$5L = 100$$
$$L = \frac{100}{5} = 20cm$$

Considering the 1[st] case;

Extension, e = 25.0 – L = 25 – 20 = 5.0cm = 0.05m

Applied force, F = 5N

From, F = Ke, force constant, $K = \frac{F}{e} = \frac{5}{0.05} = 100Nm^{-1}$

For F = 2N and K = 100N/m

Extension, $e = \frac{F}{K} = \frac{2}{100} = 0.02m = 2cm$

∴ New length of spring = Original length (L) + Extension (e)
= 20cm + 2cm
= 22cm or 0.22m

Example 5

A spiral spring of natural length 20.00cm has a scale pan hanging freely in its lower end. When an object of mass of 40g is placed in the pan, its length becomes 21.80cm. When another object of mass 60g is placed in the pan, the length becomes 22.05cm. Calculate the mass of the scale pan. *WAEC 2001*

Solution

The extension of the spring is caused by both the mass of the object and the mass of the scale pan. Let M be the mass of the scale pan.

1[st] case: extension, e = 21.80 – 20.00 = 1.8cm

mass, m = M + m = M + 40

2^{nd} case: extension, $e = 22.05 - 20.00 = 2.05$cm

mass, $m = M + m = M + 60$

$F = Ke$ or $mg = Ke$ [F=mg]

Substitute into $mg = K e$ for 1^{st} and 2^{nd} cases

1^{st} $(M + 40)g = 1.8K$ .... (1)

2^{nd} $(M + 60)g = 2.05K$............................ (2)

From (1) $g = \frac{1.8K}{M+40}$ .... (3)

From (2) $g = \frac{2.05K}{M+60}$ .... (4)

Equate (3) and (4): $\frac{1.8K}{M+40} = \frac{2.05K}{M+60}$

Rearranging: $\frac{M+40}{M+60} = \frac{1.8K}{2.05K}$

$$\frac{M+40}{M+60} = \frac{1.8}{2.05}$$

Cross multiplying, $1.8(M + 60) = 2.05(M + 40)$

$$1.8M + 108 = 2.05M + 82$$

$$2.05 - 1.8M = 108 - 82$$

$$0.25M = 26$$

$$M = \frac{26}{0.25} = 104g$$

Example 6

A force of 40N applied at the end of a wire of length 4m and diameter 2.00mm process and extension of 0.24mm. Calculate the;

(a) Stress on the wire (b) strain in the wire

(c) Young's Modulus for the material of the wire (π =3.14). *WAEC 2003*

Solution

(a) $\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{\pi r^2}$

radius, r = diameter/2 $\therefore$ r = 2.00mm/2 = 1mm = 0.001m

For F = 40N and r = 0.001m

$$\text{Stress} = \frac{40}{3.14 \times (0.001)^2} = 12.74 \times 10^6 \text{ N/m}^2$$

(b) $\text{Strain} = \frac{\text{extension}}{\text{original lenght}} = \frac{e}{l}$

$l = 4$m; $e = 0.24\text{mm} = \frac{0.24}{1000} = 2.4 \times 10^{-4}$ m

$$\therefore \text{Strain} = \frac{2.4 \times 10^{-4}}{4} = 6.0 \times 10^{-5}$$

(c) Young Modulus, $\gamma = \frac{\text{stress}}{\text{strain}} = \frac{12.74 \times 10^6}{6.0 \times 10^{-5}} = 2.12 \times 10^{11} \text{N/m}^2$

Example 7

A force of magnitude 500N is applied to the free end of a spiral spring of force constant $1.0 \times 10^4 Nm^{-1}$. Calculate the energy stored in the stretched spring. *WAEC 2005*

Solution

The energy stored in the spring is equal to the work done, W = ½ Fe or ½Ke^2

Force, F = 500N; force constant, K = $1.0 \times 10^4 Nm^{-1}$

From F = Ke, Extension, $e = \frac{F}{K}$

$$\therefore e = \frac{500}{1.0 \times 10^4} = 0.05m$$

Energy stored, $W = ½Fe = ½ \times 500 \times 0.05 = 12.5J$

or $W = ½Ke^2 = ½ \times 1.0 \times 10^4 \times 0.05^2 = 12.5J$

Example 8

A catapult used to hold a stone of mass 500g is extended by 20cm with an applied force F. If the stone leaves with a velocity of $40ms^{-1}$, the value of F is

A. 4.0×10^4N B. 4.0×10^3N C. 2.0×10^3N D. 4.0×10^2N *JAMB 2000*

Solution

The work done in stretching the catapult is equal to the kinetic energy gained by the stone. i.e. ½ Fe = ½ mv^2

Extension, e = 20cm = 0.2m; velocity, v = 40m/s; mass, m = 500g = 0.5kg.

Substituting,

$$\therefore ½ \times F \times 0.2 = ½ \times 0.5 \times 40^2$$

$$0.2F = 0.5 \times 1600$$

$$F = \frac{800}{0.2} = 4000N \text{ or } 4 \times 10^3 N$$

Example 9

On top of a spiral spring of force constant $500Nm^{-1}$ is placed a mass of $5 \times 10^{-3}kg$. If the spring is compressed downwards by a length of 0.02m and then released, calculate the height to which the mass is projected. ($g = 10m^{-2}$)

A. 8m B. 4m C. 2m D. 1m Ans: 2m *JAMB 2003*

Solution

The work done in compressing the spring is equal to the potential energy stored in the spring.

$$\therefore ½Ke^2 = mgh$$

Force constant, K = $500Nm^{-1}$; extension, e = 0.02m;

Mass, m= $5 \times 10^{-3}kg$; g = 10m/s

Substituting

$$½ \times 500 \times (0.02)^2 = 5 \times 10^{-3} \times 10 \times h$$

$$0.1 = 0.05h$$

$$\therefore h = \frac{0.1}{0.05} = 2m$$

EXERCISES 3

1. A force of 1.8N extends a wire by 0.4cm. What force will extend the wire by 1.25cm if the elastic limit is not exceeded? Ans: 5.625N *NECO 2004*

2. If a force of 50N stretches a wire from 20m to 20.01m, what is the amount of force required to stretch the same material from 20m to 20.05m
A. 100N B. 50N C. 250N D. 200N Ans: 250N *JAMB 2004*

3. The extension in a spring when 5g wt was hung from it was 0.56cm. If Hooke's law is obeyed, what is the extension caused by a load of 20g wt?
A. 1.12cm B. 2.14cm C. 2.52cm D. 2.80cm Ans: 2.24cm *JAMB 1981*

4. A 10g mass placed on the pan of a spring balance causes an extension of 5cm. If a 15g mass is placed on the pan of the same spring balance the extension is?
A. 3.3cm B. 6.5cm C. 7.5cm D. 10.8cm E. 15.0 cm
Ans: 7.5 cm *JAMB 1982*

5. A spiral spring extends from a length of 10.00cm to 10.01cm when a force of 20N is applied on it. Calculate the force constant of the spring.
Ans: $2 \times 10^5 Nm^{-1}$ *WAEC 2005*

6. The total length of a spring when a mass of 20g is hung from its end is 14cm, while its total length is 16cm when a mass of 30g is hung from the same end. Calculate the unstretched length of the spring assuming Hooke's law is obeyed.
A.9.33cm B.10.00cm C.10.66cm D.12.00cm E.15.00
Ans:10.00cm *JAMB 1983*

7. A force of 100N stretches an elastic string to a total length of 20cm. If an additional force of 100N stretches the string 5cm further, find the natural length of the spring. A.15cm B.12cm C.10cm D.8cm E.5cm Ans:15cm *JAMB 1985*

8. The spiral spring of a spring balance is 25.0cm long when 5N hangs on it and 30.0cm long when the weight is 10N. What is the length of the spring if the weight is 3N assuming Hooke's law is obeyed? A. 15.0cm B. 17.0cm C. 20.0cm D. 23.0cm Ans:23.0cm *JAMB 1991*

9. A force of 15N stretches a spring to a total length of 30cm. An additional force of 10N stretches the spring 5cm further. Find the natural length of the spring.
A. 25.0cm B. 22.5cm C. 20.0cm D. 15.0cm Ans: 22.5cm *JAMB 1994*

10. A piece of rubber 10cm long stretches 6mm when a load of 100N is hung from it. What is the strain? A.60 B.6 C.6×10^{-2} D.6×10^{-3} E. 0.6
Ans:6×10^{-2} *JAMB 1978*

11. A load of 20N on a wire of cross-sectional area $8 \times 10^{-7} m^2$, produces an extension of 10^{-4}m. Calculate Young's Modulus for the material of the wire if its length is 3m.
A. $7.0 \times 10^{11} Nm^{-2}$ B. $7.5 \times 10^{11} Nm^{-2}$ C. $8.5 \times 10^{11} Nm^{-2}$ D. $9.0 \times 10^{11} Nm^{-2}$
Ans: $7.5 \times 10^{11} Nm^{-2}$ *JAMB 1997*

12. The tendon in a man's leg is 0.01m long. If a force of 5N stretches the tendon by 2.0×10^{-5}m, calculate the strain on the muscle.
A. 5×10^6 B. 5×10^2 C. 2×10^{-3} D. 2×10^{-7} Ans: 2×10^{-3} *JAMB 1998*

13. If the stress on a wire is $10^7 Nm^{-2}$ and the wire is stretched from its original length of 10.0cm to 10.05cm. The Young's Modulus of the wire is
A. $5.0 \times 10^4 Nm^{-2}$ B. $5.0 \times 10^5 Nm^{-2}$ C. $2.0 \times 10^8 Nm^{-2}$ D. $2.0 \times 10^9 Nm^{-2}$
Ans: $2 \times 10^9 N/m^2$ *JAMB 1999*

14. A spiral spring of natural length 30.0cm and force constant of $20Nm^{-1}$ is compressed to 20.0cm. Calculate the energy stored in the spring. Ans: 0.1J *WAEC 2002*

15. An elastic string of force constant $200Nm^{-1}$ is stretched through 0.8m within its elastic limit. Calculate the energy stored in the string. Ans: 64J *WAEC 2004*

16. A spring of force constant $1500Nm^{-1}$ is acted upon by a constant force of 75N. Calculate the potential energy stored in the spring. A. 1.9J B. 3.2J C. 3.8J D. 5.0J
Ans: 1.9J *JAMB 1991*

17. When a force of 50N is applied to the free end of an elastic cord, an extension of 4cm is produced in the cord. Calculate the work done on the cord.
Ans:1.0J *WAEC 2004*

18. A spring of length 25cm is extended to 30cm by a load of 150N attached to one of its ends. What is the energy stored in the spring?
A. 3750J B. 2500J C. 3.75J D. 2.50J Ans: 3.75J *JAMB 1994*

19. The energy contained in a wire when it is extended by 0.02m by a force of 500N is
A. 5J B. 10J C. 10^3J D. 10^4J Ans: 5J *JAMB 1995*

20. A spiral spring is compressed by 0.03m. Calculate the energy stored in the spring if its force constant is $300Nm^{-1}$. Ans: 0.135J *WAEC 2004*

21. Calculate the work done to stretch an elastic string by 50cm, if a force of 12.5N produces an extension of 5cm in it. Ans: 31.250J *NECO 2007*

22. A spring of force constant $500Nm^{-1}$ is compressed such that its length shortens by 5cm. The energy stored in the spring is
A. 0.625J B. 6.250J C. 62.500J D. 625.000J *JAMB 2008* Ans: 0.625J

23. Use the data in the table below to determine the length X of a wire asumming Hooke's law is obeyed

Force applied/N	0.0	5.0	10.0	50.0
Length of wire/mm	800.0	801.0	802.0	X

NECO2008 Ans:810.0mm

24. A wire of length 5.0m and diameter 2.0mm extends by 0.25mm when a force of 50N was used to stretch it from its end. Calculate the
(a) Stress on the wire
(b) Strain in the wire (π=3.142)
Ans: a. $1.59 \times 10^7Nm^{-2}$ b. 5×10^{-5} *WAEC 2007*

25.

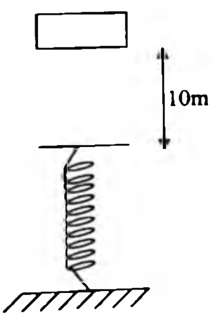


Fig. 3.1

A 50.0kg block is dropped on a spring from a point 10m above (above figure). If the force constant of the spring is $4.0 \times 10^4Nm^{-1}$, find the maximum compression of the spring [g=10m/s].
A. 1.25m B. 0.50m C. 0.25m D. 0.05m Ans: 0.5m *JAMB 1987*

26. A string of length 4m is extended by 0.02m when a load of 0.4kg is suspended at its end. What will be the length of the string when the applied force is 15N? ($g = 10ms^{-2}$) A. 6.08m B. 5.05m C. 4.08m D. 4.05m
Ans: 4.08m *JAMB 2007*

27. The diagram below shows the force extension curve of a piece of wire. The energy stored when the wire is stretched from **E** to **F** is? A. 1.5×10^{-2}J B. 7.5×10^{-1}J C. 7.5×10^{-3}J D. 2.5×10^{-3}J Ans: 2.5×10^{-3}J *JAMB 2002*

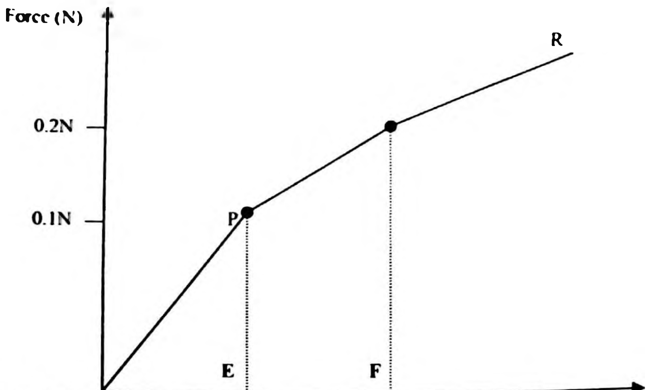


Fig. 3.2

28. If a load of mass 10 N stretches a cord by 1.2cm, what is the total work done?
A. 6.0×10^{-2}J B. 7.6×10^{-2}J C. 1.8×10^{-2}J D. 6.6×10^{-2}J

JAMB 2009[16] Ans: A

29. The diagram below represents the graph of the force applied in stretching a spiral against the corresponding extension (X). The force constant of the spring is

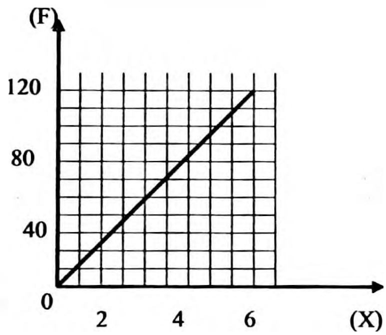


Fig. 3.3

A. 20 Nm^{-1} B. 40 Nm^{-1} C. 30 Nm^{-1} D. 10 Nm^{-1}

JAMB 2009[15] Ans: A

30. The work done in extending a spring by 40 mm is 1.52J. Calculate the elastic constant of the spring. NECO 2009[53] Ans: 1900.0Nm^{-1}

4

LINEAR, AREA AND CUBIC EXPANSIVITIES

LINEAR EXPANSIVITY

The linear expansivity or coefficient of linear expansion α of a substance is defined as the fractional increase in length of a piece of that substance per degree rise in temperature.

$$\text{Linear expansivity, } \alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{Temperature rise}}$$

$$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)} = \frac{\Delta l}{l_1 \Delta\theta}$$

where α = linear expansivity
l_1 = original length before expansion
l_2 = final length after expansion
θ_1 = initial temperature before expansion
θ_2 = final temperature after expansion
Δl = increase in length or change in length $(l_2 - l_1)$
$\Delta 0$ = temperature rise or change in temperature, $(\theta_2 - \theta_1)$

The S.I. unit of linear expansivity is per Kelvin (K^{-1}).

Example 1

Steel bars, each of length 3m at 29°C are to be used for constructing a rail line. If the linear expansivity of steel is, $1.0 \times 10^{-5}K^{-1}$, calculate the safety gap that must be left between successive bars if the highest temperature expected is 41°C. *WAEC 1989*

Solution

The safety gap to be left is found by calculating the change in length.

From $\alpha = \dfrac{\Delta l}{l_1(\theta_2 - \theta_1)}$ Change in length, $\Delta l = \alpha l_1(\theta_2 - \theta_1)$

Given: linear expansivity, $\alpha = 1.0 \times 10^{-5}K^{-1}$ Original length, $l_1 = 3m$
Initial temperature, $\theta_1 = 29°C$ Final temperature, $\theta_2 = 41°C$

$$\Delta l = 1.0 \times 10^{-5} \times 3(41 - 29)$$
$$= 0.00003(12)$$
$$= 0.00036m \text{ or } 3.6 \times 10^{-4}m$$

Example 2

A metal rod of length 50cm is heated from 40 °C to 80 °C. If the linear expansivity of the material is **α**, calculate the increase in length of the rod (in meters) in terms of **α**.

WAEC 2008'19

Solution

Given: Original length, l_1 = 50cm = 0.5m Initial temperature, $\theta_1 = 40°C$
Final temperature, $\theta_2 = 80°C$ Linear expansivity, $\alpha = \alpha$

From $\alpha = \dfrac{\Delta l}{l_1(\theta_2 - \theta_1)}$

Increase in length, $\Delta l = \alpha \times l_1(\theta_2 - \theta_1) = \alpha \times 0.5(80 - 40) = \alpha \times 0.5 \times 40 = 20\alpha$

Example 3

A metal rod of length 100cm, is heated through 100°C, calculate the change in length of the rod. (Linear expansivity of the material of the rod is $3 \times 10^{-5}K^{-1}$) *WAEC 2003*

Solution

Given: Original length, l_1 = 100m; temperature change ,$\Delta\theta_1$ = 100°C;

Linear expansivity, $\alpha = 3.0 \times 10^{-5}K^{-1}$

From $\alpha = \dfrac{\Delta l}{l_1 \Delta\theta}$

Change in length, $\Delta l = \alpha \times l_1 \times \Delta\theta$

$= 3.0 \times 10^{-5} \times 100 \times 100 = 0.3\text{cm}$

Example 4

A bridge made of steel is 600m long. What is the daily variation in its length if the night-time and day-time temperature are 10°C and 35°C respectively. The linear expansivity of steel is $0.000012°C^{-1}$

A. 0.18cm B.1.80cm C.18.0cm D.1800cm *JAMB 1992 Ans: 18.00cm*

Solution

Daily variation is the same as change in length or increase in length.

Given: Original length, l_1 = 600m; final temperature, θ_2 = 35°C

Initial temperature, θ_1 = 10°C; linear expansivity, $\alpha = 0.000012°C^{-1}$

$$\alpha = \frac{\Delta l}{l_1(\theta_2 - \theta_1)}$$

$$0.000012 = \frac{\Delta l}{600(35-10)} \quad \therefore \quad \Delta l = 0.000012 \times 600 \times 25 = 0.18\text{m} = 18\text{cm}$$

Example 5

A metal rod of length 40.00cm at 20°C is heated to a temperature of 45°C. If the new length of the rod is 40.05cm, calculate its linear expansivity. *WAEC 1994*

Solution

initial length, l_1 = 40.00cm final length, l_2 = 40.05cm

initial temperature, θ_1 = 20°C final temperature, θ_2 = 45°C

Linear expansivity, $\alpha = \dfrac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$

$$\alpha = \frac{40.05 - 40.00}{40.0(45-20)} = \frac{0.05}{40 \times 25} = 0.00005 = 5 \times 10^{-5}k^{-1}$$

Example 6

A brass rod is 2m long at a certain temperature. What is the length for a temperature rise of 100K, if the expansivity of brass is $18 \times 10^{-6}K^{-1}$

A. 2.0036m B. 2.0018m C. 2.1800m D. 2.0360m

JAMB 1991

Solution

Initial length, l_1 = 2m; temperature rise, $\Delta\theta$ = 100K; $\alpha = 18 \times 10^{-6}K^{-1}$

Substitutcinto $\alpha = \dfrac{l_2 - l_1}{l_1 \Delta\theta}$

$$18 \times 10^{-6} = \frac{l_2 - 2}{2 \times 100} \quad . \quad l_2 - 2 = 18 \times 10^{-6} \times 200 = 36 \times 10^{-4}$$

$$l_2 = 2 + 36\times10^{-4} = 2.0036m$$

Example 7

The linear expansivity of a metal P is twice that of another metal Q. When these material are heated through the same temperature change, their increase in length is the same. Calculate the ratio of the original length of P to that of Q. *WAEC 1996*

Solution

Linear expansivity of Q is α

Linear expansivity of P is 2α

Linear expansivity, $\alpha = \frac{\Delta l}{l_1 \Delta\theta}$ .... (1)

Where Δl = change in length.

l_1 = original length

$\Delta\theta$ = change in temperature

Substitute into equation (1) for Q where l_Q is original length of Q;

$$\alpha = \frac{\Delta l}{l_Q \Delta\theta} \quad \text{.................... (2)}$$

Substitute into equation (1) for P where l_P is original length of P;

$$2\alpha = \frac{\Delta l}{l_P \Delta\theta} \quad \text{...................... (3)}$$

Note that Δl and $\Delta\theta$ are the same in both cases (eqn 2 & 3) because temperature and length increase are the same.

From eqn (2), original length of Q, $l_Q = \frac{\Delta l}{\alpha\Delta\theta}$

From eqn (3), original length of P, $l_P = \frac{\Delta l}{2\alpha\Delta\theta}$

$$\therefore \quad \text{Ratio of } l_P : l_Q = \frac{\Delta l}{2\alpha\Delta\theta} : \frac{\Delta l}{\alpha\Delta\theta}$$

$$= \frac{\cancel{\Delta l}}{2\cancel{\alpha}\Delta\theta} : \frac{\cancel{\Delta l}}{\cancel{\alpha}\Delta\theta}$$

$$\frac{1}{2} \quad : \quad 1$$

Multiply both sides by 2

$\therefore$ Ratio of original length of P to that of Q = 1 : 2

Example 8

The ratio of the coefficient of linear expansion of two metals $\frac{\alpha_1}{\alpha_2}$ is 3:4. If, when heated through the same temperature change, the ratio of the increase in lengths of the two metals, $\frac{\ell_1}{\ell_2}$ is 1:2, the ratio of the original lengths $\frac{l_1}{l_2}$ is

A. ½ B. ⅜ C. ⅔ D. ⅓ JAMB 2007

Solution

	Metal 1	Metal 2
Coefficient of expansion α	$\alpha_1 = 3$	$\alpha_2 = 4$
Tempt change Δθ (Δθ is the same)	$\Delta\theta = \theta$	$\Delta\theta = \theta$
Increase in length Δl	$\Delta l = \ell_1 = 1$	$\Delta l = \ell_2 = 2$

From $\alpha = \frac{\Delta l}{l_o \Delta\theta}$ (l_o = original length)

Original length, $l_o = \frac{\Delta l}{\alpha\Delta\theta}$ $\qquad l_1 = \frac{1}{3\theta} \qquad l_2 = \frac{2}{4\theta}$

∴ Ratio of original length, $\frac{l_1}{l_2} = \frac{1}{3\theta} \div \frac{2}{4\theta} = \frac{1}{3\theta} \times \frac{4\theta}{2} = \frac{2}{3}$

AREA AND CUBIC EXPANSIVITIES

Area or superficial expansivity (β) of a substance is defined as the fractional increase in area of a piece of that substance per degree rise in temperature.

Area expansivity, $\beta = \frac{\text{increase in area}}{\text{original area} \times \text{temperature rise}}$

$$\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)} = \frac{\Delta A}{A_1(\Delta\theta)}$$

Also, area expansivity, (β) = 2 × linear expansivity (α)

That is, β = 2α

Volume or cubic expansivity (γ) of a substance is defined as the fractional increase in volume of a piece of that substance per degree rise in temperature.

Cubic expansivity, $\gamma = \frac{\text{increase in volume}}{\text{original volume} \times \text{temperature rise}}$

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} = \frac{\Delta V}{V_1(\Delta\theta)}$$

Also, cubic expansivity (γ) = 3 × linear expansivity (α)

That is, γ = 3α

Where A_1 = original area or area at temperature θ_1

A_2 = final area or area at temperature θ_2

ΔA = change in area or increase in area $(A_2 - A_1)$

V_1 = original volume or volume at initial temperature θ_1

V_2 = final volume or volume at final temperature θ_2

ΔV = change in volume or increase in volume $(V_2 - V_1)$

Both area and volume expansivity are measured in K^{-1}

REAL AND APPARENT EXPANSIVITIES

Apparent cubic expansivity (γ_a) of a liquid is the increase in volume per unit volume per degree rise in temperature when the expansion of the vessel is not considered.

Apparent volume expansivity, $\gamma_a = \dfrac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temperature}}$

Also,

$$\gamma_a = \frac{\text{mass of liquid expelled}}{\text{Mass of liquid remaining} \times \text{rise in temperature}}$$

Or

$$\gamma_a = \frac{\text{volume of liquid expelled}}{\text{volume of liquid remaining} \times \text{rise in temperature}}$$

Real or absolute cubic expansivity of a liquid (γ_r) is defined as the actual increase in volume per unit volume per degree rise in temperature, when the expansion of the vessel is taken into consideration.

Real cubic expansivity (γ_r) of the liquid	=	Apparent cubic expansivity of liquid, γ_a	+	Cubic expansivity of the container or vessel γ

i.e. $$\gamma_r = \gamma_a + \gamma$$

Example 9

A solid metal cube of side 10cm, is heated from 10°C to 60°C. If the linear expansivity of the metal is $1.2 \times 10^{-5}K^{-1}$, calculate the increase in its volume. WAEC 1993

Solution

Original length, l_1 = 10cm, :. original volume, $V_1 = l_1 \times l_1 \times l_1 = 10 \times 10 \times 10 = 1000cm^3$

Initial temperature, θ_1 = 10°C; final temperature, θ_2 = 60°C; $\alpha = 1.2 \times 10^{-5}K^{-1}$

Cubic expansivity, $\gamma = 3\alpha = 3(1.2 \times 10^{-5})$
$= 3.6 \times 10^{-5}\ K^{-1}$

$\gamma = \dfrac{\Delta V}{V_1(\theta_2 - \theta_1)}$:. Increase in volume, $\Delta V = \gamma \times V_1(\theta_2 - \theta_1)$

$= 3.6 \times 10^{-5} \times 1000 \times (60 - 10)$
$= 3.6 \times 10^{-2} \times 50$
$= 1.8\ cm^3$

Example 10

A piece of brass of mass 170kg has its temperature raised from 0°C to 30°C. Calculate its increase in volume, given the density of brass at 0°C as $8.5 \times 10^3 kgm^{-3}$ and its cubic expansivity as $5.7 \times 10^{-5}K^{-1}$ *WAEC 1998*

Solution

Original temperature, θ_1 = 0°C; Final temperature, θ_2 = 30°C;
Cubic expansivity, $\gamma = 5.7 \times 10^{-5}K^{-1}$

The original volume, V_1 is calculated from this equation

$Density = \dfrac{\text{mass}}{\text{volume}}$ $\therefore$ $volume, (V_1) = \dfrac{mass}{density}$

Given: mass = 170kg; density = $8.5 \times 10^3 kgm^{-3}$

$$V_1 = \frac{170}{8.53 \times 10^3} = 0.02m^3$$

$\gamma = \dfrac{\Delta V}{V_1(\theta_2 - \theta_1)}$:. Increase in volume, $\Delta V = \gamma \times V_1(\theta_2 - \theta_1)$

$= 5.7 \times 10^{-5} \times 0.02 \times (30 - 0)$
$= 3.42 \times 10^{-5}m^3$

Example 11

The linear expansivity of brass is 2 x $10^{-5}C^{-1}$. If the volume of a piece of brass is 15.00cm^3 at 0°C, what is the volume at 100°C? *JAMB1998*

A. 16.03cm^3 B. 16.00cm^3 C.15.09cm^3 D. 15.03cm^3

Solution

Given: $\alpha = 2 \times 10^{-5}C^{-1}$; :. Cubic expansivity, $\gamma = 3\alpha = 3(2 \times 10^{-5}) = 6 \times 10^{-5}$ °C

Original volume, V_1 = 15.00cm^3; Initial temperature, θ_1 = 0°C;

final temperature, θ_2 = 100°C; final volume V_2 = ?

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)}$$

$$6\times10^{-5} = \frac{V_2 - 15}{15(100-0)}$$

$$V_2 - 15 = 6\times10^{-5}\times15\times100$$

$$V_2 - 15 = 0.09$$

$$V_2 = 15 + 0.09 = 15.09cm^3$$

Example 12

A rectangular metal block of volume $10^{-6}m^3$ at 273K is heated to 573K. If its coefficient of linear expansion is 1.2 x $10^{-5}K^{-1}$, the percentage change of its volume is? *JAMB 1994*

Solution

original volume,V_1 = $10^{-6}m^3$; initial temperature,θ_1 = 273K; final temperature, θ_2 = 573K; $\alpha = 1.2 \times 10^{-5}K^{-1}$:. Cubic expansivity, $\gamma = 3\alpha$; Change in volume, ΔV = ?

From $$\gamma = \frac{\Delta V}{V_1(\theta_2 - \theta_1)}$$

change in volume, $$\Delta V = \gamma \times V_1(\theta_2 - \theta_1)$$
$$= 3.6 \times 10^{-5} \times 10^{-6} (573 - 273)$$
$$= 3.6 \times 10^{-11} \times 300$$
$$= 1.08 \times 10^{-8}m^3$$

$$\text{Percentage change in volume} = \frac{\text{change in volume}}{\text{original volume}} \times 100$$

$$= \frac{\Delta V}{V_1} \times 100$$

$$= \frac{1.08\times10^{-8}}{10^{-6}} \times 100$$

$$= 1.08 \approx 1.1\%$$

Example 13

A density glass bottle contains 44.25g of a liquid at 0°C and 42.02 at 50°C. Calculate the real cubic expansivity of the liquid (linear expansivity of glass = 1.0 x $10^{-5}K^{-1}$)

Solution

The linear expansivity, $\alpha = 1.2 \times 10^{-5}K^{-1}$

:. The cubic expansivity, $\gamma = 3\alpha = 3 \times (1.0 \times 10^{-5}K^{-1}) = 3 \times 10^{-5}K^{-1}$

Next the apparent expansivity, γ_a, is found first.

$$\gamma_a = \frac{\text{mass of liquid expelled}}{\text{mass of liquid remaining} \times \text{temperature rise}}$$

Mass of liquid expelled = mass of liquid at initial tempt – mass of liquid at final tempt.

Mass of liquid remaining is the mass of liquid at the final temperature.

$$\therefore \quad \gamma_a = \frac{44.25 - 42.02}{42.02 \times (50 - 0)} = \frac{2.23}{2101} = 1.062 \times 10^{-3}$$

Real expansivity of the liquid, $\gamma_r = \gamma + \gamma_a$

$$= 3 \times 10^{-5} + 1.062 \times 10^{-3}$$
$$= 1.0913 \times 10^{-3} K^{-1}$$

Example 14

A relative density bottle of volume 50cm^3 is completely filled with a liquid at 30°C. It is then heated to 80°C such that 0.75cm^3 of the liquid is expelled. Calculate the apparent cubic expansivity of the liquid. WAEC 2007[23]

Solution

Apparent cubic expansivity, $\gamma_a = \dfrac{\text{volume of liquid expelled}}{\text{volume of liquid remaining} \times \text{rise in temperature}}$

$$\gamma_a = \frac{0.75}{(50 - 0.75) \times (80 - 30)}$$

$$\gamma_a = \frac{0.75}{49.25 \times 50} = 0.0003 K^{-1}$$

Example 15

A metal cube of linear expansivity α is heated through a temperature rise of *t*. If the initial volume of the cube is *v*, what is the increase in the volume of the cube? *NECO 2007*[19]

Solution

Linear expansivity, α = α; temperature rise, Δθ = *t*; initial volume, $V_1 = v$

Cubic expansivity, $\gamma = 3\alpha$

$$\gamma = \frac{\Delta V}{V_1(\Delta \theta)}$$

Increase in volume, $\Delta V = \gamma \times V_1 \Delta \theta$

Substituting above values, we obtain, $\Delta V = 3\alpha\, vt$

EXERCISES 4.

1. A brass rod is 2m long at a certain temperature. Calculate the linear expansion of the rod for a temperature change of 100K [Take the linear expansivity of brass as $1.8 \times 10^{-5} K^{-1}$] *WAEC 1995* Ans: 3.6×10^{-3}m
2. Steel bars each of length 3m at 28°C are to be used for constructing a rail line. If the linear expansivity of steel is $1.0 \times 10^{-5} K^{-1}$, what is the safety gap that must be left between successive bars, if the highest temperature expected is 40°C? *WAEC 1997* Ans: 3.6×10^{-4}m
3. An iron rod of length 30cm is heated through 50 Kelvin. Calculate its increase in length °C [Linear expansivity of iron = $1.2 \times 10^{-5} K^{-1}$] *WAEC 2000* Ans: 1.8×10^{-2}cm
4. Metal rods of length 20m each are laid end to end to form a bridge at 25°C. What gap will be provided between consecutive rails for the bridge to withstand 75°C?

A. 0.22m B. 0.25m C. 0.02m D. 0.20m

[Linear expansivity of the material = $2.0 \times 10^{-5}K^{-1}$] *JAMB 2004* Ans: 0.02m

5. A wire, 20m long is heated from a temperature of 5°C to 55°C, if the change in length is 0.020m, calculate the linear expansivity of the wire. *WAEC 1991* Ans: $2.0 \times 10^{-5}K^{-1}$

6. The length of a zinc rod at 23°C is 200m. Calculate the increase in length of the rod when its temperature rises to 33°C. [Linear expansivity of the Zinc = $2.6 \times 10^{-5}K^{-1}$]
WAEC 2008[E12] Ans: 0.052m

7. If the cubic expansivity of brass between 27°C and 100°C is $5.7 \times 10^{-5}K^{-1}$, what is its linear expansivity? *WAEC 1998* Ans: $1.9 \times 10^{-5}K^{-1}$

8. A metal rod 800mm long is heated from 10°C to 95°C. If it expands by 1.36mm, the linear expansivity of the metal is? *WAEC 1993* Ans: $2 \times 10^{-5}\,°C$

9. An iron rod of length 50m and at a temperature of 60°C is heated to 70°C. Calculate its new length. [Linear expansivity of iron = $1.2 \times 10^{-5}K^{-1}$] *WAEC 1999* Ans: 50.006m

10. An iron rod is 2.58m long at 0°C. Calculate the length of a brass rod at 0°C if the difference between the lengths of the two rods must remain the same at all temperatures [Linear expansivity of iron = $1.2 \times 10^{-5}K^{-1}$, linear expansivity of brass = $1.9 \times 10^{-5}K^{-1}$].
NECO *2006* Ans: 1.629m

11. On a fairly cool rainy day when the temperature is 20°C, the length of a steel rail road track is 20m. What will be its length on a hot dry day when the temperature is 40°C?
A. 20.13m B. 20.009m C. 20.004m D. 20.002m
[Coefficient of linear expansion of steel = $11 \times 10^{-6}K^{-1}$] *JAMB 2003* Ans: 20.0044m

12. A steel plug has a diameter of 5cm at 30°C. At what temperature will it fit exactly into a hole of constant diameter 4.997cm? [Coefficient of linear expansion of steel is $11 \times 10^{-6}C^{-1}$) A. 27.3°C B. -27.3°C C. -2.7°C D. 7.32°C E. 2.7°C
No correct option *JAMB 1978* Ans: -24.58°C

13. The ratio of the linear expansivity of copper to that of iron is approximately 1.5. A specimen of iron and a specimen of copper expand by the same amount per unit rise in temperature. The ratio of their length is A. 3 B. 1.5 C. 1.32 D. 1 E. 0.67
JAMB 1980 Ans: 0.67

14. A solid material of volume $100cm^3$ is heated through a temperature difference of 40°C. Calculate the increase in the volume of the material if its linear expansivity is $2.0 \times 10^{-6}K^{-1}$. *WAEC 1997* Ans: $2.4 \times 10^{-2}cm^3$

15. When a metal ball is heated through 30°C, its volume becomes $1.0018cm^3$. If the linear expansivity of the material of the ball is $2.0 \times 10^{-5}K^{-1}$, calculate its original volume.
WAEC 2004 Ans: $1.00cm^3$

16. The linear expansivity of a substance is $1.2 \times 10^{-4}K^{-1}$. A cube of this substance has a volume of $8.0 \times 10^3cm^3$ at 30°C. Calculate the increase in its volume at 80°C.
WAEC 2005 Ans: $1.44 \times 10^2cm^3$

17. A brass cube of side 10cm is heated through 30°C. If the linear expansivity of brass is $2.0 \times 10^{-5}K^{-1}$, what is the increase in its volume? *NECO 2002* Ans: $1.80cm^3$

18. The linear expansivity of brass is $2 \times 10^{-5}\,°C^{-1}$. If the volume of a piece of brass is $10cm^3$ at 0°C, what will be its volume at 100°C? A. $10.02cm^3$ B. $10.04cm^3$ C. $10.06\,cm^3$ D. $10.20cm^3$ E. $102.00cm^3$
JAMB 1983 Ans: $10.06cm^3$

19. The length of a side of metallic cube at 20°C is 5.0cm. Given that the linear expansivity of the metal is $4.0 \times 10^{-5}K^{-1}$, find the volume of the cube at 120°C.
A. $126.50\,cm^3$ B. $126.25\,cm^3$ C. $126.00\,cm^3$ D. $125.00\,cm^3$
JAMB 1987 Ans: $126.5cm^3$

20. If L, S and V are the linear, area and volume expansivities of a given metal respectively, which of the following equation is correct? A. L - S = 0 B. V − 2S = 0 C. S − 2L = 0 D. 2S − L = 0 E. 3V − L = 0 *WAEC 1998* Ans: C

21. The temperature of glass vessel containing $100cm^3$ of mercury is raised from 10°C to 100°C. Calculate the apparent cubic expansion of the mercury. [Real cubic expansivity of mercury = $1.8 \times 10^{-4}K^{-1}$] [Cubic expansivity of glass = $2.4 \times 10^{-5}K^{-1}$]
WAEC 1995 Ans: $1.422cm^3$

22. The cubic expansivity of mercury is $1.8 \times 10^{-4}K^{-1}$ and the linear expansivity of glass is $8.0 \times 10^{-6}K^{-1}$, calculate the apparent expansivity of mercury in a glass container.

WAEC 2000 Ans: $1.5 \times 10^{-4}K^{-1}$

23. A cube made of a metal of linear expansivity is warmed through a temperature of t. If the initial volume of the cube is V, what is the increase in volume of the cube? A. $^1/_3\alpha$ Vt B. ½ αVt C. αVt D. 2 αVt E. 3αVt *WAEC 1990* Ans: E

24. A thin aluminum plate has a surface area of $1.500m^2$ at 20°C. What will be its surface area when it is cooled to -20°C? [Take the linear expansivity of aluminum to be $2.5 \times 10^{-5}K^{-1}$] A. $1.503\ m^3$ B. $1.500\ m^3$ C. $1.498\ m^3$ D. $1.497\ m^3$ E. $1.490\ m^3$

JAMB 1985 Ans: $1.497m^2$

25. A cube made of metal of linear expansivity α is heated through a temperature θ. If the initial volume of the cube is V_o, the correct expression for the increase in volume of the cube is A. $\frac{1}{3}\alpha V_o\theta$ B. $\frac{1}{2}\alpha V_o\theta$ C. $2\alpha V_o\theta$ D. $3\alpha V_o\theta$ WAEC 2007[22] Ans: D

26. A blacksmith heated a metal whose cubic expansivity is $6.3 \times 10^{-6}K^{-1}$. The area expansivity is A. $4.2 \times 10^{-6}K^{-1}$ B. $2.0 \times 10^{-6}K^{-1}$ C. $6.3 \times 10^{-6}K^{-1}$ D. $2.1 \times 10^{-6}K^{-1}$

JAMB 2007 [12] Ans: $4.2 \times 10^{-6}K^{-1}$

27. A rod of initial length 2m at a temperature of 25°C is heated to 80°C. Calculate the increase in length of the rod if its linear expansivity is $4.0 \times 10^{-3}K^{-1}$

WAEC 2009[18] Ans: 0.44m

5

WORK, ENERGY AND POWER

WORK

Work is defined as the product of a force and the distance in the direction of the force.

Work done, W = force × distance (displacement)

$$W = F \times s \quad \text{or}$$
$$W = mg \times s$$

However, if the force applied is at an angle θ to the horizontal as shown below, the work done depends on what direction the objects moves.

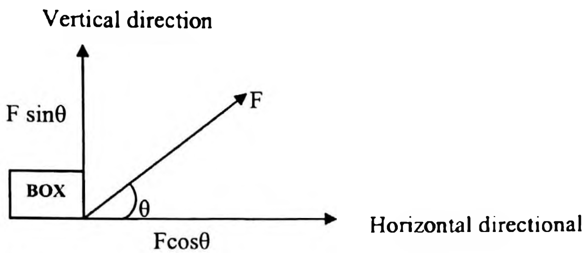


Fig 5.1

If the box moves in the horizontal direction, the work done(W) is;

$$W = F\cos\theta \times s$$

Or $$W = mg\cos\theta \times s$$

If the box moves in the vertical direction, the work done(W) is;

$$W = F\sin\theta \times s$$

Or $$W = mg\sin\theta \times s$$

Where F = Force in Newton (N)
s = distance or displacement in meter, m
m = mass in kilogram, kg
g = acceleration due to gravity, m/s^2

Work is measured in Joules.

ENERGY

Energy is the capacity to do work. Kinetic Energy (KE) is the energy possessed by a body in motion.

$$KE = ½mv^2$$

Potential energy (PE) is the energy possessed by a body at rest or at a height

$$PE = mgh$$

Where m = mass in kilogram, kg
v = velocity in meter per second, m/s
g = acceleration due to gravity, m/s^2
h = height in meter (m)

POWER

Power is defined as the rate of doing work or the rate of energy transfer.

$$\text{Power}(P) = \frac{\text{Work done (W)}}{\text{time taken (t)}}$$

$$= \frac{\text{Force} \times \text{Distance}}{\text{time taken}} = \text{Force} \times \frac{\text{Distance}}{\text{time}} = \text{Force} \times \text{Velocity}$$

$$P = \frac{W}{t} = \frac{F \times s}{t} = \frac{mg \times s}{t} = F \times v$$

Power is measured in Joules per second (J/s) or watts (W)

1 horse power (hp) = 746W = 0.746kW.

Exai ɔle 1

A boy drags a bag of rice along a smooth horizontal floor with a force of 2N applied at an angle 60° to the floor. The work done after a distance of 3m is

A. 6J B. 3J C. 4J D. 5J *JAMB 2007*

Solution

Force F = 2N; $\theta = 60^\circ$; Distance s = 3m

$$\text{Work done, } W = F\cos\theta \times s$$
$$= 2\cos 60 \times 3$$
$$= 2 \times 0.5 \times 3 = 3J$$

Example 2

A constant force of 40N acting on a body initially at rest gives an acceleration of $0.1 m/s^2$ for 4s. Calculate the work done by the force.

A. 8J B. 10J C. 32J D. 160J *JAMB 1987*

Solution

Given: Force, F = 40N; initial velocity, u =0; acceleration, a = $0.1 m/s^2$; time, t = 4s.

Distance, s is obtained from 2nd equation of motion,

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 0 \times 4 + \tfrac{1}{2} \times 0.1 \times 4^2$$
$$s = 0 + 0.05 \times 16$$
$$s = 0.8m$$

Work done = F $\times$ s = 40 x 0.8 = 32J

Example 3

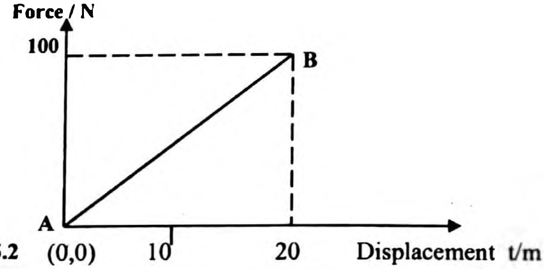


Fig. 5.2

Using the force-displacement diagram shown above, calculate the work done. *WAEC 2000*

Solution

Generally, in a force – displacement (F-d) or force distance graph, the work done is equal to the area of the shape formed by the F-d graph. If the area under the F-d graph is a trapezium or semicircle, then the work done is given by the area of trapezium [½(a + b) h] or area of a semicircle $\frac{1}{2}(\pi r^2)$.

In the above case, the area under the F-d graph is a rectangle, therefore,

Work done = 100 × 20 = 2000J

Example 4

An engine raises 100kg of water through a height of 60m in 20s. What is the power of the engine? Take $g=10m/s^2$ *WAEC 1989*

Solution

Given: mass, m = 100kg; $g = 10m/s^2$; distance, s = 60m; time, t = 20s

$$\text{Power} = \frac{\text{work done (w)}}{\text{time taken (s)}} = \frac{\text{force} \times \text{Distance}}{\text{time taken}} = \frac{mg \times s}{t}$$

$$P = \frac{mg \times s}{t} = \frac{100 \times 10 \times 60}{20} = 3000W$$

Example 5

A machine of efficiency 80% is used to raise a body of mass 75kg through a vertical height of 3m in 30s. Calculate the power input. $[g=10m/s^2]$ *WAEC 2000*

Solution

Given: efficiency, E = 80%; mass, m = 75kg;

distance (height), s = 3m; time, t =30

$$\text{Efficiency, } E = \frac{\textit{Power output, } P_o}{\text{Power input, } P_i} \times 100$$

$$\text{Power output, } P_o = \frac{mg \times s}{t}$$

$$\therefore \quad P_o = \frac{75 \times 10 \times 3}{30} = \frac{2250}{30} = 75W$$

$$\text{Power input, } P_i = \frac{\text{Power output}}{\text{efficiency}}$$

$$\therefore \quad P_i = \frac{75 \times 100}{80} = 93.75W$$

Example 6

The engine of a train produces a force of 3000N when moving at 30m/s. Calculate the power of the engine. *WAEC 2006*

Solution

Given: force = 3000N; speed = 30m/s

$$\text{Power} = \frac{\text{Force} \times \text{Distance}}{\text{time taken}} \qquad \left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$$

P = force × speed (velocity)

$\therefore$ P = 3000N × 30m/s = 90000W = 9.0×10^4W

Example 7

How long will it take a 60kg man to climb a height of 22m if he expended energy at the rate of 0.25kW? $[g=10m/s^2]$ A. 5.3s B. 34.5s C. 41.6s D. 52.8s *JAMB 1988*

Solution
Given: mass, m=60kg; height, s = 22m; $g=10m/s^2$;

Power, P = 0.25kW = 250W

$$\text{Power, } P = \frac{F\times s}{t} = \frac{mg\times s}{t}$$

$$\therefore \text{time, } t = \frac{mg\times s}{P} = \frac{60\times 10\times 22}{250} = 52.8s$$

Example 8
A car of mass 800kg attains a speed of $25ms^{-1}$ in 20 seconds. The power developed in the engine is A. 1.25×10^4W B. 2.50×10^4W C. 1.25×10^6 W D. $2.50 \times 10^6 W$ *JAMB 1999*

Solution
Given: mass, m=800kg; speed or velocity, $v = 25ms^{-1}$; time, t = 20s.

$$\text{Acceleration, } a = \frac{\text{Velocity}}{\text{Time}} = \frac{25}{20} = 1.25m/s^2$$

Force, $F = ma = 800 \times 1.25 = 1000N$

Power, $P = \text{Force} \times \text{Velocity}$

$= 1000 \times 25$

$= 25000W$ or 25KW

Example 9
A body of mass 5kg falls from a height of 10m above the ground. What is the kinetic energy of the body just before it strikes the ground? [Neglect energy losses and take g as $10m/s^2$] *WAEC 1992*

Solution
The question is solved using the equation, KE = $\frac{1}{2}mv^2$. However, in this and many other cases the velocity is usually not given and has to be calculated using any of the three equations of motion and their variations for gravitational acceleration or motion under gravity. (See the chapter on speed, velocity and acceleration).

The final velocity, v before it strikes the ground is found using the 2nd equation of motion, $v^2 = u^2 + 2gh$, for a body falling downwards or released from height.
Given: initial velocity, u=0; height, h=10m; $g=10m/s^2$; mass, m = 5kg

$$v^2 = u^2 + 2gh$$

$$v^2 = 0^2 + 2 \times 10 \times 10$$

$$v^2 = 200$$

$$v = \sqrt{200} = 14.14m/s$$

$\therefore$ KE = $\frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 14.14^2 = 500J$

Example 10
A body of mass 4kg is acted on by a constant force of 12N for 3 seconds. The kinetic energy gained by the body at the end of the time is
A. 162 J B. 144 J C. 72 J D. 81 J *JAMB 2004*

Solution
Mass, m=4kg; Force, F = 12N; time, t=3s

$F = ma$ $\therefore$ acceleration, $a = F/m = 12/4 = 3m/s^2$

Also, acceleration, $a = \frac{velocity\,(v)}{time\,(t)}$ $\therefore$ Velocity, $v = a \times t$

$= 3m/s^2 \times 3s = 9m/s$

$\therefore$ $KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 9^2 = 162J$

Example 11

A ball of mass 200g falls from a height of 5m on to a hard floor and rebounds to a height of 3m. What energy is lost by the ball as a result of the impact on the floor [$g=10ms^{-2}$]

NECO 2006

Solution

Given: mass, m=200g = 0.2kg; $g=10m/s^2$

Potential energy at height, h = 5m, $PE_5 = mgh$

$= 0.2 \times 10 \times 5$

$= 10J$

Potential energy at height, h = 3m, $PE_3 = mgh$

$= 0.2 \times 10 \times 3$

$= 6J$

Energy lost $= PE_5 - PE_3 = 10 - 6 = 4J$

Example 12

A body rolls down a slope from a height of 100m. Its velocity at the foot of the slope is 20m/s. What percentage of its potential energy is converted into kinetic energy? [$g=10m/s^2$]. A. 40% B. 35% C. 20% D. 15%

JAMB 1987

Solution

Given: height, h=100m; velocity, v=20m/s; $g=10m/s^2$

PE at height, h=100m, $mgh = m \times 10 \times 100$

$= 1000m\ J$

KE at foot of slope, $\frac{1}{2}mv^2 = \frac{1}{2} \times m \times 20^2$

$= 200m\ J$

Percentage of PE converted to KE $= \frac{KE \times 100}{PE} = \frac{200m\,J \times 100}{1000m\,J} = 20\%$

Example 13

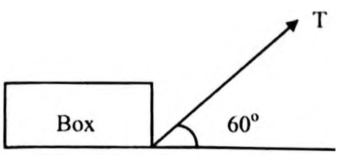


Fig. 5.3

A box of mass 40kg is being dragged along the floor by a rope inclined at 60° to the horizontal. The frictional force between the box and the floor is 100N and the tension on the rope is 300N. How much work is done in dragging the box through a distance of 4m?

A. 680 J B. 400 J C. 200 J D. 100 J

JAMB 1995

Solution

Work done = Force x distance = F x s

Effective horizontal component of tension force, $T = T\cos\theta = 300\cos60 = 150N$

Effective forward dragging force, F = Tension force - Frictional force.

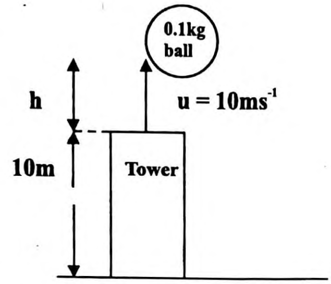
$$F = 150 - 100 = 50N$$

$\therefore$ $W = F \times s = 50N \times 4m = 200J$

Example 14

A ball of mass 0.1kg is thrown vertically upwards with a speed of $10ms^{-1}$ from the top of a tower 10m high. Neglecting air resistance, its total energy just before hitting the ground is [$g=10m/s^2$] *JAMB 1999*

Solution



The height reached, h is Calculated from $v^2 = u^2 - 2gh$.

Fig. 5.4

Initial velocity, $u = 10m/s$; final velocity, $v = 0$; $g=10m/s^2$

$$\therefore \text{ Height, } h = \frac{u^2 - v^2}{2g} = \frac{10^2 - 0^2}{2\times 10} = \frac{100}{20} = 5m$$

Total height from ground level = $h = 10 + 5 = 15m$

PE at maximum height = KE just before hitting the ground.

$PE = mgh = 0.1 \times 10 \times 15 = 15J$

Example 15

If a body of mass 5kg is thrown vertically upwards with velocity *u*, at what height will the potential energy equal to the kinetic energy?

A. $h = \frac{u^2}{g}$ B. $h = \frac{u^2}{4g}$ C. $h = \frac{2u^2}{g}$ D. $h = \frac{u^2}{2g}$

JAMB 2008

Solution

Equate potential energy (mgh) with kinetic energy ($\frac{1}{2}mu^2$)

$$mgh = \frac{1}{2}mu^2$$

$$h = \frac{1}{2}\frac{mu^2}{mg}$$

$$h = \frac{u^2}{2g}$$

EXERCISE 5.

1.

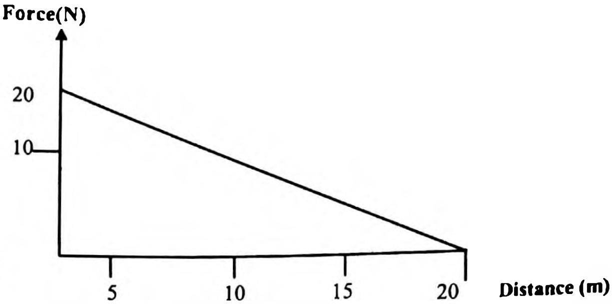


Fig. 5.5

A force varying linearly with the distance acts on a body as shown above. The work done on the body by the force during the first 10 meters of motion is
A. 100J B. 150J C. 200J D. 300J E. 600J *JAMB 1984 Ans: 100J*

2.

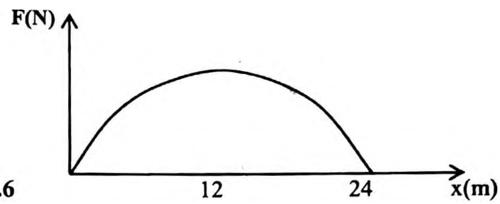


Fig. 5.6

A body is under the action of a force **F** such that the force-displacement graph of the body is semicircle as shown above. The work done on the body by the force in moving through 24 meters is A. 36πJ B. 72π J C. 144π J D. 288π J
JAMB 1991 Ans: *72π J* or *226.3J*

3. A load of mass 120kg is raised vertically through a height of 2m in 30s by a machine whose efficiency is 100%. Calculate the power generated by the machine [$g=10m/s^2$].
WAEC 2002 Ans: 80W

4. A girl whose mass is 20kg climbs up 25 steps each of height 15cm in 10 seconds. Calculate the power expended [$g=10m/s^2$] *NECO 2005* Ans: 75W

5. A man of mass 50kg ascends a flight of stairs 5m high in 5 seconds. If acceleration due to gravity is $10ms^{-2}$, the power expended is
A. 100W B. 200W C. 250W D. 400W E. 500W *JAMB 1983* Ans: 500W

6. If a water pump at Kainji dam is capable of lifting 1000kg of water through a vertical height of 10m in 10s, the power of the pump is A. 1.0kW B. 10.0kW C. 12.5kW D. 15.0kW E. 20.0kW [$g=10m/s^2$] *JAMB 1984* Ans: 10..0 kW

7. A man weighing 800N climbs up a flight of stairs to a height of 15m in 12.5secs. What is the man's average power output?
A. 667W B. 810W C. 960W D. 15000W *JAMB 1992* Ans: 960W

8. An electric water pump rated 1.5kW lifts 200kg of water through a vertical height of 6m in 10secs. What is the efficiency of the pump?
A. 90.0% B. 85.0% C. 80.0% D. 65.0% *JAMB 1997* Ans: 80%

9. A man whose mass is 80kg climbs a staircase in 20s and expends a power of 120W. Find the height of the staircase [$g=10m/s^2$]
A. 1.8m B. 2.0m C. 2.5m D. 3.0m *JAMB 1998 Ans*: 3m

10. A stone of mass 2.0kg is thrown vertically upward with a velocity of $20.0ms^{-1}$. Calculate the initial kinetic energy of the stone. *WAEC 2006* Ans: 400J

11. A 500kg car which was initially at rest traveled with an acceleration of 5m $sec,^{-2}$ its kinetic energy after 4 second was A. 10^5J B. 2.5 x 10^3J C. 2 x 10^3J D. 5 x 10^3J E. 5 x 10^5J *JAMB1979* Ans: 10^5J

12. An object of mass 50kg is released from a height of 2m. Find the kinetic energy just before it strikes the ground.
A. 250J B. 1000J C. 10,000J D. 100,000J *JAMB 1994* Ans: 1000J

13. An object of mass 100g projected vertically upwards from the ground level has a velocity of $20ms^{-1}$ at a height of 10m. Calculate its initial kinetic energy at the ground level. [$g=10m/s^2$, neglect air resistance].
A. 10J B. 20J C. 30J D. 50J *JAMB 1997* Ans: 30J

14. An object of mass 1000kg is dropped from a height of 10m. Calculate its energy when it strikes the ground. [$g=10ms^{-2}$) *WAEC 2005* Ans: 100000J or 10^2kJ

15. The efficiency of a machine is 80%. Calculate the work done by a person using the machine to raise a load of 300kg through a height of 4m [$g=10ms^{-2}$]
*WAEC 2006*Ans: 15000J

16. A bead travelling on a straight wire is brought to rest at 0.2m by friction. If the mass of the bead is 0.01kg and the coefficient of friction between the bead and the wire is 0.1, determine the work done by the friction. [$g=10ms^{-2}$] Hint: w = F x s = μR x s
A. 2 x 10^{-4}J B. 2 x 10^{-3}J C. 2 x 10^1J D. 2 x 10^2J *JAMB 2003* Ans:2 x 10^{-3}J

17.

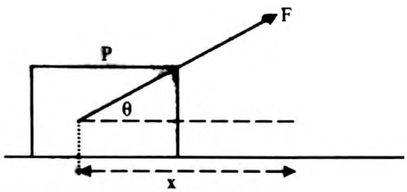


Fig. 5.7

A force, F is applied to a body, P as shown in the diagram above. If the body **P** moves through a distance **x**, which of the following represents the work done?

A. Fx B. $\frac{F\cos\theta}{x}$ C. Fxtanθ D. Fxsinθ E. Fxcosθ *WAEC 1990* Ans:E

18. A box is pulled a distance, s along a smooth horizontal floor by a force of magnitude F, inclined at an angle, θ to the horizontal. The work done is? *NECO 2005*Ans: Fscosθ

19. A force **F** is applied to a body **P** as shown in the diagram above. If the body moves through a distance, **a**, which of the following represents the work done?

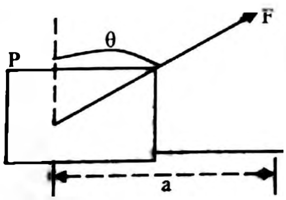


Fig. 5.8

A. Fa B. Facosθ C. $\frac{Fa}{\cos\theta}$ D. Fasinθ E. $\frac{Fa}{\sin\theta}$ *NECO 2000*Ans: D

20. A car travelling with uniform velocity of $30ms^{-1}$ along a horizontal road overcomes a constant frictional force of 600N. Calculate the power of the engine of the car.
WAEC 2008[6] Ans:18000W or 18kW

21. An object of mass 0.25kg moves at a height h above the ground with a speed of $4ms^{-1}$. If its mechanical energy at this height is 12J, determine the value of h.
[$g = 10ms^{-2}$] *WAEC 2008[16] Ans: 4.8m*

22. If a cage containing a truck of coal weighing 750kg is raised to a height of 90m in 1minute, what is the total power expended? ($g = 10 ms^{-2}$)
A. 11.50kW B. 12.60kW C. 11.25kW D. 12.10kW JAMB 2009[10] Ans: C

23. A stone of mass 0.5kg is dropped from a height of 1.2m. Calculate its maximum kinetic energy. ($g = 10ms^{-2}$) WAEC 2009[15] Ans: 60.0J

24. A ball of mass 100g falls from a height of 5m onto a concrete floor and rebounds to a height of 3m. Calculate the energy lost. ($g = 10ms^{-2}$) NECO 2009[13] Ans: 2J

25. A steam engine of efficiency 70% burns 20g of coal to produce 10kJ of energy. If it burns 200g of coal per second, calculate its output power. NECO 2009[17] Ans: 70.0kW

6

CURRENT ELECTRICITY

ELECTRIC CURRENT

Electric current is defined as the rate of flow of charge along a conductor.

$$\text{Electric current (I)} = \frac{\text{Quantity of charge (Q)}}{\text{Time (t)}}$$

That is, $I = \frac{Q}{t}$

The unit of electric current is ampere (A).

Example 1

Calculate the quantity of charge flowing through a conductor if a current of 10A passes through a conductor for 10s. *WAEC 1995*

Solution

$$I = 10A; \quad t = 10s$$

$$\text{Current (I)} = \frac{\text{Quantity of charge}}{\text{Time}} = \frac{Q}{t}$$

$\therefore$ Quantity of charge, $Q = It = 10 \times 10 = 100C$

OHM'S LAW: Ohm's law states that the current flowing through a metallic conductor is directly proportional to the potential difference across its ends, provided temperature and other physical conditions of the conductor are kept constant.

That is, $V \propto I$

$$\therefore \quad V = IR \quad \text{or} \quad I = \frac{V}{R} \quad \text{or} \quad R = \frac{V}{I}$$

Where I = current in Amperes, A
V = potential difference in Volts, V
R = resistance in Ohm, Ω

SERIES AND PARALLEL ARRANGEMENT OF RESISTORS AND CELLS

Resistors in Series:

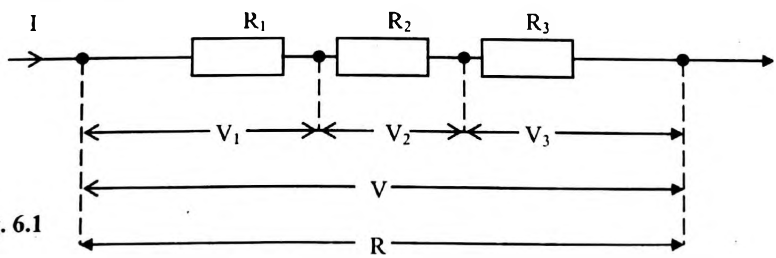


Fig. 6.1

The combined, total, equivalent or effective resistance for resistors connected in series as shown above is $R = R_1 + R_2 + R_3$

Generally, in series connection of resistors;

1. The <u>same</u> current passes through each resistor.
2. The potential difference across each resistor depends on the value of its resistance.
3. The potential difference, V, across the whole series connection is equal to the sum of

the individual potential differences across each resistor; $V = V_1 + V_2 + V_3$

4. The total resistance R is the sum of the individual resistance; $R = R_1 + R_2 + R_3$

Resistors in Parallel

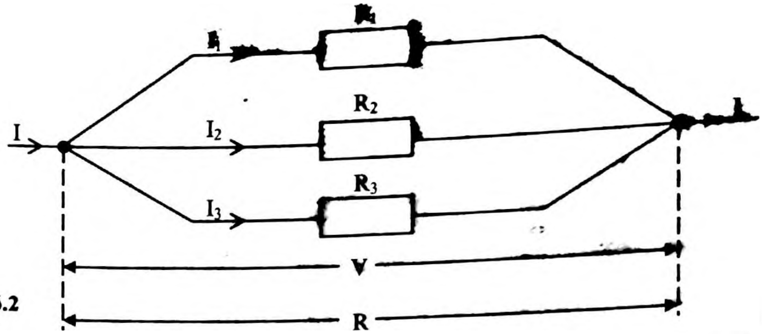


Fig. 6.2

The combined, total, equivalent or effective resistance for resistors connected in parallel is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

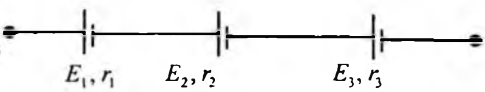
If there are two resistors, R_1 and R_2, the combined resistance, $R = \dfrac{R_1R_2}{R_1 + R_2}$

Generally, in parallel connection of resistors;

1. The potential difference across each resistor is the same and is equal to the potential difference across the whole connection, $V = V_1 = V_2 = V_3$
2. The total current is equal to the sum of the currents flowing through each resistor;
   $I = I_1 + I_2 + I_3$
3. The combined resistance is always less than the least individual resistance.

Cells in Series

Fig. 6.3



For cells connected in series the total e.m.f, $E = E_1 + E_2 + E_3$

The total internal resistance, $r = r_1 + r_2 + r_3$

Cells in Parallel

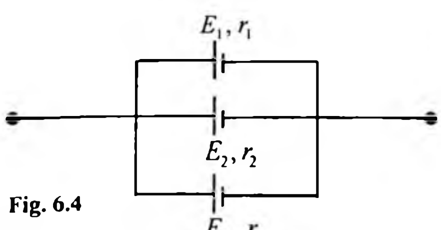


Fig. 6.4

For cells connected in parallel, the total e.m.f, $E = E_1 = E_2 = E_3$

The total internal resistance, $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

For two internal resistors, r_1 and r_2 in parallel, effective resistance, $r = \frac{r_1 r_2}{r_1 + r_2}$.

Example 2

What is the effective resistance of the circuit below.

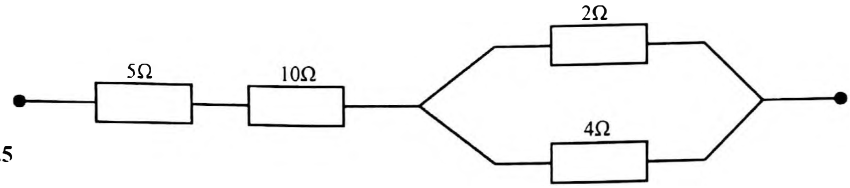


Fig. 6.5

Solution

The 5Ω and 10Ω are in series, therefore the combined resistance,
$R = R_1 + R_2 = 5 + 10 = 15\Omega$.
The circuit is redrawn as follows.

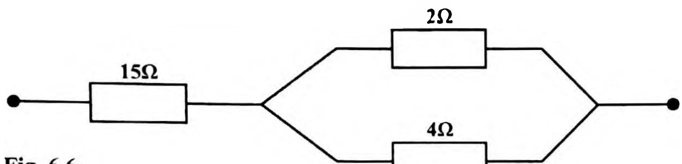
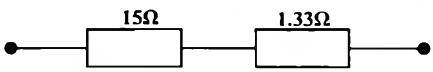


Fig. 6.6

The 2Ω and 4Ω are in parallel, therefore the combined resistance,
$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{2\times 4}{2+4} = \frac{8}{6} = 1.33\Omega$

The circuit is drawn again as follows.

Fig. 6.7



The 15Ω and 1.33Ω are in series, therefore the combined resistance,

$R = R_1 + R_2$

$R = 15 + 1.33 = 16.33\Omega$

Example 3

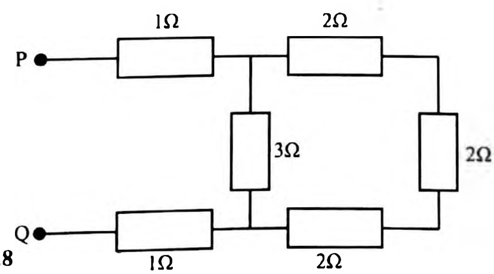


Fig. 6.8

The total resistance measured at PQ in the diagram above is?

A. 18.0 Ω B. 11.0 Ω C. 4.0 Ω D. 2.0 Ω *JAMB 1998*

Solution

The three 2Ω resistors are in series, therefore the total resistance,

$R = R_1 + R_2 + R_3$ $\therefore$ $R = 2 + 2 + 2 = 6\Omega$. The circuit is redrawn as follows.

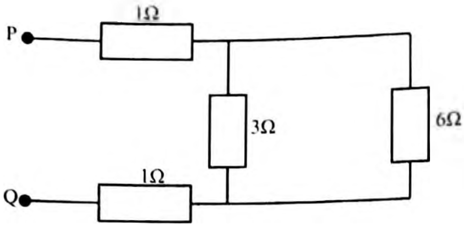


Fig. 6.9

The 3Ω and 6Ω resistors are in parallel, therefore the combined resistance,

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$

The circuit is redrawn once again as follows:

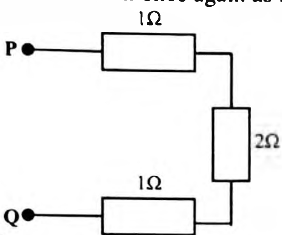


Fig. 6.10

The three resistors (1Ω, 1Ω, and 2Ω) are in series, therefore equivalent resistance,

$R = R_1 + R_2 + R_3$

$= 1 + 1 + 2 = 4\Omega$

∴ The total resistance measured at PQ is 4 Ω

Example 4

A car fuse is marked 15A and operates normally on a 12V battery. Calculate the resistance of the fuse wire. *WAEC 1994*

Solution

Current, I = 15A; p.d, V = 12V

From V = IR. $R = \frac{V}{I} = \frac{12}{15} = 0.8\Omega$

Example 5

The sum of the current I_1 and I_2 in the figure below is

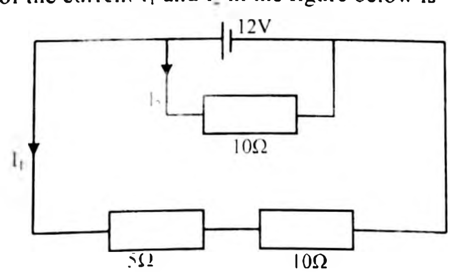


Fig. 6.11

JAMB 1979

A. 1.2A B. 0.6A C. 0.8A D. 2A E. 1A

Solution

Given: V = 12V; R=10Ω

From V = IR. $I_2 = \frac{V}{R} = \frac{12}{10} = 1.2A$

For V = 12V and R=5 + 10 = 15Ω (series connection)

$$I_1 = \frac{V}{R} = \frac{12}{15} = 0.8A \qquad \therefore \quad I_1 + I_2 = 1.2 + 0.8 = 2.0A$$

Alternatively, this could be solved as follows.

(i) The 5Ω and 10Ω resistor through which I_1 flows are in series. Therefore, effective resistance, R = 5 + 10 = 15Ω.

(ii) The 15Ω resistor is connected in parallel with the 10Ω resistor. Therefore the effective resistance,

$$R = \frac{R_1 R_2}{R_1 + R_2} \text{ where } R_1 = 15\Omega;\ R_2 = 10\Omega$$

$$R = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6\Omega$$

(iii) Total circuit current $(I_1 + I_2)$, $I = \dfrac{\text{Potential difference}}{\text{total circuit resistance}}$

$$I = \frac{V}{R} = \frac{12}{6} = 2A$$

Example 6

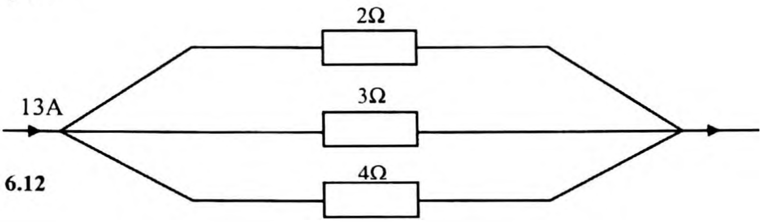


Fig. 6.12

Calculate the current in the 3Ω resistor shown in the diagram above. *WAEC 1990*

Solution

Equivalent resistance, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{2} + \dfrac{1}{3} + \dfrac{1}{4}$

$$\frac{1}{R} = \frac{6 + 4 + 3}{12} = \frac{13}{12} \qquad \therefore R = \frac{12\Omega}{13}$$

Voltage or p.d across the parallel connection, V = IR

$$V = 13 \times \frac{12}{13} = 12V$$

Current in the 3Ω resistor, $I = \dfrac{V}{R} = \dfrac{12}{3} = 4A$

Example 7

An electrical circuit is connected up as shown below. What is the value of the current through the 1 ohm resistance?

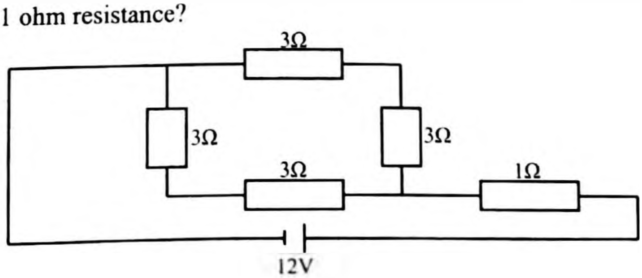


Fig. 6.13

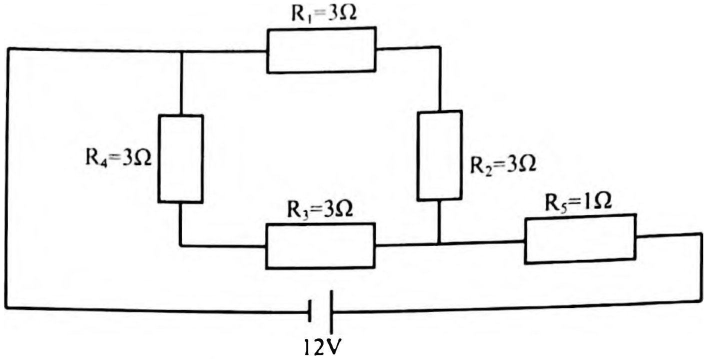


Fig. 6.14

(i) R_1 and R_2 are in series $\therefore$ Combined resistance, $R_{1.2} = 3 + 3 = 6\Omega$
(ii) R_3 and R_4 are in series $\therefore$ Combined resistance, $R_{3.4} = 3+3 = 6\Omega$
(iii) The circuit can be redrawn as follows.

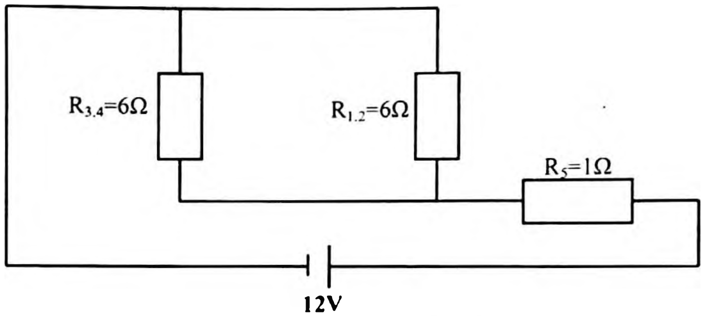


Fig. 6.15

(iv) $R_{1,2}$ and $R_{3,4}$ are connected in parallel

$$\therefore \text{ Effective resistance, } R_6 = \frac{R_{1,2}R_{3,4}}{R_{1,2}+R_{3,4}} = \frac{6\times 6}{6+6} = \frac{36}{12} = 3\Omega$$

(v) The circuit now becomes

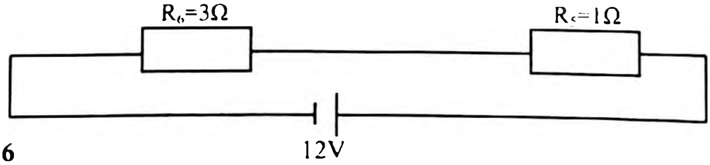


Fig. 6.16

R_6 and R_5 are in series $\therefore$ Total resistance, $R = R_5 + R_6 = 3 + 1 = 4\Omega$
The current through the 1Ω resistor is the same as the current flowing through the whole circuit.

From $V = IR$, $I = \frac{V}{R}$ Substitute $V=12V$; $R=4\Omega$

$$\therefore \quad I = \frac{12}{4} = 3A$$

Example 8
In the figure below, a voltage V is applied across the terminals P and Q. The voltage across the 1Ω resistor is

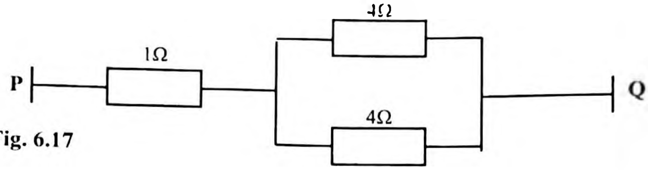


Fig. 6.17

A. $\frac{V}{8}$ B. $\frac{V}{4}$ C. $\frac{V}{3}$ D. $\frac{V}{2}$ *JAMB 1987*

Solution

The effective resistance across the parallel combination,

$$R = \frac{R_1R_2}{R_1 + R_2} = \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$$

The circuit becomes

Fig. 6.18

Total circuit resistance, $R = 1 + 2 = 3\Omega$

Total circuit voltage, $V = V$

$\therefore$ Total circuit current, $I = \frac{V}{R} = \frac{V}{3}$

From $V = IR$

Voltage across the 1Ω resistor is (where $R = 1$)

$$V = \frac{V}{3}\times 1 = \frac{V}{3}$$

Example 9

If a resistance is halved in value and the p.d across it is tripled, then the ratio of the new current to the old is A. 1:6 B. 1:3 C. 2:1 D. 6:1 *JAMB 1995*

Solution

If a resistance is halved in value, $R = \frac{1}{2}R$; p.d. across it is tripled, $V = 3V$.

$I = \frac{V}{R}$ $\therefore$ The new current, $I = \frac{3V}{\frac{1}{2}R} = \frac{3V}{0.5R} = \frac{6V}{R}$

The old current, $I = \frac{V}{R}$

So, the ratio of the new to old current is $\frac{6V}{R} : \frac{V}{R} = 6 : 1$

ELECTROMOTIVE FORCE, INTERNAL RESISTANCE, TERMINAL POTENTIAL DIFFERENCE AND LOST VOLT

The electromotive force (e.m.f.), E of a cell is defined as the total work done in driving one coulomb of electricity round a circuit. The e.m.f of a cell is also defined as the potential difference between the terminals of a cell when it is an open circuit, i.e. not delivering current to an external resistance.

The internal resistance, r, of a cell is the opposition to current flow offered by the cell when it is discharging current to a circuit.

Terminal potential difference, V, is defined as the potential difference between the terminals of a cell when it is delivering current to an external resistance, R.

The lost volt or voltage, v is the p.d across the internal resistance, r, of a cell.

E.m.f = p.d across external resistance + p.d across internal resistance.

$$E = V + v \qquad \text{.................................} \qquad (1)$$

Applying ohm's law to eqn (1) we get

$$E = IR + Ir \qquad (V = IR,\ v = Ir)$$

$$E = I(R + r)$$

$$I = \frac{E}{R + r} \qquad \text{..} \qquad (2)$$

Example 10

A battery of e.m.f 24V and internal resistance 4Ω is connected to a resistor of 32Ω. What is the terminal p.d of the battery? *WAEC 1998*

Solution

Whether you are asked to or not, always draw a circuit diagram before solving any question on current electricity.

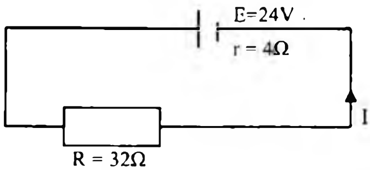


Fig. 6.19

First, calculate the circuit current, I

$$I = \frac{E}{R + r} = \frac{24}{32 + 4} = \frac{24}{36} = \frac{2}{3} A \text{ or } 0.67A$$

∴ Terminal p.d, $V = IR = 0.67 \times 32 = 21.3V$

Alternatively, the answer is gotten from the equation,

$$E = V + v \quad \text{or} \quad E = V + Ir$$

∴ Terminal pd, $V = E - Ir$

$$V = 24 - 0.67 \times 4$$

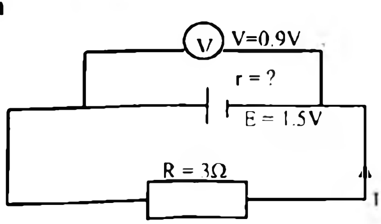
$$= 24 - 2.67$$

$$= 21.3V$$

Example 11

A cell of e.m.f 1.5V is connected in series with a resistor of resistance 3Ω. A high resistance voltmeter connected across the cell registers only 0.9V. Calculate the internal resistance of the cell. *WAEC 1996*

Solution



E = 1.5V
V = 0.9V
R = 3Ω
r = ?

Fig. 6.20

Current flowing in the circuit, $I = \frac{E}{R+r} = \frac{1.5}{3+r}$

p.d across cell, $v = E - V = 1.5 - 0.9 = 0.6V$

$v = 0.6$

Substitute $v = Ir$ in the above

$Ir = 0.6$

Substitute $I = \frac{1.5}{3+r}$ into above equation to obtain, $\frac{1.5}{3+r} \times r = 0.6$

Cross multiply;

$$1.5r = 0.6(3+r)$$
$$1.5r = 1.8 + 0.6r$$
$$1.5r - 0.6r = 1.8$$
$$0.9r = 1.8 \qquad \therefore \text{ Internal resistance, } r = \frac{1.8}{0.9} = 2\Omega$$

Example 12

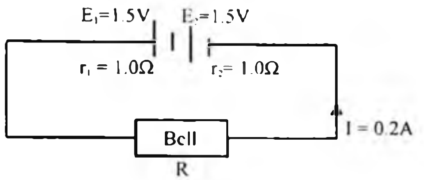
An electric bell takes a current of 0.2A from a battery of two dry cells connected in series. Each cell has an e.m.f of 1.5V and an internal resistance of 1.0Ω.

(i) Calculate the effective resistance of the bell.

(ii) What current would the bells take if the cells were arranged in parallel?

Solution

(i)



Let R be the resistance of the bell

Fig. 6.21

Total e.m.f, $E = E_1 + E_2 = 1.5 + 1.5 = 3V$

Total internal resistance, $r = r_1 + r_2 = 1 + 1 = 2\Omega$; Current, $I = 0.2A$

Substitute the above into $I = \frac{E}{R+r}$

$$0.2 = \frac{3}{R+2}$$

Cross multiplying we have

$$0.2(R+2) = 3$$
$$0.2R + 0.4 = 3$$
$$0.2R = 3 - 0.4$$
$$0.2R = 2.6 \qquad \therefore \quad R = \frac{2.6}{0.2} = 13\Omega$$

(ii)

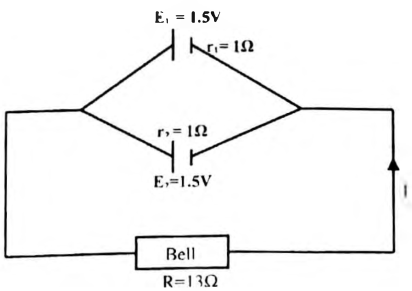


Fig. 6.22

E.m.f, $E = E_1 = E_2 = 1.5V$

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{1\times 1}{1+1} = \frac{1}{2} = 0.5\Omega \qquad and \qquad R = 13\Omega$$

$$\therefore \text{Current, } I = \frac{E}{R+r} = \frac{1.5}{13+0.5} = \frac{1.5}{13.5} = 0.11A$$

Example 13
Two cells, each of e.m.f. 2V and internal resistance 0.5Ω, are connected in series. They are made to supply current to a combination of three resistors; one of the resistances 2Ω is connected in series to a parallel combination of two other resistors each of resistance 3Ω. Draw the circuit diagram and calculate:
(i) Current in the circuit (ii) Potential difference across the parallel combination of the resistor. (iii) Lost volts of the battery. *WAEC 1998*
Solution

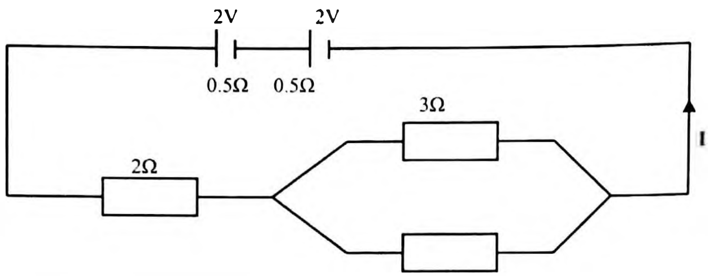


Fig. 6.23

(i) Total e.m.f, $E = 2 + 2 = 4V$

Effective internal resistance, $r = 0.5 + 0.5 = 1.0\Omega$

Effective external resistance, $R = 2 + \frac{3\times 3}{3+3} = 2 + \frac{9}{6} = 2 + 1.5 = 3.5\Omega$

$$\therefore I = \frac{E}{R+r} = \frac{4}{3.5+1} = \frac{4}{4.5} = 0.89A$$

(ii) p.d across parallel connection, $V = IR$

(Total parallel resistance, $R = \frac{3\times 3}{3+3} = 1.5\Omega$)

$\therefore V = 0.89 \times 1.5 = 1.3V$

(iii) Lost volt = p.d across the battery

$= Ir = 0.89 \times 1 = 0.9V$

Example 14
An electric cell with nominal voltage, E has a resistance of 3Ω connected across it. If the voltage falls to 0.6E, the internal resistance of the cell is? *JAMB 2002*
Solution

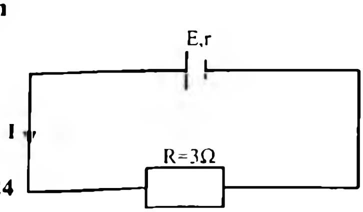


Fig. 6.24

The nominal voltage, E is the e.m.f of the cell.

The terminal p.d, $V = 0.6E$

The resistance, $R = 3\Omega$

The e.m.f of a cell, E, is equal to the sum of the terminal p.d, V, and the lost volt, v

i.e $\quad E = V + v$ .... (1)

or $E = IR + Ir$

Terminal p.d, $V = IR$

$$0.6E = 3I \quad \text{...} \quad (2)$$

From Eqn(1), $E = V + v$

The lost voltage, $v = E - V$

$$v = E - 0.6E$$

$$v = 0.4E$$

$$\therefore \quad Ir = 0.4E \quad \text{...} \quad (3)$$

From eqn (2), $I = \dfrac{0.6E}{3}$ ....(4)

From eqn (3), $I = \dfrac{0.4E}{r}$ .... (5)

Equate eqns (4) and (5)

$$\frac{0.6E}{3} = \frac{0.4E}{r}$$

Re-arranging, $r = \dfrac{0.4E \times 3}{0.6E} = \dfrac{1.2E}{0.6E} = 2$ $\quad\therefore\quad$ Internal resistance, $r = 2\Omega$

Example 15

A cell can supply currents of 1.2A and 0.4A through a 4Ω and 14Ω resistors respectively. Calculate the internal resistance of the cell.

Solution

This problem involves two instances (6.25:a & b) as shown below) and two unknown quantities, that is, the e.m.f E and the internal resistance, r.

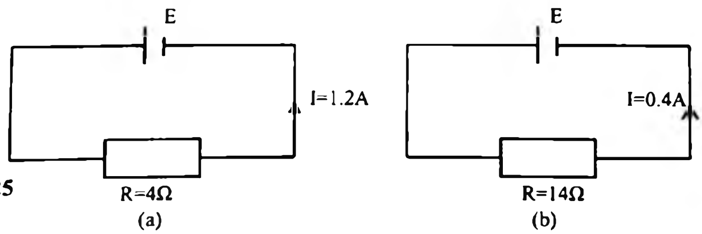


Fig. 6.25

Substitute each of the above case into $I = \dfrac{E}{R+r}$

1st case: $\quad 1.2 = \dfrac{E}{4+r} \quad \therefore \quad E = 1.2\,(4 + r)$ .... (1)

2nd case: $\quad 0.4 = \dfrac{E}{14+r} \quad \therefore \quad E = 0.4\,(14 + r)$ .... (2)

Equations 1 and 2 are equated and then solved simultaneously.

$$1.2\,(4 + r) = 0.4\,(14 + r)$$

$$4.8 + 1.2r = 5.6 + 0.4r$$

$$5.6 - 4.8 = 1.2r - 0.4r$$

$$0.8 = 0.8r$$

$$r = \frac{0.8}{0.8} = 1.0\Omega$$

Worthy of note is the fact that the e.m.f, E can be calculated by substituting the internal resistance, $r = 1.0\Omega$ into eqn 1 or 2 as follows

(i) $E = 1.2(4+r)$

$E = 1.2(4+1) = 1.2(5) = 6V$

(ii) $E = 0.4(14 + r)$

$E = 0.4(14+1) = 0.4(15) = 6V$

Example 16

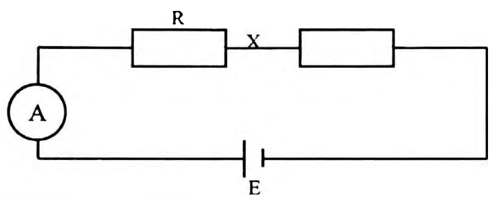


Fig. 6.26

In the circuit diagram above, the ammeter reads a current of 3A when R is 5Ω and reads 6A when R is 2Ω. The value of the unknown resistance X is

A. 1 B. 2 C. 3 D. 4 *JAMB 1988*

Solution

From $I = \dfrac{E}{R+r}$ we modify and apply it to the diagram above, making r = X.

$$\therefore \quad I = \frac{E}{R+X} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

1st instance: I = 3A, R = 5Ω substituting in (1), we obtain

$$3 = \frac{E}{5+X} \qquad \therefore \quad E = 3(5+X) = 15 + 3X \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

2nd instance: I = 6A, R=2Ω substituting in (1), we obtain

$$6 = \frac{E}{2+X} \qquad \therefore \quad E = 6(2+X) = 12 + 6X \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Equate equation (2) and (3)

$$15 + 3X = 12 + 6X$$

$$15 - 12 = 6X - 3X$$

$$3 = 3X$$

$$X = {}^{3}/_{3} = 1\Omega$$

Example 17

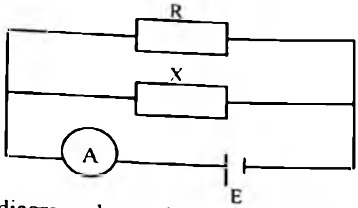


Fig. 6.27

In the circuit diagram above, the ammeter reads a current of 3A, when R is 5Ω and 6A when R is 2Ω. Determine the value of X.

A. 8Ω B. 2Ω C. 10Ω D. 4Ω *JAMB 2001*

Solution

R and X are resistors in parallel. The effective resistance is obtained from the equation of two resistors in parallel.

$$R = \frac{R_1R_2}{R_1 + R_2}$$

Let R_1 be the given resistance and R_2 the unknown resistance, X

First instance: $R = \frac{5X}{5+X}$ for $R_1 = 5\Omega$, $R_2 = X$, $I = 3A$

Second instance: $R = \frac{2X}{2+X}$ for $R_1 = 2\Omega$, $R_2 = X$, $I = 6A$

From Ohm's law, V = IR

Substitute each case into $I = {}^V/_R$

1st instance

$$3 = \frac{V}{\frac{5X}{5+X}} \quad \therefore \quad 3 = \frac{V(5+X)}{5X}$$

Make V the subject of formula,

$$V = \frac{15X}{5+X} \qquad \text{.............................. (1)}$$

2nd instance

$$6 = \frac{V}{\frac{2X}{2+X}} \quad \therefore \quad 6 = \frac{V(2+X)}{2X}$$

Making V the subject of formula,

$$V = \frac{12X}{2+X} \qquad \text{................ (2)}$$

Equate (1) and (2)

$$\frac{15X}{5+X} = \frac{12X}{2+X}$$

Cross multiply

$$30X + 15X^2 = 60X + 12X^2$$

$$60X - 30X = 15X^2 - 12X^2$$

$$30X = 3X^2$$

$$30 = 3X$$

$$X = \frac{30}{3} = 10\Omega$$

Example 18

A cell of internal resistance, r supplies current to a 6.0Ω resistor and its efficiency is 75%. Find the value of r. A. 4.5Ω B. 1.0Ω C. 8.0Ω D. 2.0Ω *JAMB 2001*

Solution

Efficiency of a cell, $E_f = \frac{\text{Power output}}{\text{power input}} \times 100$

$$E_f = \frac{I^2R}{I^2(R+r)} \times 100$$

$$E_f = \frac{R}{(R+r)} \times 100$$

Also, $E_f = \frac{\text{P.d across external resistor}}{\text{e.m.f of the cell}} \times 100$

R is external resistance(s) and r is internal resistance of the cell. Depending on the question, any of the above formula can be used to calculate efficiency of a cell.

From the question, efficiency, $E_f = 75\%$; External resistance, R = 6.0Ω.

$$E_f = \frac{R}{(R+r)} \times 100$$

$$75 = \frac{6}{(6+r)} \times 100$$

$$\frac{75}{100} = \frac{6}{(6+r)}$$

$$0.75 = \frac{6}{(6+r)}$$

$$0.75(6+r) = 6$$

$$4.5 + 0.75r = 6$$

$$0.75r = 6 - 4.5$$

$$0.75r = 1.5$$

$$r = \frac{1.5}{0.75} = 2\Omega$$

Alternatively, we make r the subject of equation before substituting.

$$r = \frac{R(100 - E_f)}{E_f}$$

$$r = \frac{6(100 - 75)}{75}$$

$$r = \frac{6 \times 25}{75}$$

$$r = 2\Omega$$

Example 19

A resistor of resistance, R is connected across a cell. If the terminal p.d of the cell is reduced to one-quarter of its e.m.f., express the internal resistance, r of the cell in terms of R. *NECO 2004*

Solution

e.m.f, E = V + v or E = IR + Ir

Internal resistance $r = \frac{E - IR}{I}$

but terminal p.d, V = ¼E (“*...terminal p.d... is... one quarter of e.m.f...*”)

$$E = 4V$$

Substituting, $r = \frac{4V - IR}{I}$ (V = IR)

$$r = \frac{4IR - IR}{I} = \frac{3IR}{I} = 3R$$

EXERCISE 6.

1. A current of 100mA passes through a conductor for 2 minutes. The quantity of electricity transported is? A. 200C B. 50C C. 12C D. 0.02C

JAMB 1993 Ans: 12C

2.

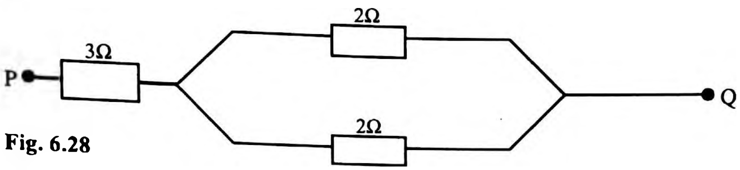


Fig. 6.28

Calculate the effective resistance between P and Q in the diagram shown above.

WAEC 2007 Ans: 4.00Ω

3. A parallel combination of 3Ω and 4Ω resistors is connected in series with a resistor of 4Ω and a battery of negligible resistance. Calculate the effective resistance in the circuit.

WAEC 2000 Ans: 5.7Ω

4.

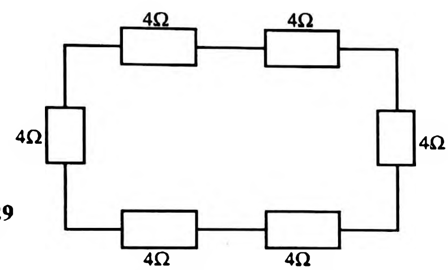


Fig. 6.29

Find the effective resistance in the diagram above.

A. 6Ω B. 12Ω C. 18Ω D. 24Ω *JAMB 2008 Ans: 24 Ω*

5.

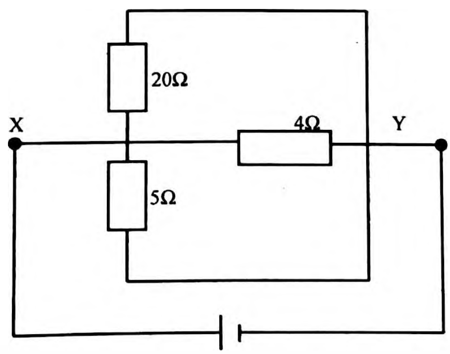


Fig. 6.30

Calculate the effective resistance between point X and Y in the diagram above.

WAEC 1994 Ans: 2Ω

6. Two resistors of resistance 6Ω and 4Ω are connected in parallel. A third resistor of 8Ω is then connected in series with the parallel connection. Calculate the effective resistance of the arrangement. *WAEC 2005* Ans: 10.4 Ω

7.

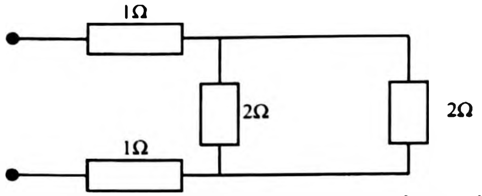


Fig. 6.31

The effective resistance of the circuit shown in the figure above is ?

NECO 2006 Ans: 3Ω

8. What is the resistance in the diagram below?

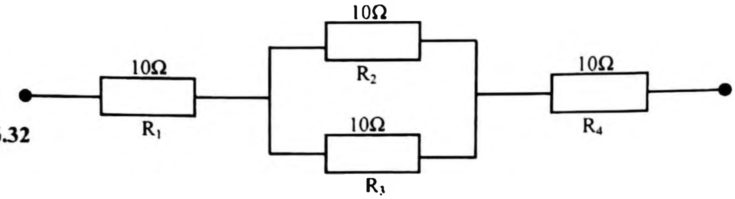


Fig. 6.32

JAMB 1980 Ans: 25Ω

9. Which of the circuits illustrated below will give a total resistance of 1Ω

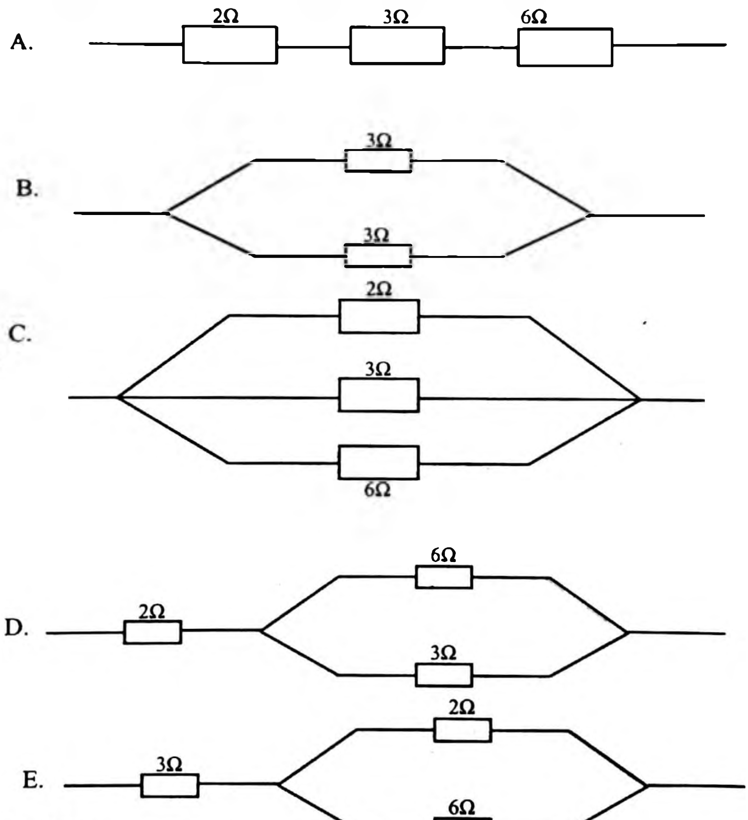


Fig. 6.33

JAMB 1982 Ans: C

10.

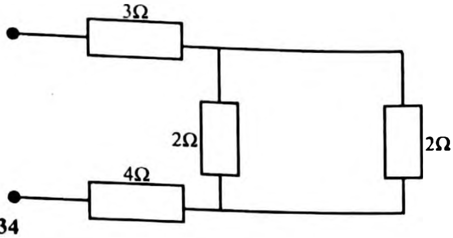


Fig. 6.34

What is the resultant resistance of the circuit given above?

A. 11Ω B. 8Ω C. 4Ω D. 3.6Ω E. 4.3Ω *JAMB 1982* Ans: 8Ω

11. What is the resistance of the circuit shown below?

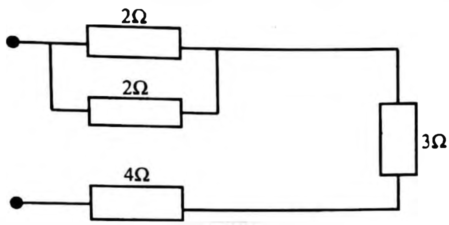
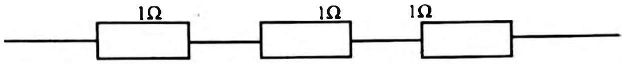


Fig. 6.35

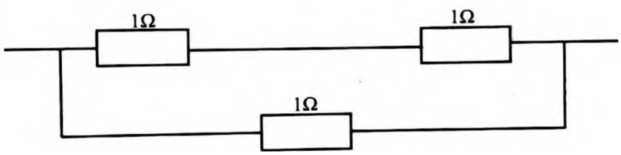
A. 4Ω B. 11Ω C. $\frac{19}{5}\Omega$ D. $\frac{17}{4}\Omega$ E. 8Ω *JAMB 1984* Ans: 8Ω

12. Which of the following arrangement will produce an equivalent resistance of 1.5Ω from three 1Ω resistors.

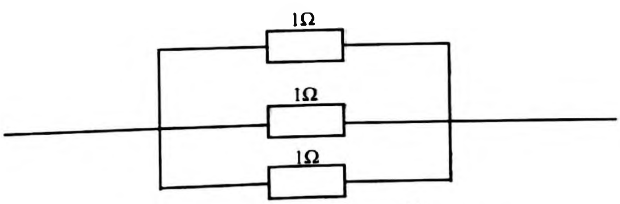
A.



B.



C.



D.

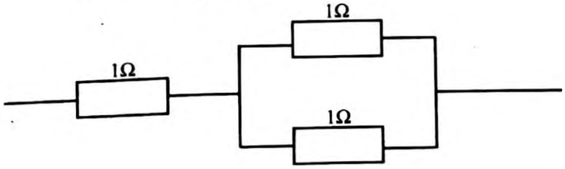


Fig. 6.36

JAMB 1986 Ans: D

13.

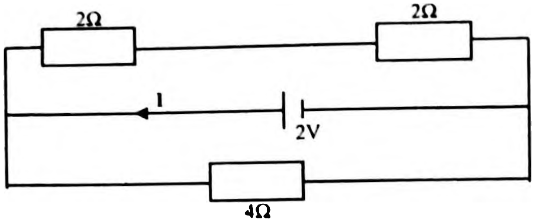


Fig. 6.37

Calculate the current I in the diagram shown above (neglect the internal resistance of the cell). *WAEC 1993* (Ans: 1A)

14. Using the data on the diagram below. Calculate the p.d across the 20Ω resistor. (Neglect the internal resistance of the cell)

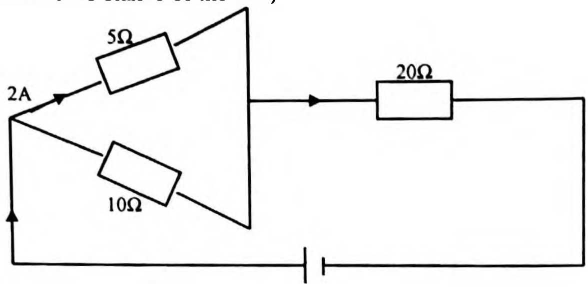


Fig. 6.38

WAEC 1999 Ans: 60V

15. A car fuse marked 3A operates optimally on a 12V battery, calculate the resistance of the fuse. *WAEC 2004* Ans: 4.00Ω

16. A 12V battery supplies a current of 0.4A to a lamp. Calculate the resistance of the lamp. *NECO 2005* Ans: 30.00Ω

17. Three 5 ohm resistors connected in parallel have a potential difference of 60V applied across the combination. The current in each resistor is

A. 4A B. 36A C. 12A D. 24A E. 10A *JAMB 1978* Ans: 12A

18.

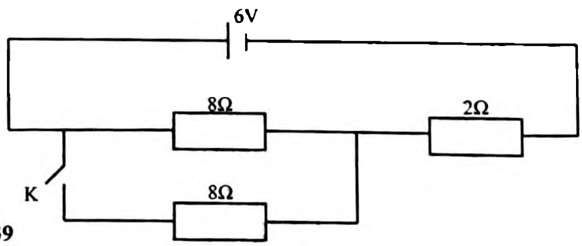


Fig. 6.39

The effect of closing the key K in the circuit shown above would be to

A. increase the current by 0.6A
B. reduce the current by 0.6A
C. reduce the current by 0.4A
D. increase the current by 0.4A
E. keep the current unchanged.

JAMB 1981 Ans: D

19.

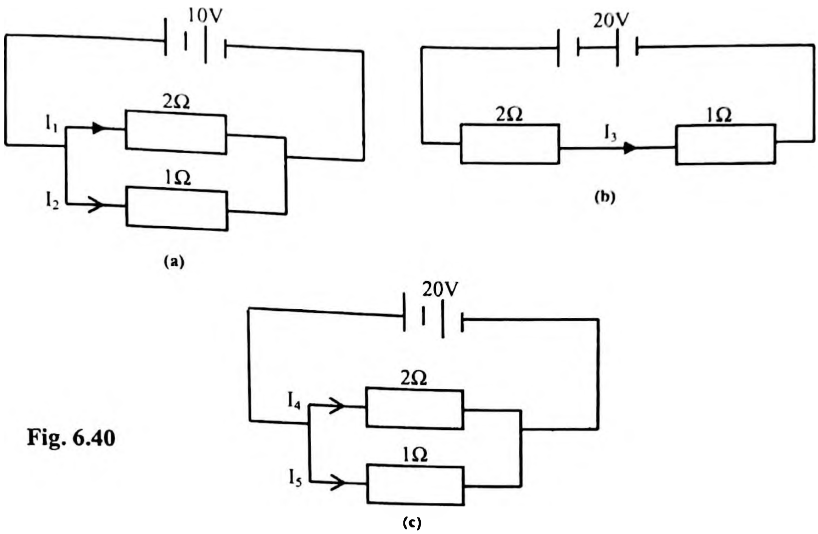


Fig. 6.40

The diagrams above show three circuits. The internal resistances of the batteries are negligible. Which of the currents is the largest?

A. I_1 B. I_2 C. I_3 D. I_4 E. I_5 *JAMB 1983* Ans: I_5

20. A 24V potential difference is applied across a parallel combination of four 6 resistors. The current in each resistor is?

A. 1A B. 4A C. 16A D. 18A E. 36A *JAMB 1983* Ans: 16A

21.

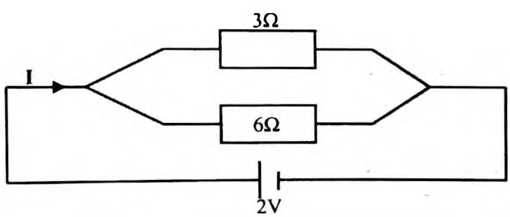


Fig. 6.41

Calculate the current I in the diagram above (neglect the internal resistance of the cell).

WAEC 1994 Ans: I = 1.0A

22.

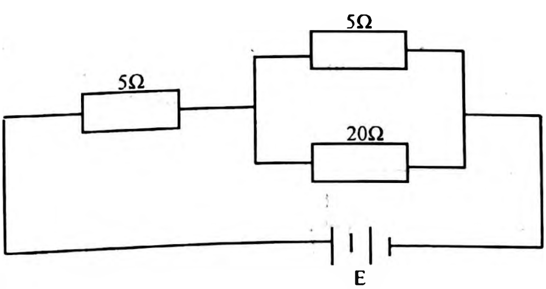


Fig. 6.42

In fig. 6.42, E is an accumulator with negligible internal resistance. If the e.m.f is 9.0V, then the total current is

A. 0.3A B. 0.8A C. 1.0A D. 1.8A *JAMB 1986* Ans: 1.0A

23. Three resistors, with resistances 250Ω, 500Ω and 1kΩ are connected in series. A 6V battery is connected to either end of the combination. Calculate the potential difference between the ends of the 250Ω resistor.

A. 0.20V B. 0.86V C. 1.71V D. 3.43V *JAMB 1989* Ans: 0.86V

24.

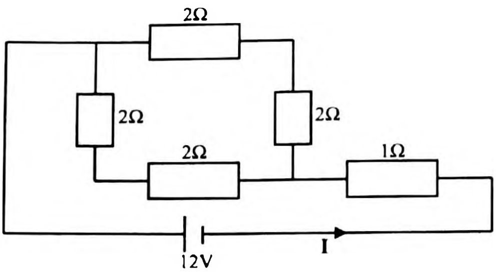


Fig. 6.43

The current I in the figure above is

A. 4.00A B. 1.30A C. 0.80A D. 0.75A *JAMB 1993* Ans: 4.0A

25.

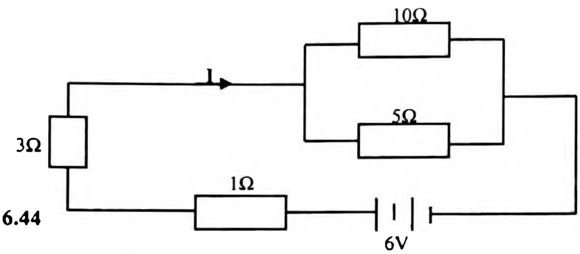


Fig. 6.44

In the diagram above, the current is

A. $\frac{3}{8}$ B. $\frac{9}{11}$ C. $\frac{11}{9}$ D. $\frac{8}{3}$ *JAMB 2001* Ans: $^9/_{11}$ or 0.82A

26.

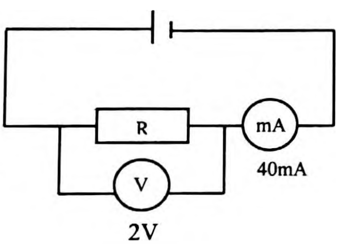


Fig. 6.45

Using the data in the circuit illustrated above, calculate the value of R.

WAEC 1991 Ans: 5Ω

27. A battery of e.m.f E and negligible internal resistance supplies a current I to the combination of two resistances R_1 and R_2 as shown in the diagram below. Calculate the current through R_2.

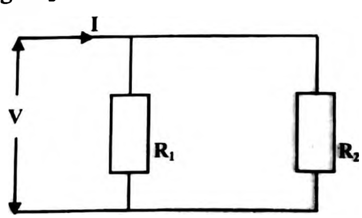


Fig. 6.46

WAEC 1994 Ans: $\frac{IR_1}{R_1 + R_2}$

28. A cell of e.m.f 1.5V and internal resistance of 2.5Ω is connected in series with an ammeter of resistance 0.5Ω and a resistor of resistance 7.0Ω. Calculate current in the circuit. *WAEC 1990* Ans: 0.15A

29. A battery of e.m.f 10V and internal resistance 2Ω is connected to a resistance of 6Ω. Calculate the p.d across the terminals. *WAEC 1994* Ans: 7.5V

30. What is the potential difference between X and Y in the diagram below if the battery is of negligible internal resistance? Calculate the current in the 3Ω resistor.

WAEC 1995 Ans: 3V 1.0A

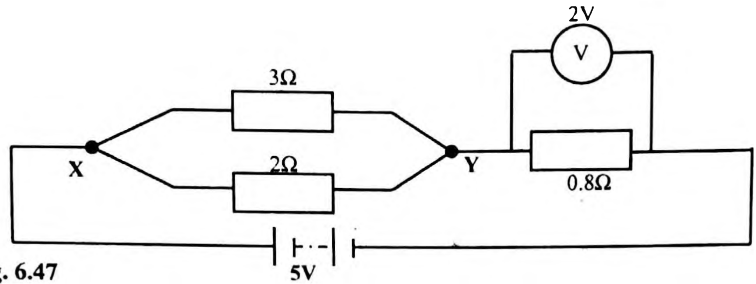


Fig. 6.47

31. Calculate the terminal p.d across 20Ω resistor connected to a battery of e.m.f 15V and internal resistance 5Ω *WAEC 1997* Ans: 12V

32. Two identical cells each of e.m.f 2V and internal resistance 1.0Ω are connected in parallel. The combination is connected to an external load of 1.5Ω, calculate the current in the circuit. *WAEC 2003* Ans: 1.0A

33. Four identical cells each of e.m.f and internal resistance r are connected in series with a resistance R. Write down the formula for the current in the circuit.

WAEC 1989 Ans: $\frac{4E}{R+4r}$

34.

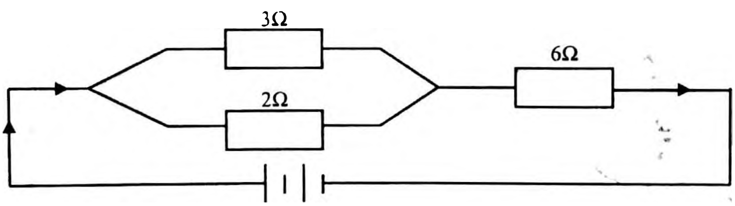


Fig. 6.48

In the diagram above, the current passing through the 6Ω resistor is 1.5A. Calculate the (i) current in the 3Ω resistor (ii) terminal p.d. of the battery.

WAEC 2000 Ans: (i) 0.6A (ii) 10.8V

35. In the circuit shown below, the cell **P** has an e.m.f 1.5V and an unknown internal resistance r while the cell **Q** has an e.m.f 2.0V and an internal resistance 1Ω. If the ammeter reads 50mA, then r is equal to? A. 0.5 ohms B. 0.62 ohms C. 1.0 ohms D. 2.0 ohms E. 3.0 ohms *JAMB 1979* Ans: 2Ω

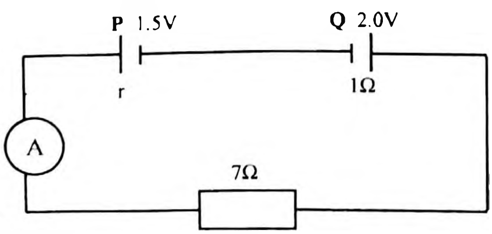


Fig. 6.49

36. An electric cell has an internal resistance of 2Ω. A current of 0.5A is found to flow when a resistor of 5Ω is connected across it. What is the e.m.f. of the cell?
A. 5 volts B. 3.5 volts C. 2.5 volts D. 1 volts E. 10 volts

JAMB 1979 Ans: 3.5V

37. A cell of e.m.f 2V and internal resistance 1Ω supplies a current of 0.5A to a resistance whose value is
A. 0,5Ω B. 1 Ω C. 2 Ω D. 2.5 Ω E. 3 Ω *JAMB 1981* Ans: 3Ω

38. A high-resistance voltmeter reads 3.0V when connected across the terminals of a battery on open circuit and 2.6V when the battery supplies a current of 0.2A through a lamp. The resistance of the lamp is A. 2.00 Ω B. 13.00 Ω C. 0.52 Ω D. 0.13 Ω E. 1.50 Ω *JAMB 1982* Ans: 13.00Ω

39. The difference of potential between the terminals of a cell is 2.2 volts. When a 4Ω resistor is connected across the terminals of this cell, the p.d is 2 volts. What is the internal resistance of the cell? A. 0.10 ohms B. 0.25 ohms C. 0.40 ohms D. 2.50 ohms E. 4.00 ohms *JAMB 1983* Ans: 0.4Ω

40. Two cells, each of e.m.f 1.5V and an internal resistance 2Ω are connected in parallel. Calculate the current flowing when the cells are connected to a 1Ω resistor.
A. 0.75 Ω B. 1.5 Ω C. 0.5 Ω D. 1.0 Ω E. 0.6 Ω *JAMB 1984* Ans: 0.75Ω

41. Three cells of e.m.f 1.5V and an internal resistance of 1.0Ω are connected in parallel across a load resistance of 2.67Ω. Calculate the current in the load.
A. 0.26A B. 0.41A C. 0.50A D. 0.79A *JAMB 1990* Ans: 0.5A

42.

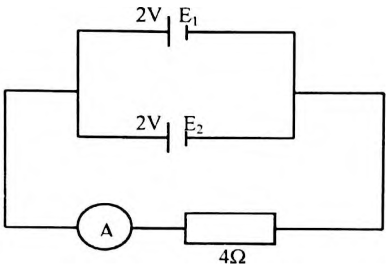


Fig. 6.50

The internal resistance of each of the cells E_1 and E_2 shown in the figure above is 2Ω. Calculate the total current in the circuit.
A. 0.80A B. 0.50A C. 0.40A D. 0.004A *JAMB 1991* Ans: 0.4A

43.

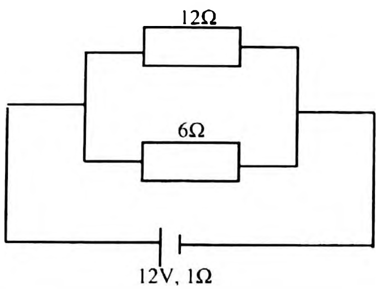


Fig. 6.51

In the circuit diagram above, calculate the current in the 12Ω resistor if the cell has an e.m.f of 12V and an internal resistance of 1Ω.
A. 0.8A B. 1.0A C. 1.6A D. 2.4A *JAMB 1994* Ans: 0.8A

44. A 12V battery has an internal resistance of 0.5Ω. If a cable of 1.0Ω resistance is connected across the two terminal of the battery, the current drawn from the battery is
A. 16.0A B. 8.0A C. 0.8A D. 0.4A *JAMB 1995* Ans: 8.0A

45. Three electric cells each of e.m.f 1.5V and internal resistance 1.0Ω are connected in parallel across an external resistance of $^2/_3$Ω. Calculate the value of the current in the resistor. A. 0.5A B. 0.9A C. 1.5A D. 4.5A
JAMB 1997 Ans: 0.9A

46. Four cells each of e.m.f 1.5V and internal resistance of 4Ω are connected in parallel. What is the effective e.m.f and internal resistance of the combination?

A. 6.0V, 16 Ω B. 6.0V, 1Ω C. 1.5V, 4 Ω D. 1.5V, 1Ω

JAMB 1999 Ans: 1.5V, 1.0Ω

47.

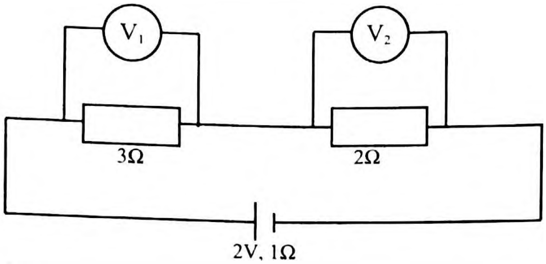


Fig. 6.52

In the diagram above, the values of V_1 and V_2 are respectively?

JAMB 2000 Ans: 1V, $^2/_3$V

48. A radio is operated by eight cells each of e.m.f 2.0V connected in series. If two of the cell are wrongly connected, the net e.m.f of the radio is

A. 16V B. 12V C. 10V D. 8V *JAMB 2000* Ans: 12V

49. A cell whose internal resistance is 0.5Ω delivers a current of 4A to an external resistor. The lost voltage of the cell is

A. 1.250V B. 8.000V C. 0.125V D. 2.000V *JAMB 2004* Ans: 2.0V

50. A resistance **R** is connected across the terminal of an electric cell of internal resistance 2Ω and the voltage was reduced to $\frac{3}{5}$ of its nominal value. The value of R is

A. 3 Ω B. 2 Ω C. 1 Ω D. 6 Ω *JAMB 2001* Ans: 3Ω

51. A cell supply current of 0.8A and 0.4A through a 2Ω and 5Ω resistor respectively. Calculate the internal resistance of the cell. *WAEC 1988* Ans: 1.0Ω

52. A cell supplies current of 0.6A and 0.2A through 1.0Ω and 4.0Ω resistors respectively. Calculate the internal resistance of the cell. *WAEC 2005* Ans: 0.5Ω

53. A cell gives a current of 0.15A through a resistance of 8Ω and 0.3A when the resistance is changed to 3Ω the internal resistance of the cell is

A. 0.05 Ω B. 1.00 Ω C. 1.50 Ω D. 2.00 Ω E. 2.50 Ω *JAMB 1985* Ans: 2Ω

54. The terminal voltage of a battery is 4.0V when supplying a current of 2.0A and 2.0V when supplying a current of 3.0A. The internal resistance of the battery is

A. 0.5 Ω B.1.0 Ω C. 2.0 Ω D. 4.0 Ω *JAMB 1993* Ans: 2Ω

55. A cell can supply currents of 0.4A and 0.2A through a 4.0Ω and 10.0Ω resistors respectively. The internal resistance of the cell is

A. 2.0 Ω B. 1. 0 Ω C. 2.5 Ω D. 1.5 Ω *JAMB 2001* Ans: 2Ω

56.

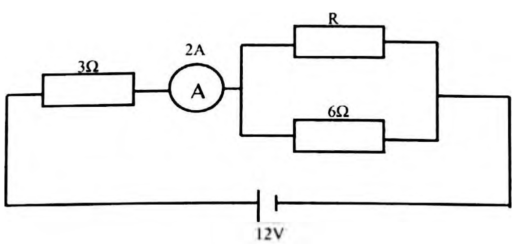


Fig. 6.53

What is the value of R in the diagram above?

A. 3 Ω B. 4 Ω C. 5 Ω D. 6 Ω *JAMB 1986* Ans: 6Ω

57.

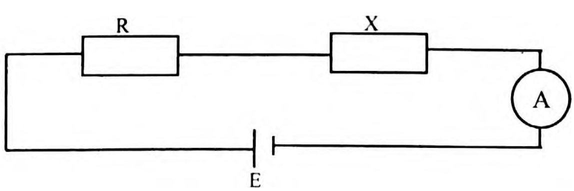


Fig. 6.54

In the circuit above, the ammeter reads a current of 5.0A when R=8Ω and reads 7.0A when R = 5Ω. The value of the unknown resistance X is
A. 10.0 Ω B. 7.5 Ω C. 5.0 Ω D. 2.5 Ω *JAMB 1999* Ans: 2.5Ω

58.

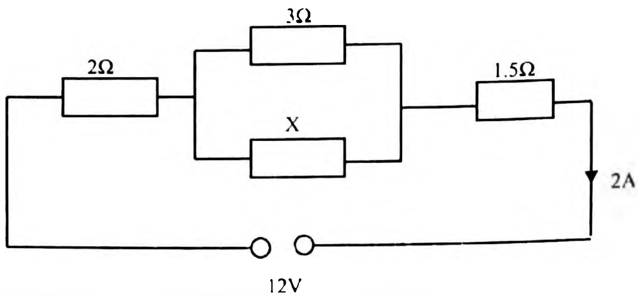


Fig. 6.55

From the diagram above, determine the value of the resistance X.
A. 9 Ω B. 12 Ω C. 15 Ω D. 6 Ω *JAMB 2002* Ans: 15Ω

59. A cell of internal resistance 1Ω supplies current to an external resistor of 3Ω. The efficiency of the cell is? A. 75% B. 50% C. 33% D. 25% *JAMB 2000* Ans: 75%

60. A cell of internal resistance 2Ω supplies current to a 6Ω resistor. The efficiency of the cell is A. 12.0% B. 25.0% C. 33.3 % D. 75.0 % *JAMB 1988* Ans: 75%

61. A chemical cell of internal resistance 1Ω supplies electric current to an external resistance 3Ω, calculate the efficiency of the cell. *WAEC 2005* Ans: 75%

62. A cell of e.m.f 1.5V and internal resistance 1.0Ω is connected to two resistors of resistance 2.0Ω and 3.0Ω in series. Calculate the current through the resistors.
WAEC 2006 Ans: 0.25A

63. Three identical cells each of e.m.f 1.5V and internal resistance 1.0Ω are connected in parallel across an internal load of resistance 2.67Ω. Calculate the current in the load.
NECO 2007 Ans: 0.50A

64. A cell supplies a current of 0.6A through a 2 Ω-resistor and a current of 0.2 A through a 7 Ω-resistor. Calculate the internal resistance of the cell.
NECO 2008 Ans: 0.5 Ω

65.

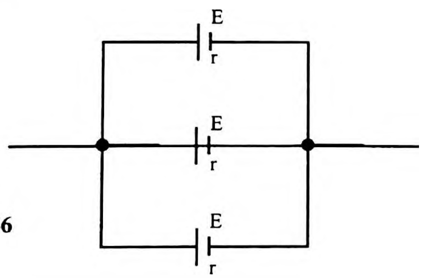


Fig. 6.56

The cells each of e.m.f 1.5V and internal resistance of 2.5Ω are connected as shown in the diagram above. Find the net e.m.f and the internal resistance.
A. 4.5V, 0.83Ω B. 4.5V, 7.50 Ω C. 1.5V, 0.83 Ω D. 1.5V, 7.50 Ω
JAMB 2007 *Ans. C*

66. A resistor of resistance *R* is connected to a battery of negligible internal resistance. If a similar resistor is connected in series with it the
A. effective resistance of the circuit is halved.
B. total power dissipated is doubled.
C. total current in the circuit is halved.
D. terminal voltage is halved WAEC 2009[38] Ans: C

67.

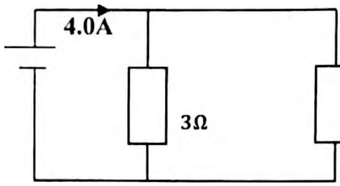


Fig. 6.57

Calculate the e.m.f of the cell in the above circuit if its internal resistance is negligible.

A. 12V B. 8V C. 2V D. 36V JAMB 2009[40] Ans: B

68. A cell of *e.m.f* 1.5V is connected in series with a resistor of resistance 3.0Ω. A voltmeter connected across the cell registers 0.9V. Calculate the internal resistance of the cell. WAEC 2009[39] Ans: 2Ω

69. A battery of *e.m.f* 12.0V and internal resistance 0.5 Ω is connected to 1.5Ω and 4.0 Ω series resistors. Calculate the terminal voltage of the battery.

WAEC 2009[41] Ans: 11V

70. When a resistor of resistance R is connected across a cell, the terminal potential difference of the cell is reduced to the three-quarters of its e.m.f. The cell's internal resistance in terms of R is

A. $R/4$ B. $R/3$ C. $R/2$ D. $2R/3$ E. R

NECO 2009[43] Ans:B

71. Determine the p.d across the load in the diagram below, if the ammeter is of negligible internal resistance. NECO 2009[44] Ans: 10.7V

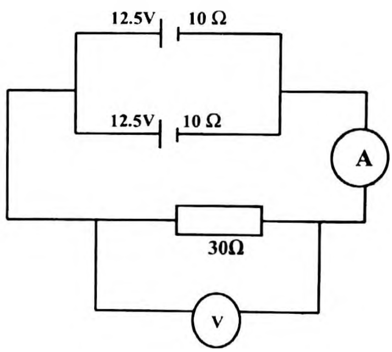


Fig. 6.58

72. A cell of electromotive force E and internal resistance r is connected in series with an ammeter and an external load of resistance R. Derive an expression for the power dissipated by the external load in terms of the parameters.

NECO 2009[E7] Ans: $\frac{RE^2}{(R+r)^2}$

7

ELECTRICAL ENERGY AND POWER

ELECTRICAL ENERGY

Electrical energy (W) = quantity of charge (Q) × Potential difference (V)

$$W = QV \quad \ldots\ldots (1)$$

Substituting $Q = It$ into (1) we have

$$W = IVt \quad \ldots\ldots (2)$$

Substituting $V = IR$ into (2) we have

$$W = I^2Rt \quad \ldots\ldots (3)$$

Substituting $I = {}^V/_R$ into (3) we have

$$W = \frac{V^2t}{R} \quad \ldots\ldots (4)$$

So, Electrical Energy can be calculated using any of these equations.

$$W = QV = IVt = I^2Rt = \frac{V^2t}{R}$$

ELECTRICAL POWER

$$\text{Electrical Power (P)} = \frac{\text{Electrical energy transferred (W)}}{\text{Time taken (t)}}$$

$$P = \frac{W}{t} \quad \ldots\ldots (5)$$

Substituting $W = QV$ into (5) we have

$$P = \frac{QV}{t} \quad \ldots\ldots (6)$$

Substituting $W = IVt$ into (5) we have

$$P = \frac{IVt}{t} = IV \quad \ldots\ldots (7)$$

Substituting $W = I^2Rt$ into (5) we have

$$P = \frac{I^2Rt}{t} = I^2R \quad \ldots\ldots (8)$$

Substituting $I = {}^V/_R$ into (8) we have

$$P = \frac{V^2R}{R^2} = \frac{V^2}{R} \quad \ldots\ldots (9)$$

So, Electrical Power can be calculated using any of these equations.

$$P = \frac{W}{t} = \frac{QV}{t} = IV = I^2R = \frac{V^2}{R}$$

Where, W = electrical energy in Joules (J)
P = electrical power in watts (W)
I = current in amperes (A)
V = potential difference in volts (V)
R = resistance in ohms (Ω)
Q = quantity of charge in coulomb (C)
t = time in seconds (s)

ELECTRICAL HEAT ENERGY

Electrical Heat energy, $H = I^2Rt$. This is in agreement with Joule's law of electrical heating which states that the heat energy produced in a wire as a result of electrical current passing through it is proportional to

(i) the resistance of the wire, R
(ii) the square of the current, I^2
(iii) the time (t)

So, electrical heat energy can be calculated using any of the three equations

$$H = I^2Rt = IVt = \frac{V^2t}{R} = Pt$$

Example 1

A work of 30 Joules is done in transferring 5 milli-coulombs of charge from a point B to a point A in an electric field. What is the potential difference between B and A? *WAEC 1990*

Solution

Energy or work done in an electric field, W = QV

Given: work done, W = 30J; charge, $Q = 5mC = 5 \times 10^{-3}C$

$$\therefore \text{ Potential difference, } V = \frac{W}{Q} = \frac{30}{5\times10^{-3}} = 6.0\times10^{3}V$$

Example 2

Calculate the resistance of the filament of a lamp rated 240V, 40W. *WAEC 1990*

Solution

Given; potential difference, V = 240V Power, P = 40W

$$P = \frac{V^2}{R}, \quad \therefore \text{ Resistance, } R = \frac{V^2}{P}$$

$$\therefore R = \frac{240^2}{40} = \frac{57600}{40} = 1440\Omega$$

Example 3

The maximum power dissipated by a 100Ω resistor in a circuit is 4W. Calculate the voltage across the resistor. *WAEC 2003*

Solution

Given: Power, P = 4W; resistance, R = 100Ω

$$P = \frac{V^2}{R} \quad \therefore \quad V^2 = P\times R$$

$$\therefore \quad V = \sqrt{P\times R} = \sqrt{4\times100} = \sqrt{400} = \underline{20V}$$

Example 4

An electric iron is rated at 1000watts, 250V. The corresponding maximum resistance and accompanying current is A. 62.5A, 4.0Ω B. 16.0A, 62.5 Ω C. 62.5A, 16.0 Ω D. 4.0A, 62.5 Ω E. 4.0A, 250 Ω *JAMB 1982 Ans: D*

Solution

Given: Power, P = 1000W, voltage, R = 250V.

$$P = IV \quad . \quad \text{current, } I = \frac{P}{V} = \frac{1000}{250} = 4A$$

$$P = I^2R \quad . \quad \text{resistance, } R = \frac{P}{I^2} = \frac{1000}{4^2} = \frac{1000}{16} = 62.5\Omega \qquad Ans: 62.5\Omega, 4A$$

Example 5

When connected to a mains of 250V, the fuse rating in the plug of an electric device of 1kW is A. 4A B. 2A C. 5A D. 3A *JAMB 2002 Ans: 4A*

Solution

The fuse rating is the maximum safe current a fuse can take before breaking.

Given: Power, P = 1kW = 1000W, p.d., V = 250V

$$P = IV \quad \therefore \quad \text{current, } I = \frac{P}{V} = \frac{1000}{250} = 4A$$

Example 6

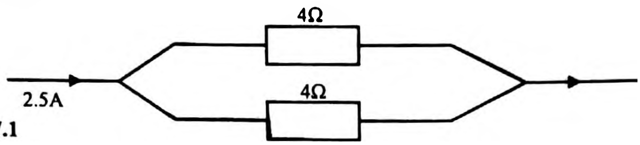


Fig. 7.1

Calculate (i) the rate of energy consumption; (ii) the total heat energy developed in 15 minutes; in the circuit illustrated by the diagram above.

Solution

(i) Rate of energy consumption = Power used

Power, $P = IV = I^2R$

Current, I = 2.5A, Effective resistance, $R = \frac{4 \times 4}{4 - 4} = 2\Omega$

$$\therefore \quad P = I^2R = 2.5^2 \times 2 = 12.5W$$

(ii) Given, I = 2.5A, R= 2Ω, time, t = 15 × 60 = 900sec.

Heat energy, $H = I^2Rt$

$$\therefore \quad H = 2.5^2 \times 2 \times 900 = 11250J$$

Example 7

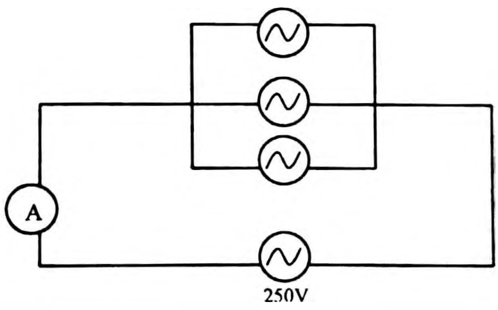


Fig. 7.2

In the diagram above, three identical lamps each of 100W are connected in parallel across a p.d of 250V. Calculate the reading of the ammeter. *WAEC 1993*

Solution

Potential difference, V = 250V; Power, P = 100W each across each lamp.

From P = IV

$$\text{Current through each lamp, } I = \frac{P}{V} = \frac{100}{250} = 0.4A$$

The ammeter reads the total current from the 3 lamps, therefore total current is

$$3 \times 0.4A = 1.2A$$

Example 8

In the diagram below, X and Y are resistance 4Ω and 6Ω respectively. If power dissipation in X is 10W, then what is the power dissipation in Y?

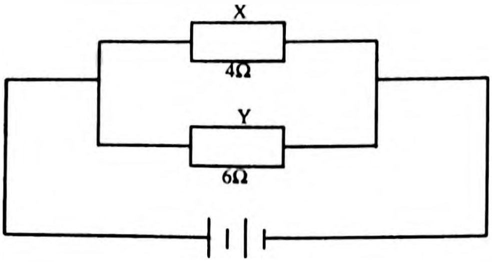


Fig. 7.2

A. 2.4W B. 4.0W C. 6.0W D. 6.7W E. 15.0W *JAMB1984 Ans:6.7W*

Solution

$$\text{Power} = \frac{\text{Voltage}^2}{\text{Resistance}} \qquad \text{i.e. } P = \frac{V^2}{R}$$

For resistor X, P = 10W and R = 4Ω

From $P = \frac{V^2}{R}$, the p.d across the 4Ω resistor is

$$V^2 = P \times R = 10 \times 4$$
$$V^2 = 40$$
$$V = \sqrt{40} = 6.32V$$

The p.d across Y is also 6.32V because the resistors are connected in parallel.
For resistor Y, V = 6.32V and R = 6Ω

From $P = \frac{V^2}{R}$, Power dissipated in Y is

$$P = \frac{6.32^2}{6} = 6.7W$$

Example 9

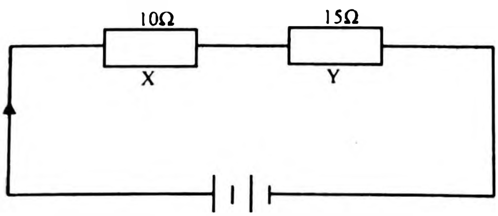


Fig. 7.4

In the diagram above, X and Y are resistance 10Ω and 15Ω respectively. If the power dissipation in X is 40W, then what is the power dissipation in Y?

Solution

From $P = I^2R$, the current through the circuit is, $I^2 = {}^{P}/_{R}$

For resistor X; P = 40W and R = 10Ω

$$\therefore \quad I^2 = \frac{40}{10} = 4$$

Circuit current, $I = \sqrt{4} = 2A$

For resistor Y, $R = 15\Omega$ and $I=2A$

Power dissipation in Y is $P= I^2R = 2^2 \times 15$

$= 60W$

Example 10

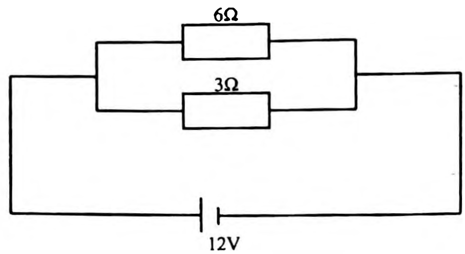


Fig. 7.5

In the diagram above, the ratio of the electric power dissipated in the 6Ω and 3Ω resistors respectively is A. 2:3 B. 1:2 C. 1:3 D. 2:1

JAMB 2004 Ans: 1:2

Solution

Effective circuit resistance, $R = \frac{R_1R_2}{R_1+R_2} = \frac{3\times6}{3+6} = 2\Omega$

Current through 3Ω resistor, $I = \frac{V}{R} = \frac{12}{3} = 4A$

Power dissipated in 3Ω resistor, $P = I^2R$

$= 4^2 \times 3$

$= 48W$

Current through 6Ω resistor, $I = \frac{V}{R} = \frac{12}{6} = 2A$

Power dissipated in 6Ω resistor, $P = I^2R$

$= 2^2 \times 6$

$= 24W$

∴ Ratio of power dissipation in 6Ω and 3Ω is 24: 48 or 1: 2

Example 11

A portable generator is connected to six 100W lamps and a 600W amplifying system. How much energy is consumed if the generator runs for 6 hours? *WAEC 1989*

Solution

Power, $P = \frac{\text{Energyconsumed}(W)}{\text{Time taken}(t)}$

∴ Energy consumed, $W = P \times t$

Power consumed by lamps = 6 x 100W = 600W

Power consumed by amplifier = 600W

Total power consumed, P = 600 + 600 = 1200W

Time taken, t = 1 hr

$\therefore$ $W = P \times t = 1200 \times 1 = 1200\text{ Wh} = 1.2\text{kWh}$

Example 12
An electric kettle, connected to a 240V mains produces 6.0×10^5J of heat energy to boil a quantity of water in 5 minutes. Find the resistance of the kettle.
A. 14.4Ω B. 28.8Ω C. 144Ω D. 288Ω E. 2880Ω *JAMB 1985 Ans:28.8Ω*
Solution
Given: $H = 6.0 \times 10^5$J; $V = 240$V; $t = 5\text{min} = 5 \times 60 = 300\text{sec}$.

Electrical heat energy, $H = IVt = \dfrac{V^2t}{R}$

$$H = \frac{V^2t}{R} \quad \therefore \quad \text{Resistance, } R = \frac{V^2t}{H} = \frac{240^2 \times 300}{6.0 \times 10^5} = \frac{1.728 \times 10^7}{6.0 \times 10^5} = 28.8\Omega$$

Example 13
Calculate the time in which 4.8kJ of energy would be expended when an electric heater of resistance 1.8×10^3 Ω is used on a 240V mains supply. *WAEC 1992*
Solution
Energy, W = 4.8kJ = 4800J; resistance, $R = 1.8 \times 10^3$ Ω; Potential difference, V = 240V

$$\text{From } P = \frac{W}{t}, \text{time}, t = \frac{W}{P} = \frac{W}{\frac{V^2}{R}} \quad \left(P = \frac{V^2}{R}\right)$$

$$t = \frac{WR}{V^2} = \frac{4800 \times 1.8 \times 10^3}{240^2} = 150s$$

Alternatively, we could use $H = \dfrac{V^2t}{R}$

$$\therefore \text{ time, } t = \frac{HR}{V^2} = \frac{4800 \times 1.8 \times 10^3}{240^2} = 150s$$

Example 14
A working electric motor takes a current of 1.5A when the p.d across it is 250V. If its efficiency is 80%, the power output is
A.300.0W B. 469.0W C. 133.0W D. 4.8W *JAMB 2001 Ans: 300W*
Solution
Given: E = 80%, current, I = 1.5A, voltage, V = 250V
$\therefore$ $P_i = IV = 1.5 \times 250V = 375W$

$$Efficiency = \frac{power\ output}{power\ input} \times 100$$

$$E = \frac{P_o}{P_i} \times 100$$

$$\text{Power output, } P_o = \frac{E \times P_i}{100} = \frac{80 \times 375}{100} = 300W$$

Example 15
An electric lamp is rated 240V, 40W. What is the cost of running the lamp for 72hr if the electricity authority charges ₦2.50 per kWh? *WAEC 2005*
Solution
Given: Power, P = 40W; time, t = 72hr

$\therefore$ $W = 40 \times 72 = 2880\text{Wh} = 2.880\text{kWh}$

$P = \frac{W}{t}$ ∴ Energy consumed, $W = P \times t$

Cost of electrical energy (C_E) = cost per kWh × electrical energy consumed in kWh

∴ Cost of running lamp = ₦2.50/kWh × 2.880kWh

= ₦7.20

Example 16
An electric heater takes 4A when operated on a 250V supply. What is the cost of the electricity consumed at 10k per kWh when the heater is used for 5 hrs? *NECO 2002*
Solution
Given: I = 4A; V = 250V; time, t = 5hr

P = IV ∴ Power, P = 250 × 4 = 1000W

Energy consumed, W = P × t = 1000 × 5 = 5000Wh = 5kWh

$$\text{Cost of electricity} = \frac{10k}{kWh} \times 5kWh = 50k$$

Example 17
A land lord has eight 40W electric light bulbs, four 60W bulb and two 100W bulb in his house. If he has all the points on for five hours daily and if NEPA charges 5k per unit, his bill for 30 days is
A. ₦5.70 B. ₦7.25 C. ₦3.65 D. ₦8.05 E. ₦4.50 *JAMB 1979 Ans: ₦5.70*
Solution
Total power consumed = (8 × 40W) + (4 × 60W) + (2 × 100W)

= (320 + 240 + 200) W

= 760W

Total time taken = 5hr/day × 30 days = 150hrs

Electrical energy used (kWh) = Power consumed × time taken

= 760W × 150hr

= 114000Wh

= 114kWh

$$\text{Cost of electricity} = 5\text{k/kWh} \times 114\text{kWh} = \frac{5\text{k}}{\text{kWh}} \times 114\text{kWh}$$

= 570k
= ₦5.70k

EXERCISE 7
1. Find the work done in moving a 2C charge between two point X and Y in an electric field if the potential difference is 100Volts.
A. 50J B. 100J C. 200J D. 400J *JAMB 1998* Ans: 200J
2. Calculate the power delivered by a 3-phase line if its voltage and current are 132kV and 60A respectively. *WAEC 1996* Ans: 7.92×10^6W

3. An electric bulb is rated 60W, 220V. Calculate the resistance of its filament when it is operating normally. *WAEC 2001* Ans: 806.67Ω

4. A lamp is rated 240V, 60W, calculate the resistance of its filament. *WAEC 2003* Ans: 960Ω

5. Calculate the resistance of the filament rated 240V, 60W. *WAEC 1998* Ans: 960Ω

6. An electric kettle contains a 720W heating unit, calculate the current it takes from a 240V mains. *NECO 2004* Ans: 3A

7. Two lamps rated 40W and 220V each are connected in series. The total power dissipated in both lamps is A.10W B. 20W C. 40W D. 80W E. none of the above *JAMB 1978* Ans: 80W

8. The bulb of a motor cycle head lamp is marked 40W, 6V. The resistance of the filament when it is switched on is?
A. $6^2/40^2$ ohms B. $40/6^2$ ohms C. 40/6 ohms D. 6x40 ohms E. $6^2/40$ ohms
JAMB 1979 Ans: 0.9Ω or $6^2/_{40}$ ohms

9. The resistance of a 240V, 60watts electric filament bulb is
A. 0.25Ω B. 480Ω C. 60Ω D. 240Ω E. 960Ω *JAMB 1981* Ans: 960Ω

10. Which of the following is most suitable for protecting the circuit of a 2000W electric iron connected to a 250V mains?
A. 13A B. 8A C. 5A D. 3A *JAMB 1990* Ans: 8A

11. A lamp is rated 240V, 60W. The resistance of the filament is?
A. 960Ω B. 16Ω C. 15Ω D. 4Ω *JAMB 1990* Ans: 960Ω

12. An equipment whose power is 1500W and resistance is 375 ohms would draw a current of A. 0.10A B. 2.00A C. 4.00A D. 77.5A *JAMB 1991* Ans: 2A

13. Which of the following apparatus will require the smallest fuse rating for its protection? A. 60W, 240V B. 60W, 40V C. 40W, 12V D. 40W, 5V
JAMB 1992 Ans: A

14. A 40W instrument has a resistance of 90Ω. On what voltage should it be operated normally? A. 60V B. 150V C. 225V D. 3600 V
JAMB 1993 Ans: 60V

15. If the maximum voltage across a 100 ohm resistor is 20V, then the maximum power it can dissipate is A. 5.00W B. 4.00W C. 2.00W D. 0.25W
JAMB 1995 Ans: 4W

16. A 3000W electric cooker is to be used on a 200V mains circuit. Which of the fuses below can be used safely with the cooker? A. 2A B. 5A C. 10A D. 20A
JAMB 1999 Ans: 20A

17. An electric generator with a power output of 3.0kW at a voltage of 1.5kV distributes power along cables of total resistance 20.0Ω. The power in loss the cable is A. 0.1W B. 10.0W C. 40.0W D. 80.0W
JAMB 2000 Ans: 80.0W

18. An electric iron is rated 1000W, 230V. What is the resistance of its element?
A. 57.6Ω B. 55.9Ω C. 52.9Ω D. 51.9Ω. *JAMB 2002* Ans: 52.9Ω

19. Calculate the total heat energy developed in 5 minutes by the system below

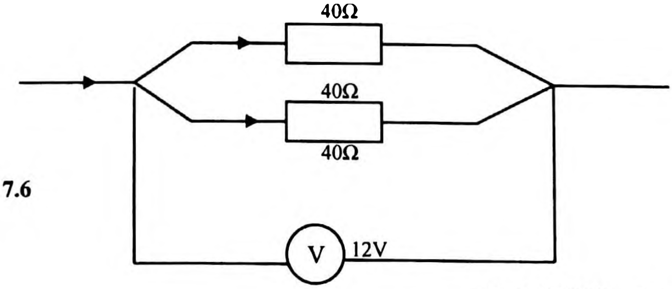


Fig. 7.6

WAEC 1988 Ans: 8640J

20.

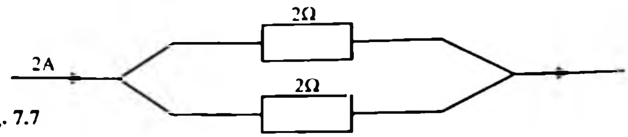


Fig. 7.7

Calculate the rate at which energy is used up in the circuit illustrated by the diagram above. *WAEC 1992* Ans: 2W

21. Three identical lamps each of power 100W are connected in parallel across a potential difference of 250V. Calculate the current in the circuit.
WAEC 2001 Ans: 1.2A

22.

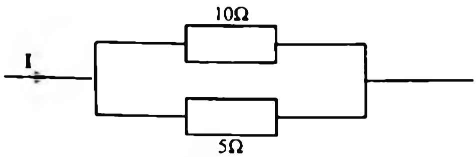


Fig. 7.8

In the diagram above, current (I) passes through the parallel combination. If the power dissipated in the 5Ω resistor is 40W, then the power dissipated in the 10Ω resistor is
A. 10W B. 20W C. 40W D. 80W E. 100W *JAMB 1985* Ans: 20W

23.

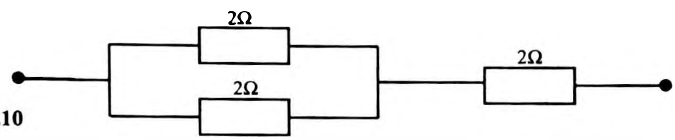


Fig. 7.10

In the diagram above, if each of the resistors can dissipate a maximum of 18W without becoming excessively heated, what is the maximum power that circuit can dissipate?
A. 27W B. 18W C. 9W D. 5W *JAMB 2003* Ans: 27W

24.

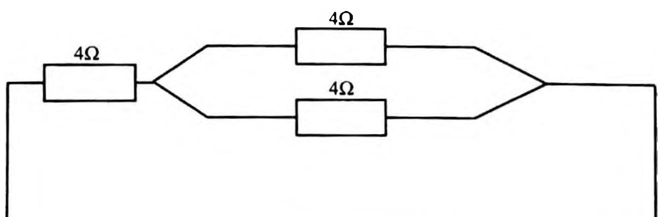


Fig. 7.9

The total power drawn from the cell in the circuit diagram above is
A. 12W B. 24W C. 32W D. 40W *JAMB 1989* Ans: 24W

25. Two resistors, R_1 = 4Ω and R_2 = 5Ω are connected in parallel across a potential difference. If P_1 and P_2 represent the power dissipated in R_1 and R_2 respectively, then the ratio P_1:P_2 is A. 4:5 B. 5:4 C. 16:25 D. 25:16
JAMB 1994 Ans: 5 : 4

26. A lamp is marked 220V 60W. Calculate the energy it would consume when connected to a 220V source for 1 hour. *WAEC 1988* Ans: 216000J

27. A potential difference of 6V is used to produce a current of 5A, for 200s through a heating coil. The heat produced is A. 4800 cal B. 6000 cal C. 2400J D. 240k cal E. 6000J *JAMB 1978* Ans: 6000J

28. A stand by generator is connected to fifteen 40W lamps and a musician's 600W amplifying system. How much energy is used if the generator runs for 6 hours? A. 3.84kwh B. 7.20kwh C. 8.40kwh D. 12.00kwh E. 14.56kwh

JAMB 1980 Ans: 7200Wh =7.2kWh

29. A current of 0.5A flows through a resistor when connected to a 40V battery. How much energy is dissipated in 2 minutes?
A. 1200J B. 1500 J C. 2400 J D. 96000 J *JAMB 1987* Ans: 2400J

30. An electric current of 2 amperes flows in a heating coil of resistance 50 ohms for 3 minutes 20 seconds. Determine the heat produced.
A. 0.5kJ B. 8.0kJ C. 20.0kJ D. 40.0kJ *JAMB 1995* Ans: 40.0kJ

31. What is the total electrical energy consumed by using an electric cooker rated 1000W for 5hrs? A. 5.3×10^3J B. 6.5×10^3J C. 1.8×10^7J D. 2.3×10^7J

JAMB 1997 Ans: 1.8×10^7J

32. Electricity is supplied to a school along a cable of total resistance 0.5Ω with the maximum current drawn from the mains as 100A. The maximum energy dissipated as heat for 1hr is A. 3.6×10^3J B. 5.0×10^3J C. 3.0×10^5J D. 1.8×10^7J

JAMB 1998 Ans: 1.8×10^7J

33. A bread toaster uses a current of 4A plugged in a 240 volts line. It takes one minute to toast slices of bread. What is the energy consumed by the toaster?
A. 5.76×10^4J B. 1.60×10^4J C. 3.60×10^3J D. 1.60×10^2J

JAMB 2001 Ans: 5.76 . 0^4J

34. An electric iron is rated 500W,200V calculate the :
(i) current required to operate the iron,
(ii) heat generated in 30 minutes. *NECO 2006* Ans: (i) 2.5A (ii) 9.0×10^5J

35. A lamp marked 100W, 250V is lit for 10 hours. If it operates normally and 1kWh of electrical energy cost 2k, what is the cost of lighting the lamp?

WAEC 1994 Ans: 2k

36. A man uses a 900W electric iron to press his cloth for an average of 4 hours a week for 5 weeks. If the cost of electrical energy is 30 kobo per unit, calculate the cost of energy for pressing. *WAEC 2005* Ans: 540k

37. An electric iron rated 250V has a coil of resistance 125Ω. What is the cost of using it for 1 hour at 10k per kWh. *NECO 2000* Ans: 5.00k

38. Find the cost of running a 60W lamp for 24hrs, if 1kWh cost 5 Naira

WAEC 2004 Ans: ₦7.20

39. An electric heater takes 4A when operated on a 250V supply. What is the cost of the electricity consumed at 10k per kWh when the heater is used for 5 hours.

NECO 2002 Ans: 50k

40. If NEPA charges 5k per kWh, what is the cost of operating for 24hours a lamp requiring 1A on a 200V line?
A. 24k B. 55k C. 40k D. 26k E. 32k *JAMB 1978* Ans: 24k

41. What is the cost of running five 50W lamps and four 100W lamps for 10 hours if electrical energy costs 2 kobo per kWh? A. ₦0.65 B. ₦0.13 C. ₦ .90 D. ₦39.00 E. ₦234.0 *JAMB 1984* Ans: ₦0.13 or 13k

42. A household refrigerator is rated 200 watts. If electricity cost 5k per kWh, what is the cost of operating it for 20 days.
A. ₦4.80 B. ₦48.00 C. ₦480.00 D. ₦4800.00

JAMB 1988 Ans: ₦4.80

43. Which of the following instruments consumes the highest current?

	Instruments	Voltage Rating	Power Rating
A	Electric iron	250V	1kW
B	Television set	220V	110W
C	Torch light	6V	30W
D	Immersion heater	110V	500W

JAMB 1988 Ans: C

44. The cost of running five 60W lamps and four 100W lamps for 20 hours if electrical energy cost ₦10.00 per kWh is A. ₦280.00 B. ₦160.00 C. ₦120.00 D. ₦140.000 *JAMB 2001* Ans: ₦140.00

45. An electric lamp rated 120V is used on a 240 $V_{r.m.s}$, calculate the resistance of its filament. *WAEC 2006. Ans: 480Ω*

46. A steady current of 2A flows in a coil of e.m.f. 12V for 0.4s. A back e.m.f of 3V was induced during this period. The stored energy in the loop that can be utilized is: A. 7.2 J B. 12.0 J C. 2.4 J D. 9.6 J *JAMB 2004*[42] *Ans. 7.2 J*

Hint: $E = I(V_1 - V_2)t$

47. A bulb marked 240V, 40W is used for 30 minutes. Calculate the heat generated. *WAEC 2007*[43] *Ans. 72000J*

48. Five 80-W and three 100-W lamps are run for 8 hours. If the cost of energy is ₦5.00 per unit, calculate the cost of running the lamps. [1unit = kWh]

WAEC 2007 Ans. ₦28.00

49. An electric generator has an e.m.f of 240V and an internal resistance of 1Ω. If the current supplied by the generator is 20A when the terminal voltage is 220V, find the ratio of the power supplied to the power dissipated.

A. 11:1 B. 1:11 C. 12:11 D. 11:12 *JAMB 2008 Ans: 12:11* Hint: E:V

50. A generator is on daily use and in the process ten 60W and five 40W tungsten bulbs are on for the same time interval. The energy consumed daily is

A. 0.9kWh B. 1.92 kWh C. 9.60 kWh D. 19.20 kWh

JAMB 2008 Ans: 19.20 kWh

51. A man has five 60W bulbs and a 240W water heater in his apartment. If the bulbs and the water heater are switched on for four hours daily and the cost of electricity is ₦ 1.20 per kWh, calculate his bill for 30 days. *NECO 2008 Ans: ₦ 77.76*

52. An electric lamp marked 240V, 60W is left to operate for an hour. How much energy is generated by the filament? A. 3.86×10^5J B. 3.56×10^5J C. 1.80×10^4J D. 2.16×10^5J *JAMB 2009*[41] *Ans: D*

1

SCALARS AND VECTORS

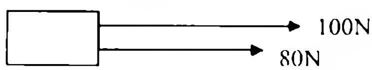
Scalar quantities are those quantities which are described by their size or magnitude only. They have no direction. Examples are temperature, speed, distance, time, mass etc.

On the other hand, vector quantities are those quantities which can be described by their size or magnitude and direction. Examples are force, velocity, acceleration, pressure, displacement etc.

VECTOR ADDITION

While scalars can be added or subtracted easily (e.g. 40kg + 50kg = 90kg or 3m – 2m = 1m), vector addition takes into consideration not only the magnitude but the direction of the vectors. The method of calculation also depends on whether the vectors are acting in a straight line or at an angle to each other, illustrated by the following typical cases.

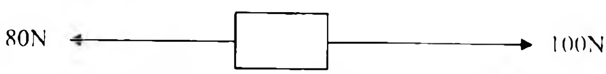
A. **(i) Two Parallel Forces Acting in the Same Direction.**



The resultant force, R = 100 + 80 = 180N

Fig 1.1

(ii) Two Forces Acting In Opposite Direction in a Straight Line



The resultant force, R = 100 – 80 = 20N

Fig 1.2

B. **Two Forces Acting Perpendicularly (At Right Angle Or 90^0) To Each Other**

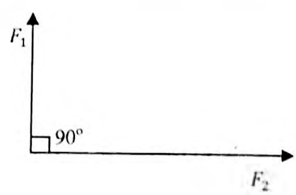
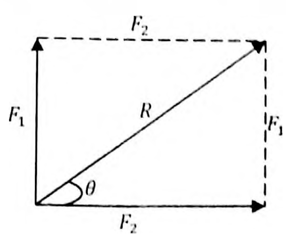


Fig. 1.3

The resultant (R) is the diagonal of the rectangle (If F_1 and F_2 are not equal to each other) or the square (If $F_1 = F_2$) as shown below.

Fig. 1.4



The resultant R, is found using Pythagoras theorem as follows.

$$R^2 = F_1^2 + F_2^2 \qquad \text{Or} \qquad R = \sqrt{F_1^2 + F_2^2}$$

The direction (to the horizontal) at which the resultant act is angle θ and is found using trigonometrical formular as follows;

$$\tan\theta = \frac{F_1}{F_2} \qquad \therefore \qquad \theta = \tan^{-1}\left|\frac{F_1}{F_2}\right|$$

C. Angle Between the Two Acting Forces is Acute (Between 0^0 And 90^0) or Obtuse (Between 90^0 and 180^0).

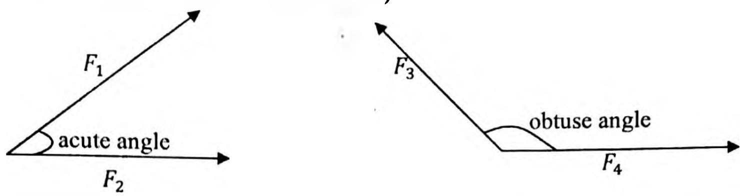


Fig. 1.5 (a) (b)

Assume that F_1 and F_2 in Fig. 1.5a acts at 55^0 to each other and F_3 and F_4 in Fig. 1.5b act at 135^0 to each other. Therefore, to find the resultant for each case, a parallelogram is drawn as shown below.

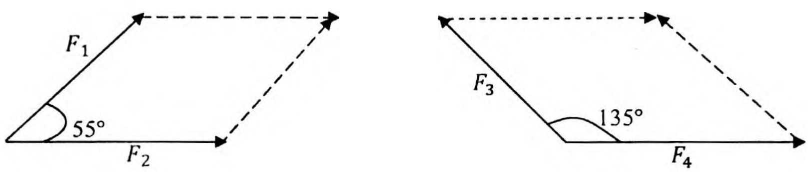


Fig. 1.6 (a) (b)

Always remember the following characteristic of a parallelogram,

1. The opposite sides are equal.
2. The opposite sides are parallel.
3. The opposite angles are equal.

On applying the three characteristics of a parallelogram, Fig. 1.6a and Fig. 1.6b becomes;

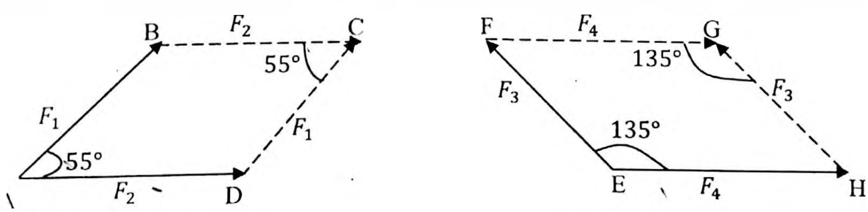


Fig. 1.7 (a) (b)

The sum of all the angles in a parallelogram is 360^0.

From Fig. 1.7a; $ADC + ABC = 360^0 - (55^0 + 55^0)$

$= 360^0 - 110^0 = 250^0$

ABC and ADC are equal. $\therefore$ $ABC = ADC = \frac{250°}{2} = 125°$

Similarly in Fig. 1.7b;

$$EFG + EHG = 360^0 - (135^0 + 135^0) = 360^0 - 270^0 = 90^0$$

EFG and EHG are equal. $\therefore$ $EFG = EHG = \frac{90°}{2} = 45°$

With the above information, Fig. 1.7a and Fig. 1.7b are redrawn as follows.

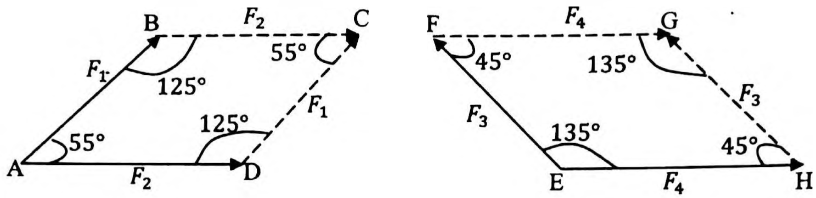


Fig. 1.8 (a) (b)

The resultant, R of the forces F_1 and F_2 in Fig. 1.8a is the <u>diagonal</u> of the parallelogram ABCD. Note, the diagonal is drawn from the point of intersection of forces F_1 and F_2. Similarly, the resultant, R of the forces F_3 and F_4 in Fig. 1.8b is the <u>diagonal</u> of the parallelogram EFGH. Again, note that the diagonal is drawn from the point of intersection of forces F_3 and F_4.

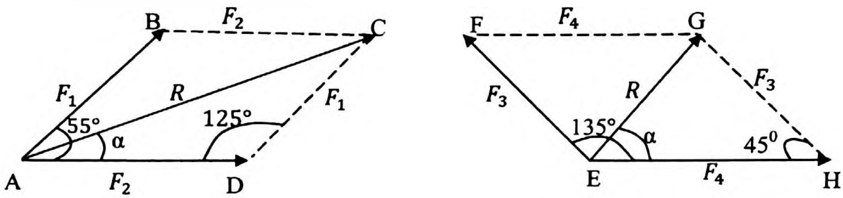


Fig. 1.9 (a) (b)

The resultant (R) and the angle (α) it forms can be calculated using cosine rule and sine rule respectively. We can apply cosine rule to triangle ACD in Fig. 1.9a to find the resultant. Assume F_1 = 10N and F_2 = 12N.

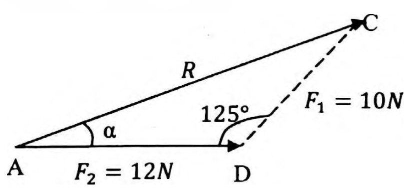


Fig. 1.10

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos D$$
$$R^2 = 10^2 + 12^2 - 2 \times 10 \times 12\cos 125$$
$$R^2 = 100 + 144 - 240 \times (-0.5736)$$
$$= 244 - (-137.66)$$
$$= 244 + 137.66$$
$$= 381.658$$
$$R = \sqrt{381.658}$$
$$R = 19.53N$$

The direction of the resultant vector (19.53N) is the angle α it forms with the F_2 vector as shown in Fig. 1.10. Applying sine rule, we obtain,

$$\frac{AC}{\sin D} = \frac{CD}{\sin A} = \frac{AD}{\sin C}$$

$$\frac{R}{\sin 125} = \frac{F_1}{\sin \alpha} = \frac{F_2}{\sin C}$$

$$\frac{19.53}{\sin 125} = \frac{10}{\sin \alpha} = \frac{12}{\sin C}$$

We pick the first two and ignore $\frac{12}{\sin C}$ because it is not relevant, except if we are asked to find angle C.

$$\frac{19.53}{\sin 125} = \frac{10}{\sin \alpha}$$

Cross multiplying, $\sin \alpha \times 19.53 = \text{Sin}125 \times 10$

$$\sin \alpha = \frac{\sin 125 \times 10}{19.53} = \frac{8.19}{19.53} = 0.419$$

$$\sin \alpha = 0.419$$
$$\therefore \alpha = \sin^{-1} 0.419 = 24.8^0$$

If we substitute R = 19.53N and $\alpha = 24.8^0$ into Fig. 1.9a, we obtain the following

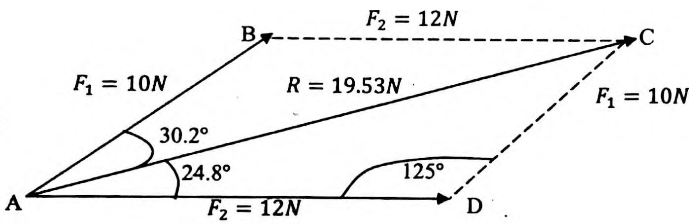


Fig 1.11

Note, BAD = 55°, CAD = 24.80° $\therefore$ BAC = 55° – 24.80° = 30.20°

The magnitude and direction of the resultant is 19.53N at 24.80^0 to the 12N force or 19.53N at 30.20^0 to the 10N force.

The above approach or procedures above can be applied to Fig. 1.9b as follows, assuming F_3 = 10N and F_4 = 12N.

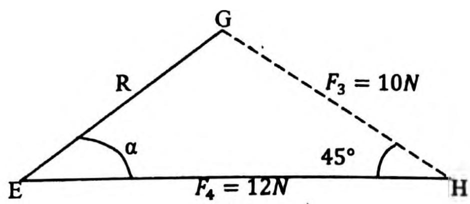


Fig 1.12

We apply cosine rule to obtain the resultant in Fig. 1.12.

$R^2 = F_3^2 + F_4^2 - 2F_3F_4 \cos H$

$R^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos 45°$

$R^2 = 244 - 169.70 = 74.29$

$R = \sqrt{74.29}$

$R = 8.62N$

We use sine rule to calculate the angle α

$$\frac{R}{\sin H} = \frac{F_3}{\sin E}$$

$$\frac{8.62}{\sin 45} = \frac{10}{\sin \alpha}$$

Cross multiplying, $\sin \alpha \times 8.62 = 10 \times \sin 45$

$$\sin \alpha = \frac{10 \times \sin 45}{8.62} = 0.8203$$

$$\alpha = \sin^{-1} 0.8203 = 55.12^0$$

Substitute R = 8.62N and α = 55.12^0 into Fig. 1.9b, to get the following:

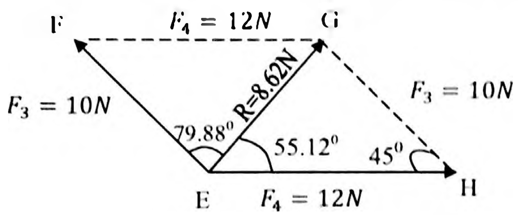


Fig. 1.13

Note, FEH = 135^0, GEH = 55.12^0 $\therefore$ FEG = 135^0 – 55.12^0 = 79.88^0

The magnitude and direction of the resultant of vector F_3 (10N) and vector F_4 (12N) is 8.62N at 55.12^0 to the 12N force or 8.62N at 79.88^0 to the 10N force.

D. More Than Two Forces Acting at a Point

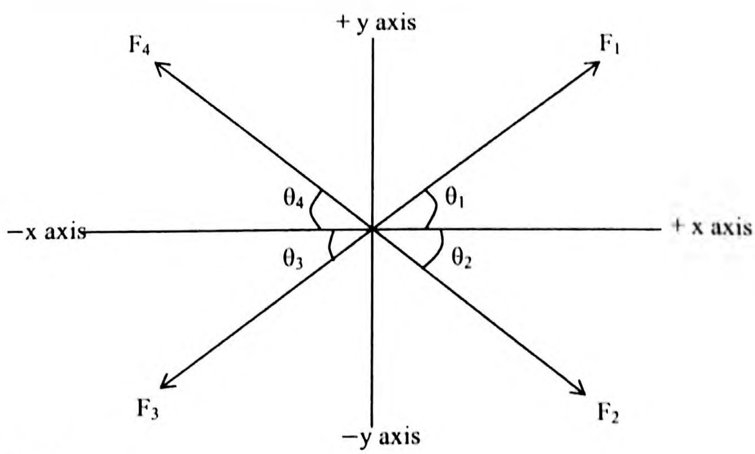


Fig. 1.14

The resultant (R) and the angle (θ) it makes with the horizontal is found as follows.

1. Resolve all the forces in the x (horizontal) and y (vertical) directions taking note of the positive and negative x and y axes.

Horizontal x components:	Vertical y components:
$F_1\cos\theta_1$	$F_1\sin\theta_1$
$F_2\cos\theta_2$	$-F_2\sin\theta_2$
$-F_3\cos\theta_3$	$-F_3\sin\theta_3$
$-F_4\cos\theta_4$	$F_4\sin\theta_4$

2. The x and y components are added separately and algebraically.

$X = F_1\cos\theta_1 + F_2\cos\theta_2 - F_3\cos\theta_3 - F_4\cos\theta_4$

$Y = F_1\sin\theta_1 - F_2\sin\theta_2 - F_3\sin\theta_3 + F_4\sin\theta_4$

If a vector is directly aligned with the x axis, its angle of inclination, $\theta = 0$. In contrast, if a vector is directly aligned with the y axis, its angle of inclination, $\theta = 90^o$.

The following trigonometric ratios can be very useful when solving problems in vectors resolution.

$\tan 60° = \sqrt{3} = 1.73$ $\qquad$ $\tan 30° = \frac{1}{\sqrt{3}} = 0.58$

$\sin 60° = \cos 30° = \frac{\sqrt{3}}{2} = 0.87$ $\qquad$ $\cos 60° = \sin 30° = \frac{1}{2} = 0.5$

Example 1

An object is acted upon by two forces of 5N and 12N. Calculate the resultant of the two forces if

(i) the forces act perpendicular to each other.

(ii) The two forces act at an angle of 40^0 to each other.

(iii) The two forces act at an angle of 125^0 to each other.

Solution

(i)

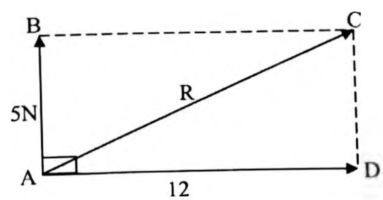


Fig 1.15

Because the angle between the two forces is 90^0, we use the pythagoras theorem as follows.

$R^2 = 5^2 + 12^2 = 25 + 144 = 169$

$R = \sqrt{169} = 13N$

(ii)

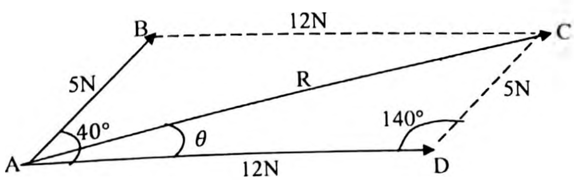


Fig 1.16

Applying cosine rule, $R^2 = 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 140$

$$R^2 = 169 - 120(-0.766)$$

$$R^2 = 169 + 91.93 = 260.93$$

$$R = \sqrt{260.93} = 16.15N$$

Using sine rule, we have;

$$\frac{R}{\sin 140} = \frac{CD}{\sin\theta}$$

$$\frac{16.15}{\sin 140} = \frac{5}{\sin\theta}$$

$$\therefore \sin\theta = \frac{5 \times \sin 140}{16.15} = \frac{5 \times 0.643}{16.15} = 0.199$$

$$\theta = \sin^{-1} 0.199 = 11.48°$$

The resultant is 16.15N at 11.48^0 to the 12N force.

(iii)

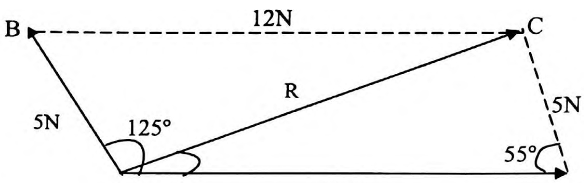


Fig 1.17

Resultant, $R^2 = 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 55$

$$= 169 - 120 \times 0.574 = 100.1$$

$$R = \sqrt{100.12} = 10N$$

We apply sine rule to find the direction

$$\frac{R}{\sin 55} = \frac{CD}{\sin\theta}$$

$$\frac{10}{\sin 55} = \frac{5}{\sin\theta}$$

$$\therefore \sin\theta = \frac{5 \times \sin 55}{10} = \frac{5 \times 0.819}{10} = 0.4095$$

$$\theta = \sin^{-1} 0.4095 = 24.17°$$

The resultant is 10N at 24.17^0 to the 12N force.

Example 2

Two forces, whose resultant is 100N are perpendicular to each other. If one of them makes an angle of 60^0 with resultant, calculate its magnitude ($\sin 60^0 = 0.8660$, $\cos 60^0 = 0.5000$). *WAEC 1991*

Solution

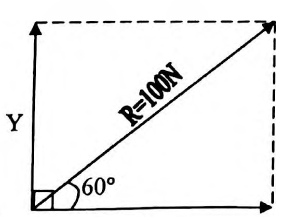


Fig 1.18

Let X and Y be the two perpendicular forces

It is the force X that makes an angle of 60^0 with the resultant as shown above and is calculated as follows:

$$\cos 60 = \frac{X}{100}$$

$$\therefore \quad X = 100 \times \cos 60 = 100 \times 0.5000 = 50\text{N}$$

Example 3

A body of mass 20kg is set in motion by two forces 3N and 4N acting at right angles to each other. Determine the magnitude of its acceleration. *WAEC 1997*

Solution

Mass, m = 20kg; acceleration, a = ?

The resultant, F, of the two forces, 3N and 4N is found as follows:

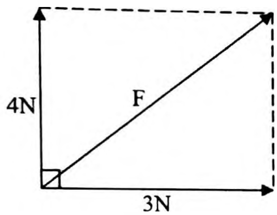


Fig 1.19

$$F^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$F = \sqrt{25} = 5N$$

$$\text{Force} = \text{mass x acceleration} \quad \therefore \quad \text{acceleration, a} = \frac{\text{F}}{\text{m}} = \frac{5}{20} = 0.25\text{ms}^{-2}$$

Example 4

A motorcyclist, passing a road junction, moves due west for 8s at a uniform speed of 5m/s. He then moves due north for another 6s with the same speed. At the end of the 6s his displacement from the road junction is 50m in what direction? *WAEC 1999*

Solution

Distance covered due west is 5m/s x 8s = 40m

Distance covered due north is 5m/s x 6s = 30m

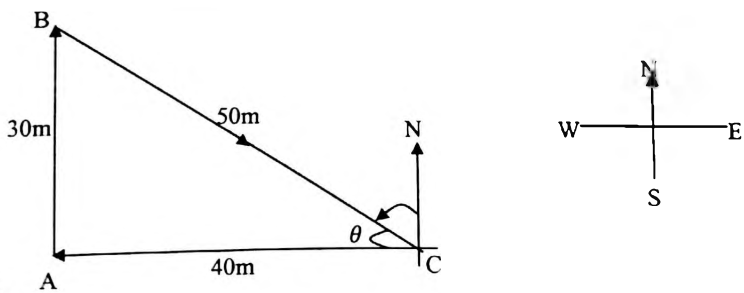


Fig 1.20

$$\tan\theta = \frac{30}{40} = \frac{3}{4} = 0.75 \quad \therefore \quad \theta = \tan^{-1} 0.75 = 37°$$

Therefore, the direction is, $90 - 37 = 53^0$

Because the displacement is in the North-West direction, the direction is stated thus; $N53^0W$ or 53^0 West of North.

Example 5

A boy travels 12km eastwards to a point B and then 5km southwards to another point C. Calculate the difference between the magnitude of displacement of the boy and the distance travelled by him. *NECO 2006*

Solution

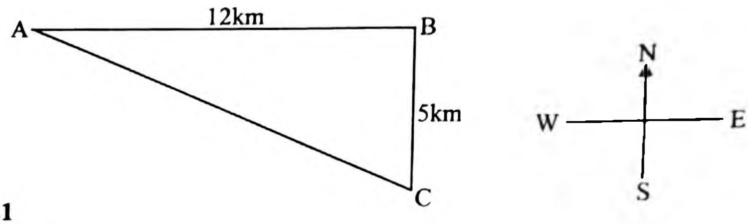


Fig 1.21

The displacement is the distance AC and is found using Pythagoras theorem.

$$AC^2 = AB^2 = BC^2$$
$$AC^2 = 12^2 + 5^2 = 144 + 25 = 169$$
$$AC = \sqrt{169} = 13km$$

The distance covered = AB + BC = 12km + 5km = 17km

Difference between distance and displacement = 17 – 13 = 4km.

Example 6

A tug boat is travelling from Asaba to Onitsha across the River Niger with a resultant velocity of 20 knots. If the river flows at 12 knots, the direction of motion of the boat relative to the direction of water flow is

$A.\ 36.87°$ $B.\ 53.13°$ $C.\ 90°$ $D.\ 136.87°$ $E.\ 143.13°$ *JAMB 1980*[27]

Solution

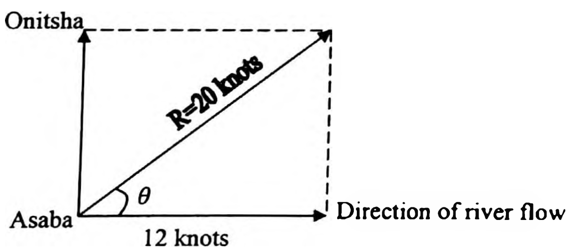


Fig 1.22

Angle θ is the direction of motion of the boat relative to the direction of water as shown above.

Applying trigonometric ratio, we have; $\cos\theta = \dfrac{12}{20} = 0.6$

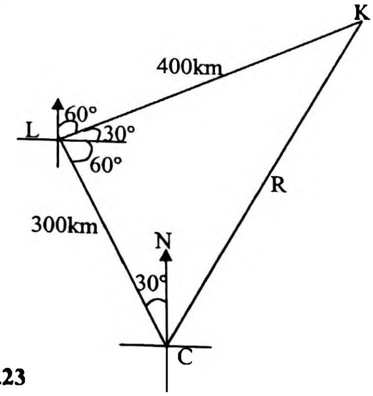
$$\theta = \cos^{-1} 0.6 = 53.13°$$

Example 7

An aircraft travelled from Calabar to Kano as follows: It flew first to Ilorin covering a distance of 300km, 30^0 West of North, and then flew 400km, 60^0 East of North to Kano. What is the resultant displacement?

A. 567km B. 410km C. 594km D.500km E. 600km *JAMB 1981*

Solution



C, L, K represent Calabar, Ilorin and Kano respectively.

Fig 1.23

The above figure is a right angle triangle (angle L is 90^0). Therefore the resultant displacement is found using Pythagoras theorem.

$CK^2 = CL^2 + KL^2$

$CK^2 = 300^2 + 400^2 = 90000 + 160000$

$CK^2 = 250000$

$CK = \sqrt{250000} = 500km$

Example 8

A stream is flowing at 0.75m/s and a boat heading perpendicular for the stream landed at the opposite bank at an angle 30^0. Calculate the velocity of the boat.

A. $0.65ms^{-1}$ B. $0.86ms^{-1}$ C. $1.00ms^{-1}$ D. $1.50\ ms^{-1}$ *JAMB 2000*

Solution

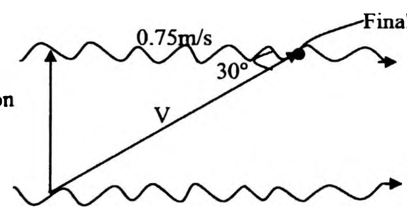


Fig 1.24

V is the velocity of the boat. Using trigonometric ratio, we have

$$\cos 30 = \frac{0.75}{V} \quad \therefore \quad V = \frac{0.75}{\cos 30} = 0.86ms^{-1}$$

Example 9

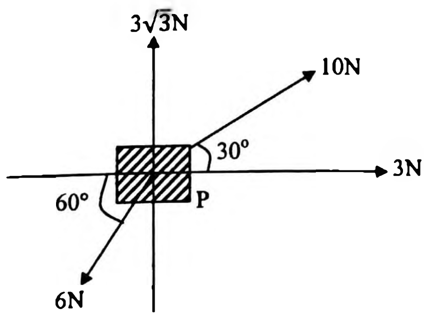


Fig 1.26

The figure above shows four forces 3N, 10N, $3\sqrt{3}N$ and 6N acting on a particle P. The resultant of the four forces is

A. 10N B. $10\sqrt{3}$ N C. 5N D. $5\sqrt{3}$ N *JAMB 2007*[14]

Solution

$\theta_1 = 90°$; $\theta_2 = 30°$; $\theta_3 = 0$; $\theta_4 = 60°$

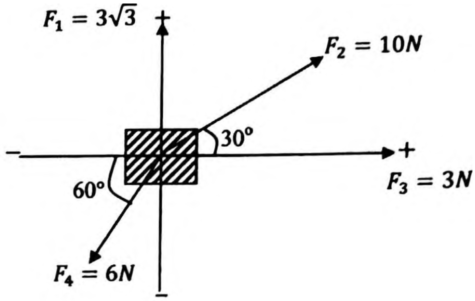


Fig 1.27

Horizontal x components:

$$F_1 \cos\theta_1 = 3\sqrt{3} \times \cos 90 = 3\sqrt{3} \times 0 = 0$$

$$F_2 \cos\theta_2 = 10 \times \cos 30 = 10\frac{\sqrt{3}}{2} = 5\sqrt{3}$$

$$F_3 \cos\theta_3 = 3 \times \cos 0 = 3$$

$$-F_4 \cos\theta_4 = -6 \times \cos 60 = -6 \times 0.5 = -3$$

Sum of x components, $X = F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 - F_4 \cos\theta_4$

$$= 0 + 5\sqrt{3} + 3 - 3 = 5\sqrt{3}$$

Horizontal y components:

$$F_1 \sin\theta_1 = 3\sqrt{3} \times \sin 90 = 3\sqrt{3} \times 1 = 3\sqrt{3}$$

$$F_2 \sin\theta_2 = 10 \times \sin 30 = 10 \times \frac{1}{2} = 5$$

$$F_3 \sin\theta_3 = 3 \times \sin 0 = 3 \times 0 = 0$$

$$-F_4 \sin\theta_4 = -6 \times \sin 60 = -6 \times \frac{\sqrt{3}}{2} = -3\sqrt{3}$$

Sum of y components, $Y = F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 - F_4 \sin\theta_4$

$$= 3\sqrt{3} + 5 + 0 - 3\sqrt{3}$$

$$= 3\sqrt{3} - 3\sqrt{3} + 5$$

$$= 5$$

The resultant of X and Y is found as follows.

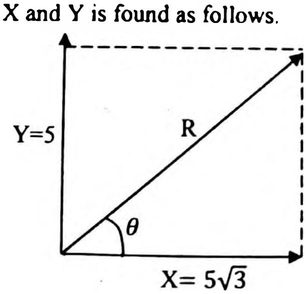


Fig 1.25

Apply Pythagoras theorem,

$$R^2 = 5^2 + (5\sqrt{3})^2 = 25 + \left[5^2(\sqrt{3})^2\right]$$
$$R^2 = 25 + 25 \times 3 = 25 + 75$$
$$R^2 = 100$$
$$R = \sqrt{100} = 10N$$

The direction of the resultant, $\theta = \tan^{-1}\left(\frac{Y}{X}\right)$

$$\theta = \tan^{-1}\left(\frac{5}{5\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \tan^{-1} 0.577 = 30°$$

VECTOR RESOLUTION

A single vector V, inclined at angle θ to the horizontal can be resolved into two perpendicular components. The horizontal (V_x) and vertical (V_y) components are shown below.

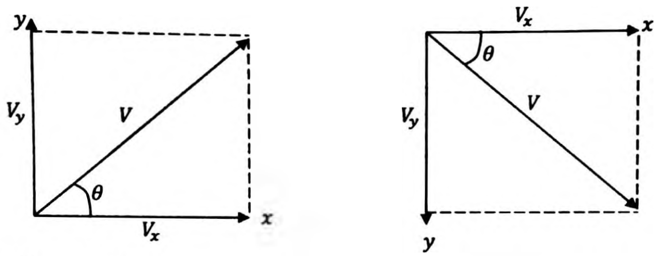


Fig 1.28 (a) (b)

Vertical component of V is, $V_y = V \sin\theta$

Horizontal component of V is, $V_x = V\cos\theta$

In some cases, the vector (V) might be inclined at angle θ to the vertical as shown below.

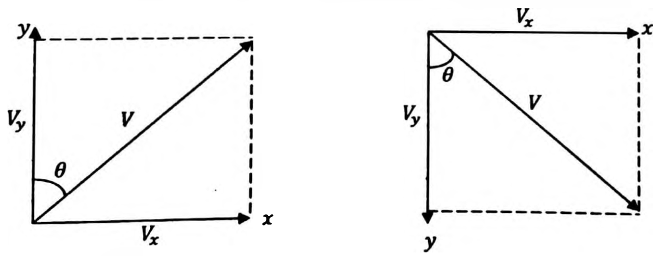


Fig 1.29 (a) (b)

Vertical component of V is, $V_y = \cos\theta$

Horizontal component of V is, $V_x = V\sin\theta$

Example 10

A wheel barrow inclined at 30° to the horizontal is pushed with a force of 150N.

(i) What is the vertical component of the applied force?

(ii) What is the horizontal component of the applied force?

(iii) In what direction will the wheel barrow move and why?

Solution

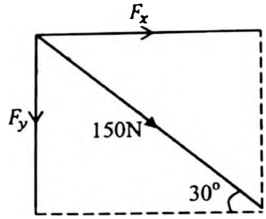


Fig 1.30

(i) Vertical component, $F_y = F\sin\theta = 150 \times \sin 30 = 150 \times 0.5 = 75N$

(ii) Horizontal component, $F_x = F\cos\theta = 150 \times \cos 30 = 150 \times 0.87 = 130N$

(iii) It will move in the horizontal direction because the horizontal component of the force is greater than the vertical component.

Example 11

A force of 10N is applied to a block of wood as illustrated below.

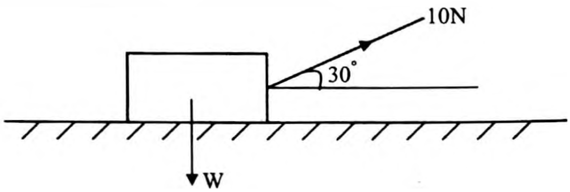


Fig 1.31

What is the vertical component of the force? *NECO 2007*[6]

Solution

Force, F = 10N; Angle of inclination $\theta = 30°$

Vertical component of force = $F\sin\theta = 10 \times \sin 30 = 10 \times 0.5 = 5.0N$

Example 12

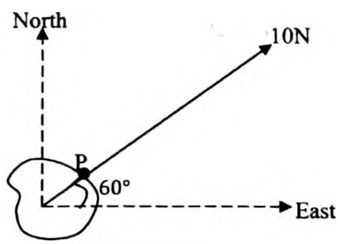


Fig 1.32

A body on the ground is acted on by a force of 10N at a point P as shown in the diagram above. What force is needed to stop the body from moving eastward?

A. 5N in the direction of East B. 5N in the direction of West

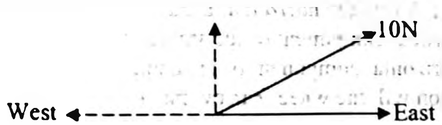
C. $5\sqrt{3}N$ in the direction of West D. 10N in the Southwest direction *JAMB 1998*

Solution

The horizontal (eastward) component of the 10N force is = 10 cos60 = 10 x 0.5 = 5N. The equilibrant force needed to stop the body moving eastward must act in exactly the opposite direction.

Therefore, the needed force is *5N in the direction of West* as shown below.

Fig 1.33



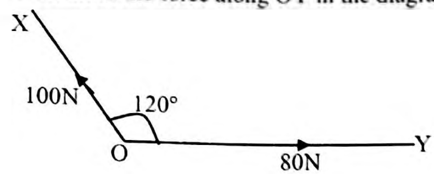
EXERCISES 1.

1. The driver of a car moving with uniform speed of 40m/s observes a truck approaching in the opposite direction with a speed of 20m/s. Calculate the speed of the car relative to that of the truck. *WAEC 2000 Ans: 60m/s*

2. Two forces 3N and 4N act on a body in directions due North and due East respectively. Calculate their equilibrant.
A. 5N 53^0 East of North. B. 5N 53^0 West of South C. 5N 37^0 North of East.
D. 7N 37^0 West of North E. 7N 37^0 South of West. *WAEC 1997 Ans: B*

3. What is the resultant of the force along OY in the diagram below?

Fig 1.34



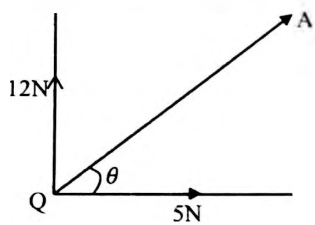
NECO 2002 Ans: 91.65N

5. What is the magnitude of the resultant of two forces 20N and 15N inclined at 90^0 to each other? *NECO 2006 Ans: 25N*

6. A man walks 1km due East and then 1km due North. What is his displacement? *WAEC 1978 Ans: $\sqrt{2}$ N45^{0}E*

7. A body of mass 5kg initially at rest is acted upon by two mutually perpendicular forces 12N and 5N as shown in the figure below. If the particle moves in the direction of QA, calculate the magnitude of the acceleration.
A. 0.40ms^{-2} B. 1.40ms^{-2} C. 0.26ms^{-2} D. 2.60ms^{-2} E. 3.40ms^{-2}

Fig 1.35



JAMB 1984 Ans: 2.6m/s^2

8. A block of mass 2.0kg resting on a smooth horizontal plane is acted upon simultaneously by two forces, 10N due North and 10N due East. The magnitude of the acceleration produced by the forces on the block is A. 0.10ms^{-2} B. 7.07ms^{-2}
C. 10.00ms^{-2} D. 14.10ms^{-2} E. 20.00ms^{-2} *JAMB 1985 Ans: 7.07m/s^2*

9. A man walks 8km North and then 5km in a direction 60^0 East of North. Find the distance from his starting point. A. 11.36km B. 12.36km C. 13.00km
D. 14.36km *JAMB 1987 Ans: 11.36km*

10. Two forces each of 10N act on a body, one towards the North and the other towards the East. What is the magnitude and direction of the resultant force?
A. 10 $\sqrt{2}$ N, 45^{0}E B. 10 $\sqrt{2}$ N,45^{0}W C. 20N, 45^{0}W D. 20N, 45^{0}E
JAMB 2002 Ans: 10$\sqrt{2}$ N45^{0}E

11. A lorry travels 10km northwards, 4km eastwards, 6km southwards and 4km westwards to arrive at a point T. What is the total displacement? A. 6km south
B. 4km north C. 6km north D. 4km east *JAMB 1988 Ans: 4km North*

12. Two forces whose resultant is 100N are at right angles to each other. If one of them makes an angle of 30^0 with the resultant, determine its magnitude. A. 8.66N
B. 50.0N C. 57.7N D. 86.60N *JAMB 1988 Ans: 86.60N*

13. Two forces of magnitude 7N and 3N act at right angles to each other. The angle θ between the resultant and the 7N force is given by: A. cos θ = $^3/_7$ B. sin θ = $^3/_7$
C. tan θ = $^3/_7$ D. cot θ = $^3/_7$ *JAMB 1993 Ans: C*

14. A boat travels due East at a speed of 40m/s across a river flowing due South at 30m/s. What is the resultant speed of the boat? A. 1.3ms^{-1} B. 10.0ms^{-1}

C. $50.0ms^{-1}$ D. $70.0ms^{-1}$ *JAMB 1994 Ans: 50m/s*

15. A ball is moving at 18m/s in a direction inclined at 60^0 to the horizontal. The horizontal component of its velocity is A. $9\sqrt{3}ms^{-1}$ B. $6\sqrt{3}ms^{-1}$ C. $6\sqrt{3}ms^{-1}$ D. $9\ ms^{-1}$ *JAMB 1998 Ans: 9m/s*

16.

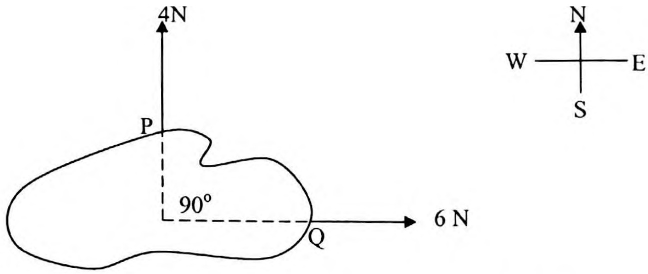


Fig 1.36

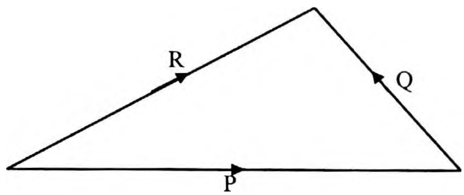
A body is acted on by forces applied at points P and Q as shown above. What is the resultant?

A. $2\sqrt{5}$ N in a direction N56°E B. $2\sqrt{5}$ N in a direction N56°W
C. $2\sqrt{13}$ N in a direction N56°E D. $2\sqrt{13}$ N in a direction N56°W
E. $\sqrt{10}$ N in a direction N56°E. *JAMB 1982 Ans: C*

17. Calculate the distance between the points P(2,4) and Q(-5,3).
NECO 2007 Ans: 8.00 units

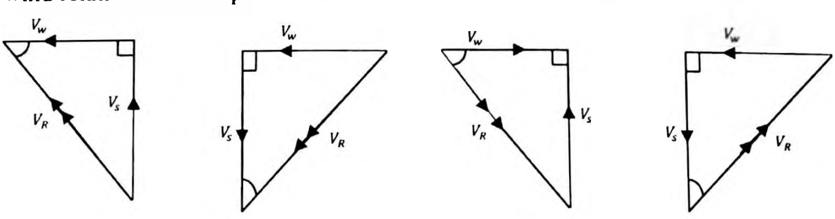
18.

Fig 1.37



The figure above illustrates three vectors P, Q and R. Which of the following is correct about the vectors? A. P+Q+R=0 B. R = P – Q C. R = P + Q D. R = P.Q E. Q = P. R *NECO 2007 Ans: A*

19. A ship moving northward with speed V_s is acted upon by a wind blowing due west at a speed V_w. Which of the following diagrams correctly indicates the velocity V_R of the wind relative to the ship?



A B C D

Fig 1.38 *WAEC 2008[10] Ans: A*

20. The resultant of two forces 12N and 5N is 13N. What is the angle between the two forces? A. 0 B. 45° C. 90° D. 180° *JAMB 2008[3] Ans: 90°*

21. An aircraft attempts to fly due north at 100 kmh^{-1}. If the wind blows against it from east to west at 60kmh^{-1}, its resultant velocity is
A. 117 kmh^{-1}, N 31° E B. 127 kmh^{-1}, N 31° E
C. 117 kmh^{-1}, N 31° W D. 127 kmh^{-1}, N 31° W *JAMB 2009[3] Ans: A*

22. Which of the vector diagrams below represents the following position vectors?
$\vec{V_1} = \binom{1}{1}$, $\vec{V_2} = \binom{2}{1}$, $\vec{V_3} = \binom{2}{-2}$ and $\vec{V_4} = \binom{1}{-2}$ *WAEC 2009[10] Ans: C*

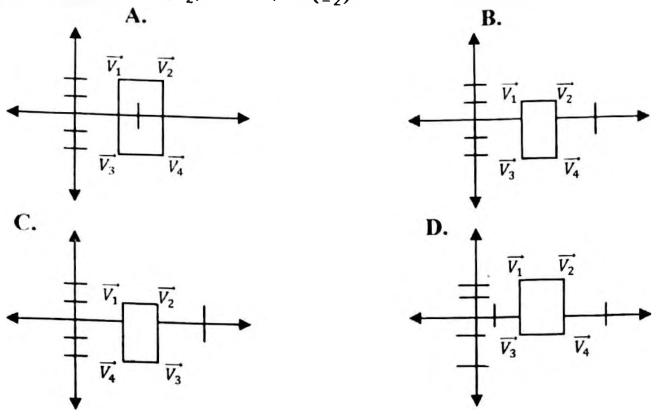


Fig 1.39

23. Two forces 6N and 8N, act eastwards and northwards respectively on a body. Calculate the magnitude of their equilibrant. WAEC 2009[11] Ans: 10N

24. Two forces of magnitudes 8N and 4N act at right angle to each other. What is the angle between the resultant and the 8N force? NECO 2009[5] Ans: 26.6°

25. Two forces **A** and **B** act a point. If their resultant is given by (B-A) in the direction of **B**, then

A. **A** and **B** are equal
B. **A** is greater than **B**
C. The angle between **A** and **B** is 0°
D. The angle between **A** and **B** is 90°
E. The angle between **A** and **B** is 180° NECO 2009[6] Ans: E

26. Two forces of magnitudes 4N and 5N act on a body in the directions due North and East respectively. Calculate their equilibrant. NECO 2009[10] Ans: 6.4N, S51.3°W

2

PROJECTILES

A projectile is any object thrown or released into space that travels along a curved or parabolic path. At any point in time a projectile motion is characterized by:

(i) horizontal motion at constant velocity where linear acceleration, $a = 0 m/s^2$

(ii) vertical motion (up or down) with constant acceleration where gravitational acceleration, $g = 9.8 m/s^2$

Calculations in projectile motion are basically in two forms viz;

(a) Objects Projected Horizontally From a Height.

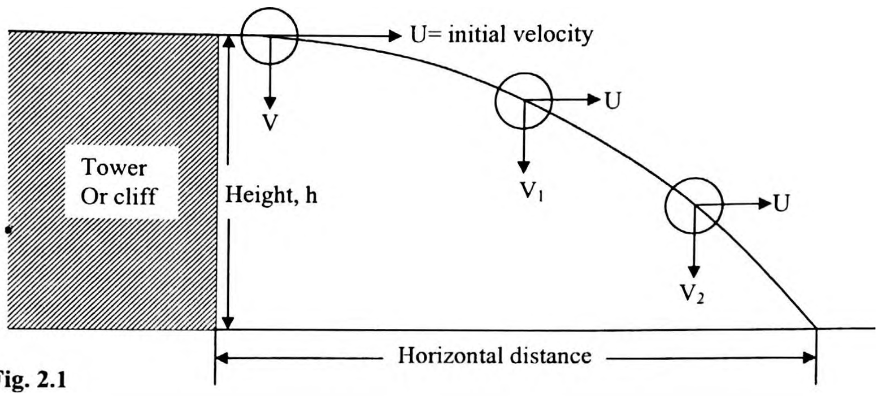


Fig. 2.1

Note: The initial horizontal velocity, U does not change. It remains constant during the duration of motion.

The initial vertical velocity, V = 0. However, the subsequent vertical velocities, V_1, V_2 ...... V_n changes with time.

Equations of uniformly accelerated motion ($v = u + at$, $v^2 = u^2 + 2as$, $s = ut + \frac{1}{2}at^2$) are used in such cases as shown in the diagram above.

You are advised to review the topic **motion under gravity** in Chapter 1 of Book One.

(b) Objects Projected at an Angle, θ, to the Horizontal.

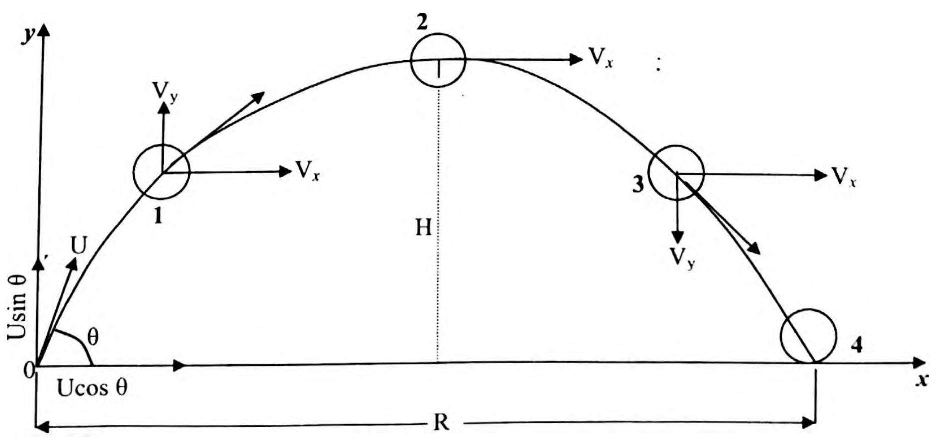


Fig 2.2

At the point of projection (t=0), the initial velocity U, can be resolved into the vertical component, V_y = Usin θ, and the horizontal component, V_x = Ucos θ.

The horizontal component of the initial velocity, Ucos θ **remains the same** throughout the time of flight (from t=0 to t=4s).

In contrast, the vertical component of the initial velocity (Usin θ) after projection **changes with time**. After time t the velocity becomes V_y = Usinθ − gt, during upward motion or V_y = Usin θ + gt, during downward motion.

This shows that the value of V_y depends on how much time (t) the projectile has spent in the air. However, at the maximum height or exactly midway through the time of flight, the velocity of the projectile has no vertical component, therefore, V_y=0.

So, *the velocity or speed of a projectile at its maximum height is Ucosθ.*

After projection into space, the motion of the projectile is governed by the resultant (V) of the vertical and horizontal components of the initial velocity as illustrated below.

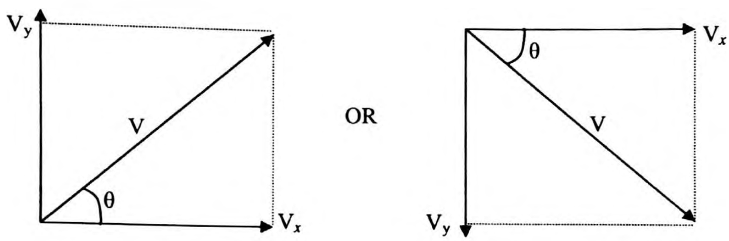


Fig 2.3 (i) During upward motion (ii) During downward motion

In whatever case, the resultant velocity, V is found using the Pythagoras theorem.

$$V^2 = V_x^2 + V_y^2$$

∴ Resultant velocity, $V = \sqrt{V_x^2 + V_y^2}$

The resultant velocity forms an angle θ with the horizontal plane and is calculated as follows.

$$\tan\theta = \frac{V_y}{V_x} \quad \therefore \quad \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

For a projectile in motion as shown in fig 2.2, the following applies.

Time taken to reach maximum height, $t = \dfrac{U\sin\theta}{g}$

Time of flight, $T = \dfrac{2U\sin\theta}{g}$

Maximum height, $H = \dfrac{U^2\sin^2\theta}{2g}$ *or* $= \dfrac{U^2(\sin\theta)^2}{2g}$

Horizontal Range, $R = \dfrac{U^2\sin 2\theta}{g}$

Example 1

An arrow is horizontally projected from the top of a tower with an initial velocity of 15m/s. If it reaches the ground 7 seconds later, calculate the height of the tower. (g=10m/s^2)

Solution

Given: initial velocity, u=0; time, t = 7s; g=10m/s^2.

Equation of motion under gravity is used to find the height; $h = ut + \frac{1}{2}gt^2$

Note that the 15m/s given in the question was not used as the initial velocity because 15m/s represents the initial *horizontal* velocity and not the initial *vertical* velocity.

Height of tower, $h = ut + \frac{1}{2}gt^2$

$$= 0 \times 7 + \frac{1}{2} \times 10 \times 7^2 = 245\text{m}.$$

Example 2

A ball is projected horizontally from a height of 20m above the ground with an initial velocity of 0.4m/s. Calculate the horizontal distance moved by the ball before hitting the ground. (g=10m/s^2) *WAEC 2005*

Solution

Given: height, h = 20m; initial horizontal velocity, u = 0.4m/s;
initial vertical velocity, u = 0

The time taken to reach the ground is governed by gravity, therefore, equation of motion under gravity is used and also initial velocity, u = 0.

Substituting into , $h = ut + \frac{1}{2}gt^2$

$$20 = 0 \times t + \frac{1}{2} \times 10 \times t^2$$

$$20 = 5t^2$$

$$\frac{20}{5} = t^2$$

$$t^2 = 4 \quad \therefore \quad t = \sqrt{4} = 2s$$

After calculating the time taken to reach the ground, the horizontal distance, s, can then be calculated using equation of uniformly accelerated motion and remembering that acceleration, a = 0 and u = 0.4m/s

Substituting into $s = ut + \frac{1}{2}at^2$

$$= 0.4 \times 2 + \frac{1}{2} \times 0 \times 2^2$$

$$= 0.8 + 0$$

$$= 0.8\text{ m}$$

Example 3

An object is projected horizontally from the top of a cliff with a velocity of $7ms^{-1}$ and lands on the ground level at a point 56m from the base of the cliff. Calculate the height of cliff.

Solution

Horizontal velocity $V = 7ms^{-1}$; horizontal distance $d = 56m$

From velocity $= \frac{\text{distance}}{\text{time}}$, time taken by object to land on the ground is

$$t = \frac{\text{horizontal distance}}{\text{horizontal velocity}} = \frac{56}{7} = 8s$$

Equation of motion under gravity is used to find the height; $h = ut + \frac{1}{2}gt^2$

Substitute initial velocity $u = 0ms^{-1}$(***not*** $7ms^{-1}$); $g = 10ms^{-1}$; time $t = 8s$

$\therefore \quad h = 0 \times 8 + \frac{1}{2} + 10 \times 8^2 = 0.5 \times 64 = 32m$

Example 4

A stone is projected upwards at an angle of 30^0 to the horizontal from the top of a tower of height 100m and it hits the ground at a point Q. If the initial velocity of projection is $100ms^{-1}$, calculate the;

(i) maximum height of the stone above the ground.
(ii) Time it takes to reach this height;
(iii) Time of flight.
(iv) Horizontal distance from the foot of the tower to the point Q.
(Neglect air resistance and take g as $10ms^{-2}$) *WAEC 1995*

Solution

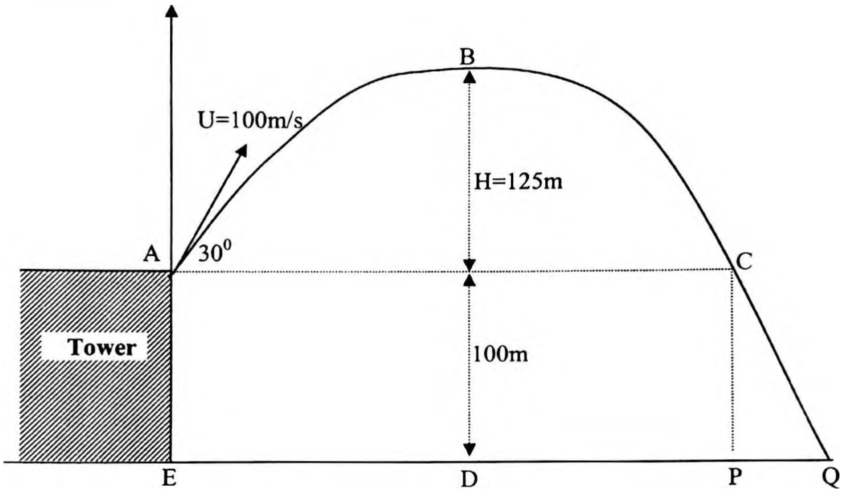


Fig. 2.4

(i) The maximum height of the stone above the ground, S, is equal to the maximum height, H from the point of projection and the height of the tower, 100m ie. S = 100 + H.

Given: initial velocity, $u = 100ms^{-1}$; angle of projection, $\theta = 30^0$

$$\text{Maximum height}, H = \frac{U^2\sin^2\theta}{2g} = \frac{100^2 \times \sin^2 30}{2 \times 10}$$

$$H = \frac{10000 \times 0.5^2}{20} = 500 \times 0.25 = 125m$$

$$\therefore \quad S = 100 + H = 100 + 125 = 225m$$

(ii) Time taken to reach highest height, $t = \frac{U\sin\theta}{g} = \frac{100 \times \sin 30}{10}$

$$= \frac{100 \times 0.5}{10} = 5s$$

(iii) Time of flight, $T = \frac{2U\sin\theta}{g} = \frac{2 \times 100 \times \sin 30}{g} = 10s$

(iv) The total time the projectile travels from A to Q (see the diagram above) must be found before the horizontal distance EQ can be calculated.

Applying equation of motion under gravity we can find the time taken to travel from the highest point, B, to the ground, Q.

At the highest height, BD or h = 225m, and considering the vertical motion of the projectile, the initial velocity, u=0. substituting into $h = ut + \frac{1}{2}gt^2$ we have

$$225 = 0 \times t + \tfrac{1}{2} \times 10 \times t^2$$

$$225 = 5t^2$$

$$t^2 = \frac{225}{5} = 45 \quad \therefore \quad t = \sqrt{45} = 6.71\text{s}$$

6.71 s is the time taken to travel from the highest point B, to the ground, Q.

From (ii) and (iii) above, the time taken to travel from A to B is 5 sec; the time taken to travel from B to C is also 5 s.

Therefore the time taken to travel from C to Q is 6.71 – 5 = 1.71sec and the total time taken to travel from the point of projection, A, to the ground Q is

5 + 6.71 = 11.71 s.

The diagram below shows time duration for each section of the flight.

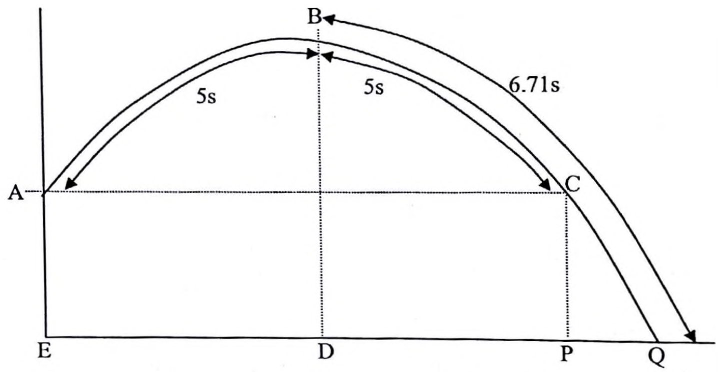


Fig 2.5

Horizontal distance (EQ) = horizontal component of initial velocity x total time taken to reach the ground from point of projection.

$$= u\cos\theta \times 11.71$$
$$= 100\cos 30 \times 11.71 = 100 \times 0.866 \times 11.71$$
$$= 1014.12\text{m}$$

Alternatively, the horizontal distance can be calculated as follows. The distance EP (see above diagrams) is the range, R of the projectile.

$$R = \frac{U^2 \sin 2\theta}{g} = \frac{100^2 \sin 2 \times 30}{10} = 866.025\text{m}$$

The time taken to travel the horizontal distance PQ is1.71s. The horizontal component of the velocity is $u\cos\theta$ = 100 × cos30 = 86.60m/s.

$$\text{Velocity} = \frac{\text{distance}}{\text{time}} \quad \therefore \text{ distance} = \text{velocity x time}$$

Distance, PQ = 86.60 × 1.71 = 148.1m

Horizontal distance EQ = Range(EP) + PQ

$= 866.025 + 148.1 = 1014.12m$

Example 5

Two bodies X and Y are projected on the same horizontal plane, with same initial speed but at angles 30^0 and 60^0 respectively to the horizontal. Neglecting air resistance, the ratio of the range of X to that of Y is

A. 1:1 B. 1:2 C. $\sqrt{3}:1$ D. $1:\sqrt{3}$ *JAMB 1997*

Solution

$$\text{Range}, R = \frac{U^2 \sin 2\theta}{g}$$

For X, $\theta = 30°$, $\therefore$ $\text{Range}, R_X = \frac{U^2 \sin 2 \times 30}{g} = \frac{U^2 \sin 60}{g}$

For Y, $\theta = 60°$, $\therefore$ $\text{Range}, R_Y = \frac{U^2 \sin 2 \times 60}{g} = \frac{U^2 \sin 120}{g}$

$\therefore$ Ratio of range X(R_X) to range Y(R_Y) is

$$\frac{R_X}{R_Y} = \frac{U^2 \sin 60}{g} \div \frac{U^2 \sin 120}{g}$$

$$= \frac{U^2 \sin 60}{g} \times \frac{g}{U^2 \sin 120}$$

$$= \frac{\sin 60}{\sin 120} = \frac{0.866}{0.866} = 1$$

$$\therefore \ Rx : Ry = 1:1$$

Example 6

A body is projected from the ground at an angle θ to the horizontal with a velocity of 30m/s. it reached a maximum height of 11.25m. Calculate

(i) The value of θ.

(ii) The time taken to strike the ground.

(iii) The range.

(iv) Its velocity 2s after projection. *NECO 2004*

Solution

(i) Initial velocity of projection, U = 30m/s; maximum height, H = 11.25m; $g=10m/s^2$

Substituting into, $H = \frac{U^2 \sin^2\theta}{2g}$

we have, $11.25 = \frac{30^2(\sin\theta)^2}{2 \times 10} = \frac{900 \times (\sin\theta)^2}{20}$

rearranging, $(\sin\theta)^2 \times 900 = 11.25 \times 20$

$$(\sin\theta)^2 = \frac{11.25 \times 20}{900} = 0.25$$

$$\sin\theta = \sqrt{0.25}$$
$$\sin\theta = 0.5$$
$$\theta = \sin^{-1} 0.5$$
$$\theta = 30°$$

(ii) The time taken to strike the ground is the time of flight, $T = \dfrac{2U\sin\theta}{g}$

$$T = \frac{2 \times 30 \times \sin 30}{g} = \frac{60 \times 0.5}{10} = 3s$$

(iii) The Range, $R = \dfrac{U^2 \sin 2\theta}{g} = \dfrac{30^2 \times \sin 2 \times 30}{10} = \dfrac{900 \times \sin 60}{10} = 77.94\text{m}$

(iv) The velocity of the body after 2 seconds of projection is found by calculating the resultant of the horizontal (V_x) and vertical (V_y) components of the initial velocity after 2s.

Initial horizontal component of the velocity,

$V_x = U\cos\theta$. Substitute U = 30m/s and θ = 30°

$V_x = 30\cos 30 = 25.98\text{m/s}$.

V_x does NOT CHANGE during the time of flight.

Initial vertical component of the velocity,

$V_y = U\sin\theta$

V_y changes with time, therefore after time t,

$V_y = U\sin\theta - gt$

Substitute $U = 30m/s$, $\theta = 30°$ and $t = 2s$

$V_y = 30 \times \sin 30 - 10 \times 2$

$= 15 - 20 = -5\text{m/s}$

The vertical, horizontal and resultant velocities can be illustrated below.

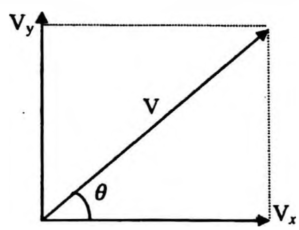


Fig. 2.6

The resultant velocity, $V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(25.98)^2 + (-5)^2}$

$= \sqrt{674.96 + 25}$

$= \sqrt{699.96} = 26.46m/s$

This resultant velocity makes an angle, θ with the horizontal given by

$$\tan\theta = \frac{V_y}{V_x} = \frac{-5}{25.98} = -0.1925$$

$$\theta = \tan^{-1}(-0.1925) = -10.9°$$

Example 7
A missile is projected with a velocity of $250ms^{-1}$ at an angle of θ to the vertical. If the total time of flight of the missile is 25s, the value of θ is
A. 30° B. 60° C. 45 D. 50
Solution

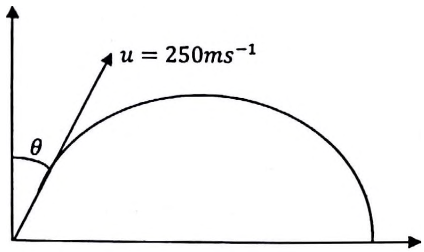


Fig 2.7

$u = 250ms^{-1}$
$T = 25s$
$g = 10ms^{-1}$

$$\text{Time of flight } T = \frac{2u\sin\theta}{g}$$

$$25 = \frac{2 \times 250 \times \sin\theta}{10}$$

$$25 = 50 \times \sin\theta$$

$$\sin\theta = \frac{25}{50} = 0.5$$

$$\theta = \sin^{-1} 0.5 = 30^\circ$$

From the given options, your answer will be option A, isn't it? YOU ARE ABSOLUTELY WRONG!!
Read the question again and look at the diagram once more. "*... at an angle of θ to the vertical..*".
In all the formulae in projectile motion, θ is defined as the angle of projection to the *horizontal*. Therefore when the given angle of projection θ is to the vertical, two approaches can be applied.

(i) The angle of projection θ to the horizontal is obtained by subtracting the given or calculated angle of projection to the vertical from 90°. That is, angle of projection to thehorizontal, $\theta = 90 - 30 = 60^\circ$

(ii) Alternatively, change $\sin\theta$ to $\cos\theta$ in the formula to be used

Therefore, $T = \dfrac{2u\sin\theta}{g}$ becomes $T = \dfrac{2u\cos\theta}{g}$

Substitute $u = 250sm^{-1}$; $T = 25s$ $g = 10ms^{-2}$

$$25 = \frac{2 \times 250 \times \cos\theta}{10}$$

$$25 = 50 \times \cos\theta$$

$$\cos\theta = \frac{25}{50} = 0.5$$

$$\theta = \cos^{-1} 0.5 = 60^\circ$$

EXERCISES 2

1.

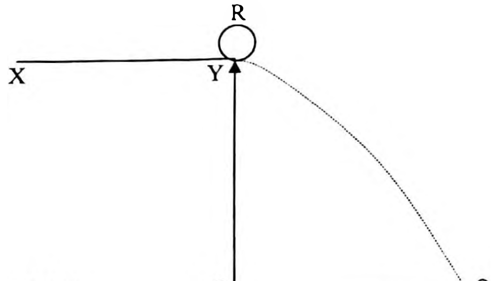


Fig 2.8

An object R leaves a platform XY with a horizontal velocity of 7m/s and lands at Q. if it takes the same object 0.3sec to fall freely from Y to P, calculate the distance PQ (Take $g=10m/s^2$). *WAEC 1990 Ans: 2.1m*

2. A ball is projected horizontally from the top of a hill with a velocity of 20m/s. If it reaches the ground 4 seconds later, what is the height of the hill? ($g=10m/s^2$).
WAEC 1991 Ans: 80m

3. An object is projected with a velocity of 100m/s from the ground level at an angle θ to the vertical. If the total time of flight of the projectile is 10s, calculate θ. ($g=10m/s^2$)
WAEC 1993 Ans: 60^0

4. An object is projected with a velocity of 100m/s at an angle of 60^0 to the vertical. Calculate the time taken by the object to reach the highest point. (Take $g=10m/s^2$).
WAEC 1998 Ans: 5s.

5. A stone is projected horizontally from the top of a tower with a speed of 5m/s. It lands on the ground level at a horizontal distance of 20m from the foot of the tower. Calculate the height of the tower. ($g=10m/s^2$). *WAEC 1999 Ans: 80m*

6. In his first attempt, a long jumper took off from the spring board with a speed of 8m/s at 30^0 to the horizontal. He makes a second attempt with the same speed at 45^0 to the horizontal. Given that the expression for the horizontal range of a projectile is $U^2\sin2\theta/g$, where all the symbols have their usual meanings, show that he gains a distance of 0.8576m in his second attempt. ($g=10m/s^5$).
WAEC 2004 Ans: $\frac{8^2\sin(2\times45°)}{10} - \frac{8^2\sin(2\times30°)}{10} = 0.8575m$

7. A stone thrown horizontally from the top of a vertical wall with a velocity of 15m/s, hits the horizontal ground at a point 45m from the base of the wall. Calculate the height of the wall. ($g=10m/s^2$). *WAEC 2005 Ans: 45m*

8. A stone is projected horizontally with a velocity of 10m/s from the top of a cliff of height 45m. Calculate the horizontal distance covered by the stone from the foot of the cliff. ($g=10m/s^2$) *NECO 2003 Ans: 30m*

9. An object is projected with a velocity of 50m/s from ground level at an angle 0 to the vertical. If the total time of flight of the projectile is 5s, what is the value of θ? ($g=10m/s^2$). *NECO 2002 Ans: 60^0*

10. A ball is kicked with a velocity of 8m/s at an angle of 30^0 to the horizontal. Calculate the time of flight of the ball. ($g=10m/s^2$). *NECO 2000 Ans: 0.8s*

11. A stone, Q is thrown with velocity U at angle of 75^0 to the horizontal. Another stone R is thrown with the same velocity U but at an angle of 15^0 to the horizontal. The ranges covered by the stones will be

A. greater for Q B. greater for R C. the same for Q and R
D. greater for heavier of two stones. *JAMB 1992 Ans: C*

12. An object is projected with a velocity of 80m/s at an angle of 30^0 to the horizontal. The maximum height reached is A. 20m B. 80m C. 160m D. 320m
JAMB 1994 Ans: 80m

13. A particle is projected horizontally at 10m/s from a height of 45m. Calculate the horizontal distance covered by the particle before hitting the ground. ($g=10m/s^2$).

WAEC 2006 Ans: 30m

14. An object is projected from a height of 80m above the ground with a velocity of 40m/s at an angle of 30^0 to the horizontal. What is the time of flight?

A. 4s B. 16s C. 10s D. 8s *JAMB 2005 Ans: 4s*

15. A shooter wants to fire a bullet in such a way that its horizontal range is equal to three times its maximum height. At what angle should he fire the bullet to achieve this?

A. 53° B. 68° C. 30° D. 45° *JAMB 2006 Ans:* 53^0

16. A particle is projected at an angle of 30° to the horizontal with a speed of $250ms^{-1}$. Calculate the

(a) total time of flight of the particle

(b) speed of the particle at its maximum height [$g=10ms^{-2}$]

WAEC 2007 Ans: (a) 25s (b) $216.5ms^{-1}$

17. A stone projected horizontally from the top of a tower with a speed of $4ms^{-1}$ lands on the level ground at a horizontal distance 25m from the foot of the tower. Calculate the height of the tower. [$g=10ms^{-2}$] *WAEC 2007 Ans: 195.3m*

18. A tennis ball is projected horizontally from a height of 30m above the ground with an initial speed of $20ms^{-1}$. Calculate the horizontal distance travelled by the ball when it hits the ground. [$g=10ms^{-2}$] NECO *2007 Ans: 48.(8m*

19. What is the total time of flight of an object projected vertically upwards with a speed of $30ms^{-1}$? [$g=10ms^{-2}$] NECO *2007 Ans: 6.0s*

20. The horizontal component of the initial speed of a particle projected at 30° to the horizontal is $50ms^{-1}$. If the acceleration of free full due to gravity is $10ms^{-2}$, determine its (a) initial speed

(b) speed at the maximum height reached

WAEC 2008[E3] Ans: (a) $57.74ms^{-1}$ (b) $50ms^{-1}$ Hint: $V=\sqrt{V_x^2+V_y^2}$

21. A particle is projected horizontally at $15ms^{-1}$ from a height of 20m. Calculate the horizontal distance covered by the particle just before hitting tho ground. ($g = 10ms^{-2}$) WAEC 2009[E2] Ans: 30m

22. A projectile attains its maximum height when the angle of projection is

A. 30° B. 45° C. 60° D. 75° D. 90°

NECO 2009[7] Ans: E

23. A stone is thrown with a speed of $10ms^{-1}$ at an angle of 30° to the horizontal. Calculate the time of flight. ($g = 10ms^{-2}$) NECO 2009[8] Ans: 1.0s

3

EQUILIBRIUM OF FORCES

MOMENT OF A FORCE

The moment of a force about a point is the product of the force and the perpendicular distance of its line of action from the point.

Moment of a force = Force (N) x Perpendicular distance(m)

The principle of moments states that when a body is in equilibrium, the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments about the same point. In addition, the sum of the forces in one direction is equal to the sum of the forces in the opposite direction.

Example 1

A body of mass 58g is suspended at the 20cm mark of a uniform meter rule. The meter rule is adjusted on a pivot until it settles horizontally at the 40cm mark. Determine the mass of the meter rule. *WAEC 1992*

Solution

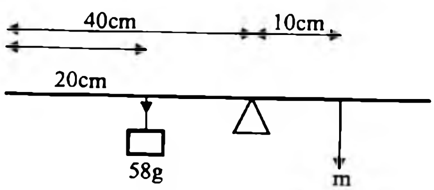


Fig 3.1

Because the meter rule is uniform the centre of gravity, C of g is at the 50cm mark or 10cm from the pivot. Let the mass of the meter rule be m.

Taking moment about the pivot

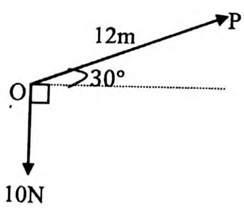
$m \times 10cm = 58g\,(40 - 20)cm$

$m \times 10cm = 58g \times 20cm$

$$m = \frac{58g \times 20cm}{10cm} = \frac{1160}{10} = 116g$$

Example 2

Using the diagram below, calculate the moment of the force of 10N about point P.



WAEC 1989

Fig 3.2

Solution

The distance OP has to be resolved into its perpendicular distance, OQ as follows.

$$\cos 30 = \frac{OQ}{OP} = \frac{OQ}{12}$$

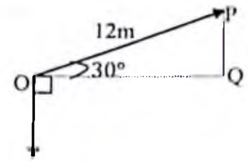


Fig 3.3

$$OQ = 12\cos 30 = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

moment of force 10N = force(10N) × perpendicular distance OQ

$$= 10 \times 6\sqrt{3} = 60\sqrt{3} \;\; or \;\; 104\, Nm$$

Example 3
What is the moment of the force F about point P in the diagram shown below?

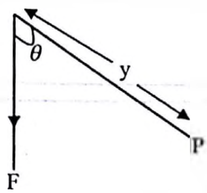


Fig 3.4 *WAEC 1994*

Solution
The perpendicular distance of y is found by resolving the distance y into its horizontal component as follows:

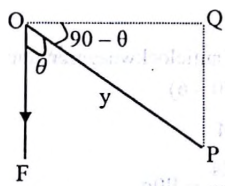


Fig 3.5

$$\cos(90 - \theta) = \frac{OQ}{y}$$

$$OQ = y\cos(90 - \theta)$$

$$but \quad \cos(90 - \theta) = \sin\theta$$

$$\therefore \; OQ = y\sin\theta$$

Moment of F about P = $F_y \sin\theta$

Example 4
A uniform bar 15m long is balanced on a pivot placed at its midpoint. A boy of mass 55kg sits on one arm of the bar at a point 5m away from the pivot. What mass can be placed 2m away from the other end of the bar to keep the bar horizontally? *WAEC 1997*
Solution

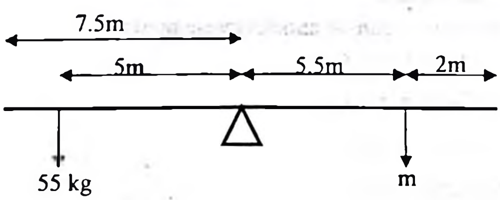


Fig 3.6

Sum of anticlockwise moment = sum of clockwise moment. Taking moment about pivot where m is the placed mass.

$$m \times 5.5 = 55 \times 5$$

$$m = \frac{55 \times 5}{5.5} = \frac{275}{5.5} = 50 \text{ kg}$$

Example 5

A meter rule is found to balance horizontally at the 48cm mark. When a body of mass 60g is suspended at the 6cm mark, the balance point is found to be at the 30cm mark. Calculate the

(i) mass of the meter rule.

(ii) distance of the balance point from the zero end, if the body were moved to the 13cm mark. *WAEC 2007*

Solution

(i)

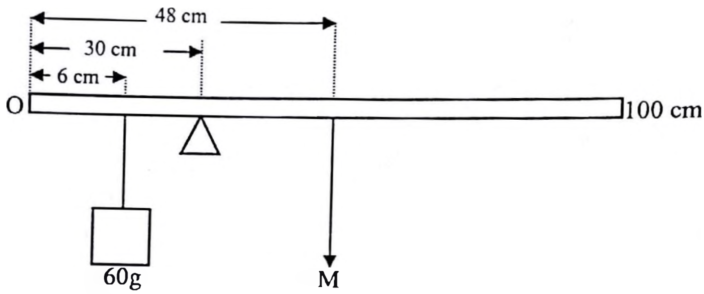


Fig 3.7

Let M be the mass of meter rule.

Taking moments about the fulcrum,

sum of clockwise moments = sum of anticlockwise moments

$$M \times (48 - 30) = 60 \times (30 - 6)$$

$$18M = 60 \times 24$$

$$M = \frac{60 \times 24}{18} = 80g$$

(ii)

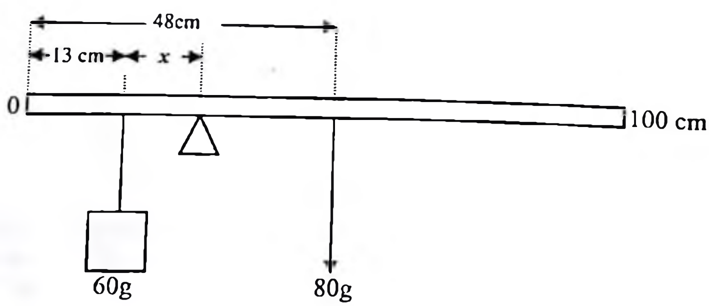


Fig 3.8

Taking moment about the fulcrum,

Sum of clockwise moments = sum of anticlockwise moments

$$80 \times [48 - (13 + x)] = 60 \times x$$

$$80 \times [48 - 13 - x] = 60 \times x$$

$$80 \times [35 - x] = 60 \times x$$

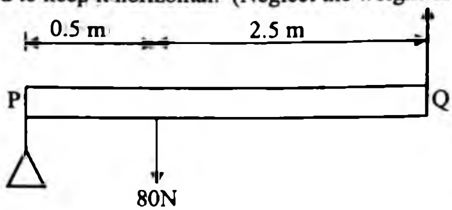
$$2800 - 80x = 60x$$

$$2800 = 80x + 60x$$
$$2800 = 140x$$
$$x = \frac{2800}{140} = 20cm$$

Therefore, the distance from the zero end is $13 + x = 13 + 20 = 33$cm.

Example 6

A beam PQ pivoted at P carries a load of 80N as shown above. Calculate the effort E, required to keep it horizontal. (Neglect the weight of the beam).



WAEC 2001

Fig 3.9

Solution

Taking moments about P

$$E \times (2.5 + 0.5) = 80 \times 0.5$$
$$E \times 3 = 40$$
$$E = \frac{40}{3} = 13.3N$$

Example 7

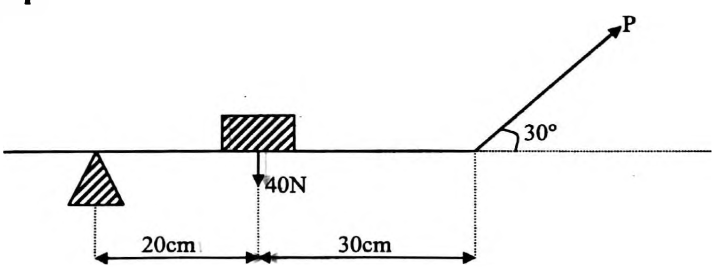


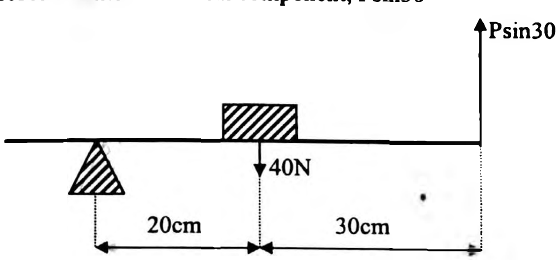
Fig 3.10

Determine the magnitude of P in the diagram above. *WAEC 2006*

Solution

The force P has to be resolved into its vertical component, Psin30

Fig 3.11



Taking moment about the pivot,

$$P \sin 30 \times 0.5m = 40 \times 0.2m \qquad (30+20 = 50 \text{ cm} = 0.5 \text{ m}; \ 20 \text{ cm} = 0.2 \text{ m})$$

$$0.5P \times 0.5 = 8$$

$$0.25P = 8$$

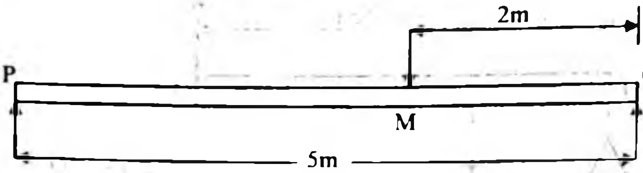
$$P = \frac{8}{0.25} = 32N$$

Example 8

A uniform beam PQ of mass 20kg and length 5m is balanced as shown below. A man of weight 800N stands at M such that QM = 2m. The reactions at P and Q respectively are

A. 300N and 500N   B. 420N and 580N   C. 220N and 380N

D. 580N and 420N   E. 380N and 220N   ($g=10m/s^2$)



JAMB 1982

Fig 3.12

Solution

Because the bar is uniform its weight ($w = mg = 20kg \times 10 = 200N$) act at the middle of the bar.

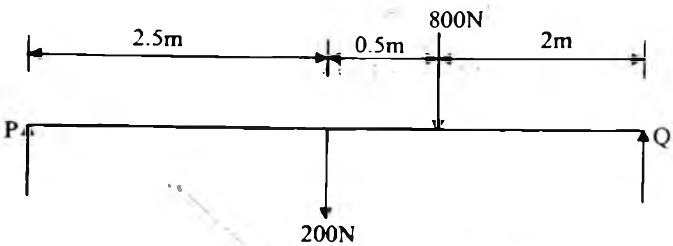


Fig 3.13

Taking moment about P

$$Q \times 5 = (200 \times 2.5) + 800 \times (2.5 + 0.5)$$

$$5Q = 500 + 800 \times 3 = 500 + 2400$$

$$5Q = 2900$$

$$Q = \frac{2900}{5} = 580N$$

Sum of downward force = sum of upward forces

$$200 + 800 = P + Q$$

$$1000 = P + 580$$

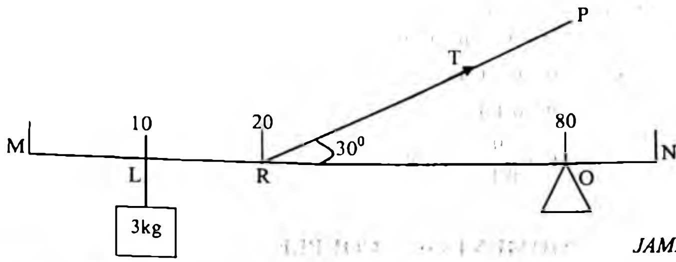
$$P = 1000 - 580 = 420N$$

The reaction at P and Q respectively are 420N and 580N.

Example 9

In the figure 3.14 below, MN is a light uniform meter rule pivoted at O, the 80cm mark. A load of mass 3.0kg is suspended on the rule at L, the 10cm mark. If the rule is kept in equilibrium by a string RP, fixed at P and attached to the rule at R, the 20cm mark, what is the tension, T in the string?

A. 3.0 N   B. 3.4 N   C. 3.5 N   D. 6.0 N   E. 7.0 N



JAMB 1985

Fig 3.14

Solution

The tension force T is resolved into its perpendicular (vertical) component, $T\sin\theta$

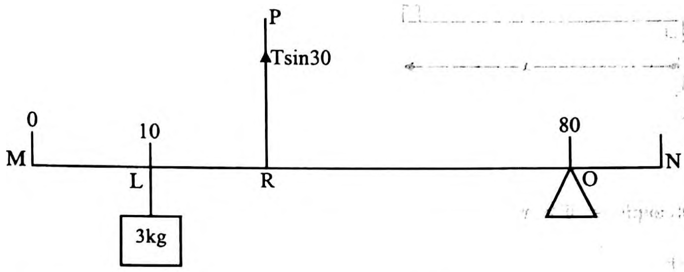


Fig 3.15

Taking moment about the pivot O

$$T\sin 30 \times (0.80 - 0.20) = 3 \times (0.8 - 0.10)$$

$$0.5T \times 0.6 = 3 \times 0.70$$

$$0.3T = 2.1$$

$$T = \frac{2.1}{0.3} = 7\ \text{N}$$

Example 10

A 90cm uniform lever has a load of 30N suspended at 15cm from one of its ends. If the fulcrum is at the center of gravity, what is the force that must be applied at its other end to keep it in horizontal equilibrium? *JAMB 2003*

Solution

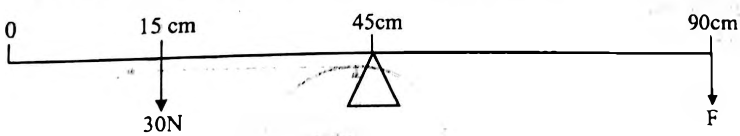


Fig 3.16

Taking moment about the fulcrum,

$$30 \times (0.45 - 0.15) = F \times (0.90 - 0.45)$$

$$30 \times 0.30 = F \times 0.45$$

$$9 = 0.45F$$

$$F = \frac{9}{0.45} = 20 \text{ N}$$

MOMENT OF A COUPLE

The moment of a couple is defined as the product of one of the forces and the perpendicular distance between the lines of action of the two forces.

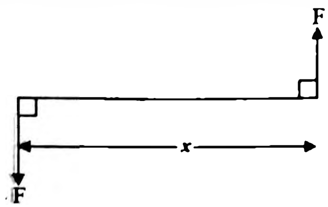


Fig 3.17

Moment of couple = $F \times x$

Example 11

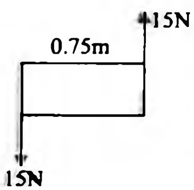


Fig 3.18

Calculate the magnitude of the couple in the figure above.

Solution

Moment of the couple = One force x perpendicular distance between the forces.

$$= F \times x$$

$$= 15 \times 0.75 = 11.25\text{Nm}$$

Example 12

Two forces, each of magnitude 25N, act in opposite directions at the opposite ends of a circular ring. If the diameter of the ring is 65cm, calculate the magnitude of the couple acting on the ring.

Solution

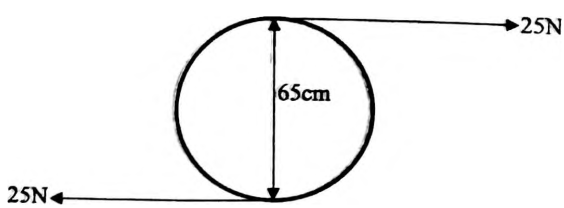


Fig 3.19

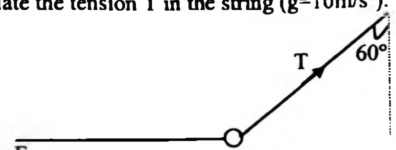
Moment of a couple = Force × perpendicular distance
= 25N × 0.65m
= 16.25Nm.

THE TRIANGLE OF FORCES

The principle of triangle of forces is applied to solve problems in which three forces act on a body to keep it in equilibrium. It states that when a body is in equilibrium under the action of three forces, the three forces can be represented in magnitude and in direction by the three sides of a triangle taken in order.

The following examples illustrate how the triangle of forces is applied to solve problem in equilibrium.

Exampl 13
The body P shown in the diagram below is in equilibrium. If the mass of the body is 10kg, calculate the tension T in the string ($g=10m/s^2$).



WAEC 1990

Fig 3.20

Solution
The weight of the body P acts downward and F acts in the horizontal direction as shown below. W = mg = 10 × 10 = 100N.

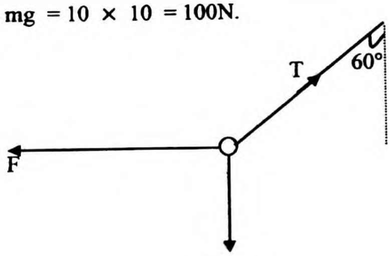
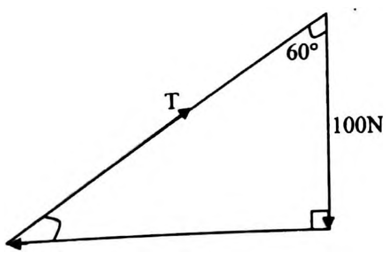


Fig 3.21

Next a triangle is formed by taking the magnitude and direction of each force in sequence.



The triangle formed is a right angle triangle and trigonometric ratio can be applied to find T.

Fig 3.22

$$\cos 60 = \frac{100}{T} \qquad \therefore \qquad T = \frac{100}{\cos 60} = \frac{100}{0.5} = 200N$$

Example 14

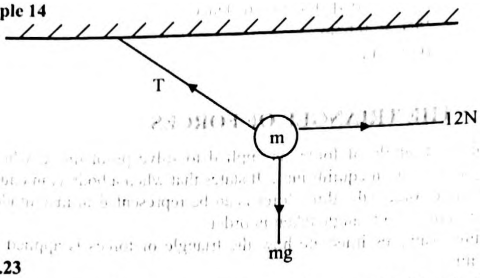


Fig 3.23

A body of mass m=0.5kg is suspended by a string and pulled by a horizontal force of 12N as shown in the diagram above. Calculate the tension T in the string if the body is in equilibrium (Take g=10m/s^2) *WAEC 1993*

Solution

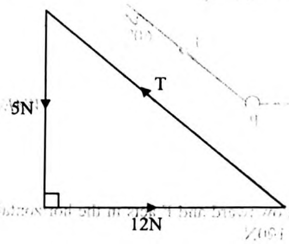


Fig 3.24

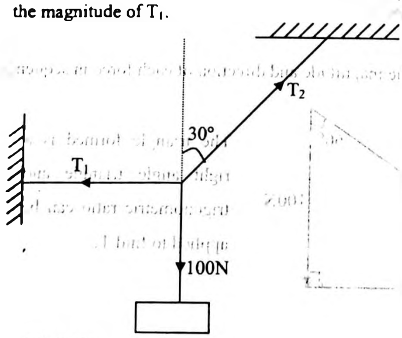
Pythagoras theorem can be applied to find T

$$T^2 = 12^2 + 5^2 = 144 + 25 = 169$$

$$T = \sqrt{169} = 13\ N$$

Example 15

The diagram below illustrates three forces T_1, T_2, and 100N in equilibrium. Determine the magnitude of T_1.



WAEC 2000

Fig 3.25

Solution

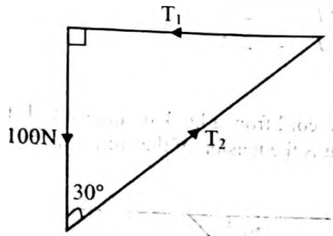


Fig 3.26

Using trigonometric ratio, T_1 can be found as follows:

$$\tan 30° = \frac{T_1}{100}$$

$$\therefore \quad T_1 = 100 \tan 30 = 57.74\ N$$

Example 16

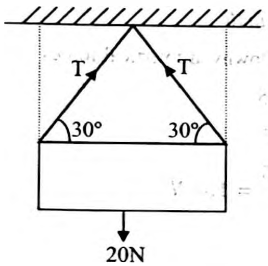


Fig 3.27

What's the value of T in the figure above? A. 10.0 N B. 11.8 N C. 20.0N D. 40.0 N *JAMB 1995*

Solution

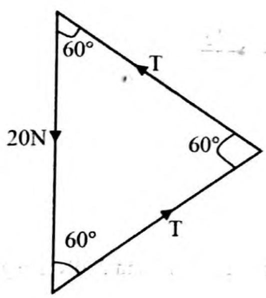


Fig 3.28

Note that it is the angle the string makes with the vertical ($90^0 - 30^0 = 60^0$) that is used to draw the triangle of forces and used for the calculation. Because the triangle formed is not a right angle triangle, we use Sine rule to find T.

$$\frac{20}{\sin 60} = \frac{T}{\sin 60} \quad \therefore \quad T = \frac{20 \times \sin 60}{\sin 60} = 20N$$

Alternatively, we could find T by resolving the two Ts into their vertical component and applying one of the conditions for equilibrium: Sum of upward forces = sum of downward forces.

Therefore, $T\sin 30 + T\sin 30 = 20N$

$$0.5T + 0.5T = 20N$$

$$T = 20N$$

Example 17

A mirror of weight 75N is hung by a cord from a hook on the wall. If the cord makes an angle of 30^0 with the horizontal, what is the tension of the cord? A. 150N B.100N C. 1500N D. 250N *JAMB 2006*

Solution

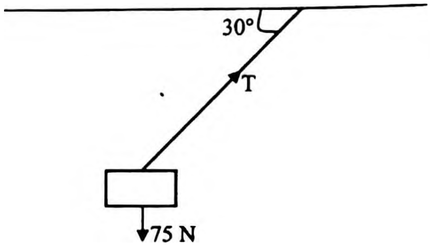


Fig 3.29

The vertical component of $T = T\sin 30$

Upward vertical forces = Downward vertical forces

$$T\sin 30 = 75$$

$$0.5T = 75$$

$$T = \frac{75}{0.5} = 150\,N$$

Example 18

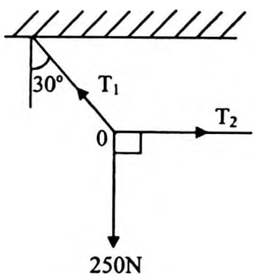


Fig 3.30

The diagram above illustrates three forces T_1, T_2 and 250N in equilibrium. Calculate the magnitude of T_2. *NECO* 2008[7]

Solution

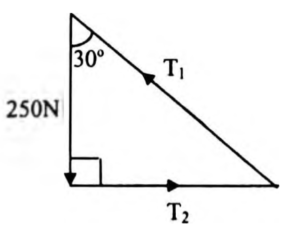


Fig 3.31

Using trigonometric ratio, T_2 can be found as follows:

$$\tan 30^\circ = \frac{T_2}{250} \quad \therefore \quad T_2 = \tan 30 \times 250 = 0.577 \times 250$$

$$= 144.34N$$

EXERCISE 3.

1. A meter rule is found to balance at the 48cm mark. When a body of mass 60g is suspended at the 6cm mark the balance point is found to be at the 30cm mark. Calculate, i) the mass of the meter rule ii) the distance of the balance point from the zero end, if the body were moved to the 13cm mark. *WAEC 1988 Ans (i) 80g (ii) 33cm*

2. uniform meter rule AB is balanced on a knife edge which is 55cm from B. If a mass of 10g is hung at P, which is 10cm from A. Calculate the mass of the meter rule. *WAEC 1994 Ans: 70g*

3. A uniform meter rule of mass 90g is pivoted at the 40cm mark. If the rule is in equilibrium with an unknown mass m placed at the 10cm mark and a 72g mass at the 70cm mark, determine m. *WAEC 2000 Ans: 102g*

4.

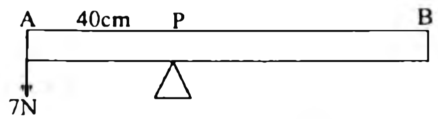
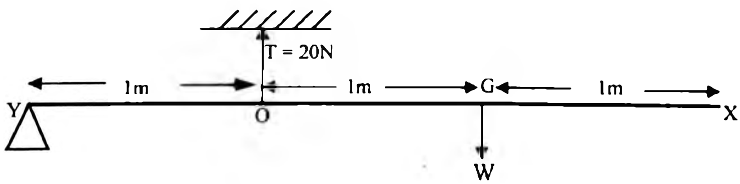


Fig 3.32

In the diagram above, AB represents a uniform rod of length 1.50m which is in equilibrium on a pivot at P. If AP = 40cm, calculate the mass of the rod. ($g=10m/s^2$) *WAEC 2001 Ans: 0.8kg*

5. Masses m_1 and m_2 at the 20cm and 65cm marks respectively of a uniform meter rule freely suspended at its centre of gravity. If the meter rule balances horizontally, determine the ratio $m_2:m_1$. *WAEC 2001 Ans: 2:1*

6. In the diagram below, what is the value of W.



JAMB 1979 Ans: 10N

Fig 3.33

7.

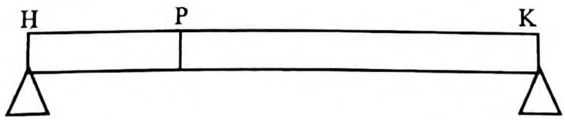


Fig 3.34

A uniform beam HK of length 10m and weighing 200N is supported at both ends as shown in the figure above. A man weighing 1000N stands at a point P on the beam. If the reactions at H and K are respectively 800N and 400N, what is the distance HP? *JAMB 1979 Ans: 3m*

8. The figure 3.35 below shows a see-saw which is exactly in balance. The weights of the people sitting on it are 50kg, X kg and 15kg as shown. They sit at distances of 1.5, 1.5 and 2m from the pivot respectively. What is the value of X?

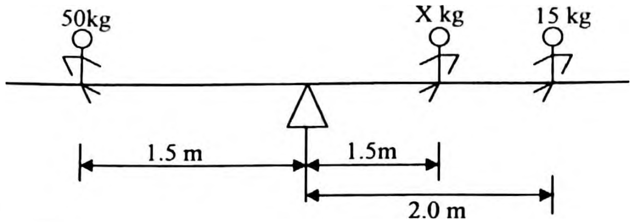


Fig 3.35

JAMB 1979 Ans:26.67kg

9. In the figure below, PT is a uniform meter rule pivoted at R, the 70cm mark. Two forces 0.1N and 0.4N are applied at Q, the 60cm mark and S, the 85cm mark. If the meter rule is kept in equilibrium by the forces and its weight, calculate the weight of the meter rule. A. 0.25N B. 0.30N C. 0.35N D. 0.50N E. 0.56N *JAMB 1984 Ans: 0.25N*

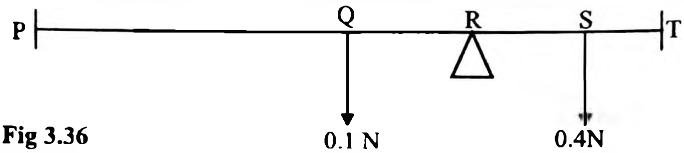


Fig 3.36

10. Two masses 40g and 60g respectively are attached firmly to the ends of a light meter rule. The centre of gravity of the system is A. At the midpoint of the meter rule B. 40cm from the lighter mass C. 40cm from the heavier mass D. 60cm from the heavier mass E. indeterminate because the meter-rule is light. *JAMB 1985 Ans: C*

11.

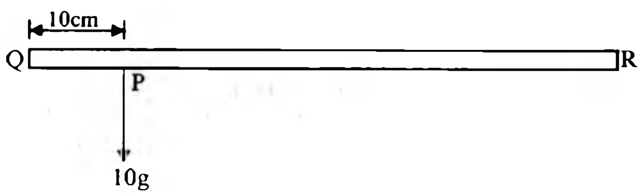


Fig 3.37

A uniform meter rule QR is balanced on a knife edge which is 55cm from R. When a mass of 10g is hung at P as shown above, what is the mass of the meter rule?
A. 550g B. 350g C. 70g D. 35g *JAMB 1986 Ans: 70g*

12.

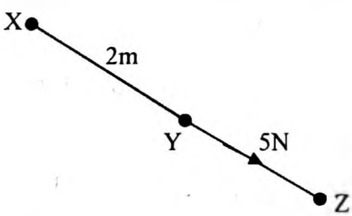


Fig 3.38

A force of 5N acts at a point Y on a rod XYZ as shown in the diagram above. If XY is 2m, what is the moment of the force about point X? A. 0 Nm B. 3 Nm C. 7 Nm D. 10 Nm *JAMB 1990 Ans: 0*

13.

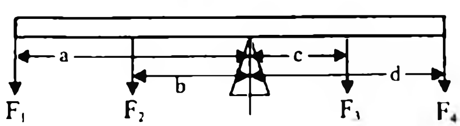


Fig 3.39

A uniform light rod is kept in horizontal equilibrium under the influence of four forces as shown below. Which of the following equations correctly represents the condition of equilibrium for the rod?

A) $F_1 + F_2 = F_3 + F_4$ B) $F_1 + F_2 - F_3 + F_4 = 0$ C) $(F_1 + F_2)ab = (F_3 + F_4)cd$
D) $F_1a + F_2b - F_3c - F_4d = 0$ *JAMB 1992* *Ans: D*

14.

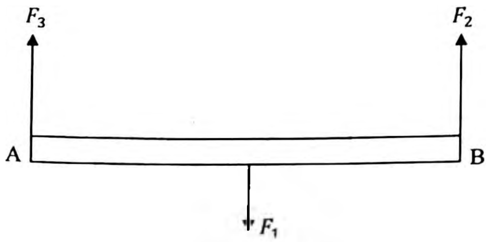


Fig. 3.40

The diagram above shows three forces F_1, F_2 and F_3 which keep the bar AB in horizontal equilibrium. Which of the following equation is correct?

A. $F_3 = F_1 + F_2$ B. $F_2 = F_1 + F_3$ C. $F_1 = F_2 - F_3$ D. $F_1 = F_2 + F_3$
WAEC 2008[11] Ans: *D*

15.

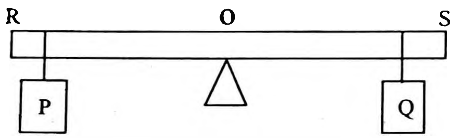


Fig. 3.41

The diagram above shows a plank RS pivoted at its centre of gravity O and is in equilibrium with the weights **P** and **Q**. If a weight **2P** is added to **P**, the plank will be in equilibrium again by

A. moving Q nearer to O
B. moving P nearer to O
C. adding a weight Q to Q
D. moving **P** further away form O *JAMB* 2008[8] Ans: *B*

16.

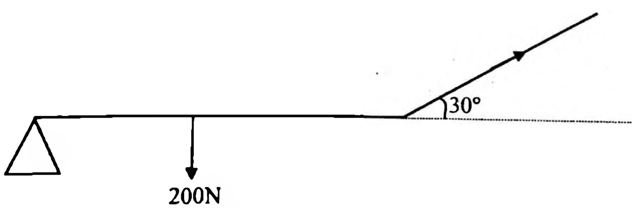


Fig. 3.42

The diagram above shows a uniform wood of weight 200N and length 50m. It is pivoted at one end and suspended by a cord at the other end at an angle of 30^0 to the wood. Calculate the tension in the cord if the wood is horizontal. *JAMB 1994* *Ans: 200N*

17. A uniform rod PQ of length 1m and mass 2kg is pivoted at the end P. If a load of 14N is placed at the centre of the rod, find the force that should be applied vertically upwards at Q to maintain the rod in equilibrium horizontally. A. 68 N B. 28N C. 17 N D. 7 N ($g=10m/s^2$). *JAMB 1995* *Ans: 17N*

18.

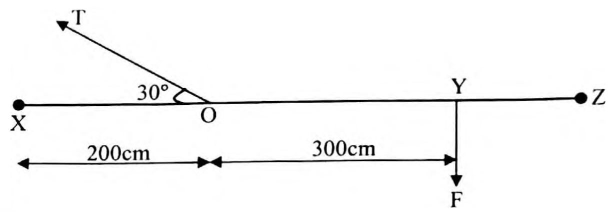


Fig. 3.43

A uniform light beam XZ is hinged at X and kept in equilibrium by the forces T and F as shown in the diagram above. If XO = 20cm and OY = 30cm, express T in terms of F.

A. $T = \frac{2\sqrt{3}}{3}F$ B. $T = 2F$ C. $T = \frac{5\sqrt{3}}{3}F$ D. $T = 5F$ *JAMB 1997 Ans: $T = 2F$*

19. A uniform meter rule weighing 0.5N is to be pivoted on a knife-edge at the 30cm mark. Where will a force of 2N be placed from the pivot to balance the meter rule?

A. 95cm B. 25cm C. 20cm D. 5cm

JAMB 1998 Ans: 25cm mark or 5cm from the pivot

20. Two forces each of 4N act on the opposite sides of a rectangular plate as shown in the diagram below. Calculate the magnitude of the couple acting on the plate.

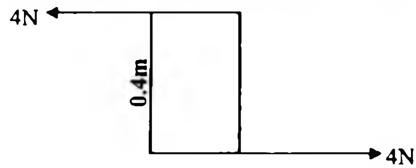


Fig. 3.44

WAEC 1990 Ans: 1.6Nm

21. Two forces, each of magnitude 4N, act in opposite directions at the ends of a rod. If the length of the rod is 40cm, calculate the magnitude of the couple acting on the rod.

NECO 2006 Ans: 1.6Nm

22.

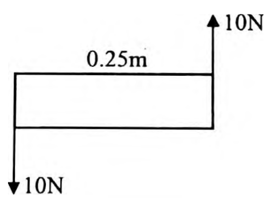


Fig. 3.45

In the figure above, calculate the magnitude of the couple. *NECO 2005 Ans: 2.50Nm*

23. Two forces forming a couple are separated by a distance of 25cm. If one of the forces equals 40N, what is the moment of the couple? *NECO 2000 Ans: 10Nm*

24. In the diagram below, MN is perpendicular to ON and MP. What is the difference between the moment about N of the force of 20N applied along MP and its moment about O?

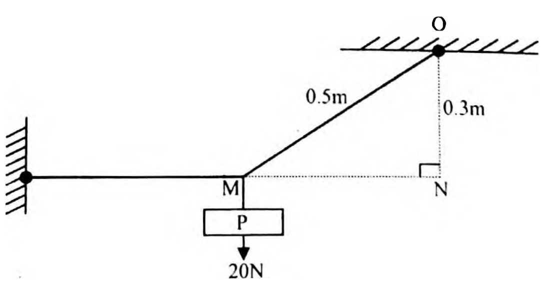


Fig. 3.46

A. Zero B. 0.2Nm C. 0.4Nm D. 0.8Nm E. 0.6Nm *JAMB 1980 Ans: 0*

25. A boy pulls a nail from the wall with a string tied to the nail. The string is inclined at an angle of 60^0 to the wall. If the tension in the string is 4N, what is the effective force used in pulling the nail? *WAEC 1989 Ans: $2\sqrt{3}N$*

26.

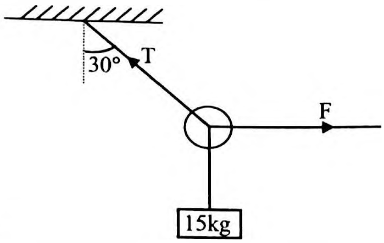


Fig. 3.47

A 15kg mass suspended from a ceiling is pulled aside with a horizontal force, F, as shown in the diagram above. Calculate the value of the tension, T. ($g=10m/s^2$) *WAEC 1999 Ans: 173.2N*

27.

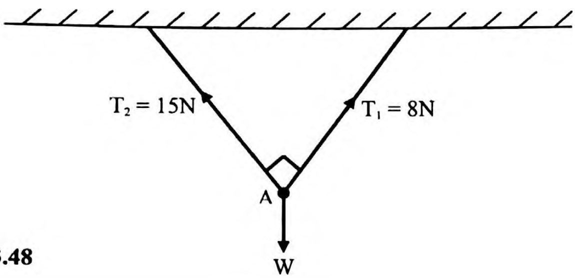


Fig. 3.48

An object A, is held in equilibrium as illustrated in the diagram above. Using the data on the diagram, determine the magnitude, W, of the weight of A. *WAEC 2002 Ans: 17N*

28.

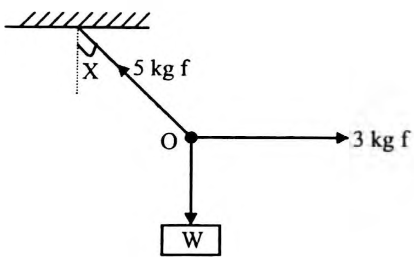


Fig. 3.49

In the given diagram above, what are the values of the vertically suspended weight W and the angle X? A. 2kgf, 30.9^0 B. 4kgf, 36.9^0 C. 6kgf, 53.1^0 D. 8kgf, 73.8^0 E. 12kgf, 106.2^0 *JAMB 1982 Ans: 4kgf, 36.9^0*

29. Find the tension T_1 in the diagram below if the system is in equilibrium.

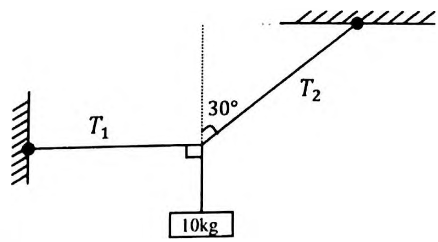


Fig. 3.50

A. $\frac{200}{\sqrt{3}}N$ B. $\frac{100}{\sqrt{3}}N$ C. $\frac{300}{\sqrt{3}}N$ D. 100 $(g = 10ms^{-2})$

JAMB 1991 Ans: 57.74N or $\frac{100}{\sqrt{3}}N$

30.

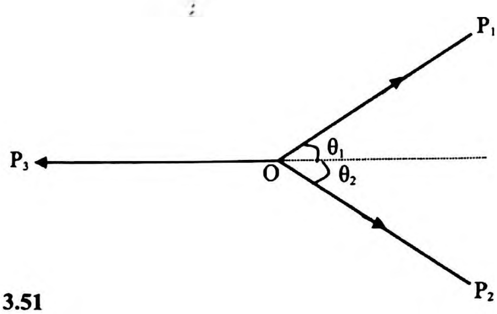


Fig. 3.51

Consider the three forces acting at O and in equilibrium as shown above. Which of the following equations is/are correct?

(i) $P_1\cos\theta_1 = P_1\cos\theta_2$ (ii) $P_3 = P_1\cos\theta_1 + P_2\cos\theta_2$

(iii) $P_1\sin\theta_1 = P_2\sin\theta_2$

A. (i) only B. (ii) only C. (iii) only D. (ii) and (iii) only E. (i) and (iii) only

JAMB 1983 Ans: D

31.

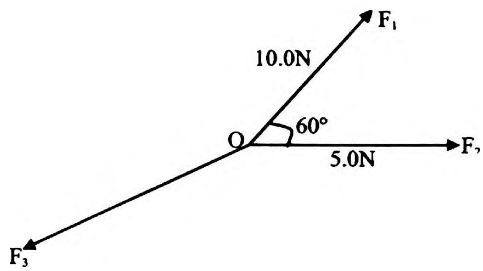


Fig. 3.52

In the figure above, the three forces F_1, F_2, F_3 acting at O are in equilibrium. If the magnitude of F_1 is 10.0N and the magnitude of F_2 is 5.0N, find the magnitude of F_3.

A. 26.4N B. 15.0N C. 13.2N D. 10.0N *JAMB 1986 Ans: 13.23N*

32. The diagram below shows forces 4N, 6N, 10N and 8N which act at a point O in the directions indicated. What is the horizontal force? A. $7\sqrt{3}$ N B. 17 N C. $\sqrt{3}$ N D. 13 N *JAMB 2004 Ans: 13N*

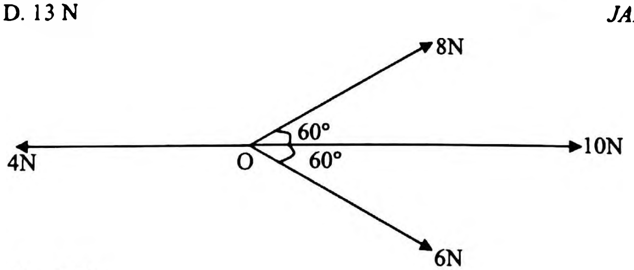


Fig. 3.53

33. Two horizontal forces, 10N and 8N and other force F, inclined at 30^0 to the vertical acting as shown in the diagram below, keep the body P in equilibrium. What is the weight of the body? A. $\frac{2\sqrt{3}}{3}N$ B. $\sqrt{3}N$ C. $\frac{4\sqrt{3}}{3}N$ D. $2\sqrt{3}N$

JAMB 1990 Ans: 3.46N or $2\sqrt{3}$

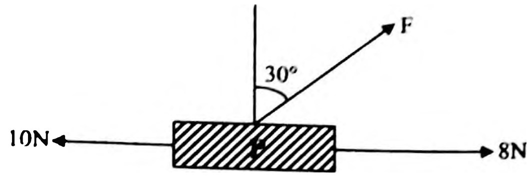


Fig. 3.54

34. What is the value of F in the figure below when it is in equilibrium?

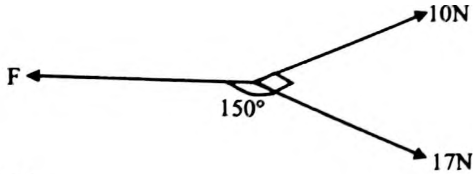


Fig. 3.55

A. 10 N B. 12 N C. 27 N D. 20 N *JAMB 2005 Ans: 20N*

35. For what value of θ are the forces in the diagram below in equilibrium?

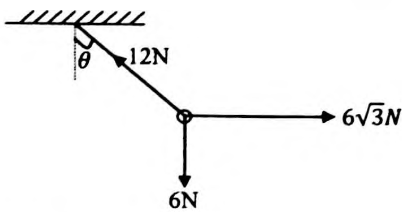


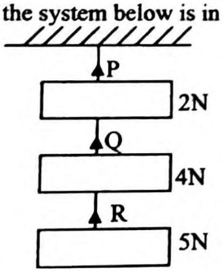
Fig. 3.56

A. 15° B. 30° C. 45° D. 60° *JAMB 1997 Ans: 60°*

36. A hand bag containing some load weighing 162N is carried by two students each holding the handle of the bag next to him. If each handle is pulled 60^0 to the vertical, find the force on each student's arm. A. 324N B. 162N C. 121N D. 81N

JAMB 2000 Ans: 162N

37. If the system below is in equilibrium, what is the tension of the string Q?



A. 6N
B. 4N
C. 11N
D. 9N

Fig. 3.57

JAMB 2005 Ans: 9N

38. A particle is under the action of three forces in equilibrium. Two of the forces are as shown below. What is the third force?

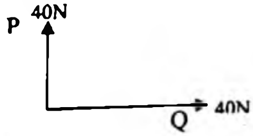


Fig. 3.58

A. 80N along P B. 160N at 60° to P C. $40\sqrt{3}$ N at 45° to either P or Q
D. 0N E. $40\sqrt{2}$ N at 30° to Q *JAMB 1981 Ans: C*

39. In the diagram below, a rod 50cm long of uniform cross-section is suspended horizontally on a fulcrum, F, by the action of two forces. What is the weight of the rod?

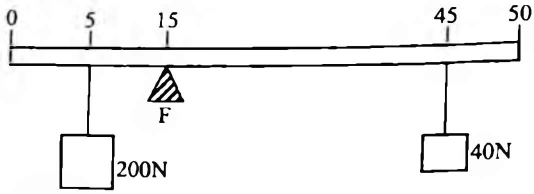


Fig. 3.59

A. 200N B. 120N C. 80N D. 40N *JAMB 2007 Ans: 80N*

40.

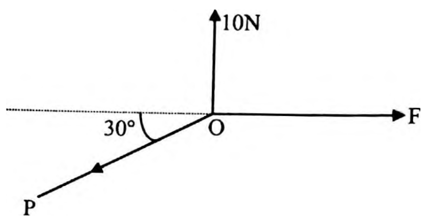


Fig. 3.60

The diagram above illustrates three forces acting on an object at point O. If the object is in equilibrium, determine the magnitude of the force P. *WAEC 2007 Ans: 20.0N*

41.

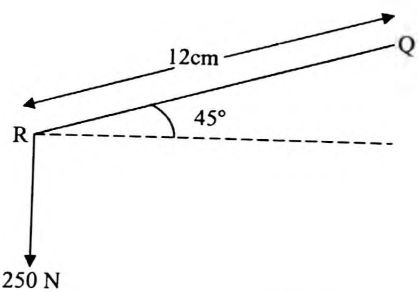


Fig. 3.61

Using the diagram above, calculate the magnitude of the moment of the force of 250N about the point Q. *NECO* 2008[E2] Ans: 21.21Nm

4

EQUILIBRIUM OF BODIES IN LIQUIDS

ARCHIMEDES PRINCIPLE:

The equilibrium of bodies in liquid is governed by Archimedes principle and principle of floatation. Closely related to these, are the densities and relative densities of objects and fluids.

Archimedes Principle states that when an object is completely or partially immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced by the object.

Principle of floatation states that an object will float when the upthrust exerted upon it by the fluid in which it floats is equal to the weight of the body.

Upthrust of object in fluid, = Weight of object in air − Weight of object in fluid,

$$U_F = W_A - W_F$$

Note that the fluid could be water, gas or any other liquid, therefore U_F and W_F could be U_W, W_W and U_L, W_L.

Upthrust of object in fluid = Weight of displaced fluid

= Mass of displaced fluid × g

= Density of fluid × volume of displaced fluid × g

= Density of fluid × volume of object × g

$$U_F = \rho \times V \times g$$

Density, ρ, is defined as the mass per unit volume of a substance, measured in kg/m^3.
Relative density, R.d, of a substance is the ratio of the mass or weight of any volume of it to the mass or weight of an equal volume of water.

$$\text{R. d of solids} = \frac{\text{mass of substance}}{\text{Mass of equal volume of water}}$$

$$= \frac{\text{weight of substance}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{density of substance}}{\text{density of water}}$$

$$= \frac{\text{weight of object in air}}{\text{weight of equal volume of water}}$$

$$= \frac{\text{weight of object in air}}{\text{upthrust of object in water}}$$

$$\text{R. d of liquids} = \frac{\text{upthrust of object in liquid}}{\text{upthrust of object in water}}$$

$$= \frac{\text{volume of liquid displaced by object}}{\text{volume of water displaced by same object}}$$

$$= \frac{\text{apparent loss of weight of a solid in liquid}}{\text{apparent loss of weight of same solid in water}}$$

Example 1

An object weighs 10.0N in air and 7.0N in water. What is its weight when immersed in a liquid of relative density 1.5? *WAEC 1990*

Solution

Given: Relative density of liquid = 1.5 Weight of object in air, $W_A = 10\ N$

Weight of object in water, $W_W = 7N$

Problem: Weight of object in liquid, W_L

∴ Upthrust on object by water, $U_W = W_A - W_W = 10 - 7 = 3N$

The appropriate formula is

$$\text{R. d of liquid} = \frac{\text{upthrust of object in liquid } (U_L)}{\text{upthrust of object in water } (U_W)}$$

$$\text{Substituting, } 1.5 = \frac{U_L}{3} \quad \therefore \quad U_L = 3 \times 1.5 = 4.5\text{N}$$

Generally, upthrust = weight in air − weight in fluid (liquid)

∴ Weight of object in liquid = weight in air − upthrust in liquid

$$W_L = W_A - U_L$$
$$W_L = 10 - 4.5 = 5.5N$$

Example 2

A block of material of volume $2 \times 10^{-5}m^3$ and density $2.5 \times 10^3kgm^{-3}$ is suspended from a spring balance with half the volume of the block immersed in water. What is the reading of the spring balance? (Density of water = $1.0 \times 10^3kgm^{-3}$, $g=10m/s^2$) *WAEC 1993*

Solution

Given: Volume of object $V = 2 \times 10^{-5}m^3$; Density of object, $\rho = 2.5 \times 10^3 kgm^{-3}$

$$\text{Volume of object immersed} = \tfrac{1}{2}\text{V} = \tfrac{1}{2}(2 \times 10^{-5}) = 1 \times 10^{-5}\text{m}^3$$

$$\text{Density of water} = 1.0 \times 10^3\ kgm^{-3}$$

Problem: Spring balance reading = weight of object in water, W_W

$$\text{Density } (\rho) = \frac{\text{Mass}}{\text{Volume(V)}} \quad \therefore \quad \text{Mass of object} = \rho\text{ V} = 2 \times 10^{-5} \times 2.5 \times 10^3 = 0.05kg$$

$$\text{Weight of object in air, } W_A = \text{mass of object} \times \text{g} = 0.05 \times 10 = 0.5\text{N}$$

$$\text{Upthrust of object in water, } \text{U}_W = \text{Density of water} \times \text{Volume of object immersed} \times \text{g}$$
$$= \rho \times V \times g = 1 \times 10^{-5} \times 1.0 \times 10^3 \times 10 = 0.1N$$

$U_W = W_A - W_W$ ∴ weight of object in water, $W_W = W_A - U_W = 0.5 - 0.1 = 0.4\text{N}$

Example 3

A rectangular block of wood floats in water with two-third of its volume immersed. When placed in another liquid, it floats with half of its volume immersed. Calculate the relative density of the liquid. *WAEC 1998*

Solution

Given: Volume of block immersed in water $= V_W = \frac{2}{3}V$

Volume of block immersed in liquid $= V_L = \frac{1}{2}V$

Problem: Relative density of liquid

$$\text{R. d of liquid} = \frac{\text{volume of liquid displaced by object } (V_L)}{\text{volume of water displaced by same object } (V_W)}$$

Note: The volume of fluid (liquid or water) displaced is equal to the volume of object immersed in fluid.

$$\therefore \quad R.d = \frac{V_L}{V_W} = \frac{\frac{1}{2}V}{\frac{2}{3}V} = \frac{1}{2}V \div \frac{2}{3}V = \frac{1}{2}V \times \frac{3}{2V} = \frac{3}{4} = 0.75$$

Example 4

A solid plastic cube of side 0.2m is submerged in a liquid of density $0.8kgm^{-3}$. Calculate the upthrust of the liquid on the cube ($g=10m/s^2$). *WAEC 1997*

Solution

Given: Length of cube, $l = 0.2m$ Density of liquid, $\rho = 0.80kgm^{-3}$

Problem: Upthrust on the cube

Volume of cube $= V = l^3 = (0.2)^3 = 0.008m^3$

Upthrust = density of liquid × volume of object × g

$= 0.8 \times 0.008 \times 10 = 0.064N$

Example 5

The density of water is $1g/cm^3$ while that of ice is $0.9g/cm^3$, calculate the change in volume when 90g of ice is completely melted. *WAEC 1994*

Solution

Given: density of water, $\rho_w = 1gcm^{-3}$ density of ice, $\rho_i = 0.9gcm^{-3}$

mass of ice or water, m = 90g

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \therefore \quad \text{Volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{Volume of ice} = V_i = \frac{m}{\rho_i} = \frac{90}{0.9} = 100cm^3$$

$$\text{Volume of water} = V_w = \frac{m}{\rho_w} = \frac{90}{1} = 90cm^3$$

$$\text{Change in volume} = V_i - V_w = 100cm^3 - 90cm^3 = 10\ cm^3$$

Example 6

What volume of alcohol with a density of $8.4 \times 10^2kgm^{-3}$ will have the same mass as $4.2m^3$ of petrol whose density is $7.2 \times 10^2kg/m^3$. *NECO 2006*

Solution

Given: density of alcohol, $\rho_a = 8.4 \times kgm^{-3}$; Density of petrol, $\rho_p = 7.2 \times 10^2\ kgm^{-3}$

Volume of petrol, $V_p = 4.2m^3$

Problem: Volume of alcohol, V_a

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \therefore \quad \text{Mass} = \text{density} \times \text{volume}$$

$$\text{mass of petrol}, M_p = \rho_p \times V_p = 7.2 \times 10^2 \times 4.2 = 3024\text{kg}$$

Therefore, mass of alcohol, $m_a = 3024\text{kg}$

$$\therefore \quad \text{Volume of alcohol}, V_a = \frac{m_a}{\rho_a} = \frac{3024\text{kg}}{8.4 \times 10^2\ kgm^{-3}} = 3.6\text{m}^3$$

Example 7

A solid weighs 5.0N in air, 4.2N in water and 4.4N in liquid Y. Calculate the relative density of liquid Y. *NECO 2000*

Solution

Given: Weight of solid in air, $W_A = 5.0\text{N}$ Weight of solid in water, $W_W = 4.2\text{N}$

Weight of solid in liquid Y, $W_L = 4.4\text{N}$

Problem: Relative density of liquid Y.

Upthrust of solid in liquid, $U_L = W_A - W_L = 5.0 - 4.4 = 0.6\ N$

Upthrust of solid in water, $U_W = W_A - W_W = 5.0 - 4.2 = 0.8\ N$

$$\text{R. d of liquid} = \frac{\text{upthrust of object in liquid }(U_L)}{\text{upthrust of object in water }(U_W)} = \frac{W_A - W_L}{W_A - W_W} = \frac{0.6\ N}{0.8\ N} = 0.75$$

Example 8

40m^3 of liquid is mixed with 60m^3 of another liquid Q. If the density of P and Q are 1.00kg/m^3 and 1.6kg/m^3 respectively. What is the density of the mixture? A. $0.05\ kgm^{-3}$ B. $1.25\ kgm^{-3}$ C. $1.30\ kgm^{-3}$ D. $1.36\ kgm^{-3}$ *JAMB 1990*

Solution

Given: Volume of liquid P, $V_P = 40m^3$ Volume of liquid Q, $V_Q = 60m^3$

Density of liquid P, $\rho_P = 1.0kgm^{-3}$ Density of liquid Q, $\rho_Q = 1.6\ kgm^{-3}$

Problem: Density of mixture.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \therefore \quad \text{Mass} = \text{density} \times \text{volume}$$

Mass of liquid P, $M_P = \rho_P \times V_P = 1.0 \times 40 = 40kg$

Mass of liquid Q, $M_Q = \rho_Q \times V_Q = 1.6 \times 60 = 96kg$

Total mass of mixture, $M = M_P + M_Q = 40 + 96 = 136kg$

Total volume of mixture, $V = V_P + V_Q = 40 + 60 = 100m^3$

$$\text{Density of mixture}, \rho = \frac{M}{V} = \frac{136}{100} = 1.36kgm^{-3}$$

Example 9

The mass of a stone is 15.0g when completely immersed in water and 10.0g when completely immersed in a liquid of relative density 2.0, what is the mass of the stone in air? A. 5.0g B. 12.0g C. 20.0g D. 25.0g *JAMB 1991*

Solution

Given: mass of stone in water, $M_W = 15.0\text{g} = 0.015\text{kg}$

Mass of stone in liquid, $M_L = 10.0g = 0.010kg$
Relative density of liquid, r.d $= 2.0$
Problem: mass of stone in air.

$$\text{R.d of liquid } (\rho) = \frac{\text{upthrust in liquid}}{\text{upthrust in water}} = \frac{\text{Weight in air} - \text{Weight in liquid}}{\text{Weight in air} - \text{Weight in water}}$$

From W = mg, weight of stone in water, $W_W = 0.015kg \times 10 = 0.15N$
Weight of stone in liquid, $W_L = 0.010 \times 10 = 0.1N$
Weight of stone in air $= W_A$

$$\text{R.d of liquid} = \frac{W_A - W_L}{W_A - W_W} \quad \therefore \quad 2 = \frac{W_A - 0.1}{W_A - 0.15}$$

Cross multiplying, $2(W_A - 0.15) = W_A - 0.1$

$$2W_A - 0.3 = W_A - 0.1$$
$$2W_A - W_A = 0.3 - 0.1$$
$$W_A = 0.2N$$

$$W = mg \quad \therefore \text{ mass of stone in air}, m = \frac{W_A}{g} = \frac{0.2}{10} = 0.02kg = 20g$$

Example 10
The mass of a specific gravity bottle is 15.2g when it is empty. It is 24.8g when filled with kerosene and 27.2g when filled with distilled water. Calculate the relative density of kerosene. A. 1.25 B. 1.10 C. 0.90 D. 0.80 *JAMB 1994*
Solution
Given: mass of empty bottle, $m = 15.2g$
Mass of empty bottle with kerosene, $M_K = 24.8g$
Mass of empty bottle with water, $M_W = 27.2g$
Problem: Relative density of kerosene.

$$\text{R.d} = \frac{\text{mass of liquid}}{\text{mass of equal volume of water}} = \frac{M_K - m}{M_W - m} = \frac{24.8 - 15.2}{27.2 - 15.2} = \frac{9.6}{12} = 0.8$$

Example 11
If a spherical metal bob of radius 3cm is fully immersed in a cylinder containing water and the water rises by 1cm. What is the radius of the cylinder?
A. 12 cm B. 1 cm C. 3 cm D. 6cm *JAMB 2001*
Solution
Given: Radius of bob, r = 3cm
Height by which water rises in cylinder, h = 1cm
Problem: Radius of cylinder

$$\text{Volume of metal bob} = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \times 3^3 = \frac{4\pi}{3} \times 27 = 36\pi$$

In general, volume of liquid displaced = volume of object (metal bob)
In this case, volume of liquid displaced = volume of water rise in cylinder
∴ Volume of water rise in cylinder = volume of metal bob

$$\pi r^2 h = \frac{4}{3}\pi r^3$$
$$\pi \times r^2 \times 1 = 36\pi$$
$$\pi r^2 = 36\pi$$

$$r^2 = 36 \quad \therefore \quad r = \sqrt{36} = 6cm$$

Example 12

Consider a balloon of mass 0.030kg being inflated with a gas of density 0.54 kgm^{-3}. What will be the volume of the balloon when it just begins to rise in air of density 1.29 kgm^{-3}? (g = 10ms^{-2}) WAEC 2008[E11]

Solution

Let V be the volume of the balloon

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \quad \therefore \quad \text{mass} = \text{density} \times \text{volume}$$

Mass of air in balloon = V ×density of gas

$$= 0.54V$$

Mass of balloon $= 0.030kg$

Mass of air displaced = volume of balloon × density of air

$$=1.29V$$

For the balloon to begin to rise, the mass of the air displaced must be equal to the mass of the balloon *and* the air inside it

$$\therefore \quad 1.29V = 0.54V + 0.030$$

$$1.29V - 0.54V = 0.030$$

$$0.75V = 0.030$$

$$V = \frac{0.030}{0.75} = 0.04m^3$$

EXERCISES 4.

1. A block of material of volume 20cm^{-3} and density of 2.5gcm^{-3} is suspended from a spring balance with half of the volume of the block immersed in water. What is the reading of the spring balance? *WAEC 1989 Ans: 0.4N or 40g*
2. A solid weighs 0.04N in air and 0.024N when fully immersed in a liquid of density 800 kgm^{-3}. What is the volume of the solid? (g=10m/s^2) *WAEC 1992 Ans: $2 \times 10^{-6}m^3$*
3. A block of material of volume $2 \times 10^{-5}m^3$ and density $2.5 \times 10^{-3}kg/m^3$ is suspended from a spring balance with half volume of the block immersed in water. What is the reading of the spring balance? (Density of water = 1000kg/m^3, g=10m/s^2). *WAEC 1993 Ans: 0.40N*
4. A body weighing 10N in air is partially immersed in water. It displaces water of mass 0.3kg. What is the upthrust on the body? (g=10m/s^2) *WAEC 1995 Ans: 3N*
5. An object weighs 2.7N in air and 1.2N when completely immersed in water. Calculate its relative density. *WAEC 1997 Ans: 1.80*
6. The apparent weight of a body wholly immersed in water is 32N and its weight in air is 96N. Calculate the volume of the body. (Density of water = 1000kg/m^3, g=10m/s^2) *WAEC 2000 Ans: $6.4 \times 10^{-3}m^3$*

7. A piece of brass of mass 20.0g is hung on a spring balance from a rigid support and completely immersed in kerosene of density $8.0 \times 10^2 kgm^{-3}$. Determine the reading on the spring balance ($g=10m/s^2$, density of brass = $8.0 \times 10^3\ kgm^{-3}$)

WAEC 2000Ans: 0.2N

8. An object of weight 10N immersed in a liquid displaces a quantity of the liquid. If the liquid displaced weighs 6N, determine the upthrust on the object.

WAEC 2001 Ans:6N

9. The apparent weight of a body fully immersed in water is 32N and its weight in air is 96N. Calculate the volume of the body. (Density of water = $1000kg/m^3$, $g=10m/s^2$). *WAEC 2001 Ans: $6.4 \times 10^{-3}m^3$*

10. A piece of metal of relative density 5.0 weighs 60N in air. Calculate its weight when fully immersed in water. *WAEC 2003 Ans: 48N*

11. A solid weighs 45N and 15N respectively in air and water. Determine the relative density of the solid. *WAEC 2002 Ans: 1.5*

12. A uniform cylindrical hydrometer of mass 20g and cross sectiona[illegible] $0.54cm^2$ float upright in a liquid. If 25cm of its length is submerged, calculate the r[illegible]e density of the liquid (Density of water = $1g/cm^3$) *WAEC 2000 Ans: 1.48*

13. An object weighs 60.0N in air, 48.2N in a certain liquid X, and 44.9N in water. Calculate the relative density of X. *WAEC 2004 Ans: 0.782*

14. Calculate the change in volume when 90g of ice is completely melted. (dens[illegible]y of water = $1g/cm^3$, density of ice = $0.9\ g/cm^3$) *WAEC 2004 Ans: [illegible]*

15. A block of volume $3 \times 10^{-5}m^3$ and density $2.5 \times 10^3kgm^{-3}$ is suspended [illegible] a spring balance with $^2/_3$ of its volume immersed in a liquid of density $900kgm^{-3}$. Determine the reading of the spring balance. ($g=10m/s^2$) *WAEC 2006 Ans: 0.57N*

16. A piece of metal of density $3.9 \times 10^3kgm^{-3}$ weighs 10N in air. Calculate the apparent weight of the metal when completely immersed in a liquid of dens[illegible] [illegible]1 x 10^2kgm^{-3} ($g=10m/s^2$). *NECO 2005 Ans: 8.95N*

17. A body has masses of 0.013kg and 0.012kg in oil and water respectively, if the relative density of oil is 0.875, calculate the mass of the body. *NECO 2004 Ans: 0.02kg*

18. An object weighing 8N in air is partially immersed in water. It displaces water of mass 0.3Kg. What is the upthrust on the body? ($g=10m/s^2$) *NECO 2003 Ans: 3N*

19. A body of weight 20N displaces 12N of the liquid in which it is immersed. What is the upthrust of the liquid on the body? *NECO 2002 Ans: 12N*

20. A piece of stone weighs 180N in air and 150N in a beaker containing 160N of methylated spirit. What is the upthrust exerted on the stone? *NECO 2000 Ans: 30N*

21. The density of $400cm^3$ of palm oil was $0.9g/cm^3$ before frying. If the density of the oil was $0.6gcm^{-3}$ after frying, assuming no loss of oil due to spilling, what was its new volume? A. $360cm^3$ B. $600cm^3$ C. $240cm^3$ D. $800cm^3$ E. $450cm^3$

JAMB 1979 Ans: $600cm^3$

22. An object of mass m and volume V is totally immersed in a liquid of density ρ. What will be the tension of a spring holding the object? A. mg B. $(m + \rho V)g$ C. $(m - \rho V)g$ D. ρVg E. $mg/(m - \rho V)$ *JAMB 1980 Ans: C*

23. The relative densities of zinc, brass, copper, gold and silver are respectively 7.1, 8.5, 8.9, 19.3 and 10.5. A metal ornament which weighs 0.425kg and can displace $50 \times 10^{-6}m^3$ of water is made up of which of the above listed metals? A. Zinc B. brass C. copper D. gold E. silver *JAMB 1980 Ans: Brass (8.5)*

24. A cube of side 10cm and mass 0.5kg floats in a liquid with only $^1/_5$ of its height above the liquid surface. What is the relative density of the liquid? A. 0.125 B. 0.250 C. 0.625 D. 2.500 *JAMB 1987 Ans: 0.625*

25. A copper cube weighs 0.25N in air, 0.17N when completely immersed in paraffin oil and 0.15N when completely immersed in water. What is the ratio of upthrust in oil to upthrust in water? A. 4:5 B. 3:5 C. 13:10 D. 7:10

JAMB 2002 Ans: 4:5

26. What volume of alcohol with a density of $8.4 \times 10^2 kgm^{-3}$ will have the same mass as $4.2m^3$ of petrol whose density is $7.2 \times 10^2 kgm^{-3}$? A. $1.4m^3$ B. $3.6m^3$ C. $4.9m^3$ D. $5.0m^3$ E. $5.8m^3$ *JAMB 1984 Ans: 3.6m³*

27. A test tube of radius 1.0cm is loaded to 8.8g. If it is placed upright in water, find the depth to which it would sink. ($g=10m/s^2$, density of water= $1000kgm^{-3}$). A. 2.8cm B. 5.2cm C. 25.5cm D. 28.0cm *JAMB 2003 Ans: 2.8cm*

28. The change in volume when 450kg of ice is completely melted is? (Density of ice = $900kg/m^3$, density of water = $1000kg/m^3$) A. $0.05m^3$ B. $0.45m^3$ C. $4.50m^3$ D. $0.50m^3$ *JAMB 2004 Ans: 0.05m³*

29. A body whose mass is 2kg and has a volume of $500cm^3$ just floats when completely immersed in a liquid. Calculate the density of the liquid. A. $4.0 \times 10^2 kgm^{-3}$ B. $4.0 \times 10^3 kgm^{-3}$ C. $1.0 \times 10^3 kgm^{-3}$ D. $1.0 \times 10^6 kgm^{-3}$ *JAMB 1988 Ans:* $4.0 \times 10^3 kgm^{-3}$

30. If a plastic sphere floats in water (density = $1000kgm^{-3}$) with 0.5 of its volume submerged and floats in oil with 0.4 of its volume submerged, what is the density of the oil? A. $800\ kgm^{-3}$ B. $1200\ kgm^{-3}$ C. $1250\ kgm^{-3}$ D. $2000kgm^{-3}$ *JAMB 1992 Ans:* $1250\ kgm^{-3}$

31. An object of mass 400g and density of $600kg/m^3$ is suspended with a spring so that half of it is immersed in a paraffin of density $900kg/m^3$. What is the tension in the spring? A. 1.0N B. 3.0N C. 4.0N D. 5.0N *JAMB 1993 Ans: 1N*

32. If it takes 5.0hrs to drain a container of $540.0m^3$ of water, what is the flow rate of water from the container in kg/s. (Density of water = $1000kgm^{-3}$). A. 32.5 B. 31.5 C. 30.8 D. 30.0 *JAMB 1994 Ans: 30kg/s*

33. If a solid X floats in liquid P of relative density 2.0 and in liquid Q of relative density 1.5, it can be inferred that the

A. Weight of P displaced is greater than that of Q
B. Weight of P displaced is less than that of Q
C. Volume of P displaced is greater than that of Q
D. Volume of P displaced is less than that of Q

JAMB 1994 Ans: D

34. A cube of sides 0.1m hangs freely from a spring. What is the upthrust on the cube when totally immersed in water? (Density of water is $1000kgm^{-3}$, $g=10m/s^2$). A. 1000N B. 700N C. 110N D. 10N *JAMB 1997 Ans: 10N*

35. A solid of weight 0.60N is totally immersed in oil and water respectively. If the upthrust in oil is 0.210N and the relative density of oil is 0.875, find the upthrust in water. A. 0.600N B. 0.360N C. 0.240N D. 0.180N *JAMB 1998 Ans: 0.24*

36. A solid weighs 10.0N in air, 6.0N when fully immersed in water and 7.0N when fully immersed in certain liquid X. Calculate the relative density of the liquid. A. $5/3$ B. $4/3$ C. $3/4$ D. $7/10$ *JAMB 1999 Ans: 0.75* or $3/4$

37. Two liquids L_1 and L_2 are contained in a U-tube. The height and the density of L_1 are 8cm and $10^3 kgm^{-3}$ respectively. If the density of L_2 is $800kg/m^3$, what is its height measured from the same level? A. 8cm B. 16cm C. 12cm D. 10cm *JAMB 2005 Ans: 10cm.*

38. A $3m^3$ volume of liquid W of density $200kgm^{-3}$ is mixed with another liquid L of volume $7m^3$ and density $150kgm^{-3}$. What is the density of the mixture? A. $265\ kgm^{-3}$ B. $165\ kgm^{-3}$ C. $100\ kgm^{-3}$ D. $350\ kgm^{-3}$ *JAMB 2005 Ans: 165kgm⁻³*

39. If the relative density of gold is 19.2, what is the volume of 2.4kg of gold? (Density of water = $10^3 kgm^{-3}$). A. $1.92 \times 10^{-4} m^3$ B. $4.61 \times 10^{-3} m^3$ C. $8.00 \times 10^{-3} m^3$ D. $1.25 \times 10^{-4} m^3$ *JAMB 2006 Ans:* $1.25 \times 10^{-4} m^3$

40. A density bottle has a mass of 50.0g when empty. When it is filled with water, its mass is 100.0g. Calculate the mass of the bottle when it is filled with kerosene. (Relative density of kerosene = 0.8) *NECO 2007 Ans: [illegible]*

41. A liquid of volume $2.00m^3$ and density $1.00 \times 10^3 kgm^{-3}$ is mixed with 3.0m³ of another liquid of density $8.00 \times 10^3 kgm^{-3}$. Calculate the density of the mixture. [Assume there is no chemical reaction] *WAEC 2007 Ans: 5.20× [illegible]*

42. A piece of iron weighs 250N in air and 200N in a liquid of density $1000 kgm^{-3}$. The volume of the iron is A. $2.0 \times 10^{-3} m^3$ B. $5.0 \times 10^{-3} m^3$ C. $4.5 \times 10^{-3} m^3$ D. $2.5 \times 10^{-3} m^3$ [$g = 10 m/s^2$] JAMB 2007 Ans: $5.0 \times 10^{-3} m^3$

43. An empty density bottle weighs 2N. If it weighs 5N when filled with water and 4N when filled with olive oil, the relative density of olive oil is
A. 1/3 B. 2/3 C. 1/5 D. 2/5 *JAMB* 2008[18] *Ans*: 2/3

44. A piece of stone weighs 0.8N in air, 0.60N when completely immersed in water and 0.66N when completely immersed in a liquid X. Calculate the relative density of X.
NECO 2008[10] *Ans* : 0.70

45. A plastic sphere floats in water with 50% of its volume submerged. If it floats in glycerine with 40 % of its volume submerged, the density of the glycerine is
A. $1400 kgm^{-3}$ B. $1250 kgm^{-3}$ C. $500 kgm^{-3}$ D. $1000 kgm^{-3}$
(Density of water = $1000 kgm^{-3}$) JAMB 2009[17] Ans: $800 kgm^{-3}$

46. A body of volume $0.046 m^3$ is immersed in a liquid of density $980 kgm^{-3}$ with ¾ of its volume submerged. Calculate the upthrust on the body. ($g = 10 ms^{-2}$)
WAEC 2009[4] Ans: 338.10N

5

CIRCULAR AND SIMPLE HARMONIC MOTIONS

CIRCULAR MOTION

Circular motion is defined as the motion of an object round a circle or circular path. An object is said to undergo uniform circular motion when it is moving along a circular path at constant speed. Circular motion has three basic characteristics.

1. constant speed
2. changing or variable velocity
3. centripetal acceleration.

Centripetal acceleration is experienced by any object undergoing circular motion and is always directed towards the centre of the circular path.

$$\text{Centripetal acceleration,} \quad a = \frac{V^2}{r}$$

where V = speed or velocity
r = radius of circular path

Centripetal force is defined as the force that keeps an object moving along a circular path at constant speed.

Remember, force = mass(m) × acceleration(a).

Therefore, centripetal force = mass × centripetal acceleration

That is, Centripetal force, $F = \dfrac{mV^2}{r}$

Example 1

An object of weight 150N moves with a speed of 4.5m/s in a circular path of radius 3m. Calculate its centripetal acceleration and the magnitude of the centripetal force. [Take g as $10m/s^2$]

Solution

Weight of object, W = 150N; Speed or velocity, V = $4.5ms^{-1}$; radius, r = 3m

$$F = W = mg \quad \therefore \quad \text{mass of object, m} = \frac{W}{g} = \frac{150N}{10} = 15kg$$

$$\text{Centripetal acceleration, a} = \frac{V^2}{r} = \frac{4.5^2}{3} = 6.75ms^{-2}$$

$$\text{Centripetal force, F} = \frac{mV^2}{r} = \frac{15 \times 4.5^2}{3} = 101.25N$$

Example 2

If an object of mass 4kg goes round a circle of radius 0.5m in 3.142s, what is the force towards the center? [$\pi = 3.142$]

Solution

The force towards the centre is the centripetal force, $F = \dfrac{mV^2}{r}$

The distance round a circle is the circumference of a circle, 2π
Time taken to go round circle = 3.142s

$$\therefore \quad \text{Velocity of object} = \frac{\text{distance}}{\text{time}} = V = \frac{2\pi}{3.142}$$

$$= \frac{2 \times 3.142}{3.142} = 2ms^{-1}$$

$$\therefore \quad F = \frac{mV^2}{r} = \frac{4 \times 2^2}{0.5} = 32\text{N}$$

Example 3
A stone of mass 500g tied to a rope 50cm long is whirled at an angular velocity of 12.0 rads^{-1}. Calculate the centripetal force.
Solution
Mass, m = 500g = 0.5kg; radius, r = 50cm = 0.5m
Angular velocity (NOT linear velocity), ω = 12 rads^{-1}

Linear velocity (V) is related to angular velocity i.e. $V = \omega r$

Substitute $V = \omega r$ into centripetal force, $F = \frac{mV^2}{r}$

$$F = \frac{m(\omega r)^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r$$

$$\therefore \quad F = m\omega^2 r = 0.5 \times 12.0^2 \times 0.5 = 36\text{N}$$

Example 4
A body moves along a circular path with uniform angular speed of 0.6 rads^{-1} and at a constant speed of 3.0ms^{-1}. Calculate the acceleration of the body towards the centre of the circle. *WAEC 1993*
Solution
Angular speed or velocity, ω = 0.6rads^{-1}; Linear speed, V = $3.0ms^{-1}$
Acceleration towards the centre of the circle is same as centripetal acceleration, a $= \frac{V^2}{r}$

From $V = \omega r$, the radius (r) of the circular path can be found

$$r = \frac{V}{\omega} = \frac{3}{0.6} = 5m$$

$$\therefore \quad \text{Centripetal acceleration, a} = \frac{V^2}{r} = \frac{3^2}{5} = 1.8ms^{-2}$$

Example 5
A force F is required to keep a 5kg mass moving round a cycle of radius 3.5km at a speed of $7ms^{-1}$. What is the speed, if the force is tripled? A. $4.0ms^{-1}$ B. $6.6ms^{-1}$ C. $12.1ms^{-1}$ D. $21.0ms^{-1}$ JAMB 2008[5]
Solution
Mass , m = 5kg; speed, v = $7ms^{-1}$; radius, r = 3.5m

The force in action is centripetal force

$$\therefore \quad F = \frac{mV^2}{r} = \frac{5 \times 7^2}{3.5} = \frac{5 \times 49}{3.5} = 70N$$

"*...if the force is tripled...*", the centripetal force F becomes 3F or 3 × 70 = 210N

Substitute F = 210N; m = 5kg; r = 3.5m

$$F = \frac{mV^2}{r}$$

$$210 = \frac{5 \times V^2}{3.5}$$

$$\therefore \quad V^2 = \frac{210 \times 3.5}{5} = 147$$

$$V = \sqrt{147} = 12.1ms^{-1}$$

Example 6
If a wheel 1.2m in diameter rotates at one revolution per second, calculate the velocity of the wheel. A. $3.6ms^{-1}$ B. $3.8ms^{-1}$ C. $4.0ms^{-1}$ D. $7.5ms^{-1}$
JAMB 2008[6]

Solution

$$\text{radius} = \frac{\text{diameter}}{2} = \frac{1.2m}{2} = 0.6m$$

convert angular velocity (*"...at one revolution per second..."*) to radian per second as follows:

1 revolution, $\boldsymbol{\theta = 360° = 2\pi}$ **radians**; time for 1 rev, $\mathbf{t = 1s}$

$$\text{Angular velocity, } \omega = \frac{\theta}{t} = \frac{2\pi}{1} = 2 \times 3.14 = 6.28 \text{ rads}^{-1}$$

$$\textbf{Linear velocity, } \boldsymbol{V = r\omega = 0.6 \times 6.28 = 3.8ms^{-1}}$$

SIMPLE HARMONIC MOTION

A body is said to undergo simple harmonic motion if:

(i) its acceleration is always directed toward a fixed point.

(ii) Its acceleration is directly proportional to its distance from the point.

Amplitude of a simple harmonic motion is maximum displacement of a body from its central or equilibrium position.

Period of a simple harmonic motion is the time taken to complete one cycle, oscillation or vibration.

$$\text{Period,} \quad T = \frac{\text{time taken}}{\text{number of oscillations}} = \frac{t}{n}$$

Frequency of a simple harmonic motion is the number of complete oscillation per second.

$$\text{Frequency,} \quad f = \frac{\text{number of oscillations}}{\text{time taken}} = \frac{n}{t}$$

Frequency is measured in cycle per second (s^{-1}) or Hertz.

The following equations can be applied in problems involving simple harmonic motion.

1. $\omega = \dfrac{\theta}{t}$
2. $V = \omega r \quad or \quad V = \omega A$
3. $\omega = 2\pi f$
4. $T = \dfrac{2\pi}{\omega}$
5. $a = \omega^2 r \quad or \quad a = \omega^2 A$
6. $a = \alpha r$

7. $V = \omega\sqrt{A^2 - x^2}$

8. $f = \dfrac{\omega}{2\pi}$

9. $360° = 2\pi\, rad$

10. $T = \dfrac{1}{f}$

11. $f = \dfrac{1}{T}$

Where ω = angular velocity ($rads^{-1}$)
θ = angle turned by body (rad)
A, r = amplitude or radius (m)
f = frequency (s^{-1}) or Hertz (Hz)
α = angular acceleration ($rads^{-2}$)
x = displacement of object from centre of motion
t = time in seconds (s)
V = linear velocity (m/s)
a = linear acceleration m/s^2
T = period (s)

The following examples will illustrate how any, or a combination of these equations can be applied to solving problems.

Example 7
An object of mass 0.40kg attached to the end of a string is whirled round in a horizontal circle of radius 2.0m with a constant speed of $8ms^{-1}$. Calculate the angular velocity of the object. *WAEC 1997*

Solution
Radius, r = 2.0m; linear speed, V = $8ms^{-1}$; mass, m = 0.40kg

Linear velocity (V) is related to angular velocity (ω) by, $V = \omega r$

$$\therefore \quad \omega = \frac{V}{r} = \frac{8}{2} = 4\ rads^{-1}$$

Example 8
The period of oscillation of a particle executing simple harmonic motion is 4π seconds. If the amplitude of oscillation is 3.0m, calculate the maximum speed of the particle. *WAEC 2000*

Solution
Period, T = 4π sec; amplitude, r = 3.0m; speed, V = ?

$$T = \frac{2\pi}{\omega} \quad \therefore \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5s^{-1}$$

Substitute into $V = \omega r$, to obtain
$V = 0.5 \times 3 = 1.5ms^{-1}$

Example 9
A body moving with simple harmonic motion in a straight line has velocity, V and acceleration, a, when the instantaneous displacement, x in cm, from its maximum position is given by $x = 2.5 \sin 0.4\pi t$, where t is in seconds. Determine the magnitude of the maximum; (i) velocity (ii) acceleration *WAEC 2005*

Solution
The equation $x = A\sin\omega t$ represents the motion of an object undergoing simple harmonic motion and can be compared with the given equation as follows.
$x = A\sin\omega t$
$x = 2.5\sin 0.4\pi t$
Therefore, the amplitude or radius, r = 2.5cm = 0.025m

Angular speed, $\omega = 0.4\pi\ rads^{-1}$

(i) velocity, $V = \omega r = 0.4\pi \times 0.025 = 3.14 \times 10^{-2} ms^{-1}$

(ii) acceleration, $a = \omega^2 r = (0.4\pi)^2 \times 0.025 = 3.95 \times 10^{-2} ms^{-2}$

Example 10

A body is rotating in a horizontal circle of radius 2.5m with an angular speed of 5 $rads^{-1}$. Calculate the magnitude of the radial acceleration of the body. *WAEC 2006*

Solution

$$\text{acceleration, } a = \omega^2 r = (5)^2 \times 2.5 = 62.5 ms^{-2}$$

Example 11

A particle moves in a circular orbit of radius 0.02m. If the speed of the particle is 0.88ms^{-1}, calculate its frequency in cycles per second. *JAMB 1984*

A. 2.0 B. 7.0 C. 8.8 D. 14.0 E. 17.6

Solution

Radius, r = 0.02m; linear speed, $V = 0.88 ms^{-1}$

$$V = \omega r \quad \therefore \quad \text{angular speed, } \omega = \frac{V}{r} = \frac{0.88}{0.02} = 44 rads^{-1}$$

$$\omega = 2\pi f, \quad \therefore \quad \text{frequency, } f = \frac{\omega}{2\pi} = \frac{44}{2\pi} = 7.00 s^{-1}$$

SIMPLE PENDULUM

The motion of a simple pendulum is a typical example of a simple harmonic motion. All equations given above can be applied.

The period of a simple pendulum is given as $T = 2\pi\sqrt{\frac{l}{g}}$ 1

Where T = period of the pendulum in seconds (s)

l = length of pendulum in meter (m)

g = acceleration due to gravity in ms^{-2}

The period T_1 and length l_1 of a particular pendulum can be compared with the period T_2 and length l_2 of the same or different pendulum in the same location. The length of a pendulum is its main distinguishing factor from other pendulums.

Square both sides of $T = 2\pi\sqrt{\frac{l}{g}}$ to obtain

$$T^2 = \frac{4\pi^2 l}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

For the 1st pendulum, equation (2) becomes

$$T_1^2 = \frac{4\pi^2 l_1}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Rearranging, $\frac{4\pi^2}{g} = \frac{T_1^2}{l_1}$ 4

$\frac{4\pi^2}{g}$ is a fixed constant for all pendulum of whatever length in a particular location.

For the 2[nd] pendulum, equation (2) becomes

$$T_2^2 = \frac{4\pi^2 l_2}{g}$$

Rearranging, $\frac{4\pi^2}{g} = \frac{T_2^2}{l_2}$ 5

Equate equation (3) and equation (4) to obtain

$$\frac{T_1^2}{l_1} = \frac{T_2^2}{l_2}$$

Rearranging, $\frac{T_1^2}{T_2^2} = \frac{l_1}{l_2}$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$$

$$\frac{T_1}{T_2} = \frac{\sqrt{l_1}}{\sqrt{l_2}}$$

Example 12

What is the period and frequency of a simple pendulum that makes 40 oscillations in 35 seconds.

Solution

Period, $T = \frac{\text{time of oscillation}}{\text{number of oscillations}} = \frac{t}{n} = \frac{35}{40} = 0.875s$

Frequency, $f = \frac{\text{number of oscillations}}{\text{time of oscillation}} = \frac{n}{t} = \frac{40}{35} = 1.14Hz$

or $f = \frac{1}{T} = \frac{1}{0.875} = 1.14Hz$

Example 13

A boy timed 30 oscillations of a certain pendulum thrice and obtained 1min 10s, 1min. 12s and 1 min. 7s respectively. The mean period of oscillation of the pendulum is

A. 0.14s B. 0.43s C. 2.32s D. 6.97s *JAMB 1991*

Solution

Convert all times into seconds and substitute into $T = \frac{t}{n}$

$$T_1 = \frac{1\ min. 10s}{30} = \frac{70s}{30} = 2.33s$$

$$T_2 = \frac{1\ min. 12s}{30} = \frac{72s}{30} = 2.4s$$

$$T_3 = \frac{1\ min. 7s}{30} = \frac{67s}{30} = 2.23s$$

$$\text{Mean period} = \frac{T_1 + T_2 + T_3}{3} = \frac{2.33 + 2.40 + 2.23}{3} = \frac{6.96}{3} = 2.32s$$

Alternatively, $$\text{mean time}, t = \frac{t_1 + t_2 + t_3}{3} = \frac{70 + 72 + 67}{3} = \frac{209}{3} = 69.6s$$

$$\text{Mean period} = \frac{\text{mean time}}{\text{number of oscillations}} = \frac{69.6}{30} = 2.32s$$

Example 14
The bob of a simple pendulum takes 0.25s to swing from its equilibrium position to one extreme end. Calculate its period. *WAEC 2000*
Solution

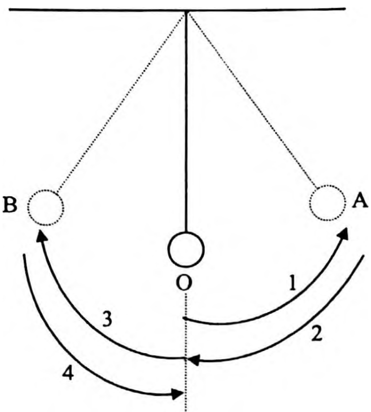


Fig 5.1

If it takes 0.25s for the bob to swing from O to A, it will then take 4 × 0.25s to undergo one oscillation, i.e. 1-2-3-4 or O-A-O-B-O.

$$\therefore \text{ Period, } \quad T = \frac{t}{n} = \frac{4 \times 0.25s}{1} = \frac{1s}{1} = 1.00s$$

Example 15
A simple pendulum 0.64m long has a period of 1.2s. Calculate the period of a similar pendulum 0.36m long in the same location. *NECO 2005*
Solution
Substitute both instances into

$$T^2 = \frac{4\pi^2 l}{g}$$

<u>1st instance</u>

$$1.2^2 = \frac{4\pi^2 \times 0.64}{g}$$

<u>2nd instance</u>

$$T^2 = \frac{4\pi^2 \times 0.36}{g}$$

Make g the subject in each case and then equate both g's

$$g = \frac{4\pi^2 \times 0.64}{1.2^2} \qquad g = \frac{4\pi^2 \times 0.36}{T^2}$$

$$\frac{4\pi^2 \times 0.64}{1.2^2} = \frac{4\pi^2 \times 0.36}{T^2}$$

Cross multiply and make T^2 subject

$$4\pi^2 \times 0.64 \times T^2 = 1.2^2 \times 4\pi^2 \times 0.36$$

$$T^2 = \frac{1.2^2 \times 4\pi^2 \times 0.36}{4\pi^2 \times 0.64}$$

$$T^2 = \frac{1.2^2 \times 0.36}{0.64} = 0.81$$

$$T = \sqrt{0.81} = 0.9s$$

The above method is long and more difficult, so you are advised to use the following quicker and easier method for this and other similar questions.
Period of first pendulum, $T_1 = 1.2s$
Length of first pendulum, $l_1 = 0.64$
Period of second pendulum, $T_2 = ?$
Length of second pendulum, $l_2 = 0.36$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} \quad \therefore \quad T_2 = T_1\sqrt{\frac{l_2}{l_1}}$$

$$T_2 = 1.2 \times \sqrt{\frac{0.36}{0.64}} = 1.2 \times \sqrt{\frac{36}{64}} = 1.2 \times \frac{\sqrt{36}}{\sqrt{64}} = 1.2 \times \frac{6}{8} = \frac{7.2}{8} = 0.9s$$

Example 16
A simple pendulum with a period of 2.0s has its length doubled. Its new period is,
A. 1.00s B. 1.41s C. 0.35s D. 2.83s E. 4.00s *JAMB 1985*
Solution
Period of first pendulum, $T_1 = 2.0s$
Length of first pendulum, $l_1 = l$
Period of second pendulum, $T_2 = ?$
Length of second pendulum, $l_2 = 2l$ (.... *length doubled*...)

$$\frac{T_1^2}{T_2^2} = \frac{l_1}{l_2} \quad \therefore \quad T_2^2 = \frac{T_1^2 \times l_2}{l_1}$$

$$T_2^2 = \frac{2^2 \times 2l}{l} = \frac{4 \times 2l}{l} = \frac{8l}{l} = 8$$

$$\therefore \; \boldsymbol{T = \sqrt{8} = 2.83s}$$

Example 17
A simple pendulum has a period of 4.50s when the length of the pendulum is shortened by 1.2m the period is 2.50s. Calculate the original length of the pendulum.
Solution
Period of first pendulum, $T_1 = 4.5s$
Length of first pendulum, $l_1 = l$
Period of second (shortened) pendulum, $T_2 = 2.50s$
Length of second (shortened) pendulum, $l_2 = l - 1.2$

Substitute into $\frac{T_1^2}{T_2^2} = \frac{l_1}{l_2}$ to obtain

$$\frac{4.5^2}{2.50^2} = \frac{l}{l - 1.2}$$

$$\frac{20.25}{6.25} = \frac{l}{l - 1.2}$$

$$20.25l - 24.3 = 6.25l$$

$$20.25l - 6.25l = 24.3$$

$$14l = 24.3$$

$$l = \frac{24.3}{14} = 1.74m$$

ENERGY OF SIMPLE HARMONIC MOTION

The energy of a simple harmonic motion is derived when the motion of a loaded spiral spring is considered. The following equations, in addition to those given before, can be applied to solving problems involving loaded spiral spring and energy of simple harmonic motion.

1. $T = 2\pi\sqrt{\frac{m}{K}}$ or $T^2 = \frac{4\pi^2 m}{K}$
2. $F = Ke$ or $mg = Ke$
3. $T = 2\pi\sqrt{\frac{e}{g}}$ or $T^2 = \frac{4\pi^2 e}{g}$
4. $\omega = \sqrt{\frac{K}{m}}$
5. $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$ or $f^2 = \frac{K}{4\pi^2 m}$
6. $W = \frac{1}{2}KA^2 = \frac{1}{2}m\omega^2 A^2$

Where T = period in seconds (s)
m = mass of body in kilogram (kg)
K = force constant of spring in Nm^{-1}
e = extension of spring in meter (m)
g = acceleration due to gravity (ms^{-2})
F = force, load or weight in Newton (N)
ω = angular speed or velocity (m/s)
f = frequency of motion in s^{-1} or Hz
W = total work done by spring or energy stored in spring (J)
A = amplitude of motion (m)

Example 18

A body of mass 500g suspended from the end of a spiral spring which obeys Hooke's law, produced an extension of 10cm. If the mass is pulled down a distance of 5cm and released, calculate;

(i) The force constant of the spring
(ii) The frequency of oscillation
(iii) The period of oscillation
(iv) The angular speed of the body

Solution

Mass, m = 500g = 0.5kg, extension, e = 10cm = 0.1m

(i) From Hooke's law, F = Ke,

$$\text{force constant}, K = \frac{F}{e} = \frac{mg}{e} = \frac{0.5 \times 10}{0.1} = 50Nm^{-1}$$

(ii) Frequency,
$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{50}{0.5}} = \frac{1}{2\pi}\sqrt{100} = \frac{1}{2\pi} \times 10 = 1.59s^{-1}$$

(iii)
$$\text{Period}, T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{0.5}{50}} = 2\pi\sqrt{0.01} = 2\pi \times 0.1 = 0.628s$$

$$\text{or} \quad T = \frac{1}{f} = \frac{1}{1.59} = 0.628s$$

(iv)
$$\text{Angular speed}, \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{50}{0.5}} = \sqrt{100} = 10\, rads^{-1}$$

EXERCISES 5.

1. A stone tied to a string is made to revolve in a horizontal circle of radius 4m with an angular speed of 2 radians per second. With what tangential velocity will the stone move off the circle if the strings cuts. *WAEC 1989 Ans: 8m/s*

2. A body weighing 100N moves with a speed of $5ms^{-1}$ in a horizontal circular path of radius 5m. Calculate the magnitude of the centripetal force acting on the body. [$g=10ms^{-2}$] *WAEC 1999 Ans: 50N*

3. An object of mass 5kg moves round a circle of radius 6m. If the period of the motion is π s, calculate the force towards the centre. *NECO 2004 Ans: 120N*

4. A particle of mass 10^{-2}kg is fixed to the tip of a fan blade which rotates with angular velocity of 100 $rads^{-1}$. If the radius of the blade is 0.2m, the centripetal force is:
A. 2N B. 20N C. 200N D. 400N *JAMB 1999 Ans: B. 20N*

5. A body moves along a circular path with uniform angular speed of 0.6 $rads^{-1}$ and at a constant speed of $3.0ms^{-1}$. Calculate the acceleration of the body towards the centre of the circle. *NECO 2005 Ans: $1.8m/s^2$*

6. A body executing a simple harmonic motion has an angular velocity of 40 radians per second. If it has a maximum displacement of 6cm, calculate its linear speed. *NECO 2008[12] Ans: $2.40ms^{-1}$*

7. The angular speed of an object describing a circle of radius 4m with a linear constant speed of $10ms^{-1}$ is? *WAEC 1995 Ans: $2.50rads^{-1}$*

8. A body executing a simple harmonic motion has an angular velocity of 22radians per second. If it has a maximum displacement of 10cm, what is its linear velocity?
NECO 2002 Ans: 2.2ms^{-1}

9. A particle in circular motion performs 30 oscillations in 6 seconds. It's angular velocity is? A. 10π rads^{-1} B. 5πrads^{-1} C. 6rads^{-1} D. 5rads^{-1}
JAMB 2002 Ans: 10π rads^{-1}

10. The amplitude of a particle executing simple harmonic motion is 5cm while its angular frequency is 10rads^{-1}. Calculate the magnitude of the maximum acceleration of the particle. *WAEC 2001 Ans: 5m/s^2*

11. A loaded spring performs simple harmonic motion with an amplitude of 5cm. If the maximum acceleration of the load is 20cms^{-2}, calculate the angular frequency of the motion. *WAEC 2004 Ans: 0.32s^{-1} Hint: $a = \omega^2 A$ and $f = {}^{\omega}/_{2\pi}$*

12. A simple pendulum makes 50 oscillations in one minute. What is its period of oscillation? *WAEC 1990 Ans: 1.2s*

13. A boy timed 20 oscillations of a certain pendulum three times and obtained 44.3s, 45.5s and 45.7s respectively. Calculate the mean period of oscillation of the pendulum.
WAEC 1993 Ans: 2.26s

14. A student found out from a simple pendulum experiment that 20 oscillations were completed in 38 seconds. What is the period of oscillation of the pendulum?
WAEC 1995 Ans: 1.9s

15. Two simple pendula *x* and y makes 400 and 500 oscillations respectively in equal time. If the period of oscillation of *x* is 1.5 seconds, what is the period of oscillation of y?
WAEC 1996 Ans: 1.2s Hint: $T_1 n_1 = T_2 n_2$

16. A pendulum bob executing simple harmonic motion has 2cm and 12Hz as amplitude and frequency respectively. Calculate the period of the motion.
WAEC 1999 Ans: 0.083s

17. The length of a simple pendulum is increased by a factor of four. By what factor is its period increased? *NECO 2005 Ans: 2*

18. The time t_1, t_2 and t_3 for 20 complete oscillations of a simple pendulum experiment are 32.0s, 34.6s and 35.5s respectively. Calculate the mean period of the pendulum. *NECO 2004 Ans: 1.70s*

19. Calculate the length of a simple pendulum that oscillates with a frequency of 0.4Hz [g=10ms^{-2}, π=3.142] *NECO 2004 Ans: 1.44m*

20. A simple pendulum with a period of 2.0s has its length doubled. What is its new period? *NECO 2003 Ans: 2.83s*

21. In a simple pendulum experiment, a boy observed that the times for 30 oscillations are 70.0s, 72.0s and 67.0s respectively. Calculate the mean period of oscillation of the pendulum. *NECO 2000 Ans: 2.32s*

22. If in a simple pendulum experiment the length of the inextensible string is increased by a factor of four, its period is increased by a factor of
A. 4 B. $^{\pi}/_2$ C. ¼ D. 2π E. 2 *JAMB 1979* Ans: 2

23. A simple pendulum, 0.6m long, has a period of 1.5s. What is the period of a similar pendulum 0.4m long in the same location?
A. $1.5\sqrt{\frac{2}{3}}s$ B. $1.5\sqrt{\frac{3}{2}}s$ C. 2.25s D. 1.00s E. 2.00s *JAMB 1984 Ans: A*

24. The length of a displaced pendulum bob which passes its lowest point twice every second is A. 0.25m B. 0.45m C. 0.58m D. 1.00m [g=10ms^{-1})
JAMB 1995 Ans: 0.25m

25. A simple pendulum has a period of 17.0s. When the length is shortened by 1.5m, its period is 8.5s. Calculate the origin of length of the pendulum.
A. 1.5m B. 2.0m C. 3.0m D. 4.0m *JAMB 2000 Ans: 2.0m*

26. An oscillating pendulum has a velocity of 2ms^{-1} at the equilibrium position O and velocity at same point P. Using the diagram below in fig 5.2, calculate the height h of P above O. [Take g=10ms^{-2}].

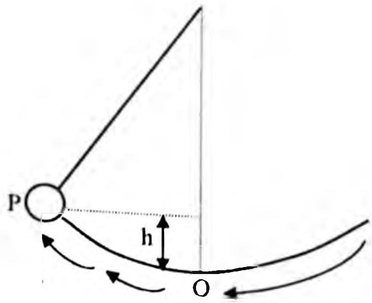


Fig 5.2

27. *WAEC 1990 Ans: 0.2m* Hint: $\frac{1}{2}mv^2 = mgh$

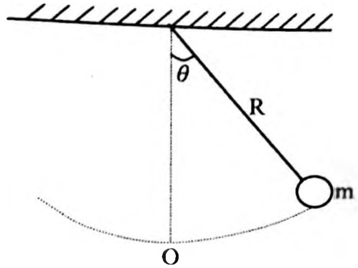


Fig 5.3

A simple pendulum of mass m moves along an arc of a circle radius R in a vertical plane as shown in the figure above. What is the work done by gravity in a downward swing through the angle θ to O?

A. $mgR\sin\theta$ B. $mgR(1-\cos\theta)$ C. mgR D. $mgR(1-\sin\theta)$ *JAMB 1989 Ans: B*

28. A mass m attached to a light spiral spring is caused to perform simple harmonic motion of frequency, $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$, where K is the force constant of the spring. If m = 0.30kg, K = 30Nm^{-1} and the maximum displacement of the mass from the equilibrium position is 0.015m, calculate the maximum

(i) kinetic energy of the system;

(ii) tension in the spring during the motion [g = 10ms^{-2}, π = 3.142]

WAEC 2005 Ans: (i) $3.38\times10^{-3}J$ (ii) 3.45N

29. When a mass is hung on a spring, the spring stretches 6cm. Determine its period of vibration if it is then pulled down a little and released. [Take π = $^{22}/_7$ and g = 10m/s^2]

NECO 2003 Ans: 0.49s. *Hint.* $T = 2\pi\sqrt{\frac{e}{g}}$

30. A car of mass 1500kg goes round a circular curve of radius 50m at a speed of 40ms^{-1}. The magnitude of the centripetal force on the car is

A. 1.2×10^3N B. 4.8×10^4N C. 1.2×10^2N D. 4.8×10^3N *JAMB 2007 Ans:* 4.8×10^4N

31. A gramophone record takes 5s to reach its constant angular velocity of 4π$rads^{-1}$ from rest. Find its constant angular acceleration.

A. 0.8π $rads^{-2}$ B. 20.0π $rads^{-2}$ C. 0.4π $rads^{-2}$ D. 1.3π $rads^{-2}$

JAMB 2007 Ans: 20.0π $rads^{-2}$

32. The frequency of vibration of an oscillator is 4Hz. If it makes 12 vibrations in t seconds, calculate the value of t. *NECO 2007'12 Ans: 3.00s*

33. A mass attached to a string is moving in a circular path. If the speed is doubled, the tension in the string will be

A. doubled B. halved C. four time as great

D. one-fourth as much *WAEC 2008'12 Ans: C*

34. What is the frequency of vibration if the balance wheel of a wrist watch makes 90 revolutions in 25s? A. 0.01Hz B. 0.04Hz
C. 2.27Hz D. 3.60Hz *JAMB 2008[10] Ans: 3.60Hz*

35. A block of mass 4.0kg causes a spiral spring to extend by 0.16m from its unstretched position. The block is removed and another body of mass 0.50kg is hung from the same spiral spring. If the spiral spring is then stretched and released, what is the angular frequency of the subsequent motion? ($g = 10ms^{-2}$)

WAEC 2008[14] Ans: $10\sqrt{5}\, rads^{-1}$

36. The period of a simple pendulum of length 80.0cm was found to have doubled when the length was increased by X. Calculate X. *WAEC 2009[12] Ans: 240.0cm*

37. A body executing simple harmonic motion has an angular speed of 2π radians. What is the period of oscillation? ($\pi = 3.14$) *NECO 2009[12] Ans: 1.0s*

38. Calculate the magnitude of the centripetal force on a particle of mass $5.0 \times 10^{-6}kg$ revolving round the earth with radial acceleration of $6.0 \times 10^{7}ms^{-2}$.

NECO 2009[E1] Ans:300N

6

LINEAR MOMENTUM

NEWTON'S LAWS OF MOTION

Newton's first law of motion states that, every body continue in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Newton's second law of motion states that, the rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's third law of motion states that whenever a force acts on one body, an equal and opposite force acts on some other body. In other words, to every action there is an equal and opposite reaction.

FORCE, IMPULSE AND MOMENTUM

Momentum of a body is defined as the product of its mass and its velocity. The S.I unit is $kgms^{-1}$ or Ns.

Momentum = *mass (m)* × *velocity (v)*

From Newton's 2nd law, we can derive equation for force and impulse.

a) Force. If an object of mass m, with initial velocity u, is acted upon by a force, F, it will attain a final velocity, v in time t.

Therefore, initial momentum of object = mu

Final momentum of object = mv

Change in momentum = final momentum − initial momentum

$$= mv - mu$$

$$\text{Rate of change in momentum} = \frac{mv - mu}{t}$$

By Newton's 2nd law, the rate of change of momentum is proportional to the applied force,

That is, $$F \propto \frac{mv - mu}{t} \quad = \quad F \propto \frac{m(v-u)}{t}$$

Remember, $$\text{acceleration}, a = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t}$$

Therefore, $F \propto ma \quad \therefore \quad F = ma$

Force (N) = mass (kg) × acceleration (ms^{-2}). $kgms^{-2}$ is also a unit of force.

b) Impulse: Impulse is defined as the product of a force and the time during which it acts on a body. It is also equal to the change in momentum produced by the force. From Newton's 2nd law,

$$\text{Force} = \frac{\text{change in momentum}}{\text{time}}$$

$$F = \frac{mv - mu}{t} \quad \therefore \quad Ft = mv - mu$$

Also, $\quad \text{Force} = \dfrac{\text{mass} \times \text{change in velocity}}{\text{time}}$

$$F = \frac{m(v - u)}{t} \quad \therefore \quad Ft = m(v - u)$$

$\therefore \quad \text{Impulse} = Ft = mv - mu = m(v - u)$

Impulse = force × time = mass × acceleration × time

The unit for impulse is Ns or $kgms^{-1}$

Example 1
A net force of magnitude 0.6N acts on a body of mass 40g, initially at rest. Calculate the magnitude of the resulting acceleration. *WAEC 1999*
Solution
Net force, F = 0.6N; mass of body, m = 40g = 0.4kg

Force (F) = mass (m) × acceleration (a)

$$\therefore \quad a = \frac{F}{m} = \frac{0.6}{0.04} = 15ms^{-2}$$

Example 2
A truck of mass 3200kg moving with a velocity of $10ms^{-1}$ increases its velocity to $20ms^{-1}$ in 5s. Calculate the magnitude of the force exerted on the engine. NECO 2008[14]
Solution
Initial velocity, $\boldsymbol{u = 10ms^{-1}}$; final velocity, $\boldsymbol{v = 20ms^{-1}}$; time taken, $\boldsymbol{t = 5s}$
Mass, $\boldsymbol{m = 3200kg}$

From Newton's 2nd law, $\quad \boldsymbol{F = \dfrac{m(v - u)}{t}}$

$$\therefore \quad F = \frac{3200(20 - 10)}{5} = \frac{3200 \times 10}{5} = 6400N$$

Example 3
A body of mass 20kg is set in motion by two forces 3N and 4N acting at right angles to each other. Determine the magnitude of its acceleration. *WAEC 1997*
Solution
The two forces, 3N and 4N, must be resolved into a single net force as follows.

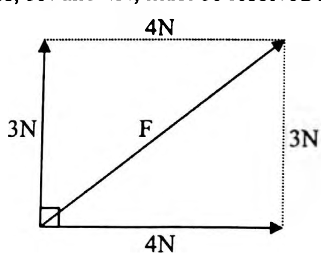


Fig 6.1

Apply Pythagoras theorem, $F^2 = 3^2 + 4^2 = 9 + 16 = 25$

$$\therefore \quad F = \sqrt{25} = 5N$$

mass, m = 20kg

$F = ma \quad \therefore \quad \text{acceleration}, a = \dfrac{F}{m} = \dfrac{5}{20} = 0.25ms^{-2}$

Example 4

A moving car of mass 800kg experiences a frictional force of 200N. If it accelerates at $2ms^{-2}$, calculate the magnitude of the force applied to the car. *WAEC 2006*

Solution

Acceleration, a = $2ms^{-2}$; mass, m = 800kg; frictional force = 200N.

Net force required to accelerate, F = ma = 800×2 = 1600N.

Net force = Force applied – Frictional force

∴ Force applied = Net force + frictional force

= 1600 + 200N

= 1800N

Example 5

A body of mass 2kg moving vertically upwards has its velocity increased uniformly from $10ms^{-1}$ to $40ms^{-1}$ in 4s. Neglecting air resistance, calculate the upward vertical force acting on the body. [$g=10ms^{-2}$]

A. 15N B. 20N C. 35N D. 45N *JAMB 1997*

Solution

Mass of body, m = 2kg; initial velocity, u = $10ms^{-1}$

Final velocity, v = $40ms^{-1}$; time, t = 4s

Substitute into $F = \frac{m(v-u)}{t}$

$$F = \frac{2(40-10)}{4} = \frac{2 \times 30}{4} = \frac{60}{4} = 15N$$

Example 6

Two bodies have masses in the ratio 3:1. They experience forces which impart to them accelerations in the ratio 2:9 respectively. Find the ratio of the forces the masses experience. A. 1:4 B. 2:1 C. 2:3 D. 2:5 *JAMB 1999*

Solution

	1st body		2nd body
mass of bodies, m =	3m	:	1m
acceleration of body, a =	2a	:	9a
force, F = ma	3m × 2a	:	1m × 9a
divide each by 3ma	6ma	:	9ma
ratio of forces =	2	:	3

Ans: 2:3

Example 7

A rope is being used to pull a mass of 10kg vertically upward. Determine the tension in the rope if, starting from rest, the mass acquires a velocity of $4ms^{-1}$ in 8s. [$g=10m/s^2$]

A. 105N B. 95N C. 50N D. 5N *JAMB 2000*

Solution

Initial velocity, u = 0; final velocity, v = $4ms^{-1}$ g=$10ms^{-2}$

Mass, m= 10kg; time, t = 8s;

The tension (T) in the rope is the difference between the weight (W) of the object and the upward pulling force (F) i.e. T = W – F

$$\text{Upward pulling force, } F = \frac{m(v-u)}{t} = \frac{10(4-0)}{8} = \frac{40}{8} = 5N$$

weight, W = mg = 10 × 10 = 100N

Therefore, Tension, T = W – F = 100 – 5 = 95N

Example 8

A ball of mass 5.0kg hits a smooth vertical wall normally with a speed of $2ms^{-1}$ and rebound with the same speed. Determine the impulse experienced by the ball. *WAEC 1996.*

Solution

Mass of ball, m = 5.0kg; initial velocity, u = $2ms^{-1}$

Final velocity, v = $-2ms^{-1}$ ("*...the direction changes on rebound...*")

$$\text{Impulse} = mv - mu = m(v-u) = 5[2-(-2)] = 5(2+2) = 5 \times 4 = 20kgms^{-1}$$

Example 9

A ball of mass 0.15kg is kicked against a rigid vertical wall with a horizontal velocity of $50ms^{-1}$. If it rebounded with a horizontal velocity of $30ms^{-1}$, calculate the impulse of the ball on the wall. A.3.0Ns B. 4.5Ns C. 7.5Ns D. 12.0Ns *JAMB 1998*

Solution

Mass, m = 0.15kg; initial velocity, u = $50ms^{-1}$;

final velocity, v = $-30ms^{-1}$ (*on rebounding, direction changes*)

$$\text{Impulse} = \text{mass} \times \text{change in velocity} = m(v-u)$$
$$= 0.15 \times [50-(-30)] = (50+30) \times 0.15 = 0.15 \times 80 = 12Ns$$

Example 10

A force of 100N is used to kick a football of mass 0.8kg. Find the velocity with which the ball moves if it takes 0.8s to be kicked.

A. $32ms^{-1}$ B. $50ms^{-1}$ C. $64ms^{-1}$ D. $100ms^{-1}$ *JAMB 2003*

Solution

Force, F = 100N; mass, m = 0.8kg; time, t = 0.8s

Initial velocity, u = 0; final velocity, v = ?

Substitute into $$F = \frac{m(v-u)}{t}$$

$$100 = \frac{0.8(v-0)}{0.8} = \frac{0.8v}{0.8}$$

$$\therefore \quad v = 100ms^{-1}$$

Example 11

A bullet of mass 120g is fired horizontally into a fixed wooden block with a speed of $20ms^{-1}$. The bullet is brought to rest in the block in 0.1s by a constant resistance. Calculate the (i) magnitude of the resistance.

(ii) distance moved by the bullet in the wood *WAEC 1999*

Solution

Mass, m = 120g = 0.12kg; initial velocity, u = $20ms^{-1}$

Final velocity, v = $0ms^{-1}$ ("... brought to rest..."); time, t = 0.1s

(i) The magnitude of the resistance is equivalent to the deceleration force the bullet experiences

From 1st equation of motion, v = u + at

$\therefore$ Deceleration, $a = \frac{(v - u)}{t} = \frac{0 - 20}{0.1} = \frac{-20}{0.1} = 200ms^{-2}$

Resistance = deceleration force = F = ma = 0.12 × 200 = 24N

(ii) From 3rd equation of motion, $v^2 = u^2 + 2as$,

$$\therefore \quad \text{distance,} \quad s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2 \times (-200)} = \frac{-400}{-400} = 1m$$

Example 12

A 0.05kg bullet travelling at 500ms^{-1} horizontally strikes a thick vertical wall. It stops after penetrating through the wall a horizontal distance of 0.25m. What is the magnitude of the average force the wall exerts on the bullet?

A. 25N B. 50N C. 250N D. 5000N E. 25000N *JAMB 1985*

Solution

Mass, m = 0.05kg; initial velocity, u = 500ms^{-1}

Final velocity, v = 0; distance penetrated, s = 0.25m

From 3rd equation of motion, $v^2 = u^2 + 2as$,

$$\therefore \quad \text{deceleration,} \quad a = \frac{v^2 - u^2}{2s} = \frac{0^2 - 500^2}{2 \times 0.25} = \frac{-250000}{0.5} = 5 \times 10^5 ms^{-2}$$

(The negative sign indicates deceleration)

The average force = decelerating force, F = ma

= $0.05 \times 5 \times 10^5$ = 25000N

CONSERVATION OF LINEAR MOMENTUM

A combination of Newton's 2nd and 3rd law results in the law of conservation of linear momentum which states that, when two or more bodies act upon one another, their total momentum remains constant, provided no external forces are acting.

The following examples shows how the momentum, velocity and kinetic energy of colliding bodies are calculated.

Example 13

An object of mass 5.0kg moving with a velocity of 12ms^{-1} due north hits a stationary body of mass 7.0kg. If they stick together after collision and move with velocity, v due north calculate the magnitude of v. *NECO 2005*

Solution

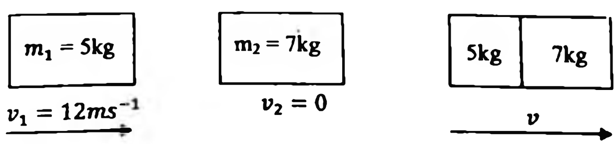


Fig 6.2

momentum before collision = momentum before collision

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$5 \times 12 + 7 \times 0 = (5 + 7)v$$

$$60 + 0 = 12v$$

$$12v = 60$$

$$v = \frac{60}{12} = 5ms^{-1}$$

Example 14
A ball P of mass 0.25kg, losses one-third of its velocity when it makes a head on collision with an identical ball Q at rest. After the collision, Q moves off with a speed of $2ms^{-1}$ in the original direction of P. Calculate the initial velocity of P. *WAEC 2000*

Solution

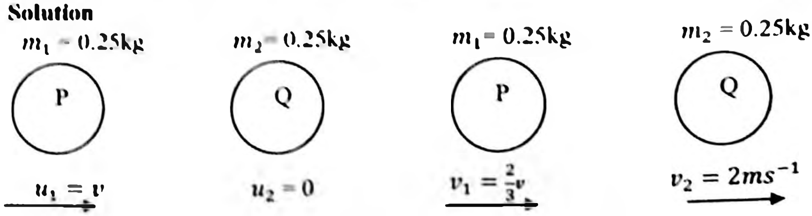


Fig 6.3

Let initial velocity of P be $v\ ms^{-1}$. After collision the velocity of P. $v_1 - \frac{1}{3}v = \frac{2}{3}v$

Momentum before collision = momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

P and Q have the same mass, therefore, $m_1 = m_2 = m$

$$mu_1 + mu_2 = mv_1 + mv_2$$

$$m(u_1 + u_2) = m(v_1 + v_2)$$

Dividing both sides by m to obtain

$$u_1 + u_2 = v_1 + v_2$$

Substituting, $v + 0 = \frac{2}{3}v + 2$

$$v - \frac{2}{3}v = 2$$

$$\frac{1}{3}v = 2$$

$$v = 2 \times 3 = 6ms^{-1}$$

Example 15
A tractor of mass 5.0×10^3kg is used to tow a car of mass 2.5×10^3kg. The tractor moved with a speed of $3.0ms^{-1}$ just before the towing rope becomes taut. Calculate the
(i) speed of the tractor immediately the rope becomes taut.
(ii) loss in K.E of the system just after the car has started moving.
(iii) impulse in the rope when it jerks the car into motion. *WAEC 2003*

Solution
Mass of tractor, $m_1 = 5.0 \times 10^3$kg; initial velocity of tractor, $u_1 = 3.0ms^{-1}$
Mass of car, $m_2 = 2.5 \times 10^3$kg; initial velocity of car, $u_2 = 0ms^{-1}$
Let v be the common velocity of tractor and car.

(i) The speed of the tractor immediately the rope becomes taut is the common velocity of tractor and car when they both began to move.

Momentum before movement = momentum after movement

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$5.0 \times 10^3 \times 3 + 2.5 \times 10^3 \times 0 = (5.0 \times 10^3 + 2.5 \times 10^3)v$$

$$1.5 \times 10^4 = 7.5 \times 10^3 v$$

$$v = \frac{1.5 \times 10^4}{7500} = 2ms^{-1}$$

(ii) Loss in K.E = final K.E − Initial K.E

Initial K.E = K.E of the moving tractor $= \frac{1}{2}m_1u_1^2$ (from KE $= \frac{1}{2}mv^2$)

$$= \frac{1}{2} \times 5.0 \times 10^3 \times 3^2 = 2.25 \times 10^4 \text{ J}$$

final K.E = K.E of the moving tractor and car $= \frac{1}{2}(m_1 + m_2)v^2$

$$= \frac{1}{2}(5.0 \times 10^3 + 2.5 \times 10^3) \times 2^2$$

$$= \frac{1}{2}(7500) \times 4 = 15000 = 1.5 \times 10^4 \text{ J}$$

loss in K.E $= 1.5 \times 10^4 - 2.25 \times 10^4 = -7500\text{J}$

$= 7500\text{J}$

The negative sign signifies loss in K.E

(iii) Impulse = change in momentum

= final momentum − initial momentum

$$= (m_1 + m_2)v - (m_1u_1 + m_2u_2)$$

$$= 7500 \times 2 - 1.5 \times 10^4 + 0$$

$$= 1.5 \times 10^4 - 1.5 \times 10^4$$

$= 0\text{Ns}$ or $0 \text{ kg}ms^{-1}$

Example 16

An arrow of mass 0.3kg is fired with a velocity of 100m/s into a wooden block of mass 0.7kg. Calculate the final K.E after impact, given that the wooden block can freely move.

Solution

Mass of arrow, $m_1 = 0.3$kg; velocity of arrow, $v_1 = 100$m/s;

Mass of block, $m_2 = 0.7$kg

Let v be the common velocity of arrow and block.

Momentum of arrow = momentum of arrow and block

$$m_1v_1 = (m_1 + m_2)v$$

$$0.3 \times 100 = (0.3 + 0.7)v$$

$$30 = 1v$$

$$v = 30ms^{-1}$$

K.E after impact $= \frac{1}{2}(m_1 + m_2)v^2$

$$= \frac{1}{2}(0.3 + 0.7) \times 30^2 = \frac{1}{2} \times 1 \times 900$$

$$= 450\text{J}$$

EXPLOSION: RECOIL OF A GUN

When a gun is fired the following statement and equation applies.

Momentum of bullet = momentum of gun

Mass of bullet × bullet's velocity = mass of gun × recoil velocity

$$m_1 v_1 = MV$$

where m_1, v_1 are mass and velocity of bullet respectively and M, V are mass and recoil velocity of gun respectively.

Example 17

A sub machine gun of mass 20kg fires a bullet of mass 100g due South with a velocity of $250ms^{-1}$. What is the recoil velocity of the gun?

Solution

Mass of bullet, m_1 = 100g = 0.1kg

Velocity of bullet, v_1 = 250m/s

Mass of gun, M = 20kg

Recoil velocity of gun, V = ?

Momentum of bullet = momentum of gun

$$m_1 v_1 = MV$$

$$0.1 \times 250 = 20 \times V$$

$$V = \frac{0.1 \times 250}{20} = 1.25ms^{-1}$$

The recoil velocity is always in the opposite direction of the bullet's velocity (due south), therefore the submachine gun recoil velocity is 1.25m/s due North.

JET AND ROCKET PROPULSION

A jet engine or rocket burns and expels a stream of hot gas at extremely high velocity. According to Newton's 3rd law, a momentum, equal and opposite to the expelled gas is experienced by the rocket or aircraft. Thus,

Rate of momentum of expelled gas = Force (thrust) experienced by rocket or aircraft

Mass of gas expelled or consumed per second x velocity = Force on rocket

$$\therefore \quad M \times V = F$$

Example 18

Fuel was consumed at a steady rate of 5.0×10^{-2}kg per second in a rocket engine and ejected as a gas with a speed of $4 \times 10^3 ms^{-1}$. Determine the thrust on the rocket.

WAEC 2005

Solution

Thrust (force) = rate of change of momentum

F = mass of fuel per second × velocity

F = $5.0 \times 10^{-2} \times 4 \times 10^3$ = 200N

Example 19

A jet engine develops a thrust of 270Ns when the velocity of the exhaust gases relative to the engine is $300ms^{-1}$. What is the mass of the material ejected per second?

A. 81.00kg B. 9.00kg C. 0.90kg D. 0.09kg *JAMB 1987*

Solution

Velocity, $v = 300ms^{-1}$; force, F = 270N

Mass of gas ejected per second × velocity = thrust on engine

$$M \times V = F$$

$$M = \frac{F}{V} = \frac{270}{300} = 0.9kg$$

WEIGHT OF A BODY IN A LIFT

The following instances and their equations exist.

1. If a lift is at rest or moving up or down with *uniform velocity*, the weight of the body is the same as when it is at rest on the earth's surface.

$$W = F = mg$$

2. If a lift is accelerating upward (ascending), the weight of the body is

$$F = mg + ma = m(g + a)$$

3. If a lift is accelerating downwards (descending) with acceleration a, less than g, the weight of the body is

$$F = mg - ma = m(g - a)$$

4. If a lift is moving downward with acceleration a greater than g, the weight of the body is

$$F = ma - mg$$

5. If a lift falls freely, when, $a = g$ then the weight of the object is

$$F = mg - mg = 0N$$

Example 20

A spring balance which is suspended from the roof of a lift, carries a mass of 1kg at its free end. If the lift accelerates upward at $2.5ms^{-2}$, determine the reading on the spring balance [$g=10m/s^2$]. *WAEC 2001*

Solution

Mass, m = 1kg; $g=10m/s^2$; acceleration of lift, $a = 2.5ms^{-2}$

The reading on the spring balance, $F = mg + ma = m(g + a)$

$$\therefore \quad F = 1(10 + 2.5) = 1 \times 12.5 = 12.5N$$

Example 21

A 1000kg elevator is descending vertically with an acceleration of $1.0ms^{-2}$. If the acceleration due to gravity is $10.0ms^{-2}$, the tension in the suspending cable is

A. 1.0N B. 10.0N C. 9000.0N D. 11000.0N *JAMB 1986*

Solution

$g=10ms^{-2}$; mass, m = 1000kg; $a = 1.0m/s^2$

For a descending lift the force (tension), $F = mg - ma = m(g - a) = 1000(10 - 1)$

$$= 1000 \times 9 = 9000N$$

Example 22

A mass of 5kg is suspended from the ceiling of a lift with a light inextensible string. As the lift moves upward with an acceleration of $2ms^{-2}$, what's the tension in the string? [$g = 10ms^{-2}$] *NECO 2004*

Solution

Mass, m = 5kg; $g = 10ms^{-2}$; acceleration of lift, a = $2ms^{-2}$

For ascending lift, force (tension), $F = mg + ma = m(g + a) = 5(10 + 2) = 60N$

Example 23

An elevator of mass 4800kg is supported by a cable which can safely withstand a maximum tension of 60 000N. The maximum upward acceleration the elevator can have is A. $2.5ms^{-2}$ B. $5.0ms^{-2}$ C. $7.5ms^{-2}$ D. $10.0ms^{-2}$ [$g=10ms^{-2}$] *JAMB 1987*

Solution

Force or tension, F = 60 000N; mass, m = 4800kg; $g=10m/s^2$;
upward acceleration, a = ?

Substitute into equation of ascending lift

$$F = mg + ma$$

$$60,000 = 4800 \times 10 + 4800 \times a$$

$$60,000 = 48,000 + 4800a$$

$$4800a = 60,000 - 48,000$$

$$4800a = 12000$$

$$a = \frac{12000}{4800} = 2.5ms^{-2}$$

EXERCISES 6.

1. A net force of 15N acts upon a body of mass 3kg for 5s, calculate the change in the speed of the body. *WAEC 2003 Ans: 25m/s*
2. A stationary object of mass 4kg is set in motion by a net force of 50N. If the object attains a speed of $5ms^{-1}$ in time, calculate the value of t. *WAEC 1996 Ans: 0.40sec*
3. A constant force acts on a body of mass 50kg and changes its speed from $20ms^{-1}$ to $90ms^{-1}$ in 10 sec. Calculate the magnitude of the force applied. *NECO 2005 Ans: 350N*
4. A body of mass 4kg is accelerated from rest by a steady force of 9N. What is its speed when it has travelled a distance of 8m? *NECO 2002 Ans: 6m/s Hint: $a = F/m$; $v^2 = u^2 + 2as$*
5. A force of 16N applied to a 40kg block that is at rest on a smooth, horizontal surface. What is the velocity of the block at t = 5 seconds? A. 4m/s B. 10m/s C. 20m/s D. 50m/s E. 80m/s *JAMB 1983 Ans: 20m/s*
6. A constant force of magnitude F acts on an object of mass 0.04kg initially at rest at a point O. If the speed of the object when it has moved 50m from O is $500ms^{-1}$, What is the value of F? A. 0.4N B. 100.0N C. 250.0N D. 1000.0N *JAMB 1986 Ans: 100N Hint $F = ma$, $v^2 = u^2 + 2as$*
7. A force of 200N acts between two objects at a certain distance apart. The value of the force when the distance is halved is
A. 100N B. 200N C. 800N D. 400N *JAMB 2005 Ans: 100N*
8. An engine of a car of power 80KW moves on a rough road with a velocity of $32ms^{-1}$. The force required to bring it to rest is A. 2.50×10^6N B. 2.56×10^6N C. 2.50×10^3N D. 2.80×10^3N *JAMB 2006 Ans: 2.50×10^3N* Hint: P = Fv

9. A constant force acts on a body of mass 50kg and reduces its speed from $90ms^{-1}$ to $20ms^{-1}$ in 20s. Calculate the magnitude of the force. *NECO 2006 Ans: 175N*

10. An object of mass 2kg moves with a uniform speed of $10ms^{-1}$ for 5s along a straight path. Determine the magnitude of its acceleration. *WAEC 2007 Ans: $4ms^{-2}$*

11. An external force of magnitude 100N acts on a particle of mass 0.15kg for 0.03s. Calculate the change in the speed of the particle. *WAEC 2007 Ans: $20ms^{-1}$*

12. A car of mass 800kg moves from rest on a horizontal track and travels 60m in 20s with uniform acceleration. Assuming there were no frictional forces, calculate the accelerating force. *WAEC 2007 Ans: 240.00N*

13. A body of mass 5kg initially at rest is acted upon by two mutually perpendicular forces 12N and 5N as shown below. If the particle moves in the direction OA, calculate the magnitude of the acceleration.

A. $0.40ms^{-2}$ B. $1.40ms^{-2}$ C. $0.26ms^{-2}$ D. $2.60ms^{-2}$ E. $3.40ms^{-2}$

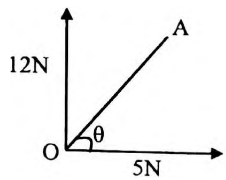


Fig 6.4

JAMB 1984 Ans: $2.60ms^{-2}$

14. A ball of mass 0.1kg approaching a tennis player with a velocity of $10ms^{-1}$, is hit back in the opposite direction with a velocity of $15ms^{-1}$. If the time of impact between the racket and the ball is 0.01s, calculate the magnitude of the force with which the ball is hit. *WAEC 1997 Ans: 250N*

15. A ball of mass 5.0kg hits a smooth vertical wall normally with a speed of $2ms^{-1}$ and rebounds with the same speed. Determine the impulse experienced by the ball. *WAEC 1998 Ans: $20kgms^{-1}$*

16. A ball of mass 5.0kg hits a smooth vertical wall normally with a speed of $2ms^{-1}$. Determine the magnitude of the resulting impulse. *WAEC 2003 Ans: $20kgms^{-1}$*

17. A force acting on a body causes a change in the momentum of the body from $12kgms^{-1}$ to $16kgms^{-1}$ in 0.2s. Calculate the magnitude of the impulse. *WAEC 2006 Ans: $4kgms^{-1}$*

18. What change in velocity would be produced on a body of mass 4kg, if a constant force of 16N acts on it for 2s? *WAEC 1992 Ans: $8.0ms^{-1}$*

19. A force acts on a body for 0.5s changing its momentum from $16.0kgms^{-1}$ to $21.0kgms^{-1}$. Calculate the magnitude of the force. *WAEC 2003 Ans: 10N*

20. A force acting on a body causes a change in the momentum of the body from $12kgms^{-1}$ to $16kgms^{-1}$ in 0.2s. Calculate the magnitude of the force. *WAEC 2004 Ans: 20N*

21. When taking a penalty kick, a footballer applies a force of 30.0N for a period of 0.05s. If the mass of the ball is 0.075kg, calculate the speed with which the ball moves off. A. $4.50ms^{-1}$ B. $11.25ms^{-1}$ C. $20.0ms^{-1}$ D. $45.00ms^{-1}$

JAMB 1988 Ans: 20.00 ms^{-1}

22. A body of mass 100g moving with a velocity of $10.0ms^{-1}$ collides with a wall. If after the collision, it moves with a velocity of $2.0ms^{-1}$ in the opposite direction, calculate the change in momentum.

A. 0.8Ns B. 1.2Ns C. 12.0Ns D. 80.0Ns *JAMB 1991 Ans: 1.2Ns*

23. A constant force of 5N acts for 5s on a mass of 5kg initially at rest. Calculate the final momentum of the mass. *NECO 2003 Ans: $25kgms^{-1}$*

24. A body of mass M_1 moving with a velocity, U collides with a stationary body of mass M_2 and both move with a common velocity, V. If linear momentum is conserved, which expression correctly represents V?

A. $\frac{M_1 + M_2}{M_1 U}$ B. $\frac{M_2 U}{M_1 - M_2}$ C. $\frac{M_1 U}{M_1 - M_2}$ D. $\frac{M_1 U}{M_1 + M_2}$ E. $\frac{M_2 U}{M_1 + M_2}$

NECO 2002 Ans: D

25. A ball of mass 0.5kg moving at 10m/s collides with another ball of equal mass at rest. If the two balls move off together after impact, calculate their common velocity. *WAEC 1988 Ans: $5ms^{-1}$*

26. A body of mass 4.2kg moving with $10ms^{-1}$ due east, hits a stationary body of mass 2.8kg. If they stick together after collision and move with velocity v due east, calculate the value of v. *WAEC 1996 Ans: $6ms^{-1}$*

27. A trolley of mass 4kg moving on a smooth horizontal platform with a speed of $1.0ms^{-1}$ collides perfectly with a stationary trolley of the same mass on the same platform. Calculate the total momentum of the two trolleys immediately after the collision.

WAEC 1999 Ans: 4.0Ns or $4.0kgms^{-1}$

28. A body of mass 5kg moving with a velocity of $10ms^{-1}$ collides with a stationary body of mass 6kg. If the two bodies stick together and move in the same direction after the collision, calculate their common velocity. *WAEC 2005 Ans: $4.55ms^{-1}$*

29. A ball of mass 100g travelling with a velocity of $100ms^{-1}$ collides with another ball of mass 400g moving at $50ms^{-1}$ in the same direction. If they stick together, what will be their common velocity? *NECO 2000 Ans: $60ms^{-1}$*

30. An arrow of mass 0.1kg moving with a horizontal velocity of $15ms^{-1}$ is shot into a wooden block of mass 0.4kg lying at rest on a smooth horizontal surface. Their common velocity after impact is

A. $15.0\ ms^{-1}$ B. $7.5ms^{-1}$ C. $3.8ms^{-1}$ D. $3.0ms^{-1}$ *JAMB 1997 Ans: $3ms^{-1}$*

31. A lead bullet of mass 0.05kg is fired with a velocity of $200ms^{-1}$ into a lead block of mass 0.95kg. Given that the lead block can move freely, the final K.E after impact is

A. 50J B. 100J C. 150J D. 200J *JAMB 1999 Ans: 50J*

32. A bullet of mass 120g is fired horizontally into a fixed wooden block with a speed of $20ms^{-1}$. If the bullet is brought to rest in the block in 0.1s by a constant resistance, calculate the

(i) magnitude of the resistance

(ii) distance moved by the bullet in the wood.

WAEC 2006 Ans: (i) 24N (ii) 1.0m

33. A bullet fired at a wooden block of thickness 0.15m manages to penetrate the block. If the mass of the bullet is 0.0025kg and the average resisting force of the wood is $7.5 \times 10^{3}N$, calculate the speed of the bullet just before it hits the wooden block.

A. $450ms^{-1}$ B. $400ms^{-1}$ C. $300ms^{-1}$ D. $250ms^{-1}$ *JAMB 1998 Ans: $300ms^{-1}$*

34. The driver in a motor car, of which the total mass is 800kg and which is travelling at $20ms^{-1}$, suddenly observes a stationary dog in his path 50m ahead. If the car brakes can exert a force of 2000N, what will most likely happen?

A. The car will be able to stop immediately the driver notices the dog.

B. The car will stop 30m after hitting the dog.

C. The car will stop 20m in front of the dog.

D. The driver will quickly reverse the car.

E. The car will stop 5m beyond the dog. *JAMB 1981 Ans: B*

35. A machine gun with a mass of 5kg, fires a 50g bullet at a speed of $100ms^{-1}$. The recoil speed of the machine gun is A. 0.5m/sec B. 1.5m/sec C. 1m/sec D. 2m/sec E. 4m/sec *JAMB 1978 Ans: C*

36. A gun of mass 2.0kg fires a bullet of mass $1.6 \times 10^{-2}kg$ due East. If the bullet leaves the nozzle of the gun with a velocity of $150ms^{-1}$, what is the recoil velocity of the gun? A. $150ms^{-1}$ due West B. $1.2 \times 10^{-4}ms^{-1}$ due West C. $1.2ms^{-1}$ due West D. $1.2ms^{-1}$ due East E. $150ms^{-1}$ due East *JAMB 1984 Ans: C*

37. A gun of mass 3kg fires a bullet of mass 20g with a velocity of $500ms^{-1}$. Calculate the recoil velocity of the gun. *NECO 2004 Ans: $-3.33ms^{-1}$*

38. A rocket burns 0.01kg of fuel each second and ejects it as a gas with a velocity of 5,000m/s. What force does the gas exert on the rocket? A. 500,000N B. 500N C. 50N D. 5,000N E. 50,000N *JAMB 1982 Ans: 50N*

39. A rocket burns fuel at the rate of $10kgs^{-1}$ and ejects it with a velocity of

$5 \times 10^3 ms^{-1}$. The thrust exerted by the gas on the rocket is A. $2.5 \times 10^7 N$ B. $5.0 \times 10^4 N$ C. $5.0 \times 10^2 N$ D. $2.0 \times 10^3 N$ *JAMB 1993* *Ans: $5.0 \times 10^4 N$*

40. A girl whose mass is 55kg stands on a spring weighing machine inside a lift. When the lift starts to ascend, its acceleration is $2ms^{-2}$. What will be the reading on the machine? [Take $g=10m/s^2$] *WAEC 1990* *Ans: 66kg*

41. A body of mass 2kg is suspended from the ceiling of a lift with a light inextensible string. If the lift moves upwards with acceleration of $2ms^{-2}$, calculate the magnitude of the tension in the string. [$g=10ms^{-2}$] *WAEC 1996* *Ans: 24N*

42. A force of 21.0N acts on a particle of mass 3.0kg in a medium whose resistance is $2.0Nkg^{-1}$. Calculate the magnitude of the acceleration of the particle.

NECO 2007[16] *Ans: $5.00ms^{-2}$*

43. A bullet of mass m is fired from a gun of mass M with a velocity v. The recoil velocity of the gun is expressed as

A. $\frac{Mv}{m}$ B. $\frac{Mv}{M-m}$ C. $\frac{Mv}{M+m}$ D. $\frac{mv}{M-m}$ E. $\frac{mv}{M}$ *NECO 2007* *(Ans: E)*

44. An object of mass 2kg moving with a velocity of $3ms^{-1}$ collides head-on with another object of mass 1kg moving in the opposite direction with a velocity of $4ms^{-1}$. If the objects stick together after collision, calculate their common speed.

WAEC 2008[13] *Ans: $0.67ms^{-1}$*

45. A body of mass 4kg resting on a smooth horizontal plane is simultaneously acted upon by two perpendicular forces 6N and 8N. Calculate the acceleration of the motion.

A. $2.52ms^{-2}$ B. $3.0ms^{-2}$ C. $4.0ms^{-2}$ D. $4.5ms^{-2}$

JAMB 2008[7] *Ans: $2.5ms^{-2}$*

46. A body of mass 80kg moving with a velocity of $6ms^{-1}$ hits a stationary body of mass 40kg. If the two bodies stick together after the collision, calculate the magnitude of the velocity with which they move. *NECO 2008[15]* *Ans: $4ms^{-1}$*

47. A body of mass 12 kg travelling at $4.2 ms^{-1}$ collides with a second body of mass 18kg at rest. Calculate their common velocity if the two bodies coalesce after collision.

A. $1.5 ms^{-1}$ B. $1.4 ms^{-1}$ C. $2.1 ms^{-1}$ D. $1.7 ms^{-1}$ *JAMB 2009[7]* *Ans: D*

48. A body of mass 5kg with a velocity of $30ms^{-1}$ due East is suddenly hit by another body and changes its velocity to $50ms^{-1}$ in the same direction. Calculate the magnitude of the impulse received. *WAEC 2009[13]* *Ans: 100Ns*

49. A ball of mass 0.1kg moving with velocity of $20ms^{-1}$ is hit by a force which acts on it for 0.02s. If the ball moves off in the opposite direction with a velocity of $25ms^{-1}$, calculate the magnitude of the force. *NECO 2009[16]* *Ans: 25N*

50. A body of mass 4.0kg moving with a speed of $2.5ms^{-1}$ collides with a stationary body of mass 0.2kg. If the two bodies stick together after collision, calculate the magnitude of the velocity with which they move. *NECO 2009[E3]* *Ans: $2.38ms^{-1}$*

7

MACHINES

A machine is any device or tool by means of which a force or effort(E) applied at one point can be used to overcome a force or load(L) at some other point. The following terms and formulae are common to all types of machines.

A. Mechanical Advantage = Force Ratio = $\frac{\text{Load}}{\text{Effort}} = \frac{\text{Output force}}{\text{Input force}}$

That is, $M.A. = F.R = \frac{L}{E}$

B. Velocity Ratio = $\frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{e}{l}$

$$V.R. = \frac{\text{distance between effort and fulcrum}}{\text{distance between load and fulcrum}}$$

Also, $$\frac{1}{\text{Velocity Ratio}} = \frac{1}{V.R} = \frac{\text{distance moved by load}}{\text{distance moved by effort}} = \frac{l}{e}$$

C. Efficiency = $\frac{\text{work output}}{\text{work input}} \times 100\%$

The relationship between mechanical advantage (M.A.), velocity ratio (V.R) and efficiency (Ɛ) is derived as follows.

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

Generally, work = force (load or effort) × distance

Hence, $$\text{efficiency} = \frac{\text{load} \times \text{distance moved by load}}{\text{effort} \times \text{distance moved by effort}} \times 100\%$$

$$= \frac{L \times l}{E \times e} \times 100$$

$$= \frac{L}{E} \times \frac{l}{e} \times 100\%$$

But $M.A. = \frac{L}{E}$ *and* $\frac{1}{V.R} = \frac{l}{e}$

$\therefore$ $$\text{Efficiency} = \text{M.A} \times \frac{1}{\text{V.R}} \times 100\%$$

$$\text{Efficiency} = \frac{\text{M.A}}{\text{V.R}} \times 100\%$$

$$\text{Efficiency} = \frac{\text{mechanical advantage}}{\text{velocity ratio}} \times 100\%$$

$$\mathcal{E} = \frac{\text{M.A}}{\text{V.R}} \times 100\%$$

$$\varepsilon = \frac{L/E}{e/l} \times 100\%$$

Where ε = efficiency in percentage (%)
M.A. = mechanical advantage
V.R = velocity ratio
L = load in Newton (N) or kilograms (kg)
E = effort in Newton or kilograms (kg)
e = distance moved by effort in meter (m)
l = distance moved by load in meter (m)
F.R = force ratio

Example 1

A machine has an efficiency of 80%. If the machine is required to overcome a load of 60N with a force of 40N, calculate its velocity ratio. *NECO 2003*[13]

Solution

Efficiency ε= 80%; Load, L = 60N; Effort, E = 40N Velocity ratio, V.R. = ?

$$\text{Mechanical Advantage,} \quad M.A = \frac{\text{load}}{\text{effort}} = \frac{L}{E} = \frac{60N}{40N} = 1.5$$

$$\text{Velocity Ratio,} \quad V.R. = \frac{M.A}{\varepsilon} \times 100\% = \frac{1.5}{80} \times 100 = 1.875$$

Example 2

The efficiency of a machine is 80%. Determine the work done by a person using this machine to raise a load of 200kg through a vertical distance of 3.0m (Take g = $10ms^{-2}$) *WAEC 1988*[e1]

Solution

Efficiency, ε = 80%; Load, L = 200kg = (200 x 10)N = 2000N;
Distance moved by load, l = 3m

$$\text{Efficiency} = \frac{\text{work done by machine}}{\text{Work done on machine or by person}} \times 100\%$$

$$\varepsilon = \frac{L \times l}{\text{Work done by person}} \times 100\%$$

$$\text{Work done by person} = \frac{L \times l \times 100}{\varepsilon} = \frac{2000 \times 3 \times 100}{80} = \frac{600000}{80} = 7500\,J$$

Example 3

The velocity ratio of a machine is 5 and its efficiency is 75%. What effort would be needed to lift a load of 150N with the machine?

A. 50N B. 40N C. 30N D. 20N *JAMB 2000*[10]

Solution

Velocity ratio V.R = 5; efficiency, ε = 75%; Load, L = 150N; effort, E = ?

$$\varepsilon = \frac{M.A}{V.R} \times 100\% \quad \therefore \quad M.A. = \frac{\varepsilon \times V.R}{100}$$

$$M.A. = \frac{75 \times 5}{100} = 3.75$$

$Also, \quad M.A = \frac{L}{E}$

$$\therefore \quad Effort, E = \frac{L}{M.A} = \frac{150}{3.75} = 40N$$

Example 4
A load of mass 120kg is raised vertically through a height of 2m in 30s by a machine whose efficiency is 100%. Calculate the power generated by the machine. *WAEC 2002*[16]
[g = 10ms^{-2}]

Solution
Mass, m = 120kg; height (distance) s = 2m; time, t = 30s; efficiency, ε = 100%.

$$\text{Power} = \frac{\text{Force} \times \text{distance}}{\text{Time}} = \frac{F \times s}{t} = \frac{mg \times s}{t}$$

$$\text{Efficiency}, \mathcal{E} = \frac{\text{Power output (P.O)}}{\text{Power input (P.I)}} \times 100\%$$

$$\mathcal{E} = \frac{\text{P.O}}{\text{P.I}} \times 100\%$$

If $\mathcal{E}$ = 100% then the above equation becomes

$$100 = \frac{\text{P.O}}{\text{P.I}} \times 100$$

$$\frac{100}{100} = \frac{\text{P.O}}{\text{P.I}}$$

$$1 = \frac{\text{P.O}}{\text{P.I}}$$

Rearranging, P.I = P.O

Simply put, if a machine has 100% efficiency then the power input is equal to power output.

$$\text{Therefore, power output by machine} = \frac{mg \times s}{t} = \frac{120 \times 10 \times 2}{30} = 80W$$

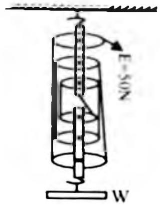
TYPES OF MACHINES

These include the levers, the inclined plane, the screw or screw jack, the wheel and axle, and gear wheels.

1. The Pulley or Block and Tackle System.

In a block and tackle or pulley system, the velocity ratio (V.R) is equal to the number of pulleys in the system.
That is V.R = number of ropes supporting the pulley
= number of pulleys in the system.

Example 5
The diagram below represents a block and tackle pulley system on which an effort of [illegible] N is just able to lift a load of weight W. If the efficiency of the machine is 40%, find the value of W.



WAEC 1991[18]

Fig 7.1

Solution

Effort, E = 50N; Load, L = W; Efficiency, $\varepsilon = 40\%$

From diagram, velocity ratio, V.R = number of pulleys = 6

$$\varepsilon = \frac{M.A}{V.R} \times 100\% \quad \therefore \quad M.A. = \frac{\varepsilon \times V.R}{100} = \frac{40 \times 6}{100} = 2.4$$

$$Also, \quad M.A = \frac{L}{E} \quad \therefore \quad L = W = M.A \times E = 2.4 \times 50 = 120N$$

Example 6

The velocity ratio and efficiency of a system of pulleys are 6 and 80% respectively. How much effort is required to lift a load of mass 120kg with this system [$g = 10ms^{-2}$]

WAEC 1996

Solution

Velocity ratio = V.R = 6; Efficiency, $\varepsilon = 80\%$; Mass, m = 120kg

$\therefore$ L = mg = 120 x 10 = 1200N

$$\varepsilon = \frac{M.A}{V.R} \times 100\% \quad \therefore \quad M.A. = \frac{\varepsilon \times V.R}{100} = \frac{80 \times 6}{100} = 4.8$$

$$Also, \quad M.A = \frac{L}{E} \quad \therefore \quad Effort, E = \frac{L}{M.A} = \frac{1200}{4.8} = 250N$$

Example 7

A block and tackle system is used to lift a load of 20N through a vertical height of 10m. If the efficiency of the system is 40%, how much work is done against friction?

JAMB 1989

Solution

Load, L = 20N; Distance moved by load, l = 10m

Efficiency, $\varepsilon = 40\%$; Work output = L × l = 20 x 10 = 200N

$$Efficiency = \frac{work\ output}{work\ input} \times 100\% = \frac{load \times distance\ moved\ by\ load}{work\ input} \times 100\%$$

$$\therefore \quad Work\ input = \frac{load \times distance\ moved\ by\ load}{efficiency} \times 100\%$$

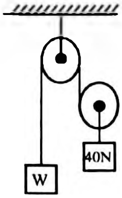
$$Work\ input = \frac{20 \times 10 \times 100}{40} = \frac{20000}{40} = 500N$$

Work output = work input − frictional forces

$\therefore$ Work done against friction = Work input − work output

= 500 − 200 = 300N

Example 8



The figure 7.2 represents a frictionless pulley system in which a weight W is in equilibrium with a weight of 40N. Find the value of W.

A. 13.3N B. 20.0N C. 40.0N

D. 80.0N *JAMB 1999*[4]

Fig 7.2

Solution

Velocity ratio, V.R = number of pulleys = 2; for a frictionless pulley, V.R = M.A.
Therefore, M.A = 2. Effort, E = W; load, L = 40N.

$$M.A = \frac{L}{E} \quad \therefore \quad E = W = \frac{L}{M.A} = \frac{40}{2} = 20N$$

2. The Inclined Plane

The inclined plane is used to lift heavy load into vans or trucks. Example are, a slope, a hill, a ramp e.t.c.

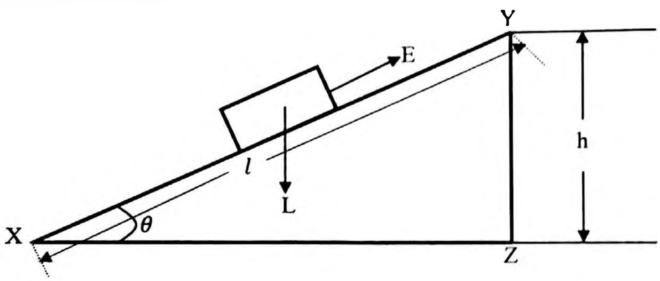


Fig 7.3

Velocity ratio of an inclined plane is given by:

$$V.R = \frac{\text{distance moved by effort}}{\text{distance moved by load}} = \frac{\text{length of inclined plane}}{\text{height of inclined plane}} = \frac{l}{h}$$

From figure 7.3, $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{l}$

Rearranging, $\frac{1}{\sin\theta} = \frac{l}{h}$

Therefore, $\mathrm{V.R} = \frac{l}{h} = \frac{1}{\sin\theta}$

For a Frictionless inclined plane, $\mathrm{V.R} = \mathrm{M.A} = \frac{1}{\sin\theta}$

Furthermore, Work output = load × distance moved by load

= load × height of inclined plane

= $L \times h$

Work input = Effort × distance moved by effort

$= \text{effort} \times \text{length of inclined plane}$

$= E \times l$

Efficiency of inclined plane, $\mathcal{E} = \dfrac{\text{work output}}{\text{work input}} \times 100\%$

$$\mathcal{E} = \frac{L \times h}{E \times l} \times 100\% = \frac{L}{E} \times \frac{h}{l} \times 100\%$$

Example 9
An inclined plane of angle 15° is used to raise a load of 4500N through a height of 2m. If the plane is 75% efficient, calculate
(i) velocity ratio of the plane (ii) work done on the load *WAEC 1999*[e2]
Solution
Angle of inclination $\theta = 15^\circ$; load, L = 4500N; height of plane, h = 2m;
Efficiency, $\mathcal{E} = 75\%$

(i) Velocity ratio, $V.R = \frac{1}{\sin\theta} = \frac{1}{\sin 15} = 3.86$

(ii) Work done on the load is the work input

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

$$\therefore \quad \text{Work input} = \frac{\text{work output}}{\text{Efficiency}} \times 100\% = \frac{\text{load} \times \text{height of plane}}{\text{efficiency}} \times 100\%$$

$$= \frac{L \times h}{\mathcal{E}} \times 100 = \frac{4500 \times 2 \times 100}{75} = 12000J$$

Example 10
A plane inclined at 30° to the horizontal has an efficiency of 50%. Calculate the force parallel to the plane required to push a load of 120N uniformly up the plane.
WAEC 2007[17]

Solution
Angle of inclination $\theta = 30^\circ$; $\therefore$ $V.R = \frac{1}{\sin\theta} = \frac{1}{\sin 30} = 2$;
load, L = 120N; Efficiency, $\mathcal{E} = 50\%$

The force parallel to the plane required to push the load is the effort E.

First, we obtain M.A from $\mathcal{E} = \dfrac{M.A}{V.R} \times 100\%$

$$\therefore \quad M.A = \frac{\mathcal{E} \times V.R}{100} = \frac{50 \times 2}{100} = \frac{100}{100} = 1$$

$$Also, \quad M.A = \frac{L}{E} \quad \therefore \quad E = \frac{L}{M.A} = \frac{120}{1} = 120N$$

Example 11
A man pulls up a box of mass 70kg using an inclined plane of effective length 5m onto a platform 2.5m high at uniform speed. If the frictional force between the box and the plane is 100N, draw the diagram of all the forces acting on the box when in motion and calculate the

(i) minimum effort applied in pulling up the box.

(ii) Mechanical advantage of the plane

(iii) Velocity ratio of the plane
(iv) Efficiency of the plane
(v) Energy lost in the system
(vi) Work output of the man
(vii) Total power developed by the man

given that the time taken to raise the box onto the platform is 50s, ($g=10ms^{-2}$) *WAEC 1997*

Solution

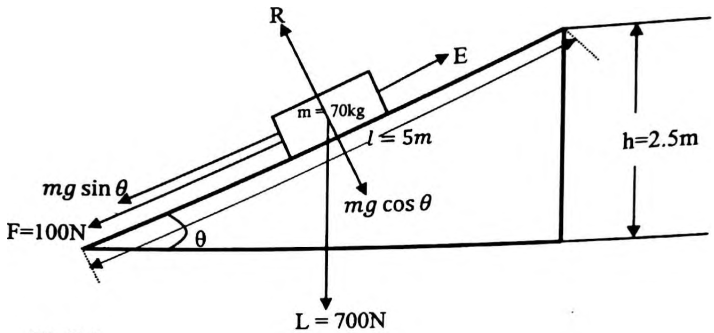


Fig 7.4

Mass of box, m =70kg; frictional force, F = 100N; $g=10ms^{-2}$;
Height of plane, h = 2.5m; length of plane, l = 5m.

The load or weight of the block acts downwards and has two components.

a) $mg\sin\theta$, which is parallel to the plane
b) $mg\cos\theta$, which is perpendicular to the plane.

$$\therefore \sin\theta = \frac{h}{l} = \frac{2.5}{5} = 0.5$$

$$\theta = \sin^{-1} 0.5 = 30°$$

(i) For the object to be pulled up the plane, it must overcome the opposing frictional force, F and the parallel component of the load, $mg\sin\theta$. Therefore, the minimum effort, $E = mg\sin\theta + F$.

$$E = 70 \times 10 \times \sin 30 + 100 = 700 \times 0.5 + 100 = 350 + 100$$
$$= 450N$$

(ii) Mechanical Advantage or Force ratio $= \frac{\text{load}}{\text{effort}}$

$$M.A = \frac{L}{E} = \frac{mg}{E} = \frac{70 \times 10}{450} = \frac{700}{450} = 1.56$$

(iii) Velocity ratio, $V.R = \frac{l}{h} = \frac{5}{2.5} = 2$

Alternatively, $V.R = \frac{1}{\sin\theta} = \frac{1}{\sin 30} = \frac{1}{0.5} = 2$

(iv) Efficiency $= \frac{M.A}{V.R} \times 100\% = \frac{1.56 \times 100}{2} = \frac{156}{2} = 78\%$

(v) Energy lost in the system = work input − work output
= **Effort × distance moved by effort − load ×** distance moved by load
= **$(E \times l) - (L \times h)$**
= **$(450 \times 5) - (700 \times 2.5)$**

$= 2250 - 1750$

$= 500J$

(vi) Work output of the man $=$ force (load) $\times$ height

$= mg \times h$

$= 70 \times 10 \times 2.5 = 1750J$

(vii) Total power developed $= \dfrac{\text{work output}}{\text{time taken}} = \dfrac{1750}{50} = 35W$

3. The Screw

Examples of screw machines are bolts and nut, car screw jack, Engineer's vice e.t.c. All screws have thread. The distance between successive thread on a screw is called its pitch. For one complete revolution, the screw travels a distance that is equal to its pitch.

For a screw, work done = work ouput

Effort $\times$ distance moved by effort = load $\times$ distance moved by load

Effort $\times$ circumference of handle = load $\times$ pitch

$$E \times 2\pi r = L \times p$$

Hence, the velocity ratio, $V.R = \dfrac{2\pi r}{p}$

For a perfect or frictionless screw, M.A = V.R

Hence, $M.A = V.R = \dfrac{2\pi r}{p}$

For a screw jack, r is the length of the handle or Tommy bar of the screw jack.

Example 12

A screw jack whose pitch is 4.4mm is used to raise a body of mass 8000kg through a height of 20cm. The length of the Tommy bar of the jack is 70cm. If the efficiency of the jack is 80%, calculate

(i) the velocity ratio of the jack

(ii) mechanical advantage of the jack

(iii) effort required in raising the body

(iv) work done by the effort in raising the body [$=10ms^{-2}$; $\pi = {}^{22}/_7$] *WAEC 1993*[EI]

Solution

Pitch, p = 4.4mm = 0.0044m; mass, m=8000kg; $g=10m/s^2$

$\therefore$ Load, L= mg = 8000 $\times$ 10 = 80000N; distance load moved, l = 20cm = 0.2m;

length of Tommy bar, r = 70cm = 0.70m; efficiency, $\mathcal{E}$ = 80%

(i) The velocity ratio, $V.R = \dfrac{2\pi r}{p} = \dfrac{2 \times {}^{22}/_7 \times 0.70}{0.0044} = \dfrac{4.4}{0.0044} = 1000$

(ii) From $\mathcal{E} = \dfrac{M.A}{V.R} \times 100\%$ we obtain

Mechanical advantage, $M.A = \dfrac{\mathcal{E} \times V.R}{100} = \dfrac{80 \times 1000}{100} = 800$

(iii) $M.A = \frac{\text{load}}{\text{effort}} = \frac{L}{E}$

$\therefore$ Effort $= \frac{L}{M.A} = \frac{80000}{800} = 100N$

(iv) Work done by effort = work input

$$\text{Efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

$$\therefore \text{ Work input} = \frac{\text{work output}}{\text{efficiency}} \times 100\%$$

$$= \frac{\text{load} \times \text{distance moved by load}}{\text{efficiency}} \times 100\%$$

$$= \frac{\text{load} \times \text{distance moved by load}}{\text{efficiency}} \times 100\%$$

$$= \frac{L \times l \times 100}{\varepsilon} = \frac{80000 \times 0.2 \times 100}{80}$$

$$= \frac{1600000}{80} = 20000J = 20KJ$$

Example 13

Two spanners X and Y of lengths 15cm and 20cm respectively are used in turn to give a screw of pitch 2mm one complete rotation. If R_x and R_y are the respective velocity ratios of the spanners, what is the ratio R_x: R_y? *WAEC 1996*[7]

Solution

	X		Y
Length of spanner, r =	15cm	:	20cm
Pitch, p =	2mm	:	2mm
Convert mm to cm (×10)	20cm	:	20cm
Velocity ratio, V.R =	R_x	:	R_y
Substitute into V. R	$\frac{2\pi r}{p}$	:	$\frac{2\pi r}{p}$
	$\frac{2\pi \times 15}{20}$	:	$\frac{2\pi \times 20}{20}$
Divide both sides by 2π	$\frac{2\pi \times 3}{4}$	:	2π
Multiply both sides by 4	$\frac{3}{4}$	:	1
	$\frac{3 \times 4}{4}$	:	1×4
	3	:	4

Therefore, ratio $R_x : R_y$ is 3 : 4

Example 14

A screw jack with a Tommy bar of length 12cm is used to raise a car through a vertical height of 25cm by turning the Tommy bar through 50 revolutions. Calculate the approximate velocity ratio of the jack. [$\pi = 3.14$] *WAEC 2001*[14]

Solution

Length of Tommy bar, r = 12cm; number of revolution, n = 50;

Height raised or distance load moved, h = 25cm;

$$\text{Therefore, pitch, } p = \frac{h}{n} = \frac{25\text{cm}}{50} = 0.5\text{cm}$$

velocity ratio, $$V.R = \frac{2\pi r}{p} = \frac{2 \times 3.14 \times 12\text{cm}}{0.5\text{cm}} = \frac{75.36\text{cm}}{0.5\text{cm}} = 150.72 \cong 151$$

4. The wheel and Axle

The wheel and axle is a simple machine used to raise water or sand from very deep hole or pit. For a wheel of radius R and axle of radius, r; it follows that:

Work input (work done by effort) = work output (work done by load)

Effort × distance moved by effort = load × distance moved by load

Effort × circumference of wheel = load × circumference of axle

$$E \times 2\pi R = L \times 2\pi r$$

$$\therefore \quad \text{Velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$V.R = \frac{2\pi R}{2\pi r} = \frac{R}{r}$$

For a frictionless wheel and axle, $$M.A = V.R = \frac{R}{r}$$

$$\text{Efficiency of wheel and axle} = \frac{\text{work output}}{\text{work input}} \times 100\%$$

$$= \frac{L \times 2\pi r}{E \times 2\pi R} \times 100\%$$

$$\mathcal{E} = \frac{L}{E} \times \frac{r}{R} \times 100$$

Example 15

A wheel and axle is used to raise a load of 500N by the application of an effort of 250N. If the radii of the wheel and the axle are 0.4cm and 0.1cm respectively, the efficiency of the machine is? A. 60% B. 50% C. 40% D. 20% *JAMB 2002*[6]

Solution

Load, L = 500N; E = 250N; Radius of wheel, R = 0.4cm; Radius of axle, r = 0.1cm

Efficiency, $$\mathcal{E} = \frac{L}{E} \times \frac{r}{R} \times 100\%$$

$$\mathcal{E} = \frac{500}{250} \times \frac{0.1}{0.4} \times 100\% = \frac{5000}{100} = 50\%$$

Example 16
In a wheel and axle mechanism, the diameter of the wheel and axle are 40cm and 8cm respectively. Given that the machine is 80% efficient, what effort is required to lift a load of 100N? A. 20N B. 25N C. 50N D. 80N *JAMB 1998*[12]

Solution

Radius $= \frac{\text{diameter}}{2}$, radius of wheel, $R = \frac{40}{2} = 20\text{cm}$ Load, $L = 100N$;

Radius of axle, $r = \frac{8\text{cm}}{2}$; efficiency, $\mathcal{E} = 80\%$ Effort, $E = ?$

$$\mathcal{E} = \frac{L}{E} \times \frac{r}{R} \times 100\% \quad \therefore \text{ Effort}, E = \frac{L \times r \times 100}{\mathcal{E} \times R}$$

$$E = \frac{100 \times 4 \times 100}{80 \times 20} = \frac{40000}{1600} = 25N$$

5. Gear Wheels

Gear wheels are used in cars and bikes. In gear wheels, the effort and load are applied to the shafts of the gears.
Therefore:

$$\text{Velocity ratio} = \frac{\text{number of teeth in driven wheel}}{\text{number of teeth in driving wheel}}$$

Example 17
A 20-toothed gear wheel drives a 60 toothed one. If the angular speed of the smaller wheel is 120 rev s^{-1}, the angular speed of the larger wheel is?
A. 3 rev s^{-1} B. 40 rev s^{-1} C. 360 rev s^{-1} D. 2400 rev s^{-1} *JAMB 1988*[8]

Solution

$$\frac{\text{Number of teeth of driving wheel}}{\text{Number of teeth of driven wheel}} = \frac{\text{speed of driving wheel}}{\text{speed of driven wheel}}$$

$$\frac{20}{60} = \frac{120}{\text{speed of driven wheel}}$$

$$\therefore \text{ Speed of driven or larger wheel} = \frac{60 \times 120}{20} = \frac{7200}{20} = 360\ rev\ s^{-1}$$

EXERCISES 7.

1. A machine has an efficiency of 60%. If the machine is required to overcome a load of 30N with a force of 20N, calculate its mechanical advantage. *WAEC 1995*[9] *Ans: 15*
2. A simple machine with an efficiency of 75% lifts a load of 5000N when a force of 500N is applied to it. Calculate the velocity ratio of the machine.
WAEC 2001[13] *Ans: 13.33*

3. A machine has a velocity ratio of 6 and an efficiency of 75%. Calculate the effort needed to raise a load of 90N. *NECO 2002 Ans: 20N*

4. A machine whose efficiency is 60% has a velocity ratio of 5. If a force of 500N is applied to lift a load P, what is the magnitude of P?
A. 750N B. 4166N C. 500N D. 1500N *JAMB 2004[11] Ans:1500N*

5. A machine has a velocity ratio of 4. If it requires 800N to overcome a load of 1600N, what is the efficiency of the machine?
A. 2% B. 40% C. 60% D. 50% *JAMB 2007[29] Ans:50%*

6. A block and tackle system has 6 pulleys. If the efficiency of the machine is 60%, determine its mechanical advantage. *WAEC 2002[17] Ans: 3.6*

7. A block and tackle system has six pulleys. A force of 50N applied to it lifts a load of weight W. If the efficiency of the system is 40%, calculate W.
WAEC 2006[21] Ans:120N

8. A block system of five pulleys has an efficiency of 70%. Calculate the effort that will be required to raise a load of 42.00N. *NECO 2000[15] Ans: 12N*

9. The figure 7.5 represent a block and tackle pulley system on which an effort of W newtons supports a load of 120N. IF the efficiency of the machine is 40%, then the value of W is
A. 28.0N B. 48.0N C. 50.0N
D. 288.0N *JAMB 1986[12] Ans: 50.0N*

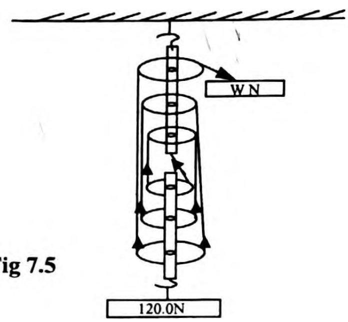


Fig 7.5

10. The diagram in Fig 7.6 is a block-and-tackle pulley system in which an effort of 80N is used to lift a load of 240N. The efficiency of the machine is?
A. 60% B. 50% C. 40%
D. 33% *JAMB 2001[4] Ans: 50%*

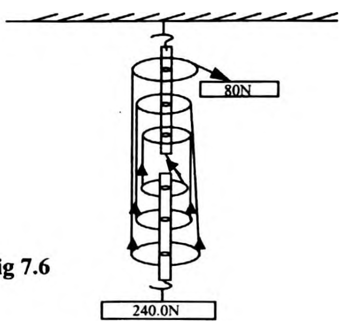


Fig 7.6

11. A plane inclined at an angle of 30^o to the horizontal has an efficiency of 50%. The force parallel to the plane required to push a load of 120N uniformly up the plane is?
WAEC 1990[5] Ans: 120N

12. A body of mass 7.5kg is to be pulled up along a plane which is inclined at 30^o to the horizontal. If the efficiency of the plane is 75%, what is the minimum force required to pull the body up the plane? [$g=10ms^{-2}$]. *WAEC 1995[4] Ans: 50N*

13. What is the velocity ratio of a plane which is inclined at an angle of 30^o to the horizontal? *NECO 2004[14] Ans: 2*

14. A plane inclined at an angle of 30^o to the horizontal has an efficiency of 60%. The force parallel to the plane required to push a load of 120N uniformly up the plane is
A. 60NB. 100N C. 120N D. 200N E. 240N *JAMB 1984[20] Ans: 100N*

15. An inclined plane which makes an angle of 30^o with the horizontal has a velocity ratio of? A. 2 B. 1 C. 0.866 D. 0.50 *JAMB 1990[12] Ans: 2*

16. A plane inclined at angle θ has a velocity ratio of 10:1. The inclination of the plane to the horizontal is given by A. $\tan\theta = {}^{1}/_{10}$ B. $\cot\theta = {}^{1}/_{10}$ C. $\cos\theta = {}^{1}/_{10}$ D. $\sin\theta = {}^{1}/_{10}$ *JAMB 1993*[9] *Ans: $\sin\theta = {}^{1}/_{10}$*

17. A 20kg mass is to be pulled up a slope inclined at 30° to the horizontal. If the efficiency of the plane is 75%, the force required to pull the load up the plan is
A. 13.3N B. 73.5N C. 133.3N D. 533.2N [$g = 10ms^{-2}$]
JAMB1991[10] *Ans: 133.3N*

18. A man pulls up a box of mass 70kg using an inclined plane of effective length 5m onto a platform 2.5m high at a uniform speed. If the frictional force between the box and the plane is 1000N;

(i) draw a diagram to illustrate all the forces acting on the box while in motion. (see Example 11)

(ii) Calculate the

I. minimum effort applied in pulling up the box.

II. Velocity ratio of the plane if it is inclined at 30° to the horizontal

III. Force ratio of the plane

WAEC 2007[x11] *Ans: I. 1350N II. 2 III. 0.52*

19. A screw jack, 25% efficient and having a screw pitch 0.4cm is used to raise a load through a certain height. If in the process, the handle turns through a circle of radius 40cm, calculate :

(i) velocity ratio of the machine

(ii) mechanical advantage of the machine

(iii) effort required to raise a load of 1000N with the machine.

[Take $\pi = 3.14$] *WAEC 2002*[x11] *Ans: (i) 628 (ii) 157(iii) 6.37N*

20. Calculate the velocity ratio of a screw jack of pitch 0.3cm if the length of the Tommy bar is 21cm. A. ${}^{1}/_{140\pi}$ B. 14π C. 70π D. 140π
JAMB 1994[12] *Ans: 140π*

21. A wheel and axle of radii 800mm and 200mm respectively is used to raise a body of weight 800N by the application of 250N. Calculate the efficiency of the machine.
WAEC 2005[7] *Ans: 80%*

22. A wheel and axle have radii 80cm and 10cm respectively. If the efficiency of the machine is 0.85, an applied force of 1200N to the wheel will raise a load of
A. 8.0N B. 6.8N C. 8160.0N D. 9600.0N *JAMB 1991*[8] *Ans: 8160N*

23. In an ideal wheel and axle system, R stands for the radius of the wheel and r is the radius of the axle. The mechanical advantage is A. ${}^{R}/_{r}$ B. $({}^{r}/_{R})^2$ C. ${}^{r}/_{R}$ D. $({}^{R}/_{r})^2$ *JAMB 2005*[27] *Ans: ${}^{R}/_{r}$*

24. Which of the following levers has the greatest mechanical advantage.

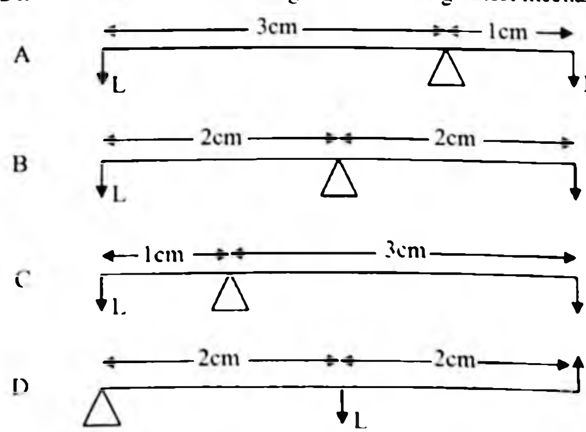


Fig 7.7

JAMB 1995 *Ans: C*

25. A particle of weight 120N is placed on a plane inclined at an angle 30^0 to the horizontal. If the plane has an efficiency of 60%, what is the force required to push the weight uniformly up the plane?

A. 175N B. 50N C. 210N D. 100N *JAMB 2007 Ans: 100N*

26. Calculate the velocity ratio of a plane inclined at an angle of 60% to the horizontal. *NECO 2007[c3] Ans: 1.15*

27. Which of the following equation for the efficiency of a machine is correct?

A. $Efficiency = \frac{velocity\ ratio}{mechanical\ advantage} \times 100\%$ B. $Efficiency = \frac{input}{output} \times 100\%$

C. $Efficiency = \frac{load\ distance}{effort\ distance} \times 100\%$ D. $Efficiency = \frac{mechanical\ advantage}{velocity\ ratio} \times 100\%$

WAEC 2008[17] Ans: D

28. The efficiency of the pulley systems shown in Fig 7.8 is 80%. Find the effort E required to lift a load of 1200N.

A. 275N B. 325N C. 375N
D. 573N *JAMB2008[14] Ans: 375N*

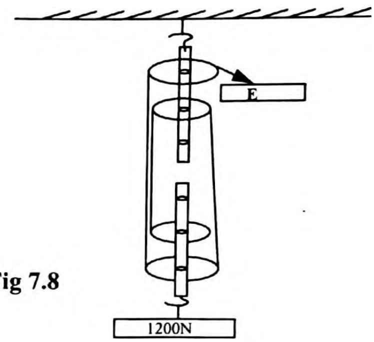


Fig 7.8

29. If a heavy barrel is rolled up a plane inclined at 30° to the horizontal, its velocity ratio will be

A. 3.0 B. 3.1 C. 3.2 D. 2.0 JAMB 2009[12] Ans: D

30. A machine of efficiency 80% is used to lift a box. If the effort applied by the machine is twice the weight of the box, calculate the velocity ratio of the machine.

WAEC 2009[16] Ans: 0.63

31. An effort ***P*** applied at one end of a crowbar just overcomes the resistance ***W*** at the lid of the tin. The mechanical advantage of the crowbar is expressed as

A. $W + P$ B. $P - W$ C. W/P D. P/W E. WP

NECO 2009[14] Ans:C

8

TEMPERATURE AND ITS MEASUREMENT

TEMPERATURE SCALES

Temperature is the degree of hotness or coldness of a body and is a measure of the average kinetic energy of the molecules in a body. Temperature is measured by different types of thermometers in any of these scales.

1. Absolute or Kelvin scale.
2. Celsius or centigrade scale.
3. Fahrenheit scale.

No matter the scale used, every thermometer has an upper fixed point (steam point), a lower fixed point (ice point) and a fundamental interval as shown in figure 8.1.

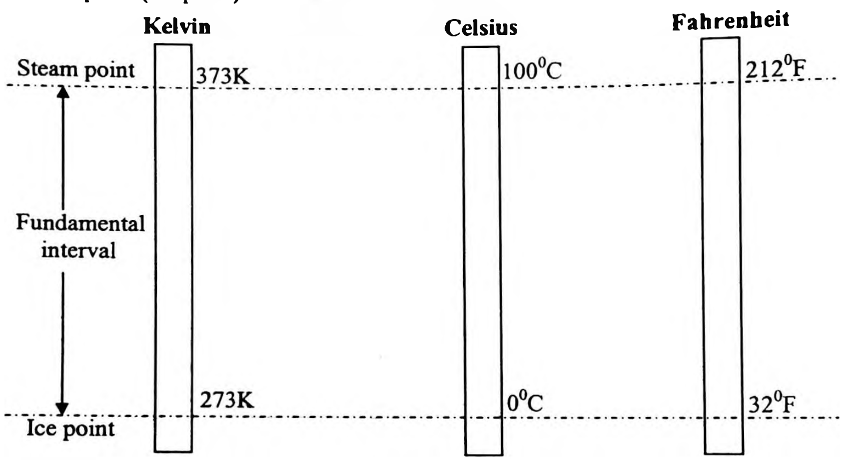


Fig 8.1

Fundamental interval = steam point − ice point.
E.g fundamental interval for Kelvin scale is 373 – 273 = 100K

Conversion Between Temperature Scales:

(i) To convert from Fahrenheit (°F) to Celsius (°C), use this formula.

$$°C = \frac{5(°F - 32)}{9}$$

Example 1

What is the equivalent of a temperature of 120^0F in degree Celsius?

$$°F = 120°F \quad \therefore \quad °C = \frac{5(120 - 32)}{9} = \frac{5 \times 88}{9} = 48.89°C$$

(ii) To convert from degree centigrade (°C) to degree Fahrenheit (°F), use this formula.

$$°F = \frac{9°C}{5} + 32 \quad or \quad °F = 1.8°C + 32$$

Example 2

The freezing point of alcohol is −115°C. What is this temperature in degree Fahrenheit?

$$°C = -115°C \quad \therefore \quad °F = 1.8 \times (-115) + 32$$

$$= -207 + 32$$
$$= -175°F$$

(iii) To convert Celsius (^{0}C) to Kelvin and Kelvin (K) to Celsius, use these formulas;

$$K = °C + 273 \qquad \text{and} \qquad °C = K - 273$$

Example 3
The temperature of a body increases from 30^{0}C to 70^{0}C, what is the temperature change of the body in Kelvin?

Temperature change = 70°C − 30°C = 40°C

°C = 40°C $\therefore$ temperature in Kelvin, $K = 40 + 273 = 313K$

THERMOMETRIC CALCULATIONS

Liquid –in-Glass Thermometers

Even if a liquid-in-glass thermometer is ungraduated, faulty or it's graduated in millimeter instead of a particular scale (°C, °F, K), it can still be used to measure temperature if its reading is compared ("corresponded") with the reading of a scaled functioning thermometer. The following examples will illustrate how.

Example 4
A liquid-in-glass thermometer is graduated in millimeters. The upper and lower fixed points of the thermometer are 230mm and 80mm respectively. What is the temperature in degree centigrade when the thermometer reads 95mm.

Solution
Because the answer is required in the Celsius or Centigrade scale, the thermometer is "corresponded" or compared with the Celsius scale as follows.
Let T be the recorded temperature.

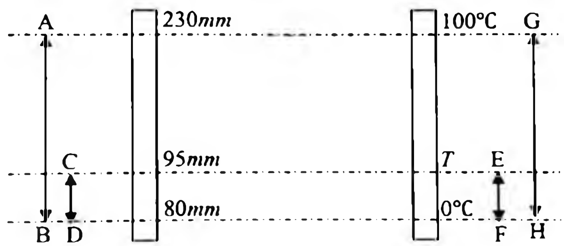


Fig 8.2

Take proportions as follows from fig 8.2

$$\frac{CD}{AB} = \frac{EF}{GH}$$

$$\frac{C - D}{A - B} = \frac{E - F}{G - H}$$

Substituting letters for figures we have,

$$\frac{95mm - 80mm}{230mm - 80mm} = \frac{T - °C}{100°C - 0°C}$$

$$\frac{15mm}{150mm} = \frac{T}{100°C}$$

Cross multiplying, we have

$$150mm \times T = 15mm \times 100$$

$$T = \frac{15mm \times 100°C}{150mm} = 10°C$$

Example 5

A temperature scale has an upper fixed point of 260mm and a lower fixed point of 50mm. What will be the reading on this scale when a thermometer reads 125^0F.

Solution

In this case, the temperature scale is compared with or 'corresponded' to the Fahrenheit scale.

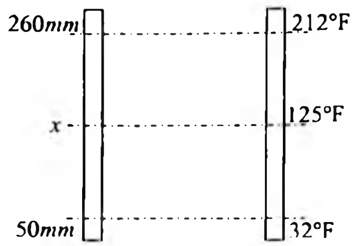


Fig 8.3

Taking proportions in similar manner with example 4, we have

$$\frac{x - 50mm}{260mm - 50mm} = \frac{125°F - 32°F}{212°F - 32°F}$$

$$\frac{x - 50mm}{210mm} = \frac{93°F}{180°F}$$

$$180(x - 50) = 210 \times 93$$

$$180x - 9000 = 19530$$

$$180x = 19530 + 9000$$

$$180x = 28530$$

$$x = \frac{28530}{180} = 158.5mm$$

Example 6

The ice point and steam point of a mercury-in-glass thermometer are marked X and 180mm respectively. During an experiment, the mercury meniscus in the thermometer reads 45mm. If the corresponding reading on a Kelvin scale is 283K, calculate the value of X.

Solution

The thermometer scale is corresponded with the Kelvin scale.

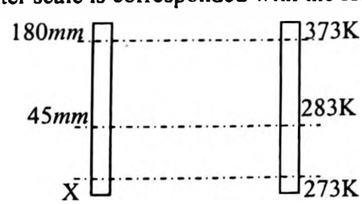


Fig 8.4

Taking proportions, $\dfrac{45mm - X}{180mm - X} = \dfrac{283K - 273K}{373K - 273}$

$$\frac{45 - X}{180 - X} = \frac{10}{100}$$

Cross multiplying,

$$100(45 - X) = 10(180 - X)$$
$$4500 - 100X = 1800 - 10X$$
$$4500 - 1800 = 100X - 10X$$
$$2700 = 90X$$
$$X = \frac{2700}{90}$$
$$X = 30mm$$

Example 7

The ice and steam points of a certain thermometer are -20^0 and 100^0 respectively. Calculate the Celsius temperature corresponding to 70^0 on the thermometer. *WAEC 2002*

Solution

The thermometer scale is corresponded with the Celsius scale.

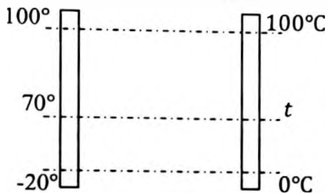


Fig 8.5

Taking proportions; $\dfrac{70° - (-20°)}{100° - (-20°)} = \dfrac{t - 0°C}{100°C - 0°C}$

$$\frac{70 + 20}{100 + 20} = \frac{t}{100}$$
$$\frac{90}{120} = \frac{t}{100}$$
$$120t = 90 \times 100$$
$$t = \frac{9000}{120}$$
$$t = 75°C$$

Example 8

The ice and steam points of a faulty mercury-in-glass thermometer are 0.3^0C and 99.2^0C respectively. When used to measure the temperature of a medium, it recorded 47.5^0C. What is the correct temperature of the medium?

Solution

The fixed point and recorded reading of the faulty or inaccurately calibrated thermometer is corresponded with a correct or accurately calibrated Celsius thermometer as follows.

Let θ be the correct temperature when the faulty thermometer reads 47.5^0C.

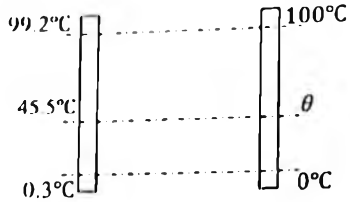


Fig 8.6

Taking proportions;

$$\frac{47.5°C - 0.3°C}{99.2 - 0.3°C} = \frac{\theta - 0°C}{100°C - 0°C}$$

$$\frac{47.2}{98.9} = \frac{\theta}{100}$$

$$\therefore \quad \theta \times 98.9 = 47.2 \times 100$$

$$\theta = \frac{4720}{98.9} = 47.7°C$$

Example 9

The fundamental interval of a Celsius thermometer is 250mm. What is the temperature when the mercury level is 45mm above the ice point.

Solution

Remember that the fundamental interval of a thermometer is the difference between the steam point and the ice point. The thermometer is corresponded with the Celsius scale.

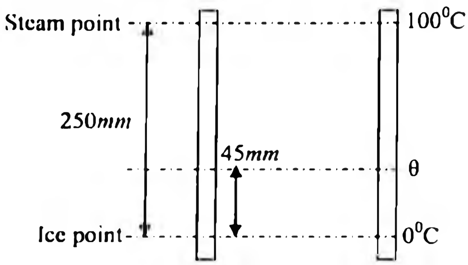


Fig 8.7

Taking proportions;

$$\frac{45mm}{250mm} = \frac{\theta - 0°C}{100°C - 0°C}$$

$$\frac{45}{250} = \frac{\theta}{100°C}$$

$$\therefore \quad \theta = \frac{45mm \times 100°C}{250mm} = 18°C$$

Resistance Thermometer

There is a proportionally direct relation between the resistance of metallic conductor (e.g platinum) and its temperature. Because there is a corresponding relationship between resistance and temperature, we can apply the same procedure used in liquid-in-glass thermometer. In addition we could use the following equation to find temperature θ when the resistance is R.

$$\theta = \frac{R - R_0}{R_{100} - R_0} \times 100°C$$

Where R_{100} and R_0 are the resistance at the upper fixed point (100^0C) and the lower fixed point (0^0C) respectively.

Example 10

A resistance thermometer has a resistance of 10Ω at 0^0C and 70Ω at 100^0C. If it records 55Ω in a certain medium, calculate the temperature of the medium.

Solution

The resistance thermometer can be corresponded with the Celsius scale as follows.

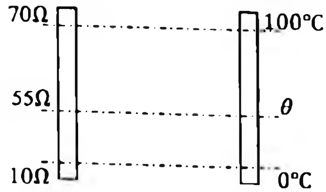


Fig 8.8

Taking proportions;

$$\frac{55\Omega - 10\Omega}{70\Omega - 10\Omega} = \frac{\theta - 0°C}{100°C - 0°C}$$

$$\frac{45}{60} = \frac{\theta}{100}$$

Cross multiplying,

$$\theta \times 60 = 45 \times 100$$

$$\theta = \frac{4500}{60} = 75°C$$

Alternatively, we could use the equation for resistance thermometer.

$$\theta = \frac{R - R_0}{R_{100} - R_0} \times 100°C$$

$R = 55\Omega$, $R_0 = 10\Omega$, $R_{100} = 70\Omega$

$$\therefore \quad \theta = \frac{55 - 10}{70 - 10} \times 100°C$$

$$= \frac{45}{60} \times 100 = 75°C$$

Example 11

A platinum resistance thermometer records a resistance of 4Ω at 32^0F and 10Ω at 212^0F. If resistance changes uniformly with temperature, what is the resistance of the thermometer when the temperature is 75^0F.

Solution

The resistance thermometer is corresponded with the Fahrenheit scale as follows

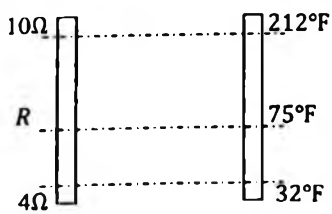


Fig 8.9

Taking proportions; $$\frac{R - 4\Omega}{10\Omega - 4\Omega} = \frac{75°F - 32°F}{212°F - 32°F}$$

$$\frac{R - 4}{6} = \frac{43}{180}$$

Cross multiplying, $$180(R - 4) = 6 \times 43$$

$$180R - 720 = 258$$

$$180R = 258 + 720 = 978$$

$$R = \frac{978}{180} = 5.43\Omega$$

Example 12
The lower fixed point and upper fixed point of a resistance thermometer are 0.5Ω and 3.5Ω respectively. If it records 2.5Ω in a certain environment, what is the temperature of the medium in Kelvin?
Solution
The resistance thermometer is corresponded with the Kelvin scale because the temperature is required in Kelvin.

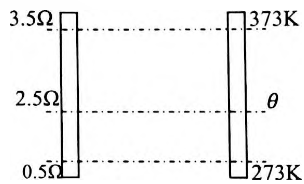


Fig 8.10

Taking proportions; $$\frac{2.5\Omega - 0.5\Omega}{3.5\Omega - 0.5\Omega} = \frac{\theta - 273K}{373K - 273K}$$

$$\frac{2}{3} = \frac{\theta - 273}{100}$$

Cross multiplying, $$3(\theta - 273) = 2 \times 100$$

$$3\theta - 819 = 200$$

$$3\theta = 200 + 819 = 1019$$

$$\theta = \frac{1019}{3} = 339.67K$$

Example 13
What is the temperature gradient across a copper rod of thickness 0.02m, maintained at two temperature junctions of 20^0C and 80^0C respectively. A. $3.0 \times 10^2 Km^{-1}$
B. $3.0 \times 10^3 Km^{-1}$ C. $5.0 \times 10^3 Km^{-1}$ D. $3.0 \times 10^4 Km^{-1}$ *JAMB 1999*[26]
Solution

$$\text{Temperature gradient} = \frac{\text{Difference in temperature}}{\text{Difference in length}}$$

$$= \frac{80 - 20}{0.02} = \frac{60}{0.02} = 3 \times 10^3 °Cm^{-1} \quad or \quad 3 \times 10^3 Km^{-1}$$

Gas Thermometers

There is a linear or directly proportional relation between the temperature of a constant volume gas thermometer and its pressure. As temperature increases, the pressure of the gas increases. Also, as temperature decreases, the pressure of the gas decreases.

Calculations involving gas thermometers are done in similar ways like those of liquid-in-glass and resistance thermometers.

Example 14

A constant volume gas thermometer records a pressure of 320mmHg at the ice point and 400mmHg at the steam point. What would the temperature be in degree Celsius when the gas thermometer records a pressure of 360mmHg?

Solution

The fixed points pressures (320mmHg and 400mmHg) and the recorded pressure (360mmHg) on the gas thermometer is "corresponded" with the Celsius scale as shown below.

Let θ be the recorded temperature.

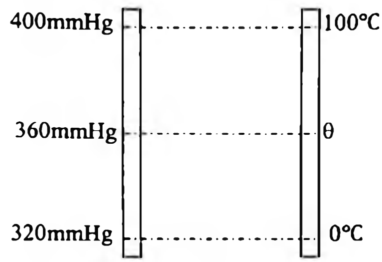


Fig 8.11

Taking proportions;
$$\frac{360\text{mmHg} - 320\text{mmHg}}{400\text{mmHg} - 320\text{mmHg}} = \frac{\theta - {}^\circ\text{C}}{100^\circ\text{C} - 0^\circ\text{C}}$$

$$\frac{40}{80} = \frac{\theta}{100}$$

Cross multiplying,
$$80 \times \theta = 40 \times 100 = 400$$

Therefore,
$$\theta = \frac{400}{80} = 50^\circ\text{C}$$

Example 15

A constant volume gas thermometer is used to measure the room temperature during an experiment. If the gas pressure at 273K is 220mmHg and 290mmHg at 373K, what is the room temperature if the thermometer records a pressure of 234mmHg?

Solution

The lower and upper fixed points pressures (220 and 290mmHg) and the recorded pressure (234mmHg) of the gas thermometer is "corresponded" with the Kelvin scale.

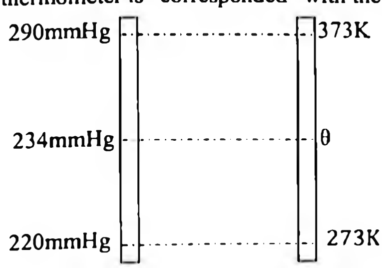


Fig 8.12

Taking proportions; $\frac{234mmHg - 220mmHg}{290mmHg - 220mmHg} = \frac{\theta - 273K}{373K - 273K}$

$$\frac{14}{70} = \frac{\theta - 273}{100}$$

Cross multiplying, $70(\theta - 273) = 14 \times 100$

$$70\theta - 19110 = 1400$$

$$70\theta = 1400 + 19110 = 20510$$

$$\theta = \frac{20510}{70} = 293K$$

EXERCISES 8.

1. A thermometer has its stem marked in millimeter instead of degree Celsius. The lower fixed point is 30mm and the upper fixed point is 180mm. Calculate the temperature in degree Celsius when the thermometer reads 45mm. *WAEC 1989 Ans: 10^0C*
2. A mercury-in-glass thermometer reads -20^0C at the ice point and 100^0 at the steam point. Calculate the Celsius temperature corresponding to 70^0 on the thermometer. *WAEC 1995 Ans: 75^0C*
3. A platinum-resistance thermometer has a resistance of 5Ω at 0^0C and 9Ω at 100^0C. Assuming that resistance changes uniformly with temperature, calculate the resistance of the thermometer when the temperature is 45^0C. *WAEC 1996 Ans: 6.8Ω*
4. An object is heated from 30^0C to 57^0C. What is the increase in its temperature on the Kelvin scale? *WAEC 1998 Ans: 300K*
5. The lower and upper fixed points of a mercury-in-glass thermometer are marked Y and 180mm respectively. On a particular day the mercury meniscus in the thermometer rises to 45mm. If the corresponding reading on a Celsius scale is 10^0C, calculate the value of Y. *WAEC 1998 Ans: 30mm*
6. The ice point of an ungraduated mercury-in-glass thermometer is X, while its steam point is 90^0. This thermometer reads 60^0 when the true temperature is 40^0C. Calculate the value of X. *WAEC 1999 Ans: 40^0*
7. A resistance thermometer has a resistance of 20Ω at 0^0C and 85Ω at 100^0C. If its resistance is 52Ω in a medium, calculate the corresponding temperature. *WAEC 2001 Ans: 49.2^0C*
8. Which of the following value on the absolute scale of temperature is the ice point?
A. 0K B. 32K C. 100K D. 273K *WAEC 2004 Ans: 273K*
9. A platinum resistance thermometer has a resistance of 4Ω at 0^0C and 10Ω at 100^0C. Assuming the resistance changes uniformly with temperature, calculate the resistance of the thermometer when the temperature is 45^0C. *WAEC 2003 Ans: 6.7Ω*
10. The lower and upper fixed points of a thermometer are 30mm and 180mm respectively. Calculate the temperature in degree Celsius when the thermometer reads 45mm. *WAEC 2005 Ans: 10^0C*
11. A platinum-resistance thermometer has a resistance of 10Ω at 0^0C and 18Ω at 100^0C. Assuming that resistance changes uniformly with temperature, what is the resistance of the thermometer when the temperature is 45^0C. *NECO 2002 Ans: 13.6^0C*
12. What is the equivalent of a temperature of 20^0C in degree Fahrenheit? A. 36^0F
B. 68^0F C. 11.1^0F D. 43.1^0F E. 25^0F *JAMB 1978 Ans: 68^0F*
13. The ice and steam points of a mercury-in-glass thermometer of centigrade scale and of uniform bore correspond respectively to 3cm and 19cm lengths of the mercury thread. When the length is 12cm, what will the temperature be? A. 32^0C B. 48^0C
C. 56^0C D. 65^0C E. 75^0C *JAMB 1981 Ans: 56^0C*

14. The distance between the fixed points of a centigrade thermometer is 20cm. What is the temperature when the mercury level is 45cm above the lower mark?
A. 22.5^0C B. 29.0^0C C. 90.0^0C D. 100.0^0C *JAMB 1987 Ans: 22.5^0C*

15. The resistance of a platinum wire at the ice and steam points are 0.75Ω and 1.05Ω respectively. Determine the temperature at which the resistance of the wire is 0.90Ω.
JAMB 1987 Ans: 22.5^0C

16. The resistance of a platinum wire at the ice and steam points are 0.75Ω and 1.05Ω respectively. Determine the temperature at which the resistance of the wire is 0.9Ω
A. 43.0^0C B. 50.0^0C C. 69.9^0C D. 87.0^0C *JAMB 1990 Ans: 50^0C*

17. A thermometer with an arbitrary scale, S, of equal division registers -30^0S at the ice point and $+90^0S$ at the steam point. Calculate the Celsius temperature corresponding to 60^0S. A. 25.0^0C B. 50.0^0C C. 66.7^0C D. 75.0^0C *JAMB 1991 Ans: 75^0C*

18. A platinum resistance thermometer wire has a resistance of 5Ω at 0^0C and 5.5Ω at 100^0C. Calculate the temperature of the wire when the resistance is 5.2Ω.
A. 80^0C B. 60^0C C. 40^0C D. 10^0C *JAMB 1992 Ans: 40^0C*

19. The melting point of naphthalene is 78^0C. What is this temperature in Kelvin?
A. 100K B. 351K C. 378K D. 444K *JAMB 1994 Ans: 351K*

20. A temperature scale has a lower fixed point of 40mm and an upper fixed point of 200mm. What is the reading on this scale when a thermometer reads 60^0C?
A. 33.3mm B. 36.0mm C. 96.0mm D. 136.0mm *JAMB 1995 Ans: 136mm*

21. A platinum resistance thermometer records 3.0Ω at 0^0C and 8.0Ω at 100^0C. If it records 6.0Ω in a certain environment, what is the temperature of the medium?
A. 80^0C B. 60^0C C. 50^0C D. 30^0C *JAMB 1998 Ans: 60^0C*

22. A platinum-resistance thermometer has a resistance of 4Ω at 0^0C and 12Ω at 100^0C. Assuming that the resistance changes uniformly with temperature, calculate the resistance of the thermometer when the temperature is 45^0C. *NECO 2003 Ans: 7.6Ω*

23. The electrical resistances of the element in a platinum resistance thermometer at 100^0C, 0^0C and room temperature are 75.000, 63.000 and 64.992Ω respectively. Use these data to determine the room temperature. *WAEC 2004 Ans: 16.6^0C*

24. A temperature scale has a lower fixed point of 40mm and an upper fixed point of 200mm. What is the reading on this scale when a thermometer reads 60^0C.
JAMB 1995 Ans: 136.0mm

25. A platinum-resistance thermometer has a resistance of 4Ω at 0^0C and 12Ω at 100^0C. Assuming that the resistance changes uniformly with temperature, calculate the resistance of the thermometer when the temperature is 45^0C. *NECO 2003 Ans: 7.6Ω*

26. The ice and steam points on a mercury-in-glass thermometer are found to be 90.0mm apart. What temperature is recorded in degree Celsius when the length of the mercury thread is 33.6mm above the ice point mark. *WAEC 1994 Ans: 37.33^0C*

27. The height of the mercury thread in a mercury-in-glass thermometer when in melting ice and then in steam are 3cm and 18cm respectively. What would be the height of the thread at a temperature of 60^0C? A. 7.5cm B. 9cm C. 10.8cm D. 12cm E. 12.6cm *JAMB 1979 Ans: 12cm*

28. The lower and upper fixed points marked on a mercury-in-glass thermometer are 210mm apart. The end of the mercury column in the tube is 49mm above the lower fixed point in a room. What is the temperature of the room in degree Celsius? A. 55.3^0C B. 23.3^0C C. 49.0^0C D. 16.1^0C E. 76.7^0C *JAMB 1984 Ans: 23.3^0C*

29. The pressure on the gas of constant gas thermometer at the ice point is 325mm of mercury and at the steam point 875mm of mercury. Find the temperature when the pressure of the gas is 490mm of mercury. A. 30K B. 243K C. 300K D. 303K *JAMB 1989 Ans:303K*

30. The length of mercury thread when it is at 0^0C, 100^0C and at an unknown temperature θ is 25mm, 225mm and 175mm respectively. What is the value of θ?
A. 85.0^0C B. 80.0^0C C. 75.0^0C D. 70.0^0C *JAMB 1997 Ans: 75^0C.*

31. The readings on the pressure scale at the steam and ice points are 800mmHg and 300mmHg respectively. Determine the corresponding temperature in degree Celsius when it reads 450mmHg. *NECO 2007*[22] *(Ans: 30°)*

32. The ice and steam points on a mercury-in-glass thermometer are 10cm and 30cm respectively. Calculate the temperature in degree Celsius, when the mercury meniscus is at the 14cm mark. *WAEC 2007 (Ans: 20°C)*

33. The lower and upper fixed points of a thermometer are X_0 *and* X_{100} respectively. A temperature t on the Celsius scale corresponding to X_t on the scale is given by

A. $\frac{100(X_{100}-X_0)}{X_t-X_0}$ B. $\frac{X_t-X_0}{100(X_{100}-X_0)}$ C. $\frac{X_t-X_0}{X_{100}-X_0}$ D. $\frac{100(X_0-X_t)}{(X_{100}-X_t)}$

E. $\frac{100(X_t-X_0)}{X_{100}-X_0}$ *NECO 2008*[17] *Ans: E*

34. A faulty thermometer registers 102.5°C at 100°C. If the thermometer has no zero error, what will it register at 55°C? WAEC 2009[17] Ans: 56.4°C

9

MEASUREMENT OF HEAT ENERGY

SPECIFIC HEAT CAPACITY AND HEAT CAPACITY

When heat is added to or removed from a body, the temperature change experienced depends on the mass of the body and its specific heat capacity as illustrated by this equation:

$$Q = mc(\theta_2 - \theta_1) \qquad or \qquad Q = mc\Delta\theta$$

Where Q = quantity of heat energy in Joule(J)
m = mass of substance in kilograms (kg)
θ_2 = final temperature in degree Celsius(°C) or Kelvin (K)
θ_1 = initial temperature in degree Celsius(°C) or Kelvin (K)
c = specific heat capacity in $Jkg^{-1}K^{-1}$ or $Jkg^{-1}°C^{-1}$

The heat capacity (thermal capacity), C, of a substance is defined as the heat required to raise the temperature of the substance by 1°C or 1K. The S.I. unit is $J°C^{-1}$ or JK^{-1}.

The specific heat capacity, c of a substance is defined as the heat required to raise the temperature of a unit mass of the substance by 1K or 1°C. The S.I. unit is $Jkg^{-1}K^{-1}$ or $Jkg^{-1}°C^{-1}$

Heat capacity (C) is related to specific heat capacity (c) as follows:

Heat capacity = mass × specific heat capacity

$$C = mc$$

Quantity of heat (Q) is also related to heat capacity as follows:

Quantity of heat = heat capacity × temperature change

$$Q = C(\theta_2 - \theta_1) \qquad \text{or} \qquad Q = C\Delta\theta$$

Example 1
What is the quantity of heat that will be given out if a bar of brass of mass 350g is cooled from 95°C to 20°C. [specific heat capacity of brass = 380 $Jkg^{-1}K^{-1}$]
Solution
Mass, m = 350g = 0.35kg; c = 380 $Jkg^{-1}K^{-1}$; Tempt change, $\Delta\theta$ = 95 – 20 = 75°C

Heat energy given out = mass of substance × specific heat capacity × temperature change

$$Q = mc\Delta\theta$$

$$Q = 0.35 \times 380 \times 75 = 9975\,J$$

Example 2
How much heat energy will be needed to change the temperature of 275g of paraffin oil by 75K [specific heat capacity of paraffin oil = 2130$Jkg^{-1}K^{-1}$]
Solution
Mass, m = 275g = 0.275kg; temperature change, $\Delta\theta$ = 75K
Specific heat capacity, c = 2130 $Jkg^{-1}K^{-1}$;
Substituting into $Q = mc\Delta\theta$, we have

$$Q = 0.275 \times 2130 \times 75 = 43931.25\,J$$

Example 3

Calculate the final temperature of 2kg of alcohol at 25°C when 20160 Joules of heat energy is added to it. [specific heat capacity of alcohol = 2520 $Jkg^{-1}K^{-1}$]

Solution

Quantity of heat, Q = 20160J; mass of alcohol, m = 2kg; final tempt., θ_2 = ?
initial temperature, θ_1 = 25°C; specific heat capacity, c = 2520 $Jkg^{-1}K^{-1}$;

Substituting into $Q = mc(\theta_2 - \theta_1)$, we have

$$20160 = 2 \times 2520(\theta_2 - 25)$$

$$20160 = 5040(\theta_2 - 25)$$

$$20160 = 5040\theta_2 - 126000$$

$$20160 + 126000 = 5040\theta_2$$

$$146160 = 5040\theta_2$$

$$\theta_2 = \frac{146160}{5040} = 29°C$$

Example 4

500g of water is heated so that its temperature rises from 30°C to 72°C in 7 minutes. Calculate the heat supplied per minute. (Specific heat capacity = 4200 $Jkg^{-1}K^{-1}$)

WAEC 1994

Solution

Mass of water, m = 500g = 0.5kg; initial temperature, θ_1 = 30°C;
final temperature, θ_2 = 72°C; specific heat capacity, c = 4200 $Jkg^{-1}K^{-1}$;

Substituting into $Q = mc(\theta_2 - \theta_1)$

$$= 0.5 \times 4200 \times (72 - 30)$$

$$= 0.5 \times 4200 \times 42 = 88200\,J$$

88200J is the total heat supplied in the 7 minutes, therefore heat supplied per minute is $\frac{88200}{7} = 12600J$

Example 5

The temperature of a piece of metal of mass 9g is raised from 10°C to 110°C when it absorbs 108J of heat energy. Determine the specific heat capacity of the metal in $Jkg^{-1}K^{-1}$

WAEC 1999

Solution

Mass of metal, m = 9g = 0.009kg; initial temperature, θ_1 = 10°C;
final temperature, θ_2 = 110°C; heat absorbed, Q = 108J.

$$Q = mc(\theta_2 - \theta_1) \quad \therefore \text{ Specific heat capacity, } c = \frac{Q}{m(\theta_2 - \theta_1)} = \frac{108}{0.009(110 - 10)}$$

$$= \frac{108}{0.009 \times 100} = 120\,Jkg^{-1}K^{-1}$$

Measurement of specific heat capacity is done by two methods viz; method of mixtures and electrical method. Therefore, the methods of calculation are also two:

Method of Mixtures

When a hotter substance and a colder substance (solid/solid, solid/liquid or liquid/liquid) come in contact with each other, the hotter substance loses heat to the colder substance. The transfer of heat from the hotter to the colder substance will continue until the two substances attain the same temperature.

Therefore, we can state the following:

Heat given out by hotter substance = Heat gained by colder substance

That is, $Q_H = Q_C$

Let θ_H be the temperature of the hotter substance

θ_C be the temperature of the colder substance

θ be the final or equilibrium temperature of both substances or mixture.

∴ The temperature change of the hotter substance is:

$$\Delta\theta_H = \theta_H - \theta$$

The temperature change of the colder substance is

$$\Delta\theta_C = \theta - \theta_C$$

Heat given out by hotter substance, $Q_H = mc\Delta\theta_H$

$$Q_H = mc(\theta_H - \theta)$$

Heat gained by colder substance, $Q_C = mc\Delta\theta_C$

$$Q_C = mc(\theta - \theta_C)$$

If heat given out = Heat gained

Then $Q_H = Q_C$

$$mc\Delta\theta_H = mc\Delta\theta_C$$

$$mc(\theta_H - \theta) = mc(\theta - \theta_C)$$

The masses and specific heat capacities of the hotter and colder substances could be different or the same.

Example 6

Calculate the final or equilibrium temperature of the mixture of 250g of water at 65°C is added to 150g of colder water at 5°C. Neglect heat absorbed by the surrounding. [specific heat capacity of water = 42000 $Jkg^{-1}K^{-1}$].

Solution

Mass of hot water, m = 250g = 0.25kg,
Temperature of hot water, θ_H = 65°C.
SHC of water, c = 4200 $Jkg^{-1}K^{-1}$,

mass of cold water, m = 150g = 0.15kg
Temperature of cold water, θ_C = 5°C
SHC of water, c = 4200 $Jkg^{-1}K^{-1}$

Let θ be the final or equilibrium temperature.

Heat given out by hot water = Heat gained by cold water

$$Q_H = Q_C$$

$$mc(\theta_H - \theta) = mc(\theta - \theta_C)$$

$$0.25 \times 4200 \times (65 - \theta) = 0.15 \times 4200 \times (\theta - 5)$$

Divide both sides by 4200 to obtain

$$0.25(65 - \theta) = 0.15(\theta - 5)$$

$$16.25 - 0.25\theta = 0.15\theta - 0.75$$

$$16.25 + 0.75 = 0.15\theta + 0.25\theta$$

$$17.0 = 0.4\theta$$

$$\theta = \frac{17.0}{0.4} = 42.5°C$$

Example 7

Hot water is added to four times its mass of water at 25°C and thoroughly stirred. If the final temperature of the mixture is 40°C, calculate the initial temperature of the hot water.

Solution

Mass of hot water = m

mass of cold water, = 4m

Temperature of hot water, θ_H

Temperature of cold water, θ_C = 25°C

SHC of hot water = c

SHC of cold water = c

Final temperature of mixture, θ = 40°C.

Heat given out by hot water = Heat gained by cold water

$$Q_H = Q_C$$

$$mc(\theta_H - \theta) = mc(\theta - \theta_C)$$

$$mc(\theta_H - 40) = 4mc(40 - 25)$$

Divide both sides by mc to obtain

$$\theta_H - 40 = 4 \times 15$$

$$\theta_H - 40 = 60$$

$$\theta_H = 60 + 40 = 100°C$$

Example 8

A tap supplies water at 15°C while another supplies water at 90°C. If a man wishes to bathe with water at 30°C, calculate the ratio of the mass of cold water to the mass of hot water.

Solution

Mass of hot water = m_H

mass of cold water, = m_C

Temperature of hot water, θ_H = 90°C

Temperature of cold water, θ_C = 15°C

SHC of water = c

SHC of water = c

Final temperature of mixture of cold/hot water, θ = 30°C.

Heat given out by hot water = Heat gained by cold water

$$Q_H = Q_C$$

$$m_H c(\theta_H - \theta) = m_C c(\theta - \theta_C)$$

Substituting and dividing both sides by c we have

$$m_H(90 - 30) = m_C(30 - 15)$$

$$60m_H = 15m_C$$

Because we are asked to find ratio of cold water to hot water, we make m_C the subject of equation.

$$m_C = \frac{60m_H}{15}$$

Rearrange as follows

$$\frac{m_C}{m_H} = \frac{60}{15} = \frac{4}{1}$$

$\therefore$ Ratio of mass of cold water to mass of hot water is 4:1

On the other hand, if you were asked to calculate the ratio of mass of hot water to mass of cold water, you would proceed as follows:

From $60m_H = 15m_C$, make m_H subject of the equation

$$m_H = \frac{15m_C}{60}$$

Rearrange; $$\frac{m_H}{m_C} = \frac{15}{60} = \frac{1}{4}$$

Therefore ratio of mass of hot water to mass of cold water is 1:4

Example 9

When 100g of liquid L_1 at 78°C was mixed with X_g of liquid L_2 at 50°C, the final temperature was 66°C. Given that the specific heat capacity of L_2 is half that of L_1, find X. A. 50g B. 100g C. 150g D. 200g *JAMB 1994*

Solution

Mass of L_1, $m_H = 100g = 0.1kg$ mass of L_2, $m_C = X$

Temperature of L_1, $\theta_H = 78°C$ Temperature of L_2, $\theta_C = 50°C$

SHC of $L_1 = c$ SHC of $L_2 = \frac{1}{2}c$

Final temperature of mixture of L_1 and L_2 is $\theta = 66°C$.

Heat given out by liquid, L_1 (Q_H) = Heat gained by liquid L_2 (Q_C)

$$Q_H = Q_C$$

$$m_Hc(\theta_H - \theta) = m_Cc(\theta - \theta_C)$$

$$0.1 \times c \times (78 - 66) = X \times \frac{1}{2}c \times (66 - 50)$$

$$0.1 \times c \times 12 = X \times \frac{1}{2}c \times 16$$

$$1.2c = X \times 8c$$

Divide through by c

$$1.2 = X \times 8$$

$$\therefore \quad X = \frac{1.2}{8} = 0.15kg \text{ or } 150g$$

Example 10

When two objects **P** and **Q** are supplied with the same quantity of heat, the temperature change in **P** is observed to be twice that in **Q**. If the masses of **P** and **Q** are the same, calculate the ratio of the specific heat capacities of **P** to **Q**. *WAEC 1994.*

Solution

P : **Q**

Same quantity of heat :. Q : Q

Temperature change, $\Delta\theta$	2θ	:	θ
P and Q have same mass	m	:	m

Specific heat capacity, $c = \dfrac{Q}{m\Delta\theta}$

Substituting into c for each	$\dfrac{Q}{2m\theta}$	:	$\dfrac{Q}{m\theta}$
	$\dfrac{1}{2}\left(\dfrac{Q}{m\theta}\right)$	:	$1\left(\dfrac{Q}{m\theta}\right)$
	$\dfrac{1}{2}$	:	1
multiply both sides by 2	1	:	2

Ratio of specific heat capacities of **P** to **Q** is 1 : 2

Example 11

Two metals **A** and **B** lose the same quantity of heat when their temperatures drop from 20°C to 15°C. If the specific heat capacity of **A** is thrice that of **B**, calculate the ratio of the mass of **A** to that of **B**. *NECO 2005*

Solution

	A	:	B
Same quantity of heat lost, Q	Q	:	Q
Same temperature change, $\Delta\theta = 20 - 15 = 5°C$ ∴	5	:	5
Specific heat capacity, c	3c	:	c

$Q = mc\Delta\theta \quad \therefore \quad m = \dfrac{Q}{c\Delta\theta}$

substituting, we have	$\dfrac{Q}{3c \times 5}$	:	$\dfrac{Q}{c \times 5}$
	$\dfrac{Q}{15c}$	:	$\dfrac{Q}{5c}$
	$\dfrac{1}{15}\left(\dfrac{Q}{c}\right)$	:	$\dfrac{1}{5}\left(\dfrac{Q}{c}\right)$
	$\dfrac{1}{15}$	:	$\dfrac{1}{5}$
multiplying both sides by 15	$15\left(\dfrac{1}{15}\right)$	:	$15\left(\dfrac{1}{5}\right)$
	1	:	3

∴ Ratio of mass of **A** to that of **B** is 1:3

Example 12

A piece of copper of mass 0.1kg is heated to 100°C and then transferred to a well lagged copper can of mass 0.05kg containing 0.2kg of water at 10°C. Calculate the final steady temperature of the mixture. [Specific heat capacity of water = $4.2\times10^3\,Jkg^{-1}K^{-1}$, Specific heat capacity of copper = $4.0\times10^2\,Jkg^{-1}K^{-1}$]. *NECO 2005*

Solution

Heat lost by copper = Heat gained by calorimeter + water

Let the final steady temperature be θ

Heat lost by copper, $Q_C = mc\Delta\theta = 0.1 \times 4.0 \times 10^2 \times (100 - \theta)$

Heat gained by calorimeter, $Q_{calorimeter} = mc\Delta\theta = 0.05 \times 4.0 \times 10^2(\theta - 10)$

Heat gained by water, $Q_{water} = mc\Delta\theta = 0.2 \times 4.2 \times 10^3(\theta - 10)$

$$\therefore \quad Q_C = Q_{calorimeter} + Q_{water}$$

$$0.1 \times 4.0 \times 10^2 \times (100 - \theta) = 0.05 \times 4.0 \times 10^2(\theta - 10) + 0.2 \times 4.2 \times 10^3(\theta - 10)$$

$$40(100 - \theta) = 20(\theta - 10) + 840(\theta - 10)$$

$$4000 - 40\theta = 20\theta - 200 + 840\theta - 8400$$

$$4000 + 8400 + 200 = 20\theta + 840\theta + 40\theta$$

$$12600 = 900\theta$$

$$\theta = \frac{12600}{900} = 14°C$$

Example 13

A metal of mass 0.5kg is heated to 100°C and then transferred to a well-lagged calorimeter of heat capacity $80JK^{-1}$ containing water of heat capacity $420JK^{-1}$ at 15°C. If the final steady temperature of the mixture is 25°C, find the specific heat capacity of the metal. A. $92\,Jkg^{-1}K^{-1}$ B. $133\,Jkg^{-1}K^{-1}$ C. $286\,Jkg^{-1}K^{-1}$ D. $877\,Jkg^{-1}K^{-1}$ *JAMB 2000*

Solution

Heat energy gained or lost = heat capacity × temperature change

$$Q = C\Delta\theta$$

Heat lost by metal = Heat gained by calorimeter + water

Heat lost by metal, $Q = C\Delta\theta = C \times (100 - 25)$

Heat gained by calorimeter, $Q_C = C\Delta\theta = 80 \times (25 - 15)$

Heat gained by water, $Q_W = C\Delta\theta = 420 \times (25 - 15)$

$$\therefore \quad Q = Q_C + Q_W$$

$$C \times 75 = (80 \times 10) + (420 \times 10)$$

$$75C = 800 + 4200$$

$$75C = 5000$$

$$C = \frac{5000}{75} = 66.67JK^{-1}$$

Heat capacity (C) = mass (m) × specific heat capacity (c)

Mass of metal, m = 0.5kg

$\therefore$ Specific heat capacity of metal, $c = \frac{C}{m} = \frac{66.67JK^{-1}}{0.5kg} = 133.33Jkg^{-1}K^{-1}$

Electrical Method

A quick revision of electrical energy and power in chapter 7 of Book 1 will be very helpful and relevant to calculation of specific heat capacity by electrical method.

When an electrical heater is used to heat up a substance or substances (liquid, solid/liquid or solid), the heat energy (Q) gained by the substance(s) is equal to the electrical energy (H) supplied by the heater. That is

Heat gained by substance = heat supplied by electrical heater

$$Q = H$$

$$mc\Delta\theta = IVt \quad \textbf{Or} \quad mc(\theta_2 - \theta_1) = IVt$$

$$mc\Delta\theta = Pt \quad \textbf{Or} \quad mc(\theta_2 - \theta_1) = Pt$$

$$mc\Delta\theta = I^2Rt \quad \textbf{Or} \quad mc(\theta_2 - \theta_1) = I^2Rt$$

$$mc\Delta\theta = \frac{V^2t}{R} \quad \textbf{Or} \quad mc(\theta_2 - \theta_1) = \frac{V^2t}{R}$$

Where m = mass of substance in kilograms (kg)
c = specific heat capacity in $Jkg^{-1}K^{-1}$ or $Jkg^{-1}{}^{\circ}C^{-1}$
$\Delta\theta$ = change in temperature in K or °C
θ_2 = final temperature after heating in K or °C
θ_1 = initial temperature before heating in K or °C
I = current in Amperes (A)
V = potential difference or voltage in volts (V)
t = duration of current flow in seconds (s)
P = IV = electrical power (power rating) in watt(W)
H = electrical heat energy in Joules (J)
R = resistance of heating coil in ohm (Ω)

Any of the above equations can be used to solve problems but this will depend on the data given in the question.

Example 14

How long will it take to heat 3kg of water from 28°C to 88°C in an electric heater taking a current of 6A from an e.m.f. source of 220V?
[specific heat capacity of water =4200 $Jkg^{-1}K^{-1}$]. *WAEC 1990*

Solution

Mass of water, m = 3kg; initial temperature, θ_1 = 28°C
Final temperature, θ_2 = 88°C; current, I = 6A
Voltage, V = 220V; c = 4200 $Jkg^{-1}K^{-1}$

Heat energy supplied by heater = heat gained by water

$$IVt = mc(\theta_2 - \theta_1)$$

$$6 \times 220 \times t = 3 \times 4200(88 - 28)$$

$$1320 \times t = 12600 \times 60$$

$$t = \frac{756000}{1320} = 572.7s$$

Example 15

How long does it take a 750W heater operating at full rating to raise the temperature of 1kg of water from 40°C to 70°C? [Take the specific heat capacity of water as 4200$Jkg^{-1}K^{-1}$ and neglect heat losses]. *WAEC 1993*

Solution

Mass of water, m = 1kg; initial temperature, $\theta_1 = 40°C$; final temperature, $\theta_2 = 70°C$; power rating of heater, P = 750W; specific heat capacity, $c = 4200\,Jkg^{-1}K^{-1}$

Heat supplied by heater = heat gained by water

$$Pt = mc(\theta_2 - \theta_1)$$

$$750 \times t = 1 \times 4200(70 - 40)$$

$$750 \times t = 4200 \times 30$$

$$t = \frac{126000}{750} = 168s$$

Example 16

A 500W heater is used to heat 0.6kg of water from 25°C to 100°C in t_1 second. If another 1000W heater is used to heat 0.2kg of water from 10°C to 100°C in t_2 seconds, find t_1/t_2

A. 50 B. 5 C. 5/4 D. 1/5 *JAMB 1998*

Solution

From $Pt = mc(\theta_2 - \theta_1)$ we make t subject of the equation.

$$t = \frac{mc(\theta_2 - \theta_1)}{P}$$

$$\text{1st instance, } t_1 = \frac{0.6 \times c \times (100 - 25)}{500} = \frac{0.6c \times 75}{500} = 0.09c$$

$$\text{2nd instance, } t_2 = \frac{0.2 \times c \times (100 - 10)}{1000} = \frac{0.2c \times 90}{1000} = 0.018c$$

$$\text{Therefore, } \frac{t_1}{t_2} = \frac{0.09c}{0.018c} = \frac{0.09}{0.018} = \frac{90}{18} = 5$$

Example 17

An immersion heater rated 400W, 220V is used to heat a liquid of mass 0.5kg. If the temperature of the liquid increases uniformly at the rate of 2.5°C per second, calculate the specific heat capacity of the liquid (Assume no heat is lost). *WAEC 2005*

Solution

Power rating of heater, P = 400W; temperature change, $\Delta\theta = 2.5°C$;
Mass of liquid, m = 0.5kg; time, t = 1 sec. (.... *rate of 2.5°C per second*);
Voltage, V = 220V.

Heat energy supplied by heater = heat gained by liquid

$$Pt = mc\Delta\theta$$

$$400 \times 1 = 0.5 \times c \times 2.5$$

$$c = \frac{400}{0.5 \times 2.5} = 320Jkg^{-1}K^{-1}$$

Example 18

An electric current of 3A flowing through an electric heating element of resistance 20Ω embedded in 1,000g of an oil raises the temperature of the oil by 10°C in 10 sec. What is the specific heat capacity of the liquid? A. 1.8 J/g B. 0.6J/g
C. 0.18$J/g°C$ D. 1.8 $J/g°C$ *E. 0.06* $J/g°C$ *JAMB 1983*

$$\text{Heat supplied by heater} = \text{heat gained by oil}$$

$$I^2Rt = mc\Delta\theta$$

$$\text{Specific heat capacity, } c = \frac{I^2Rt}{m\Delta\theta} = \frac{3^2 \times 20 \times 10}{1000 \times 10} = 0.18\,Jg^{-1}{}^{\circ}C^{-1}$$

Example 19

Calculate the power rating of an immersion heater used for 10 minutes to increase the temperature of 10kg of water by 15K [specific heat capacity of water $= 4200Jkg^{-1}K^{-1}$].

WAEC 1999

Solution

Time, t = 10min = 10 × 60sec = 600s; mass of water, m = 10kg;

Temperature change, $\Delta\theta$ = 15K; specific heat capacity of water, $c = 4200Jkg^{-1}K^{-1}$

$$mc\Delta\theta = Pt$$

$$\text{Therefore,} \quad P = \frac{mc\Delta\theta}{t} = \frac{10 \times 4200 \times 15}{600} = \frac{630000}{600} = 1050W$$

Example 20

A heating coil of resistance 20Ω connected to a 220V source is used to boil a certain quantity of water in a container of heat capacity $100JK^{-1}$ for 2 minutes. If the initial temperature of the water is 40°C, calculate the mass of the water in the container.

[specific heat capacity of water = $4.2 \times 10^3\ Jkg^{-1}K^{-1}$; assume boiling point of water = 100°C].

WAEC 2002

Solution

Note that that generated by the heater will be absorbed by the water and the container, therefore:

Heat generated by heater = heat absorbed by water + heat absorbed by container

Resistance of heater, R = 20Ω;

voltage, V = 220V;

heat capacity of container, $C = 100\,JK^{-1}$;

time, t = 2min = 2 x 60s = 120s;

final tempt., θ_2 = 100°C (boiling point of water).

initial tempt., θ_1 = 40°C;

Specific heat capacity of water = $4200Jkg^{-1}K^{-1}$

Let m be the mass of water

$$\text{Substitute into } \frac{V^2t}{R} = [mc(\theta_2 - \theta_1)]_{water} + [C(\theta_2 - \theta_1)]_{container}$$

(Remember: quantity of heat (Q) = heat capacity(C) × temperature change($\Delta\theta$))

$$\frac{220^2}{20} \times 120 = m \times 4200\,(100 - 40) + 100(100 - 40)$$

$$290400 = m \times 4200 \times 60 + 100 \times 60$$

$$290400 = m \times 252000 + 6000$$

$$290400 - 6000 = 252000m$$

$$284400 = 252000m$$

$$m = \frac{284400}{252000} = 1.13kg$$

Example 21

A 1.2KW immersion heater is used to supply energy for 75s. The energy supplied is used to heat 800g of paraffin in calorimeter of heat capacity 42.5 JK^{-1} from 20°C to 66°C. Calculate the specific heat capacity of the paraffin. NECO 2008[21]

Solution

Power rating of heater, P = 1.2KW = 1200W; time taken, t = 75s

mass of paraffin, m = 800g = 0.8kg heat capacity of calorimeter, C = 42.5 JK^{-1}

initial tempt., $\theta_1 = 20°C$ final tempt., $\theta_2 = 66°C$

specific heat capacity of the paraffin, c =?

$$Pt = [mc(\theta_2 - \theta_1)]_{paraffin} + [C(\theta_2 - \theta_1)]_{calorimeter}$$

$$1200 \times 75 = 0.8 \times c \times (66 - 20) + 42.5(66 - 20)$$

$$90000 = 0.8 \times c \times 46 + 42.5 \times 46$$

$$90000 = 36.8c + 1955$$

$$90000 - 1955 = 36.8c$$

$$88045 = 36.8c$$

$$c = \frac{88045}{36.8} = 2392.53 \cong 2393Jkg^{-1}K^{-1}$$

TEMPERATURE DIFFERENCE IN A WATERFALL

When a mass of water falls from the top of a waterfall to the bottom, the potential energy it possesses at the top is transformed to kinetic energy and internal molecular energy. These energy transformation eventually result in a slight increase in temperature at the bottom of the waterfall.

Example 22

A waterfall is 1260m high. Calculate the change in temperature of a quantity of water that falls from the top to bottom of the water fall. (Neglect heat losses to surrounding, take g as 10m/s^2 and specific heat capacity as 4200 $Jkg^{-1}K^{-1}$) *WAEC 1998*

Solution

Height of waterfall, h = 1260m; g = 10m/s^2; specific heat capacity, c = 4200 $Jkg^{-1}K^{-1}$

The following assumption can be made. All the potential energy (PE) the water possesses at the top is converted to heat energy at the bottom. Therefore,

$$\text{Potential energy} = \text{heat energy}$$

$$mgh = mc\Delta\theta$$

substituting we have, $m \times 10 \times 1260 = m \times 4200 \times \Delta\theta$

divide through by m, $10 \times 1260 = 4200 \times \Delta\theta$

$$\Delta\theta = \frac{10 \times 1260}{4200} = 3°C$$

Example 23
A piece of substance of specific heat capacity $450Jkg^{-1}K^{-1}$ falls through a vertical distance of 20m from rest. Calculate the rise in temperature of the substance on hitting the ground when all energies are converted into heat. A. 2/9 °C B. 4/9 °C C. 9/4 °C D. 9/2 °C *JAMB 1999*

Solution
Vertical distance, h = 20m; $g = 10m/s^2$; specific heat capacity, $c = 450Jkg^{-1}K^{-1}$

$$\text{Potential energy} = \text{heat energy}$$

$$mgh = mc\Delta\theta$$

substituting we have, $m \times 10 \times 20 = m \times 450 \times \Delta\theta$

divide through by m, $10 \times 20 = 450 \times \Delta\theta$

$$\Delta\theta = \frac{10 \times 20}{450} = \frac{200}{450} = \frac{4}{9}°C = 0.44°C$$

LATENT HEAT

Latent heat is defined as the heat absorbed or given out by a substance during a change of state at constant temperature.

Latent heat of fusion

Specific latent heat of fusion (l) of a substance is the quantity of heat (Q) required to change a unit mass (m) of the substance at its melting point to a liquid without a change in temperature.

That is, $Q = ml$ or $l = \frac{Q}{m}$

where Q = quantity of heat in joules (J)
m = mass of the substance in kg
l = specific latent heat of fusion in Jkg^{-1}

However, when the heat supplied (Q) is from an electric heater, the following equation apply:

From $Q = ml$ we have

$$IVt = ml$$

$$Pt = ml$$

$$I^2Rt = ml$$

$$\frac{V^2t}{R} = ml$$

See page 112 for the definition of the term on the left hand side of the above equations.

Example 24
Calculate the quantity of heat required to change 200g of ice to water at 0°C. [specific latent heat of fusion of ice = $336Jg^{-1}$]. *NECO 2006g*

Solution
Mass of ice, m = 200g; specific latent heat of fusion, $l = 336Jg^{-1}$

Quantity of heat, $Q = ml = 200 \times 336 = 67200J$.

Example 25
A 90W immersion heat is used to supply energy for 5min. The energy supplied is used to completely melt 160g of a solid at its melting point. Calculate the specific latent heat of fusion of the solid. *WAEC 1991*

Solution
Power rating of heat, P = 90W; time, t = 5mm = 5 × 60 = 300s;
mass of solid, m = 160g = 0.16kg.

$$Pt = ml \qquad \therefore \quad l = \frac{P \times t}{m} = \frac{90 \times 300}{0.16} = 168750Jkg^{-1} \; or \; 168.75Jg^{-1}$$

Example 26
What amount of current would pass through a 10Ω coil if it takes 21s for the coil to just melt a lump of ice of mass 10g at 0°C if there are no heat losses? [latent heat of fusion of ice = 336 Jg^{-1}]. *WAEC 1993*

Solution
Resistance of heating coil, R = 10Ω; time, t = 21s;
mass of ice, m = 10g; Latent heat of fusion, l = 336 Jg^{-1}

From $I^2Rt = ml$ we obtain $I^2 = \frac{ml}{Rt}$

$$\therefore \quad I = \sqrt{\frac{ml}{Rt}} = \sqrt{\frac{336 \times 10}{10 \times 21}} = \sqrt{\frac{3360}{210}} = \sqrt{16} = 4A$$

Example 27
An electric heater is used to melt a block of ice, mass 1.5kg. If the heater is powered by a 12V battery and a current of 20A flows through the coil, calculate the time taken to melt the block of ice at 0°C. [Specific latent heat of fusion of ice = 336 x 103 Jg^{-1}]

A. 76.0 min B. 35.0 min C. 21.0 min D. 2.9 min

JAMB 1989

Solution
Mass of ice, m = 1.5kg; potential difference, V = 12V; current, I = 20A

From $IVt = ml$ we derive,

$$\text{Time taken}, t = \frac{ml}{IV} = \frac{336 \times 10^3 \times 1.5}{20 \times 12} = \frac{504000}{240} = 2100s = 35mins$$

Latent Heat of Vaporization

Specific latent heat of vaporization (L) of a substance is the quantity of heat (Q) required to change a unit mass (m) of the substance from liquid at its boiling point to vapor without a corresponding change in temperature.

That is, $Q = mL$ or $L = \frac{Q}{m}$

where Q = quantity of heat in Joules(J)
m = mass of substance in kg
L = specific latent heat of vapourization in Jkg^{-1}

If the heat (Q) supplied from an electric heater, the following equations apply.

From $Q = mL$ we have

$$IVt = mL$$

$$Pt = mL$$

$$I^2Rt = mL$$

$$\frac{V^2t}{R} = mL$$

See page 112 for the definition of the term on the left hand side of the above equations.
The equation used for a question will depend on the data given in the question.

Example 28
A quantity of steam at 100°C condenses to water at the same temperature by releasing 6.9 × 10^4J of energy. Calculate the mass of the condensed steam [specific latent heat of vapourization of water = 2.3 x 106 Jkg^{-1}]

WAEC 2006

Solution
Quantity of heat released by steam, Q = 6.9 × 10^4J
Latent heat of vapourization, L = 2.3 × $10^6 Jkg^{-1}$

From $Q = mL$ we derive

Mass of condensed steam, $m = \frac{Q}{L} = \frac{6.9 \times 10^4}{2.3 \times 10^6} = 0.03kg$

Example 29
10^6J of heat is required to boil off completely 2kg of a certain liquid. Neglecting heat lost to the surrounding, what is the latent heat of vapourization of the liquid?
A. 5.0 × $10^5 Jkg^{-1}$ B. 2.0 × $10^5 Jkg^{-1}$ C. 5.0 × $10^6 Jkg^{-1}$ D. 2.0 × $10^6 Jkg^{-1}$

JAMB 2005

Solution
Quantity of heat supplied, Q = 10^6J; mass of substance, m = 2kg

From $Q = mL$ we obtain

Latent heat of vapourization, $L = \frac{Q}{m} = \frac{10^6}{2} = 5 \times 10^5\ Jkg^{-1}$

Example 30
A heating coil rated at 1000W is used to boil off 0.5kg of boiling water. What is the time taken to boil off the water? [Specific latent heat of vapourization of water = 2.3 × $10^6 Jkg^{-1}$] A. 1.15 × $10^9 s$ B. 1.15 × $10^7 s$ C. 1.15 × $10^5 s$ D. 1.15 × $10^3 s$

JAMB 1997

Solution
Power rating, P = 1000W; mass of water, m = 0.5kg; L = 2.3 × $10^6 Jkg^{-1}$]

$Pt = mL$ ∴ Time taken to boil off water, $t = \frac{mL}{P} = \frac{0.5 \times 2.3 \times 10^6}{1000}$

$= 1150$sec or 19.17min

Example 31
All the heat generated by a current of 2A passing through a 6Ω resistor for 25s is used to evaporate 5g of a liquid at its boiling point. What is the specific latent heat of the liquid?
WAEC 1989

Solution
Current, I = 2A; mass of liquid, m = 5g = 0.005kg
resistance, R = 6Ω; time, t = 25s;

$$I^2Rt = mL \quad \therefore \text{ specific latent heat of vapourization, } L = \frac{I^2Rt}{m} = \frac{2^2 \times 6 \times 25}{0.005}$$

$$= \frac{600}{0.005} = 1.2 \times 10^5 Jkg^{-1} \ or \ 120Jg^{-1}$$

The problems treated so far involve only one of these for each question; (a) specific heat capacity (b) specific latent heat of vapourization (c) specific latent heat of fusion. However, there are many other instances that a combination of two or three of the specific heats could be used in solving problems as illustrated by the following examples.

Example 32
What is the total amount of heat energy required to convert 0.5kg of ice at −15°C into steam at 100°C? [Specific heat capacity of ice = 2100 $Jkg^{-1}K^{-1}$, specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, specific latent heat of ice = 336000 Jkg^{-1}, specific latent heat of steam = 2.26 x 10^6 Jkg^{-1}]
Solution
Quantity of heat required to convert ice at −15°C to ice at 0°C

$$Q_1 = mc\Delta\theta = 0.5 \times 2100 \times 15 = 1.575 \times 10^4 J$$

Quantity of heat required to convert ice at 0°C to water at 0°C

$$Q_2 = ml = 0.5 \times 336000 = 1.68 \times 10^5 J$$

Quantity of heat required to convert water at 0°C to water at 100°C

$$Q_3 = mc\Delta\theta = 0.5 \times 4200 \times 100 = 2.10 \times 10^5 J$$

Quantity of heat required to convert water at 100°C to steam at 100°C

$$Q_4 = mL = 0.5 \times 2.26 \times 10^6 = 1.13 \times 10^6 J$$

Total quantity of heat, $Q = Q_1 + Q_2 + Q_3 + Q_4$

$$Q = 1.575 \times 10^4 + 1.68 \times 10^5 + 2.10 \times 10^5 + 1.13 \times 10^6$$

$$= 1.52 \times 10^6 J$$

Example 33
0.5kg of water at 10°C is completely converted to ice at 0°C by extracting 188000J of heat from it. If the specific heat capacity of water is 4200 $Jkg^{-1}{}^{\circ}C^{-1}$, calculate the specific latent heat of fusion of ice.
WAEC 1988
Solution
Mass of water, m = 0.5kg; heat extracted, Q = 188000J
Conversion of water at 10°C to ice at 0°C involves two different processes:

1. Conversion of water at 10°C to water at 0°C. Because there is a change in temperature, the specific heat capacity of water is used.

$$mc\Delta\theta = 0.5 \times 4200 \times 10 = 2100 J$$

2. Conversion of water at 0°C to ice at 0°C. Because there is NO change of temperature, the specific latent heat of ice is used.

$$ml = 0.5 \times l$$

Therefore total heat extracted to form ice is equal to heat extracted during processes 1 and 2.

$$Q = mc\Delta\theta + ml$$
$$188000 = 21000 + 0.5l$$
$$188000 - 21000 = 0.5l$$
$$167000 = 0.5l$$
$$l = \frac{167000}{0.5} = 334000 Jkg^{-1}$$

Example 34

A cup containing 100g of pure water at 20°C is placed in a refrigerator. If the refrigerator extracts heat at the rate of 840J per minute, calculate the time taken for the water to freeze. [Neglect heat capacity of the material of the cup] [specific heat capacity of water = $4.2 Jkg^{-1}K^{-1}$] [specific latent heat of fusion = $336 Jg^{-1}$]

Solution

Mass of substance, m = 100g; $c = 4.2Jg^{-1}K^{-1}$;
$l = 336Jg^{-1}$; temperature change, $\Delta\theta = 20°C$

Total energy required for freezing, $Q = mc\Delta\theta + ml$

$$Q = 100 \times 4.2 \times 20 + 336 \times 100 = 8400 + 33600 = 42000 J$$

If 840J is extracted in 1 minutes

42000J will be extracted in x minutes.

$$\therefore \quad x = \frac{42000}{840} = 50\text{min}$$

Example 35

A copper calorimeter of mass 30g contains 50g of oil at 20°C. Some dried ice at 0°C is added to the oil. If the final steady temperature of the calorimeter and its contents is 0°C, calculate the mass of ice added. [Specific heat capacity of copper = $400 Jkg^{-1}K^{-1}$] [Specific heat capacity of oil = $2400 Jkg^{-1}K^{-1}$] [Specific latent heat of fusion of ice = $336000 Jkg^{-1}$]

NECO 2002

Solution

From the question we deduce the following:

Mass of calorimeter, m_c = 30g = 0.03kg

Mass of oil, m_o = 50g = 0.05kg

Temperature change for calorimeter & oil, $\Delta\theta = 20 - 0 = 20°C$

Specific heat capacity of calorimeter, $c_c = 400Jkg^{-1}K^{-1}$

Specific heat capacity of oil, $c_o = 2400Jkg^{-1}K^{-1}$

Latent heat of fusion of ice, $l = 336000Jkg^{-1}$

Let m be the mass of ice added

Heat gained by ice = heat lost by calorimeter + heat lost by oil

$$(ml)_{ice} = (mc\Delta\theta)_{calorimeter} + (mc\Delta\theta)_{oil}$$

$$\therefore \quad ml = m_c c_c \Delta\theta + m_o c_o \Delta\theta$$

$$m \times 336000 = 0.03 \times 400 \times 20 + 0.05 \times 2400 \times 20$$

$$336000m = 240 + 2400$$

$$336000m = 2640$$

$$m = \frac{2640}{336000} = 0.00786\text{kg or } 7.86\text{g}$$

Example 36

How many grams of water at 17°C must be added to 42g of ice at 0°C to melt the ice completely? [Specific latent heat of fusion of ice = $3.4 \times 10^5 Jkg^{-1}$; specific heat capacity of water = $4200 Jkg^{-1}$] A. 200g B. 300g C. 320g D. 400g

JAMB 1998

Solution

Heat absorbed by ice = heat lost by water

$$m_i l = mc\Delta\theta$$

Mass of ice, $m_i = 42\text{g} = 0.042\text{kg}$

Temperature change of water, $\Delta\theta = 17 - 0 = 17°C$

Latent heat of fusion of ice, $l = 3.4 \times 10^5 Jkg^{-1}$

Specific heat capacity, $c = 4200 Jkg^{-1}K^{-1}$

Let m be the mass of water

Substituting into $m_i l = mc\Delta\theta$ we have,

$$0.042 \times 3.4 \times 10^5 = \text{m} \times 4200 \times 17$$

$$m = \frac{0.042 \times 3.4 \times 10^5}{4200 \times 17} = \frac{14280}{71400} = 0.2kg \text{ or } 200g$$

Example 37

A piece of copper of mass 300g at a temperature of 950°C is quickly transferred to a vessel of negligible thermal capacity containing 250g of water at 25°C. If the final steady temperature of the mixture is 100°C, calculate the mass of the water that will boil away. [Specific heat capacity of copper = $4.0 \times 10^2 Jkg^{-1}K^{-1}$] [Specific heat capacity of water = $4.2 \times 10^3 Jkg^{-1}K^{-1}$] [Specific latent heat of vapourization of steam = $2.26 \times 10^6 Jkg^{-1}$]

WAEC 1990

Solution

Heat lost = heat gained

Heat given out by copper = heat absorbed by changing water at 25°C to water at 100°C + heat absorbed by changing water at 100°C to steam at 100°C

$$m_c c_c \Delta\theta_c = m_w c_w \Delta\theta_w + mL$$

mass of copper, $m_c = 300\text{g} = 0.3\text{kg}$

specific heat capacity of copper, $c_c = 4.0 \times 10^2 Jkg^{-1}K^{-1}$

temperature change of copper, $\Delta\theta_c = 950 - 100 = 850°C$

mass of water, $m_w = 250\text{g} = 0.25\text{kg}$

Specific heat capacity of water, $c_w = 4.2 \times 10^3 Jkg^{-1}K^{-1}$

Temperature change of water, $\Delta\theta_w = 100 - 25 = 75°C$

Latent heat of vapourization of steam, $L = 2.26 \times 10^6 Jkg^{-1}$

Let m be the mass of water that will boil away

Substituting into $m_c c_c \Delta\theta_c = m_w c_w \Delta\theta_w + mL$

$$0.3 \times 4.0 \times 10^2 \times 850 = 0.25 \times 4.2 \times 10^3 \times 75 + m \times 2.26 \times 10^6$$

$$102000 = 78750 + 2.26 \times 10^6 \times m$$

$$m = \frac{23250}{2.26 \times 10^6} = 0.0103\text{kg} = 10.3\text{g}$$

Example 38

What is the difference in the amount of heat given out by 4kg of steam and 4kg of water when both are cooled from 100°C to 80°C? [Specific latent heat of steam = 2,260,000 $Jkg^{-1}K^{-1}$] [Specific capacity of water = 4200 Jkg^{-1}] *WAEC 1996*

Solution

Heat given out by steam $= mL + mc\Delta\theta$

$= 4 \times 2{,}260{,}000 + 4 \times 4200\,(100 - 80)$

$= 9040000 + 336000 = 9376000\text{J}$

Heat given out by water $= mc\Delta\theta$

$= 4 \times 4200 \times 20 = 336000\text{J}$

Difference in heat given out $= 9376000 - 336000$

$= 9040000\text{J}$

RELATIVE HUMIDITY

$$\text{Relative humidity} = \frac{\text{mass of water vapour in a given volume of air(m)}}{\text{mass of water vapour required to saturate same volume of air at the same temperature (M)}} \times 100\%$$

$$= \frac{\text{saturation vapour pressure of water at the dew point}}{\text{saturation vapour pressure(S.V.P)of water at the original air temperature}} \times 100\%$$

$$= \frac{\text{vapour pressure (partial pressure) of the air (V.P)}}{\text{S.V.P at the temperature of the air}} \times 100\%$$

$$\text{Relative humidity} = \frac{\text{m}}{\text{M}} \times 100\% = \frac{V.P}{S.V.P} \times 100\%$$

Example 39

The mass of water vapour in a certain volume of air is 0.20g at 30°C. What is the relative humidity of the air if the mass of water vapour required to saturate it at the same temperature is 0.35g.

Solution

Mass of water vapour, m = 0.20g;

mass of water vapour required for saturation, M = 0.35g

$$\text{Relative humidity} = \frac{m}{M} \times 100\%$$

$$= \frac{0.20}{0.35} \times 100\% = 57.14\%$$

Example 40

The table below shows the saturation vapour pressure against in a particular city. If on a certain day, the vapour pressure in this city at 30°C is 22.0mmHg, what is the relative humidity?

T°C	0	10	20	30	40	50	60
S.V.P(mmHg)	4.6	9.2	17.5	31.8	55.1	92.5	149.0

Solution

Vapour pressure of air at 30°C, V.P = 22.0mmHg

Saturation vapour pressure of air at 30°C, S.V.P = 31.8mmHg

$$\therefore \text{ Relative humidity} = \frac{V.P}{S.V.P} \times 100\% = \frac{22.0}{31.8} \times 100\% = 69.2\%$$

Example 41

On a certain day, the dew point is 10°C when the temperature of the air is 32°C. Calculate the relative humidity of the air. (S.V.P. of water at 32°C and 10°C are 16.2mm and 9.8mm of mercury respectively). NECO 2008[24]

Solution

S.V.P. at dew point (10°C) = 9.8mmHg

S.V.P. at air temperature (32°C) = 16.2mmHg

$$\therefore \text{ Relative humidity} = \frac{\text{S. V. P at dew point}}{\text{S. V. P at air temperature}} \times 100\% = \frac{9.8}{16.2} \times 100\% = 60.5\%$$

EXERCISES 9.

1. A tap supplies water at 26°C while another supplies at 82°C. If a man wishes to bath with water at 40°C, what is the ratio of the mass of hot water to that of cold water required? *WAEC 1991 Ans: 1:3*

2. A tap supplies water at 30°C while another supplies water at 86°C. If a man wishes to bath with water at 44°C, calculate the ratio of the mass of hot water to that of cold water required? *WAEC 1994 Ans: 1:3*

3. Two thermos flasks of volume V_x and V_y are filled with liquid water at an initial temperature of 0°C. After some time the temperatures were found to be θ_x, θ_y respectively. Given $V_x/V_y = 2$ and $\theta_x/\theta_y = \frac{1}{2}$. What is the ratio of heat flow into the flasks. A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 4 D. 1 E. 2 *JAMB1985 Ans: 1:1* or 1

4. A tap supplies water at 25°C while another supplies water at 75°C. If a man wishes to bath with water at 40°C, what is ratio of the mass of hot water to that of cold water required? A. 1:3 B. 15:8 C. 7:3 D. 3:1 *JAMB 1988 Ans: 7:3*

5. Hot water at a temperature of t is added to twice that amount of water at a temperature of 30°C. If the resulting temperature of the mixture is 50°C, calculate t. *WAEC 1995 Ans: 90°C*

6. 400g of cold water is added to 200g of water at 70°C. If they are properly mixed and the temperature of the mixture is 30°C, calculate the initial temperature of the cold water. [Neglect the heat absorbed by the container] *NECO 2000 Ans: 10°C*

7. Two liquids, P at a temperature of 20°C and Q at a temperature of 80°C have specific heat capacities of $1.0Jkg^{-1}\,°C^{-1}$ and $1.5Jkg^{-1}\,°C^{-1}$ respectively. If equal masses of P and Q are mixed in a lagged calorimeter, what is the equilibrium temperature. A. 44°C B. 50°C C. 56°C D. 60°C E. 70°C *JAMB 1985 Ans: 56°C*

8. Hot water is added to three times its mass of water at 10°C and the resulting temperature is 20°C. What is the initial temperature of the hot water? A. 100°C B. 80°C C. 50°C D. 40°C *JAMB 1987 Ans: 50°C*

9. 200g of water at 90°C is mixed with 100°C of water at 30°C. What is the final temperature? A. 50°C B. 60°C C. 70°C D. 80°C *JAMB1993 Ans: 70°C*

10. What mass of water at 100°C should be added to 15g of water at 40°C to make the temperature of the mixture 50°C. [Neglect heat losses to the surrounding] *NECO 2006 Ans: 3g*

11. Water of mass 120g at 50°C is added to 200g of water at 10°C and the mixture is well stirred. Calculate the temperature of the mixture. [Neglect heat losses to surrounding] *Ans: 25°C WAEC 2002*

12. How much heat is given out when a piece of iron of mass 50g and specific heat capacity $460Jkg^{-1}K^{-1}$ cools from 85°C to 25°C? *WAEC 1990 Ans: $1.38 \times 10^3 J$*

13. A piece of copper of mass 30g loses 60J of heat energy. If the specific heat capacity of copper is $400\ Jkg^{-1}K^{-1}$, calculate the change in temperature of the copper. *WAEC 1997 Ans: 5K*

14. How much heat is emitted when a body of mass 200g cools from 37°C to 31°C? [Specific heat capacity of the body = $0.4\ Jg^{-1}K^{-1}$] *WAEC 1998 Ans: 480J*

15. A piece of metal of mass 50g is cooled from 80°C to 20°C. Calculate the amount of heat lost. [Specific heat capacity of the material of metal = $450\ Jkg^{-1}K^{-1}$) *WAEC 2000 Ans: 1350J*

16. A body of mass, m has a specific heat capacity, s and a heat capacity, C. If the temperature of the body changes by θ°C, which of the following equations is correct? A. $ms\theta = mc^{-1}$ B. $ms = c$ C. $ms = s\theta$ D. $ms = c\theta$ *WAEC 2005 Ans: B*

17. A copper ball of heat capacity $400JK^{-1}$ is heated from 20°C to 100°C. Calculate the quantity of heat absorbed. *NECO 2005 Ans: $3.2 \times 10^4 J$ or 32KJ*

18. A body of mass 40g loses 80J of heat energy. If the specific heat capacity of the body is $400Jkg^{-1}K^{-1}$, calculate the change in temperature of the body. *NECO 2004 Ans: 5.0K*

19. How much heat is emitted when a body of mass 50g cools from 80°C to 20°C? [Specific heat capacity of the body = $460\ Jkg^{-1}K^{-1}$]. *NECO 2002 Ans: 1380J*

20. 22000J of heat is required to raise the temperature of 1.5kg of paraffin from 20°C to 30°C. Calculate the specific heat capacity of paraffin. A. $1466.7\ Jkg^{-1}\,°C^{-1}$ B. $2933\ Jkg^{-1}\,°C^{-1}$ C. $4400\ Jkg^{-1}\,°C^{-1}$ D. $5866\ Jkg^{-1}\,°C^{-1}$ *JAMB 1987 Ans: $1466.7\ Jkg^{-1}\,°C^{-1}$*

21. How much heat is absorbed when a block of copper of mass 0.05kg and specific heat capacity $390Jkg^{-1}K^{-1}$ is heated from 20°C to 70°C? A. $3.98 \times 10^{-1} J$ B. $9.75 \times 10^2 J$ C. $3.98 \times 10^3 J$ D. $9.75 \times 10^3 J$ *JAMB 1992 Ans: 975J*

22. When two objects P and Q are supplied with the same quantity of heat, the temperature change in P is observed to be twice that in Q. If the masses of P and Q are the same, calculate the ratio of the specific heat capacities of Q to P. *WAEC 1995 Ans: 2:1*

23. When two objects P and Q are supplied with the same quantity of heat, the temperature change in P is observed to be twice that in Q. The mass of P is half that of Q. What is the ratio of the specific heat capacities of Q to P? *WAEC 1986 Ans: 1:1*

24. A mass of liquid at 30°C is mixed with a mass of the same liquid at 70°C and the temperature of the mixture is 45°C. Find the ratio of the mass of the cold liquid to the mass of the other liquid. A. 3:5 B. 5:3 C. 3:7 D. 7:3 *JAMB 1997 Ans: 5:3*

25. A piece of copper ball of mass 20g at 200°C is placed in a copper calorimeter of mass 60g containing 50g of water at 30°C, ignoring heat losses, calculate the final steady temperature of the mixture. [Specific heat capacity of water = $4.2Jg^{-1}K^{-1}$, Specific heat capacity of copper = $0.4Jg^{-1}K^{-1}$] *WAEC 1994 Ans: 35.6°C*

26. An iron rod of mass 2kg and at a temperature of 280°C is dropped into some quantity of water initially at a temperature of 30°C. If the temperature of the mixture is 70°C, calculate the mass of the water. [Neglect heat losses to the surrounding]. [Specific heat capacity of iron = $460Jkg^{-1}K^{-1}$, specific heat capacity of water = 4200 Specific heat capacity of water = $4.2Jkg^{-1}K^{-1}$] *WAEC 1995 Ans: 1.15kg*

27. A piece of copper block of mass 24g at 230°C is placed in a copper calorimeter of mass 60g containing 54g of water at 31°C. Assuming heat losses are negligible, calculate the final steady temperature of the mixture. [Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$. Specific heat capacity of copper = $400 Jkg^{-1}K^{-1}$] *WAEC 2001 Ans: 38.34°C*

28. A calorimeter of thermal capacity 80J contains 20g of water at 25°C. Water at 100°C is added so that the final temperature of the set up is 50°C. What is the amount of water added? [Heat capacity of water = $4.18 Jg^{-1}°C^{-1}$] A. 20g B. 25g C. 45g D. 50g E. 100g *JAMB 1980 Ans: 20g*

29. 250g of lead at 170°C is dropped into 100g of water at 0°C. If the final temperature is 12°C, what is the specific heat capacity of lead? [Specific heat capacity of water $4200 Jkg^{-1}°C^{-1}$] A. $39.5 Jkg^{-1}°C^{-1}$ B. $50.4 Jkg^{-1}°C^{-1}$ C. $127.6 Jkg^{-1}°C^{-1}$ D. $154.6 Jkg^{-1}°C^{-1}$ E. $173.4 Jkg^{-1}°C^{-1}$ *JAMB 1982 Ans: 127.6 Jkg^{-1}*

30. Calculate the time taken to heat 2kg of water from 50°C to 100°C in an electric kettle taking 5A, from a 210V supply. [Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$] *WAEC 1994 Ans: 400s or 6.67mins*

31. An immersion heater is rated 120W. How long does it take the heater to raise the temperature of 1.2kg of water by 15°C. [Assume heat lost to the surrounding is negligible. Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$] *WAEC 2000 Ans: 10.5 minutes or 630 seconds*

32. How long will it take to heat 3kg of water from 28°C to 88°C in an electric kettle taking 6A from a 220V supply? [Specific heat capacity of water = $4180 Jkg^{-1}K^{-1}$] *WAEC 2006 Ans: 570s or 9.5 min.*

33. How long does it take a 750W heater to raise the temperature of 1kg of water from 20°C to 50°C. [Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$] A. 84s B. 112s C. 168s D. 280s *JAMB 1991 Ans: 168s or 2.8 min*

34. An electric kettle with negligible heat capacity is rated at 2000W. If 2.0kg of water is put in it, how long will it take the temperature of the water to rise from 20°C to 100°C? [Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$] A. 420s B. 336s C. 168s D. 84s *JAMB 1995 Ans: 336s or 5.6 min*

35. An immersion heater rated 2.0A, 240V is used to boil water from temperature 52°C to 100°C. If the mass of water is 2.5kg, determine the time taken to boil the water. [Specific heat capacity of water = $4.2 \times 10^3 Jkg^{-1}K^{-1}$] A. 1.05×10^4s B. 1.05×10^5s C. 1.05×10^3s D. 1.05×10^2s *JAMB 2006 Ans: 1050sec or 17.5 min*

36. A metal of mass 1.5kg was heated from 27°C to 47°C in 4 minutes by a boiling ring of 75W rating. Calculate the specific heat capacity of the metal. [Neglect heat loses to the surrounding] *WAEC 1992 Ans: $6.0 \times 10^2 Jkg^{-1}°C^{-1}$*

37. A 400W immersion heat is used to heat a liquid of mass 0.5kg. If the temperature of the liquid increases by 2.5°C in one second, calculate the specific heat capacity of the liquid. [Neglect heat losses to the surrounding] *WAEC 1996 Ans: 320 $Jkg^{-1}K^{-1}$*

38. Heat is supplied uniformly at the rate of 100W to 1.0×10^{-2}kg of a liquid for 20 sec. If the temperature of the liquid rises by 5°C, then what is the specific heat capacity of the liquid? A. $2.0 \times 10^2 Jkg^{-1}K^{-1}$ B. $2.0 \times 10^2 Jkg^{-1}$ C. $4.0 \times 10^4 Jkg^{-1}K^{-1}$ D. $4.0 \times 10^4 Jkg^{-1}$ E. $8.4 \times 10^3 Jkg^{-1}K^{-1}$ *JAMB 1984 Ans: $4.0 \times 10^4 Jkg^{-1}K^{-1}$*

39. A 2000W electric heater is used to heat a metal object of mass 5kg initially at 10°C. If a temperature rise of 30°C is obtained after 10min, what is the heat capacity of the material? A. $6.0 \times 10^4 J°C^{-1}$ B. $4.0 \times 10^4 J°C^{-1}$ C. $1.2 \times 10^4 J°C^{-1}$

D. $8.0 \times 10^3 J/^\circ C^{-1}$ *JAMB 2003 Ans: $4.0 \times 10^4 J^\circ C^{-1}$*

40. A 50W, electric heater is used to heat a metal block of mass 5kg. If in 10 minutes a temperature rise of $12^\circ C$ is achieved, what is the specific heat capacity of the metal? A. 500 $Jkg^{-1}K^{-1}$ B. 130 $Jkg^{-1}K^{-1}$ C. 390 $Jkg^{-1}K^{-1}$ D. 400 $Jkg^{-1}K^{-1}$
JAMB 2004 Ans: $500Jkg^{-1}K^{-1}$

41. A waterfall is 420m high. Calculate the difference in temperature of the water between the top and bottom of the waterfall. Neglect heat losses. [$g=10m/s^2$, specific heat capacity of water = 4.2×10^3 $Jkg^{-1}K^{-1}$] *WAEC 1991 Ans: $1.0^\circ C$*

42. A body of specific heat capacity 450$Jkg^{-1}K^{-1}$ falls to the ground from rest through a vertical height of 20m. Assuming conservation of energy, calculate the change in temperature of the body on striking the ground level [$g=10m/s$] *WAEC 1995 Ans: $0.44^\circ C$*

43. Water falls through a height of 50m. Determine the temperature rise of the water at the bottom of the fall. [Neglect energy losses, specific heat capacity of water is 4200$Jkg^{-1}K^{-1}$, $g=10m/s^2$]. *WAEC 2005 Ans: $0.119^\circ C$*

44. A waterfall is 1260m high. Calculate the difference in temperature of the water between the top and the bottom of the water fall. [Neglect heat losses. $g=10m/s^2$, specific heat capacity of water = 4.20×10^3 $Jkg^{-1}K^{-1}$] *NECO 2006 Ans: $3.0^\circ C$ or 276K*

45. How much heat is required to convert 20g of ice at $0^\circ C$ to water at the same temperature? [Specific latent heat of ice = 336Jg^{-1}] *WAEC 1995 (Ans: $6.72 \times 10^3 J$)*

46. The latent heat of fusion of ice is 80 cal g^{-1}. How much heat is required to change 10g of ice at $0^\circ C$ into water at the same temperature? A. 80 cal B. 10 cal C. 8 cal D. 800 cal E. 400 cal *JAMB 1978 Ans: 800 cal*

47. The melting point of a solid is given as $80^\circ C$. If $10^5 J$ of heat energy is required at this temperature to melt 10g of the solid, what is the specific latent heat of fusion of the solid? A. $1.00 \times 10^3 Jkg^{-1}$ B. $1.25 \times 10^5 Jkg^{-1}$ C. $1.00 \times 10^7 Jkg^{-1}$ D. $8.00 \times 10^8 Jkg^{-1}$ *JAMB 1993 Ans: $1.00 \times 10^7 Jkg^{-1}$*

48. A 90W immersion heater is used to supply energy for 5 minutes. The energy supplied is used to completely melt 180g of a solid at its melting point. Neglecting energy losses to the surrounding, calculate the specific latent heat of fusion of the solid.
WAEC 1998 Ans: $150Jg^{-1}$

49. A 10Ω coil takes 21s to melt 10g of ice at $0^\circ C$. Assuming no heat losses, determine the current in the coil. [Specific latent heat of fusion of ice = 336Jg^{-1}]
WAEC 1999 Ans: 4A

50. Heat is supplied to a test tube containing 100g of ice at its melting point. The ice melts completely in 1 min. What is the power rating of the source of heat? [Latent heat of fusion of ice = 336Jg^{-1}] A. 336W B. 450W C. 560W D. 600W
JAMB 1994 Ans: 560W

51. Calculate the quantity of heat released when 100g of steam at $100^\circ C$ condenses to water. [Take the specific latent heat of vapourization of water as $2.3 \times 10^6 Jkg^{-1}K^{-1}$]
WAEC 1997 Ans: $2.3 \times 10^5 J$

52. How much heat is required to convert 50g of water at $100^\circ C$ to steam at the same temperature? [Specific latent heat of vapourization of water = 2260Jg^{-1}].
NECO 2003 Ans: $1.13 \times 10^5 J$

53. All the heat generated in a 5Ω resistor by 2A flowing for 30 seconds is used to evaporate 5g of a liquid at its boiling point. What is the correct value of the specific latent heat of the liquid? A. 120 J B. 60 Jg^{-1} C. 120 Jg^{-1} D. 1500 J E. 1500 Jg^{-1}
JAMB 1979 Ans: $120Jg^{-1}$

54. A heater marked 50W will evaporate 0.005kg of boiling water in 50 seconds. What is the specific latent heat of vapourization of water in J/kg? A. 5.0×10^6 B. 1.0×10^6 C. 2.5×10^6 D. 2.5×10^5 E. 5.0×10^5
JAMB 1982 Ans: $5 \times 10^5 Jkg^{-1}$

55. All the heat generated in a 5Ω resistor by 2A flowing for 30s is used to evaporate 5g of a liquid at its boiling point. What is the specific latent heat of vapourization of the

liquid? A. 120 J B. 60 Jg^{-1} C. 120 Jg^{-1} D. 1500 J E. 1500 Jg^{-1}
JAMB 1984 Ans: 120Jg^{-1}

56. Steam at 100°C, is passed into a container of negligible heat capacity, containing 20g of ice and 100g of water at 0°C, until the ice is completely melted. Determine the total mass of water in the container. [Specific latent heat of steam = 2.3 x $10^3 Jg^{-1}$, specific latent heat of ice = 3.4 x $10^2 Jg^{-1}$, specific heat capacity of water = 4.2 $Jg^{-1}K^{-1}$]
WAEC 2005 Ans: 122.5g

57. Calculate the heat required to convert 20g of ice at 0°C to water at 16°C. [Specific latent heat of fusion of ice = 336Jg^{-1}, specific heat capacity of water = 4.2 $Jg^{-1}K^{-1}$]
WAEC 1994 Ans: 8064J

58. Calculate the heat energy required to change 0.1kg of ice at 0°C to water boiling at 100°C. [Specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, specific latent heat of fusion of ice = 336,000Jkg^{-1}]
WAEC 1999 Ans: 75,600J

59. The specific heat of a substance in the solid state is C_1; its specific heat in the liquid state is C_2 and its latent heat of fusion is L. If a mass M of the substance is changed from the solid state at temperature T, to the liquid state, also at temperature T, the amount of heat required will be:
A. $M(C_1 + C_2) \times T$ B. ML C. L D. $M(C_1 + C_2) \times T + ML$
E. $ML\left[\frac{C_1+C_2}{2}\right] \times T$
NECO 2000 Ans: B

60. Calculate the amount of heat required to convert 2kg of ice at −2°C to water at 0°C. [Specific heat capacity of ice = 2090Jkg^{-1} °C^{-1}, specific latent heat of fusion of ice = 333Jkg^{-1}] A. 666 J B. 8360 J C. 666000 J D. 674360
JAMB 1987 Ans: 674360J

61. 1kg of copper is transferred quickly from boiling water to a block of ice. Calculate the mass of ice melted, neglecting heat loss. [Specific heat capacity of copper = 400$Jkg^{-1}K^{-1}$ and latent heat of fusion of ice = 333 x $10^3 Jkg^{-1}$] A. 60g B. [illegible]g
C. 120g D. 133g
JAMB 1990 Ans: 120g or 0.12kg

62. What is the difference in the amount of heat given out by 4kg of steam and 4kg of water when both are cooled from 100°C to 80°C. [Specific latent heat of steam = 2,260,000Jkg^{-1}, specific heat capacity of water is 4200$Jkg^{-1}K^{-1}$]. A. 4200 J
B. 2,260,000 J C. 9,040,000 J D. 9,380,000 J *JAMB 1991 Ans: 9,040,000J*

63. Calculate the mass of ice that would melt when 2kg of copper is quickly transferred from boiling water to a block of ice without heat loss. [[Specific heat capacity of copper = 400 $Jkg^{-1}K^{-1}$, latent heat of fusion of ice = 3.3 x $10^5 Jkg^{-1}$] A. $\frac{8}{33}kg$
B. $\frac{33}{8}kg$ C. $\frac{80}{33}kg$ D. $\frac{33}{8}kg$ *JAMB 1999 Ans: $^8/_{33}kg$ or 0.24kg*

64. Calculate the heat energy required to vapourize 50g of water initially at 80°C if the specific heat capacity of water is 4.2$Jg^{-1}K^{-1}$. [Specific latent heat of vapourization of water = 2260Jg^{-1}]
WAEC 1989 Ans: 117200J

65. An electric heater immersed in some water raises the temperature of the water from 40°C to 100°C in 6 minutes. After another 25 minutes, it is noticed that half the water has boiled away. Neglecting heat losses to surrounding, calculate the specific latent heat of vapourization of water.
WAEC 2000 Ans: 2.1 × $10^6 Jkg^{-1}$

66. Calculate the energy required to vapourize 50g of water initially at 80°C. [Specific heat capacity of water = 4.200$Jg^{-1}K^{-1}$, specific latent heat of vapourization of water = 2260Jg^{-1}]
WAEC 2002 Ans: 117,200J

67. The mass of water vapour in a given volume of air is 0.05g at 20°C, while the mass of water vapour required to saturate it at the same temperature is 0.15g. Calculate the relative humidity of the air.
WAEC 2000 Ans: 33.33%

68. The table below shows the saturation vapour pressure against temperature in a certain town. If the vapour pressure in this town at 20°C is 10mmHg, what is the relative humidity?

T°C	0	5	10	15	20	40	60
S.V.P(mmHg)	4.58	6.51	8.94	12.67	17.50	55.10	149.00

A. 170.0 % B. 57.0% C. 17.5% D. 10.0% *JAMB 2003 Ans: 57.0%*

69. A body of mass 200g and specific heat capacity $0.4 Jg^{-1}K^{-1}$ cools from 37°C to 31°C. Calculate the quantity of heat released by the body. *WAEC 2007 Ans: 480J*

70. A piece of copper of mass 300g at a temperature of 950°C is quickly transferred into a vessel of negligible thermal capacity containing 250g of water at 25°C. If the final steady temperature of the mixture is 100°C, calculate the mass of water that will boil away. [Specific heat capacity of copper = $4.0 \times 10^2 Jkg^{-1}K^{-1}$]
[Specific heat capacity of water = $4.2 \times 10^3 Jkg^{-1}K^{-1}$]. [Specific latent heat of vaporization of steam = $2.26 \times 10^6 Jkg^{-1}$] *WAEC 2007 Ans: 0.0102876kg or 10.3g*

71. 2kg of water is heated with a heating coil which draws 3.5A from a 200V mains for two minutes. What is the increase in temperature of the water?
A. 10°C B. 15°C C. 25°C D. 30°C
[Specific heat capacity of water = $4200 Jkg^{-1}K^{-1}$] *JAMB 2007 Ans: 10°C*

72. The temperatures of water from tap A and tap B are 25°C and 75°C respectively. If a mixture of water at 40°C is required, calculate the ratio of the mass of water from tap A to that from tap B. *NECO 2007 Ans: 7:3*

73. A 420W electric heater is used to heat water of mass 50kg from 25°C to its boiling point. How long, in hours, does the electric heater work? [Specific heat capacity of water = $4.2 \times 10^3 Jkg^{-1}C^{-1}$] *NECO 2007 Ans: 10.4hr*

74. A well lagged copper calorimeter of mass 120g contains 70g of water and 10g of ice both at 0°C. Dry steam at 100°C is passed in until the temperature of the mixture is 40°C. Calculate the mass of the steam condensed.
[Specific latent heat of fusion of ice = $3.2 \times 10^2 Jg^{-1}$]
[Specific latent heat of vaporization of steam = $2.2 \times 10^3 Jg^{-1}$]
[Specific heat capacity of copper = $4.0 \times 10^{-1} Jg^{-1}K^{-1}$]
[Specific heat capacity of water = $4.2 Jg^{-1}K^{-1}$] *NECO 2007[b12] Ans: 7.56g*

75. Water of mass 1.5kg is heated from 30°C to 80°C using an electric kettle which is rated 5A, 230V. Calculate the time taken to reach the final temperature. (Specific heat capacity of water = $4200 Jg^{-1}K^{-1}$) *WAEC 2008[31] Ans: 4.57 min*

76. A block of aluminium is heated electrically by a 25W heater. If the temperature rises by 10°C in 5 minutes, the heat capacity of the aluminium is A. $850 JK^{-1}$ B. $750 JK^{-1}$ C. $650 JK^{-1}$ D. $500 JK^{-1}$ *JAMB 2008[22] Ans: 750 JK^{-1}*

77. If the partial pressure of water vapour at 27°C is 18mmHg and the saturated vapour pressure of the atmosphere at the same temperature is 24 mmHg, the relative humidity at this temperature is A. 25% B. 33% C. 75% D. 82%
JAMB 2008[24] Ans: 75%

78. An electric heater rated 220V, 1000W is immersed into a bucket full of water. Calculate the mass of water if the temperature changes from 30°C to 100°C and the current flows for 300 seconds.
A. 4.28 kg B. 42.86kg C. 1.02kg D. 7.14kg
(specific heat capacity of water = $4200Jkg^{-1}K^{-1}$) *JAMB 2009[21] Ans: C*

79. A heating coil rated 1000W is used to boil off completely 2kg of boiling water. The time required to boil off the water is A. $1.15 \times 10^4 s$
B. $1.15 \times 10^3 s$ C. $4.6 \times 10^4 s$ D. $1.15 \times 10^4 s$ (Specific latent heat of vapourization of water = $2.3 \times 10^6 Jkg^{-1}$) *JAMB 2009[21] Ans: D*

80. It takes 4 minutes to boil a quantity of water using an electrical heating coil. How long will it take to boil the same quantity of water using the same heating coil if the current is doubled? [Neglect any external heat losses]
WAEC 2009[20] Ans: 2 mins.

81. An immersion heater is rated 1500W. How long does it take to raise the temperature of 1kg of water by 30°C? (Assume heat lost to the surrounding is negligible, specific heat capacity of water = $4200Jkg^{-1}K^{-1}$) *NECO 2009[21] Ans: 84s*

82. Calculate the heat energy lost when 10g of boiling water changes to ice at 0°C [Specific latent heat of ice = $336Jg^{-1}$, specific heat capacity of water = $4.2Jg^{-1}K^{-1}$]
NECO 2009[31] Ans: 7560J

10

GAS LAWS

Gases are described by three basic properties; volume, temperature and pressure. The pressure of a gas is the force the gas exerts perpendicularly per unit area. That is

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\text{mass} \times \text{acceleration due to gravity}}{\text{Area}}$$

$$= \frac{\text{density} \times \text{volume} \times g}{\text{Area}} \qquad \left(\text{Density} = \frac{\text{mass}}{\text{volume}}\right)$$

$$= \frac{\text{density} \times \text{Area} \times \text{height} \times g}{\text{Area}} \qquad (\text{Volume} = \text{Area} \times \text{height})$$

Pressure = density × height × g

P = ρhg is known as the pressure formula.

Pressure is measured in N/m^2

$1.013 \times 10^5 N/m^2$ = 1 atmosphere = $1.013 \times 10^5 Pa$ = 760mmHg

GAS PRESSURE MEASUREMENT

The pressure of a gas is measured using a manometer which could be a water or mercury manometer as shown below.

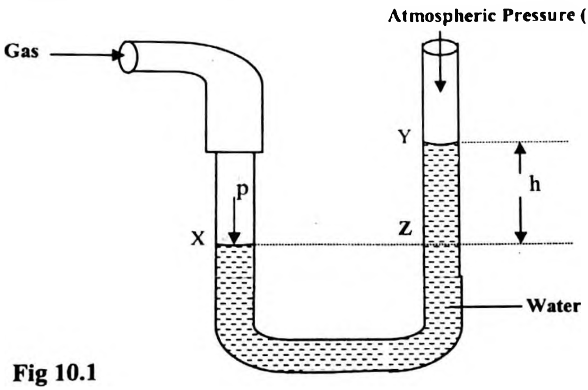


Fig 10.1

The gas pressure (p) at X = pressure in the liquid at Z

= Atmospheric pressure + pressure due to water column YZ

∴ Gas pressure, p = P + h

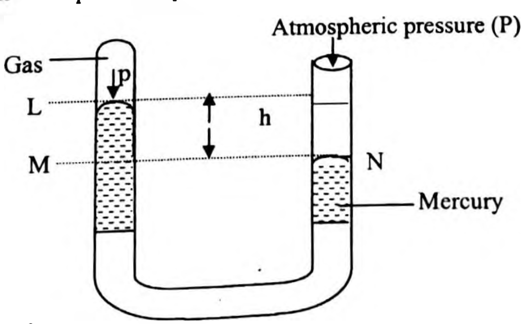


Fig 10.2

Atmospheric pressure (P) acting at N = pressure in the liquid at M

= pressure of gas(p) + pressure due to mercury column LM

$$P = p + h$$

Therefore, pressure of gas, $p = P - h$

Example 1

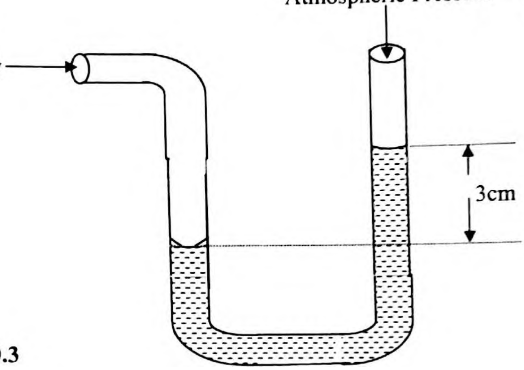


Fig 10.3

The water manometer shown above is measuring the pressure of the gas supply. If the specific gravity of mercury is 13.6 and the atmospheric pressure is 70cm of mercury, what is the total pressure of the gas supply in cm of water.

A. 67cm B. 73cm C. 949cm D. 952cm E.955 cm *JAMB 1982*

Solution

Specific gravity is the same as relative density.

$$\text{Relative density} = \frac{\text{density of substance(Hg)}}{\text{density of water}}$$

Therefore, density of mercury = Relative density of Hg × density of water

= 13.6 × 1000

= $13600 kgm^{-3}$

The equivalent of the atmospheric pressure (70cmHg) in cm of water is calculated by equating pressure formular for water and mercury as follows.

$$(\rho hg)_{water} = (\rho hg)_{mercury}$$

density of water × height of water × g = density of Hg × height of Hg × g

$$1000 \times h \times 10 = 13600 \times 70 \times 10$$

$$h = \frac{13600 \times 70 \times 10}{1000 \times 10} = \frac{952000}{1000} = 952cm$$

∴ The atmospheric pressure, P = 952cm of water

Total gas pressure, = Atmospheric pressure (P) + pressure due to 3cm of water

= 952 + 3

= 955cm of water

Example 2

If the atmospheric pressure is 760mmHg, what is the pressure of the gas supply in the diagram above?

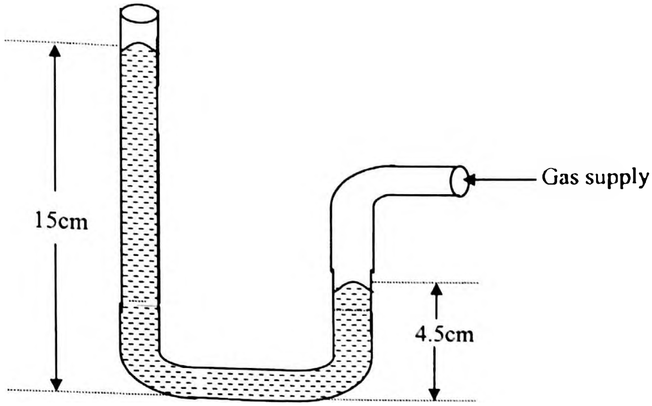


Fig 10.4

Solution

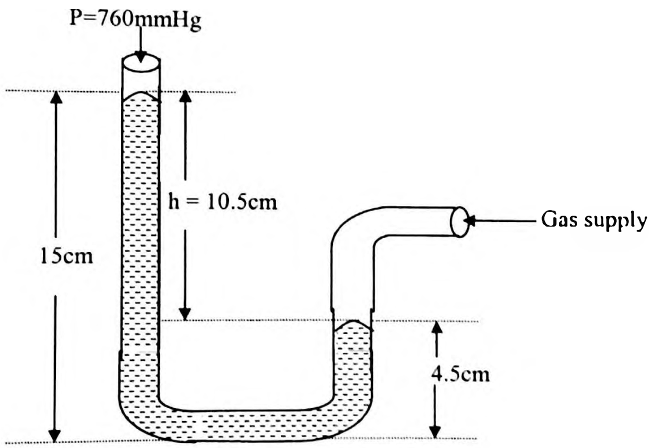


Fig 10.5

h = difference in levels of mercury in the two arms of the manometer

(15 – 4.5 = 10.5cm)

10.5cm = 105mm

Gas pressure = Atmospheric pressure + pressure due to 105mm of mercury

$P = p + h$

$= 760 + 105 = 865\text{mmHg}$

BOYLE'S LAW

Boyle's law states that the volume (V) of a fixed mass of gas is inversely proportional to its pressure (P), provided the temperature remains constant.

i.e $V \propto \frac{1}{P}$ or $V = \frac{K}{P}$ or $PV = K$

$$\therefore \quad P_1V_1 = P_2V_2$$

Example 3

A fixed mass of gas at a pressure of 650mmHg occupies a volume of $30cm^3$. What will be the volume of the gas at 760mmHg if the temperature remains constant?

Solution

Initial gas pressure, P_1 = 650mmHg

Final gas pressure, P_2 = 760mmHg

Initial gas volume, V_1 = $30cm^3$

From $P_1V_1 = P_2V_2$, final gas volume, $V_2 = \frac{P_1V_1}{P_2}$

$$V_2 = \frac{650mmHg \times 30cm^3}{760mmHg} = 25.66cm^3$$

Example 4

A gas at a volume V_o in a container at pressure P_o is compressed to one fifth of its volume. What will be its pressure if it maintains its original temperature T?

A. $P_o/5$ B. $\frac{4}{5}P_o$ C. P_o D. $5P_o$ *JAMB 1999*

Solution

Initial gas pressure, P_1 = P_o

Initial gas volume, V_1 = V_o

Final gas volume, V_2 = $^1/_5V_o$

From $P_1V_1 = P_2V_2$, final gas pressure, $P_2 = \frac{P_1V_1}{V_2}$

$$\therefore \quad P_2 = \frac{P_oV_o}{\frac{1}{5}V_o} = P_oV_o \div \frac{1}{5}V_o = P_oV_o \times \frac{5}{V_o} = 5P_o$$

Application of Boyle's Law to Gas Trapped in a Cylindrical Tube by a Column of Mercury

When a tube of uniform cross-section closed at one end contains a fixed mass of gas or air which is sealed by a column of mercury of length *X*cm, the pressure exerted on the gas depends on the plane of inclination of the tube, as shown below.

(a) Vertically, with closed end at the bottom.

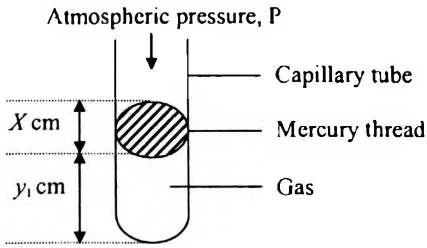


Fig 10.6

The pressure, p, of the gas is due to the ***sum*** of pressures exerted by *X*cm column of mercury and the atmospheric pressure, P.

Therefore, p = P + *X* (cm Hg)

(b) Inverted, with open end underneath

The gas pressure, p is due to the ***difference*** of the atmospheric pressure (P) and the pressure exerted by the *X* cm column of mercury.

Therefore, p = P − *X* (cm Hg)

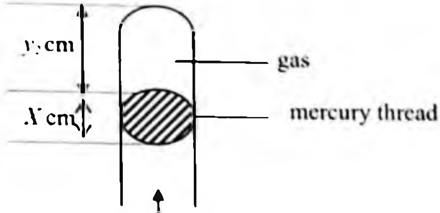


Fig 10.7

(c) Horizontally

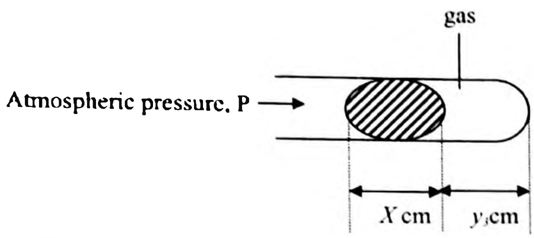


Fig 10.8

The gas pressure. p is due ONLY to the atmospheric pressure P, because when the tube is in the horizontal position, the column of mercury exert very negligible pressure the gas.

Therefore, p = P (cm Hg)

Generally, the gas volume = length of column of gas(y) x cross sectional area (A) of the tube.
Therefore, the gas volumes for figures a, b and c are y_1A, y_2A and y_3A cm^3 respectively. However, because the cross sectional area of the tube is uniform and the same in each case, the volume of the gas is taken as the length of the column of gas. That is, volume V is proportional to length y.

So, the volume of gas in Fig 10.6 is y_1cm^3

The volume of gas in Fig 10.7 is y_2cm^3

The volume of gas in Fig 10.8 is y_3cm^3

Boyle's law ($P_1V_1 = P_2V_2$) can be applied in deriving a relationship between fig 10.6 and fig 10.7 as follows

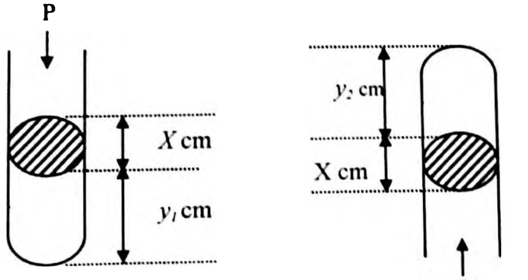


Fig 10.9

(a) Vertical (b) Inverted

From Fig.10.9a. Initial or vertical pressure, $P_1 = P + X$

Initial or vertical volume, $V_1 = y_1A$

From Fig.10.9b, Final or inverted pressure, $P_2 = P - X$

Final or inverted volume, $V_2 = y_2A$

Substituting into Boyle's Law, $P_1V_1 = P_2V_2$ we have

$$(P + X)y_1A = (P - X)y_2A$$

divide through by A to obtain

$$(P + X)y_1 = (P - X)y_2$$

Boyle's law can also be applied in obtaining a relationship between Fig. 10.6 and Fig. 10.8

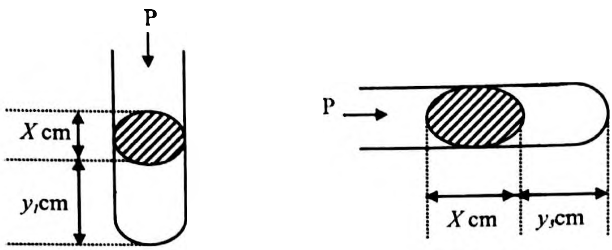


Fig 10.10 a. Vertical b. Horizontal

From Fig 10.10a, Initial or vertical pressure, $P_1 = P + X$

Initial or vertical volume, $V_1 = y_1A$

From Fig 10.10b, Final or horizontal pressure, $P_3 = P$

Final or horizontal volume, $V_3 = y_3A$

Applying Boyle's Law, $P_1V_1 = P_3V_3$ we obtain

$$(P + X)\,y_1\,A = P\,y_3\,A$$

Dividing both sides by A, we have

$$(P + X)\,y_1 = P\,y_3$$

Similarly, Boyle's law can be applied for Fig. 10.7 and Fig. 10.8 as shown below.

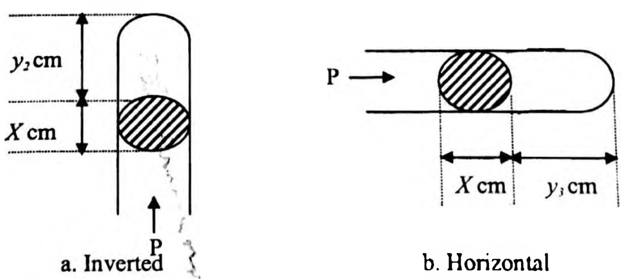


Fig 10.11 a. Inverted b. Horizontal

From Fig 10.11a, Initial or inverted pressure, $P_2 = P - X$

Initial or inverted volume, $V_2 = y_2A$

From Fig 10.11b, Final or horizontal pressure, $P_3 = P$

Final or horizontal volume, $V_3 = y_3A$

Applying Boyle's Law, $P_2V_2 = P_3V_3$ we obtain

$$(P - X)\,y_2 A = P\,y_3 A$$

Dividing both sides by A, we have

$$(P - X)y_2 = Py_3$$

Please note that the length of gas column or column of mercury could be in units other than centimeter (cm).

Finally, Boyle's law can be applied to derive a general equation connecting Fig 10.6, 10.7 and 10.8.

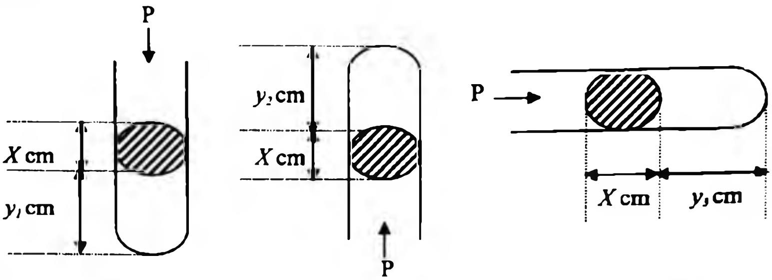


Fig 10.12 a. Vertical . b. Inverted c. Horizont...

From Boyle's Law, $P_1V_1 = P_2V_2 = P_3V_3$ we obtain for the three diagram above,

$$(P + X)\,y_1 A = (P - X)\,y_2 A = P\,y_3 A$$

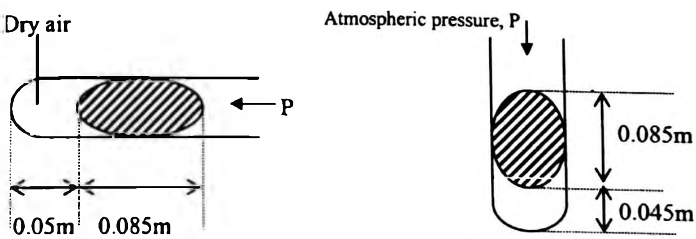
Divide through by A to obtain

$$(P + X)\,y_1 = (P - X)\,y_2 = P\,y_3$$

Example 5

A uniform capillary tube, closed at one end contained dry air trapped by a thread of mercury 8.5×10^{-2}m long. When the tube was held horizontally, the length of the air column was 5.0×10^{-2}m, when it was held vertically with the closed end downwards, the length was 4.5×10^{-2}m. Determine the value of the atmospheric pressure. [$g=10m/s^{-2}$, density of mercury = $1.36 \times 10^4 kgm^{-3}$]. *WAEC 2004*

Solution



a. Horizontal b. Vertical

Fig 10.13

At horizontal position, gas pressure is due only to atmospheric pressure, P.

From Fig 10.13 a. above, horizontal gas pressure, $P_1 = P$

Horizontal gas volume, $V_1 = 0.05A$

Where A is cross sectional area.

At vertical position, gas pressure is due to atmospheric pressure + pressure due to mercury column.

∴ From Fig 10.13 b. above, vertical gas pressure, $P_2 = P + 0.085$

vertical gas volume, $V_2 = 0.045A$

Substituting into Boyle's law, $P_1V_1 = P_2V_2$ we obtain

$$P \times 0.05A = (P + 0.085) \times 0.045A$$

Divide both sides by A

$$P \times 0.05 = (P + 0.085) \times 0.045$$

$$0.05P = 0.045P + 0.003825$$

$$0.05P - 0.045P = 0.003825$$

$$0.005P = 0.003825$$

$$P = \frac{0.003825}{0.005} = 0.765\text{mHg}$$

The atmospheric pressure (0.765mHg) so obtained is in length of mercury and has to be converted to N/m^2 or Pa using the pressure formula, $P = \rho gh$

Given values: density of mercury, $\rho = 1.36 \times 10^4 kgm^{-3}$; $g = 10m/s^2$
Calculated value: height of mercury, h = 0.765mHg

∴ Atmospheric pressure, $P = \rho gh$

$$= 1.36 \times 10^4 \times 10 \times 0.765$$

$$= 104040Nm^{-2} \text{ or } 1.04 \times 10^5 Pa$$

Example 6
A thread of mercury of length 12cm is used to trap some air in a capillary tube with uniform cross-sectional area and closed at one end. With the tube vertical and the closed end at the bottom, the length of the trapped air is 25cm. Calculate the length of the air column when the tube is held

(a) horizontally
(b) vertically with the closed end of the top.
(Atmospheric pressure = 76cm of mercury)

Solution

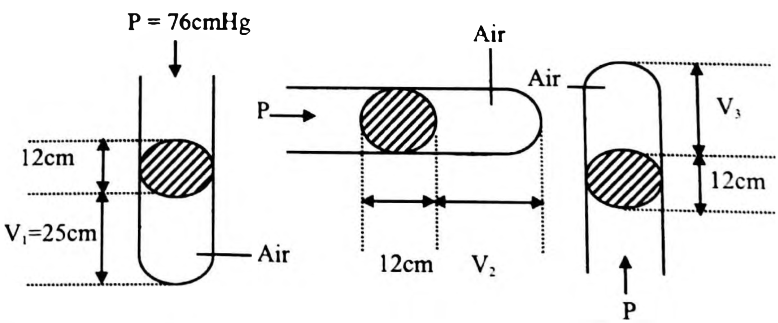


Fig 10.14 (i) (ii) (iii)

Let P be atmospheric pressure

Let volume be proportional to length.

(a) From Fig 10.14 (i) above, vertical air pressure, $P_1 = 76 + 12 = 88$ cmHg

Vertical air volume, V_1 = 25cm

From Fig 10.14 (ii) above, Horizontal air pressure, P_2 = 76 cmHg

Horizontal air volume, V_2 = ?

Applying Boyle's Law, $P_1V_1 = P_2V_2$ and substituting above values, we have

$$88 \times 25 = 76 \times V_2$$

$$\therefore V_2 = \frac{88 \times 25}{76} = 28.95\text{cm}$$

(b) Considering Fig 10.14 (i) and (iii) above, we have

Vertical air pressure, P_1 = 88cmHg   Vertical air volume, V_1 = 25cm

Inverted air pressure, $P_3 = 76 - 12 = 64$cm   Inverted air volume, V_3 = ?

Applying Boyle's Law, $P_1V_1 = P_3V_3$ we obtain

$$88 \times 25 = 64 \times V_3$$

$$\therefore V_3 = \frac{88 \times 25}{64} = 34.38\text{cm}$$

Example 7

An air column 10cm in length is trapped into the sealed end of a capillary tube by a 15cm column of mercury with the tube held vertically as shown below. On inverting the tube, the air column becomes 15cm long. What is the atmospheric pressure during the experiment? A. 90cm B. 76cm C. 75cm D. 60cm E. 50cm *JAMB 1979*

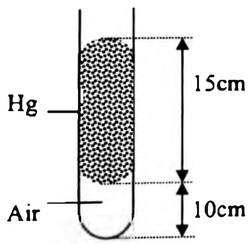


Fig 10.15

Solution

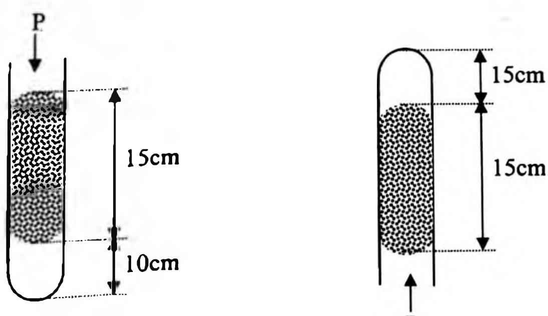


Fig 10.16 (i) Vertical (ii) Inverted

Let P be the atmospheric pressure

From (i) Vertical air pressure, $P_1 = (P + 15)$cm

Vertical air volume, $V_1 = 10cm$

Assuming volume is proportional to length.

From (ii) above, inverted air pressure, $P_2 = (P - 15)cm$

Inverted air volume, $V_2 = 15cm$

Applying Boyle's Law, $P_1V_1 = P_2V_2$ we obtain

$$(P + 15) \times 10 = (P - 15) \times 15$$

$$10P + 150 = 15P - 225$$

$$15P - 10P = 225 + 150$$

$$5P = 375$$

$$P = \frac{375}{5} = 75cmHg$$

Application of Boyle's Law to Air Bubbles in a Water Body

An air bubble at the bottom of a body of water (swimming pool, dam, drum of water or sea) is under the influence of two pressures.

(i) Atmospheric pressure, P

(ii) Pressure due to height of water, H

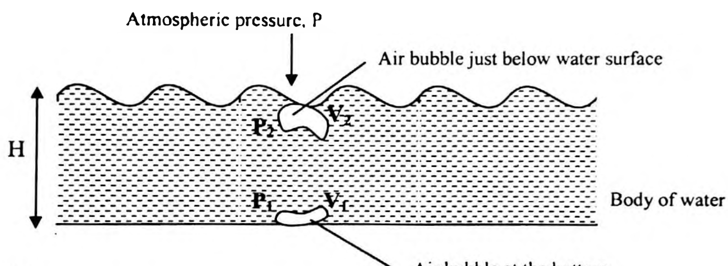


Fig 10.17

In contrast, an air bubble just below the surface of water is under the influence of atmospheric pressure ONLY.

The equivalent of atmospheric pressure (760mmHg = 76cmHg = 0.76mHg) in height of water (H) is obtained by equating the pressure formular for water and mercury.

$$(\rho hg)_{water} = (\rho hg)_{mercury}$$

i.e, density of mercury × height of mercury × g = density of water × height of water × g

Let height of water, h = H

$$\therefore \; 13600 \times 0.76 \times 10 = 1000 \times H \times 10$$

$$H = \frac{13600 \times 0.76 \times 10}{1000 \times 10} = 10.34m$$

One atmosphere of pressure therefore supports a height of about 10m of water.

The pressure, P_1 of the air bubble at the bottom is $P_1 = P + H$ or $P_1 = 10 + H$

The pressure, P_2 of the air bubble just below the surface is $P_2 = P$

Boyle's law can be applied to air bubble rising from the bottom to top of a water b[illegible] as follows:

From $P_1V_1 = P_2V_2$ we obtain

$$(P + H)V_1 = PV_2$$

$$(10 + H)V_1 = PV_2$$

Where P = atmospheric pressure in height of water (10m)
H = depth or height of water body
V_1 = volume of air bubble at the bottom of water
V_2 = volume of air bubble just below water surface

Example 8
An air bubble of volume $6cm^3$ rises from the bottom to the top of a swimming pool which is 15m deep. What will be the volume of the air bubble just below the pool's surface, if the atmospheric pressure is equivalent to 10m of water?
Solution

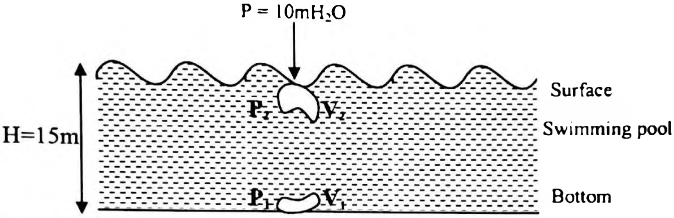


Fig 10.18

At the bottom: $P_1 = P + H = 10 + 15 = 25mH_2O$

$V_1 = 6cm^3$

At the top: $P_2 = P = 10mH_2O$

$V_2 = ?$

Applying Boyle's Law, $P_1V_1 = P_2V_2$ we have

$$25mH_2O \times 6cm^3 = 10mH_2O \times V_2$$

$$V_2 = \frac{25mH_2O \times 6cm^3}{10mH_2O} = \frac{150}{10} = 15cm^3$$

Example 9
The volume of air bubble changed from $3cm^3$ to $12cm^3$ when it rise from the bottom to the top of a hydroelectric power dam. Calculate the depth of water in the dam if the atmospheric pressure is equivalent to 10m of water.
Solution

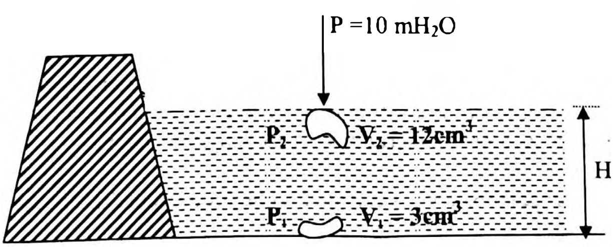


Fig 10.19

At the bottom: $P_1 = P + H = 10 + H$

$V_1 = 3cm^3$

At the top: $P_2 = P = 10mH_2O$

$V_2 = 12cm^3$

Applying Boyle's Law, $P_1V_1 = P_2V_2$ we have

$$(10 + H) \times 3 = 10 \times 12$$

$$30 + 3H = 120$$

$$3H = 120 - 30$$

$$3H = 90$$

$$H = \frac{90}{3} = 30m$$

CHARLES LAW

Charles's law states that the volume of a given mass of gas at a constant pressure is directly proportional to its absolute temperature.

That is, $V \alpha T$ or $\frac{V}{T} = \text{constant}$

Therefore, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

Where V_1 and T_1 are initial volume and temperature, while V_2 and T_2 are final volume and temperature of the gas, respectively.

Always convert temperature in °C to K when using Charles's law by adding 273.

Example 10

The volume of a given mass of gas is $40cm^3$ at 27°C. What is its volume at 90°C if its pressure remains constant? *NECO 2003*

Solution

Initial gas volume, $V_1 = 40cm^3$

Initial gas temperature, $T_1 = 27°C = (27 + 273) = 300K$

Final gas temperature, $T_2 = 90°C = (90 + 273) = 363K$

From $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, final gas volume, $V_2 = \frac{V_1T_2}{T_1} = \frac{40 \times 363}{300} = 48.40cm^3$

Example 11

Dry oxygen is trapped by a pellet of mercury in a uniform capillary tube which is sealed at one end. The length of the column of oxygen at 27°C is 50cm. If the pressure of the oxygen is constant, at what temperature will the length be 60cm. *WAEC 1998*

Solution

The volume of the gas (oxygen) is assumed to be proportional to the length of the column of gas, i.e. $V \alpha l$

Initial gas volume, $V_1 = 50cm$

Initial gas temperature, $T_1 = 27°C = (27 + 273) = 300K$

Final gas volume, $V_2 = 60cm$

From $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, final gas temperature, $T_2 = \frac{T_1V_2}{V_1} = \frac{300 \times 60}{50} = 360K$ or 87^oC

Example 12
A fixed mass of gas of volume $600cm^3$ at a temperature of 27^oC is cooled at constant pressure to a temperature of 0^oC. What is the change in volume? *WAEC 1991*
Solution
Initial gas volume, $V_1 = 600cm^3$
Initial gas temperature, $T_1 = 27^oC = (27 + 273) = 300K$
Final gas temperature, $T_2 = 0^oC = (0 + 273) = 273K$

From $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, final gas volume, $V_2 = \frac{V_1T_2}{T_1} = \frac{600 \times 273}{300}$
$= 546cm^3$

Change in volume = volume at 27^oC – volume at 0^oC

i.e. $\Delta V = V_1 - V_2 = 600 - 546 = 54cm^3$

Example 13
A gas occupies a certain volume at 27^oC. At what temperature will its volume be doubled assuming that its pressure remains constant? *NECO 2000*
Solution
Initial gas temperature, $T_1 = 27^oC = (27 + 273) = 300K$ Initial gas volume, $V_1 = V$
Final gas volume, $V_2 = 2V$ (…… *its volume be doubled* ……)

Substitute above values into $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ to obtain

$$\frac{V}{300} = \frac{2V}{T_2} \quad \therefore \quad T_2 = \frac{300 \times 2V}{V} = 300 \times 2 = 600K \text{ or } 327^oC$$

Example 14
At what value of x, y and z is the equation, $P^xV^yT^z$ = constant, a statement of
(i) Charles's law (ii) Boyle's law (iii) Pressure law (iv) General gas law
Solution
The following laws of indices can be applied to a problem of this kind.
a. $^1/_x = x^{-1}$ b. $x = x^1$ c. $x^0 = 1$

(i) Charles's law; $\frac{V}{T}$ = constant

$$\frac{V}{T} = \frac{V^1}{T^1} = V^1T^{-1}$$

$\therefore$ $P^xV^yT^z$ becomes $P^oV^1T^{-1}$ or $1 \times V^1T^{-1} = \frac{V}{T}$

Therefore, for x = 0, y = 1 and z = -1, $P^xV^yT^z$ = constant is Charles's law.
(Any term P, V or T which is not present in the equation is assigned a superscript of zero, hence P^o).

(ii) Boyle's law: PV = constant

$$PV = P^1V^1$$

$\therefore$ $P^xV^yT^z$ becomes $P^1V^1T^o$ or $P^1V^1 \times 1 = PV$

Therefore, for x = 1, y = 1 and z = 0, $P^xV^yT^z$ = constant is Boyle's law.

(iii) Pressure law: $\frac{P}{T}$ = constant

$$\frac{P}{T} = \frac{P^1}{T^1} = P^1T^{-1}$$

$\therefore$ $P^xV^yT^z$ becomes $P^1V^0T^{-1}$ or $P^1 \times 1 \times T^{-1} = \frac{P}{T}$

Therefore, for x=1, y=0 and z= -1, $P^xV^yT^z$ = constant is Pressure law.

(iv) General gas law: $\frac{PV}{T}$ = constant

$$\frac{PV}{T} = \frac{P^1V^1}{T^1} = P^1V^1T^{-1}$$

$\therefore$ $P^xV^yT^z$ becomes $P^1V^1T^{-1} = \frac{PV}{T}$

Therefore for x = 1, y = 1 and z = -1, $P^xV^yT^z$ = constant is general gas law.

PRESSURE LAW

Pressure law states that the pressure of a fixed mass of gas at constant volume is proportional to the absolute temperature of the gas.

That is, $P \propto T$ or $\frac{P}{T}$ = constant

Therefore, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

Where P_1 and T_1 are initial gas pressure and temperature, while P_2 and T_2 are final gas pressure and temperature, respectively.

Always convert temperature in degree Celsius (°C) to Kelvin (K) when applying pressure law.

Example 15

Before starting a journey, the tyre pressure of a car was $3 \times 10^5 Nm^{-2}$ at 27°C. At the end of the journey, the pressure rose to $4 \times 10^5 Nm^{-2}$. Calculate the temperature of the tyre after the journey assuming the volume is constant.

A. 400°C B. 300°C C. 273°C D. 127°C *JAMB 1997*

Solution

Initial gas pressure, $P_1 = 3 \times 10^5 Nm^{-2}$

Initial gas temperature, T_1 = 27°C = (27 + 273) = 300K

Final gas pressure, $P_2 = 4 \times 10^5 Nm^{-2}$

From , $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ final temperature, $T_2 = \frac{P_2T_1}{P_1}$

$$\therefore T_2 = \frac{4 \times 10^5 \times 300}{3 \times 10^5} = 400K \quad \text{or} \quad (400 - 273)°C = 127°C$$

Example 16

A gas at pressure P Nm^{-2} and temperature 27°C is heated to 77°C at constant volume.

What is the new pressure? A. $0.85PNm^{-2}$ B. $0.86PNm^{-2}$ C. $1.16PNm^{-2}$ D. $1.18PNm^{-2}$ E. $2.85PNm^{-2}$ *JAMB 1978*

Solution

Initial gas pressure, $P_1 = P\ Nm^{-2}$

Initial gas temperature, $T_1 = 27^oC = (27 + 273) = 300K$

Final gas temperature, $T_2 = 77^oC = (77 + 273) = 350K$

From $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, final pressure, $P_2 = \frac{P_1T_2}{T_1} = \frac{P \times 350}{300} = 1.16P\ Nm^{-2}$

Example 17

A closed inexpansible vessel contains air saturated with water vapour at 77 ^oC. The total pressure in the vessel is 1007 mmHg. Calculate the new pressure in the vessel if the temperature is reduced to 27 ^oC . (The S.V.P. of water at 77 ^oC and 27 ^oC respectively are 314mmHg and 27mmHg. Treat the air in the vessel as an ideal gas). WAEC2008[E12]

Solution

Based on the law of partial pressures, the pressure of air alone in the vessel is equal to the *difference* between the total pressure in the vessel and the S.V.P. of water at the same temperature.

. initial air pressure, $P_1 = 1007 - 314 = 693mmHg$

Final air pressure, $P_2 = ?$

Initial temperature, $T_1 = 77\ ^oC = 77 + 273 = 350K$

Final temperature, $T_2 = 27\ ^oC = 27 + 273 = 350K$

Because the vessel is *inexpansible*, its volume does not change, therefore pressure law can be applied.

Substitute into $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ to obtain

$$\frac{693}{350} = \frac{P_2}{300}$$

$$P_2 = \frac{693 \times 300}{350} = 594\,mmHg$$

The pressure in the vessel at 27 ^oC is equal to the sum of the pressure of air and the S.V.P. of water at 27 ^oC.

Vessel pressure at 27 ^oC = air pressure + S.V.P. of water at 27 ^oC.

= 594 mmHg + 27 mmHg

= 621 mmHg

GENERAL GAS LAW

A combination of Boyle's, Charles's and Pressure laws results in the general gas law for ideal gas.

That is, $\frac{PV}{T}$ = constant

Therefore, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Example 18

The volume and pressure of a given mass of gas at 27°C are $76cm^3$ and 80cm of mercury respectively. Calculate its volume at s.t.p. *WAEC 2000*

Solution

0°C (273K) and 760mmHg ($1.013 \times 10^5\ Nm^{-2}$) are called standard temperature and pressure, usually abbreviated, s.t.p.

Initial gas volume, $V_1 = 76cm^3$ Initial gas pressure, $P_1 = 80cmHg$

Final gas volume, $V_2 = ?$ Initial gas temperature, $T_1 = 27°C = (27 + 273) = 300K$

Final gas temperature, $T_2 = 273K$ Final gas pressure, $P_2 = 760mmHg = 76cmHg$

Substituting above values into the ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \text{we have,}$$

$$\frac{80cmHg \times 76cm^3}{300K} = \frac{76cmHg \times V_2}{273K}$$

$$\therefore \quad V_2 = \frac{80cmHg \times 76cm3 \times 273K}{76cmHg \times 300K} = 72.8cm^3$$

Example 19

A given mass of an ideal gas occupies a volume V at a temperature T and under a pressure P. If the pressure is increased to 2P and the temperature reduced to ½T, then what is the percentage change in the volume of the gas?

A 0% B. 25% C. 75% D. 300% E. 400% *JAMB 1984*

Solution

Initial gas volume, $V_1 = V$ Initial gas temperature, $T_1 = T$

Initial gas pressure, $P_1 = P$ Final gas temperature, $T_2 = ½T = 0.5T$

Final gas pressure, $P_2 = 2P$ Final gas volume, $V_2 = ?$

Substituting into $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ we obtain

$$\frac{P \times V}{T} = \frac{2P \times V_2}{0.5T}$$

$$\therefore \quad V_2 = \frac{P \times V \times 0.5T}{2P \times T} = \frac{V \times 0.5}{2} = \frac{0.5V}{2} = 0.25V$$

$$\text{Percentage change in volume} = \frac{V_1 - V_2}{V_1} \times 100$$

$$= \frac{V - 0.25V}{V} \times 100$$

$$= \frac{0.75V}{V} \times 100 = 0.75 \times 100 = 75\%$$

Example 20

The pressure of a given mass of a gas changes from $300Nm^{-2}$ to $120Nm^{-2}$ while the temperature drops from 127°C to −73°C. What is the ratio of the final volume to the initial volume? A. 2:5 B. 5:4 C. 5:2 D. 4:5 *JAMB 2001*

Solution

Initial gas volume, $P_1 = 300Nm^{-2}$ Initial gas volume, $V_1 = V_1$

Initial gas temperature, $T_1 = 127°C = (127 + 273) = 400K$

Final gas pressure, $P_2 = 120Nm^{-2}$ Final gas volume, $V_2 = V_2$

Final gas temperature, $T_1 = -73°C = (-73 + 273) = 200K$

Substitute into $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$\frac{300 \times V_1}{400} = \frac{120V_2}{200}$$

Rearranging, we have $\frac{300 \times 200}{400 \times 120} = \frac{V_2}{V_1}$

$$\frac{60000}{48000} = \frac{V_2}{V_1}$$

$$\frac{60}{48} = \frac{V_2}{V_1}$$

Divide both numerator and denominator by 12

$$\frac{5}{4} = \frac{V_2}{V_1}$$

∴ The ratio of final volume (V_2) to the initial volume (V_1), $V_2 : V_1 = 5:4$

WORK DONE BY AN EXPANDING GAS AT CONSTANT PRESSURE

Work done = Force x distance (length) ie. $W = F \times l$ .... (1)

$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$ i.e. $P = \frac{F}{A}$ .... (2)

∴ Force = Pressure × Area i.e. $F = PA$ .... (3)

Substitute equation (3) into equation (1)

If $F = PA$

Then $W = F \times l$ becomes

$W = PAl$ ....(4)

Volume = Area × length i.e. $V = Al$ ....(5)

Substitute equation (5) into equation (4)

If $V = Al$

Then $W = PAl$ becomes

$W = PV$ ....(6)

When a gas expands the volume (V) changes.

Therefore work done by expanding gas = Pressure × Change in volume.

Equation (6) becomes, $W = P\Delta V$ or $W = P(V_2 - V_1)$

Where W = work done by expanding gas in Joules (J)
$\Delta V = (V_2 - V_1)$ = change in volume
V_2 = final volume after expansion
V_1 = initial volume before expansion

Example 21
The volume of a fixed mass of gas increased from $0.04m^3$ to $0.32m^3$ while the pressure remains constant at $1.5 \times 10^5 Nm^{-2}$. How much work was done by the gas?
Solution
Initial volume, $V_1 = 0.04m^3$ Final volume, $V_2 = 0.32m^3$
Constant pressure, $P = 1.5 \times 10^5 Nm^{-2}$

Work done = constant pressure × change in volume

$$W = P(V_2 - V_1)$$

$$W = 1.5 \times 10^5 (0.32 - 0.04)$$

$$W = 1.5 \times 10^5 \times 0.28 = 4.2 \times 10^4 \text{Joule}$$

Example 22
The work done by a gas when it expand to eight times its initial volume of $0.5 \times 10^{-4}m^3$ is 35.5J. At what constant pressure did the expansion occur?
Solution
Work done, $W = 35.5J$ Initial volume, $V_1 = 0.5 \times 10^{-4}m^3$
Final volume, $V_2 = 8 \times (0.5 \times 10^{-4}) = 4 \times 10^{-4}m^3$
From $W = P(V_2 - V_1)$

$$\text{Constant pressure, } P = \frac{W}{V_2 - V_1} = \frac{35.5}{4\times10^{-4} - 0.5\times10^{-4}}$$

$$= \frac{35.5}{0.00035} = 1.014\times10^5 Nm^{-2}$$

CUBIC AND PRESSURE EXPANSIVITY OF A GAS

Cubic Expansivity of a gas at constant pressure is the fraction of its volume at 0°C by which the volume of a given mass of gas expands per Kelvin change in temperature.

$$\text{Cubic expansivity, } \gamma = \frac{\text{change in volume from } 0°C}{\text{volume at } 0°C \times \text{change in temperature}}$$

$$\text{i.e. } \gamma = \frac{V_\theta - V_o}{V_o \times \theta}$$

Where γ = cubic or volume expansivity of gas measured in $0°C^{-1}$ or K^{-1}
V_θ = volume of gas at θ°C
V_o = volume of gas at 0°C
θ = change in temperature of gas

Because the length of a gas column in a tube of uniform cross-sectional area can be taken as being proportional to its volume (i.e. $V \alpha l$), cubic expansivity can also be written as

$$\gamma = \frac{l_\theta - l_o}{l_o \times \theta}$$

Where l_θ = length of gas column at θ°C
l_o = length of gas column at 0°C
θ = change in temperature of gas

Example 23

The volume of air in a capillary tube is $56cm^3$ at 0°C and $75cm^3$ at 100°C. Calculate the cubical expansivity of the air at constant pressure.

Solution

Volume of air at 0°C, $V_o = 56cm^3$ Volume of air at 100°C, $V_0 = 75cm^3$

Change in temperature, $\theta = 100°C$

$$\therefore \text{ Cubic expansivity, } \gamma = \frac{V_0 - V_o}{V_o \times \theta} = \frac{75-56}{56 \times 100} = \frac{19}{5600}$$

$$\therefore \quad \gamma = 3.39 \times 10^{-3}K^{-1} \text{ or } 3.39 \times 10^{-3}{}^{o}C^{-1}$$

Example 24

The cubic expansivity of a fixed mass of gas trapped in a capillary tube of uniform cross-sectional area is $3.68 \times 10^{-3}K^{-1}$. If the gas column at 0°C is $13.69 \times 10^{-2}m$ long, calculate its length at 100°C.

Solution

Cubic expansivity, $\gamma = 3.68 \times 10^{-3}K^{-1}$ Length of gas column at 0°C, $l_o = 13.69 \times 10^{-2}m$

Length of gas column at 100°C, $l_\theta = ?$ Change in temperature, $\theta = 100°C$

Substitute into $\gamma = \dfrac{l_\theta - l_o}{l_o \times \theta}$ to obtain

$$3.68 \times 10^{-3} = \frac{l_0 - 13.69 \times 10^{-2}}{13.69 \times 10^{-2} \times 100}$$

$$l_0 - 13.69 \times 10^{-2} = 3.68 \times 10^{-3} \times 13.69 \times 10^{-2} \times 100$$

$$l_\theta - 13.69 \times 10^{-2} = 0.0503792$$

$$l_\theta = 0.0503792 + 13.69 \times 10^{-2}$$

$$l_0 = 0.1873m \text{ or } 18.73 \times 10^{-2}m$$

Example 25

The cubic expansivity of a certain gas at constant pressure is $\frac{1}{273}K^{-1}$. If a given mass of the gas is held at constant pressure and its volume at 0°C if $273m^3$, determine the volume of the gas at 273°C. *WAEC 2008*

Solution

Cubic expansivity, $\gamma = \frac{1}{273}K^{-1}$ Volume of gas at 0°C, $V_o = 273m^3$

Volume of gas at 273°C, $V_\theta = ?$ Change in temperature, $\theta = 273°C = (273 + 273)K = 546K$

The change in temperature θ given in centigrade (°C) had to be converted to Kelvin (K) because the cubic expansivity is given in K^{-1}.

Substitute into $\gamma = \dfrac{V_\theta - V_o}{V_o \times \theta}$ to obtain

$$\frac{1}{273} = \frac{V_0 - 273}{273 \times 546}$$

$$\text{Rearranging, } \frac{273 \times 546}{273} = V_0 - 273$$

$$546 = V_0 - 273$$

$$\therefore \text{ Volume of gas at } 273°C, \quad V_0 = 546 + 273 = 819m^3$$

Pressure Expansivity of a gas at constant volume is the fraction of its pressure at 0°C by which the pressure of a given mass of gas increases per Kelvin change in temperature.

$$\text{Pressure expansivity, } \beta = \frac{\text{change in pressure from } 0°C}{\text{pressure at } 0°C \times \text{change in temperature}}$$

$$\text{Pressure expansivity, } \beta = \frac{P_\theta - P_o}{P_o \times \theta}$$

Where P_o = gas pressure at 0°C
P_θ = gas pressure at θ°C
θ = change in temperature of gas
β = pressure expansivity of gas

Example 26

What is the pressure expansivity of a gas whose pressure changes from 83mmHg at 0°C to 105mmHg at 70°C?
Solution

Gas pressure at 0°C, P_o = 83.6mmHg
Gas pressure at 70°C, P_θ = 105mmHg
Change in temperature, θ = 70°C

$$\text{Pressure expansivity, } \beta = \frac{P_\theta - P_o}{P_o \times \theta} = \frac{105 - 83.6}{83.6 \times 70} = \frac{21.4}{5852} = 3.66 \times 10^{-3}\ {}^oC^{-1}$$

EXERCISE 10

1.

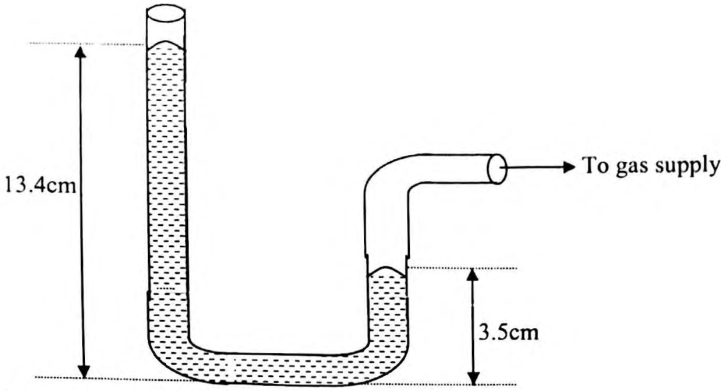


Fig 10.20

When one arm of a U-tube manometer is connected to a gas supply, the levels of mercury in the two arms of the U-tube are as shown in the diagram above. If the atmospheric pressure is 76.0cmHg, what is the gas pressure?

A. 62.6cmHg B. 72.5cmHg C. 79.5cmHg D. 85.9cmHg *JAMB 1988 Ans:85.9cmHg*

2.

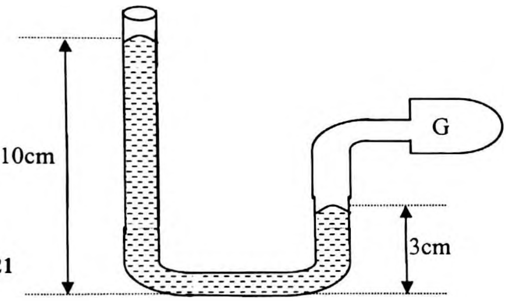


Fig 10.21

In the diagram above, if the atmospheric pressure is 760mm, what is the pressure in the chamber G?

A 660mm B. 690mm C. 830mm D. 860mm *JAMB 1994 Ans: 830mm*

3. A thread of mercury of length 15cm is used to trap some air in a capillary tube with uniform cross-sectional area and closed at one end. With the tube vertical and the open end uppermost, the length of the trapped air column is 20cm. Calculate the length of the air column when the tube is held

(i) horizontally

(ii) vertically with the open end underneath.

(Atmospheric pressure = 76cm of mercury) *WAEC 1988 Ans: (i) 23.95cm* (ii) 29.84cm

4.

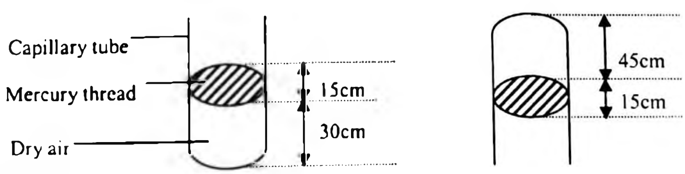


Fig 10.22

The set-up illustrated above shows a capillary tube of uniform cross sectional area in two different arrangements. Using the data in the diagrams, calculate the pressure of the atmosphere. *WAEC 1991 Ans: 75cm of mercury*

5. A thread of mercury of length 16cm is used to trap some air in a capillary tube of uniform cross-sectional area and closed at one end. When the tube is held vertically, with the closed end at the bottom, the length of the trapped air column is 30cm. Calculate the length of the air column when the tube is held,
   (i) horizontally
   (ii) vertically with the open end underneath. [Atmospheric pressure = 76cm of mercury] *NECO 2006 Ans: (i) 36.32cmHg (ii) 46cmHg*

6.

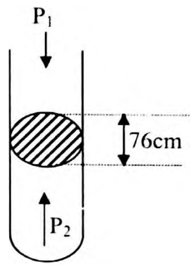


Fig 10.23

In the figure above, a thread of mercury 76cm long is used to trap some dry air in a capillary tube of uniform cross-section closed at one end. Calculate the ratio of the pressure P_2/P_1 [Atmospheric pressure is 76cm of mercury] *NECO 2004*

Ans: $\frac{P_2}{P_1} = \frac{76+76}{76} = \frac{152}{76} = 2$

7.

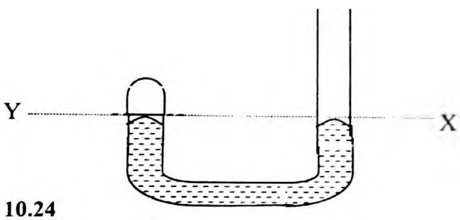


Fig 10.24

In the J-tube above, Y and X are on the same horizontal level and $30cm^3$ of air is trapped above Y when the atmospheric pressure is 75cmHg. Calculate the volume of air trapped above Y when 15cmHg is now poured into the limb above X.

A. $15cm^3$ B. $25cm^3$ C. $35cm^3$ D. $45cm^3$ *JAMB 1999 Ans: 25cm³*

8. An air bubble of volume $2cm^3$ is formed 20m under water. What will be its volume when it rises to just below the surface of the water if the atmospheric pressure is equivalent to a height of 10m of water? *WAEC 1995 Ans: 6cm³*

9. An air bubble of volume $4cm^3$ is formed 20m under water. What will be its volume when it rises to just below the surface of the water if the atmospheric pressure is equivalent to a height of 10m of water? *NECO 2002 (Ans: 12cm³)*

10. An air bubble rises from the bottom to the top of a water dam which is 40m deep. The volume of the bubble just below the surface is $2.5cm^3$. Find its volume at the bottom of the dam, if atmospheric pressure is equivalent to 10m of water.

A. $10.0cm^3$ B. $2.0cm^3$ C. $1.6cm^3$ D. $0.625cm^3$ E. $0.5cm^3$ *JAMB 1979 Ans: 0.5 cm³*

11. The pressure of a fixed mass of gas is $2.0\times10^5 Nm^{-2}$ at a known temperature. Assuming that the temperature remains constant, what will be the pressure of the gas if its volume is halved? *WAEC 1996 (Ans: 4.0 × 10⁵ Nm⁻²)*

12. A fixed mass of gas occupies a volume of $20cm^3$ at a pressure of 700mmHg. Assuming that the temperature remains constant, what will be the volume of the gas at 750cmHg? *WAEC 1997 (Ans: $18.7cm^3$)*

13. When the pressure of a fixed mass of gas is doubled at constant temperature, the volume of the gas is;
A. increased four times B. doubled C. unchanged D. halved *WAEC 2003Ans: D*

14. Dry hydrogen is trapped by pellet of mercury in a uniform capillary tube closed at one end. If the length of the column of hydrogen at 27°C is 1.0m, at what temperature will the length be 1.20m? *WAEC 1990 Ans: 360K or 87°C*

15. A gas which obeys Charles's law exactly has a volume of $283cm^3$ at 10°C. What is its volume at 30°C? *WAEC 1992 Ans: $303cm^3$*

16. A gas has a volume of $546cm^3$ at 0°C. What is the volume of the gas at 100°C if its pressure remains constant? *WAEC 1993 Ans: $746cm^3$*

17. A gas occupies a certain volume at 27°C. At what temperature will its volume be three times the original volume assuming that its pressure remains constant?
WAEC 1994 Ans: 900K or 627°C

18. A balloon containing $546cm^3$ of air is heated from 0°C to 10°C. If the pressure is kept constant, what will be its volume at 10°C? *WAEC 1995 Ans: $566cm^3$*

19.

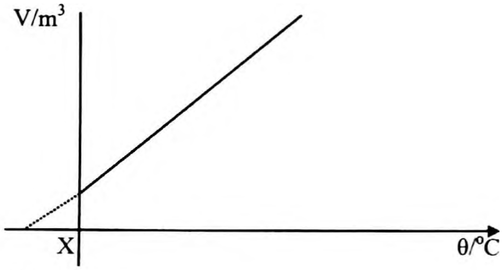


Fig 10.25

The diagram above shows the variation of volume V of a gas with temperature θ in a Charles's law experiment. What is the value of the temperature at the point X?
WAEC 2000 Ans: -273°C

20. The volume of a given mass of gas is $273cm^3$ at 0°C. What is the volume at 273°C if its pressure remains constant? *NECO 2006 Ans: $546cm^3$*

21. A gas occupies a volume of $300cm^3$ at a temperature of 27°C. What is its volume at 54°C when the pressure is constant?
A. $150cm^3$ B. $273cm^3$ C. $327cm^3$ D. $600cm^3$ *JAMB 1986 Ans: $327cm^3$*

22. A column of air 10.0cm long is trapped in a tube at 27°C. What is the length of the column at 100°C?
A. 12.4cm B. 13.7cm C. 18.5cm D. 37.0cm *JAMB 1989 Ans: 12.43cm*

23. The equation $P^xV^yT^z$ = constant, is Charle's law when
A. x = 1, y = -1, z = 1 B. x = 0, y = 1, z = -1
B. x = 1, y = 0, z = -1 D. x = 0, y = 1, z = 1 *JAMB 1995 Ans: B*

24. A gas has a volume of $100cm^3$ at 27°C. If it's heated to temperature T until a final volume of $120cm^3$ is attained, calculate T.
A. 33°C B. 60°C C. 87°C D. 114°C *JAMB 1998 Ans: 360K or 87°C*

25. A given mass of gas has a pressure of $80Nm^{-2}$ at a temperature of 47°C. If the temperature is reduced to 27°C with the volume remaining constant, what is the new pressure? *WAEC 1988 Ans: $75Nm^{-2}$*

26. The pressure of air in a tyre is $22.5Nm^{-2}$ at 27°C. If the air in the tyre heats up to 47°C, calculate the new pressure of the air, assuming that no air leaks out and that the change in volume of the air can be neglected. *WAEC 1994 Ans: $24Nm^{-2}$*

27. A given mass of an ideal gas has a pressure of $500Nm^{-2}$ at −13°C. If its volume remains constant, calculate its pressure at 247°C. *WAEC 2001 Ans: $1000Nm^{-2}$*

28. The tyre pressure of a car was found to be 150cmHg in the morning at a temperature of 27°C. Hard driving in the afternoon raised the temperature of the tyres to 57°C. The tyre pressure had by afternoon;
A. increased by 15.0cmHg B. decreased by 15.0cmHg C. increased by 22.50cmHg D. decreased by 22.50cmHg E. remained constant. *JAMB 1982 Ans: A*

29. A motor tyre is inflated to pressure of $2.0 \times 10^5 Nm^{-2}$ when the temperature of air is 27°C. What will be the pressure in it at 87°C assuming that the volume of the tyre does not change? A. $2.6 \times 10^5 Nm^{-2}$ B. $2.4 \times 10^5 Nm^{-2}$ C. $2.2 \times 10^5 Nm^{-2}$ D. $1.3 \times 10^5 Nm^{-2}$
JAMB 1994 Ans: $2.4 \times 10^5 Nm^{-2}$

30. A given mass of gas at a temperature of 30°C is trapped in a tube of volume V. Calculate the temperature of the gas when the volume is reduced to two-third of its original value by applying a pressure twice the original value.
WAEC 1992 Ans: 404K or 131°C

31. $500cm^3$ of a gas is collected at 0°C and at a pressure of 72.0cm of mercury. What is the volume of the gas at the same temperature and at a pressure of 76.0cm of mercury?
A. $\frac{76 \times 500cm^3}{72}$ B. $\frac{72 \times 500cm^3}{76}$ C. $\frac{72 \times 76cm^3}{500}$ D. $\frac{76cm^3}{72 \times 500}$ E. $\frac{72cm^3}{76 \times 500}$
WAEC 1994 Ans: B

32. The volume of an ideal gas at a pressure of 77cmHg and temperature 60°C is $240cm^3$. If the temperature and pressure are increased to 98°C and 81cmHg respectively, calculated the new volume of the gas. *WAEC 2001 Ans: $254.2cm^3$*

33. The volume of a given mass of an ideal gas at 327K and $9.52 \times 10^4 Pa$ is $40cm^3$. Calculate the volume of the gas at 273K and $1.034 \times 10^5 Pa$. *WAEC 2003 Ans: $30.75cm^3$*

34. The temperature of $900cm^3$ of an ideal gas at a pressure of 114cmHg is 27°C. Calculate its volume at 76cmHg and 0°C. *WAEC 2004 Ans: $1228.5cm^3$*

35. A quantity of sungas at -133°C occupies $^1/_3 m^3$ under a pressure $1.4 \times 10^5 Nm^{-2}$. If the gas occupies $31m^3$ at 37°C, what will its pressure be?
A. $1.26 \times 10^3 Nm^{-2}$ B. $10^4 Nm^{-2}$ C. $1.35 \times 10^4 Nm^{-2}$ D. $2.2 \times 10^4 Nm^{-2}$ E. $10^3 Nm^{-2}$
JAMB 1980 Ans: $3.3 \times 10^3 Nm^{-2}$ .no correct option

36. If the pressure on $100cm^3$ of an ideal gas is doubled while its Kelvin temperature is halved, what then will become the new volume of the gas?
A. $25cm^3$ B. $50cm^3$ C. $100cm^3$ D. $200cm^3$ E. $400cm^3$ *JAMB 1983 Ans: $25cm^3$*

37. A quantity of gas occupies a certain volume when the temperature is −73°C and the pressure is 1.5 atmospheres. If the pressure is increased to 4.5 atmospheres and the volume is halved at the same time, what will be the new temperature of the gas?
A. 573°C B. 327°C C. 300°C D. 110°C E. 27°C *JAMB 1985 Ans: 300K or 27°C*

38. A mass of gas at 7°C and 70cm of mercury has a volume of $1200cm^3$. Determine its volume at 27°C and pressure of 75cm of mercury.
A. $1200cm^3$ B. $1378cm^3$ C. $4320cm^3$ D. $4629cm^3$ *JAMB 1989 Ans: $1200cm^3$*

39. A gas with initial volume $2 \times 10^{-6} m^3$ is allowed to expand to six times its initial volume at constant pressure of $2 \times 10^5 Nm^{-2}$ what's the work done?
A. 2.0J B. 4.0J C. 12.0J D. 1.2J *JAMB 2001 Ans: 2.0J*

40. A small circular membrane is 10cm below the surface of a pool of mercury when the barometric height is 76cm of mercury. If the density of mercury is $13600kgm^{-3}$, what is the pressure on the membrane in Nm^{-2}? [$g=10m/s^2$] *WAEC 1992 Ans: $1.17 \times 10^5 Nm^{-2}$*

41. As a result of air at the top of a barometer the height of the mercury column is 73.5cm when it should be 75.0cm; the volume of the space above the mercury is $8.0cm^3$. Calculate the correct barometric height when the barometer reads 74.0cm and the volume of the space above the mercury is $6.0cm^3$.
A.72.0cm B.74.5cm C.75.1cm D.76.0cm *JAMB 1987 Ans: 76cm*

42. The pressure of 3 moles of an ideal gas at a temperature of 27°C having a volume of $10^{-3} m^3$ is? [$R=8.3J\ molK^{-1}$] A. $2.49 \times 10^6 Nm^{-2}$ B. $7.47 \times 10^6 Nm^{-2}$ C. $2.49 \times 10^5 Nm^{-2}$ D. $7.47 \times 10^5 Nm^{-2}$
JAMB 2002 Ans: $7.47 \times 10^6 Nm^{-2}$ Hint. use PV = nRT

43. A uniform capillary tube of negligible expansion sealed at one end, contains air trapped by a pellet of mercury. The trapped air column is 13.7cm long at 0°C and 18.7cm long at 100°C. Calculate the cubical expansivity of the air at constant pressure.

WAEC 1993 Ans: $3.65 \times 10^{-3}K^{-1}$

44. The volume of a given mass of gas is $40cm^3$ at 27°C. What is its volume at 90°C if its pressure remain constant? *NECO 2003 Ans: $48.4cm^3$*

45.

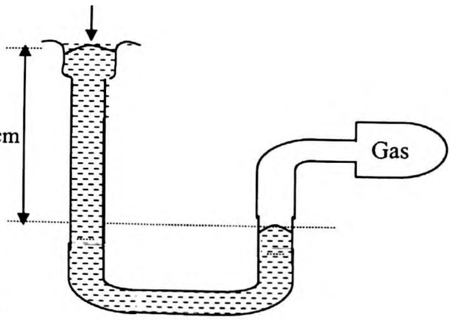


Fig 10.26

In the diagram above, what is the pressure of the gas?

A. 96cm of mercury B. 86cm of mercury C. 66cm of mercury D. 76cm of mercury

JAMB 2006 Ans: 86cmHg

46. The pressure of a fixed mass of an ideal gas at 27°C is 3Pa. The gas is heated at a constant volume until its pressure is 5Pa. Determine the new temperature of the gas.

WAEC 2007[27] (Ans: 227°C)

47.

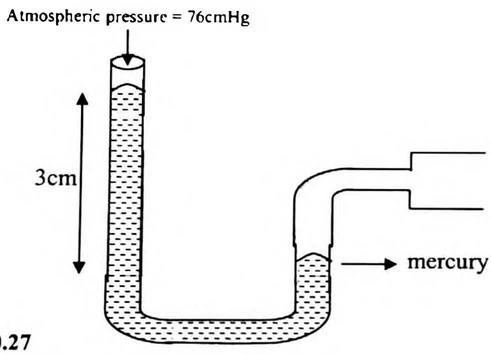


Fig 10.27

Using the above diagram, calculate the pressure of the gas supply.

NECO 2007[21] Ans: 79cmHg

48. A sealed flask contains $600cm^3$ of air at 27°C and is heated at 35°C at constant pressure. The new volume is

A. $508cm^3$ B. $516cm^3$ C. $608cm^3$ D. $616cm^3$ *JAMB 2008 Ans: $616cm^3$*

49. The pressure of two moles of an ideal gas at a temperature of 27°C and volume 10^{-2} m^3 is

A. $4.99 \times 10^5 Nm^{-2}$ B. $9.80 \times 10^3 Nm^{-2}$ C. $4.98 \times 10^3 Nm^{-2}$

D. $9.80 \times 10^5 Nm^{-2}$ [R = 8.313 $J\,mol^{-1}K^{-1}$] *JAMB 2009[20] Ans: A*

50. A thread of mercury of length 20cm is used to trap some air in a capillary tube with uniform cross-sectional area and closed at one end. With the tube vertical and the open end uppermost, the length of the trapped air column is 15cm. Calculate the length of the air column when the tube is held

(i) horizontally (ii) vertically with the open end underneath.

[Atmospheric pressure = 76cmHg] *WAEC 2009[E12] Ans: (i) 18.95cm (ii) 25.71cm*

51. The equation $P^x V^y T^z$ = Constant is Charles' if

A. $x = 0, \ y = 1 \ and \ z = -1$

B. $x = 0, \ y = -1 \ and \ z = 1$

C. $x = 1, \ y = 1 \ and \ z = 0$

D. $x = 1 \ y = 1 \ and \ z = -1$

E. $x = 1, \ y = 1 \ and \ z = 1$ *NECO 2009[17] Ans: A*

52. The pressure of a certain mass of gas at constant volume increases from 250mmHg at 20°C to 400mmHg at 50°C. Calculate the pressure expansivity of the gas.

NECO 2009[E12] Ans: $0.02°C^{-1}$

11

PRESSURE IN FLUIDS

Pressure is defined as the force acting perpendicularly per unit area.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \qquad \text{That is,} \qquad P = \frac{F}{A}$$

Also,

$$\text{Pressure} = \frac{\text{Weight}}{\text{Area}} \qquad \text{That is,} \qquad P = \frac{W}{A}$$

Note: $F = W = mg$. Where m is mass (kg) and g acceleration due to gravity (m/s^2).
Pressure is measured in Nm^{-2}. $10^5Nm^{-2} = 10^5Pa = 1bar$

Example 1
A rectangular water tank of weight 4.5×10^3N measures 2.0m by 1.5m by 1.2m. Calculate the minimum pressure it can exert when resting on a horizontal surface.
NECO 2006.

Solution
Minimum pressure is obtained when the tank rest on the greatest area.
From, *2.0m by 1.5m by 1.2m*, the greatest area, $A = 2 \times 1.5 = 3.0m^2$
Weight, $W = F = 4.5 \times 10^3N$

$$\therefore \quad \text{Pressure} = \frac{F}{A} = \frac{4.5 \times 10^3}{3.0} = 1.5 \times 10^3 Nm^{-2}$$

Example 2
A rectangular block of dimensions $2.0m \times 1.0m \times 0.5m$ weighs 200N. Calculate the maximum pressure exerted by the block on a horizontal floor. *WAEC 2008*

Solution
Maximum pressure is obtained when the block rest on the least area.
From, "$2.0m \times 1.0m \times 0.5m$" the least area, $A = 1.0m \times 0.5m = 0.5m^2$
Weight of block $W = F = 200N$

$$\therefore \text{Pressure exerted} \quad P = \frac{F}{A} = \frac{200N}{0.5} = 400Nm^{-2}$$

Example 3
A rectangular tank contains water to a depth of 2m. If the base is 4m x 3m calculate the force on the base. (Density of water $= 10^3kgm^{-3}$, $g=10ms^{-2}$) A. $2.4 \times 10^4 N$ B. $2.4 \times 10^4 N$ C. $2.0 \times 10^4 N$ D. $1.7 \times 10^3 N$
JAMB 1986.

Solution
The force (F) on the base of the tank will depend on the weight (W) of the water.

Volume of tank $= 4 \times 3 \times 2 = 24m^3$

Density of water $= 1000kgm^{-3}$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \qquad \therefore \text{Mass of water}, m = \text{Density} \times \text{Volume}$$

$$m = 1000 \times 24 = 24000kg$$

Weight of water, $W = F = mg = 24000 \times 10 = 2.4 \times 10^5N$.

FLUID PRESSURE

Pressure in fluids (liquids or gases) depends on the depth (height) and density of the fluid.

$$P = \frac{F}{A}$$

$$P = \frac{mg}{A}$$

$$P = \frac{\rho \times V \times g}{A}$$

$$P = \frac{\rho \times A \times h \times g}{A}$$

$$P = \rho hg$$

Where P = pressure (Nm^{-2}), F = force (N)
m = mass (kg), g = acceleration due to gravity(m/s^2)
A = area (m^2) ρ = density (kgm^{-3})
h = height or depth(m)

Example 3
A reservoir is filled with a liquid of density $2000kgm^{-3}$. Calculate the depth at which the pressure in the liquid will be equal to $9100Nm^{-2}$ ($g=10m/s^2$) *WAEC 2002*
Solution
Density of liquid, $\rho = 2000kgm^{-3}$; Pressure, $P = 9100Nm^{-2}$; $g = 10m/s^2$

$$P = \rho hg, \quad \therefore \quad \text{depth}, h = \frac{P}{\rho g} = \frac{9100}{2000 \times 10} = 0.455m$$

Example 4
A 5m x 4m x 3m vessel of negligible weight is filled with a liquid of density $2500kgm^{-3}$. If the vessel is placed on a flat surface, what is the maximum pressure it can exert?
Solution
The maximum pressure is exerted when the vessel rests on its least area.
From 5m x 4m x 3m, the least area, $A = 4 \times 3 = 12m^2$.
Volume of vessel, $V = l \times b \times h = 5 \times 4 \times 3 = 60m^3$
Density of liquid $\rho = 2500kgm^{-3}$

From $\rho = {}^m/_V$, mass, $m = \rho \times V = 2500 \times 60 = 150000kg$
$W = F = mg = 1.5 \times 10^5 kg \times 10 = 1.5 \times 10^6 N$

$$P = \frac{E}{A} = \frac{1.5 \times 10^6 N}{12} = 12500 = 1.2 \times 10^5 N/m^2$$

Alternatively, we could apply the pressure formular, $P = \rho hg$ because pressure in a liquid depends on depth or height. If the vessel rests on its least area 4 x 3, then the height(h) becomes 5m.

$$P = \rho hg = 2500 \times 5 \times 10 = 1.25 \times 10^5 N/m^2$$

ATMOSPHERIC PRESSURE

At sea level the atmospheric pressure is 760mmHg or $1.013 \times 10^5 N/m^2$. Pressure reduces as you go higher into the atmosphere (above sea level). Pressure increases as you go lower (below sea level). The pressure formular can be applied in both cases.

Example 5

Calculate the length of the liquid in a barometer tube that would support an atmospheric pressure of 3.06 x $10^5 Nm^{-2}$ if the density of the liquid is 1.36 x $10^4 kgm^{-3}$ ($g=10m/s^2$).

Solution

Density of liquid, $\rho = 1.36 \times 10^4 kgm^{-3}$

Pressure, $P = 3.06 \times 10^5 Nm^{-2}$; ($g=10m/s^2$).

From pressure formula, $P = \rho hg$, Lenght of liquid, $h = \frac{P}{\rho g} = \frac{3.06\times10^5}{1.36\times10^4\times10} = 2.25m$

Example 6

A pilot records the atmospheric pressure outside his plane as 63cm of Hg while a ground observer records a reading of 75cm of Hg with his barometer. Assuming that the density of air is constant, calculate the height of the plane above the ground. (Take the relative densities of air and mercury as 0.00136 and 13.6 respectively). *WAEC 1998*

Solution

Pressure change for air = pressure change for mercury

$$\rho \times H \times g = \rho \times h \times g$$

or $$\rho \times H = \rho \times h$$

ie. $$\rho_{air} \times H = \rho_{Hg} \times h$$

Where ρ_{air} and ρ_{Hg} are densities or relative densities of air and mercury respectively.

$$\therefore\ 0.00136 \times H = 13.6 \times (75 - 63) = 13.6 \times 12$$

$$\text{Height above ground, } H = \frac{13.6\times12}{0.00136} = 12000cm = 1200m$$

Example 7

The atmospheric pressure due to water is 1.3 x $10^6 Nm^{-2}$. What is the total pressure at the bottom of an ocean 10m deep? (Density of water = $1000kgm^{-3}$, $g=10m/s^2$)

A. $1.3\times10^7 Nm^{-2}$ B. $1.4\times10^6 Nm^{-2}$ C. $1.4\times10^5 Nm^{-2}$ D. $1.0\times10^5 Nm^{-2}$ *JAMB1998*

Solution

Atmospheric pressure, $P = 1.3 \times 10^6 Nm^{-2}$.

Depth of ocean, $h = 10m$

Density of water, $\rho = 1000kgm^{-3}$; $g = 10m/s^2$

Total pressure $= P + \rho hg$

$= 1.3 \times 10^6 + 1000 \times 10 \times 10$

$= 1.4 \times 10^6 Nm^{-2}$

PASCAL PRINCIPLE

Pascal principle states that when a fluid completely fills a vessel and a pressure is applied to it at any part of the surface, that pressure is transmitted equally throughout the whole of the enclosed fluid and the walls of the containing vessel.

Pascal principle is applied in a hydraulic press as follows.

$$\text{Pressure, } P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \frac{A_2F_1}{A_1}$$

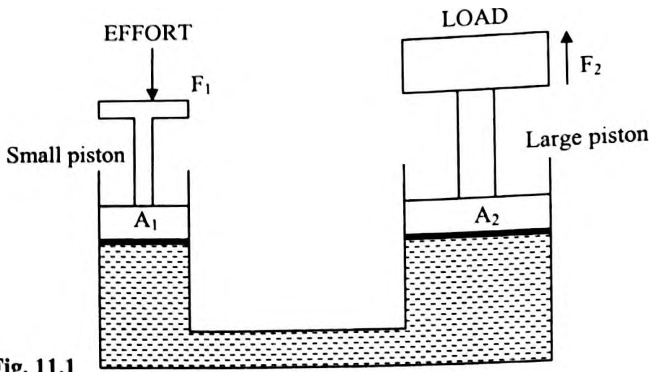


Fig. 11.1 Hydraulic Press

Where F_1 = effort or force applied on smaller piston
A_1 = cross sectional area of small piston
F_2 = load or force applied on larger piston
A_2 = cross-sectional area of larger piston

The velocity ratio (V.R.) of an hydraulic press is

$$X_1A_1 = X_2A_2 \quad \text{or} \quad \frac{X_1}{X_2} = \frac{A_2}{A_1}$$

Where X_1 = distance moved by the small piston
X_2 = distance moved by the larger piston

Example 8
A hydraulic press has a large circular piston of radius 0.8m and a circular plunger of radius 0.2m. A force of 500N is exerted by the plunger. Find the force exerted on the piston. A. 8000N B. 4000N C. 2000N D. 31N *JAMB 1995*

Solution

$r_1 = 0.2$m; $r_2 = 0.8$m; $F_1 = 500$N

Area of large piston, $A_2 = \pi r_2^2 = \pi \times 0.8^2 = 0.64\pi$
Area of small piston, $A_1 = \pi r_1^2 = \pi \times 0.2^2 = 0.4\pi$
Force exerted on small piston, $F_1 = 500$N

Substitute into $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ to obtain

$$\frac{500}{0.04\pi} = \frac{F_2}{0.64\pi}$$

Force on larger piston, $F_2 = \frac{500 \times 0.64\pi}{0.04\pi} = 8000$N

HARE'S APPARATUS

The densities of two liquids can be compared by means of Hare's apparatus as shown. Applying the pressure formular on the figure below, the following equations are obtained.

$$h_1\rho_1 g = h_2\rho_2 g$$

$$h_1\rho_1 = h_2\rho_2$$

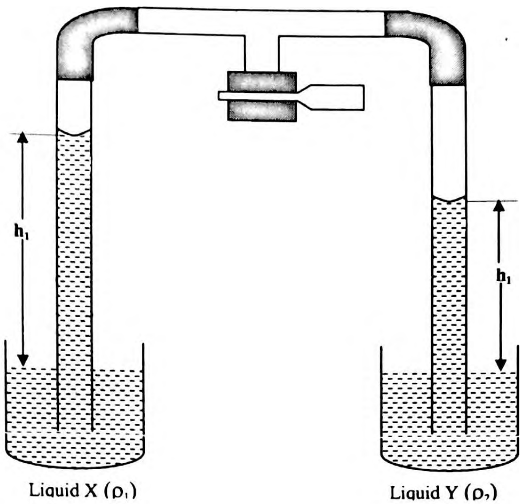


Fig. 11.2

or $$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$$

$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$ = relative density of ρ_1 (liquid X) if ρ_2 (liquid Y) is the density of water.

Example 9

In an experiment using Hare's apparatus, the height of water column was 10.5cm and that of glycerine is 8.1cm. (a) Calculate the density of the glycerine if the density of water is 1000kgm^{-3}. (b) If the density of was not given in (a) above, show how you would calculate the relative density of glycerine.

Solution

(a) Height of glycerine column, h_1 = 8.1cm
Density of glycerine column, ρ_1 = ?
Height of water column, h_2 = 10.5cm
Density of water column, ρ_2 = 1000kgm^{-3}

Substitute into $\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$

$$\frac{\rho_1}{1000} = \frac{10.5}{8.1}$$

$$\therefore \quad \rho_1 = \frac{1000 \times 10.5}{81} = 1296.3 \approx 1300\text{kgm}^{-3}$$

(b) Height of water column, h_2 = 10.5
Height of glycerine column, h_1 = 8.1

$\frac{h_2}{h_1}$ = relative density of glycerine, if h_2 is the height of water column.

Relative density, $\frac{h_2}{h_1} = \frac{10.5}{8.1} \approx 1.3$

EXERCISE 11

1. The stylus of a phonograph record exert a force of 7.7×10^{-2}N on a groove of radius 10^{-5}m, compute the pressure exerted by the stylus on the groove.
A. $2.45\times10^{8} Nm^{-2}$ B. $3.45\times10^{8} Nm^{-2}$ C. $4.90\times10^{8} Nm$ D. $2.42 \times 10^{8}Nm^{-2}$
JAMB 2003 Ans: $2.45 \times 10^{8}Nm^{-2}$

2. A man exerts a pressure of 2.8×10^{3} Nm^{-2} on the ground and has 4×10^{-2}m^{2} of his feet in contact with the ground. What is the weight of the man?
A. 112N B. 140N C. 102 D. 70N *JAMB 2004 Ans: 112N*

3. The horizontal door of a submarine at a depth of 500m has an area of 0.4^{2}. Calculate the force exerted by the sea water on the door at this depth. [Relative density of sea water = 1.03] [Atmospheric pressure = 1.0×10^{5}Nm^{-2}] [Density of pure water = 1000kgm^{-3}] [g=10m/s^{2}]. *WAEC 2002 Ans: $2.06 \times 10^{6}N$*

4. A diver is 5.2m below the surface of water density 1000kgm^{-3}. If the atmospheric pressure is 1.02×10^{5}Pa, calculate the pressure on the diver (g=10m/s^{2}).
WAEC 2004 Ans: $1.54 \times 10^{5}Pa$

5. Two divers G and H are at depths 20m and 40m respectively below the water surface in a lake. The pressure on G is P_1 while the pressure on H is P_2. If the atmospheric pressure is equivalent to 10m of water, what is the value of P_2/P_1?
WAEC 1985 Ans: $^{5}/_{3}$

6. A liquid of mass 1.0×10^{3}kg fills a rectangular tank of length 2.5m and width 2.0m. If the tank is 4m high what is the pressure at the middle of the tank? [g=10m/s^{2}]
A. $1.0\times10^{4} Nm^{-2}$ B. $2.0\times10^{3} Nm^{-2}$ C. $1.5\times10^{3} Nm^{-2}$ D. $1.0\times10^{3} Nm^{-2}$
JAMB 1997 Ans: $1 \times 10^{3}Nm^{-2}$

7. A weightless vessel of dimensions 4m x 3m x 2m is filled with a liquid of density 1000kgm^{-3} and sealed. What is the maximum pressure this container can exert on a flat surface? [g=10m/s^{2}] A. $9\times10^{4} Nm^{-2}$ B. $4\times10^{4} Nm^{-2}$ C. $3\times10^{4} Nm^{-2}$ D. $2\times10^{4} Nm^{-2}$
JAMB 1993 Ans: $4 \times 10^{4}Nm^{-2}$

8. Normal atmospheric pressure at seal level is 10^{5}Nm^{-2} and the acceleration due to gravity is approximately 10m/s. If the atmosphere has a uniform density of 1kgm^{-3}, what is its height? Hint: use $P = \rho hg$ A.100m B.1000m C.10,000m D.100,000m E.1000,000m *JAMB 1984 Ans:* 10,000m or $10^{4}m$

9. A faulty barometer reads 72.6cmHg when the atmospheric pressure is 75.0cmHg. Calculate the atmospheric pressure when this barometer reads 72.0cmHg.
WAEC 2004 Ans: 74.38cmHg

10. Calculate the length of the liquid in a barometer tube that would support an atmospheric pressure of 204000Nm^{-2}, if the density of the liquid is 5200kgm^{-3} [g=10m/s^{2}] *NECO 2004 Ans: 3.92m*

11. What is the length of the liquid in a barometer tube that would support an atmospheric pressure of 102000Nm^{-2} if the density of the liquid is 2600kgm^{-3} [g=10m/s^{2}]
A. 0.75m B. 0.76m C. 3.92m D. 39.23m *JAMB 1990 Ans: 3.92m*

12. A pilot records the atmospheric pressure outside his plane as 63cm of Hg while a ground observer records a reading of 75cm of Hg for the atmospheric pressure on the ground. Assuming that the density of the atmosphere is constant, calculate the height of the plane above the ground. (Relative density of Hg = 13.6 and that of air = 0.00013)
A.1200m B. 6300m C.7500m D.13,800m *JAMB 1991 Ans:120,000cm = 1200m*

13. The hydrostatic blood pressure difference between the head and feet of a boy standing straight is 1.65×10^{4}Nm^{-2}. Find the height of the boy. [Density of blood = 1.1×10^{3}kgm^{-3}, g=10m/s^{2}] Hint: $P = \rho hg$ A. 0.6m B. 0.5m C. 2.0m D.1.5m
JAMB 2002 Ans: 1.5m

14. The areas of the effort and load pistons of a hydraulic press are 0.5m^{2} and 5m^{2} respectively. If a force F_1 of 100N is applied on the effort piston, what is the force F_2 on the load. A.10N B.100N C.500N D.1000N E.5000N *JAMB 1985 Ans:1000N*

15. In a hydraulic press, a force of 40N is applied on the effort piston of area $0.4m^2$. If the force exerted on the load is 400N, what is the area of the large piston?
A. $8m^2$ B. $4m^2$ C. $2m^2$ D. $1m^2$ *JAMB 2004 Ans: $4m^2$*

16.

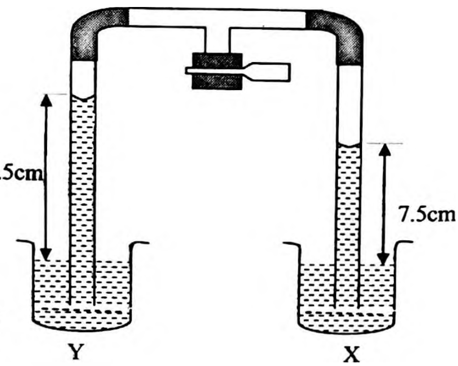


Fig. 11.3

The figure above shows the heights of two liquids X and Y when some air is sucked out of the apparatus through the pump P. The diameter of the tube in X is twice that of the tube in Y. What is the relative density of liquid X with respect to liquid Y.
A. 1/3 B. 2/3 C. 3 D. 6 *JAMB 1992 Ans: ⅓*

17. A reservoir 500m deep is filled with a fluid of density $850kgm^{-3}$. If the atmospheric pressure is $1.05 \times 10^5Nm^{-2}$, the pressure at the bottom of the reservoir is
A. $4.28\times10^6Nm^{-2}$ B. $4.72 \times 0^6Nm^{-2}$ C. $4.25\times10^6Nm^{-2}$ D. $4.36\times10^6Nm^{-2}$
$[g=10ms^{-2}]$ *JAMB 2007 Ans: $4.36\times10^6Nm^{-2}$*

18. In the Hare's apparatus, water rises to a height of 26.5cm in one limb. If a liquid rises to a height of 20.4cm in the other limb, what is the relative density of the liquid?
A. 0.8 B. 1.1 C. 1.2 D. 1.3 *JAMB 2008 Ans: 1.3*

19.

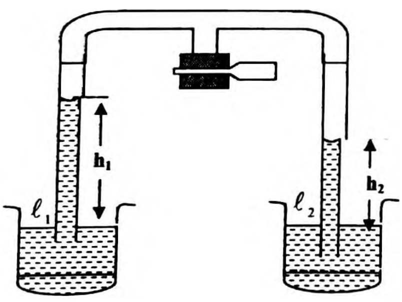


Fig. 11.4

The diagram above illustrates a Hare's apparatus. $\ell_1 \ell_2$ represent densities and h_1, h_2 heights of column of liquids. Which of the following equations is correct?

A. $h_1 = \frac{h_2\ell_1}{\ell_2}$ B. $h_1 = \frac{h_2\ell_2}{\ell_1}$ C. $h_1 = \frac{\ell_1\ell_2}{h_2}$ D. $h_1 = \frac{\ell_1}{h_2\ell_2}$

WAEC 2008 Ans: B

19. In a hydraulic press, a force of 40N is applied to the smaller piston of area $10cm^2$. If the area of the larger piston is $200cm^2$, calculate the force obtained.
WAEC 2009[3] Ans: 800N

12

WAVES

GENARAL CHARACTERISTICS OF WAVES

A wave is a disturbance which travels through a medium and transfers energy from one point to another, without any permanent displacement of the medium.
A typical wave is defined by the following terms:

a. **Amplitude(A)**. This is the maximum displacement of the particles of a wave from its mean or equilibrium position. It's measured in meter.

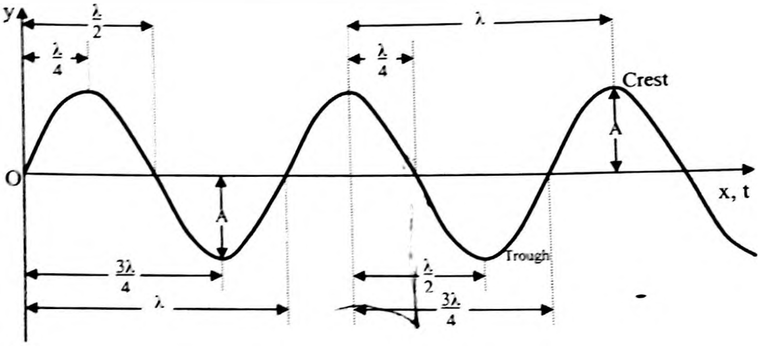


Fig. 12.1

b. **Wavelength (λ)**. This is the distance between two successive troughs or two successive crests. It is also the distance covered by a wave when it completes one cycle, vibration or oscillation. It is measured in meter. A wavelength can be subdivided into half ($\lambda/2$), three-quarters ($3\lambda/4$) and one-quarter ($\lambda/4$).

c. **Period (T)**. This is the time taken by the particle of a wave to perform one complete cycle or oscillation. It is also the time required by a wave to travel one wavelength. It is measured in seconds and can be stated thus:

$$T = \frac{\text{time taken}}{\text{number of cycles}} = \frac{\text{time taken}}{\text{number of wavelengths}}$$

d. **Frequency (f)**. It is the number of cycles or oscillations which the wave completes in one second. Frequency is measured in Hertz(Hz) or s^{-1}.

$$f = \frac{\text{number of cycles}}{\text{time taken}} = \frac{\text{number of wavelengths}}{\text{time taken}}$$

Frequency and period are related as follows:

$$T = \frac{1}{f} \qquad \text{or} \qquad f = \frac{1}{T}$$

e. Wave Speed or Velocity (V). $\frac{\text{wavelenght}}{\text{period}} = \frac{\lambda}{T}$

or $\quad V = \text{wavelength} \times \text{frequency} = \lambda f$

i.e $\quad V = \dfrac{\lambda}{T} \quad$ or $\quad V = \lambda f$

Example 1

The diagram below represents part of a wave motion in air. If the waves travels with a speed of $300ms^{-1}$, calculate the frequency of the wave. *WAEC 1990*[26]

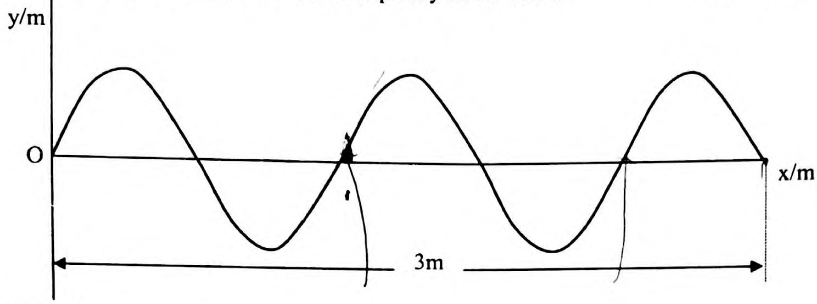


Fig. 12.2

Solution

Wave velocity, $V = 300ms^{-1}$; frequency, $f = ?$

For a particular wave, the number (n) of wavelength and their displacement (d) are related by

$$\frac{n_1}{d_1} = \frac{n_2}{d_2}$$

where n_1, d_1 = one wavelength and its corresponding distance

n_2, d_2 = any number of wavelength and its corresponding distance.

The diagram shows 2½ or 2.5 wavelengths with a corresponding length of 3m. Therefore, $n_2 = 2.5\lambda$ and $d_2 = 3m$. For one wavelength, $n_1 = 1\lambda$ and $d_1 = ?$

Substitute into $\dfrac{n_1}{d_1} = \dfrac{n_2}{d_2}$ to obtain

$$\frac{1\lambda}{d_1} = \frac{2.5\lambda}{3m}$$

cross multiplying, $d_1 \times 2.5\lambda = 3m \times 1\lambda$

$$d_1 = \frac{3m \times 1\lambda}{2.5\lambda} = \frac{3m}{2.5} = 1.2m$$

From $V = \lambda f$, frequency, $f = \dfrac{V}{\lambda} = \dfrac{300}{1.2} = 250Hz$

Example 2

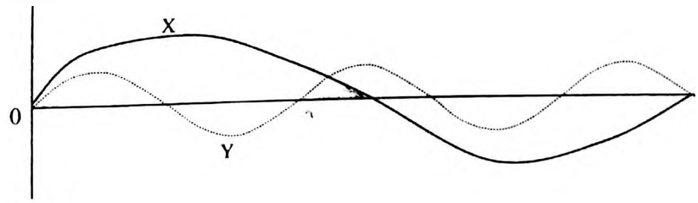


Fig. 12.3

The diagram above shows two waveforms X and Y. If the frequency of X is 30Hz, what is the frequency of Y? A.12Hz B. 19Hz C. 60Hz D.75Hz *JAMB 1988*[22]

Solution

$$\text{Wave frequency, } f = \frac{\text{number of wavelength}}{\text{time taken}}$$

X has one wavelength and frequency of 30Hz. Therefore time taken by X is

$$= \frac{\text{number of wavelenght}}{\text{frequency}} = \frac{1}{30}\text{s}$$

From the diagram, wave Y with 2.5 wavelengths also takes $^1/_{30}$ sec, same as X.

$$\text{Therefore, frequency of Y} = \frac{\text{number of wavelenghts}}{\text{time taken}} = \frac{2.5}{1/30}$$

$$= 2.5 \times 30 = 75\text{Hz}$$

Alternatively, we could derive an equation relating the frequency (f_x) and number of wavelengths (n_x) of wave X with the frequency (f_y) and number of wavelength (n_y) of wave Y, as follows:

$$\frac{n_x}{f_x} = \frac{n_y}{f_y}$$

substituting,
$$\frac{1}{30\text{Hz}} = \frac{2.5}{f_y}$$

frequency of Y, $f_y = 30\text{Hz} \times 2.5 = 75\text{Hz}$

Example 3

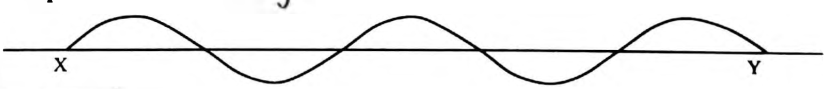


Fig. 12.4

The diagram above shows a waveform in which energy is transferred from X to Y in a time of 2.5×10^{-3}s. What is the frequency of the wave? *NECO 2002*[34]

Solution

Number of wavelengths = 2½ or 2.5 Time taken $= 2.5\times 10^{-3}$s

$$\text{Frequency} = \frac{\text{number of wavelenghts}}{\text{time taken}} = \frac{2.5}{2.5\times10^{-3}\text{s}} = 1.0\times10^{3}\text{Hz}$$

Example 4

A wave travels a distance of 20cm in 3s. The distance between successive crests of the wave is 4cm. What is the frequency of the wave? *NECO 2000*[14]

Solution

Wavelength, λ = 4cm; wave distance = 20cm; time taken = 3s.

$$\text{wavespeed} = \frac{\text{wavedistance}}{\text{timetaken}} = \frac{20\text{cm}}{3\text{s}} = 6.67\text{cm}^{-1}$$

$$\text{From } V = f\lambda,\ \text{frequency, } f = \frac{V}{\lambda} = \frac{6.67\text{cm}^{-1}}{4\text{cm}} = 1.67\text{Hz}$$

Example 5

The wavelength of ultraviolet radiation is 400nm. If the speed of light in air is $3 \times 10^8 ms^{-1}$, then the frequency of the ultraviolet radiation is A. 1.3×10^{15}Hz B. 7.5×10^5Hz C. 1.2×10^{11}Hz D. 7.5×10^{14}Hz *JAMB 1986*[49]

Solution

Wavelength, $\lambda = 400nm = 400 \times 10^{-9}m$; speed, $V = 3 \times 10^8 ms^{-1}$

From $V = f\lambda$, frequency, $f = \frac{V}{\lambda} = \frac{3 \times 10^8}{400 \times 10^{-9}} = 7.5 \times 10^{14}$ Hz

Example 6

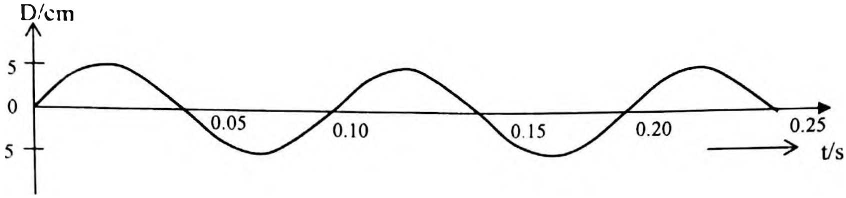


Fig. 12.5

The diagram above represents the displacement D versus time, t, graph of a progressive wave. Deduce the frequency of the wave. *WAEC 2003*[18]

Solution

From the graph, number of wavelength = 2.5λ; time taken by the 2.5 wavelengths = 0.25s.

$$\text{Frequency} = \frac{\text{number of wavelenghts}}{\text{time taken}} = \frac{2.5}{0.25} = 10\text{Hz}$$

Example 7

A radio station broadcast at a frequency of 400KHz. If the speed of the wave is $3 \times 10^8 ms^{-1}$, calculate the wavelength of the radio wave. *NECO 2005*[25]

Solution

Frequency, f = 400KHz = 400000Hz; Speed, $V = 3 \times 10^8 ms^{-1}$

From $V = \lambda f$, wavelength $\lambda = \frac{V}{f} = \frac{3 \times 10^8}{400000} = 7.5 \times 10^2 m$

Example 8

The diagram below illustrates a variation of the displacement y of a wave particle with time t. If the velocity of wave is $250ms^{-1}$, calculate the distance between two successive particles which are in phase. *WAEC 1996*[40]

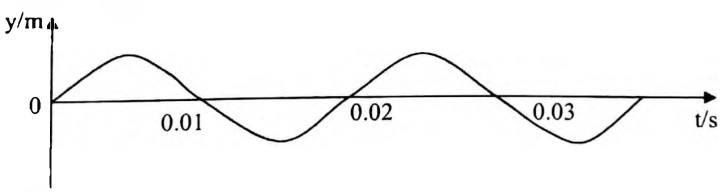


Fig. 12.6

Solution

The distance between two successive particles in phase is equal to the wavelength λ.
Wave speed, V = 250m/s.

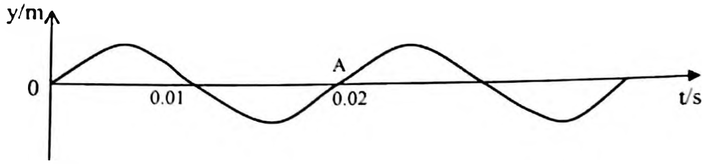


Fig. 12.7

The distance OA is equal to one wavelength. The time taken to travel OA is 0.02s.

$$\text{Frequency} = \frac{\text{number of wavelenghts}}{\text{time taken}} = \frac{1}{0.02} = 50\text{Hz}$$

$$\text{From } V = \lambda f, \quad \text{wavelength } \lambda = \frac{V}{f} = \frac{250}{50} = 5.0\text{m}$$

Example 9

A periodic pulse travels a distance of 20.0m in 1.00s. If its frequency is 2.0×10^3Hz, calculate the wavelength. *WAEC 2000*[26]

Solution

$$\text{Wave speed } V = \frac{\text{distance}}{\text{time}} = \frac{20.0\text{m}}{1.00\text{s}} = 20\text{ms}^{-1}$$

$$\text{From } V = f\lambda, \quad \text{wavelength } \lambda = \frac{V}{f} = \frac{20}{2.0 \times 10^3} = 0.01 = 1.0 \times 10^{-2}\text{ m}$$

Example 10

A wave has a frequency of 2Hz and a wave length of 30cm. The velocity of the wave is.

A. 60.0ms^{-1} B. 6.0ms^{-1} C. 1.5ms^{-1} D. 0.6ms^{-1} *JAMB 1987*[27]

Solution

Frequency, f = 2Hz; wavelength, λ = 30cm = 0.30m

Velocity, $v = f\lambda = 0.30 \times 2 = 0.60\text{ms}^{-1}$

Example 11

A boat at anchor is rocked by waves whose crest are 100m apart and whose velocity is 25ms^{-1}. At what interval does the wave crest reach the boat?

A. 2500.00s B. 75.00s C. 4.00s D. 0.25s *JAMB 1987*[28]

Solution

Wavelength, λ = 100m; velocity, v = 25ms^{-1}. Interval at which boat reaches crest is period, T = ?

$$\text{From } V = \frac{\lambda}{T}, \quad \text{period, } T = \frac{\lambda}{v} = \frac{100}{25} = 4s$$

Example 12

The figure below represents a displacement – time graph of a travelling wave moving with a speed of 2ms^{-1}.

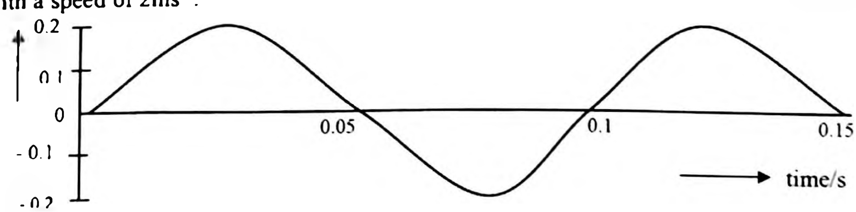


Fig. 12.8

Determine, from the graph, the (i) amplitude (ii) period (iii) wavelength of the wave.

Solution

Wave speed, $V = 2ms^{-1}$

(i) Amplitude, A = 2.0m as shown in the diagram below.

(ii) Period, T = 0.1s i.e. the time for one wavelength. M – N.

(iii) From $V = \frac{\lambda}{T}$, wavelength, $\lambda = VT = 2 \times 0.1 = 0.2ms^{-1}$

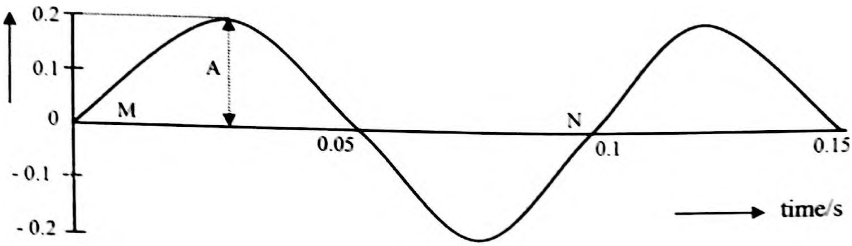


Fig. 12.9

Example 13

Infrared rays of frequency 1.0×10^{13} Hz have a wave length of 3.0×10^{-5} m in vacuum. The wavelength of X-rays of frequency 5.0×10^{16} Hz in vacuum will be

A. 6.0×10^{-23} m B. 6.0×10^{-10} m C. 6.0×10^{-7} mD. 5.0×10^{16} m

E. 3.0×10^{7} m *NECO 2008[31]*

Solution

For infrared rays: $f = 1.0 \times 10^{13}$ Hz; $\lambda = 3.0 \times 10^{-5}$ m

$$v = \lambda f = 3.0 \times 10^{-5} \times 1.0 \times 10^{13} = 3.0 \times 10^{8}\ ms^{-1}$$

For the X-ray: $f = 5.0 \times 10^{16}$ Hz; $v = 3 \times 10^{8}\ ms^{-1}$

$$v = \lambda f \quad \therefore \quad \lambda = \frac{v}{f} = \frac{3 \times 10^{8}}{5.0 \times 10^{16}} = 6.0 \times 10^{-9}\, m$$

None of the options given is correct.

STATIONARY WAVES

A stationary or standing wave is a wave that is formed when two progressive waves of equal frequency and amplitude travelling in opposite direction are combined. Stationary waves consist of nodes and antinodes as shown in fig 12.10.

A node is a point on a stationary wave where there is no movement of the medium, while an antinode is a point on a stationary wave where there is maximum displacement of the medium.

Below in Fig 12.10 is a diagram of a stationary wave.

For a stationary wave the following applies.

Distance between a node and the next antinode, N – A = $^{\lambda}/_{4}$

Distance between two successive nodes (N – N) or antinode (A – A) = $^{\lambda}/_{2}$

Distance between three successive nodes (N – N – N) or antinode (A – A – A) = λ

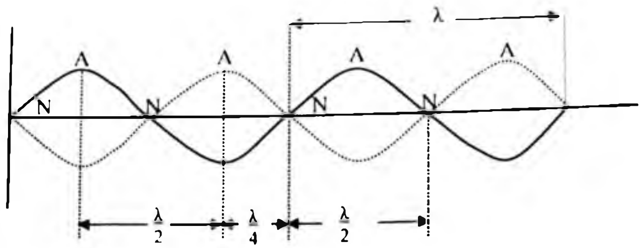


Fig. 12.10

Example 13
The wavelength of a stationary wave is 36.0cm. What is the distance between a node and the next anti-node? *NECO 2002*[36]

Solution
Wavelength $\lambda = 36.0$cm
Distance between a node and next antinode $= \frac{1}{4}\lambda$
$= \frac{1}{4} \times 36 = 9$cm.

Example 14
A wave of frequency 10Hz forms a stationary wave pattern in a medium where the velocity is $20cms^{-1}$. The distance between adjacent node is
A. 1.0cm B. 1.5cm C. 2.0cm D. 5.0cm *JAMB 1986*[34]

Solution
Frequency, f = 10Hz; wave velocity, $v = 20cms^{-1}$

From $v = \lambda f$, wavelength $\lambda = \frac{V}{f} = \frac{20}{10} = 2$m

Distance between adjacent node $= \frac{\lambda}{2} = \frac{2}{2} = 1$cm

Example 15
Radio waves emitted from an antenna are picked up by a radar after reflection from an aircraft in 4×10^{-3}s. How far is the aircraft from the antenna? ($v = 3 \times 10^{3} ms^{-1}$).
A. 6.0×10^{2}km B. 1.2×10^{3}km C. 3.0×10^{3}km D. 6.0×10^{5}km *JAMB 1993*[23]

Solution
Velocity, $v = 3 \times 10^{8} ms^{-1}$; time $= \frac{1}{2}(4 \times 10^{-3}) = 2 \times 10^{-3}$s

(The time is divided into two because the wave was reflected).

Distance = Velocity × time $= 3 \times 10^{8} \times 2 \times 10^{-3} = 6 \times 10^{5}$m or 6×10^{2}km.

MATHEMATICAL REPRESENTATION OF PROGRESSIVE WAVE MOTION

A progressive wave is any wave, transverse or longitudinal, that spreads out from a vibrating source and moves through a medium transferring energy as it travels. A graph of displacement against time for a progressive waveform is as shown in fig 12.11

Mathematically, a wave can be represented by the following equation.

$$Y = A\sin\frac{(2\pi x)}{\lambda}$$

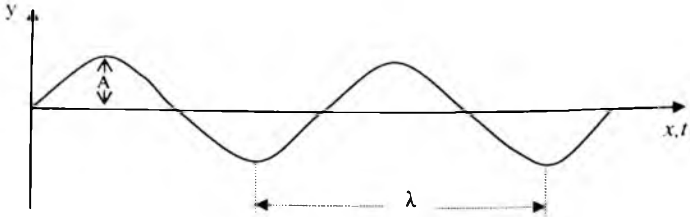


Fig. 12.11

Where Y = vertical displacement of oscillating wave particle.
λ = wavelength
A = amplitude
x = horizontal coordinate, from the origin of the oscillating wave particle.
$\frac{(2\pi x)}{\lambda}$ = phase difference of wave particle at distance x from the origin.

A wave can travel in either the positive x-direction or negative x-direction. After a particular time t, the displacement of the wave particle can be represented by the following equations: Positively travelling wave and negatively travelling wave, respectively.

1. $Y_{+ve} = A\sin\frac{2\pi}{\lambda}(x - vt)$ or $Y_{-ve} = A\sin\frac{2\pi}{\lambda}(vt - x)$

2. $Y_{+ve} = A\sin 2\pi\left(\frac{x}{\lambda} - ft\right)$ or $A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

3. $Y_{+ve} = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ or $Y_{-ve} = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

4. $Y_{+ve} = A\sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$ or $Y_{-ve} = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

5. $Y_{+ve} = A\sin\omega\left(\frac{x}{v} - t\right)$ or $Y_{-ve} = A\sin\omega\left(t - \frac{x}{v}\right)$

Where $\omega = 2\pi f$ = angular speed or velocity.
f = frequency of wave.
T = period of wave.
t = time.
v = wave velocity.

Students are advised to memorize just equations 1 and 2 as the others are basically the same equation expressed in different forms. Just remember that: $T = {}^{1}/_{f}$, $V = \lambda f$, $\omega = 2\pi f$

Example 16

A progressive wave has a wavelength of 50cm. Calculate the phase difference between two points at a distance of 20cm apart. *WAEC 1997*[35]

Solution

Phase difference of a wave between two points $= \frac{(2\pi x)}{\lambda}$

Wavelength, λ = 50cm; distance from origin, x = 20cm.

Substitute into $\frac{(2\pi x)}{\lambda} = 2\pi \times \frac{20\text{cm}}{50\text{cm}} = 2\pi \times \frac{2}{5} = \frac{4}{5}\pi$

Example 17
The distance between two points P and Q, along a wave is 0.05m. If the wavelength of the wave is 0.10m, determine the phase angle between P and Q in radians. *WAEC 2007*[19]
Solution
Wavelength, $\lambda = 0.1\text{m}$; distance between point P and Q, $x = 0.05\text{m}$.

$$\text{Phase angle} = \frac{(2\pi x)}{\lambda} = \frac{2\pi \times 0.05\text{m}}{0.1\text{m}} = \frac{0.1\pi}{0.1} = \pi$$

Example 18
A wave has an amplitude equal to 4.0m, angular speed $^1/_3\pi$ rads^{-1} and phase angle $^{2\pi}/_3$ rad. The displacement y of the wave is given as? *WAEC 2000*[27]
Solution
Amplitude $A = 4.0\text{m}$; angular speed $\omega = 1/3\pi$ rads^{-1}; Phase angle $\left(\frac{2\pi x}{\lambda}\right) = \frac{2\pi}{3}$ rad

The appropriate equation is $y = A\sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$ or $y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

Substitute given values into the two equations.

$$Y_{+ve} = A\sin\left(\frac{2\pi x}{\lambda} - \omega t\right) = 4\sin\left(\frac{2\pi}{3} = \frac{1\pi t}{3}\right) = 4\sin\frac{\pi}{3}(2-t)$$

$$Y_{-ve} = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right) = 4\sin\left(\frac{1\pi t}{3} = \frac{2\pi}{3}\right) = 4\sin\frac{\pi}{3}(t+2)$$

Both answers are correct depending on the options given. If it is an essay question and the direction of the wave is not specified, students are advised to state both answers in the same way as solution to a quadratic equation, i.e.

$$4\sin\frac{\pi}{3}(2-t) \quad \text{or} \quad \frac{4\pi}{3}(t+2)$$

Example 19
A travelling wave moving from left to right has an amplitude of 0.15m, a frequency of 550Hz and a wavelength of 0.01m. The equation describing the wave is
A. $y = 0.15\sin 200\pi(x - 5.5t)$
B. $y = 0.15\sin \pi(0.01x - 5.5t)$
C. $y = 0.15\sin 5.5\pi(x - 200t)$
D. $y = 0.15\sin \pi(550x - 0.01t)$
Solution
Frequency, $f = 550\text{Hz}$; amplitude, $A = 0.15\text{m}$; wavelength, $\lambda = 0.01\text{m}$

$\therefore$ Wave speed, $v = \lambda f = 0.01 \times 550 = 5.5\text{ms}^{-1}$
The direction of the wave is stated, "*moving from left to right*". Because it is a positively travelling wave, we substitute into $y = A\sin\frac{2\pi}{\lambda}(x - vt)$

$$Y = 0.15\sin\frac{2\pi}{0.01}(x - 5.5t)$$

$$Y = 0.15\sin 200\pi(x - 5.5t)$$

Example 20
The equation of transverse wave travelling along a string is given by $y = 0.3\sin(0.5x - 50t)$ where y and x are in cm and t is in seconds, find the maximum displacement of the particles from the equilibrium position.
A. 50.0cm B. 2.5cm C. 0.5cm D. 0.3cm *JAMB 1994*[28]

Solution

The maximum displacement of the particles from equilibrium position is the amplitude (A) of the wave. Comparing the given equation $y = 0.3\sin(0.5x - 50t)$ with one of the general equation $y = A\sin\left(\frac{2\pi x}{\lambda} - \omega t\right)$, It is obvious that the amplitude is 0.3cm.

Example 21

The equation of a wave travelling along the positive x-direction is given by $y=0.25 \times 10^{-3}\sin(500t - 0.025x)$. Determine the angular frequency of the wave motion.

A. $0.25\times10^{-3}rads^{-1}$ B. $0.25\times10^{-1}rads^{-1}$ C. $5.00\times10^{2}rads^{-1}$ D. $2.50\times10^{2}rads^{-1}$

JAMB 1999[23]

Solution

The given equation $y = 0.25\times10^{-3}\sin(500t - 0.025x)$

can be compared with $y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

Therefore, $\omega t = 500t$ (Remember, $\omega = 2\pi f$)

$2\pi ft = 500t$

$$f = \frac{500\,t}{2\pi\, t} = \frac{500}{2\pi} = \frac{250}{\pi} = 2.50\times10^{2}\,rad^{-1}$$

Example 22

A wave is represented by the equation $y = 2\sin(0.5x - 200t)$, where all distances are measured in centimeters and time in seconds. For this wave, calculate its

(i) frequency (ii) wavelength (iii) speed. *NECO 2000*[c13]

Solution

$Y = 2\sin \pi(0.5x - 200t)$

The above equation can be compared with any of the general equation (1, 2 or 3 in page 168) as long as they are arrange in such a way that it can be compared with the given equation. The three instances are shown below.

I. The first equation $y = A\sin \frac{2\pi}{\lambda}(x - vt)$ can be rearranged by taking $^2/_\lambda$ into the bracket to obtain $y = A\sin\left(\frac{2x}{\lambda} - \frac{2vt}{\lambda}\right)$-----A

comparable to $y = 2 \sin \pi\,(0.5x - 200t)$-----B

from equation **B**, $0.5x = \frac{2x}{\lambda}$ in equation **A**

cross multiplying $0.5x \times \lambda = 2x$

$$\text{wavelength } \lambda = \frac{2x}{0.5x} = \frac{2}{0.5} = 4m$$

similarly, from equation **B**, $200t = \frac{2vt}{\lambda}$ in equation **A**

cross multiplying and substituting $\lambda = 4m$ we obtain

$2vt = 200t \times 4$

$$\text{speed, } v = \frac{200t\times4}{2t} = \frac{200\times4}{2} = \frac{800}{2} = 400ms^{-1}$$

From $v = \lambda f$, frequency, $f = \frac{v}{\lambda} = \frac{400}{4} = 100Hz$

Therefore, (i) 100Hz (ii) 4m (iii) $400ms^{-1}$

II. The second equation $y = A\sin 2\pi\left(\frac{x}{\lambda} - ft\right)$ can be rearranged by taking 2 into the bracket to obtain $y = A\sin\pi\left(\frac{2x}{\lambda} - 2ft\right)$ ——— C which can be compared to

$y = 2\sin\pi\,(0.5x - 200t)$ ———— D

From equation **C**, $\frac{2x}{\lambda} = 0.5x$ in equation **D**

Cross multiplying, $0.5x \times \lambda = 2x$

Wavelength, $\lambda = \frac{2x}{0.5x} = \frac{2}{0.5} = 4m$

Similarly, from equation **C**, $2ft = 200t$ in equation **D**.

cross multiplying, frequency, $f = \frac{200t}{2t} = \frac{200}{2} = 100Hz$

Speed, $v = \lambda f = 4 \times 100 = 400ms^{-1}$

Therefore, (i) 100Hz (ii) 4m (iii) $400ms^{-1}$

III. The third equation $y = A\sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ can be rearranged by taking 2 into the bracket

to obtain $y = A\sin\pi\left(\frac{2x}{\lambda} - \frac{2t}{T}\right)$, compared with

$y = 2\sin\pi(0.5x - 200t)$

Therefore: $\frac{2x}{\lambda} = 0.5x$; wavelenght$\lambda = 4m$

Also: $200t = \frac{2t}{T}$

Cross multiplying $200t \times T = 2t$

$$T = \frac{2t}{200t} = \frac{2}{200} = \frac{1}{100} = 0.01s$$

Frequency, $f = \frac{1}{T} = \frac{1}{0.01} = 100Hz$

Speed, $v = f\lambda = 100 \times 4 = 400ms^{-1}$

(i) Frequency = 100Hz (ii) Wavelength = 4m (iii) Speed = $400ms^{-1}$

EXERCISE 12

1.

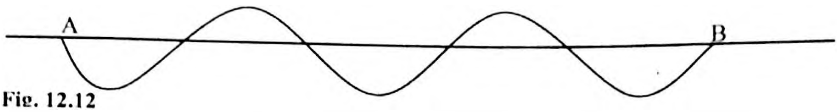


Fig. 12.12

The diagram above shows a waveform in which energy is transferred from A to B in a time of 2.5×10^{-3} s. Calculate the frequency of the wave. *WAEC 1992*[33] *Ans: 1.0×10^{3} Hz*

2.

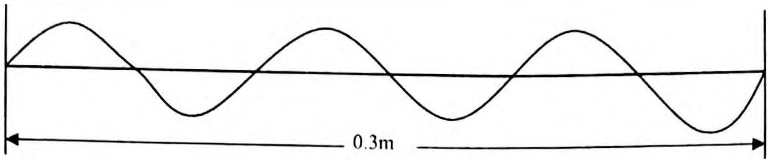


Fig. 12.13

The diagram above represents a transverse electromagnetic wave travelling with speed $3.0 \times 10^{8}\,ms^{-1}$. What is the frequency of the wave? A. 3.0×10^{7} Hz B. 9.0×10^{7} Hz C. 1.0×10^{9} Hz D. 3.0×10^{9} Hz *JAMB 1999*[25] *Ans: 3.0×10^{9} Hz*

3.

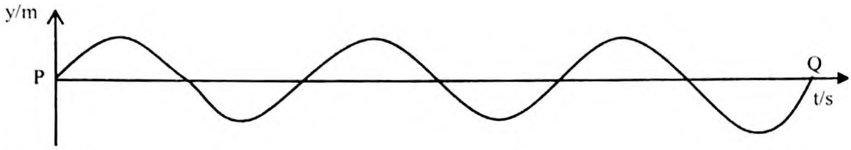


Fig. 12.14

The diagram above illustrates the profile of a progressive wave in which energy is transferred from P to Q in 3.0×10^{-3} s. Calculate the frequency of the wave.

WAEC 1997[31] *Ans: 1.0×10^{3} Hz*

4. In a ripple tank experiment, a vibrating plate is used to generate ripples in the water. If the distance between two successive troughs is 3.5cm and the wave travels a distance of 31.5cm in 1.5s, calculate the frequency of the vibrator.

WAEC 1993[22] *Ans: 6.0Hz*

5. A radio wave has a wavelength of 150m. If the velocity of radio waves in free space is $3 \times 10^{8}\,ms^{-1}$, calculate the frequency of the radio wave.

WAEC 2002[25] *Ans: 2.0×10^{6} Hz*

6. A wave of wavelength 0.30m travels 900m in 3.0s, calculate its frequency.

WAEC 2003[19] *Ans: 1000Hz*

7. The distance between two successive crests of a water wave travelling at $3.6\,ms^{-1}$ is 0.45m, calculate the frequency of the wave. *WAEC 2005*[c13] *Ans: 8Hz*

8. The distance between the successive crests of a wave travelling at $20\,ms^{-1}$ is 25cm. Calculate the frequency of the wave. *WAEC 2006*[32] *Ans: 80.0Hz*

9.

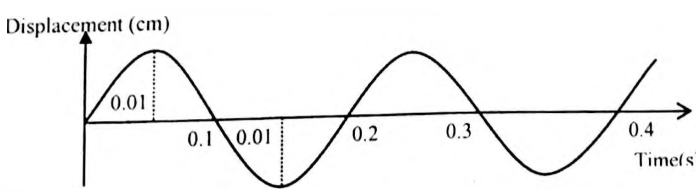


Fig. 12.15

The wave motion shown above has a frequency of
A. 0.1Hz B. 0.2Hz C. 5Hz D. 10Hz E. 100Hz *JAMB 1981*[22] *Ans: 5Hz*

10.

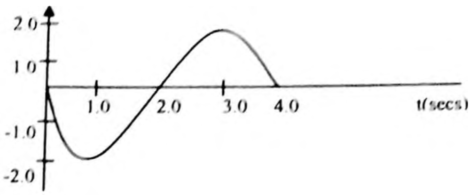


Fig. 12.16

The frequency of the wave in the diagram above is

A. 0.25Hz B. 1.57Hz C. 0.17Hz D. 1.05Hz *JAMB 2006[2] Ans: 0.25Hz*

11. The period of a wave is 0.02 second. Calculate its wavelength if its speed is $330ms^{-1}$. *WAEC 1995[32] Ans: 6.6m*

12. Calculate the wavelength of light travelling with a speed of $3\times10^8 ms^{-1}$ and a frequency of 6.25×10^{14}Hz. *NECO 2004[25] Ans: 4.8×10^{-7}m*

13. Radio waves have a velocity of $3\times10^8 ms^{-1}$. A radio station sends out a broadcast on a frequency of 800KHz. The wavelength of the broadcast is

A. 375.0m B. 267.0m C. 240.0m D. 37.5m E. 26.7m *JAMB 1984[45] Ans: 375.0m*

14. The wavelength of signals from a radio transmitter is 1500m and the frequency is 200KHz. What is the wavelength for a transmitter operating at 1000KHz.

A. 7500m B. 300m C. 75m D. 15m *JAMB 1995[28] Ans: 300m*

15. Sixty complete waves pass a particular points in 4s. If the distance between three successive troughs of the waves is 15m, calculate the speed of the wave.

WAEC 1995[28] Ans: $112.5ms^{-1}$

16. The distance between two successive crests of a water wave is 1m. If a particle on the surface of the water makes two complete vertical oscillations in one second, calculate the speed of the wave. *WAEC 1996[37] Ans: $2.0ms^{-1}$*

17. The distance between two successive crests of a water wave is 0.5m. If a particle on the surface of the water wave makes two complete vertical oscillations in 1s, calculate the speed of the wave. *NECO 2003[33] Ans: $1.0ms^{-1}$*

18. What is the frequency of a radio wave of wavelength 150m if the velocity of radio waves in free space is $3\times10^8 ms^{-1}$? *WAEC 1997[24] Ans: 2×10^6Hz*

19. The distance between two points in phase on a progressive wave is 5cm. If the speed of the wave is $0.20ms^{-1}$, calculate its period. *WAEC 2005[27] Ans: 0.25s*

20. If the wavelength of a wave travelling with a velocity of $360ms^{-1}$ is 60cm, the period of the wave is

A. 6s B. 3.6s C. 0.17s D. 0.61s E. 3s *JAMB 1979[13] Ans: 1.67×10^{-3}s*

21. A wave travelling with a speed of $360ms^{-1}$ has a wavelength of 60cm. The period of the wave in second is? *NECO 2007[25] Ans: 1.7×10^{-3}*

22. The distance between two successive crests of a wave is 0.664m. If the wave travels 43.2m in 1.2 seconds, calculate the (a) speed (b) frequency of the wave.

NECO 2007[e6] Ans: (a) $36ms^{-1}$ (b) 54.22Hz

23. In the wave equation $y = E_o\sin(200t - \pi x)$ E_o represents

A. Amplitude B. frequency C. period D. wavelength. *WAEC 2002[26] Ans: A*

24. The equation of a wave is $y = 0.005\sin[x(0.5x - 200t)]$ where x and y are in meters and t is in seconds. What is the velocity of the wave?

A. $4000ms^{-1}$ B. $400ms^{-1}$ C. $250ms^{-1}$ D. $40ms^{-1}$ *JAMB 1992[20] Ans: $250ms^{-1}$*

Hint: $y = A\sin(^{2\pi x}/_{\lambda} - \omega t)$.

25. A progressive wave equation is represented by $y = A\sin(150\pi t - {}^{\pi x}/_4)$. If the phase difference of a progressive wave is 45°, the value of x in the equation is

A. 2cm B. 1cm C. 4cm D. 3cm *JAMB 2006[5] Ans: 1cm*

26. The equation, $y = 5\sin(3x - 4t)$, where y is in millimeters, x is in meters and t is in seconds represents a wave motion. Determine the (i) frequency (ii) period and (iii) speed of the wave. *WAEC 2004[e13] Ans: (i) 0.6Hz (ii) 1.56s (iii) $1.34ms^{-1}$*

Use the figure below which shows a stationary wave in a closed tube to answer question 27 and 28. *JAMB 1993*[20, 21]

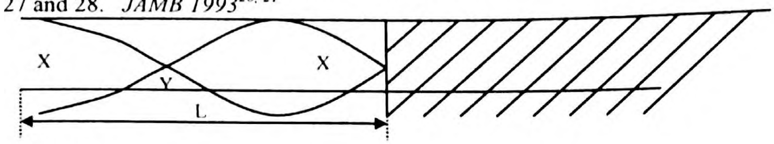


Fig. 12.17

27. The relationship between L and the wavelength λ of the stationary wave is
A. $2L = \lambda$ B. $L = \lambda$ C. $L = \lambda/3$ D. $4L = 3\lambda$ *Ans: D*

28. Determine the distance between consecutive antinodes XX if the wavelength is 60cm. A. 15cm B. 30cm C. 60cm D. 120cm *Ans: B*

29. The equation $y = A\sin(\omega t - kx)$ represents a plane wave traveling in a medium along the x – direction, y being the displacement at the point x at time t.

i. Given that x is the meters and t is in seconds, state the units of k and w.

ii. What physical quantity does $\frac{w}{k}$ represent?

iii. State whether the wave is traveling in the positive or negative direction x – direction. *WAEC 2008*

Ans: (i) k(m^{-1}), w (rad s^{-1}, s^{-1} or Hz)
(ii) w/k = Speed of wave
(iii) positive direction

30. Given the progressive wave equation $y = 5\sin(200\pi t - 0.4x)$, calculate the wavelength A. 12.4m B. 15.7m C. 12.5m D. 18.6m *JAMB 2008 Ans: 15.70m*

31. The diagram below represents a stationary wave pattern.

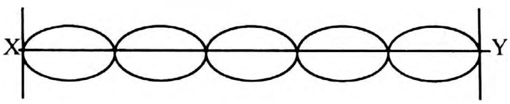


Fig 12.18

The distance XY represents
A. One and a half times the wavelength.
B. The wavelength
C. Three times the wavelength
D. Twice the wavelength
E. Two and a half times the wavelength *NECO 2008*[26] *Ans: E*

32. A progressive wave is represented by $y = 10\sin\left(100\pi t - \frac{\pi x}{34}\right)$. Two layers of the wave separated by 153cm have a phase difference of
A. 270° B. 45° C. 90° D. 180° *JAMB 2009*[27] *Ans: D*

33. If a light wave has a wavelength of 500nm in air, what is the frequency of the wave?
A. 3.0×10^{14}Hz B. 6.0×10^{14}Hz C. 6.0×10^{12}Hz
D. 2.5×10^{14}Hz $(c = 3 \times 10^{8} ms^{-1})$ *JAMB 2009*[30] *Ans: B*

34. A boat is rocked by waves of speed 30ms^{-1} whose successive crests are 10m apart. Calculate the rate at which the boat receives the waves. *WAEC 2009*[25] *Ans: 3s^{-1}*

35. A plane progressive wave is represented by the equation
$y = 0.2\sin(200\pi - 0.5x)$ where x *and* y are measured in *cm* and *t* in *s*. What is the frequency of wave? *NECO 2009*[25] *Ans: 100Hz*

13

REFLECTION OF LIGHT WAVES

Formation of images by a pinhole camera, plane and spherical mirrors depends on the facts that light travels in a straight line and can be reflected by plane surfaces and mirrors.

THE PINHOLE CAMERA: MAGNIFICATION

The magnification (M) of a pinhole camera is given as:

$$M = \frac{\text{image height}}{\text{object height}} = \frac{\text{image distance from pinhole}}{\text{object distance from pinhole}}$$

$$= \frac{\text{lenght of camera}}{\text{object distance from pinhole}}$$

Any, or a combination of the above formulas can be used when solving problems involving a pinhole camera.

Example 1

A boy 1.5m tall stands 2.0m in front of a pinhole camera of length 20.0cm. Calculate the height of his image. *NECO 2006*[27]

Solution

Object height = 1.5m; object distance from pinhole = 2.0m

Camera length = 20cm = 0.2m; height of image = ?

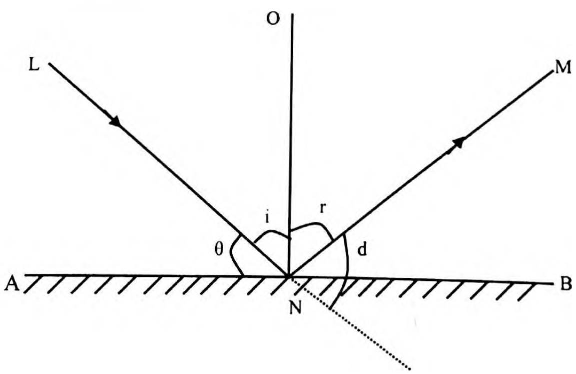
Substitute into $\frac{\text{image height}}{\text{object height}} = \frac{\text{camera length}}{\text{object distance from pinhole}}$

$$\frac{\text{image height}}{1.5\text{m}} = \frac{0.2\text{m}}{2.0\text{m}}$$

$$\therefore \text{Image height} = \frac{1.5 \times 0.2}{2.0} = 0.15\text{m or } 15\text{cm}$$

REFLECTION OF LIGHT AT PLANE SURFACES

Fig. 13.1



When a ray of light is incident upon a plane mirror AB shown above, the following are true.

1. LN is the incident ray MN is the reflected ray and ON is the normal.
2. The angle of incidence (i) is equal to the angle of reflection (r). That is, $i = r$

3. The angle (θ) between the incident ray and the plane mirror is called the glancing angle, and is always equal to the angle between the reflected ray and the plane mirror, MNB.
4. The angle of deviation $d = 180 - (i + r)$. Because $i = r$, the angle of deviation could be, $d = 180 - 2i$ or $d = 180 - 2r$.

Example 2

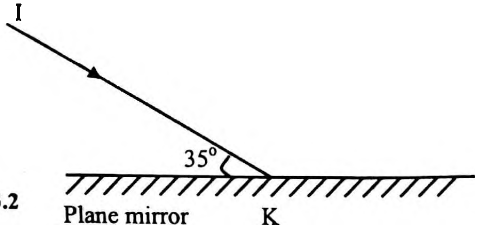


Fig. 13.2

The diagram above shows a ray of light IK incident on plane mirror at K. Calculate the angle of deviation of the ray after reflection. *WAEC 1994*[25]

Solution

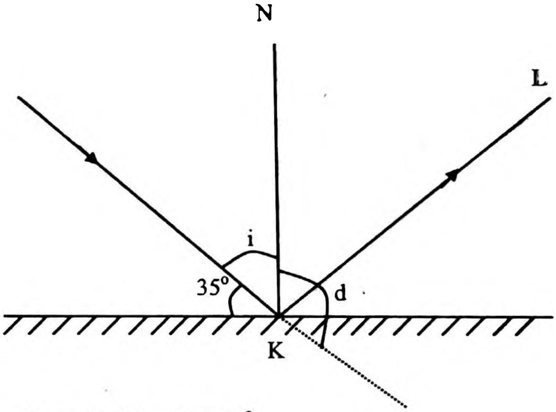


Fig. 13.3

Angle of incidence $i = 90 - 35 = 55°$

Therefore, angle of deviation, $d = 180 - 2i = 180 - 2 \times 55$

$= 180 - 110 = 70°$

Example 3

A ray of light strikes a plane mirror at a glancing angle of 55°. Calculate the angle between the incident and reflected rays. *NECO 2006*[28]

Solution

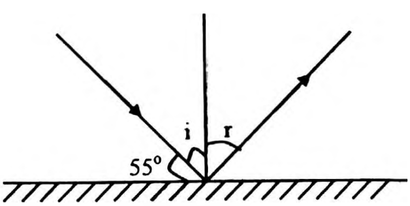


Fig. 13.4

Angle of incidence $i = 90 - 55 = 35°$

Angle of reflection $r = 35°$

Angle between incident and reflected ray is, $i + r$.

$i + r = 35 + 35 = 70°$

Example 4

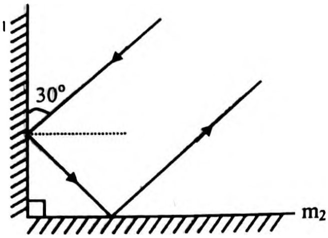


Fig. 13.5

A ray of light is incident on mirror m_1 and after reflection is incident on mirror m_2 as shown in the diagram above. Calculate the angle of reflection of the ray at mirror m_2.

WAEC 995[21]

Solution

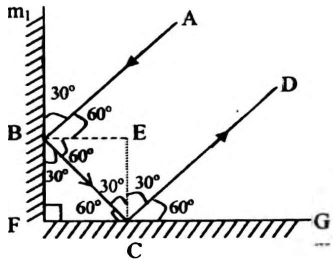


Fig. 13.6

Any time an incident ray is reflected on a plane mirror, a normal perpendicular (at 90°) to the mirror can be drawn.

EC is the normal between incident ray BC and reflected ray CD.

EB is the normal between incident ray AB and reflected ray BC.

Angle $A\hat{B}E = 90° - 30° = 60°$

Angle $E\hat{B}C = 60°$ (Angle of incidence = Angle of reflection)

Angle $C\hat{B}F = 90° - 60° = 30°$

Angle $B\hat{C}F = 180° - (90° + 30°) = 180° - 120° = 60°$ (sum of angles in a triangle is 180°)

Angle $B\hat{C}E = 90° - 60° = 30°$

Angle $E\hat{C}D = 30°$ (Angle of incidence = Angle of reflection).

Angle $D\hat{C}G = 90° - 30° = 60°$

Therefore, angle of reflection at mirror m_2 = $E\hat{C}D = 30°$

Example 5

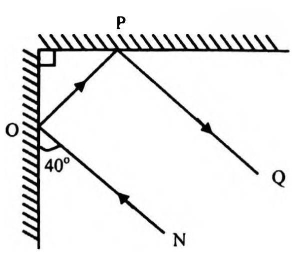


Fig. 13.7

Two mirrors of the same length are arranged as shown in the diagram above. A ray of light NO strikes the system at O and emerges along PQ. The emergent ray has been deviated through

A. 220° B. 200° C. 210° D. 180° E. 230° *JAMB 1982*[36]

Solution

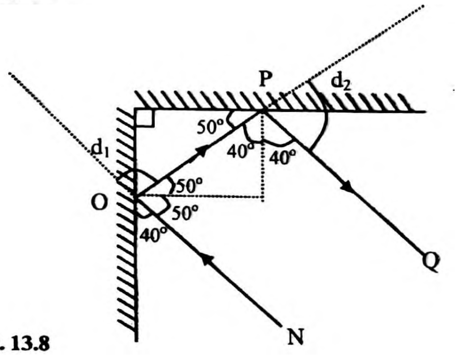


Fig. 13.8

The incident ray ON that emerges along PQ has been deviated twice; through angle d_1 and d_2 as shown above. Therefore, the emergent ray will be deviated through $d_1 + d_2 = D$.

Remember, angle of deviation, d = 180 – 2i or 180 – 2r.

The angles of incidences and angles of reflection are found in ways similar to those in example 4.

Angle of incidence (i) or ray NO is 50°. Therefore, $d_1 = 180 - 2i = 180 - 2 \times 50 = 180 - 100 = 80°$

Angle of incidence of emergent ray PQ is 40°. Therefore, $d_2 = 180 - 2i = 180 - 2 \times 40 = 180 - 80 = 100°$

Angle of deviation of the emergent ray $D = d_1 + d_2 = 80 + 100 = 180°$.

Example 6

An object is placed 10cm in front of a plane mirror. If it is moved 8cm farther away from the mirror, determine the distance of the final image from the mirror. *WAEC 2007*[31]

Solution

When an object is placed in front of a plane mirror, the size of the image formed is the same size as the object. Also, the distance (u) of the object from the mirror is always equal to the distance (v) of the image from the mirror.

That is u = v.

Object distance from mirror, u = 10 + 8 = 18cm. Therefore, image distance v = 18cm.

Example 7

A man stands 4m in front of a plane mirror. If the mirror is moved 1m towards the man, the distance between him and his new image is A. 3m B. 5m C. 6m D. 10m *JAMB 1999*[20]

Solution

When the mirror is moved 1m towards the man, the object distance becomes 4–1=3m. The image distance from the mirror is also 3m. Therefore, distance between the man and his image is 3 + 3 = 6m.

IMAGE FORMED BY INCLINED MIRRORS

When two mirrors are inclined at an angle (θ) to each other, the number (n) of image formed is given by

$$n = \frac{360}{\theta} - 1$$

Example 8

A candle is place between two mirrors which are inclined at an angle of 72° and facing each other. How many images will be observed in the two mirrors?

Solution

Angle of inclination, $\theta = 72^\circ$

Substitute into $n = \frac{360}{\theta} - 1$

Number of images, $n = \frac{360}{72} - 1$

$n = 5 - 1 = 4$

Example 9

At what angle θ will 2 plane mirrors be inclined so that 11 images will be formed when an object is placed in front of the two mirrors?

Solution

Number of image n = 11

Substitute into $n = \frac{360}{\theta} - 1$

$$11 = \frac{360}{\theta} - 1$$

$$11 + 1 = \frac{360}{\theta}$$

$$12 = \frac{360}{\theta} \qquad \therefore \qquad \theta = \frac{360}{12} = 30^\circ$$

ROTATION OF PLANE MIRRORS

When a ray of light is incident on a plane mirror and the mirror is rotated through an angle θ while keeping the incident ray constant, the following are true.

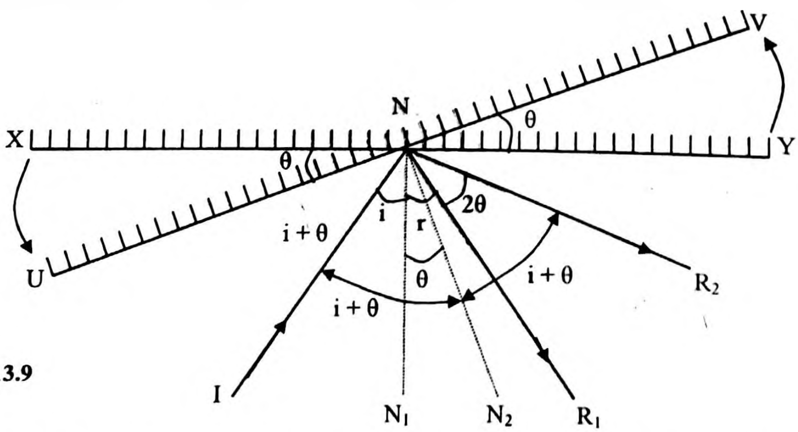


Fig. 13.9

The angle of incidence before rotation is i or INN_1. The angle of incidence after rotation is $i + \theta$ or INN_2.

1. The angle between the initial normal (NN_1) and the final normal (NN_2) is equal to the angle of rotation θ.
2. The angle of reflection before rotation is r or R_1NN_1. The angle of reflection after rotation is $i + \theta$ or R_2NN_2.
3. Angle between the incident ray and the final reflected ray is $(i + \theta) + (i + \theta) = 2(i + \theta)$.
4. The reflected ray is rotated through twice the angle of rotation, 2θ, or R_1NR_2 as shown in the diagram above.
5. The angle between the normal after rotation and the initial reflected ray is $i-\theta$ if angle of rotation is less than initial angle of incidence or $\theta - i$ if angle of rotation is greater than initial angle of incidence.

Example 10

A ray of light is incident on a plane mirror at an angle of 20° This mirror is rotated through twice this angle. In this new position, the angle between the incident ray and the reflected ray is? *WAEC 1999*[21]

Solution.

Angle of incidence $i = 20°$; angle of rotation $\theta = 2 \times 1 = 2 \times 20 = 40°$ (*...rotated through twice....*). Angle between incident ray and new reflected ray $= 2(i + \theta) = 2(20 + 40) = 2 \times 60 = 120°$

Example 11

An incident ray is reflected normally by a plane mirror onto a screen where it forms a bright spot. The mirror and screen are parallel and 1m apart. If the mirror is rotated through 5°, calculate the displacement of the spot. *WAEC 2001*[23]

Solution

Let x be the displacement of the spot. Angle of rotation $\theta = 5°$. Angle through which reflected ray rotates $= 2\theta = 2 \times 5 = 10°$.

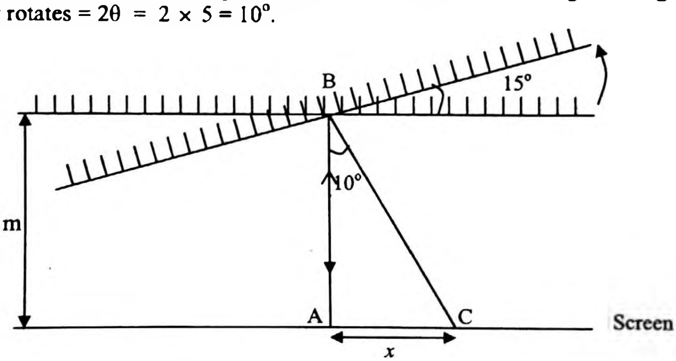


Fig. 13.10

From triangle ABC, $\text{Tan}\,10° = \frac{x}{1}$

$\therefore \quad x = \text{Tan}\ 10° \times 1 = 0.176\text{m or } 17.6\text{cm}$

Example 12

A ray of light is incident normally on a plane mirror. If the incident ray is kept fixed while the mirror is rotated through an angle of 30°, determine the initial and final angles of reflection respectively. *WAEC 2005*[31]

Solution

When a ray of light is incident normally (at 90°) on a plane mirror, the angle of incidence and the angle of reflection are zero. That is, $i = r = 0$. If the mirror is rotated through 30°, the final angle of reflection 2θ becomes $2 \times 30 = 60°$.

Therefore, the initial and final angles of reflection respectively are 0° and 60°.

Example 13
In the figure below, an incident ray XY makes an angle of 15° with a fixed line AY which is normal to the surface of a plane mirror. If the mirror is turned as indicated by the arrow through 40^0, what angle will the reflected ray make with AY? *JAMB 1982*[23]

A. 25° B. 40° C. 15° D. 55° E. 95°

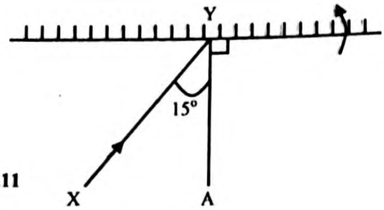


Fig. 13.11

Solution
XY is the incident ray. Angle of rotation $\theta = 40°$; angle of incidence $i = 15°$.
The angle that the new reflected ray makes with AY (normal before rotation) is,
$i + \theta + \theta$ or $i + 2\theta = 15 + 2 \times 40 = 95°$.

SPHERICAL MIRRORS: CONCAVE AND CONVEX

The following formulars, expressions and rules are used in solving concave and convex mirror problems.

I. Focal length = ½ radius of curvature. That is $f = {}^r/_2$

II. Relationship between the object distance u, image distance v and focal length f is given by mirror equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{or} \quad f = \frac{uv}{u+v}$$

III. Magnification, $M = \dfrac{\text{image height}}{\text{object height}} = \dfrac{\text{image distance from lens}}{\text{object distance from lens}}$

That is, $M = \dfrac{v}{u}$

- A convex mirror has negative focal length.
- A concave mirror has positive focal length.
- Image distance in a convex mirror is negative.
- Image distance in a concave mirror is positive except when the object distance is less than the focal length (see example 17).
- Distances of real objects and images are positive.
- Distances of virtual objects and images are negative.

Example 14
The image of an object is located 6cm behind a convex mirror. If its magnification is 0.6, calculate the focal length of the mirror. *WAEC 1999*[24]

Solution
Image distance, v = -6cm; magnification M = 0.6; focal length f = ?

Object distance u is obtained by substituting into $M = \dfrac{v}{u}$

$$0.6 = \frac{-6cm}{u} \qquad \therefore \qquad u = \frac{-6}{0.6} = -10cm$$

Substitute above value into $f = \frac{uv}{u+v}$ to obtain

Focal length $f = \frac{-10 \times -6}{-10+(-6)} = \frac{60}{-10-6} = \frac{60}{-16}$

$f = -3.75cm$

The negative sign shows it's a convex mirror.

Example 15

An object is placed in front of a concave mirror whose radius of curvature is 12cm. If the magnification of the image produced is 1.5, how far is the object from the mirror?

NECO 2005[28]

Solution

Magnification M = 1.5; radius of curvature r = 12cm; focal length $f = {}^{12}/_2 = 6cm$;
Object distance u = ?

Substitute into $M = \frac{v}{u}$ to obtain $1.5 = {}^{v}/_{u}$.

Therefore image distance, v = 1.5u

Substitute into $f = \frac{uv}{u+v}$ to obtain

$$6 = \frac{u \times 1.5u}{u+1.5u} = \frac{1.5u^2}{2.5u} = \frac{1.5u}{2.5}$$

$$6 = \frac{1.5u}{2.5}$$

$$1.5u = 6 \times 2.5 = 15$$

$$\therefore \quad u = \frac{1.5}{1.5} = 10cm$$

Example 16

A concave mirror of radius of curvature 40cm forms a real image twice as large as the object. The object distance is A. 30cm B. 40cm C. 60cm D. 10cm *JAMB 2002*[28]

Solution

Radius of curvature, r = 40cm $\therefore$ $f = {}^{r}/_2 = {}^{40}/_2 = 20cm$.

Object distance, u = ? Magnification, M = 2, because image is twice (2x) the object.

$$M = \frac{\text{image size (v)}}{\text{object size (u)}} \quad = \quad 2 = \frac{v}{u} \quad \therefore \quad \textit{image distance}, v = 2u$$

substitute into $f = \frac{uv}{u+v}$ to obtain,

$$20 = \frac{u \times 2u}{u+2u} = \frac{2u^2}{3u}$$

$$20 = \frac{2u}{3}$$

$$2u = 20 \times 3 = 60$$

$$u = \frac{60}{2} = 30cm$$

Example 17

An object is placed 10cm in front of a concave mirror of focal length 15cm. What is the position and nature of the image formed? A. 30cm and virtual B. 6cm and real C. 6cm and virtual D. 30cm and real. *JAMB 1988*[32]

Solution

Object distance, u = 10cm; focal length, f = 15cm; image distance, v = ?

Substitute into $f = \frac{uv}{u+v}$ to obtain

$$15 = \frac{10 \times v}{10 + v}$$

Cross multiplying, $15(10 + v) = 10 \times v$

$$150 + 15v = 10v$$

$$150 = 10v - 15v = -5v$$

$$v = \frac{-150}{5} = -30cm.$$

If the image distance of a concave mirror is negative, it means that the image is virtual.

Example 18

Which of the following expressions gives the linear magnification produced by a concave mirror of radius of curvature r, if u and v are the object and image distances respectively?

A. $\frac{v}{r} - 1$ B. $\frac{2v}{r} - 1$ C. $\frac{u}{r} - 1$ D. $\frac{2u}{r} - 1$ *JAMB 1993*[26]

Solution

Radius of curvature = r ∴ Focal length, f = ½ r ; Linear magnification = $^v/_u$

Substitute into $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{½} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{2}{r} = \frac{1}{u} + \frac{1}{v}$$

multiply both sides by v

$$\frac{2v}{r} = \frac{v}{u} + \frac{v}{v}$$

$$\frac{2v}{r} = \frac{v}{u} + 1$$

make $^v/_u$ subject of above equation to obtain linear magnification.

Linear magnification, $\frac{v}{u} = \frac{2v}{r} - 1$

Example 19

The driving mirror of a car has a radius of curvature of 1m. A vehicle behind the car is 4m from the mirror. Find the image distance behind the mirror.

A. $^8/_7$ B. $^4/_9$ C. $^9/_2$ D. $^4/_7$ *JAMB 2001*[14]

Solution

Driving mirrors are usually convex mirrors. Radius of curvature, r = 1m ∴ f = ½

1. The angle between the initial normal (NN_1) and the final normal (NN_2) is equal to the angle of rotation θ.
2. The angle of reflection before rotation is r or R_1NN_1. The angle of reflection after rotation is $i + \theta$ or R_2NN_2.
3. Angle between the incident ray and the final reflected ray is $(i + \theta) + (i + \theta) = 2(i + \theta)$.
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A ray of light is incident on a plane mirror at an angle of 20° This mirror is rotated through twice this angle. In this new position, the angle between the incident ray and the reflected ray is? *WAEC 1999*[21]

Solution

Angle of incidence $i = 20°$; angle of rotation $\theta = 2 \times I = 2 \times 20 = 40°$ (*...rotated through twice....*). Angle between incident ray and new reflected ray $= 2(i + \theta) = 2(20 + 40) = 2 \times 60 = 120°$

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An incident ray is reflected normally by a plane mirror onto a screen where it forms a bright spot. The mirror and screen are parallel and 1m apart. If the mirror is rotated through 5°, calculate the displacement of the spot. *WAEC 2001*[23]

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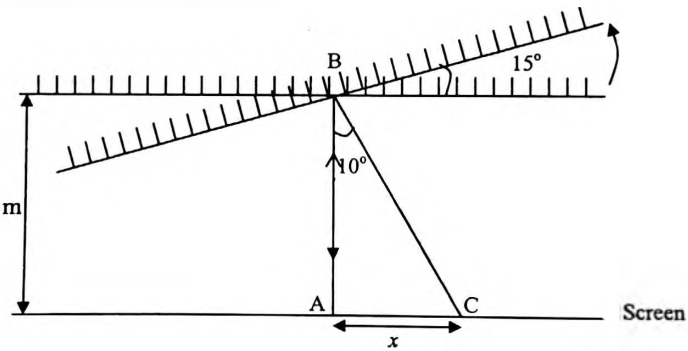

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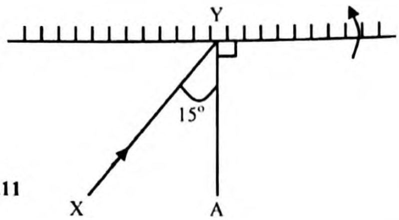

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NECO 2005[28]

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multiply both sides by v

$$\frac{2v}{r} = \frac{v}{u} + \frac{v}{v}$$

$$\frac{2v}{r} = \frac{v}{u} + 1$$

make $^v/_u$ subject of above equation to obtain linear magnification.

Linear magnification, $\frac{v}{u} = \frac{2v}{r} - 1$

Example 19

The driving mirror of a car has a radius of curvature of 1m. A vehicle behind the car is 4m from the mirror. Find the image distance behind the mirror.

A. $^8/_7$ B. $^4/_9$ C. $^9/_2$ D. $^4/_7$

JAMB 2001

Solution

Driving mirrors are usually convex mirrors. Radius of curvature, r = 1m ∴ f = ½

14

REFRACTION OF LIGHT WAVES

LAWS OF REFRACTION AND REFRACTIVE INDEX

Refraction is the bending or change in direction of a light ray as a result of change in the speed of light when it passes from one medium to another.

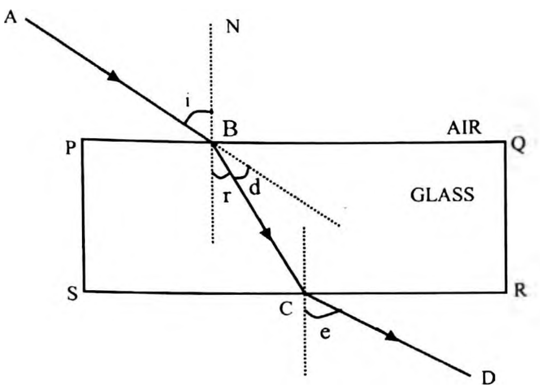


Fig. 14.1

AB = incidence ray; i = angle of incidence
BC = refracted ray; r = angle of refraction
CD = emergent ray; e = angle of emergence
d = angle of deviation of incidence ray = i – r

The laws of refraction are as follows:

1. The incident and refracted rays are on opposite sides of the normal at the point of incidence and all three are in the same plane.
2. The ratio of the sine of the angle of incidence (i) to the sine of the angle of refraction (r) is a constant for a given pair of media. This is also known as Snell's law and is stated thus.

$$\frac{\sin i}{\sin r} = \cap$$

$\cap$ = is known as the refractive index of the second medium with respect to the first medium. Air and glass are used as typical examples of a pair of media. It could be different. General equations for refractive index are as follows:

(a) Refractive index of glass, ${}_a\cap_g = \dfrac{\sin i \text{ in air}}{\sin r \text{ in glass}}$

(b) Refractive index of air, ${}_g\cap_a = \dfrac{\sin i \text{ in glass}}{\sin r \text{ in air}}$

Therefore, ${}_a\cap_g = \dfrac{1}{{}_g\cap_a}$ or ${}_g\cap_a = \dfrac{1}{{}_a\cap_g}$

(c) ${}_a\cap_g = \dfrac{\text{speed of light in air}}{\text{speed of light in glass}}$ i.e. ${}_a\cap_g = \dfrac{V_a}{V_g}$

${}_a\cap_g = \dfrac{\text{wavelength of light in air}}{\text{wavelength of light in glass}}$ i.e. ${}_a\cap_g = \dfrac{\lambda_a}{\lambda_g}$

While the speed and wavelength of light change from one medium to the other, ***frequency remains unchanged.***

Example 1

A beam of light is incident from air to water at an angle of 30^0. Find the angle of refraction if the refractive index of water is $^4/_3$. *JAMB 1993*[29]

A. 15^o B. 18^o C. 22^o D. 24^o

Solution

Angle of incidence $i = 30^o$; refractive index $\cap = {}^4/_3$; angle of refraction $r = ?$

Substitute into $\cap = \dfrac{\sin i}{\sin r}$ to obtain

$$\frac{4}{3} = \frac{\sin 30}{\sin r}$$

cross multiplying, $\sin r \times 4 = 3 \times \sin 30^o$

$$\sin r = \frac{3 \times 0.5}{4} = \frac{1.5}{4} = 0.375$$

$\therefore$ Angle of refraction $r = \sin^{-1} 0.375 = 22^o$

Example 2

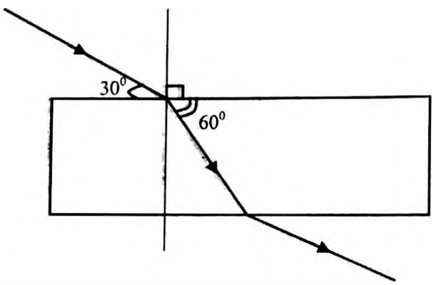


Fig. 14.2

The figure above shows the path of light passing through a glass block. Calculate the refractive index of the glass. *NECO 2000*[27]

Solution

Angle of incidence $i = 90 - 30 = 60^o$; Angle of refraction $r = 90 - 60 = 30^o$

$$\text{Refractive index } \cap = \frac{\sin i}{\sin r} = \frac{\sin 60^o}{\sin 30^o} = \frac{0.866}{0.5} = 1.73$$

Example 3

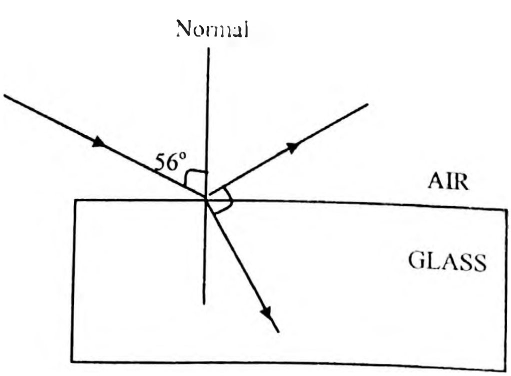


Fig 14.3

A ray of light is incident on a glass block as shown in the diagram above. If the reflected and the refracted rays are perpendicular to each other, what is the refractive index of the glass relative to air? *WAEC 1996*[30]

Solution

Angle NOB = 56° (angle of incidence AON = angle of reflection BON)

Angle of refraction COD = 180 – (90 + 56) = 34°

i = 56° and r = 34°

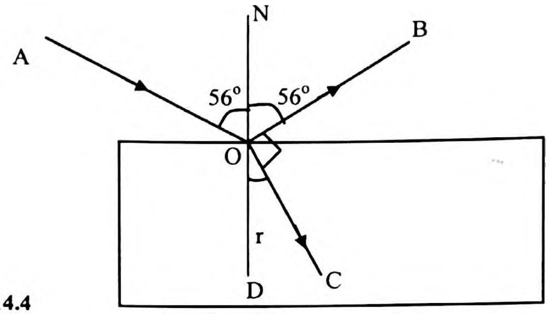


Fig. 14.4

Refractive index of glass, $_{a}n_{g} = \frac{\sin i}{\sin r} = \frac{\sin 56^\circ}{\sin 34^\circ} = \frac{0.829}{0.559} = 1.48$

Example 4

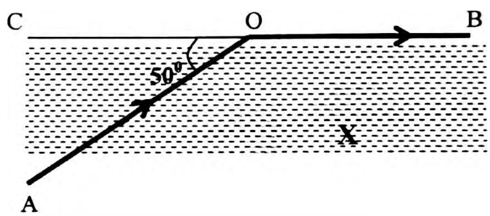


Fig. 14.5

The diagram above shows an incident ray AO inclined at an angle of 50° to the interface CB. The refractive ray OB is found to lie along the surface. What is the refractive index of the medium X with respect to air?

A. $\frac{\sin 50^\circ}{\sin 40^\circ}$ B. $\frac{\sin 40^\circ}{\sin 50^\circ}$ C. $\frac{\sin 90^\circ}{\sin 50^\circ}$ D. $\frac{\sin 40^\circ}{\sin 90^\circ}$ E. $\frac{\sin 90^\circ}{\sin 40^\circ}$ *WAEC 1990*[32]

Solution

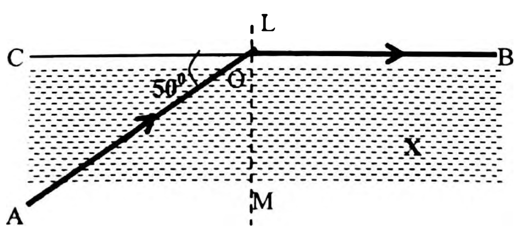


Fig. 14.6

A normal LM is drawn through O as shown in the diagram above.

Angle of incidence i = angle $A\hat{O}M$ = 90° - 50° = 40°

Angle of refraction r = angle $B\hat{O}M$ = 90°

If you were asked to find the refractive index of air, your answer will be:

$$ {}_x n_a = \frac{\text{sin of angle of incidence in meduim X}}{\text{sin of angle of refraction in air}} = \frac{\sin 40^\circ}{\sin 90^\circ} $$

However, you are asked to find the refractive index of medium X with respect to air. So, based on the principle of reversibility of light, the refracted ray becomes the incident ray and the incident ray becomes the refracted ray.

Refractive index of medium X, $ {}_a n_x = \frac{\text{sin of angle of incidence in air}}{\text{sin of angle of refraction in X}} = \frac{\sin 90^\circ}{\sin 40^\circ} $

Note that $\quad {}_x n_a = \frac{1}{{}_a n_x}$

Example 5

A ray of light is incident at an angle of 30^0 at an air-glass interface.

(i) Draw a ray diagram to show the deviation of the ray in the glass.

(ii) Determine the angle of deviation. [refractive index of glass = 1.50]

WAEC 2005

Solution

Angle of incidence i = 30°; ${}_a n_g = 1.5$

(i)

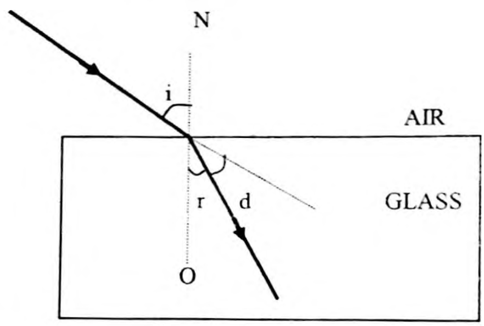


Fig. 14.7

(ii) Substitute into ${}_a n_g = {}^{\sin i}/_{\sin r}$ to find angle of refraction r.

$$ 1.5 = \frac{\sin 30^\circ}{\sin r} \qquad \therefore \qquad \sin r = \frac{0.5}{1.5} = 0.333 $$

$$ r = \sin^{-1} 0.333 = 19.5^\circ \qquad \therefore \quad \text{Angle of deviation } d = i - r = 30 - 19.5 = 10.5^\circ $$

Example 6

A ray of light is inclined at angle of 30^0 on one top surface of a parallel-sided glass of refractive index 1.5. The ray finally emerges from the low surface. What is the angular deviation of the emerged ray? A. 60^0 B. 39^0 C. 28^0 D. 0^0 *JAMB 1987*

Solution

When a light ray passes through a rectangular glass block, the incident and the emergent rays are parallel to each other. There is no deviation or change in the direction of the emergent ray when compared with the incident ray. Therefore, the angular deviation of the emergent ray with respect to the incident ray is zero. (*Ans. D*)

Example 7

The absolute refractive indices of glass and water are $^3/_2$ and $^4/_3$ respectively. The refractive index at the interface when a ray travels from water to glass is

A. ½ B. $^8/_9$ C. $^9/_8$ D. $^{17}/_{12}$ E. 2 *JAMB 198*

Solution

The absolute refractive index of a medium is the value of $\cap$ when the first medium is a vacuum. Therefore, for glass $_a\cap_g$ = 3/2; for water $_a\cap_w$ = 4/3.

$$\text{Refractive index of glass, } {}_a\cap_g = \frac{\text{speed of light in air}}{\text{speed of light in glass}}$$

$$\frac{3}{2} = \frac{3\times10^8}{V_g}$$

$$\text{Speed of light in glass, } V_g = \frac{2\times3\times10^8}{3} = \frac{3\times10^8\times2}{3}$$

$$\text{Refractive index of water, } {}_a\cap_w = \frac{\text{speed of light in air}}{\text{speed of light in water}}$$

$$\frac{4}{3} = \frac{3\times10^8}{V_w}$$

$$\text{Speed of light in water, } V_w = \frac{3\times3\times10^8}{4}$$

$$\text{Refractive index of ray from water to glass, } {}_w\cap_g = \frac{\text{speed of light in water}}{\text{speed of light in glass}} = \frac{V_w}{V_g}$$

$$= \frac{3\times3\times10^8}{4} \div \frac{3\times10^8\times2}{3}$$

$$= \frac{3\times3\times10^8}{4} \times \frac{3}{3\times10^8\times2} = \frac{3\times3}{4\times2}$$

$${}_w\cap_g = 9/8$$

Example 8

The velocities of light in air and glass are $3.0 \times 10^8 ms^{-1}$ and $1.8 \times 10^8 ms^{-1}$ respectively. Calculate the sine of the angle of incidence that will produce an angle of refraction of 30^0 for a ray of light incident on glass. *WAEC 1988*[28]

Solution

Velocity of light in air, $V_a = 3.0 \times 10^8 ms^{-1}$

Velocity of light in glass, $V_g = 1.8 \times 10^8 ms^{-1}$

Angle of refraction $r = 30^0$

Sine of angle of incidence, sin i = ?

$${}_a\cap_g = \frac{\sin i}{\sin r} = \frac{\text{velocity of light in air}}{\text{velocity of light in glass}}$$

$$= \frac{\sin i}{\sin 30°} = \frac{3\times10^8}{1.8\times10^8}$$

$$\therefore \quad \sin i = \frac{\sin 30°\times3.0\times10^8}{1.8\times10^8} = \frac{0.5\times3}{1.8} = \frac{1.5}{1.8} = 0.83$$

Example 9

The speed of light in vacuum is $3 \times 10^8 ms^{-1}$. If the refractive index of a transparent liquid is $^4/_3$, then the speed of light in the liquid is A. $0.44 \times 10^8 ms^{-1}$ B. $2.25 \times 10^8 ms^{-1}$ C. $3.0 \times 10^8 ms^{-1}$ D. $4.0 \times 10^8 ms^{-1}$ E. $4.33 \times 10^8 ms^{-1}$ *JAMB 1983*[46]

Solution

Speed of light in vacuum $V_a = 3.0 \times 10^8 ms^{-1}$ Speed of light in the liquid V_L =?

Refractive index $\cap = {}^4/_3$

Substitute into $\cap = \frac{V_a}{V_L}$ to obtain

$$\frac{4}{3} = \frac{3\times10^8}{V_L}$$

$\therefore$ Speed of light in the liquid $V_L = \frac{3\times10^8\times3}{4} = 2.25\times10^8 ms^{-1}$

Example 10

Light of velocity $3.0 \times 10^8 ms^{-1}$ is incident on a material of refractive index $\cap$. If the velocity of light is reduced to $2.4 \times 10^8 ms^{-1}$ in the material, what is $\cap$? *JAMB 1997*[35]

A. 2.33 B. 2.25 C. 1.33 D. 1.25

Solution

Velocity of light at incidence $V_a = 3.0 \times 10^8 ms^{-1}$

Velocity of light after refraction $V_g = 2.4 \times 10^8 ms^{-1}$

Refractive index $\cap = \frac{V_a}{V_g} = \frac{3.0\times10^8}{2.4\times10^8} = 1.25$

Example 11

A glass plate 0.9cm thick has a refractive index of 1.50. How long does it take for a pulse of light to pass through the plate? A. $3.0 \times 10^{-10}s$ B. $3.0 \times 10^{-11}s$ C. $4.5 \times 10^{-10}s$ D. $4.5 \times 10^{-11}s$ [$c=3.0 \times 10^8 ms^{-1}$] *JAMB 2007*[20]

Solution

Glass thickness or length = 0.9cm; $\cap = 1.5$; $V_a = 3.0 \times 10^8 ms^{-1}$

$$\cap = \frac{\text{velocity of light in air } (V_a)}{\text{velocity of light in glass } (V_g)}$$

$$1.5 = \frac{3.0\times10^8}{V_g}$$

$$V_g = \frac{3.0\times10^8}{1.5} = 2\times10^8 ms^{-1}$$

The speed of light in glass is V = 2 d $10^8 ms^{-1}$

Length (distance) of glass is d = 0.9cm = 0.009m

From velocity $= \frac{\text{distance}}{\text{time}}$, time $= \frac{\text{distance}(d)}{\text{velocity}(v)}$

Time taken to pass through glass, $t = \frac{d}{v} = \frac{0.009}{2\times10^8} = 4.5\times10^{-11}s$

Example 12

Surface waves travelling in deep water at $15ms^{-1}$ are incident at a shallow water boundary. If the angles of incidence and refraction are 45^o and 30^o respectively, calculate the speed of the waves in shallow water.

Solution

Wave speed in deep water V_D = 15m/s

Angle of incidence $i = 45^o$

Angle of refraction $r = 30^o$;

Wave speed in shallow water V_S = ?

Substitute into $\frac{\sin i}{\sin r} = \frac{V_D}{V_S}$

$$\frac{\sin 45^\circ}{\sin 30^\circ} = \frac{15}{V_S}$$

Wave speed in shallow water $V_S = \dfrac{\sin 30^\circ \times 15}{\sin 45^\circ} = 10.6\text{ms}^{-1}$

Example 13

A light wave travels from air into a medium of refractive index 1.54. If the wavelength of the light in air is 5.9 x 10^{-7}m, calculate its wavelength in the medium. *WAEC 1994*[31]

Solution

$$\text{Refractive index} = \frac{\text{wavelenght of ligth in air}}{\text{wavelenght of light in meduim}}$$

$$1.54 = \frac{5.9 \times 10^{-7}}{\text{wavelength of light in meduim}}$$

$$\text{wavelenght of light in meduim} = \frac{5.9 \times 10^{-7}}{1.54} 3.83 \times 10^{-7}\text{m}$$

Example 14

An electromagnetic wave of frequency 5.0 x 10^{14}Hz is incident on the surface of water of refractive index $^4/_3$. Taking the speed of wave in air as 3.8 x 10^8ms^{-1}, calculate the wavelength of the wave in water. *WAEC 1998*[39]

Solution

Frequency f = 5.0 × 10^{14}Hz; refractive index $\cap = {}^4/_3$; Wave speed V = 3.8 × 10^8ms^{-1}

From $V = \lambda f$, wavelength in air, $\lambda = \dfrac{V}{f} = \dfrac{3.8 \times 10^8}{5.0 \times 10^{14}} = 6.0 \times 10^{-7}\text{m}$

$${}_a\cap_w = \frac{\text{wavelength of wave in air } (\lambda_a)}{\text{wavelength of wave in water} (\lambda_w)}$$

$$\frac{4}{3} = \frac{6.0 \times 10^{-7}}{\lambda_w}$$

$$\lambda_w = \frac{6.0 \times 10^{-7}}{4} = \frac{3 \times 6.0 \times 10^{-7}}{4} = 4.50 \times 10^{-7}\text{m}$$

REAL AND APPARENT DEPTH

Refractive index ($\cap$) is related to real and apparent depth by the following equation;

$$\cap = \frac{\text{Real (actual) depth}}{\text{Apparent depth}}$$

$$\text{That is, } \cap = \frac{R}{A}$$

Lateral displacement or apparent (upward) displacement of an object placed under a glass block or in a pool of liquid is given by;

Apparent displacement = Real depth − Apparent depth

That is, $a = R - A$

For example, the apparent upward displacement of a point object placed under a rectangular glass prism of thickness d and absolute refractive index $\cap$ is derived as follows:

Real depth = d; refractive index = η

$$\text{Refractive index} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

$$\therefore \quad \text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}} = \frac{d}{\eta}$$

$$\text{Apparent displacement} = \text{Real depth} - \text{Apparent depth}$$

$$= d - \frac{d}{\eta}$$

$$= \frac{\eta d - d}{\eta} = \frac{d(\eta - 1)}{\eta}$$

Example 15

A transparent rectangular block 5.0cm thick is placed on a black dot. The dot when viewed from above is seen 3.0cm from the top of the block. Calculate the refractive index of the material of the block. *WAEC 1988*[27]

Solution

Real depth R = 5.0cm; Apparent depth A = 3.0cm

$$\text{Refractive index,} \quad \eta = \frac{R}{A} = \frac{5}{3} = 1.67$$

Example 16

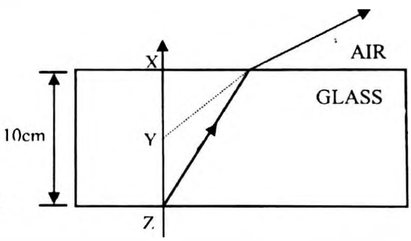


Fig. 14.8

Calculate the refractive index of the material of the glass block shown in the diagram above if YZ = 4cm. *WAEC 1991*[25]

Solution

Real depth or height (XZ), R = 10cm; YZ = 4cm

Apparent depth, A = XY = XZ – YZ = 10 – 4 = 6cm

$$\text{Refractive index,} \quad \eta = \frac{R}{A} = \frac{10\text{cm}}{4\text{cm}} = 1.67$$

Example 17

A rectangular glass prism of thickness 12cm is placed on a mark on a piece of paper resting on a horizontal bench, if the refractive index of the material of the prism is 1.5, calculate the apparent displacement of the mark. *WAEC 2000*[E13]

Solution

Real depth (thickness) R = 12cm; Refractive index η = 1.5

For a real depth R and refractive index η, the apparent displacement, a is given by

$$a = \frac{R(\eta - 1)}{\eta} = \frac{12(1.5 - 1)}{1.5} = \frac{12 \times 0.5}{1.5} = 4\text{cm}$$

Solution

Refractive index $\cap = {}^3/_2$; Prism's refracting angle $A = 60°$;

Angle of minimum deviation D_m = ?

Substitute int *o* $\cap = \dfrac{\sin \frac{1}{2}(D_m + 60°)}{\frac{A}{2}}$ *to obtain*

$$\frac{3}{2} = \frac{\sin \frac{1}{2}(D_m + 60°)}{\sin {}^{60}\!/_2}$$

$$1.5 = \frac{\sin \frac{1}{2}(D_m + 60°)}{\sin 30} = \frac{\sin \frac{1}{2}(D_m + 60°)}{0.5}$$

cross multiplying $1.5 \times 0.5 = \sin \frac{1}{2}(D_m + 60°)$

$$0.75 = \sin \tfrac{1}{2}(D_m + 60°)$$

$$\frac{0.75}{\sin} = \tfrac{1}{2}(D_m + 60°)$$

$$\sin^{-1} 0.75 = \frac{D_m}{2} + \frac{60°}{2} = \frac{D_m}{2} + 30°$$

$$48.59° = \frac{D_m}{2} + 30''$$

$$\frac{D_m}{2} = 48.59 - 30 = 18.59°$$

$$D_m = 2 \times 18.59° = 37.2''$$

An alternative method for 'Example 30' is as follows:

$A = 60°$; $\cap = {}^3/_2 = 1.5$; At minimum deviation $r = {}^A/_2 = {}^{60}/_2 = 30°$

Substitute into $\cap = \dfrac{\sin i}{\sin r}$ to obtain

$$1.5 = \frac{\sin i}{\sin 30}$$

$$\sin i = 1.5 \times \sin 30° = 1.5 \times 0.5 = 0.75$$

$$i = \sin^{-1} 0.75 = 48.75° \simeq 48.6°$$

Angle of minimum deviation $D_m = 2i - 2r$

$$= 2(48.6) - 2(30)$$

$$= 37.2^0$$

or $D_m = 2i - A$

$$= 2(48.6) - 60$$

$$= 37.2^0$$

Example 31

Calculate the refractive index of the material for the glass prism in the diagram below.

A. $\sqrt{2}$ B. ${}^3/_2$ C. $\dfrac{\sqrt{2}}{2}$ D. ${}^4/_3$ *JAMB 2002*[35]

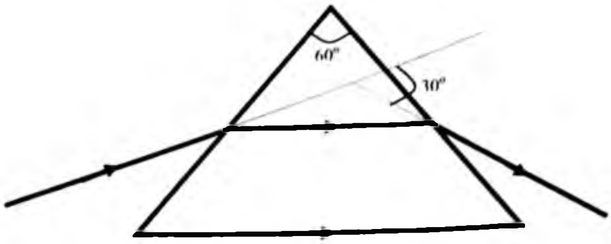


Fig. 14.10

Solution

Refracting angle of prism A = 60°; Angle of deviation D = 30°

Substitute into $\cap = \dfrac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2} A}$

$$\cap = \frac{\sin \frac{1}{2}(60 + 30)}{\sin \frac{1}{2}(60)} = \frac{\sin \frac{1}{2}(90)}{\sin 30} = \frac{\sin 45^o}{\sin 30^o}$$

From Trigonometry, $\sin 45 = \frac{1}{\sqrt{2}}$; $\sin 30^o = \frac{1}{2}$

$$\therefore \quad \frac{\sin 45}{\sin 30} = \frac{1}{\sqrt{2}} \div \frac{1}{2} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \frac{2}{\sqrt{2}}$$

multiply both numerator and denominator by $\sqrt{2}$; $\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Example 32

The figure below shows a plot of angles of deviation through a glass prism when light is incident at θ degrees on the prism. The incident angle that produces the minimum deviation is A. 25° B. 18° C. 35° D. 20° *JAMB 2005*[39]

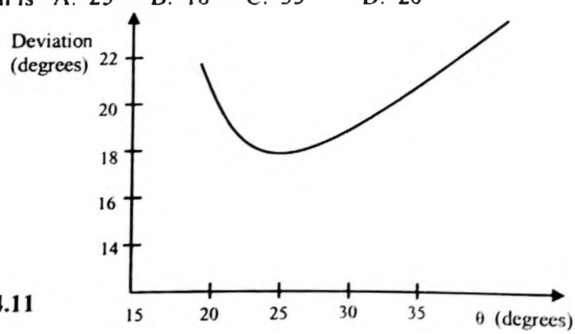


Fig. 14.11

Solution

As seen from the graph, fig 14.12, at minimum deviation of 18°, the angle of incidence is 25°

Ans: A

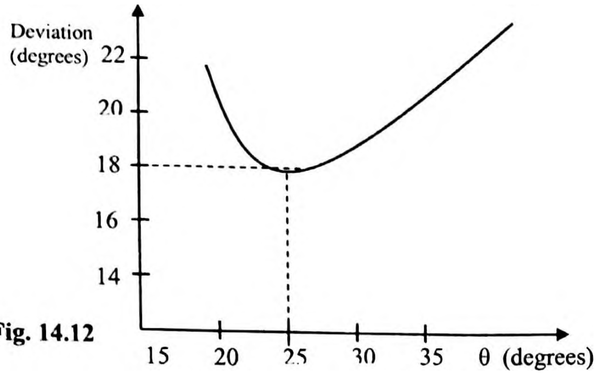


Fig. 14.12

REFRACTION THROUGH CONVEX AND CONCAVE LENSES

In contrast with convex mirror, *a convex lens converges rays* of light. Similarly, *a concave lens diverges rays* of light as opposed to concave mirror that converges rays of light.

The following formulas, expressions and rules are used in solving convex and concave lenses problems.

1. Relationship between the object distance u, image distance, v and focal length, f is given by the lens equation.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad f=\frac{uv}{u+v}$$

2. Magnification, $M=\dfrac{\text{image height}}{\text{object height}}=\dfrac{\text{image distance from lens}}{\text{object distance from lens}}$

That is, $M=\dfrac{v}{u}$

3. (i) A convex (converging) lens has a positive focal length.
   (ii) A concave (diverging) lens has a negative focal length.
   (iii) Distances of real object and real images are positive.
   (iv) Distance of virtual object and virtual images are negative.

4. The power of a lens, $P=\dfrac{1}{\text{focal length in meters}}$ That is, $P=\dfrac{1}{f}$

   The unit of lens power is the dioptre (D)

Example 33

An object is placed 36cm from a converging lens of focal length 24cm. If a real image which is 4cm high is formed, calculate the height of the object. *WAEC 1988*[30]

Solution

Object distance, u = 36cm; focal length, f = 24cm; Image height = 4cm; object height = ?

Substitute into $f=\dfrac{uv}{u+v}$ to get image distance, v

$$24=\frac{36v}{36+v}$$

Cross multiplying, $24(36 + v) = 36v$

$$24 \times 36 + 24v = 36v$$

$$24 \times 36 = 36v - 24v = 12v$$

$$v = \frac{24 \times 36}{12} = 72\text{cm}$$

$$M = \frac{\text{image height}}{\text{object height}} = \frac{\text{image distance}}{\text{object distance}}$$

substituting; $\dfrac{4\text{cm}}{\text{object height}} = \dfrac{72}{36}$

$$\therefore \quad \text{object height} = \frac{4 \times 36}{72} = \frac{144}{72} = 2\text{cm}$$

Example 34

A lens of focal length 15cm forms an upright image four times the size of an object. Calculate the distance of the image from the lens. *WAEC 1993*[20]

Solution

$f = 15\text{cm}$; $M = {}^{v}/_{u} = 4$ From ${}^{v}/_{u} = 4$; $u = {}^{v}/_{4}$; image distance, $v = ?$

Substitute into $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{{}^{v}/_{4}} = \frac{1}{v} + \frac{4}{v}$$

$$\frac{1}{15} = \frac{1+4}{v} = \frac{5}{v}$$

$$\frac{1}{15} = \frac{5}{v} \quad \therefore \quad v = 15 \times 5 = 75cm$$

Example 35

An object placed 50cm away from the focus of an emerging lens of focal length 15cm, produces a focused image on a screen. Calculate the distance between the object and the screen. *WAEC 1990*[P2]

Solution

Focal length, $f = 15\text{cm}$; object distance, $u = 50 + 15 = 65\text{cm}$

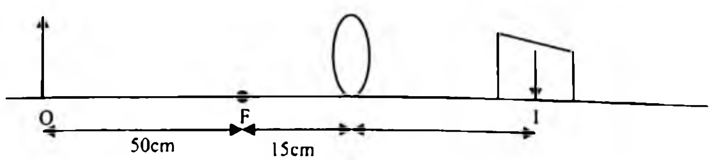


Fig. 14.13

Substitute into $f = \dfrac{uv}{u+v}$ to obtain image distance.

$$15 = \frac{65v}{65+v}$$

$$15(65 + v) = 65v$$

$$975 + 15v = 65v$$

$$975 = 65v - 15v$$

$$975 = 50v$$

$$v = \frac{975}{50} = 19.5\text{cm}$$

Object distance from screen $= u + v = 65 + 19.5 = 84.5\text{cm}$

Solution

Refractive index $\cap = {}^3/_2$; Prism's refracting angle $A = 60^\circ$;

Angle of minimum deviation $D_m = ?$

Substitute int *o* $\cap = \dfrac{\sin \frac{1}{2}(D_m + 60^\circ)}{\frac{A}{2}}$ *to obtain*

$$\frac{3}{2} = \frac{\sin \frac{1}{2}(D_m + 60^\circ)}{\sin {}^{60}/_2}$$

$$1.5 = \frac{\sin \frac{1}{2}(D_m + 60^\circ)}{\sin 30} = \frac{\sin \frac{1}{2}(D_m + 60^\circ)}{0.5}$$

cross multiplying $1.5 \times 0.5 = \sin \frac{1}{2}(D_m + 60^\circ)$

$$0.75 = \sin \tfrac{1}{2}(D_m + 60^\circ)$$

$$\frac{0.75}{\sin} = \tfrac{1}{2}(D_m + 60^\circ)$$

$$\sin^{-1} 0.75 = \frac{D_m}{2} + \frac{60^\circ}{2} = \frac{D_m}{2} + 30^\circ$$

$$48.59^\circ = \frac{D_m}{2} + 30^\circ$$

$$\frac{D_m}{2} = 48.59 - 30 = 18.59^\circ$$

$$D_m = 2 \times 18.59^\circ = 37.2^\circ$$

An alternative method for 'Example 30' is as follows:

$A = 60^\circ$; $\cap = {}^3/_2 = 1.5$; At minimum deviation $r = {}^A/_2 = {}^{60}/_2 = 30^\circ$

Substitute into $\cap = \dfrac{\sin i}{\sin r}$ to obtain

$$1.5 = \frac{\sin i}{\sin 30}$$

$$\sin i = 1.5 \times \sin 30^\circ = 1.5 \times 0.5 = 0.75$$

$$i = \sin^{-1} 0.75 = 48.75^\circ \simeq 48.6^\circ$$

Angle of minimum deviation $D_m = 2i - 2r$

$$= 2(48.6) - 2(30)$$

$$= 37.2^0$$

or $D_m = 2i - A$

$$= 2(48.6) - 60$$

$$= 37.2^0$$

Example 31

Calculate the refractive index of the material for the glass prism in the diagram below.

A. $\sqrt{2}$ B. ${}^3/_2$ C. $\dfrac{\sqrt{2}}{2}$ D. ${}^4/_3$ *JAMB 2002*[35]

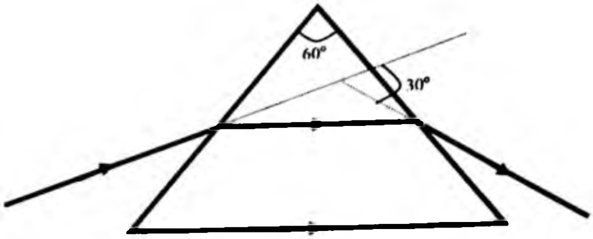


Fig. 14.10

Solution

Refracting angle of prism A = 60°; Angle of deviation D = 30°

Substitute into $\cap = \dfrac{\sin \frac{1}{2}(A+D)}{\sin \frac{1}{2}A}$

$$\cap = \frac{\sin \frac{1}{2}(60+30)}{\sin \frac{1}{2}(60)} = \frac{\sin \frac{1}{2}(90)}{\sin 30} = \frac{\sin 45^o}{\sin 30^o}$$

From Trigonometry, $\sin 45 = 1/\sqrt{2}$; $\sin 30^o = \frac{1}{2}$

$$\therefore \quad \frac{\sin 45}{\sin 30} = \frac{1}{\sqrt{2}} \div \frac{1}{2} = \frac{1}{\sqrt{2}} \times \frac{2}{1} = \frac{2}{\sqrt{2}}$$

multiply both numerator and denominator by $\sqrt{2}$; $\dfrac{2}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{\sqrt{4}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

Example 32

The figure below shows a plot of angles of deviation through a glass prism when light is incident at θ degrees on the prism. The incident angle that produces the minimum deviation is A. 25° B. 18° C. 35° D. 20° *JAMB 2005*[39]

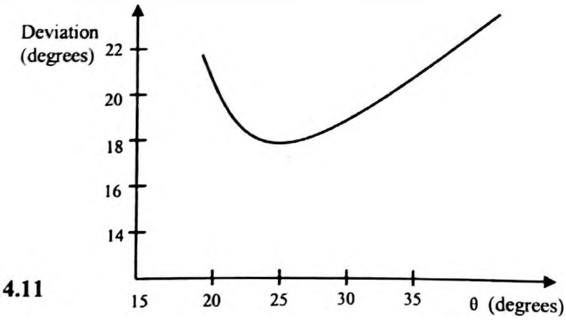


Fig. 14.11

Solution

As seen from the graph, fig 14.12, at minimum deviation of 18°, the angle of incidence is 25°

Ans: A

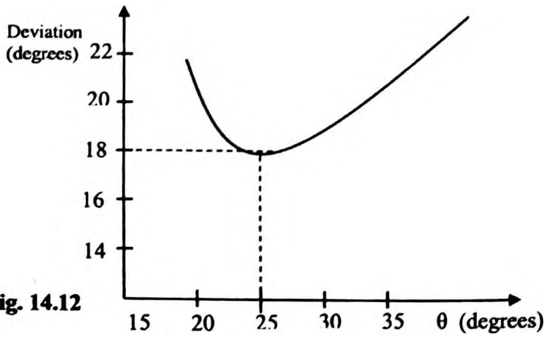


Fig. 14.12

REFRACTION THROUGH CONVEX AND CONCAVE LENSES

In contrast with convex mirror, *a convex lens converges rays* of light. Similarly, *a concave lens diverges rays* of light as opposed to concave mirror that converges rays of light.

The following formulas, expressions and rules are used in solving convex and concave lenses problems.

1. Relationship between the object distance u, image distance, v and focal length, f is given by the lens equation.

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad \text{or} \quad f=\frac{uv}{u+v}$$

2. Magnification, $M = \frac{\text{image height}}{\text{object height}} = \frac{\text{image distance from lens}}{\text{object distance from lens}}$

That is, $M = \frac{v}{u}$

3. (i) A convex (converging) lens has a positive focal length.
   (ii) A concave (diverging) lens has a negative focal length.
   (iii) Distances of real object and real images are positive.
   (iv) Distance of virtual object and virtual images are negative.

4. The power of a lens, $P = \frac{I}{\text{focal length in meters}}$ That is, $P = \frac{I}{f}$

The unit of lens power is the dioptre (D)

Example 33

An object is placed 36cm from a converging lens of focal length 24cm. If a real image which is 4cm high is formed, calculate the height of the object. *WAEC 1988*[30]

Solution

Object distance, u = 36cm; focal length, f = 24cm; Image height = 4cm; object height = ?

Substitute into $f = \frac{uv}{u+v}$ to get image distance, v

$$24 = \frac{36v}{36+v}$$

Cross multiplying. $24(36 + v) = 36v$

$$24 \times 36 + 24v = 36v$$

$$24 \times 36 = 36v - 24v = 12v$$

$$v = \frac{24 \times 36}{12} = 72\text{cm}$$

$$M = \frac{\text{image height}}{\text{object height}} = \frac{\text{image distance}}{\text{object distance}}$$

substituting; $\dfrac{4\text{cm}}{\text{object height}} = \dfrac{72}{36}$

$$\therefore \quad \text{object height} = \frac{4 \times 36}{72} = \frac{144}{72} = 2\text{cm}$$

Example 34

A lens of focal length 15cm forms an upright image four times the size of an object. Calculate the distance of the image from the lens. *WAEC 1993*

Solution

$f = 15\text{cm}$; $M = {}^{v}/_{u} = 4$ From ${}^{v}/_{u} = 4$; $u = {}^{v}/_{4}$; image distance, $v = ?$

Substitute into $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{v/4} = \frac{1}{v} + \frac{4}{v}$$

$$\frac{1}{15} = \frac{1+4}{v} = \frac{5}{v}$$

$$\frac{1}{15} = \frac{5}{v} \qquad \therefore \quad v = 15 \times 5 = 75cm$$

Example 35

An object placed 50cm away from the focus of an emerging lens of focal length 15cm. produces a focused image on a screen. Calculate the distance between the object and the screen. *WAEC 1990*

Solution

Focal length, $f = 15\text{cm}$; object distance, $u = 50 + 15 = 65\text{cm}$

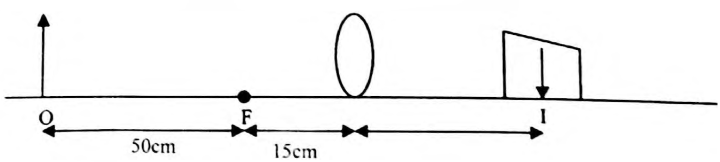


Fig. 14.13

Substitute into $f = \dfrac{uv}{u+v}$ to obtain image distance.

$$15 = \frac{65v}{65 + v}$$

$$15(65 + v) = 65v$$

$$975 + 15v = 65v$$

$$975 = 65v - 15v$$

$$975 = 50v$$

$$v = \frac{975}{50} = 19.5\text{cm}$$

Object distance from screen $= u + v = 65 + 19.5 = 84.5\text{cm}$

Example 36

A converging lens of focal length 5cm is used as a magnifying glass by a man whose near point is 35cm. Calculate the magnification given by the lens. *WAEC 1998*[29]

Solution

f = 5cm; the near point is also the image distance v = 35cm; object distance, u = ?

Substitute into $f = \dfrac{uv}{u+v}$

$$5 = \frac{35u}{35+u}$$

$$5(35+u) = 35u$$

$$175 + 5u = 35u$$

$$175 = 35u - 5u$$

$$175 = 30u$$

$$u = \frac{175}{30} = 5.83$$

Magnification, $M = \dfrac{v}{u} = \dfrac{35}{5.83} = 6$

Example 37

A lantern gives an image of 3m square of a slide 7.62cm square on a screen. If the screen is 10m from the projection lens of the lantern, calculate the focal length of lens.

WAEC 1998[32]

Solution

Image size or height, v = 3m = 300cm; object size or height, u = 7.62cm

Magnification, $M = {}^{v}/_{u}$

$$\therefore \quad M = \frac{300}{7.62} = 39.37$$

Image distance, v = 10m = 1000cm

From ${}^{v}/_{u} = 39.37$, object distance, $u = \dfrac{v}{39.37} = \dfrac{1000}{39.37} = 25.40$

Focal length, $f = \dfrac{uv}{u+v} = \dfrac{25.40 \times 1000}{25.40 + 1000} = \dfrac{25400}{1025.40} = 24.8\text{cm}$

Example 38

An object 2.5mm long is viewed through a converging lens of focal length 10.0cm held close to the eyes. A magnified image of the object is formed 30.0cm from the lens.

Calculate the (i) distance of the object from the lens

(ii) size of the image

(iii) power of the lens *WAEC 2002*[E13]

Solution

f = 10cm; image distance, v = -30cm (the final image seen by the eye is virtual); object height = 2.5mm = 0.25cm; object distance from lens u = ?

(i)' Substitute into $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$

$$\frac{1}{10} = \frac{1}{u} + \frac{1}{-30} \quad \therefore \quad \frac{1}{10} = \frac{1}{u} - \frac{1}{30}$$

$$\frac{1}{u} = \frac{1}{30} + \frac{1}{10} \quad \therefore \quad \frac{30}{u} = \frac{30}{30} + \frac{30}{10}$$

$$\frac{30}{u} = 1 + 3$$

$$\frac{30}{u} = 4 \qquad \therefore \qquad u = \frac{30}{4} = 7.5cm$$

(ii) $M = \frac{v}{u} = \frac{30}{7.5} = 4$

$$M = \frac{\text{image height}}{\text{object height}} \qquad \therefore \qquad 4 = \frac{\text{image height}}{0.25\text{cm}}$$

image height(size) = 4×0.25 = 1cm or 10mm

(iii) Lens power, D = $^1/_f = {}^1/_{0.1}$ = 10 dioptre
Note that the focal length, 10cm is converted to metres (0.1m)

Example 39
An object 40cm from a converging lens of focal length 20cm moves with a velocity of $5cms^{-1}$ towards the lens. Calculate the image position after 2s. *NECO 2004*[E6]
Solution
Object distance, u = 40cm; after 2s at 5cm/s the object moves 10cm, therefore the new object distance, u = 40 – 10 = 30cm; f = 20cm.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{20} = \frac{1}{30} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30}$$

$$\frac{1}{v} = \frac{3-2}{60}$$

$$\frac{1}{v} = \frac{1}{60} \qquad \therefore \qquad v = 60cm$$

Image distance, v = 60cm.

Example 40
By what factor will the size of an object placed 10cm from a convex lens be increased if the image is seen on a screen placed 25cm from the lens?
A. 15.0 B. 2.5 C. 1.5 D. 0.4 *JAMB 2003*[16]
Solution
Object distance, u = 10cm; image distance, v = 25cm
Magnification, M = $^v/_u = {}^{25}/_{10} = 2.5$

Example 41
If u is the object distance and v the image distance, which of the following expressions gives the linear magnification produced by a concave lens of focal length f?
A. $\frac{u}{v} + f$ B. $\frac{u}{f} - f$ C. $\frac{v}{f} - 1$ D. $\frac{v}{f} + 1$ *JAMB 2004*[7]
Solution
Magnification, M = $^v/_u$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

Multiply both sides by v

$$\frac{v}{f}=\frac{v}{u}+\frac{v}{v}$$

$$\frac{v}{u}=\frac{v}{f}-\frac{v}{v}$$

$$\frac{v}{u}=\frac{v}{f}-1$$

$\therefore$ Linear magnification ($M = {}^{v}/_{u}$) $= \frac{v}{f}-1$

Example 42

An object is placed in front of a converging lens of focal length 20cm. The image is virtual and has a magnification of 2. What is the distance of the object from the lens?

A. 5cm B. 10cm C. 30cm D. 40cm *JAMB 1994*[35]

Solution

Focal length, f = 20cm; image distance = – v (negative because it's virtual);

$M = -{}^{v}/_{u} = 2$ $\therefore$ $v = -2u$; object distance, u = ?

Substitute into $f = \frac{uv}{u-v}$

$$20=\frac{u\times(-2u)}{u+(-2u)}=\frac{-2u^2}{u-2u}=\frac{-2u^2}{-u}=2u$$

$20 = 2u$ $\therefore$ $u = {}^{20}/_{2} = 10cm$

Example 43

A concave lens of focal length 20cm form an image ½ the size of the object. The object distance is A. 100cm B. ${}^{100}/_{9}$cm C. 60cm D. ${}^{60}/_{7}$cm E.None of these values.

JAMB 1979[32]

Solution

$f = -20cm$; $M = {}^{v}/_{u} = \frac{1/2}{1}$ (…. Image ½ size of object …..)

$\therefore$ $V = ½u$; object distance, u = ?

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{-20}=\frac{1}{u}+\frac{1}{\frac{1}{2}u}=\frac{1}{u}+\frac{2}{u}=\frac{1}{u}(1+2)$$

$$\frac{-1}{20}=\frac{3}{u} \qquad \therefore \quad u = 20\times3 = 60cm$$

Example 44

The image of a pin formed by a diverging lens of focal length 10cm is 5cm from the lens. Calculate the distance of the pin from the lens. *WAEC 1992*[26]

Solution

Focal length, f = – 10cm (diverging lens); image distance, v = – 5cm;

Object distance, u = ?

Substitute into $f = \frac{uv}{u+v}$

$$-10 = \frac{-5u}{u-5}$$

Cross multiply $-10(u-5) = -5u$

$$-10u + 50 = -5u$$

$$50 = 10u - 5u$$

$$50 = 5u$$

$$u = \frac{50}{5} = 10cm$$

Example 45

The image formed by a converging lens of focal length f is at a distance of 2f from the lens. Calculate the magnification produced. *NECO 2000*[30]

Solution

Focal length, f = f; image distance, v = 2f; magnification, $M = {}^{v}/_{u} = ?$

Substitute into $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{2f}$$

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{2f}$$

$$\frac{1}{u} = \frac{2-1}{2f} = \frac{1}{2f}$$

$$\frac{1}{u} = \frac{1}{2f} \qquad \therefore \quad u = 2f$$

Magnification, $M = \frac{v}{u} = \frac{2f}{2f} = 1.5$

Example 46

A thin converging lens has a power of 4.0 diopters. Determine its focal length.

A. 0.25m B. 0.03m C. 5.00m D. 2.5m *JAMB 2005*[36]

Solution

Power = ${}^{1}/_{f}$ $\therefore$ $4 = {}^{1}/_{f}$; $f = ¼ = 0.25$

APPLICATION OF LIGHT WAVES

The Human Eye

The ***near point*** is the nearest distance (about 25cm) from the eye at which an object can be seen clearly by the eye.

The ***far point*** is the farthest point (infinity) at which an eye can see an object clearly. The least distance of distinct vision is the distance from the near point to the eye.

Long sightedness is corrected by convex lens. ***Short sightedness*** is corrected by concave lens. The lens formula is applied in problems involving the human eye as exemplified by the following:

Example 47

A far-sighted person cannot see objects that are less than 100cm away. If this person wants to read a book at 25cm, what type and focal length of lens does he need?

A. convex, 20cm B. concave, 20cm C. convex, 33cm D. concave, 33cm JAMB *1989*[38]

Solution

Object distance, u = 25cm; image distance, v = 100cm

Focal length, $f = \frac{uv}{u+v} = \frac{25\times100}{25+100} = \frac{2500}{125} = 20cm$ *Ans* A: convex, 20cm

Example 48

A patient with a sight defect has a least distance of distinct vision of 150cm. For him to be able to read a material placed at a distance of 25cm, what is the focal length of the glasses he should wear?

A. 15.0cm B. 17.6cm C. 21.4cm D. 30.0cm *JAMB 1997*[36]

Solution

Image distance, v = 150cm; object distance, u = 25cm; focal length, f = ?

Focal length, $f = \frac{uv}{u+v} = \frac{150\times25}{150+25} = \frac{3750}{175} = 21.43cm$

Example 49

A man wears convex lens glasses of focal length 30cm in order to correct his eye defect. Instead of the optimum 25cm, his least distance of distinct vision is

A. 14cm B. 28cm C. 75cm D. 150cm *JAMB 1998*[34]

Solution

Focal length, f = 30cm; image distance, v = ?; object distance, u = 25cm;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{30} = \frac{1}{25} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{25}$$

$$\frac{1}{v} = \frac{5-6}{150} = \frac{-1}{150}$$

$$\therefore \quad v = 150cm$$

Example 50

A long-sighted man cannot see clearly objects that are less than 120cm away. What is the type and focal length of lens required to make him read a book at 30cm away. *NECO 2007*[32]

Solution

Image distance, v = – 120cm (negative because image formed by the eye is virtual);
Object distance, u = 30cm.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{-120}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{120}$$

$$\frac{1}{f} = \frac{4-1}{120} = \frac{3}{120} = \frac{1}{40}$$

$\therefore$ $f = 40cm$. Because f is positive, the lens is a converging or convex lens.

MICROSCOPES AND TELESCOPES AND MAGNIFYING GLASS

In addition to the lens formular ($^1/_u + {}^1/_v = {}^1/_f$), the following are used in problems involving microscopes and telescopes.

1. Magnifying power (M) of compound microscope is $M = M_1 \times M_2$

   Where M_1 = magnification by the objective lens

   M_2 = magnification by the eye piece

2. Magnifying power of the telescope is

   $$M = \frac{f_o}{f_e}$$

   Where f_o = focal length of objective lens

   f_e = focal length of eye piece lens

3. Distance (d) between objective lens and eye piece of an astronomical telescope is

   $$d = f_o + f_e$$

4. Distance (d) between objective lens and eye piece of a Galilean telescope is

   $$d = f_o - f_e$$

Example 51

Converging lenses of focal lengths 120cm and 10cm are used to construct an astronomical telescope. What is the distance between the lenses at its normal adjustment?

NECO 2007[30]

Solution

Distance between lenses $d = f_o + f_e = 120 + 10 = 130cm$

Example 52

A simple magnifying glass is used to view an object. At what distance from the lens must the object be placed so that an image 5 times the size of the object is produced 20cm from the lens?

WAEC 1994[28]

Solution

Image distance, $v = -20cm$ (image formed by magnifying glass is virtual);

magnification, $M = \frac{v}{u} = 5$

Substitute $v = -20cm$ into $\frac{v}{u} = 5$

$$\frac{-20}{u} = 5 \qquad \therefore\ u = \frac{-20}{5} = 4cm$$

Example 53

In a compound microscope, the image formed by the objective lens is at a distance of 3.0cm from the eye lens. If the final image is at 25.0cm from the eye lens, calculate the focal length of the eye lens.

WAEC 2002[30]

Solution

Object distance, $u = 3.0cm$; image distance, $v = -25cm$ (image is virtual);
Focal length, $f = ?$

$$f = \frac{uv}{u+v} = \frac{3\times25}{3+(-25)} = \frac{75}{3-25} = \frac{75}{22} = 3.41\text{cm}$$

EXERCISE 14.

1. A ray of light is incident on a body X as shown in the diagram below. What is the refractive index of the body?

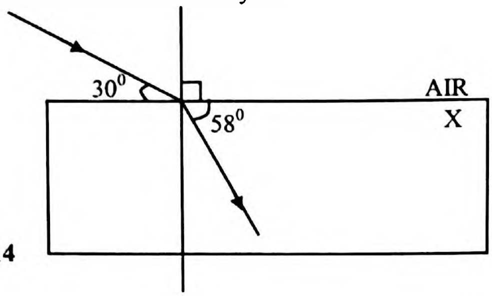


Fig. 14.14

WAEC 1995[22] *Ans: 1.63*

2. When a ray of light is incident normally on an air-glass interface, what is its angle of refraction? *WAEC 1996*[29] *Ans: 0°*

3.

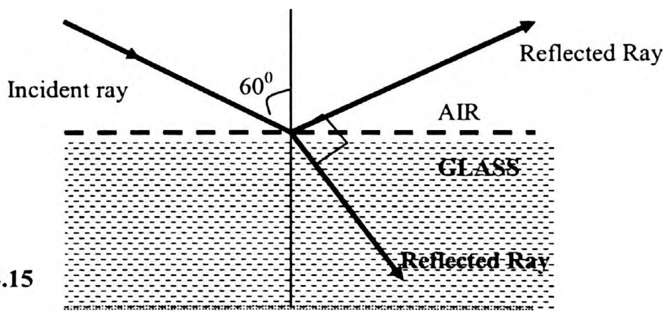


Fig. 14.15

In the figure above, a ray of light in our strikes a glass plate at an angle of incidence of 60°. The reflected ray is observed to be perpendicular to the refracted ray. What is the refractive index of the glass?

A. 1.73 B. 150 C. 0.87 D. 0.57 *JAMB 1986*[27] *Ans: 1.73*

4.

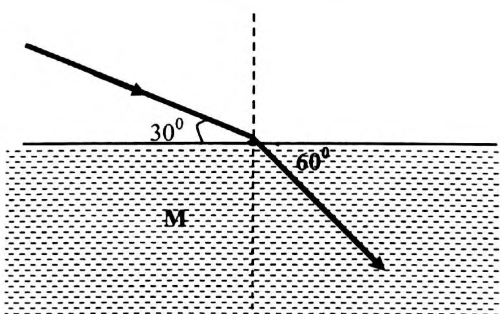


Fig. 14.16

The refractive index of the medium M in the diagram above is

A. $\frac{1}{\sqrt{3}}$ B. $2\sqrt{3}$ C. $\sqrt{3}$ D. $\frac{2}{\sqrt{3}}$ *JAMB 2004* *Ans:* $\sqrt{3}$

5. The speed of light in air is $3 \times 10^8 ms^{-1}$. If the refractive index of light from air – to water is $^4/_3$, then which of the following is the correct value of the speed of light in water? A. $4 \times 10^8 ms^{-1}$ B. $2.25 \times 10^8 ms^{-1}$ C. $^4/_9 \times 10^8 ms^{-1}$ D. $2.25 \times 10^8 ms^{-2}$ E. $4.33 \times 10^8 ms^{-1}$. *JAMB 1979*[42] *Ans: 2.25* $\times 10^8 ms^{-1}$

6. The speed of light in air is $3.00 \times 10^8 ms^{-1}$. Its speed in glass having a refractive index of 1.65 is A. $1.65 \times 10^8 ms^{-1}$ B. $3.00 \times 10^8 ms^{-1}$ C. $4.95 \times 10^8 ms^{-1}$ D. $1.82 \times 10^8 ms^{-2}$ E. $6.00 \times 10^8 ms^{-1}$. *JAMB 1982*[8] *Ans: D*

7. The refractive index of a liquid is 1.5. If the velocity of light in vacuum is $3.0 \times 10^8 ms^{-1}$, the velocity of light in the liquid is A. $1.5 \times 10^8 ms^{-1}$ B. $2.0 \times 10^8 ms^{-1}$ C. $3.0 \times 10^8 ms^{-1}$ D. $4.5 \times 10^8 ms^{-1}$ E. $9.0 \times 10^8 ms^{-1}$. *JAMB 1983*[7] *Ans: 2.0* $\times 10^8 ms^{-1}$

8. The speed of light in air is $3.0 \times 10^8 ms^{-1}$. What is the speed in glass with a refractive index of 1.50 A. $1.5 \times 10^8 ms^{-1}$ B. $3.0 \times 10^8 ms^{-1}$ C. $2.0 \times 10^8 ms^{-1}$ D. $6.0 \times 10^8 ms^{-1}$ *JAMB 1992*[28] *Ans: 2.0* $\times 10^8 ms^{-1}$

9. The velocities of light in air and glass are $3.0 \times 10^8 ms^{-1}$ and $2.0 \times 10^8 ms^{-1}$ respectively. If the angle of refraction is 30°, the sine of the angle of incidence is
A. 0.33 B. 0.50 C. 0.67 D. 0.75 *JAMB 1999*[34] *Ans: 0.33*

10. Radio waves travel in air at $3.0 \times 10^8 ms^{-1}$. If the waves enter water of refractive index $^4/_3$, calculate the speed of radio waves in water. *WAEC 2005*[28] *Ans: 2.25* $\times 10^8 ms^{-1}$

11. Light of wavelength 5000×10^{-8}cm travels in free space with a velocity of $3.0 \times 10^8 ms^{-1}$. What is its wavelength in glass of refractive index 1.5?
A. 3333×10^{-8}cm B. 5000×10^{-8}cm C. 6666×10^{-8}cm D. 7500×10^{-8}cm
JAMB 1989[24] *Ans:* 3333×10^{-8}cm

12. A light wave of frequency 5×10^{14}Hz moves through water which has a refractive index of $^4/_3$. Calculate the wavelength in water if the velocity of light in air is $3.0 \times 10^8 ms^{-1}$. A. 4.5×10^{-7}m B. 6.0×10^{-7}m C. 1.7×10^6m D. 2.2×10^6m
JAMB 1990[21] *Ans: 4.5* $\times 10^{-7}m$

13. Light of frequency 6.0×10^{14}Hz travelling in air is transmitted through glass of refractive index 1.5. Calculate the frequency of the light in the glass.
A. 4.0×10^{14}Hz B. 6.0×10^{14}Hz C. 7.5×10^{14}Hz D. 9.0×10^{14}Hz
JAMB 1994[34] *Ans:6.0* $\times 10^{14}Hz$

14. A point object placed in contact with one surface of a glass block of thickness 1.6cm and of refractive index 1.61 is viewed along the normal to the opposite surface. By how much does the point object appears to be displaced. *WAEC 1989*[P3] *Ans: 0.60cm*

15. A microscope is focused on a mark on a table. When the mark is covered by a plate of glass 3.00cm thick, the microscope has to be raised 1.18cm for the mark to be once more in focus. Calculate the refractive index of the glass. *WAEC 1995*[P2] *Ans: 1.64*

16. A rectangular glass prism of thickness 6cm and refractive index 1.5 is placed on the page of a book. The prints on the book are viewed vertically downwards from above. Determine the apparent upward displacement of the prints. *WAEC 2001*[P2] *Ans: 2cm*

17. A rectangular glass prism of thickness d and absolute refractive index $\cap$ is placed on a point object, which is viewed vertically downward from above the prism. Which of the following expressions correctly defines the apparent upward displacement of the object?
A. $^d/_\cap$ B. $d\cap$ C. $^d/_{\cap-2}$ D. $^{d(\cap-1)}/_\cap$ *WAEC 2007*[34] *Ans: D*

18. The displacement d produced in a glass block of thickness t and refractive index $\cap$ when an object is viewed through it is
A. $t - \cap$ B. $t(1 + \frac{1}{\cap})$ C. $t(1 - \frac{1}{\cap})$ D. $t(\frac{1}{\cap} - 1)$ *JAMB 1998*[32] *Ans: C*

19. A coin lies at the bottom of a tank containing water to a depth of 130cm. If the refractive index of water is 1.3, calculate the apparent displacement of the coin when viewed vertically from above. *WAEC 1992*[P2] *Ans: 30cm*

20. The horizontal floor of a water reservoir appears to be 1.0m deep when viewed vertically from above. If the refractive index of water is 1.35, calculate the real depth of the reservoir. *WAEC 2003*[24] *Ans: 1.35m*

21. A coin is placed at the bottom of a cube of glass t cm thick. If the refractive index of the glass is μ, how high does the coin appear to be raised to an observer looking perpendicularly into glass?

A. $\frac{1}{t-\mu}$ B. $\frac{t(\mu-1)}{\mu}$ C. $t\left(1+\frac{1}{\mu}\right)$ D. $\frac{t}{\mu}$ E. μt *JAMB 1984*[32] *Ans: B*

21. Two rays of light from a point below the surface of water are equally inclined to the vertical and are inclined to each other at 60° in water. What is the angle between the rays when they emerge into air? [Take the refractive index of water to be $^4/_3$]
JAMB 1985[33] *Ans: 83.6°*

22. A coin placed below a rectangular glass block of thickness 9cm and refractive index 1.5 is viewed vertically above the block. The apparent displacement of the coin is
A. 8cm B. 6cm C. 5cm D. 3cm *JAMB 2002*[14] *Ans: 3cm*

23. A trough 12.0cm deep is filled with water of refractive index 4/3. By how much would a coin at the bottom of the trough appear to be displaced when viewed vertically from above the water surface?
A. 3.0cm B. 6.0cm C. 9.0cm D. 16.0m *JAMB 1991*[27] *Ans:3.0cm*

24. The refractive index of a medium relative to air is 1.8. Calculate the critical angle for the medium to the nearest degree. *WAEC 1996*[11] *Ans: 34°*

25. Calculate the critical angle of a medium of refractive index 1.65 when light passes from the medium to air. *WAEC 2005*[P2] *Ans: 37.3°*

26. Calculate the critical angle for light travelling from water to air. [Refractive index of water = 1.33] *NECO 2003*[28] *Ans: 48.75°*

27. If the refractive index of water is 1.33, what is the critical angle for a water – air boundary? *NECO 2002*[30] *Ans: 48.8°*

28. If the refractive index of a medium is $\sqrt{2}$, what is the critical angle?
A. 45° B. 50° 12' C. 56° 25' D. 75° E. 90° *JAMB 1982*[24] *Ans: 45°*

29. If the refractive index of a medium in air is 2.0, what is the critical angle for this medium? A. 30° B. 42° C. 45° D. 50° *JAMB 1992*[29] *Ans:30°*

30. What is the approximate critical angle for total internal reflection for diamond if the refractive index of diamond is 2.42?
A. 21° B. 22° C. 23° D. 24° *JAMB 1995*[33] *Ans:24°*

31. A ray of light is incident at an angle of 30° on a glass prism of refractive index 1.5. Calculate the angle through which the ray is minimally deviated in the prism. (The medium surrounding the prism is air). *WAEC 1991*[26] *Ans: 21.1°*

32. Calculate the angle of minimum deviation for a ray which is refracted through an equiangular prism of refractive index 1.4.
A. 29° B. 60° C. 99° D. 90° *JAMB 2004*[5] *Ans:29°*

33. If the critical angle of a glass-air boundary is C and the refractive index of the glass is ∩, which of the following relationship is correct?

A. $\cap = \frac{90}{\sin C}$ B. $\cap = \frac{\sin C}{90}$ C. $\sin 90 \sin C = \cap$ D. $\sin C = \frac{1}{\cap}$ E. $\cap = \frac{\sin C}{\sin 45}$
WAEC 1994[27] *Ans: D*

34. A ray of light experience a minimum deviation when passing through an equilateral triangular glass prism. Calculate the angle of incidence of the ray. [Refractive index of glass = 1.5] *NECO 2004*[29] *Ans: 48.6°*

35. In an experiment to measure the focal length of a converging lens, object distance, u and corresponding image distance, v were measured and $^1/_u$ plotted against $^1/_v$ to obtain the type of graph illustrated below. How would f be found from this graph?

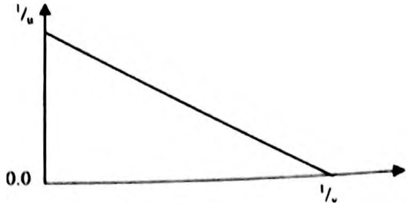


Fig. 14.17

A. f = the slope of the graph
B. f = the intercept on the $^1/_u$ – axis
C. f = the intercept on the $^1/_v$ – axis
D. • f = reciprocal of the slope
E. f = reciprocal of the intercept in either axis. *WAEC 1988*[29] *Ans: E*

36. At which of the following distances from the lens should a slide be placed in a slide projector, if f is the focal length of the projection lens?
A. Less than f B. Greater than 2f C. Greater than f but less than 2f
D. Equal to f E. Equal to 2f *WAEC 1991*[28] *Ans: C*

37. What is the image distance of an object placed at a distance of 2f from a converging lens of focal length f? *WAEC 1998*[28] *Ans: 2f*

38. A converging lens has a focal length of 5cm. Determine its power. *WAEC 2003*[25] *Ans: +20 D*

39. An object is placed a distance 10cm in front of a concave mirror of focal length 15cm. Determine the characteristic of the image formed. *WAEC 2004 Ans: Virtual, v = 30cm*

40. A converging lens of focal length 5cm forms a virtual image which is 10cm from the lens. How far from the lens is the object? *WAEC 1996*[32] *Ans: 3.3cm*

41. In an experiment to measure the focal length of a converging lens, object distance, u and corresponding image distance, v were measured and $^1/_u$ plotted against $^1/_v$ to obtain the type of graph illustrated below.

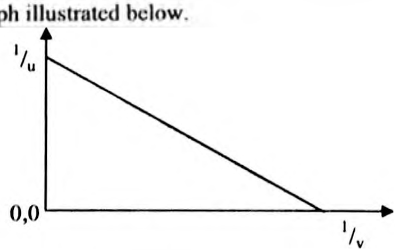


Fig. 14.18

How would f be found from this graph?
A. f = the slope of the graph
B. f = the intercept on the $^1/_u$ – axis
C. f = the intercept on the $^1/_v$ – axis
D. f = reciprocal of the slope
E. f = reciprocal of the intercept in either axis. *WAEC 1992*[25] *Ans: E*

42. A real image of an object formed by a converging lens of focal length 15cm is three times the size of the object. What is the distance of the object from the lens? *WAEC 1992*[28] *Ans: 20cm*

43. A converging lens of focal length 15cm is used to obtain a real image magnified 1½ times. Calculate the distance of the image from the lens. *WAEC 1997*[22] *Ans: 37.5cm*

44. The real image of an object, formed by a converging lens of focal length 15cm, is three times the size of the object. Calculate the object distance. *WAEC 1999*[23] *Ans: 20cm*

45. An object is placed 20cm from a lens. If an image is formed on a screen 260cm away from the lens, calculate the magnification of the image. *WAEC 2001*[25] *Ans: 13*

46. A converging lens of focal length 15cm forms a virtual image at a point 10cm from the lens. Calculate the distance of the object from the lens. *WAEC 2002*[29] *Ans: 6cm*

47. A converging lens produces an image four times as large as an object placed 25cm from the lens. Calculate its focal length. *WAEC 2003[23] Ans:20cm*

48. A converging lens forms a real image of a real object. If the magnification is 2 and the distance between the image and the object is 90.0cm, determine the

(i) focal length of the lens;

(ii) object distance for which the image would be the same size as the object.

WAEC 2006[E13] Ans: (i) 20cm (ii) 40cm

49. An object placed on the principal axis of a convex lens of focal length 10cm, produces a real image of double magnification. The image distance from the lens is

A. 30cm B. 25cm C. 20cm D. 15cm E. 10cm *JAMB 1981[46] Ans: 30cm*

50. A lens of focal length 12.0cm forms an upright image three times the size of a real object. The distance between the object and the image is A. 8.0cm B. 16.0cm C. 24.0cm D. 32.0cm *JAMB 1986[28] Ans:64cm* [incorrect options]

51. An object 3.0cm high is placed 60.0cm from a converging lens whose focal length is 20.0cm. Calculate the size of the image formed.

A. 0.5cm B. 1.5cm C. 2.0cm D. 6.0cm *JAMB 1987[34] (Ans:1.5cm)*

52. To obtain a magnification of 2.5, how far should an object be placed from the pole of thin converging lens of focal length 0.20m?

A. 0.13m B. 0.25m C. 0.28m D. 0.50m *JAMB 1988[30] (Ans: 0.28m)*

53. What must be the distance between an object and a converging lens of foc: .ngth 20cm to produce an erect image two times the object height. *JAMB 1990[27] Ans. cm*

54. If the focal length of a camera lens is 20cm, the distance from the film at which the lens must be set to produce a sharp image of an object 100cm away is

A. 17cm B. 20cm C. 25cm D. 100cm *JAMB 1993[27] Ans: 25cm*

55. A projection lantern is used to give the image of a slide on a screen. If the image is 24 times as large as the slide and the screen is 72.0m from the projecting le... what is the position of the slide from the lens?

A. 4.0m B. 3.5m C. 3.0m D. 0.3m *JAMB 2000[34] Ans:3.0m*

56. A certain long-sighted person cannot see clearly objects placed 75cm from his eye. This defect can be corrected by the use of a

A. converging lens of focal length 37.5cm
B. converging lens of focal length 75cm
C. cylindrical lens of focal length 37.5cm
D. diverging lens of focal length 37.5cm
E. diverging lens of focal length 75cm *NECO 2004[33] Ans: B*

57. A person can focus objects when they lie beyond 75cm from his eyes. The focal length of the lens required to reduce his least distance of distinct vision to 25cm is

A. 75.00cm B. 18.75cm C. 25.00cm D. 37.50cm *JAMB 2006[3] Ans:18.75cm*

58. A simple microscope forms an image twice the size of the object. If the focal length of the lens of the microscope is 20cm, how far is the object from the lens?

WAEC 1990[34] Ans: 10m

59. When an astronomical telescope is in normal adjustment, the focal length of the objective lens is 50cm and that of the eye piece is 2.5cm. What's the distance between the lenses? *WAEC 1992[30] Ans: 52.5cm*

60. At what distance from a simple microscope must an object be placed so that an image 5 times the size of the object is produced 20cm from the lens? *WAEC 1997[23] Ans: 4.0cm*

61. An astronomical telescope, having an objective of focal length 100cm and eye piece of focal length 10cm, is used in normal adjustment. Calculate the separation of the lenses. *WAEC 1998[31] Ans: 1.10m*

62. A simple microscope forms an image 10cm from an eye close to the lens. If the object is 6cm from the eye, calculate the focal length of the lens. *WAEC 2000[30] Ans:15cm*

63. Four lenses are being considered for use as a microscope objective. Which of the following focal lengths is most suitable?

A. –5mm B. +5mm C. –5cm D. +5cm *JAMB 1987[37] Ans: B*

64. A converging lens and a screen are placed 20cm and 80cm respectively from an object in a straight line so tat a sharp image of the object is formed on the screen. If the object is 3cm high, calculate the height of the image formed. WAEC 2008 Ans: 9cm

65. If a convex lens of focal length 12cm is usedd to produce a real image four times the size of the object, how far from the lens must the object be placed? *JAMB 2008 Ans: 15cm*
A. 10cm B. 15cm C. 20cm D. 25cm

66. An observer with normal eyes view an object with a magnifying glass of focal length 5cm. The angular magnification is _A. -6 B. -5 C. 5 D. 6
[least distance of distinct vision D = 25cm) JAMB 2008 Ans:

66. Calculate the critical angle in glass for the light travelling from glass to water.(Refractive index of water = 1.33, refractive index of glass = 1.50)
NECO 2008[29] Ans: 41.8°

67. If the refractive index of crown glass is 1.51, its critical angle is
A. 48.6° B. 22.5° C. 41.5° D. 45.0° *JAMB 2009[34] Ans: C*

68. When light passes through two media x and y of refractive indices 1.51 and 1.33 respectively, the speed of light in
A. x is same as in y B. x and y is same as in vacuum
C. x is higher than in y D. y is higher than in x *JAMB 2009[35] Ans: D*

69. A near-sighted student has a near point of 0.1m and a focal length of 5.0cm. what is the student's far point?
A. 0.200m B. 8.000m C. 0.125m D. 2.100m *JAMB 2009[36] Ans: A*

70. A thin lens is placed 50cm from an illuminated object. The image produced has a linear magnification of ¼. Calculate the power of the lens in dioptres.
WAEC 2009[30] Ans: 10.0D

71. A ray of light is incident on one face of an equilateral glass prism.
(i) Draw a ray diagram to show the path of the ray through the prism.
(ii) Calculate the refractive index of the glass if the angle of minimum deviation is 41°
WAEC 2009[E13] Ans: 1.54

72. An object placed 15cm from a converging lens forms a real image of magnification 3. Calculate the focal length of the lens.
NECO 2009[29] Ans: 11.25cm

15

SOUND WAVES

VELOCITY AND REFLECTION OF SOUND WAVES

Sound wave is a longitudinal wave and also a form of energy produced by vibrating bodies.

As common with waves, the velocity or speed of sound wave in any media is given by:

$$V = \lambda f \qquad \text{or} \qquad V = \frac{\lambda}{T}$$

Where V = velocity of sound wave (ms^{-1})
λ = wavelength of sound wave (m)
f = frequency of sound wave (Hz)
T = period of sound wave (s)

Also, speed of sound (V) in air is directly proportional to the square root of the absolute temperature, θ.

That is, $V = \sqrt{\theta}$

A quick review of Chapter Twelve with emphasis on stationary wave will be helpful and relevant to this Chapter.

Example 1

A source of sound produces waves in air of wavelength 1.65. If the speed of sound in air is $330ms^{-1}$, the period of vibration in second is? *WAEC 1988*[33]

Solution

Wavelength λ = 1.65m; speed of sound V = $330ms^{-1}$; period T = ?

Substitite into $V = \frac{\lambda}{T}$ to obtain

$$330 = \frac{1.65}{T} \qquad \therefore \qquad \text{Period of vibration } T = \frac{1.65}{330} = 0.005s$$

Example 2

In a sound wave in air, the adjacent rarefactions and compressions are separated by a distance of 17cm. If the velocity of the sound wave is $340ms^{-1}$, determine the frequency

A. 10Hz B. 20Hz C. 100Hz D. 5780Hz *JAMB 1989*[22]

Solution

Distance between successive rarefaction and compression is equivalent to $\lambda/2$.

$\lambda/2 = 17$ or $\lambda = 2 \times 17$, = 34cm = 0.34m; Velocity, V = $340ms^{-1}$

$$\text{From } V = \lambda f, \quad \text{frequency, } f = \frac{V}{\lambda} = \frac{340}{0.34} = \frac{34000}{34} = 1000Hz$$

Example 3

A sound wave of velocity $350ms^{-1}$ is directed towards the surface of water. If the ratio of the wavelength of sound in water to that in air is 425:100, calculate the velocity of the wave in water. *WAEC 1996*[38]

Solution

Sound wave velocity in air $V_1 = 350ms^{-1}$ Sound wave velocity in water V_2 = ?
Wavelength of sound in air $\lambda_1 = 100$ Wavelength of sound in water $\lambda_2 = 425$

The frequency of a wave does not change when it travels from one medium to another.

From $V = \lambda f$, we derive $f = \frac{V}{\lambda}$

Therefore $\frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2}$ on substitution we obtain,

$$\frac{350}{100} = \frac{V_2}{425}$$

Sound wave velocity in water, $V_2 = \frac{350 \times 425}{100} = 1487.5ms^{-1}$

Example 4

The velocity of sound wave at 27°C is 360ms^{-1}. Its velocity at 127°C is *JAMB 1998*

A. $120\sqrt{3}$ ms^{-1} B. 240 ms^{-1} C. $240\sqrt{3}$ ms^{-1} D. $720\sqrt{3}$ ms

Solution

Temperature (θ) and speed of sound in air are related by $V \propto \sqrt{\theta}$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\sqrt{\theta_1}}{\sqrt{\theta_2}}$$

$\theta_1 = 27 + 273 = 300K$; $V_1 = 360ms^{-1}$

$\theta_2 = 127 + 273 = 400K$; $V_2 = ?$

[Always remember to convert °C to Kelvin]

Substituting $\frac{360}{V_2} = \frac{\sqrt{300}}{\sqrt{400}}$

$$\frac{360}{V_2} = \frac{\sqrt{3}}{\sqrt{4}}$$

$$V_2 = \frac{360 \times \sqrt{4}}{\sqrt{3}} = \frac{2 \times 360}{\sqrt{3}} = \frac{720}{\sqrt{3}}$$

Multiply both numerator and denominator by $\sqrt{3}$

$$V_2 = \frac{720 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{720\sqrt{3}}{\sqrt{3 \times 3}} = \frac{720\sqrt{3}}{\sqrt{9}}$$

Velocity of sound wave at 127°, $V_2 = \frac{720\sqrt{3}}{3} = 240\sqrt{3}ms^{-1}$

Example 5

The speed of sound in air is 330ms^{-1}. How far from the centre of a storm is an observer who hears a thunder clap 2s after the lightning flash? [Neglect the time taken by light to travel to the observer]

Solution

Speed of sound V = 330ms^{-1}; time taken t = 2s; distance of storm's centre

From, speed $(V) = \frac{\text{distance}(x)}{\text{time}(t)}$

$$x = V \times t = 330 \times 2 = 660m$$

Echoes

Like any other wave, sound waves can be reflected ... after a sound wave has been reflected from a hard plane surf...

The time (t) taken for an echo to be heard in any me...

$$t = \frac{2x}{V}$$

Where x = distance between the source of sound and the reflecting surface (wall, cliff, sea bed, hill e.t.c.)

V = velocity of sound in the medium (air, water, glass, metal e.t.c.)

Example 6

A bat emits a sound wave at a speed of $1650.0ms^{-1}$ and receives the echoes 0.15s later. Calculate the distance of the bat from the reflector. *WAEC 1990*[33]

Solution

Speed of sound wave V = $1650ms^{-1}$; echo time t = 0.15s; Distance from reflector, x = ?

Substitute into $x = \frac{Vt}{2}$ to have

$$x = \frac{1650 \times 0.15}{2} = 123.75m$$

Example 7

A fathometer is used to send a wave down to the sea bed. The reflected wave is received after 0.5 seconds. Calculate the depth of the sea. [Speed of sound in water = $1500ms^{-1}$] *NECO 2007*[36]

Solution

Echo time t = 0.5; speed of sound V = $1500ms^{-1}$

$$\text{Depth of sea, } x = \frac{V \times t}{2} = \frac{1500 \times 0.5}{2} = 375m$$

Example 8

A man standing 510m away from a wall sounds a whistle. The echo from the wall reaches him 3s later. Calculate the velocity of sound in air. *NECO 2000*[35]

Solution

Distance from reflector (wall) x = 510m; echo time t = 3s; Sound velocity V = ?

$$t = \frac{2x}{V} \quad \therefore \quad V = \frac{2x}{t}$$

Substituting, $V = \frac{2 \times 510}{3} = 340ms^{-1}$

Example 9

How far from a cliff should a boy stand in order to hear the echo of his clap 0.9s later? [Speed of sound in air = $330ms^{-1}$]. *WAEC 1992*[36]

Solution

Speed of sound V = $330ms^{-1}$; echo time t = 0.9s; Distance from reflector (cliff) x = ?

Substitute into $x = \frac{Vt}{2} = \frac{330 \times 0.9}{2} = 148.50m$

Example 10

A sound pulse sent vertically downwards into the earth is reflected from two different layers of the earth such that echoes are heard after 1.2s and 1.4s. Assuming the speed of the pulse is $2000ms^{-1}$, calculate the distance between layers. *WAEC 1989*[32]

Solution

distance of first layer = x_1; first layer echo time t_1 = 1.2s;

distance of second layer = x_2; second layer echo time t_2 = 1.4s.

Velocity of sound pulse V = $2000ms^{-1}$;

From $t = \frac{2x}{V}$, distance from reflector (layer) is, $x = \frac{Vt}{2}$, Therefore, distance between between layers $= x_2 - x_1 = \frac{Vt_2}{2} - \frac{Vt_1}{2} = \frac{V(t_2 - t_1)}{2}$

$$x_2 - x_1 = \frac{V(t_2 - t_1)}{2}$$

$$= \frac{2000(1.4 - 1.2)}{2} = 1000 \times 0.2$$

$$= 200\text{m}$$

Example 11

A man hears his echo from a nearby hill 2s after he shouted. If the frequency of his voice is 260Hz and the wavelength is 1.29m, how far is the hill? *JAMB 1991*[31]

A. 330.0m B. 335.4m C. 660.0m D. 670.8m

Solution

Echo time t = 2s; frequency f = 260Hz; wavelength λ = 1.29m

Velocity of sound $V = \lambda f = 1.29 \times 260 = 335.4\text{m/s}$

Distance from reflector (hill) $x = \frac{Vt}{2} = \frac{335.4 \times 2}{2} = 335.4\text{ms}^{-1}$

Example 12

As a ship approaches a cliff, its siren is sounded and the echo is heard in the ship after 12 seconds. 2.1 minutes later the siren is sounded again and the echo is heard 8 seconds later. If the speed of sound in air is 340ms^{-1}, calculate the velocity at which the ship is approaching the cliff. *WAEC 2001*[E13]

Solution .

First echo time t_1 = 12s; second echo time t_2 = 8s;

Sound speed V = 340ms^{-1}; time interval between echoes T = 2.1min = 2.1 × 60 = 126s

$$x = \frac{V \times t}{2}$$

Ship distance at 1st echo $x_1 = \frac{V \times t_1}{2} = \frac{340 \times 12}{2} = 2040\text{m}$

Ship distance at 2nd echo $x_2 = \frac{V \times t_2}{2} = \frac{340 \times 8}{2} = 1360\text{m}$

Distance between 1st and 2nd echo $X = x_1 - x_2 = 2040 - 1360 = 680\text{m}$

Velocity of approach $= \frac{\text{distance between echoes}}{\text{time interval between echoes}} = \frac{X}{T} = \frac{680m}{126s} = 5.4\text{ms}^{-1}$

Example 13

A hunter 412.5m away from a cliff, moves a distance x towards the cliff and fires a gun. He hears the echo from the cliff after 2.2 seconds. Calculate the value of x. [Speed of sound in air = 330ms^{-1}] *NECO 2006*[F6]

Solution

Initial distance from cliff x_1 = 412.5; final distance from cliff x_2; echo time t = 2.2s;

Sound speed V = 330ms^{-1}

$$x_2 = \frac{Vt}{2} = \frac{330 \times 2.2}{2} = 363\text{m}$$

Distance moved toward cliff, $x = x_1 - x_2 = 412.5 - 363 = 49.5\text{m}$

Example 14

A man stands one-third of the way between two walls and clap his hands. He hears two distinct echoes 1s apart. If the speed of sound is 330ms^{-1} the distance between the walls is A. 33m B. 110m C. 220m D. 330m E. 495m *JAMB 1980*[45]

Solution

Fig. 15.1

x is the distance between two walls.

One third of x = $^x/_3$ and x - $^x/_3$ = $^{2x}/_3$

From $V = \dfrac{2x}{t}$, echo time, $t = \dfrac{2x}{V}$

Let t_1 be the echo time for $x = {}^x/_3$

Let t_2 be the echo time for $x = {}^{2x}/_3$

Substitute $t = \dfrac{2x}{V}$ to obtain

$$t_1 = \frac{2\times {}^x/_3}{330}; \qquad t_2 = \frac{2\times {}^{2x}/_3}{330}$$

Time interval between echo is $t_2 - t_1 = 1$s

$$\therefore \quad 1 = \frac{2\times {}^{2x}/_3}{330} - \frac{2\times {}^x/_3}{330} = \frac{4 {}^x/_3 \times {}^{2x}/_3}{330}$$

Cross multiplying

$$1\times 330 = \frac{4x}{3} - \frac{2x}{3}$$

$$330 = \frac{4x-2x}{3}$$

$$330 = \frac{2x}{3}$$

$$2x = 3\times 330 = 990$$

$$x = \frac{990}{2} = 495\text{m}$$

Example 15

A ship travelling towards a cliff receives the echo of its whistle after 3.5 seconds. A short while later it receives the echo after 2.5 seconds. If the speed of sound in air under the prevailing condition is 250ms^{-1}, how much closer is the ship to cliff?

A. 10m B. 125m C. 175m D. 350m E. 1000m *JAMB 1983*[10]

Solution

First echo time $t_1 = 3.5$s; second echo time $t_2 = 2.5$s; Speed of sound V = 250ms^{-1}

Distance at first echo $x_1 = \dfrac{Vt_1}{2} = \dfrac{250\times 3.5}{2} = 437.5$m

Distance at 2nd echo $x_2 = \dfrac{Vt_2}{2} = \dfrac{250\times 2.5}{2} = 312.5$m

Distance from cliff $x = x_1 - x_2 = 437.5 - 312.5 = 125$m

Example 16
A plane sound wave of frequency 85.5Hz and velocity $342ms^{-1}$ is reflected from a vertical wall. At what distance from the wall does the wave have an antinode? *JAMB 2001*[17]

A. 2m B. 4m C. 1m D. 3m

Solution
Frequency f = 85.5Hz; Velocity of sound $V = 342ms^{-1}$

From $V = \lambda f$, wavelength $\lambda = \frac{V}{f} = \frac{342}{85.5} = 4m$

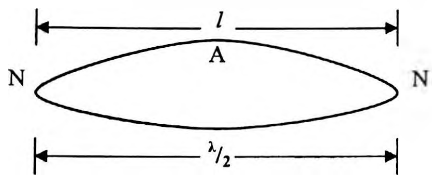
An antinode is ¼ of a wavelength

Therefore distance of antinode to wall = $\frac{1}{4}\lambda = \frac{1}{4}\times 4 = 1m$

VIBRATION OF STRINGS

Fundamental Frequency. Harmonics. Overtones

Fundamental frequency of a vibrating string is the lowest possible frequency that can be obtained from a plucked string when it vibrates in a single loop as shown below.



Fundamental frequency, f_o or first harmonic

Fig. 15.2

The length l of the string is equal to $\lambda/2$, distance between the two consecutive nodes (N – N)

That is, $l = \lambda/2$ or $\lambda = 2l$

For a sound wave, $V = \lambda f$ or Frequency, $f = \frac{V}{\lambda}$. Substitute $\lambda = 2l$ to obtain

Fundamental frquency, $f_o = \frac{V}{2l}$

Harmonics are frequencies which are whole numbers multiples of the fundamental frequency f_o.
Overtones are frequencies above the fundamental frequency which may or may not be whole numbers multiples of the fundamental frequency. Whole numbers overtones are also called harmonics.

The second harmonics or first overtone of a string is obtained from a plucked string when the string vibrate in two loops as shown below.

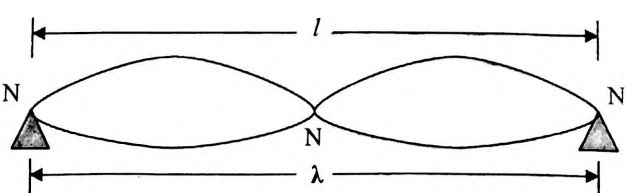


Fig. 15.3 Second harmonic or first overtones

The distance between three consecutive nodes is λ and is equal to l, the length of the string. That is $l = \lambda$

From, $V = \lambda f$, $f = \frac{V}{\lambda}$ substitute $l = \lambda$ to obtain.

First overtone/second harmonic, $f_1 = \frac{V}{l}$

The third harmonic or 2[nd] overtone of a string is obtained when the string is plucked and made to vibrate in three loop as shown below.

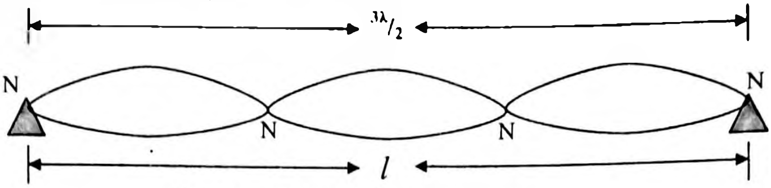


Fig. 15.4 Third harmonic or 2[nd] overtone

From the diagram, $l = \frac{3\lambda}{2}$ or $\lambda = \frac{2l}{3}$

From $V = \lambda f$, frequency, $f = \frac{V}{\lambda}$

Therefore, 2[nd] overtone or 3[rd] harmonic $f_2 = V \div \frac{2l}{3} = \frac{3V}{l}$

Thus, for a vibrating string or transverse wave,

Fundamental frequency (1[st] harmonic) $f_0 = {}^{v}/_{2l}$

First overtone (2[nd] harmonic) $f_1 = 2f_0 = {}^{v}/_{l}$

Second overtone (3[rd] harmonic) $f_2 = 3f_0 = {}^{3v}/_{2l}$

Third overtone (4[th] harmonic) $f_3 = 4f_0 = {}^{2v}/_{l}$

Example 17

A vibrator of frequency 60Hz is used in generating transverse stationary waves in a long thing wire. If the average distance between successive nodes on the wire is 45cm, find the speed of the transverse waves in the wire.

A. $27ms^{-1}$ B. $54ms^{-1}$ A. $90ms^{-1}$ D. $108ms^{-1}$ *JAMB 1989'9*

Solution

Frequency $f = 60Hz$; length, $l = 45cm = 0.45m$

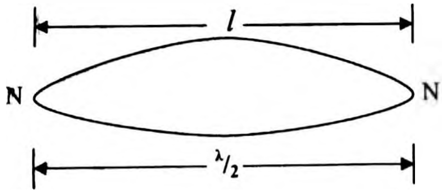


Fig. 15.5

From the diagram, $^{\lambda}/_{2} = l$ or $\lambda = 2l = 2 \times 0.45 = 0.90m$

Speed, $V = \lambda f = 0.9 \times 60 = 54ms^{-1}$

Example 18

A string is stretched tightly between two points 50cm apart. It is plucked at its centre and the velocity of the wave produced is $300ms^{-1}$. Calculate the number of vibration made by the string in one second. *WAEC 1989*[14]

Solution

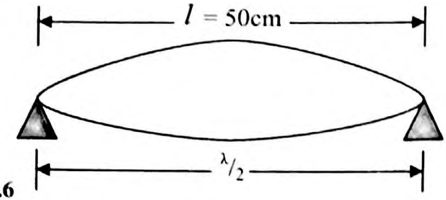


Fig. 15.6

$l = 50cm = 0.5m$

wave velocity, $V = 300ms^{-1}$

Number of vibration in one second is same as frequency.

Frequency, $f = \frac{V}{2l} = \frac{300}{2 \times 0.5} = \frac{300}{l} = 300Hz$

Example 19

Find the frequencies of the first three harmonics of a piano string of length 1.5m. If the velocity of the waves on the string is $120ms^{-1}$.

A. 40Hz, 80Hz, 120Hz B. 80Hz, 160Hz, 240Hz C. 180Hz, 360Hz, 540Hz

D. 360Hz, 180Hz, 90Hz *WAEC 2001*[12]

Solution

Length, $l = 1.5m$; velocity $V = 120ms^{-1}$

Fundamental frequency of string, $f_o = \frac{V}{2l} = \frac{120}{2 \times 1.5} = \frac{120}{3} = 40Hz$

1st harmonic $f_o = 40Hz$ 2nd harmonic $2f_o = 2 \times 40 = 80Hz$

3rd harmonic $3f_o = 3 \times 40 = 120Hz$. Ans: 40Hz, 80Hz, 120Hz

Example 20

If tension is maintained on a stretched string of length 0.6m, such that its fundamental frequency of 220Hz is excited, determine the velocity of the transverse wave in the string.

A. $66ms^{-1}$ B. $132ms^{-1}$ C. $264ms^{-1}$ D. $528ms^{-1}$ *JAMB 2002*[20]

Solution

Length $l = 0.6m$; frequency $f = 220Hz$; velocity of wave $V = ?$

Substitute into $f_o = \frac{V}{2l}$

$$220 = \frac{V}{2 \times 0.6}$$

$$V = 220 \times 2 \times 0.6 = 264ms^{-1}$$

Example 21

The lowest note emitted by a stretched string has a frequency of 40Hz. How many overtones are there between 40Hz and 150Hz? A. 1 B. 2 C. 3 D. 4 *JAMB 1987*[iii]

Solution

Overtones of fundamental frequency, f_o of a plucked string are $2f_o$, $3f_o$, $4f_o$ e.t.c.

So, $f_o = 40Hz$, $2f_o = 2 \times 40 = 80Hz$, $3f_o = 3 \times 40 = 120Hz$, $4f_o = 4 \times 40 = 160Hz$.

Therefore, there are 2 overtones between 40Hz and 150Hz. 160Hz is not included because it is greater than 150Hz.

Example 22
Calculate the wavelength of a note which is one octave lower than a note of 256Hz in a medium in which the speed of sound is $352ms^{-1}$ *WAEC 1998*[37]
Solution
An octave of a note is a note of twice the frequency.
Therefore a note which is one octave lower than a note of 256Hz is ½(256Hz) or 128Hz.

Frequency f = 128Hz; speed of sound V = $352ms^{-1}$

From, $V = \lambda f$, wavelength, $\lambda = \frac{V}{f} = \frac{352}{128} = 2.75m$

SONOMETER: FACTORS AFFECTING FREQUENCY OF VIBRATING STRING

The frequency of the notes obtained from a string instrument is studied with the help of a Sonometer.

The frequency (f) of a vibrating string of tension T, length l and mass per unit length M obeys the following laws:

1. $f \propto \frac{1}{l}$ (at constant T and M)
2. $f \propto \sqrt{T}$ (at constant l and M)
3. $f \propto \frac{1}{\sqrt{M}}$ (at constant l and T)

Combining the above three relationships, we obtain $f \propto \frac{1}{l}\sqrt{\frac{T}{M}}$ *or* $f = \frac{K}{l}\sqrt{\frac{T}{M}}$

The velocity V of a wave propagated along a string depends on its tension T and its mass per unit length M and is given by;

$$V = \sqrt{T/M}$$

At fundamental frequency, $f_o = \frac{V}{\lambda} = \frac{V}{2l}$

Therefore, f_o = fundamental frequency = 1st harmonic $= \frac{1}{2l}\sqrt{\frac{T}{M}}$

f_1 = first overtone = 2nd harmonic = $2f_o = \frac{1}{l}\sqrt{\frac{T}{M}}$

f_2 = second overtone = 3rd harmonic = $3f_o = \frac{3}{2l}\sqrt{\frac{T}{M}}$

f_3 = third overtone = 4th harmonic = $4f_o = \frac{2}{l}\sqrt{\frac{T}{M}}$

Example 23
A plucked string produces a note of 200Hz when its length is 1.50m. Determine the frequency of the note produced with a length of 0.75m. [Assume constant tension]
NECO 2006[34]

Solution
$f_1 = 200Hz$; $l_1 = 1.50$; $f_2 = ?$ $l_2 = 0.75m$

At constant tension, $f \alpha \frac{1}{l}$ $\therefore$ $\frac{f_1}{f_2} = \frac{l_2}{l_1}$

Substituting, $\frac{200}{f_2} = \frac{0.75}{1.50}$

Frequency, $f_2 = \frac{200 \times 1.50}{0.75} = \frac{300}{0.75} = 400\text{Hz}$

Example 24

A Sonometer wire under a tension of 10N produces a frequency of 250Hz when plucked. Keeping the length of the wire constant, the tension is adjusted to produce a new frequency of 350Hz. Calculate the new tension. *WAEC 1998*[35]

Solution

Original tension $T_1 = 10N$; original frequency $f_1 = 250Hz$

New frequency $f_2 = 350Hz$; new tension $T_2 = ?$

From $f \alpha \sqrt{T}$ or $f = \sqrt{T}$ we derive,

$\frac{f_1}{f_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$ *Substituting,* $\frac{250}{350} = \frac{\sqrt{10}}{\sqrt{T_2}}$

$$\sqrt{T_2} = \frac{350 \times \sqrt{10}}{250} = 1.4 \times \sqrt{10} = 4.43$$

$\sqrt{T_2} = 4.43$ Square both sides to obtain

$$(T_2)^2 = (4.43)^2$$

$$\therefore \quad T_2 = 19.6N$$

Alternative method:

$f \alpha \sqrt{T}$ or $f = K\sqrt{T}$ (K is constant)

Substitute $T = 10N$ and $f = 250Hz$ into $f = K\sqrt{T}$ *to obtain*

$$250 = K \times \sqrt{10}$$

$$K = \frac{250}{\sqrt{10}} = 79.1$$

Now, substitute $f = 359Hz$ and $K = 79.1$ to find the new tension.

$$f = K\sqrt{T}$$

$$350 = 79.1 \times \sqrt{T}$$

$$\sqrt{T} = \frac{350}{79.06} = 4.43$$

Square both sides

$$\left(\sqrt{T}\right)^2 = (4.43)^2$$

$$T = 19.6N$$

Example 25

When the tension in a Sonometer wire is doubled, the ratio of the new frequency to the initial frequency is A. $1/\sqrt{2}$ B. ½ C. $\sqrt{2}$ D. 2 *JAMB 1989*[21]

Solution

In a Sonometer, tension and frequency are related by $f = \sqrt{T}$

Let f_1 be initial frequency and f_2 the new frequency

$$\therefore \quad \frac{f_1}{f_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \qquad \text{or} \qquad \frac{f_2}{f_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}}$$

If tension is doubled, T_2 becomes $2T_2$

$$\frac{f_2}{f_1} = \sqrt{\frac{2T_2}{T_1}} \qquad (\text{note that, } T_2 = T_1 = T)$$

$$\therefore \quad \frac{f_2}{f_1} = \sqrt{\frac{2T}{T}} = \sqrt{2}$$

Example 26

A Sonometer wire of linear density $0.08 kgm^{-1}$ subjected to a tension of 800N is plucked. Calculate the speed of a pulse which moves from one end of the wire to the other.

WAEC 2003[30]

Solution

Linear density $M = 0.008 kgm^{-1}$; tension $T = 800N$

(linear density is also known as mass per unit length)

$$\text{Speed of pulse } V = \sqrt{\frac{T}{M}} = \sqrt{\frac{800}{0.08}} = \sqrt{10000} = 100 ms^{-1}$$

Example 27

Under constant tension and constant mass per unit length, the note produced by a plucked string is 500Hz when the length of the string is 0.90m. At what length is the frequency 150Hz? A. 3m B. 4m C. 5m D. 6m *JAMB 1997*[29]

Solution

$f_1 = 500Hz$; $l_1 = 0.90m$; $f_2 = 150Hz$; $l_2 = ?$

At constant M and T, $f \ \alpha \ \frac{1}{l}$ $\therefore \quad \frac{f_1}{f_2} = \frac{l_2}{l_1}$

$$\text{Substituting, } \frac{500}{150} = \frac{l_2}{0.90}$$

$$\text{Length of string } l_2 = \frac{500 \times 0.90}{150} = 3m$$

Example 28

Two strings of the same length and under the same tension give notes of frequencies in the ratio 4:1. The masses of the strings are in the corresponding ratio of

A. 2:1 B. 1:2 C. 1:4 D. 1:16 *JAMB 1988*[29]

Solution

For constant (same) length and tension $f \ \alpha \ \frac{1}{\sqrt{M}}$ or $\frac{f_1}{f_2} = \sqrt{\frac{M_2}{M_1}}$

Substitute $f_1 = 4$ and $f_2 = 1$ to obtain,

$$\frac{4}{1} = \sqrt{\frac{M_2}{M_1}}$$

Square both sides;

$$\left(\frac{4}{1}\right)^2 = \left(\sqrt{\frac{M_2}{M_1}}\right)^2 \qquad \text{or} \qquad \frac{16}{1} = \frac{M_2}{M_1} \qquad \text{or} \qquad \frac{1}{16} = \frac{M_1}{M_2}$$

$\therefore$ Ratio of $M_1 : M_2 = 1:16$

Example 29

The frequency of a plucked string is 800Hz when the tension is 8N. Calculate the

(i) Frequency when the tension is reduced to ¼ of its original value.

(ii) Tension required to produce a note of frequency 600Hz. *NECO 2005*[E13]

Solution

Frequency f = 800Hz; tension T = 8N

Substitute into $f = K\sqrt{T}$ to find constant K.

$$800 = K \times \sqrt{8}$$

$$K = \frac{800}{\sqrt{8}} = \frac{800}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{800\sqrt{8}}{\sqrt{64}}$$

$$K = \frac{800\sqrt{8}}{8} = 100\sqrt{8}$$

(i) Frequency f = ? tension T = ¼ (8N) = 2N (*..tension is ¼ of its original...*);

$K = 100\sqrt{8}$

Substitute into $f = K\sqrt{T}$ to obtain

$$f = K\sqrt{T} = 100\sqrt{8} \times \sqrt{2} = 100\sqrt{8 \times 2} = 100\sqrt{16}$$

$$f = 100 \times 4 = 400\text{Hz}$$

(ii) Tension, T = ? Frequency f = 600Hz

Substitute into $f = K\sqrt{T}$ to obtain

$$600 = 100\sqrt{8} \times \sqrt{T}$$

$$\sqrt{T} = \frac{600}{100\sqrt{8}} = \frac{6}{\sqrt{8}}$$

Square both sides

$$\left(\sqrt{T}\right)^2 = \frac{6^2}{\left(\sqrt{8}\right)^2} = \frac{36}{\sqrt{64}} \quad \therefore \quad T = \frac{36}{8} = 4.5\text{N}$$

VIBRATION OF AIR COLUMNS IN PIPES

Air column can be made to vibrate in closed pipes, open pipes or in a resonance tube.

Vibration in Closed Pipes

When an air column vibrates in a closed pipe, a node is ALWAYS formed at the closed end while an antinode is always formed at the open end as shown below.

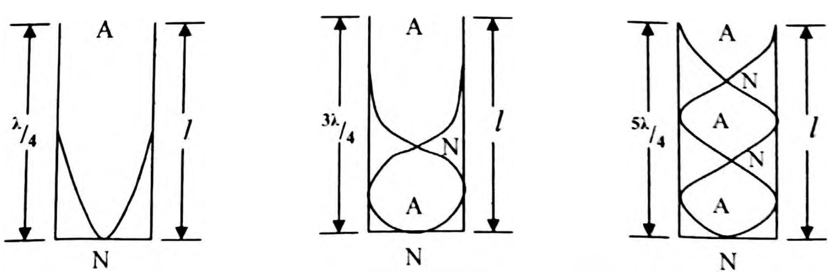


Fig. 15.7 a. Fundamental note, f_0 b. First overtone, f_1 c. Second overtone, f_2

From Fig. 15.7a, $l = \lambda/4$ (distance between node and antinode). Therefore, $\lambda = 4l$

From $V = \lambda f$, frequency, $f = \dfrac{V}{\lambda}$

$\therefore$ Fundamental frequency or 1st harmonic $f_o = V/4l$

From Fig. 15.7b, $l = 3\lambda/4$ or $\lambda = 4l/3$

From $V = \lambda f$, frequency, $f = \dfrac{V}{\lambda}$

$\therefore$ First overtone or 2nd harmonic, $f_1 = V \div \dfrac{4l}{3} = \dfrac{V \times 3}{4l}$

$$f_1 = 3V/4l$$

From Fig. 15.7c, $l = 5\lambda/4$ *or* $\lambda = 4l/5$

From $V = \lambda f$, frequency, $f = \dfrac{V}{\lambda}$

$\therefore$ Second overtone or 3rd harmonic $f_2 = V \div \dfrac{4l}{5} = \dfrac{5V}{4l}$

Thus for vibration in a closed pipe:

Fundamental frequency (1st harmonic), $f_o = V/4l$

First overtone (2nd harmonic), $f_1 = 3f_o = 3v/4l$

Second overtone (3rd harmonic), $f_2 = 5f_o = 5v/4l$

Third overtone (4th harmonic), $f_3 = 7f_o = 7v/4l$

Example 30
A pipe closed at one end is 100cm long. If the air in the pipe is set into vibration and a fundamental note is produced, calculate the frequency of the note. [Velocity of sound in air = $340ms^{-1}$] *WAEC 2007*[E13]

Solution
Length l = 100cm = 1.0m; velocity V = $340ms^{-1}$

For a closed pipe, fundamental frequency $f_o = V/4l$

$$f_o = \frac{340}{4 \times 1.0} = \frac{340}{4.0} = 85.0Hz$$

Example 31
A pipe closed at one end is 1.0m long. The air in the pipe is set into vibration and a fundamental note of frequency 85Hz is produced. Calculate the velocity of sound in the air. [Neglect end correction). *WAEC 1992*[19]

Solution
Length of pipe l = 1.0m; fundamental frequency f_o = 85Hz

For a closed pipe $f_o = v/4l$ $\therefore$ $V = f_o \times 4l$

Velocity of sound in air, $V = f_o \times 4l = 85 \times 4 \times 1$
$= 340ms^{-1}$

Example 32

The wavelength of the first overtone of a note in a closed pipe of length 33cm is *JAMB 2006*

A. 22cm B. 17cm C. 44cm D. 33cm

Solution

Length $l = 33cm$

In first overtone of a closed pipe, $l = {}^{3\lambda}/_{4}$

$\therefore$ Wavelength, $\lambda = {}^{4l}/_{3} = {}^{4}/_{3} \times 33 = 44cm$

Example 33

If the fundamental frequency of a closed pipe organ on a day when the speed of sound is $340ms^{-1}$ is 170Hz, then the length of the pipe is

A. 50cm B. 70cm C. 100cm D. 150cm E. 200cm *JAMB 1985*[29]

Solution

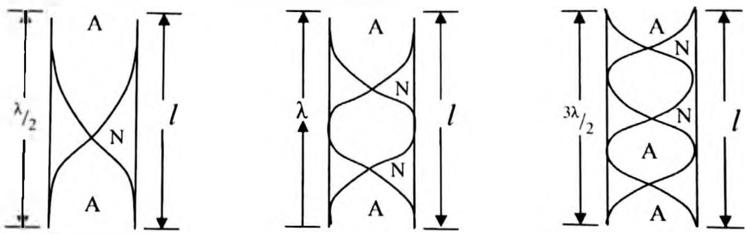
Fundamental frequency $f_0 = 170Hz$; speed of sound $V = 340ms^{-1}$

From $f_0 = \frac{V}{4l}$, lenght of pipe, $l = \frac{V}{4f_0}$

$$l = \frac{340}{4\times170} = \frac{340}{680} = 0.5m \text{ or } 50cm$$

Vibration in Open Pipes

When a column of air vibrates in an open pipe, the stationary waves formed always have antinodes at both ends as shown below.



a. Fundamental note, f_0 b. First overtone, f_1 c. Second overtone, f_2

Fig. 15.8 Vibration of open pipes.

From Fig. 15.8a, $l = \lambda/2$ *or* $\lambda = 2l$

From $V = \lambda f$, frequency, $f = \frac{V}{\lambda}$

Fundamental frequency or 1[st] harmonic $f_0 = {}^{V}/_{2l}$

From Fig. 15.8b, $l = \lambda$

First overtone/2[nd] harmonic $f_1 = \frac{V}{\lambda}$

From Fig. 15.8c, $l = {}^{3\lambda}/_{2}$ *or* $\lambda = {}^{2l}/_{3}$

Second overtone or 3[rd] harmonic, $f_2 = \frac{V}{\lambda} = \frac{3V}{2l}$

Thus for vibration in an open pipe:

Fundamental frequency (1[st] harmonic) $f_0 = \frac{V}{2l}$

First overtone (2nd harmonic) $\qquad f_1 = 2f_o = \frac{V}{l}$

Second overtone (3rd harmonic) $\qquad f_2 = 3f_o = \frac{3V}{2l}$

Third overtone (4th harmonic) $\qquad f_3 = 4f_o = \frac{2V}{l}$

Example 34 An organ pipe of length 40cm opened at both ends sounds its fundamental note. Neglecting end corrections, calculate the frequency of the note. [Speed of sound in air = $340ms^{-1}$]

NECO 2006[35]

Solution

Length of pipe l = 40cm = 0.4m; $\qquad$ speed of sound V = $340ms^{-1}$

For an open pipe, fundamental frequency, $f_o = V/2l$

$$f_o = \frac{340}{2 \times 0.4} = 425Hz$$

The Resonance Tube

A resonance tube is used to find the velocity of sound in air as shown below.

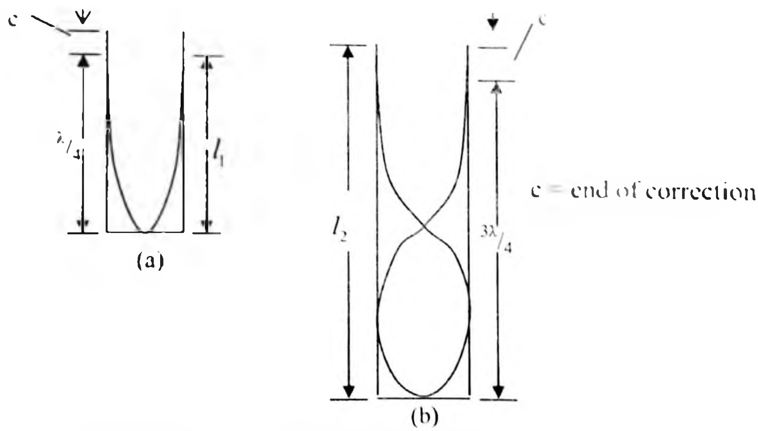


Fig. 15.9 $\qquad$ First and Second Positions of Resonance

For the first position of resonance, $l_1 + c = \frac{\lambda}{4}$

For the second position of resonance, $l_2 + c = \frac{3\lambda}{4}$

Subtracting, $l_2 + c - (l_1 + c) = \frac{3\lambda}{4} - \frac{\lambda}{4}$

$$l_2 + c - l_1 - c = \frac{2\lambda}{4}$$

$$l_2 - l_1 = \frac{\lambda}{2}$$

$$\lambda = 2(l_2 - l_1)$$

substitute the above into $V = \lambda f$ to obtain

$$V = 2f(l_2 - l_1)$$

Where V = velocity of sound in air.
f = frequency
l_2 = second resonance position
l_1 = first resonance position

Example 35
A tuning fork of frequency 600Hz is sounded over a closed resonance tube. If the first and second resonant positions are 0.130m and 0.413m respectively, calculate the speed of sound in air. *WAEC 2000*[33]

Solution
Frequency f = 600Hz; first resonant position $l_1 = 0.130$m;
Second resonant position $l_2 = 0.413$m
Speed of sound $V = 2f(l_2 - l_1)$

$$V = 2 \times 600(0.413 - 0.130)$$
$$= 1200 \times 0.283$$
$$= 339.6\text{ms}^{-1}$$

Example 36
The shortest length of the air column in a resonance tube at resonance is 0.11m and the next resonant length is 0.36m. Calculate the frequency of vibration, given that the speed of sound in air is 340ms^{-1}. *NECO 2007*[35]

Solution
First resonance position $l_1 = 0.11$m; second resonance position $l_2 = 0.36$m;
Speed of sound $V = 340\text{ms}^{-1}$

From $V = 2f(l_2 - l_1)$, frequency, $f = \dfrac{V}{2(l_2 - l_1)}$

$$f = \frac{340}{2(0.36 - 0.11)} = \frac{340}{2 \times 0.25} = \frac{340}{0.5} = 680\text{Hz}$$

Example 37
In a resonance tube experiment, if the fundamental frequency of the vibrating air column is 280Hz, the frequency of the third overtone is
A. 70Hz B. 840Hz C. 1120Hz D. 1960Hz *JAMB 1998*[28]

Solution
Fundamental frequency $f_o = 280$Hz

For a resonance tube, 1^{st}, 2^{nd}, 3^{rd} overtone are $f_1 = 3f_o$, $f_2 = 5f_o$, $f_3 = 7f_o$ e.t.c.

The third overtone $f_3 = 7f_o = 7 \times 280\text{Hz} = 1960\text{Hz}$

Example 38
A tuning fork vibrating at a frequency of 512Hz is held over the top of a jar filled with water and fitted with a tap at the bottom. If the jar is 60cm tall and the speed of sound is 350ms^{-1}, determine the possible resonance position(s). [Neglect end correction]
WAEC 2003[E13]

Solution
Frequency f = 512Hz; speed of sound $V = 350\text{ms}^{-1}$

Height of resonance jar or tube = 60cm = 0.6m

From $V = \lambda f$, wavelenght, $\lambda = \frac{V}{f} = \frac{350}{512} = 0.68\text{m}$

First resonance position, $f_o = \frac{\lambda}{4} = \frac{0.68}{4} = 0.17\text{m}$

Second resonance position, $3f_o = \frac{3\lambda}{4} = \frac{3\times 0.68}{4} = 0.51\text{m}$

Third resonance position, $5f_o = \frac{5\lambda}{4} = \frac{5\times 0.68}{4} = 0.85\text{m}$

Because the resonance jar is 0.60m, the possible resonance positions are 0.17m and 0.51m. 0.85m is NOT a possible resonance position because 0.85m is greater than the length of the jar.

Example 39
If the first position of resonance in a resonance tube is 18.0cm from the open end, the distance from the open end to the next position of resonance will be? *NECO 2000*[36]
Solution
First resonance position $f_o = {}^{\lambda}/_4 = 18\text{cm}$ $\therefore$ $\lambda = 18 \times 4 = 72\text{cm}$

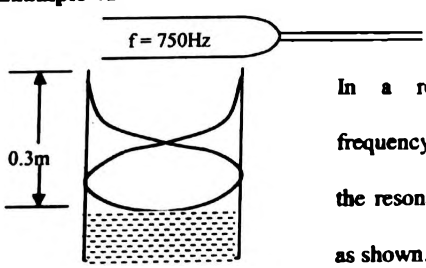
2nd resonance position, $3f_o = \frac{3\lambda}{4} = \frac{3\times 72}{4} = 54\text{cm}$

Example 40
In a resonance tube experiment, a tube of fixed length is closed at one end and several tuning forks of increasing frequency used to obtain resonance at the open end. If the turning fork with the lowest frequency which gave resonance had a frequency f_1 and the next tuning fork to give resonance had a frequency f_2, find the ratio f_2/f_1.
A. 8 B. 3 C. 2 D. ½ E. ${}^{1}/_{3}$ *JAMB 1983*[1]
Solution
In a closed tube, frequency at first resonance; $f_1 = \frac{V}{4l}$,

Frequency at 2nd or next resonance, $f_2 = \frac{3V}{4l}$

Therefore, $\frac{f_2}{f_1} = \frac{3V}{4l} + \frac{V}{4l} = \frac{3V}{4l} \times \frac{4l}{V} = 3$

Example 41



In a resonance tube experiment, the frequency of the tuning fork is 750Hz and the resonating length of air column is 0.3m as shown. The velocity of the wave is

Fig. 15.10
A. 1.85ms^{-1} B. 225ms^{-1} C. 300ms^{-1} D. $53 \times 10^{-5}\text{ms}^{-1}$ E. $2{,}500\text{ms}^{-1}$
JAMB 1982[47]

Solution

$f = 750Hz$; $l = 0.3m$

The diagram shows second resonant position. At second resonant position, $f = \frac{3V}{4l}$

Make V subject of equation to obtain velocity of sound wave

$$V = \frac{f \times 4l}{3} = \frac{750 \times 40 \times 0.3}{3} = \frac{900}{3} = 300ms^{-1}$$

Example 42

A siren has a disc of 64 holes and makes 20 revolutions per second. Calculate the frequency of the sound from the siren. *WAEC 1996*[36]

Solution

$$\text{For a disc siren, frequency} = \frac{\text{number of revolution} \times \text{number of holes}}{\text{time taken for revolution in seconds}}$$

$$= \frac{20 \times 64}{1} = 1280Hz$$

Also, frequency for a toothed wheel is given by

$$\text{Frequency, } f = \frac{\text{number of revolution} \times \text{number of teeth in wheel}}{\text{time taken in seconds}}$$

Beat Frequency

Beat frequency is a phenomenon that occurs as a result of interference of sound waves. Beats are heard whenever two sources of sound of nearly, but not exactly, the same frequency emits sound waves. If two instruments have frequencies f_1 and f_2 such that f_2 is greater than f_1, then the number of beats per second or beat frequency, f_b, is given by;

$$f_b = f_2 - f_1$$

Example 43

A Sonometer wire has a frequency of 259 Hz. It is sounded alongside a tuning fork of frequency 256Hz. Calculate the beat frequency *WAEC 2008*

Solution

Greater frequency, $f_2 = 259Hz$; lesser frequency, $f_1 = 256Hz$

Beat frequency, $f_b = f_2 - f_1$

$= 259 - 256 = 3Hz$

EXERCISE 15.

1. A note of frequency 2000Hz has a velocity of 400ms^{-1}. What is the wavelength of the note? *WAEC 1994*[33] *Ans: 0.2m*

2. The velocity of sound in air will be doubled of its absolute temperature is
A. doubled B. halved C. constant D. quadrupled *JAMB 2005*[38] *Ans:D*

3. The speed of sound in air at sea level is 340ms^{-1} while that of light is 300,000kms^{-1}. How far (to the nearest meter) from the centre of a thunderstorm is an observer who hears a thunder 2s after a lighting flash?
A. 170m B. 340m C. 600m D. 680m *JAMB 1988*[37] *Ans: 680m*

4. A note of frequency 2000Hz has a velocity of 400ms^{-1}. Calculate the wavelength of the note. *WAEC 2006*[29] *Ans: 0.2m*

5. A boy standing some distance from the feet of a tall cliff claps his hands and hears an echo 0.5s later. If the speed of sound is 340ms^{-1}, how far is he from the cliff? *WAEC 1988*[35] *Ans: 85m*

6. A man standing 300m away from a wall sounds a whistle. The echo from the wall reaches him 1.8s later. Calculate the velocity of sound in air. *WAEC 1991*[37] *Ans: 333.3m/s*

7. A man stands in front of a tall wall and produces a sound. If he receives the echo of the sound two seconds later, calculate his distance from the wall. [The speed of sound in air is 330ms^{-1}] *WAEC 1993*[34] *Ans: 330m*

8. A sound note of frequency 250Hz and wavelength 1.3m is produced at a point near a hill. If the echo of the sound is received one second later at the point, how far away is the hill from the point? *WAEC 1995*[31] *Ans: 162.50m*

9. A sound note is produced by a ringing bell and the echo of the note from a nearby wall is received 0.5s later. If the frequency of the note is 400Hz and its wavelength 1m, calculate the distance between the bell and the wall. *WAEC 1997*[29] *Ans: 100m*

10. A pulse of a sound is transmitted from a ship and the reflection from the sea bed is recorded after 0.2s. Calculate the depth of the sea. [Take speed of sound in sea water = 1560ms^{-1}] *WAEC 2002*[31] *Ans: 156m*

11. A girl stands 80m away from a tall cliff and blows a whistle. If the speed of sound in air is 330ms^{-1}, how long would it take for her to hear the echo of the sound? *WAEC 2004*[33] *Ans: 0.485s*

12. A body stands in front of a tall wall and produces a sound. If he hears the echo 3s later, calculate his distance from the wall. (Speed of sound in air = 330ms^{-1}) *NECO 2006*[33] *Ans:495m*

13. How far from a hill should a boy stand to hear the echo of his clap 1.6s later? [Speed of sound in air is 340ms^{-1}] *NECO 2004*[34] *Ans: 272m*

14. A man stands in front of a high wall and produces a sound. If he receives the echo of the sound four seconds later, what is his distance from the walls. [The speed of sound in air is 330ms^{-1}] *NECO 2002*[37] *Ans: 660m*

15. The echo of a sounder from the bottom of an ocean is heard 2.5 seconds later. If the speed of sound in water is 1400ms^{-1}, the depth of the ocean is
A. 1,750m B. 3500m C. 1400m D. 2800m *JAMB 1982*[6] *Ans: 1,750m*

16. In order to find the depth of the sea, a ship sends out a sound wave and receives an echo after one second. If the velocity of sound in water is 1500m/s, what is the depth of sea? A. 0.75km B. 1.50km C. 2.2km D. 3.00km E. 3.75km *JAMB 1985*[27] *Ans: 0.75km*

17. A man clapping his hands at regular intervals observes that the echo of a clap coincides with the next clap. If the reflecting cliff is 160m away and the speed of sound is 320ms^{-1}, what is the frequency of the clapping.
A. 1Hz B. 2Hz C. 4Hz D. 8Hz *JAMB 1986*[17] *Ans: 1Hz*

18. The sound from a source travelled to the bottom of the sea and the echo was heard 4s later. If the speed of sound in sea water is 1500ms^{-1} the depth of the sea is
A. 6000m B. 3000m C. 1500m D. 375m *JAMB 1994*[29] *Ans: 3000m*

19. A boy stands between two vertical wall and fires a rifle.
(i) Under what condition will he hear a single echo from both walls?

(ii) Calculate the distance between the walls if the single echo is heard 6s later. [Speed of sound in air = 330ms^{-1}). *NECO 2007[E13] Ans: (i) when he stands between the walls at a point equidistant from the them (ii) 1980m*

20. A wire is stretched between two points, 1m apart. If the speed of the wave generated on plucking the wire is 200ms^{-1}, what is the minimum frequency which will resonate with the wire? *WAEC 1995[27] Ans: 100Hz*

21. A note **A** from a guitar produces a wave of amplitude 4mm and frequency 1000Hz. Another note, **B** from a whistle produces a similar waveform of amplitude 2mm and frequency 2200Hz. If the two notes are compared,
A. **A** has a higher pitch than **B**. B. **A** is louder than **B**
C. **B** has a greater speed than **A** D. **B** is louder than **A**
E. **A** and **B** have the same quality *NECO 2004[35] Ans: B*

22. A steel wire of length 0.50m is stretched between 2 fixed points and its fundamental frequency is 200Hz. The speed of the wave in the wire is
A. 100ms^{-1} B. 120ms^{-1} C. 200ms^{-1} D. 250ms^{-1} *JAMB 1998[27] Ans: 200ms^{-1}*

23. A string is fastened tightly between two walls 24cm apart. The wavelength of the second overtone is A. 24cm B. 16cm C. 12cm D. 8cm *JAMB 2001[18] Ans: 16cm*

24. All of the following frequencies are overtones of 320Hz EXCEPT
A. 960Hz B. 640Hz C. 520Hz D. 1280Hz E. 1600Hz
JAMB 1981[24] Ans: 520Hz

25. Of two identical tuning forks with natural frequency 256Hz, one is loaded so that 4 beats per second are heard when they are sounded together. What is the frequency of the loaded tuning fork?
A. 260Hz B. 252Hz C. 248Hz D. 264Hz E. 258Hz *JAMB 1984[16] Ans: A*

26. The lowest note emitted by a stretched string has a frequency of 40Hz. How many overtones are there between 40Hz and 180Hz?
A. 4 B. 3 C. 2 D. 1 *JAMB 1999[19] Ans: 3*

27. A Sonometer wire of length 100cm under a tension of 10N has a frequency of 250Hz. Keeping the length of the wire constant, the tension is adjusted to produce a new frequency of 350Hz. The new tension is
A. 5.1N B. 7.1N C. 14.0N D. 19.6N *JAMB 1990[33] Ans: 19.6N*

28. A string of length 1.0m vibrates in 10 loops. If the total mass of the string is 1.0 x 10^{-3}kg and the tension in it is 10N, calculate the frequency of the vibration.
WAEC 1996[35] Ans: 50Hz

29. The fundamental frequency of a plucked wire under a tension of 400N is 250Hz. When the frequency is changed to 500Hz at constant length, the tension is
A. 160N B. 1600N C. 40N D. 400N *JAMB 2007[17] Ans:40N*

30. A transverse wave is applied to a string whose mass per unit length is 3 x 10^{-2} kgm^{-1}. If the string is under a tension of 12N, the speed of propagation of the wave is A. 40ms^{-1} B. 30ms^{-1} C. 20ms^{-1} D. 5ms^{-1} *JAMB 2004[24] Ans: 20ms^{-1}*

31. The note produced by a stretched string has a fundamental frequency of 400Hz. If the length of the string is doubled while the tension in the string is increased by a factor of 4, the frequency is
A. 200Hz B. 400Hz C. 800Hz D. 1600Hz *JAMB 1994[32] Ans: 400Hz*

32. A piano wire 0.50m long has a total mass of 0.01kg and is stretched with a tension of 800N. Calculate the frequency of the wire when it sounds its fundamental note.
A. 200Hz B. 100Hz C. 4Hz D. 2Hz *JAMB 1994[32] Ans: 400Hz*

33. A pipe closed at one end is 1m long. The air in the pipe is set into vibration and a fundamental note is produced. If the velocity of sound in air is 340ms^{-1}, calculate the frequency of the note. *WAEC 1989[E2] Ans: 85Hz*

34. An open pipe closed at one end produces its first fundamental note. If the velocity of sound in air is V and l the length of the pipe, the frequency of the note is
A. $\frac{2V}{l}$ B. $\frac{V}{5l}$ C. $\frac{V}{4l}$ D. $\frac{V}{2l}$ *JAMB 2003[18] Ans: C*

35. An organ pipe closed at one end is 80cm long. Determine the frequency of the fundamental note assuming that the speed of sound in air is 340ms^{-1}.

A. 106Hz B. 213Hz C. 318Hz D. 425Hz *JAMB 1990[12] Ans: 106Hz*

36. A pipe of length 45cm is closed at one end. Calculate the fundamental frequency of the sound wave generated in the pipe if the velocity of sound in air is 360ms^{-1}. [Neglect end corrections]

A. 5.5Hz B. 148.5Hz C. 200.0Hz D. 550.0Hz *JAMB 1994[31] Ans: 200Hz*

37. A pipe, open at both ends, produces a fundamental note. If the velocity of sound in air is V and L the length of the pipe, which of the following expresses the frequency of the note? [Neglect end corrections]

A. $\frac{2V}{L}$ B. $\frac{V}{2L}$ C. $\frac{V}{3L}$ D. $\frac{V}{4L}$ E. $\frac{V}{5L}$ *WAEC 1997[33] Ans:* $^{V}/_{2L}$

38. A tuning fork of frequency 340Hz is vibrated just above a cylindrical tube of height 1.2m. If water is slowly poured into the tube, at what minimum height will resonance occur? [Speed of sound in air = 340ms^{-1}]

A. 0.95m B. 0.6m C. 0.50m D. 0.45m *JAMB 2003[15] Ans:0.95m*

39. In a resonance tube experiment, the effective length of the air column for the first resonance is 20cm when set into vibration by a tuning fork of frequency 480Hz. Neglecting end effect, the velocity of sound in air is

A. 96ms^{-1} B. 255ms^{-1} C. 340ms^{-1} D. 384ms^{-1} *JAMB 1990[38] Ans: 384ms^{-1}*

40. The shortest length of the air column in a resonance tube at resonance is [illegible]2m and the next resonant length is 0.37m. Calculate the frequency of vibration given that the speed of sound in air is 340ms^{-1} *WAEC 1998[34] Ans: 680Hz*

41. If the position of resonance in a resonance tube is 16.50cm from the open end of the tube, calculate the distance from the open end to the next position where resonance occurs. [Neglect end correction] *WAEC 1999[29] Ans: [illegible]50cm*

42. If the distance from a point source of sound is doubled, by what factor does the intensity decreases? A. 4.00 B. 2.00 C. 0.50 D. 0.25 *JAMB 2003[24] Ans: D*

43. Two tuning forks of frequencies 256Hz and 260Hz are sounded close to each other. What is the frequency of the beats produced?

A. 2Hz B. 4Hz C. 8Hz D. 258Hz *JAMB 1991[30] Ans: B*

44.

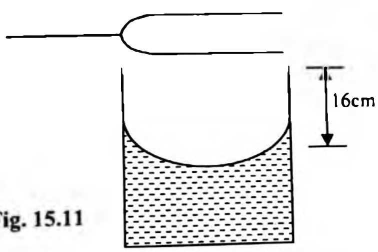


Fig. 15.11

In a resonance tube experiment which is illustrated in fig 15.11, the velocity of sound in air is 327.68ms^{-1}, the frequency of the tuning fork used is therefore

A. 128Hz B. 256Hz C. 512Hz D. 768Hz *JAMB 1979[23] Ans: 512Hz*

45.

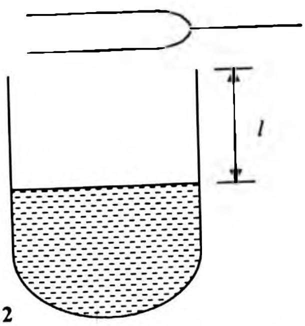


Fig. 15.12

In figure 15.12, a resonance tube experiment is performed using one tuning fork. As the water level is lowered the first resonance is obtained when the length of the air column $l = \lambda/4$. The second resonance is obtained when l equals

A. $^{\lambda}/_2$ B. $^{3\lambda}/_4$ C. λ D. $^{3\lambda}/_2$ *JAMB 1986[35] Ans: B*

46. What is the frequency of the sound made by a siren having a disc with 32 holes and making 25 revolutions per second?

A. 80Hz B. 600Hz C. 800Hz D. 1600Hz *JAMB 1992[23] Ans: 800Hz*

47. When the length of a vibrating string is reduced by one-third, its frequency becomes

A. Three times its former value B. Twice its former value
C. One-third of its former value D. One-sixth of its former value

JAMB 2008 Ans: A Hint: $f = {}^{V}/_{l}$

48. A sound wave is produced from a source and an echo is heard t seconds afterwards. If d is the distance of the reflecting source from the source, V the speed, λ the wavelength and T the period of wave, then

A. $d = \frac{2t}{\lambda}$ B. $d = \frac{\lambda t}{2T}$ C. $d = \frac{2T}{\lambda t}$ D. $d = \frac{\lambda T}{2t}$ *WAEC 2008 Ans:* $d = \frac{\lambda t}{2T}$

Hint: substitute $V = \frac{\lambda}{T}$ into 2d = Vt

49. A Sonometer wire has a frequency of 259Hz and is under a tension of 1200N. If a meter of the wire has a mass of 0.03kg, calculate the length l of the wire when it is vibrating in the fundamental mode. *WAEC 2008[E13] Ans: 0.386m* Hint: $f = \frac{1}{2l}\sqrt{T/M}$

50. A note of frequency 300Hz is produced when the length of a wire is 80cm and the tension is 40N. What is the frequency if the length of the wire is halved and the tension is doubled? *NECO 2008[34] Ans: 848.4Hz* Hint: $f = {}^{K\sqrt{T}}/_{l}$

51. In a resonance tube closed at one end, the first two lengths of the air column that vibrate in resonance with a tuning fork of frequency 316Hz are 20.6cm and 73.1cm respectively. Calculate the velocity of sound in air to 3 significant figures.

NECO 2008[35] Ans: 332ms^{-1}

52. A stretched wire of length 24cm when plucked, produces a note of the same frequency as that of a fork of frequency 256Hz. If the wire is then adjusted to 16cm and the tension kept constant, calculate the frequency of the fork which will be in tune with the wire. *NECO 2008[E13] Ans: 384Hz*

53. An observer heard the sound of thunder 5.8s after the lightning flash was seen. How far was he from the source? [speed of sound in air = 330ms^{-1}]

NECO 2009[34] Ans: 1914m

54. The first two lengths of air column that vibrate in resonance with a tuning fork of frequency f are l_1 and l_2. If the resonance tube is closed at one end, express the velocity of sound in terms of f, l_1 and l_2

A. $\frac{3}{4}f(l_2 - l_1)$ B. $f(l_2 - l_1)$ C. $2f(l_2 - l_1)$ D. $3f(l_2 - l_1)$
E. $4f(l_2 - l_1)$ *NECO 2009[35] Ans: C*

55. A sound wave from a ship is received 8.4s later. What is the debt of the sea? [speed of sound in water = 1500ms^{-1}] *NECO 2009[34] Ans: 6.30km*

56. A man, standing between two cliffs, claps his hands and hears two echoes after 3.6s and 4.2s respectively. If the speed of sound in air is 330ms^{-1}, calculate the distance between the cliffs. *NECO 2009[E6] Ans: 1287m*

57. A pipe closed at one end has a length of 15cm. Calculate the frequency of the

(i) fundamental note,
(ii) first overtone [speed of sound in air = 340ms^{-1}]

NECO 2009[E13] Ans: (i) 566.6Hz (ii) 1700Hz

BOOK THREE

1

GRAVITATIONAL FIELD

GRAVITATIONAL FORCE

Gravitational field is the region around a body having mass in which the gravitational force of the body can be experienced. Gravitational force of attraction between two bodies is governed by the *Newton's law of universal gravitation*, which states that, any two particles of matter attracts one another with a force F, which is proportional to the product of their masses m_1 & m_2 and inversely proportional to the square of their distance r, apart.

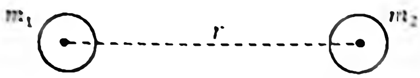


Fig. 1.1

Gravitational force between two masses is given by

$$F = \frac{Gm_1m_2}{r^2}$$

Where m_1 and m_2 are the masses of the two particles; r is the distance between masses m_1 and m_2; G is the gravitational constant $= 6.67 \times 10^{-11} Nm^2kg^{-2}$

Example 1

Two spheres of masses $100kg$ and $90kg$ respectively have their centers separated by a distance of 1.0m.Calculate the magnitude of the force of attraction between them. $[G = 6.70 \times 10^{-11} Nm^2kg^{-2}]$ *WAEC 2002*[37]

Solution

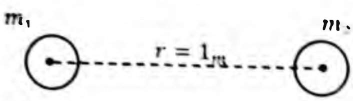


Fig. 1.2

$$\text{Force of attraction } F = \frac{Gm_1m_2}{r^2} = \frac{6.70 \times 10^{-11} \times 100 \times 90}{1^2} = 6.03 \times 10^{-7} N$$

Example 2

A force of $200N$ acts between two objects at a certain distance apart. The value of the force when the distance is halved is A. $100N$ B. $200N$ C. $800N$ D. $400N$

JAMB 2005[32]

Solution

Initial force, $F_1 = 200N$; final force (when distance is halved) $F_2 = ?$; Initial distance $r_1 = r$; final distance $r_2 = r/2$ (*- - - the distance is halved - - -*)

Gravitational force is inversely proportional to the square of the distance;

$$F \propto \frac{1}{r^2}$$

$$\therefore \quad F_1r_1^2 = F_2r_2^2$$

$$200 \times r^2 = F_2 \times (^r/_2)^2$$

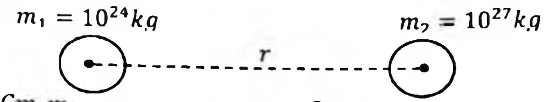
$$200 \times r^2 = F_2 \times \frac{r^2}{4}$$

$$\text{Rearranging, } F_2 = \frac{4 \times 200 \times r^2}{r^2} \qquad \therefore F_2 = 800N$$

Example 3

The gravitational force between two objects of masses 10^{24} and 10^{27} is 6.67N. Calculate the distance between them($G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}) NECO 2008[50]

Solution



$$F = \frac{Gm_1m_2}{r^2} \qquad \therefore \qquad r^2 = \frac{Gm_1m_2}{F}$$

$$r^2 = \frac{6.67 \times 10^{-11} \times 10^{24} \times 10^{27}}{6.67} = 10^{-11} \times 10^{24} \times 10^{27} = 10^{-11+24+27} = 10^{40}$$

$$r = \sqrt{10^{40}} = 1.0 \times 10^{20}m$$

Example 4

A planet has mass m_1 and is at a distance r_1 from the sun. A second planet has mass $m_2 = 10m_1$ and is at a distance of $r_2 = 2r_1$ from the sun. Determine the ratio of the gravitational forces experienced by the planets.

A. 1:5 B. 2:5 C. 3:5 D. 4:5 *JAMB* 1997[8]

Solution

First planet: mass $m = m_1$; distance $r = r_1$

Second planet: mass $m = 10m_1$; distance $r_2 = 2r_1$; mass of the sun = m_s

Substitute for each planet into gravitational force equation $F = \frac{Gm_sm}{r^2}$

1st planet

$$F_1 = \frac{G \times m_s \times m_1}{r_1^2}$$

2nd planet

$$F_2 = \frac{G \times m_s \times 10m_1}{(2r_1)^2}$$

$$= \frac{G \times m_s \times 10m_1}{4r_1^2}$$

$$\text{Ratio of gravitational force, } \frac{F_2}{F_1} = \frac{G \times m_s \times 10m_1}{4r_1^2} \div \frac{G \times m_s \times m_1}{r_1^2}$$

$$\frac{F_2}{F_1} = \frac{G \times m_s \times 10m_1}{4r_1^2} \times \frac{r_1^2}{G \times m_s \times m_1}$$

$$\text{Rearranging, } \frac{F_2}{F_1} = \frac{G \times m_s \times 10m_1 \times r_1^2}{G \times m_s \times m_1 \times 4r_1^2} = \frac{10}{4}$$

$$\frac{F_2}{F_1} = \frac{10}{4} = \frac{5}{2}$$

Therefore, $F_2 : F_1 = 5:2$ *or* $F_1 : F_2 = 2:5$ Ratio of gravitational force is 5:2 *or* 2:5

Example 5

The force of attraction between two point masses is $10^{-4}N$ when the distance between them is 0.18m. If the distance is reduced to 0.06m, calculate the force.

A. $1.1 \times 10^{-5}N$ B. $3.3 \times 10^{-5}N$ C. $3.0 \times 10^{-4}N$ D. $9.0 \times 10^{-4}N$
JAMB 1998

Solution
1^{st} *case:* $F_1 = 10^{-4}N$; $r_1 = 0.18m$
2^{nd} *case:* $F_2 = ?$; $r_2 = 0.06m$

Gravitational force (F) is inversely proportional to square of the distance (r): $F \propto \frac{1}{r^2}$
Therefore, $F_1 r_1^2 = F_2 r_2^2$ Substitute to obtain $10^{-4} \times 0.18^2 = F_2 \times 0.06^2$

$$F_2 = \frac{10^{-4} \times 0.18^2}{0.06^2} = \frac{10^{-4} \times 18^2}{6^2} = \frac{10^{-4} \times 324}{36} = 9.0 \times 10^{-4}N$$

GRAVITATIONAL POTENTIAL

Gravitational potential, V, is the potential due to the gravitational field of the earth. It is defined as the work done in taking a unit mass from infinity to the point (a particular distance form the center of the earth).

$$\text{Gravitational potential } V = -\frac{GM}{r}$$

Where $M =$ mass of the earth
$G =$ gravitational constant
$r =$ distance of the point from center of earth

The negative sign indicates that the potential decreases as the object is moved from infinity towards the earth.

Example 6
What is the gravitational potential at a point on the surface of the earth if $G = 6 \cdot 6 \times 10^{-11}\text{N}m^2kg^{-2}$ and the radius and mass of the earth are respectively $6 \cdot 4 \times 10^6 m$ and $5 \cdot 98 \times 10^{24} kg$?
Solution

$$M = 5 \cdot 98 \times 10^{24}kg\,; \quad r = 6 \cdot 4 \times 10^6 m\,; \quad G = 6 \cdot 6 \times 10^{-11}\text{N}m^2kg^{-2}$$

$$\text{Gravitational potential } V = -\frac{GM}{r} = \frac{6 \cdot 6 \times 10^{-11} \times 5 \cdot 98 \times 10^{24}}{6.4 \times 10^6}$$

$$= \frac{3 \cdot 95 \times 10^{14}}{6 \cdot 4 \times 10^6} = 6.2 \times 10^7 \text{Jkg}^{-1}$$

ACCELERATION DUE TO GRAVITY/GRAVITATIONAL FIELD INTENSITY

The acceleration due to gravity (g) is also known as the force per unit mass (m) of an object and is responsible for the weight or force an object experiences on the earth.

From $F = \frac{Gm_e m}{r_e^2}$ (m_e and r_e are mass and radius of the earth respectively)

Force per unit mass $\frac{F}{m} = \frac{Gm_e}{r_e^2}$

Acceleration due to gravity is equal to force per unit mass.

Therefore, $g = \frac{F}{m}$ *or* $g = \frac{Gm_e}{r_e^2}$

Hence, $F = mg$

Example 7

The magnitude of the gravitational attraction between the earth and a particle is 40N. If the mass of the particle is $4kg$, calculate the magnitude of the gravitational field intensity of the earth on the particle *WAEC* 2002

Solution

Force of attraction on particle, $F = 40\text{N}$; mass of particle $m = 4kg$

Gravitational field intensity, $g = \frac{F}{m} = \frac{40}{4} = 10Nkg^{-1}$

Example 8

The mass and weight of a body on earth are $8kg$ and $80N$ respectively. Determine the mass and weight of the body respectively on a planet where the pull of gravity is $\frac{1}{8}$ that on earth. *WAEC* 2005

Solution

Mass $m = 8kg$; $weight\ w = 80\text{N}$

Weight $= mass \times acceleration\ due\ togravity$

$w = mg$

Therefore, $g = {}^{w}/_{m} = {}^{80}/_{8} = 10ms^{-2}$

$\frac{1}{8}$ of $g = \frac{1}{8} \times 10 = 1 \cdot 25ms^{-2}$

$m = 8kg$ mass of objects do not change whatever their location.

From $w = mg$; $w = 8 \times 1 \cdot 25 = 10\text{N}$ Ans: $8kg, 10\text{N}$

Example 9

The earth is four times the size of the moon and the acceleration due to gravity on the earth is 80 times that on the moon. The ratio of the mass of the moon to that of the earth is

A. 1: 320 B. 1: 1280 C. 1: 80 D. 1: 4 *JAMB* 2004[17]

Solution

Acceleration due to gravity on moon $g_m = 1$

Acceleration due to gravity on earth, $g_e = 80$

radius of the moon, $r_m = 1$

radius of the earth, $r_e = 4$

Acceleration due to gravity, $g = \frac{Gm}{r^2}$

For the earth, $g_e \frac{Gm_e}{r_e^2}$. Therefore, $m_e = \frac{g_e r_e^2}{G}$

For the moon, $g_m = \frac{Gm_m}{r_m^2}$. Therefore, $m_m = \frac{g_m r_m^2}{G}$

The ratio of the mass of the moon (m_m) to that of the earth (m_e) is:

$$\frac{m_m}{m_e} = \frac{g_m r_m^2}{G} \div \frac{g_e r_e^2}{G} = \frac{g_m r_m^2}{G} \times \frac{G}{g_e r_e^2}$$

$\frac{m_m}{m_e} = \frac{g_m r_m^2}{g_e r_e^2}$ substituting, we obtain

$$\frac{m_m}{m_e} = \frac{1 \times 1^2}{80 \times 4^2} = \frac{1}{80 \times 16} = \frac{1}{1280}$$ Therefore, $m_m : m_e = 1 : 1280$

WEIGHT OF OBJECTS IN SPACE

When an object is on or near the surface of the earth, it experiences the full extent of the force of gravity or acceleration due to gravity of the earth. However, if the object (for example a satellite) is launched into outer space at a distance from the surface or center of the earth, the weight of the object reduces because the earth's gravitational force on the object diminishes in proportion to its distance from the earth. The farther the distance from the earth, the lesser the weight.

Example 10

If the weight of a satellite on the earth's surface is $2000N$,

a. What would be its weight when it is

(i) at a distance equal to the radius of the earth from the surface of the earth

(ii) at a distance of $1.28 \times 10^7 m$ from the surface of the earth?

(iii) at a distance equal to four times the radius R of the earth, from the center of the earth

b. For each case in (a) above, derive an expression for the acceleration due to gravity (g_s) in space with respect to the acceleration due to gravity (g_e) on earth.

$[Radius\ of\ earth\ R = 6 \cdot 4 \times 10^6 m]$

Solution

Let W_e = weight of satellite on earth = $2000N$

W_s = weight of satellite in space

M = Mass of the earth

m = Mass of the satellite

g_e = Acceleration due to gravity on earth

g_s = Acceleration due to gravity in space

R = Radius of the earth = $6 \cdot 4 \times 10^6 m$

a (i)

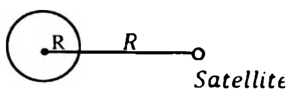


Fig. 1.3

Distance of satellite from center of earth = $R + R$

The weight of the satellite in space W_s is equal to the gravitational force of attraction between the earth and the satellite in space.

$$F = W_s = \frac{GMm}{(R+R)^2} = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$$

Rearranging, $$W_s = \frac{m}{4} \times \frac{GM}{R^2} \qquad \left[substituting\ g_e = \frac{GM}{R^2}\right]$$

$$W_s = \frac{mg_e}{4} = \quad [substitute\ W_e = mg_e]$$

Therefore, $$W_s = \frac{W_e}{4} = \frac{200\text{N}}{4} = 500N$$

This means that the weight of an object in space at a distance R (radius of earth) from the surface of the earth or a distance $2R$ from the center of the earth is one-fourth its weight on earth: $W_s = {}^{W_e}/_4$.

(ii) If the radius of the earth $R = 6 \cdot 4 \times 10^6 m$,

Then $1 \cdot 28 \times 10^7 m = 2 \times 6 \cdot 4 \times 10^6 = 2R$

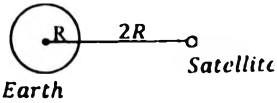


Fig.1.4

In similar manner to a (i) above;

Weight of satellite in space, $W_s = \dfrac{GMm}{(R+2R^2)} = \dfrac{GMm}{(3R)^2} = \dfrac{GMm}{9R^2}$

Rearranging $W_s = \dfrac{m}{9} \cdot \dfrac{GM}{R^2}$ $\left[\text{substitute } g_e = \dfrac{GM}{R^2}\right]$

$$W_s = \frac{mg_e}{9} \qquad [\text{substitute } W_e = mg_e]$$

$$W_s = {}^{W_e}/_9$$

Therefore, $W_s = 2000N/9 = 222.22N$

Hence, the weight of object in space at a distance $2R$ from the surface of the earth or a distance $3R$ from the center of the earth is one-ninth its weight on the earth: $W_s = \frac{W_e}{9}$

(iii)

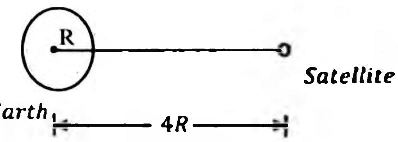


Fig.1.5

$$W_s = \frac{GMm}{(4R)^2} = \frac{GMm}{16R^2} = \frac{m}{16} \cdot \frac{GM}{R^2} = \frac{mg_e}{16} = \frac{W_e}{16}$$

$$W_s = \frac{W_e}{16} = \frac{2000}{16} = 125N$$

Therefore, at a distance $4R$ from the center of the earth or distance $3R$ from the surface of the earth, the acceleration due to gravity or the weight of an object is one-sixteenth of that on earth: $W = \frac{W_e}{16}$

b. From a (i) the equation $W_s = \frac{W_e}{4}$ can be rewritten remembering that generally, $W = mg$

$$W_s = \frac{W_e}{4}$$

$$\cancel{m}g_s = \frac{\cancel{m}g_e}{4}$$

$$g_s = \frac{g_e}{4}$$

At a distance $2R$ from the center of the earth, an object experiences $^1/_4$ (one fourth) of the earth's force of gravity i.e. acceleration due to gravity

From a (ii) $W_s = \frac{W}{\;}$

$$mg_s = \frac{mg_e}{9} \quad \therefore \quad g_s = \frac{g_e}{9}$$

From a (iii) $\quad W_s = \frac{W_e}{16}$

$$mg_s = \frac{mg_e}{16} \quad \therefore \quad g_s = \frac{g_e}{16}$$

ESCAPE VELOCITY

Escape velocity is the *minimum* velocity with which an object would just escape from the influence of the earth's gravitational field.

Escape velocity $V_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

Where g = acceleration due to gravity of the astronomical body (earth)
R = Radius of the earth (astronomical body)
M = Mass of the earth (astronomical body)

Example 11

What is the escape velocity of a satellite launched from the earth's surface? (Take g as $10ms^{-2}$ and the radius of the earth as $6 \cdot 4 \times 10^6 m$) *NECO 2002*

Solution

$g = 10ms^{-1}$; $R = 6 \cdot 4 \times 10^6 m$

Escape velocity $V_e = \sqrt{2gR} = \sqrt{2 \times 10 \times 6 \cdot 4 \times 10^6}$

$$V_e = \sqrt{1 \cdot 28 \times 10^8} = 1 \cdot 13 \times 10^4 m/s = 11.3km/s$$

Example 12

Complete the following table, assuming Newton's law of universal gravitation. $[G = 6.67 \times 10^{-11} Nm^2kg^{-2}]$

Table 1.1

	Planet A	Planet B	Planet C
Mass	$2.0 \times 10^{27} kg$		$7.4 \times 10^{22} kg$
Diameter	$1.84 \times 10^8 m$	$1.28 \times 10^7 m$	
Weight of $10kg$ ball		$98N$	$16N$
Escape velocity			

Solution

Planet A

Mass of planet $M = 2 \cdot 0 \times 10^{27} kg$

$$\text{Radius } r = \frac{\text{diameter}}{2} = \frac{1 \cdot 84 \times 10^8 m}{2} = 9 \cdot 2 \times 10^7 m$$

Substitute into $g = \frac{GM}{r^2}$ to obtain acceleration due to gravity in planet A

$$g = \frac{6 \cdot 6 \times 10^{-11} \times 2 \cdot 0 \times 10^{27}}{(9 \cdot 2 \times 10^{7})^2} = 15 \cdot 76\, m/s^2$$

Weight of $10kg$ ball, $W = mg = 10 \times 15 \cdot 76 = 157 \cdot 60N$

Escape velocity $V_e = \sqrt{2gR} = \sqrt{2 \times 15 \cdot 76 \times 9 \cdot 2 \times 10^7} = 5 \cdot 39 \times 10^4\, m/s$

Planet B

Mass of ball $m = 10kg$; weight of ball, $W = 98N$

From $W = mg$, acceleration due to gravity in planet B is $g = \frac{W}{m} = \frac{98}{10}$

$$= 9.8ms^{-2}$$

$$\text{Radius } r = \frac{\text{diameter}}{2} = 1 \cdot 28 \times 10^7/2 = 6 \cdot 4 \times 10^6 m$$

$g = \frac{GM}{r^2}$ ∴ mass of planet B, $M = \frac{gr^2}{G}$

$$M = \frac{9 \cdot 8 \times (6 \cdot 4 \times 10^6)^2}{6 \cdot 67 \times 10^{-11}} = 6 \cdot 0 \times 10^{24} kg$$

Escape velocity $= V_e = \sqrt{2gR} = \sqrt{2 \times 9 \cdot 8 \times 6 \cdot 4 \times 10^6} = 1 \cdot 12 \times 10^4 ms^{-1}$

Planet C

Mass of planet $M = 7.4 \times 10^{22} kg$

Mass of ball $m = 10kg$; weight of ball $W = 16N$

From $W = mg$, $\quad g = \frac{W}{m} = \frac{16}{10} = 1.6ms^{-2}$

From $g = \frac{GM}{r^2}$, $\quad r^2 = \frac{GM}{g}$

$$\text{Radius } r = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67 \times 10^{-11} \times 7.4 \times 10^{22}}{1.6}} = \sqrt{3.08 \times 10^{12}} = 1.76 \times 10^6 m$$

Diameter, $= 2r = 2 \times 1 \cdot 76 \times 10^6 = 3 \cdot 5 \times 10^6 m$

Escape velocity, $V_e = \sqrt{2gR} = \sqrt{2 \times 1 \cdot 6 \times 1 \cdot 76 \times 10^6} = \sqrt{5 \cdot 63 \times 10^6}$

$$= 2373ms^{-1}$$

Table 1.1 is redrawn with the calculated values for planet A, B and C as shown in table 1.2 below.

	Planet A	Planet B	Planet C
Mass	$2.0 \times 10^{27} kg$	$6.0 \times 10^{24} kg$	$7.4 \times 10^{22} kg$
Diameter	$1.84 \times 10^{8} m$	$1.28 \times 10^{7} m$	$7.4 \times 10^{22} kg$
Weight of $10kg$ ball	$157.60N$	$98N$	$16N$
Escape velocity	$5.39 \times 10^{4} m/s$	$1.12 \times 10^{4} m/s$	$2373 m/s$

Table 1.2

EXERCISE 1

1. Given that the gravitational constant is $7 \times 10^{-11} Nm^2kg^{-2}$, what is the force of the attraction between $10^6 kg$ mass of lead hanging one meter away from a $10^3 kg$ mass of iron? *WAEC* 1990[36] *Ans*: $7 \times 10^{-2} N$

2. Two object of masses $80kg$ and $50kg$ are separated by a distance $0 \cdot 2m$. If the gravitational constant is $6 \cdot 6 \times 10^{-11} Nm^2kg^{-2}$, calculate the gravitational attraction between them. *WAEC* 1991[42] *Ans*: $6 \cdot 6 \times 10^{-6} N$

3. Given that the gravitational constant is G, Newton's universal law of gravitation states that the force of attraction between two masses M_1 and M_2 separated by a distance r is A. $Gr^2M_1M_2$ B. GrM_1M_2 C. $\frac{GM_1M_2}{r^2}$ D. $\frac{GM_1M_2}{r}$ *WAEC* 2004[42] *Ans*: *C*

4. Two spheres of masses $5 \cdot 0kg$ and $10 \cdot 0kg$ are $0 \cdot 3m$ apart calculate the force of attraction between them. A. $4 \cdot 00 \times 10^2 N$ B. $3 \cdot 57 \times 10^{-2} N$ C. $3 \cdot 71 \times 10^{-8} N$ D. $3 \cdot 50 \times 10^{-10} N$ $[G = 6 \cdot 67 \times 10^{-11} Nm^2kg^{-2}]$ *JAMB* 2007[35] *Ans*: $3 \cdot 71 \times 10^{-8} N$

5. Calculate the force on a mass of $10kg$ placed on the earth's surface. (Radius of the earth= $6 \cdot 4 \times 10^6 m$, mass of the earth $= 6 \cdot 0 \times 10^{24} kg$,) $G = 6 \cdot 7 \times 10^{-11} Nm^2kg^{-2}$ *NECO* 2006[39] *Ans*: $98N$

6. Calculate the gravitational potential at a point on the earth's surface. (Radius of the earth $= 6 \cdot 4 \times 10^6 m$, mass of the earth $= 6 \cdot 0 \times 10^{24} kg$, $G = 6.7 \times 10^{-11} Nm^2kg^{-2}$) *NECO* 2006[E7] *Ans*: $6 \cdot 28 \times 10^7 Jkg^{-1}$

7. What is the gravitational potential due to a molecule of mass m at a distance r from it? (G = gravitational constant) $A. \frac{Gm^2}{r^2}$ $B. \frac{Gm}{r}$ $C. \frac{Gm^2}{r}$ $D. \frac{G^2m^2}{r}$ $E. \frac{m^2}{Gr^2}$ *WAEC* 1989[37] *Ans*: *B*

8. What is the gravitational potential due to a point mass m at a distance from it? (G = gravitatonal constant) $A. \frac{-Gm^2}{r^2}$ $B. \frac{-Gm^2}{r}$ $C. \frac{-Gm^2}{Gr^2}$ $D. \frac{-Gm}{r}$ $E. \frac{-Gm}{2}$ *WAEC* 1993[34] *Ans*: *D*

9. The force experienced by an object of mass $60 \cdot 0kg$ in the moon's gravitational field is $1 \cdot 002 \times 10^2 N$. What is the intensity of the gravitational field? A. $0.60Nkg^{-1}$ B. $1.67Nkg^{-1}$ C. $6.12 \times 10^2 Nkg^{-1}$ D. $9.81ms^{-2}$ *JAMB* 1993[7] Ans: $1.67Nkg^{-1}$

10. A missile weighing $400N$ on the earth's surface is short into the atmosphere to an altitude of $6 \cdot 40 \times 10^6 m$. taking the earth as a sphere of radius $6 \cdot 4 \times 10^6 m$ and assuming inverse square law of universal gravitation. What would be the weight of the missile at the altitude? *WAEC* 1997[7] *Ans*: $100N$

11. A body moves in a circular orbit of radius $4R$ round the earth. Express the acceleration of free fall due to gravity of the body in terms of g. [R = radius of the earth, g = accelerationof free fall due to gravity]
$A.\frac{16}{g}$ $B.\frac{g}{16}$ $C.\frac{4}{g}$ $D.\frac{g}{4}$ $E.\frac{g}{8}$ $NECO\ 2007$[38] $Ans: g/16$
12. The gravitational force of the moon is one-sixth that of the earth. If a body weighs $8\cdot 0N$ on the moon, calculate its weight on the earth. $NECO\ 2004$[41] $Ans: 48N$
13. Calculate the escape velocity for a rocket fired from the earth's surface at a point where the acceleration due to gravity is $10ms^{-2}$ and the radius of the earth is $6\times 10^6 m$ $WAEC\ 1989$[38] $Ans: 1\cdot 1\times 10^4\ m/s$
14. Calculate the escape velocity of a satellite launched from the earth's surface.(*Take g as* $10ms^{-2}$ *and the radius of the earth as* $6.4\times 10^6 m$)
$WAEC\ 1993$[33] $Ans: 1.13\times 10^4 ms^{-1}$
15. What is the escape velocity of a body on the surface of the earth of radius R if the gravitational constant is G and the mass of the earth is M?(Neglet energy losses to the surrounding) $A. \sqrt{2GR}$ $B. \sqrt{\frac{2G}{R}}$ $C. \sqrt{2GR^2}$ $D. \sqrt{\frac{2GM}{R}}$ $E. \sqrt{2GM}$
$WAEC\ 1995$[35] $Ans: D.$ $Hint\ V_e = \sqrt{2gR}\ ;\ g = \frac{GM}{R^2}$
16. A rocket is launched from the surface of the earth. If the radius of the earth is $6\cdot 4\times 10^6 m$ and the acceleration of free fall due to gravity is $10ms^{-1}$ calculate the escape velocity of the rocket. $WAEC\ 1996$[5] $Ans: 1\cdot 13\times 10^4 ms^{-1}$
17. A rocket of mass m is fired from the earth surface such that it just escapes from the earth's gravitational field. If R is the radius of the earth and g the acceleration of free fall due to gravity, the escape velocity of the rocket is expressed as
$A.\sqrt{2gR}$ $B.\sqrt{R/2g}$ $C.\sqrt{2g/R}$ $D.\sqrt{R^2/2g}$ $WAEC1999$[35] $Ans: A$
18. Calculate the escape velocity of a satellite from the earth's surface. Take g as $10ms^{-2}$ and the radius of the earth as $6\cdot 0\times 10^6 m$
$NECO\ 2000$[39] $Ans: 1\cdot 1\times 10^4 ms^{-1}$
19. Calculate the escape velocity of a satellite from the earth's attraction. [$g = 10ms^{-2}$, radius of the earth $= 6\cdot 4\times 10^6 m$] $NECO\ 2005$[40] $Ans: 1.1\times 10^4 ms^{-1}$
20. The escape velocity of a rocket at location R meters, above the surface of the earth of mass M and radius R, is given by
A. $\sqrt{2GMR}$ B. $\frac{\sqrt{2GM}}{R}$ C. $\sqrt{\frac{2GM}{R}}$ D. $\sqrt{\frac{GM}{2R}}$ E. $\sqrt{\frac{GM}{R}}$
$NECO\ 2008$[16] $Ans: C$
21. The values of x, y *and* z respectively in the expression $M^x L^y T^z$ for the universal gravitational constant G are
A. 2, -3, -2 B. -1, 3, -2 C. -1, 2, -3 D. -2, -1, 3 *JAMB 2009*[7] *Ans: B*
22. The gravitational field intensity at a location X, in space, is two-fifths of its value on the earth's surface. If the weight of an object at X is 4.80N, what is its weight on the earth? NECO 2009[39] Ans: 12.00N

2

ELECTRIC FIELD: STATIC ELECTRICITY

Static electricity or electrostatics is the study of charges at rest. A charge is an atom that has lost or gain electron. An electric field is a region or space where a system of electric charges experiences an electric force.

COULOMB'S LAW

Electric force between two charges is governed by *Coulomb's law* which states that the force of repulsion or attraction between two point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between the two charges.

That is, $F \propto \frac{q_1 q_2}{r^2}$ or $F = \frac{K q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q_1 q_2}{r^2}$

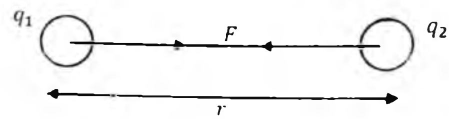


Fig. 2.1

Where F = magnitude of electric force in Newton (N)

q_1, q_2 = charges in coulombs (C)

r = Distance between charges in meter (m)

$K = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 Nm^2C^{-2}$ or $9 \times 10^9 mF^{-1}$

ε_o = Permittivity of free space = $8.85 \times 10^{-12} C^2N^{-1}m^{-2}$ or $8.85 \times 10^{-12} Fm^{-1}$

Example 1

An electron is at a distance of $5.0 \times 10^{-11} m$ away from the center of a proton. Calculate the electrostatic force on it. [Electron charge = $-1.6 \times 10^{-19} C$, proton charge = $+1.6 \times 10^{-19} C$, $\frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 Nm^2C^{-2}$]

Solution

Distance $r = 5.0 \times 10^{-11} m$; $q_1 = q_2 = 1.6 \times 10^{-19} C$, $\frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 Nm^2C^{-2}$

Electrostatic force $F = \frac{1}{4\pi\varepsilon_o} \times \frac{q_1 q_1}{r^2}$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{(5 \times 10^{-11})^2} = \frac{2.304 \times 10^{-28}}{2.5 \times 10^{-21}}$$

$$= 9.2 \times 10^{-8} N$$

$9.2 \times 10^{-8} N$ is a force of attraction

Example 2

If the distance between two point charges is increased by a factor of 4, the magnitude of the electrostatic force between them will be A. $\frac{1}{2}$ of its former value. B. $\frac{1}{4}$ of its former value. C. $\frac{1}{16}$ of its former value. D. 4 times its former value. [JAMB 2005]

Solution

The inverse square law $F = \frac{1}{r^2}$ relates electrostatic force (F) and distance (r).

$r = 4$ (- - - *distance* - - *is increased by a factor of four*)

Therefore, electrostatic force $F = \frac{1}{r^2} = \frac{1}{4^2} = \frac{1}{16}$

ELECTRIC FIELD INTENSITY

Electric field intensity (E) also known as the strength of an electric field at any point, is defined as the force (F) experienced by a unit positive test charge (q) at that point.

Electric force = Electric charge × Electric field intensity,

$$F = qE$$

$\therefore$ Electric field intensity $E = \frac{F}{q}$ *or* $E = \frac{q}{4\pi\varepsilon_o r^2}$ *or* $E = \frac{Kq}{r^2}$

The unit of electric field intensity is Newton per coulomb, NC^{-1}

Example 3

In a uniform electric field, the magnitude of the force on a charge of $0.2C$ is $4N$. Calculate the electric field intensity. *WAEC* 1999[37]

Solution

Force; $F = 4N$; charge $q = 0.2C$

$$\text{Electric field intensity } E = \frac{F}{q} = \frac{4N}{0 \cdot 2C} = 20NC^{-1}$$

Example 4

Calculate the magnitude of the electric field intensity in vacuum at a distance of $25cm$ from a charge of $5 \times 10^{-3}C$. $\left[Take\ \frac{1}{4\pi\varepsilon_o}\ as\ 9 \times 10^9 Nm^2C^{-2}\right]$

NECO 2004[42]

Solution

Distance $r = 25cm = 0.25m$; charge $q = 5 \times 10^{-3}CNm^2C^{-2}$

$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 Nm^2C^{-2}$

$$E = \frac{q}{4\pi\varepsilon_o r^2} = \frac{5 \times 10^{-3} \times 9 \times 10^9}{(0.25)^2} = 7.2 \times 10^8 NC^{-1}$$

Example 5

A charge of $1.6 \times 10^{-10}C$ is placed in a uniform electric field of intensity $2.0 \times 10^5 NC^{-1}$, what is the magnitude of the electric force exerted on the charge?

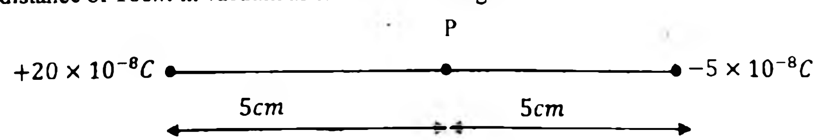
WAEC 1995[36] **Solution**

Charge $q = 1.6 \times 10^{-10}C$; $E = 2.0 \times 10^5 NC^{-1}$

Electric force $F = qE = 1.6 \times 10^{-10} \times 2.0 \times 10^5 = 3.2 \times 10^{-5}N$

Example 6

Two point charges of magnitude $+20 \times 10^{-8}C$ and $-5 \times 10^{-8}C$ are separated by a distance of $10cm$ in vacuum as shown in the diagram below.



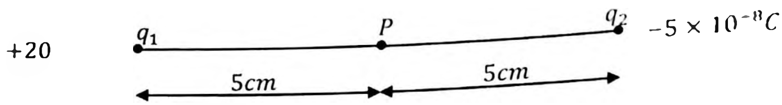
Calculate

(i) The electric field intensity at a point P, midway between the charges.

(ii) The force on a $-4 \times 10^{-8}C$ charge placed at P. NECO 2004[E14]

$\left[Take\ \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 Nm^2C^{-2}\right]$

Solution



(i) Field at P due to q_1

$$E_1 = \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{20 \times 10^{-8} \times 9 \times 10^9}{0.05^2} = 7.2 \times 10^5 NC^{-1}$$

Field at P due to q_2

$$E_2 = \frac{q_2}{4\pi\varepsilon_o r^2} = \frac{5 \times 10^{-8} \times 9 \times 10^9}{0.05^2} = 1.8 \times 10^5 NC^{-1}$$

Resultant field at P is $E = E_1 + E_2$

$E = 1.8 \times 10^5 + 7.2 \times 10^5 = 9 \times 10^5 NC^{-1}$

The field is directed towards the -5×10^{-8} Charge

$q = -4 \times 10^{-8}C;\ E = 9 \times 10^5 NC^{-1}$

Force $F = qE = 4 \times 10^{-8} \times 9 \times 10^5 = 3.6 \times 10^{-2}N$

Because the charge $(-4 \times 10^{-8}C)$ is negative, it's directed toward the positive charge $(+20 \times 10^{-8}C)$.

Example 7

A student is at a height $4m$ above the ground during a thunderstorm. Given that the potential difference between the thunder cloud and the ground is 10^7V, the electric field created by the storm is $A. 2.0 \times 10^6 NC^{-1}$ $B. 2.5 \times 10^6 NC^{-1}$ $C. 1.0 \times 10^7 NC^{-1}$ $D. 4.0 \times 10^7 NC^{-1}$ IAMB 2001[26]

Solution

Electric field intensity, $E = \frac{V}{r}$

Potential difference $V = 10^7$; distance $r = 4m$

$$E = \frac{10^7}{4} = 2.5 \times 10^6 NC^{-1}\ or\ 2.5 \times 10^6 Vm^{-1}$$

Example 8

The potential difference between two parallel plates of a capacitor placed 3.5cm apart is 5V. Calculate the magnitude of the electric field intensity between the two plates. NECO 2008[10]

Solution

Distance between plates, r = 3.5cm = 0.035m

Potential difference, $V = 5V$

Electrtic field intensity, $E = \frac{V}{r} = \frac{5V}{0.035m} = 142.86Vm^{-1}$

Example 9

A charge $50\mu C$ has an electric field strength of $360NC^{-1}$ at a certain point. The electric field strength due to another charge $120\mu C$ kept at the same distance apart and in the same medium is

$A.\ 18NC^{-1}$ $B.\ 144NC^{-1}$ $C.\ 864NC^{-1}$ $D.\ 150NC^{-1}$ JAMB 2007[44]

Solution

Charge $q = 50\mu C = 50 \times 10^{-6}C$; Electric field strength due to $50\mu C$ $E = 360NC^{-1}$

Electric force of the medium $F = qE = 50 \times 10^{-6} \times 360 = 0.018N$.

Electric field strength due to $120\mu C(120 \times 10^{-6}C)$Charge,

$$E = \frac{F}{q} = \frac{0.018}{120 \times 10^{-6}} = 150NC^{-1}$$

ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

Electric potential (V) at a point is defined as the work done by an electric field in bringing a unit positive charge from infinity to that point.

$$\text{Electric potential } V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r} \text{ or } V = K\frac{q}{r}$$

Example 10

Calculate the electric potential at a distance of $20.0cm$ from a point charge of $0.015C$ placed in air of permittivity. $\left[Take\ \frac{1}{4\pi\varepsilon_o} as\ 9.0 \times 10^9 Nm^2C^{-2}\right]$. WAEC 1995[40]

Solution

Distance $r = 20.0cm = 0.2m$; charge $q = 0.015C$; $\frac{1}{4\pi\varepsilon_o} = 9.0 \times 10^9 Nm^2C^{-2}$

$$\text{Electric potentail } V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r} = 9 \times 10^9 \times \frac{0.015}{0.2}$$

$$V = \frac{1.35 \times 10^8}{0.2} = 6.75 \times 10^8 V$$

Electric potential difference between point A and point B (V_{AB}) is defined as the work done in moving a unit positive charge from A to B.

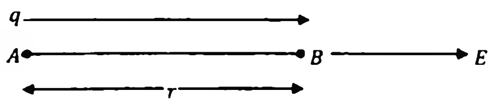


Fig. 2.2

Work done = charge $\times$ $p.d$ between A and B

$W = qV$ ------- (1)

$W = q(V_A - V_B)$

In general, work done = $force \times distance$

Therefore, work done = electric force between A and B $\times$ distance betwen A and B

$$W = F \times r$$

Remember, $F = qE$ $(E = electric\ field\ intensity)$

$W = q \times E \times r$ ------- (2)

i.e. work done = $charge \times field\ intensity \times distance$

Equating both work done in equation in 1 and 2 we obtain

$$q \times E \times r = qV$$

$$E \times r = V$$

$$E = \frac{V}{r}$$

This equation sums up the relationship between electric field intensity E, and electric potential V. Unit of field intensity is also Vm^{-1}

Example 11
If two charged plates are maintained at a *potential difference* of 3 KV, the work done in taking a charge of $600\mu C$ across the field is $JAMB$ 2007[40]
A. $1.8J$ B. $18.0J$ C. $0.8J$ D. $9.0J$
Solution
Potential difference, $V = 3KV = 3000V$; charge $q = 600\mu C = 600 \times 10^{-6} C$

Work done = *charge* × *potential difference*

$$W = qV = 600 \times 10^{-6} \times 3000 = 1.8J$$

Example 12
A work of 30 *joules* is done in transferring 5 millicoulombs of charge from a point B to a point A in an electric field. The potential difference between B and A is? $WAEC$ 1990[39]
Solution
Work done $W = 30J$; *charge* $q = 5$ *millicoulombs* $= 5 \times 10^{-3} C$;

Work done = *charge* × *p.d.*

$$W = q \times V \quad \therefore \quad V = \frac{W}{q} = \frac{30}{5 \times 10^{-3}} = 6000V$$

Example 13
A point charge of magnitude $2.5\mu C$ is moved through a distance of $0.04m$ against a uniform electric field of intensity $15Vm^{-1}$. Calculate the work done on the charge.
$NECO$ 2007[41]
Solution
Charge, $q = 2.5\mu C = 2.5 \times 10^{-6} C$; distance $r = 0.04m$;

field intensity $E = 15Vm^{-1}$

Work done $W = q \times E \times r = 2.5 \times 10^{-6} \times 15 \times 0.04 = 1.5 \times 10^{-6} J$

Example 14
Two parallel plates at a distance of $8.0 \times 10^{-3} m$ apart are maintained at a potential difference of 600 volts with the negative plate earthed. What is the electric field strength?
A. $4.8Vm^{-1}$ B. $75.0Vm^{-1}$ C. $4800.0Vm^{-1}$ D. $75000.0Vm^{-1}$ $JAMB$ 1995[37]
Solution
Potential difference, $V = 600V$; distance $r = 8.0 \times 10^{-3} m$

Electric field intensity $E = \frac{V}{r} = \frac{600}{8.0 \times 10^{-3}} = 75000Vm^{-1}$

ELECTRON VOLT
Electron volt is defined as the quantity of energy gained by an electron when it accelerates through a potential difference of 1 volt. The unit of electron volt is joules.

Work done by electron = *charge* × *potential difference*

Kinetic energy $(K.E) = q \times V$

$$\frac{1}{2}mv^2 = q \times V$$

Where m = mass of electron in kilogram (kg)

v = velocity of electron in ms^{-1}

V = potential difference

q = magnitude of electron charge

Example 15

An electron of charge $1.6 \times 10^{-19}C$ and mass $9.1 \times 10^{-31}kg$ is accelerated between 2 metal plates with a velocity of $4 \times 10^7 ms^{-1}$, the potential difference between the plate is A. 4.55×10^1V B. 9.10×10^1V C. 4.55×10^2V D. 4.55×10^3V JAMB 2000[28]

Solution

Charge $q = 1.6 \times 10^{-19}C$; $mass\ m = 9.1 \times 10^{-31}kg$; Velocity $v = 4 \times 10^7 ms^{-1}$; $p.d\ V = ?$

From, $\frac{1}{2}mv^2 = q \times V$

Potential difference, $V = \frac{\frac{1}{2}mv^2}{q} = \frac{0.5 \times 9.1 \times 10^{-31} \times (4 \times 10^7)^2}{1.6 \times 10^{-19}}$

$$= \frac{7.28 \times 10^{-16}}{1.6 \times 10^{-19}} = 4.55 \times 10^3 V$$

Example 16

An electron of mass m and charge e enters a uniform electric field between two metal plates P and Q separated by a distance d. P is maintained at a potential V while Q is earthed. Determine an expression for the magnitude of the acceleration of the electron through the field $A. \frac{eV}{md}$ $B. \frac{d}{meV}$ $C. \frac{md}{eV}$ $D. \frac{e}{Vmd}$ $WAEC$ 2006[39]

Solution

a = acceleration; F = electric force; q = charge = e

E = electric field intensity; m = mass; r = distance = d;

V = potential difference

$F = ma$ $\quad F = qE$ $\quad E = \frac{V}{r}$

From $F = ma$ and substituting above equations, we obtain

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{q \times {}^{V}/_{r}}{m}$$

Substitute q with e and r with d as given in the question.

$$a = \frac{e \times {}^{V}/_{d}}{m} = e \times \frac{V}{d} \div m = e \times \frac{V}{d} \times \frac{1}{m} = \frac{eV}{md}$$

CAPACITORS AND CAPACITANCE

Capacitors are devices for storing electric charges. The capacitance (C) of a capacitor is defined as the ratio of charge (Q) stored by the capacitor to the potential difference (V) between the plates.

$$C = \frac{Q}{V}$$

Capacitance is measured in farads(F).

1 microfarads = $1\mu F = 10^{-6}F\ or\ 1 \times 10^{-6}F$

1 nanofarads = $1nF = 10^{-9}F$ or $1 \times 10^{-9}F$
1 picofarads = $1pF = 10^{-12}F$ or $1 \times 10^{-12}F$

Capacitance (C) of parallel plate capacitor is also given by;

$$C = \frac{\varepsilon A}{d}$$

Where A = area of overlap of plates (m^2)
d = distance between plates (m)
ε = permittivity of material between plates Fm^{-1} or $C^2N^{-1}m^{-2}$

Example 17

A parallel plate capacitor of capacitance $600\mu F$ has a *potential difference* of $2000V$ between its plate. Calculate the charge on either plate of the capacitor.

Solution

Capacitance $C = 600\mu F = 600 \times 10^{-6}F$

Potential difference $V = 2000V$

From $C = \frac{Q}{V}$, $Q = CV = 600 \times 10^{-6} \times 2000 = 1.2C$

Example18

A parallel plate capacitor of area $10cm^2$ in vacuum has a capacitance of $10^{-2}\mu F$. what is the distance between the plates? $[\varepsilon_o = 9 \times 10^{-12}Fm^{-1}]$

A. $9 \times 10^{-13}m$ B. $9 \times 10^{-7}m$ C. $9 \times 10^{-3}m$ D. $9 \times 10^{7}m$ *JAMB* 1993[40]

Solution

Area $A = 10cm^2 = 10 \times 10^{-4}m^2$ permittivity $\varepsilon_o = 9 \times 10^{-12}Fm^{-1}$

Capacitance $C = 10^{-2}\mu F = 10^{-2} \times 10^{-6}F$

From $C = \frac{\varepsilon_o A}{d}$, distance, $d = \frac{\varepsilon_o A}{C} = \frac{9 \times 10^{-12} \times 10 \times 10^{-4}}{10^{-2} \times 10^{-6}} = 9 \times 10^{-7}m$

SERIES AND PARALLEL ARRANGEMENT OF CAPACITORS

Parallel Connection

When capacitors are arranged in parallel as shown below, the following apply

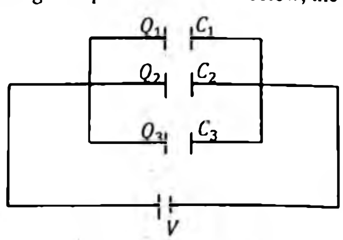


Fig. 2.3

1. The equivalent or combined capacitance C, is given by: $C = C_1 + C_2 + C_3$
2. C_1, C_2 and C_3 are all at the same potential difference V
3. Total circuit charge, $Q = Q_1 + Q_2 + Q_3$ where $Q_1 = C_1V$, $Q_2 = C_2V$, $Q_3 = C_3V$

Example 19

What is the equivalent capacitance of a $3\mu F$ capacitor and a $6\mu F$ capacitor connected in parallel? *WAEC* 2006[41]

Solution

Equivalent capacitance $C_T = C_1 + C_2 = 3\mu F + 6\mu F = 9\mu F$

Example 20

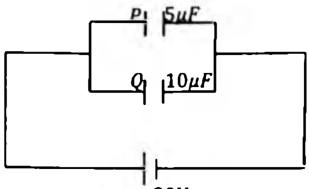


Fig. 2.4

The diagram above shows two capacitors P and Q of capacitances $5\mu F$ and $10\mu F$. Find the charges stored in P and Q respectively. A. $200\mu C$ and $100\mu C$
B. $100\mu C$ and $200\mu C$ C. $4\mu C$ and $2\mu C$ D. $2\mu C$ and $4\mu C$ *JAMB* 2000[30]

Solution

From $C = \frac{Q}{V}$, charge $Q = CV$

$Q_P = C_PV = 5\mu F \times 20 = 5 \times 10^{-6} \times 20 = 100 \times 10^{-6}C = 100\mu C$

$Q_Q = C_QV = 10\mu F \times 20 = 10 \times 10^{-6} \times 20 = 200 \times 10^{-6}C = 200\mu C$

Series Connection

When capacitors are arranged in series as shown below, the following apply

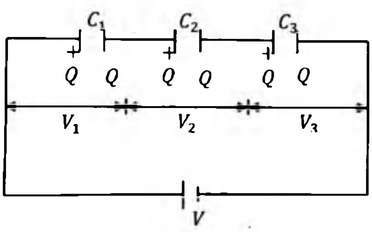


Fig. 2.5

1. The equivalent or combined capacitance (C) *is*, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Equivalent capacitance for two capacitors C_1 and C_2 is, $C = \frac{C_1C_2}{C_1 + C_2}$

2. Capacitors C_1, C_2 and C_3 all have the same charge Q.

3. Total circuit voltage, $V = V_1 + V_2 + V_3$ where $V_1 = V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}$

Example 21

Three capacitors each of capacitances $18\mu F$ are connected in series. Calculate the effective capacitance of the capacitors. *WAEC* 2002[36]

Solution

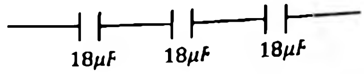


Fig 2.6

$$\frac{1}{C} = \frac{1}{18} + \frac{1}{18} + \frac{1}{18} = \frac{1+1+1}{18} = \frac{3}{18}$$

$$\frac{1}{C} = \frac{3}{18} \qquad \therefore \quad C = \frac{18}{3} = 6\mu F$$

Example 22

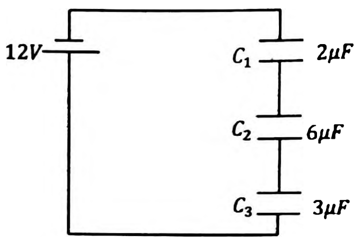


Fig. 2.7

The diagram above shows three capacitors C_1, C_2 and C_3 of capacitances $2\mu F, 6\mu F$ and $3\mu F$ respectively. The potential differences across C_1, C_2 and C_3 respectively are

A. $4V, 6V$ and $2V$ B. $2V, 6V$ and $4V$ C. $6V, 4V$ and $2V$ D. $6V, 2V$ and $4V$

JAMB 2001[25]

Solution

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = \frac{3+1+2}{6}$$

$\frac{1}{C} = \frac{6}{6} = 1$ $\therefore$ equivalent capacitance $C = 1$

Circuit charge $Q = CV = 1 \times 12 = 12C$

From $C = \frac{Q}{V}$, $\quad p.d, V = \frac{Q}{C}$

$V_1 = \frac{Q}{C_1} = \frac{12}{2} = 6V;$ $\quad V_2 = \frac{Q}{C_2} = \frac{12}{6} = 2V;$ $\quad V_3 = \frac{Q}{C_3} = \frac{12}{3} = 4V$

Ans: $6V, 2V$ and $4V$

Example 23

Two capacitance of $6\mu F$ and $8\mu F$ are connected in series. What additional capacitance must be connected in series with this combination to give a total of $3\mu F$?

A. $3\mu F$ B. $16\mu F$ C. $24\mu F$ D. $30\mu F$ *JAMB* 1987[40]

Solution

Let C_T = total capacitance = $3\mu F$, x = additional capacitance

$$\frac{1}{C_T} = \frac{1}{6} + \frac{1}{8} + \frac{1}{x}$$

$$\frac{1}{3} - \frac{1}{6} - \frac{1}{8} = \frac{1}{x}$$

$$\frac{8-4-3}{24} = \frac{1}{x}$$

$$\frac{1}{24} = \frac{1}{x} \quad \therefore x = 24\mu F$$

Combined Series And Parallel Connection

Example 24

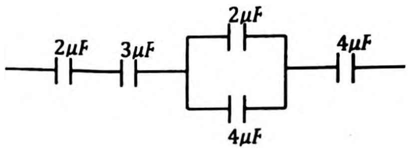


Fig. 2.8

The resultant capacitance in the figure above is A. $15.0\mu F$ B. $9.8\mu F$ C. $1.3\mu F$ D. $0.8\mu F$ *JAMB* 1995[43]

Solution

Capacitor $2\mu F, 3\mu F$ and $4\mu F$ are in series with one another and also in series with a parallel combination of capacitors $2\mu F$ and $4\mu F$.

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{(2+4)} + \frac{1}{4}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{4}$$

$$\frac{1}{C} = \frac{6+4+2+3}{12} = \frac{15}{12}$$

$$\frac{1}{C} = \frac{15}{12} \quad \therefore \quad C = \frac{12}{15} = 0.8\mu F$$

Example 25

What is the total capacitance in the circuit represented by the diagram below?

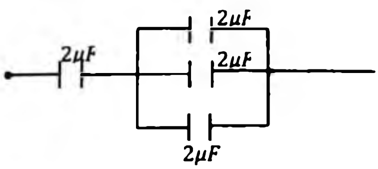


Fig. 2.9 *NECO* 2000[46]

Solution

In parallel: $2\mu F + 2\mu F + 2\mu F = 6\mu F$. $6\mu F$ in series with $2\mu F$

$$\therefore C = \frac{6 \times 2}{6+2} = \frac{12}{8} = 1.5\mu F$$

Example 26

The effective capacitance between points X and Y in the diagram below is $1.50\mu F$

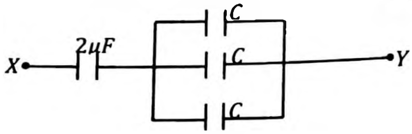


Fig. 2.10

What is the value of the capacitance C? *NECO* 2002[43]

Solution

In parallel: $C + C + C = 3C$. Then $3C$ in series with $2\mu F$

Total capacitance, $1.50 = \dfrac{2 \times 3C}{2 + 3C} = \dfrac{6C}{2 + 3C}$

Cross multiply $1.50 = \dfrac{6C}{2 + 3C}$ to obtain

$$6C = 1.5(2 + 3C)$$
$$6C = 3 + 4.5C$$
$$6C - 4.5C = 3$$
$$1.5C = 3 \qquad \therefore\ C = \frac{3}{1.5} = 2\mu F$$

Example 27

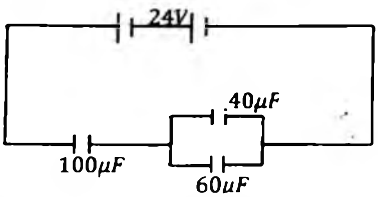


Fig. 2.11

Calculate the current in the circuit diagram above. *NECO* 2007[E8]

Solution

Combined parallel capacitance $= 40 + 60 = 100\mu F$

$100\mu F$ in series with $100\mu F$

$\therefore$ Total combined capacitance $C = \dfrac{100 \times 100}{100 + 100} = \dfrac{10000}{200} = 50\mu F$

From $C = \dfrac{Q}{V}$, charge, $Q = CV = 50 \times 10^{-6} \times 24 = 1.2 \times 10^{-3}C$

Current $I = \dfrac{Q}{t} = \dfrac{1.2 \times 10^{-3}A}{t}$

Example 28

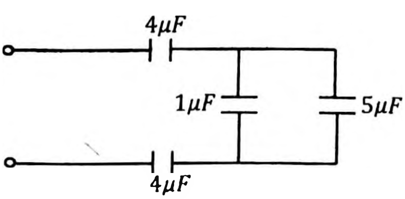


Fig. 2.12

What is the resultant capacity circuit above?

A. $1.5\mu F$ B. $18.0\mu F$ C. $6.0\mu F$ D. $6.8\mu F$ E. $8.83\mu F$ *JAMB* 1982[26]

Solution

$1\mu F$ and $5\mu F$ are in parallel, therefore combined capacitance $= 1 + 5 = 6\mu F$

$6\mu F$ is in series with $4\mu F$ and $4\mu F$

$$\therefore \frac{1}{C} = \frac{1}{6} + \frac{1}{4} + \frac{1}{4} = \frac{4 + 6 + 6}{24} = \frac{16}{24}$$

$$\frac{1}{C} = \frac{16}{24} \quad \therefore C = \frac{24}{16} = \frac{3}{2} = 1.5\mu F$$

Example 29

In the network shown below, determine the $p.d$ across the $5\mu F$ capacitor.

A. $3V$ B. $6V$ C. $12V$ D. 18 *JAMB* 1993[41]

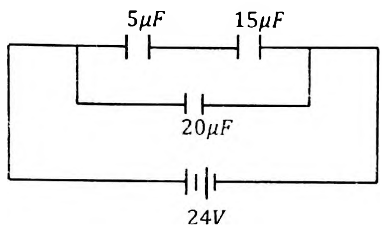


Fig. 2.13

Solution

$5\mu F$ *and* $15\mu F$ are series $\therefore \quad C = \frac{5 \times 15}{5 + 15} = \frac{75}{20} = 3.75$

Charge following through series capacitors is:

$Q = CV = 3.75 \times 24 = 90\mu C$

The same charge ($90\mu C$) flow through $5\mu F$

$\therefore$ potential difference across $5\mu F$, $\quad V = \frac{Q}{C} = \frac{90\mu C}{5\mu F} = 18V$

ENERGY STORED IN A CAPACITOR

Energy stored in a capacitor is equivalent to the work done in charging the capacitor.

Energy store in capacitor $W = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{1}{2} \cdot \frac{q^2}{C}$

Any of these equations can be used depending on the term giving in the question.

Example 30

Calculate the energy stored in a $20\mu F$ capacitor if the potential difference between the plates is $40V$. *WAEC* 2004[43]

Solution

Capacitance $C = 20\mu F = 20 \times 10^{-6} F$; potential difference $V = 40V$

$$Energy\ W = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times 40^2 = 1.6 \times 10^{-2} J$$

Example 31

The energy stored in a capacitor of capacitance $5\mu F$ is $40J$. What is the voltage applied across its terminal? *NECO 2002*[42]

Solution

capacitance $C = 5\mu F = 5 \times 10^{-6}F$ energy, $W = 40J$

$$W = \frac{1}{2}CV^2$$

$$40 = 0.5 \times 5 \times 10^{-6} \times V^2$$

$$V^2 = \frac{40}{0.5 \times 5 \times 10^{-6}} = 1.6 \times 10^7$$

$$V = \sqrt{1.6 \times 10^7} = 4 \times 10^3 V$$

Example 32

Use the diagram shown below to answer the following question.

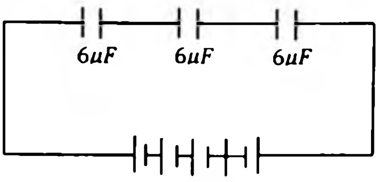


Fig. 2.14

1. What is the effective capacitance in the circuit?
2. What is the total energy stored by the capacitors?

Solution

(1). Effective capacitance $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$$\frac{1}{C} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$

$$\frac{1}{C} = \frac{1+1+1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{1}{C} = \frac{1}{2} \qquad \therefore \qquad C = 2\mu F$$

(ii) $C = 2\mu F$ potential difference, $V = 100V$

Total energy $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 100^2 = 0.01$ or $1 \times 10^{-2}J$

Example 33

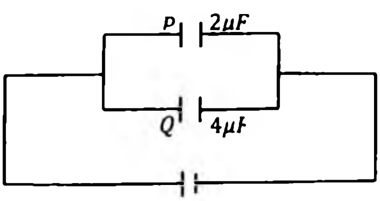


Fig. 2.15

The diagram above shows two capacitors P and Q of capacitances $2\mu F$ and $4\mu F$ respectively connected to a $d.c$ source. The ratio of energy stored in P to Q is

A. 4:1 B. 2:1 C. 1:4 D. 1:2

Solution

$W = \frac{1}{2}CV^2 \qquad \therefore \qquad W_P = \frac{1}{2}C_PV^2 \qquad W_Q = \frac{1}{2}C_QV^2$

$$\frac{W_P}{W_Q} = \frac{\frac{1}{2}C_PV^2}{\frac{1}{2}C_QV^2} = \frac{C_P}{C_Q} = \frac{2\mu F}{4\mu F} = \frac{1}{2} \qquad \therefore \qquad W_P : W_Q = 1:2$$

Example 34

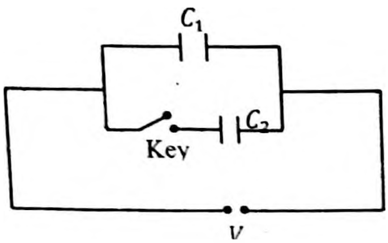


Fig. 2.16

Two capacitors C_1 and C_2 are connected as shown in the diagram above. The capacitance of C_2 is twice C_1. When the key is opened, the energy stored up in C_1 is W. If the key is later closed and the system is allowed to attain electrical equilibrium, the total energy stored in the system will be A. $\frac{1}{2}W$ B. $\frac{2}{3}W$ C. W D. $2W$ E. $3W$

WAEC 1991[43]

Solution

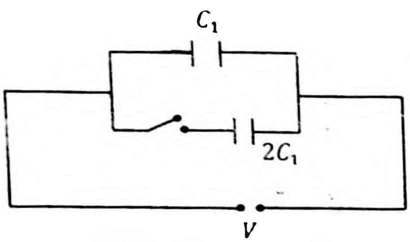


Fig 2.17

$C_2 = 2C_1$ (······ *capacitance of C_2 is twice C_1* ······)

When the key is opened, capacitor $2C_1$ is cut off.

$\therefore$ Energy in C_1 is $W = \frac{1}{2}C_1V^2$

When the key is closed both capacitors are involved.

Total capacitance $= 2C_1 + C_1 = 3C_1$

$\therefore$ Energy stored at equilibrium $= \frac{1}{2} \times 3C_1V^2$

Rearranging, we obtain, $3\left(\frac{1}{2}C_1V^2\right)$

Substituting $W = \frac{1}{2}C_1V^2$, energy stored at equilibrium $= 3W$

Example 35

The plates of a parallel plate capacitor are $5mm$ apart and $2m^2$ in area. The plates are in vacuum. A potential difference of $1000V$ is applied across the capacitor calculate the

(i) Capacitance

(ii) Charge on each plate

(iii) Electric intensity in the space between them

(iv) Energy stored in the capacitor.

(permittivity of free space $\varepsilon_o = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$) $NECO\ 2000^{E14}$

Solution

Distance apart, $d = 5mm = 0.005m$; area, $A = 2m^2$;
potential difference, $V = 1000V$; $\varepsilon_o = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$

(i) Capacitance, $C = \frac{\varepsilon_{oA}}{d} = \frac{8.85 \times 10^{-12} \times 2}{0.005} = \frac{1.77 \times 10^{11}}{0.005} = 3.54 \times 10^{-9}F$

(ii) Charge $Q = CV = 3.54 \times 10^{-9} \times 1000 = 3.54 \times 10^{-6}C$

(iii) Electric intensity $E = \frac{V}{d} = \frac{1000}{0.005} = 2 \times 10^5 Vm^{-1}$

(iv) Energy stored $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 3.54 \times 10^{-9} \times 1000^2 = 1.77 \times 10^{-3}J$

Example 36

An isolated electrically charged sphere of radius, r, and charge , Q, is supported on an insulator in air of permittivity, ε_o.

a Write down

(i) An expression for the electric field intensity on the surface of the sphere.

(ii) An expression for the electric potential at the surface of the sphere .

(iii) A relationship between the electric field intensity and the electric potential at the surface of the sphere.

b. The plates of a parallel plate capacitor, $5.0\times 10^{-3}m$ apart are maintained at a potential difference of $5.0 \times 10^4 V$. Calculate the magnitude of the

(i) Electric field intensity between the plate

(ii) Force on the electron

(iii) Acceleration of the electron

[Electric charge = $1.60 \times 10^{-19}C$]

[Mass of electron = $9.1 \times 10^{-31}kg$] $WAEC\ 2005^{E14}$

Solution

a. (i) Electric field intensity, $E = \frac{Q}{4\pi\varepsilon_o r^2}$

(ii) Electric potential, $V = \frac{Q}{4\pi\varepsilon_o r}$

(iii) $E = \frac{V}{r}$

b. Distance between plate, $r = 5.0 \times 10^{-3}m$; potential difference, $V = 5.0 \times 100^4 V$;

electron charge, $q = 1.6 \times 10^{-19}C$; mass of electron, $m = 9.1 \times 10^{-31}kg$

(i) Electric field intensity $E = \frac{V}{r} = \frac{5.0 \times 10^4}{5.0 \times 10^{-3}} = 1 \times 10^7 Vm^{-1}$

(ii) Force on electric $F = qE = 1.6 \times 10^{-19} \times 1 \times 10^7 = 1.6 \times 10^{-12}N$

(iii) From $F = ma$, acceleration of electron,

$$a = \frac{F}{m} = \frac{1.6 \times 10^{-12}}{9.1 \times 10^{-31}} = 1.76 \times 10^{18}ms^{-2}$$

Example 37

The capacitance of a parallel plate capacitor when in air is $3\mu F$ and in the presence of a dielectric material $6\mu F$. The dielectric constant is

A. 3 B. 18 C. 2 D. 9 $JAMB\ 2006^{32}$

Solution

$$\text{Dielectric constant} = \frac{\text{dielectric of medium}}{\text{dielectric of air}} = \frac{6\mu F}{3\mu F} = 2$$

EXERCISE 2

1. The force of repulsion between two point positive charges $5\mu F$ and $8\mu C$ separated at a distance of $0.02m$ apart is A. $1.8 \times 10^{-10}N$ B. $9.0 \times 10^{-8}N$ C. $9.0 \times 10^{2}N$ D. $4.5 \times 10^{3}N$ $\left[\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}Nm^{2}C^{-2}\right]$ *JAMB* 1998[36] Ans: $9.0 \times 10^{2}N$

2. A charge of 1.0×10^{-5} coulombs experiences a force of $40N$ at a certain point in space. What is the electric field intensity? *WAEC* 1990[42] Ans: $4 \times 10^{6}NC^{-1}$

3. A point charge of $1.0 \times 10^{-7}C$ experiences a force of $0.01N$ in a uniform electric field. Calculate the magnitude of the strength of the field. *WAEC* 1993[38] Ans: $1 \times 10^{5}Vm^{-1}(NC^{-1})$

4. A charge of $1.0 \times 10^{-5}C$ experiences a force of $40N$ at a certain point in space. What is the magnitude of the electric field intensity at the point in Newton per coulomb? *WAEC* 1997[38] Ans: 4.0×10^{6}

5. A charge of $1.6 \times 10^{-10}C$ is placed in a uniform electric field of intensity $2.0 \times 10^{5}NC^{-1}$. Calculate the magnitude of the electric force exerted on the charge. *WAEC* 2004[43] Ans: $3.2 \times 10^{-5}N$

6. A charge of $2.0 \times 10^{-5}C$ experiences a force of $80N$ in a uniform electric field. Calculate the magnitude of the electric field intensity *NECO* 2006[43] Ans: $4 \times 10^{6}NC^{-1}$

7. If the force on a charge of 0.2 coulomb in an electric field is $4N$, then the electric field intensity of the field is A. 0.8 B. $0.8N/C$ C. $20.0N/C$ D. $4.2N/C$ E. $20.0C/N$ *JAMB* 1983[47] Ans: $20.0N/C$

8. The electric field intensity in a place where a charge of $10^{-10}C$ experiences a force of $0.4N$ is A. $8.0 \times 10^{-12}NC^{-1}$ B. $8.0 \times 10^{9}NC^{-1}$ C. $4.0 \times 10^{9}NC^{-1}$ D. $4.0 \times 10^{-11}NC^{-1}$ *JAMB* 2005[23] Ans: $4.0 \times 10^{9}NC^{-1}$

9. A point charge of magnitude $2\mu C$ is moved through a distance of $0.2m$ against uniform field of intensity $25Vm^{-1}$. Calculate the work done on the charge. *WAEC* 1999[40] Ans: $1 \times 10^{-5}J$

10. If $8 \times 10^{-2}J$ of work is required to move $100\mu C$ of charge from a point X to a point Y in an electrical circuit, the $p.d$ between X and Y is A. $4.0 \times 10^{2}V$ B. $4.0 \times 10^{4}V$ C. $8.0 \times 10^{2}V$ D. $8.0 \times 10^{4}V$ *JAMB* 1997[42] Ans: $8.0 \times 10^{2}V$

11. Find the work done in moving a $2C$ charge between two points X and Y in an electric field if the $p.d$ is 100 volts A. $50J$ B. $100J$ C. $200J$ D. $400J$ *JAMB* 1998[41] Ans: $200J$

12. The potential difference between two points A and B situated at a distance d apart is V. Which of the following expresses the magnitude of the electric field intensity between the two points assuming the field is uniform. A. Vd B. dV^{-1} C. Vd^{-1} D. $V^{2}d$ E. $d^{2}V$ *WAEC* 1993[39] Ans: Vd^{-1}

13. What is the magnitude of the electric field intensity between two plates $30cm$ apart, if the potential difference between the plate is $4.2V$? *NECO* 2002[40] Ans: $14.0Vm^{-1}$

14. An electron of charge $1.6 \times 10^{-19}C$ is accelerated between two metal plates. If the kinetic energy of the electron is $4.8 \times 10^{-17}J$, the potential difference between the plates is A. $30V$ B. $40V$ C. $300V$ D. $400V$ *JAMB* 2003[33] Ans: $300V$

15. A $5V$ battery is connected across the plates of a $2.0\mu F$ uncharged parallel plate capacitor. The charge on the capacitor after a long time is A. $2.5\mu C$ B. $10.0\mu C$ C. $20.0\mu C$ D. $50.0\mu C$ *JAMB* 1992[39] Ans: $10.0\mu C$

16. A parallel plate capacitor has a common plate area of $5 \times 10^{-8} m^2$ and plate separation of $2 \times 10^{-3} m$. Assuming free space, what is the capacitance?
A. $2.25 \times 10^{-17} F$ B. $4.50 \times 10^{-17} F$ C. $2.25 \times 10^{-16} F$ D. $4.50 \times 10^{-16} F$
$[\varepsilon_o = 9.0 \times 10^{-12} C^2 N^{-1} m^{-2}]$ JAMB 1998[42] Ans: $2.25 \times 10^{-16} F$

17. Two $50\mu C$ parallel plate capacitors are connected in series. The combined capacitor is then connected across a 100V battery. The charge on each plate of the capacitor is A. $5.00 \times 10^{-5} C$ B. $2.50 \times 10^{-3} C$ C. $1.25 \times 10^{-3} C$
D. $1.00 \times 10^{-2} C$ JAMB 1994[44] Ans: $2.5 \times 10^{-3} C$

18.

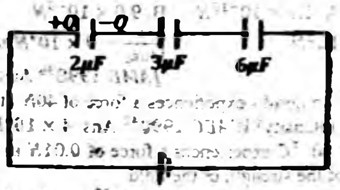


Fig. 2.18

The figure above shows three capacitors $2\mu F$, $3\mu F$ and $6\mu F$ connected in series. If the potential difference across the system is 12V, the potential difference across the $6\mu F$ capacitor is A. 4V B. 6V C. 12V D. 2V JAMB 2005[19] Ans: 2V

19.

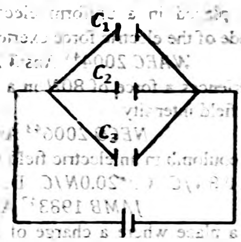


Fig. 2.19

What is the total capacitance in the represented by the diagram? A. $C_1 + C_2 + C_3$
B. $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ C. $\frac{C_1 C_2 C_3}{C_1 + C_2 + C_3}$ D. $\frac{C_2}{C_1 + C_3}$ E. $C_1 C_2 C_3$
WAEC 1994[46] Ans: $C_1 + C_2 + C_3$

20. Two capacitors of capacitance $0.4\mu F$ and $0.5\mu F$ are connected in parallel and charged to a potential difference of 50V. Determine the total charge acquired.
WAEC 2005[34] Ans: $45\mu C$

21.

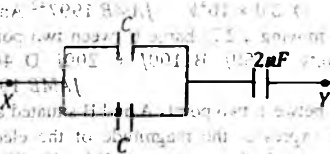


Fig. 2.20

The effective capacitance betw n points X and Y in the diagram is $1.0\mu F$. What is the value of the capacitance C, mea red in microfarad? WAEC 1995[45] Ans: $1.0\mu F$

22.

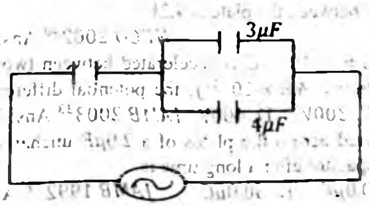


Fig. 2.21

Using the diagram above, calculate the effective capacitance of the circuit. WAEC 1999[31] Ans: 1.56

23. The diagram below represents a section of a circuit. Calculate the effective capacitance in the section.

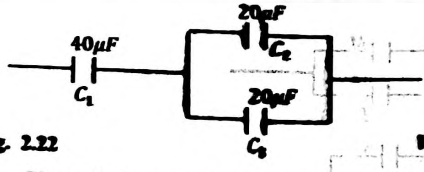


Fig. 2.22 WAEC 2000[E14] Ans: $20\mu F$

24. The equivalent capacitance for the circuit drawn below is

A. $18\mu F$ B. $4\mu F$ C. $2\mu F$ D. $12\mu F$ E. $13\mu F$

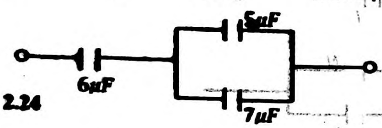


Fig. 2.24 JAMB 1978[29] Ans: $4\mu F$

25.

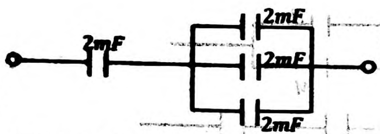


Fig. 2.25

The total capacitance of the circuit above is A. $0.25mF$ B. $0.50mF$ C. $0.75mF$ D. $1.25mF$ E. $1.50mF$ JAMB 1980[9] Ans: $1.50mF$

26.

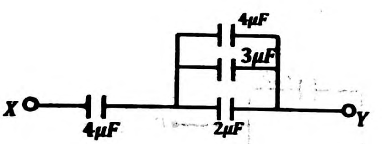


Fig. 2.26

In the circuit shown above, calculate the effective capacitance between X and Y

A. $\frac{3}{4}\mu F$ B. $2\frac{10}{13}\mu F$ C. $12\mu F$ D. $4\frac{12}{13}\mu F$ E. $13\mu F$ JAMB 1984[35] Ans: $2\frac{10}{13}\mu F$

27. a. Derive a formula for the energy W stored in a charged capacitor of capacitance C carrying a charge Q on either plate.

b. Two parallel plate capacitors of capacitance $2\mu F$ and $3\mu F$ are connected in parallel and the combination is connected to a $50V$ d.c source.

Draw the circuit diagram of the arrangement and determine the

(i) Charge on either plate of each capacitor
(ii) Potential difference across each capacitor
(iii) Combined energy of the capacitors

WAEC 1996[E3] Ans: a. $W = \frac{Q^2}{C}$ b. (i) $1 \times 10^{-4}C$, $1.5 \times 10^{-4}C$ (ii) $50V$
(iii) $6.25 \times 10^{-3}J$

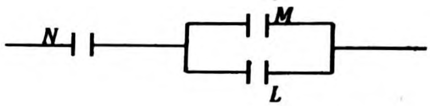
28. A series arrangement of three capacitors of values $8\mu F$, $12\mu F$ and $24\mu F$ is connected in series with a $90V$ battery.

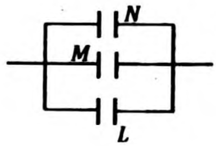
(i) Draw an open circuit diagram for this arrangement,
(i) Calculate the effective capacitance in the circuit,
(ii) On closed circuit, calculate the charge on each capacitor when fully charged.

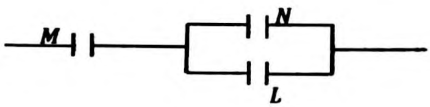
(iii) Determine the potential difference across the $8\mu F$ capacitor

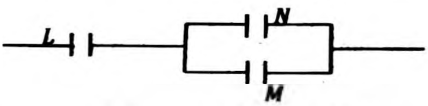
WAEC 2001[E14] Ans: (*ii*)$4\mu F$ (iii)$360\mu C$ (iv)$45V$

29. Three capacitors L, M, and N with capacitances C_1, C_2, and C_3 respectively are to be combined. If $C_3 > C_2 > C_1$, which of the following arrangements will give maximum capacitance?

A


B


C


D


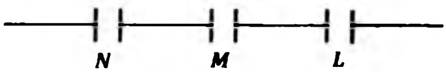
E


Fig. 2.23 *NECO* 2007[42] Ans:B

30.

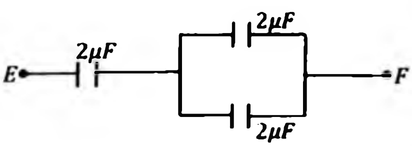


Fig. 2.28

Three $2\mu F$ capacitors are arranged as shown in the above circuit. The effective capacitance between E and F is

A. $0.75\mu F$ B. $1.33\mu F$ C. $3.00\mu F$ D. $6.00\mu F$ *JAMB* 1989[39] Ans: $1.33\mu F$

31.

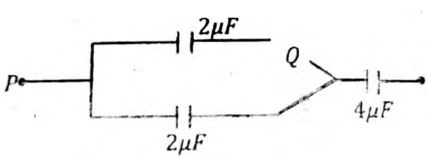


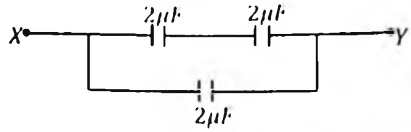
Fig. 2.29

The capacitance ... is

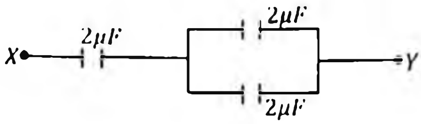
A. ... B. $6.0\mu F$ C. $4.0\mu F$ D. $2.0\mu F$ *JAMB* 1991[39] Ans: $2.0\mu F$

32. Which of the following combinations of $2\mu F$ capacitors will give an effective capacitance of $3\mu F$ across terminal XY?

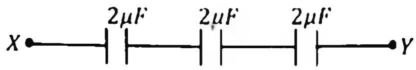
A



B.



C.



D.

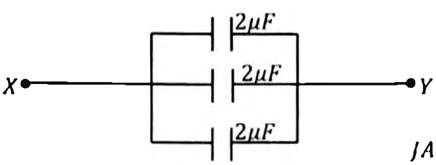


Fig. 2.27 *JAMB* 1988[42] Ans: *A*

33. Given 3 capacitors $0.3\mu F$, $0.5\mu F$ and $0.2\mu F$, the joint capacitance when arrange to give minimum capacitance is A. $0.3\mu F$ B. $1.0\mu F$ C. $0.1\mu F$ D. $0.5\mu F$

JAMB 2004[50] Ans: $0.1\mu F$

34. Calculate the energy stored in a $10\mu F$ capacitor if the potential difference between the plates is $20V$. *WAEC* 1989[41] Ans: $2 \times 10^{-3} J$

35. Calculate the energy stored in a $20\mu F$ capacitor if the potential difference between the plates is $40V$. *WAEC* 1992[49] Ans: $1.6 \times 10^{-2} J$

36. The energy stored in a capacitor of capacitor $5mF$ is $40J$. Calculate the voltage applied across its terminals? *WAEC* 1995[44] Ans: $4000V$

37.

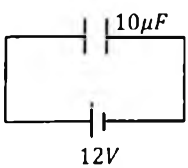


Fig. 2.30

A capacitor of $10\mu F$ is connected across a cell of 12 volts as shown above. Calculate the energy stored in the capacitor. A. $7.5 \times 10^{-4} J$ B. $1.4 \times 10^{-3} J$ C. $1.4 \times 10^{-4} J$ D. $1.2 \times 10^{-2} J$ *JAMB* 1992[41] Ans: $7.2 \times 10^{-4} J$

38. A capacitor $8\mu F$, is charged to a potential difference of $100V$. The energy stored by the capacitor is A. $1.0 \times 10^{4} J$ B. $8.0 \times 10 J$ C. $1.25 \times 10 J$ D. $4.0 \times 10^{-2} J$ *JAMB* 1994[43] Ans: $4.0 \times 10^{-2} J$

39.

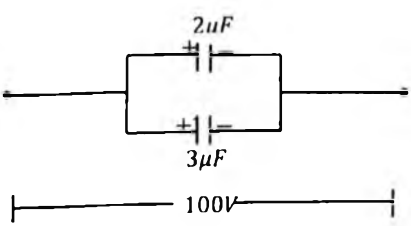


Fig. 2.31

In the circuit above, the potential across each capacitor is $100V$. The total energy stored in the two capacitor is A. $3.0 \times 10^4 J$ B. $3.0 \times 10^2 J$ C. $2.5 \times 10^{-2} J$ D. $6.0 \times 10^{-2} J$ *JAMB* 1999[38] Ans: $2.5 \times 10^{-2} J$

40. The energy stored in a capacitor of capacitance $10\mu C$ carrying a charge of $100\mu C$ is A. $4 \times 10^{-3} J$ B. $5 \times 10^{-4} J$ C. $5 \times 10^4 J$ D. $4 \times 10^2 J$ JAMB 2002[30] Ans: $5 \times 10^{-4} J$

Use the information below to answer question 41 and 42.
An isolated metal sphere of radius of radius R, carrying an electric charge, Q, is situated in a medium of permittivity, ε_r. A test charge is placed at a point P, distance r from the surface of the sphere. Let ε_o represent the permittivity of free space.

41. The electric potential at P is given by the expression

A. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r r}$ B. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r(R+r)}$ C. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r(R-r)}$ D. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r R}$

WAEC 2009[36] Ans: B

42. The magnitude of the electric field intensity at P is given by the expression

A. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r r^2}$ B. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r(R+r)^2}$ C. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r(R-r)^2}$ D. $\frac{Q}{4\pi\varepsilon_o\varepsilon_r R^2}$

WAEC 2009[37] Ans: B

43. The electric potential at a distance d from a point charge q in air of permittivity ε_o is expressed as

A. $\frac{q^2}{4\pi\varepsilon_o d}$ B. $\frac{q^2}{4\pi\varepsilon_o d^2}$ C. $\frac{q}{4\pi\varepsilon_o d^2}$ D. $\frac{q}{4\pi\varepsilon_o d}$ E. $\frac{q}{4\pi\varepsilon_o{}^2 d}$

NECO 2009[40] Ans: D

44. Calculate the work done in charging a capacitor of capacitance $20\mu F$ through a potential difference of 1000V. NECO 2009[41] Ans: 10.00J

3

CURRENT ELECTRICITY

RESISTIVITY AND CONDUCTIVITY

Resistivity of a material is defined as the resistance per unit length per unit cross sectional area of the material.

$$\text{Resistivity } \rho = \frac{RA}{l} \quad \text{or} \quad \rho = \frac{R\pi r^2}{l} \quad \text{or} \quad \rho = \frac{R\pi d^2}{4l}$$

Where
ρ = resistivity in Ωm
R = resistance in Ω
A = cross sectional area $= \pi r^2 = \frac{\pi d^2}{4}$ in (m)
l = length of wire in m
d = diameter of wire in m^2
r = radius of wire in m^2

Electrical Conductivity is the reciprocal of resistivity. It is measured in $(\Omega m)^{-1}$ or $\Omega^{-1}m^{-1}$

$$\text{Electrical conductivity } \sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{l}{R \times \pi r^2} = \frac{4l}{R \times \pi d^2}$$

Always make sure you do your calculation in uniform unit of measurement. Do not mix up m^2 and cm^2 or m and mm.
Preferably, express all measurements in S.I unit.
Note: $1mm = 0.001m$ or $1 \times 10^{-3}m$
$1cm = 0.01m$ or $1 \times 10^{-2}m$
$1mm^2 = 0.000001$ or $1 \times 10^{-6}m^2$
$1cm^2 = 0.0001$ or $1 \times 10^{-4}m^2$

Example 1
Calculate the resistivity of a wire of length $2m$ and cross sectional area $0.004cm^2$ if its resistance is 3.0 ohm. *WAEC* 1988[51]
Solution
Length $l = 2m = 200cm$; cross sectional area $A = 0.004cm^2$;
resistance $R = 3.0\Omega$

$$\text{Resistivity,} \quad \rho = \frac{RA}{l} = \frac{3 \times 0.004}{200} = \frac{0.012}{200} = 0.00006\Omega\text{cm} = 6.0 \times 10^{-5}\Omega cm$$

Resistivity in Ωcm can be converted to Ωm by dividing by 100 ($100cm = 1m$)

$$\frac{0.00006}{100} = 0.0000006 = 6.0 \times 10^{-7}\Omega m$$

Example 2
A constantan wire has a cross sectional area of $4 \times 10^{-8}m^2$ and a resistivity of $1.1 \times 10^{-6}\Omega m$. If a resistor of resistance 11Ω is to be made from this wire, calculate the length of the wire required. *WAEC* 2005[39]
Solution
$A = 4 \times 10^{-8}m^2$; $\quad l = 1.1 \times 10^{-6}\Omega m$; $\quad R = 11\Omega$

From $\rho = \frac{RA}{l}$, $\quad length, l = \frac{RA}{\rho} = \frac{11 \times 4 \times 10^{-8}}{1.1 \times 10^{-6}} = 0.4m$

Example 3

A wire of length $100cm$ and cross sectional area $2.0 \times 10^{-3}cm^2$ has a resistance of 0.10Ω. Calculate its electrical conductivity. *NECO* 2002[39]

Solution

Length $l = 100cm$; cross sectional area $= 2.0 \times 10^{-3}cm^2$; $R = 0.10\,\Omega$

Electrical conductivity $\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{100}{0.1 \times 2.0 \times 10^{-3}} = 5.0 \times 10^5\Omega^{-1}cm^{-1}$

Example 4

A $2m$ wire of resistivity $5.5 \times 10^{-7}\Omega m$ has a cross sectional area of $0.50mm^2$. Calculate its resistance. *NECO* 2004

Solution

$l = 2m$; resistivity, $\rho = 5.5 \times 10^{-7}\Omega m$;

Cross sectional area, $A = 0.5mm^2 = 0.5 \times 10^{-6}m$ $(1mm^2 = 1 \times 10^{-6}m)$

From $\rho = \frac{RA}{l}$, resistance $R = \frac{\rho l}{A} = \frac{5.5 \times 10^{-7} \times 2}{0.5 \times 10^{-6}} = \frac{1.1 \times 10^{-6}}{0.5 \times 10^{-6}} = 2.2\Omega$

Example 5

A wire of length $15m$ made of a material of resistivity $1.8 \times 10^{-6}\Omega m$ has a resistance of 0.27Ω. Determine the area of the wire. A. $1.5 \times 10^{-4}m^2$ B. $1.0 \times 10^{-4}m^2$ C. $2.7 \times 10^{-5}m^2$ D. $7.3 \times 10^{-6}m^2$ *JAMB* 1992[36]

Solution

$l = 15m$; $\rho = 1.8 \times 10^{-6}\Omega m$; $R = 0.27\Omega$

From $\rho = \frac{RA}{l}$, area, $A = \frac{\rho l}{R} = \frac{1.8 \times 10^{-6} \times 15}{0.27} = \frac{2.7 \times 10^{-5}}{0.27}$

$$= 1.0 \times 10^{-4}m^2$$

Example 6

A wire of 5Ω resistance is drawn out so that its new length is two times the original length. If the resistivity of the wire remains the same and the cross sectional area is halved, the new resistance is A. 40Ω B. 20Ω C. 10Ω D. 5Ω *JAMB* 2003[32]

Solution

	Original wire	New wire
Resistance	$R_1 = 5\Omega$	$R_2 = ?$
Length	$l_1 = l$	$l_2 = 2l$(*two times the original length*)
Resistance	$\rho_1 = \rho$	$\rho_2 = \rho$(*same resistivity*)
Area	$A_1 = A$	$A_2 = \frac{A}{2}$ (*area is halved*)
Resistivities	$\rho_1 = \frac{R_1A_1}{l_1}$	$\rho_2 = \frac{R_2A_2}{l_2}$

Equate ρ_1 and ρ_2 because they are equal

$$\frac{R_1A_1}{l_1} = \frac{R_2A_2}{l_2}$$

Cross multiplying, $R_2A_2l_1 = R_1A_1l_2$

Therefore, $R_2 = \frac{R_1A_1l_2}{A_2l_1}$

Substituting, $R_2 = \frac{5 \times A \times 2l}{\frac{A}{2} \times l} = \frac{5 \times 2 \times A \times 2l}{A \times l} = 5 \times 2 \times 2 = 20\Omega$

GALVANOMETER CONVERSION: SHUNT AND MULTIPLIER

I. ***Conversion of Galvanometer to Ammeter***. A galvanometer, also known as moving coil meter, can be converted to an ammeter by connecting a low resistance **shunt** in parallel with the galvanometer as shown below.

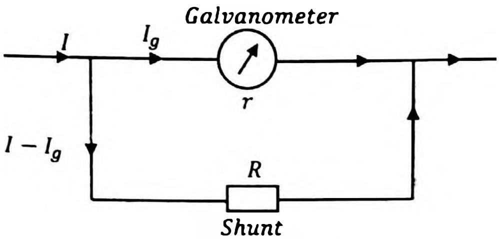


Fig 3.1

I = Ammeter reading of the converted galvanometer
I_g = Current flowing through galvanometer at full scale deflection
R = Shunt resistance
r = Galvanometer resistance
Remember, potential difference (p.d) = current × resistance
Current flowing through shunt = $I - I_g$
Current through galvanometer = I_g
Potential difference across shunt = shunt current × shunt resistance

$$= (I - I_g)R$$

Potential difference across galvanometer = galvanometer current × galvanometer resistance

$$= I_g \times r$$

Because r and R are in parallel, they have same potential difference

$\therefore$ p.d across shunt = p.d across galvanometer

$$(I - I_g)R = I_g \times r$$

$\therefore$ Shunt resistance, $R = \frac{I_g \times r}{i - i_g}$

Example 8
A galvanometer of resistance 5.0Ω has full scale defection for a current of 100mA. How would its range be extended to 1.0A? By placing a resistance of A. $\frac{5}{9}\Omega$ in parallel B. $\frac{9}{5}\Omega$ in series C. 45Ω in parallel D. 45Ω in series E. $\frac{9}{5}\Omega$ in parallel *JAMB* 1984[49]

Solution
Galvanometer resistance, $r = 5.0\Omega$ Shunt resistance R =?
Galvanometer current, $I_g = 100mA = 0.1A$ Ammeter reading $I = 1.0A$

$$\therefore \quad R = \frac{I_g \times r}{I - I_g} = \frac{0.1 \times 5}{1.0 - 0.1} = \frac{0.5}{0.9} = \frac{5}{9}$$

Ans: A resistance of $\frac{5}{9}\Omega$ in parallel

Example 7

The diagram below illustrates the conversion of a galvanometer of resistance 2Ω to an ammeter. The galvanometer gives a full scale deflection for a current of $10mA$. Calculate the value of R. *WAEC* 1996[45]

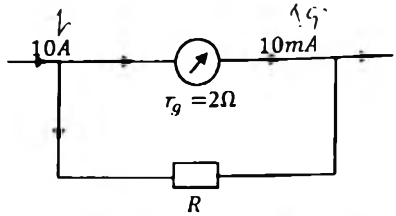


Fig 3.2

Solution

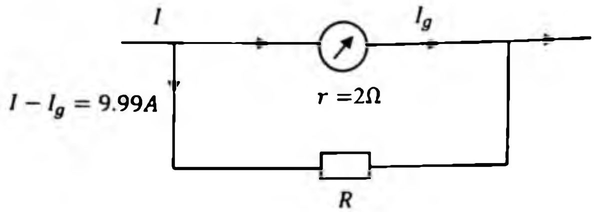


Fig 3.3

Galvanometer current $I_g = 10mA = 0.01A$ Ammeter reading $I = 10A$

Galvanometer resistance $r = 2\Omega$ Shunt resistance $R = ?$

$$R = \frac{I_g \times r}{I - I_g} = \frac{0.01 \times 2}{10 - 0.01} = \frac{0.02}{9.99} = 2.0 \times 10^{-3}\Omega$$

Example 9

A galvanometer has a resistance of 5Ω. By using a shunt wire of resistance 0.05Ω, the galvanometer could be converted to an ammeter capable of reading $2A$. What is the current through the galvanometer? A. $2mA$ B. $10mA$ C. $20mA$ D. $25mA$ JAMB 1999[41]

Solution

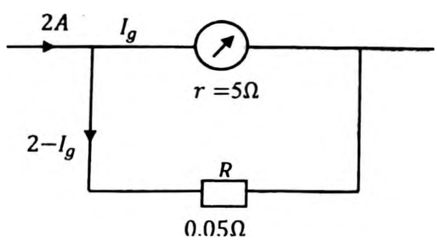


Fig 3.4

Galvanometer resistance, $r = 5\Omega$ Shunt resistance $R = 0.05\Omega$

Galvanometer current, $I_g = ?$ Ammeter reading, $I = 2$

Substitute into $R = \dfrac{I_g \times r}{I - I_g}$ to obtain $0.05 = \dfrac{I_g \times 5}{2 - I_g}$

Cross multiplying, $0.05(2 - I_g) = 5I_g$

$$0.1 - 0.05I_g = 5I_g$$

$$0.1 = 5.05I_g$$

$$I_g = \frac{0.1}{5.05} = 1.98 \times 10^{-2}A \simeq 2.0 \times 10^{-2}A = 20mA$$

Next time remember that galvanometer current, $I_g = \frac{IR}{r + R}$

II. Conversion of galvanometer to voltmeter. A galvanometer can be converted to a voltmeter by connecting a high resistance **multiplier** in series with the galvanometer as shown below.

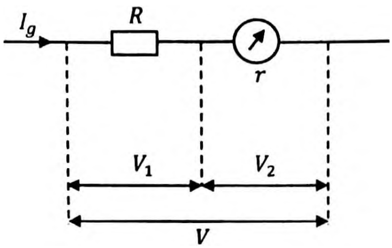


Fig 3.5

I_g = Current through galvanometer at full scale defection
R = Resistance of multiplier
r = Galvanometer resistance
V_1 = Potential difference across multiplier
V_2 = Potential difference across galvanometer
V = Voltmeter reading of the converted galvanometer (Total p.d across circuit)

Potential difference across multiplier, $V_1 = I_g \times R$

Potential difference across galvanometer $V_2 = I_g \times r$

Voltmeter reading = $p.d$ across multiplier + $p.d$ across galvanometer

$$V = V_1 + V_2$$
$$V = I_gR + I_gr$$
$$V = I_g(R + r)$$

Rearranging, $I_g(R + r) = V$

$$R + r = \frac{V}{I_g} \qquad \therefore \quad \text{Multiplier resistance } R = \frac{V}{I_g} - r$$

Example 10

A moving coil meter with an internal resistance of 100Ω has a full scale deflection when a current of $10mA$ flows through it. What value of resistance would convert it to read $10V$ at full scale deflection? *WAEC* 1995[55]

Solution

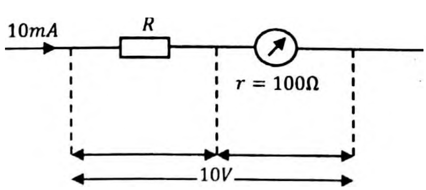


Fig 3.6

Galvanometer resistance $r = 100\Omega$; voltmeter reading, $V = 10V$

Current at full scale deflection $I_g = 10mA = 0.01A$

Multiplier resistance $R = \frac{V}{I_g} - r = \frac{10}{0.01} - 100 = 1000 - 100 = 900\Omega$

Example 11

The maximum permissible current through a galvanometer G of internal resistance 10Ω is $0.05A$. A resistance R is used to convert G into a voltmeter with a maximum reading of $100V$. Find the value of R and how it is connected to G.

A. 20,000 ohms in parallel B. 19,900 ohms in series

C. 1990 ohms in series D. 100 ohms series. *JAMB* 1992[44]

Solution

$r = 10\Omega$; $I_g = 0.05A$; $V = 100V$

Multiplier resistance $R = \frac{V}{I_g} - r$

$$R = \frac{100}{0.05} - 10 = 2000 - 10 = 1990\Omega \text{ in series}$$

POTENTIOMETER. WHEATSTONE BRIDGE. METER BRIDGE

I. *Potentiometer* A potentiometer is used to measure resistances of wire and $e.m.f$ of cells as shown below.

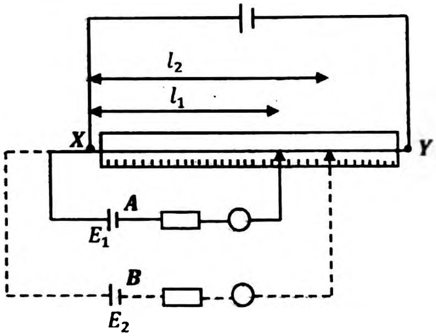


Fig 3.6

For the potentiometer above,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Where $E_1 = e.m.f$ of cell A

$E_2 = e.m.f$ of cell B

$l_1 =$ Balance point of cell A

$l_2 =$ Balance point of cell B

Example 12

The balance length of a potentiometer wire for a cell of $e.m.f$ $1.63V$ is $85cm$. If the cell is replaced by another one of $e.m.f$ $1.07V$, calculate the new balance length.

NECO 2007[44]

Solution

$E_1 = 1.63V$, $l_1 = 85cm$; $E_2 = 1.07V$, $l_2 = ?$

For a potentiometer, $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

$$\frac{1.63}{1.07} = \frac{85}{l_2} \quad \therefore \text{ New balance length } l_2 = \frac{1.07 \times 85}{1.63} = 55.80cm$$

II. *Wheatstone Bridge.* The Wheatstone bridge shown below is used to accurately measure resistance.

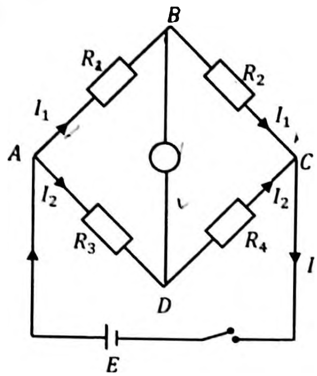


Fig 3.7

At balance point, B and D are at the same potential, therefore the potential difference between B and D is zero

For resistances R_1, R_2, R_3, and R_4 connected as shown above,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Example 13

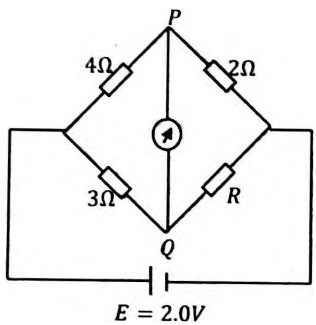


Fig 3.8

In the diagram above, the galvanometer indicates a null defection. Calculate

(i) The value of R

(ii) The potential difference between P and Q *NECO* 2005[43]

Solution

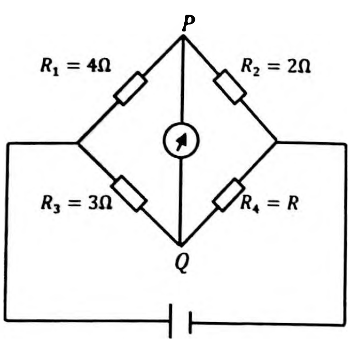


Fig 3.9

(i) From the digram, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ becomes

$$\frac{4}{3} = \frac{2}{R} \quad \therefore \quad R = \frac{3 \times 2}{4} = \frac{6}{4} = 1.5\Omega$$

(ii) The potential difference between P and Q is zero at null defection or balance point.

Example 14

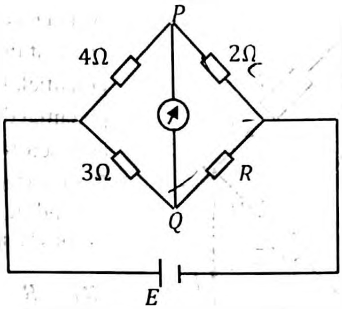


Fig 3.10

Solution
Ans: 0

In the diagram below, the galvanometer indicates a null deflection. What is the potential difference between P and Q? / *WAEC* 1993[47]

III. ***Meter Bridge.*** The meter bridge is a special application of a wheatstone bridge and is also used for measuring resistance accurately.

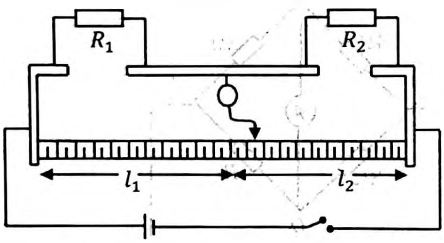


Fig 3.10

For the meter bridge above at null defection,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}$$ **If l_1 is known, then $l_2 = 100 - l_1$**

Example 15
In a meter bridge experiment, there is a zero defection of the galvanometer when $l_1 = 40cm$ as shown in the diagram below. Calculate the value of the resistance R.
WAEC 1989[45]

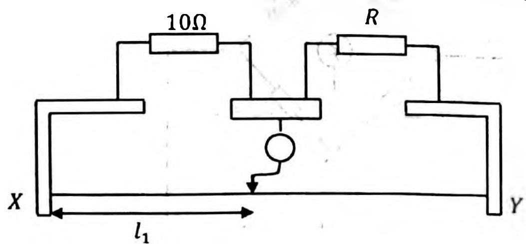


Fig 3.11

Solution

Given: $l_1 = 40cm$ ∴ $l_2 = 100 - l_1 = 100 - 40 = 60cm$

From the diagram, $\frac{R_1}{R_2} = \frac{l_1}{l_2}$ becomes

$$\frac{10\Omega}{R} = \frac{40cm}{60cm}$$

Therefore, $R = \frac{10\Omega \times 60cm}{40cm} = \frac{10\Omega \times 60}{40} = 15\Omega$

Example 16

In a meter bridge experiment, two resistors 2Ω and 3Ω occupy the left and right gaps respectively. Find the balance point from the left side of the bridge.

A. $20cm$ B. $40cm$ C. $60cm$ D. $80cm$ *JAMB* 1997[43]

Solution

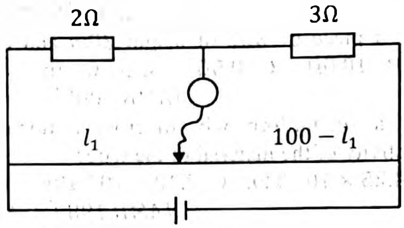


Fig 3.12

Balance point from left side of bridge.

From the diagram, $\frac{R_1}{R_2} = \frac{l_1}{l_2}$ becomes

$$\frac{2}{3} = \frac{l_1}{100 - l_1}$$

Cross multiply, $2 \times (100 - l_1) = 3l_1$

$$200 - 2l_1 = 3l_1$$

$$200 = 3l_1 + 2l_1$$

$$200 = 5l_1$$

$$l_1 = \frac{200}{5} = 40cm$$

EXERCISE 3

1. Calculate the length of a constant wire of cross sectional area $4\pi \times 10^{-8}m^2$ and resistivity $1.1 \times 10^{-6}\Omega m$ required to construct a standard resistor of resistance 21Ω. $\left(Take\ \pi\ as\ \frac{22}{7}\right)$ *WAEC* 1992[44] Ans: $2.4m$

2. A given wire of resistance 10Ω has a length of $5m$ and a cross sectional area of $4.0 \times 10^{-3}m^2$. Calculate the conductivity of the wire.
WAEC 1995[50] Ans: $1.25 \times 10^7\Omega^{-1}m^{-1}$

3. A wire, $1.0m$ long and with cross sectional area $2.0 \times 10^{-7}m^2$ has a resistance of 0.1Ω. Calculate the electrical conductivity of the wire.
WAEC 2008[38] Ans: $5.0 \times 10^7\Omega^{-1}m^{-1}$

4. A wire of length $100cm$ and cross sectional area $2.0 \times 10^{-3}cm^2$ has a resistance of 0.10Ω. Calculate its electrical conductivity.

$WAEC$ 1996[46] Ans: $2.0 \times 10^{-6}\Omega^{-1}cm^{-1}$

5. The resistivity of a given wire of cross sectional area $0.7mm^2$ is $4.9 \times 10^{-4}\Omega mm$. Calculate the resistance of a $2m$ length of the wire.

$WAEC$ 1994[PR3] Ans: 1.4Ω

6. A resistance wire of length $2m$ and of uniform cross sectional area $5.0 \times 10^{-7}m^2$ has a resistance of 2.2Ω. Calculate its resistivity. $WAEC$ 1994[34] Ans: $5.5 \times 10^{-7}\Omega m$

7. A piece of resistance wire of diameter $0.2mm$ and resistance 7Ω has resistivity of $8.8 \times 10^{-7}\Omega m$, calculate the length of the wire $\left(\pi = \frac{22}{7}\right)$

$WAEC$ 2002[PR3] Ans: $2.5 \times 10^{-7}m$

8. A 0.6Ω-resistor is made from a wire of length 4m and resistivity $2.0 \times 10^{-6}\Omega$m. What is the cross-sectional area of the wire? $NECO$ 2008[42] Ans:$1.33 \times 10^{-5}m^2$

9. A piece of resistance wire of diameter $0.2m$ and resistance 7Ω has resistivity of $8.8 \times 10^{-7}\Omega m$, Calculate the length of the wire.

$WAEC$ 2003[PR3] Ans: 2.5×10^5m

10. The resistance of a piece of wire of length $20m$ and cross sectional area $8 \times 10^{-6}m^2$ is A. 1.0Ω B. 10.0Ω C. 0.5Ω B. 5.0Ω [resistivity of the wire $= 4 \times 10^{-7}\Omega$m] $JAMB$ 2005[18] Ans: 1.0Ω

11. The resistance of a $5m$ uniform wire of cross sectional area $0.2 \times 10^{-6}m^2$ is 0.425Ω. What is the resistivity of the material of the wire?
A. $1.10 \times 10^{-6}\Omega m$ B. $4.25 \times 10^{-6}\Omega m$ C. $2.40 \times 10^{-7}\Omega m$ D. $1.70 \times 10^{-8}\Omega m$

$JAMB$ 1989[35] Ans: $1.70 \times 10^{8}\Omega m$

12. A conductor has a diameter of $1.00m$ and length $2.00m$. If the resistance of the material is 0.1Ω, its resistivity is A. $2.55 \times 10^2\Omega m$ B. $2.55 \times 10^5\Omega m$ C. $3.93 \times 10^{-8}\Omega m$ D. $3.93 \times 10^{-6}\Omega m$ $JAMB$ 2007[35] Ans: $3.93 \times 10^{-8}\Omega m$

13. A milliammeter with full scale defection of $10mA$ has an internal resistance of 5Ω. It would be converted to an ammeter with a full scale deflection of $1A$ by connectin a reisstance of A. $\frac{5}{99}$ ohm in series with it B. $\frac{5}{99}$ ohm in parallel with it C. $\frac{99}{5}$ ohm in parallel with it D. $\frac{99}{5}$ ohm in series with it E. 2 ohm in series with it

$JAMB$ 1983[20] Ans: B

14. An ammeter of resistance 0.1Ω has a full scale deflection of $50mA$. Determine the resultant full scale deflection of the meter when a shunt of 0.0111Ω is connected across its terminals. A. $400mA$ B. $450mA$ C. $500mA$ D. $550mA$

JAMB 1989[43] Ans: $500mA$

15. A 0-$10mA$ galvanometer with a coil resistance of 30Ω can be converted to a 0-$10A$ ammeter by using A. 0.03Ω series resistor B. 0.03Ω shunt resistor C. 9.99Ω shunt resistor D. 9.99Ω series resistor $JAMB$ 1990[45] Ans: B

16. A moving coil galvanometer of 300Ω resistance gives full scale deflection for $1.0mA$. The resistance, R, of the shunt that is required to convert the galvanometer into a $3.0A$ ammeter is A. 899.70Ω B. 10.00Ω C. 0.10Ω D. 0.01Ω

$JAMB$ 1991[37] Ans: 0.10Ω

17. An ammeter of resistance 5Ω has a full scale deflection when a current of $50mA$ flows in it. The value of the resistor required to adapt to measure a current of $5A$ is
A. 19.80Ω B. 5.00Ω C. 0.05Ω D. 0.25Ω

$JAMB$ 2006[39] Ans: 0.05Ω

18. A galvanometer with a full scale deflection of $1.5 \times 10^{-3}A$ has a resistance of 50Ω. Determine the resistance required to convert it into a voltmeter, reading up to $1.5V$. $WAEC$ 2001[48] Ans: 950Ω

19. A milliameter of resistance 5Ω and full scale deflection of $50mA$ is to be used to measure a potential difference of $50V$. What should be the resistance of the multiplier?
A. 95Ω B. 110Ω C. 595Ω D. 995Ω E. 1005Ω

20. The galvanometer G in the diagram below has a resistance of 5Ω and gives a full scale deflection for a current of $10mA$.

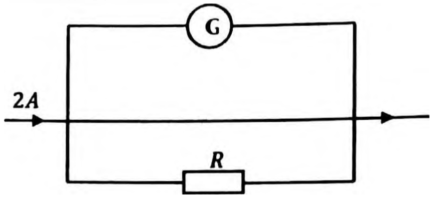


Fig 3.13

If the galvanometer is to be used as an ammeter reading up to $2A$, which of the following equations is correct? A. $2R = 20$ B. $1.99 \times R = 0.01 \times 5$
C. $2 \times R = 0.01 \times 5$ D. $1.99 \times 2R = 0.01 \times 5$ E. $1.99 \times 2R = R - 0.01$

NECO 2000[50] Ans: *B*

21. The value of the series resistance required to convert the galvanometer below into a voltmeter with a full range deflection of $15V$ is

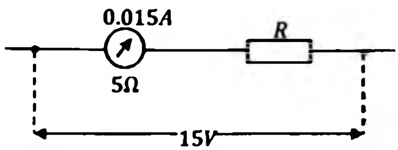


Fig 3.14

A. 985 ohms B. 990 ohms C. 995 ohms D. 1,000 ohms E. 1,005 ohms

JAMB 1982[30] Ans: 995Ω

22. A galvanometer of internal resistance 50Ω has a full scale deflection for a current of $5mA$. What is the resistance required to convert it to a voltmeter with full scale deflection of $10V$? A. 1750Ω B. 1950Ω C. 2000Ω D. 2500Ω

JAMB 1997[41] Ans: 1950Ω

23. A potentiometer wire carrying a steady current is $1m$ long with a standard cell of $e.m.f$ $1.1V$ suitably connected, a balanced length of $44.0cm$ was obtained. What is the $e.m.f$ of a cell which gives a balance length of $68.0cm$?

NECO 2000[48] Ans: $1.7V$

24. Calculate the value of R when G shows no deflection in the circuit illustrated below.

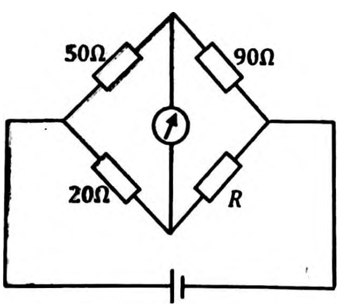


Fig 3.15

WAEC 1988[44] Ans: 36Ω

25. When a known standard resistor of 2.0Ω is connected to the $0.0cm$ end of a wire bridge, the balance point is found to be at $55.0cm$. What is the value of the unknown resistor? A. 1.10Ω B. 1.64Ω C. 2.44Ω D. 27.50Ω

JAMB 1990[88] Ans: 1.64Ω

26. In the diagram below, the galvanometer indicates a null deflection. What is the potential difference between X and Y?

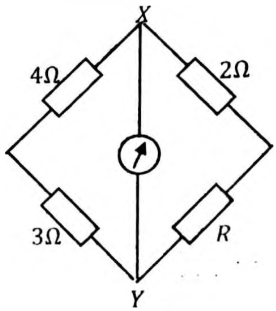


Fig 3.16

WAEC 1989[42] Ans: 0

27. What is the value of *R* when *G* shows no deflection in the circuit illustrated below.

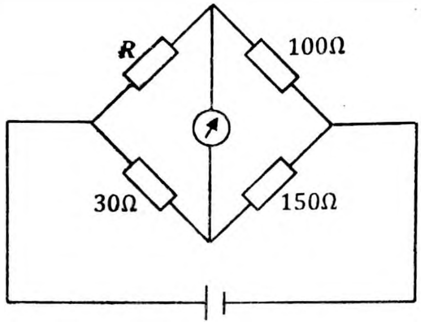
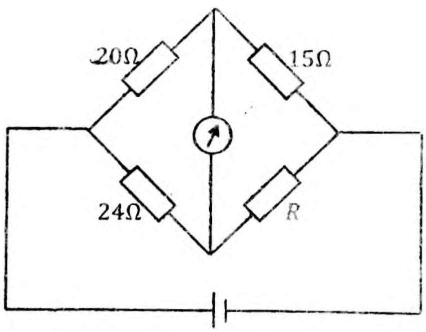


Fig 3.17

WAEC 1990[40] Ans: 20Ω

28.



NECO 2003[48]
Ans: 18Ω

Fig 3.17

Calculate the resistance *R* required to balance the bridge in the circuit above.

29.

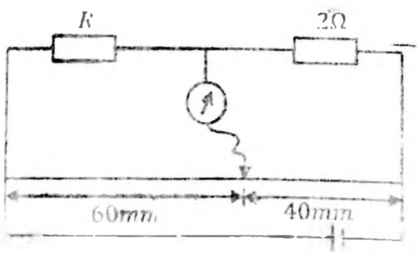


Fig 3.18

The diagram above shows a meter bridge in which two of the arms contain resistors [illegible] *R* and 2[illegible]. A balance point is obtained at 60*cm* from the [illegible]
A. 1 [illegible] B. 1.3Ω C. 3.0Ω D. 5.0Ω *JAMB* [illegible] Ans: 3.0Ω

30.

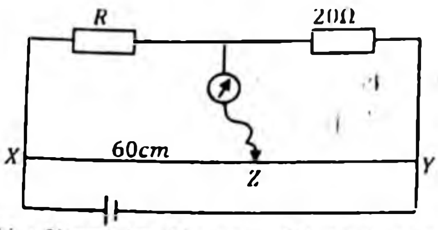


Fig 3.18

In the figure above XY is of length $1m$. The value of R at balance point Z is
A. 3.0Ω B. 13.3Ω C. 15.0Ω D. 30.0Ω *JAMB* 1994[42] Ans: 30.0Ω

31. A wire of resistivity $4.4 \times 10^{-5}\Omega cm$ has a cross-sectional area of $7.50 \times 10^{-4} cm^2$. Calculate the length of this wire that will be required to make a 4.0Ω resistor.
WAEC 2009[40] Ans: 68.18cm

4

ELECTROLYSIS, MAGNETIC AND ELECTROMAGNETIC FIELDS

ELECTROLYSIS

Electrolysis is defined as the chemical change (decomposition of compounds) in a liquid due to the flow of current.

Faraday's first law of electrolysis states that the mass (M) of a substance librated or deposited during electrolysis is directly proportional to the quantity of electricity (Q) passing through the electrolyte.

That is, $M \propto Q$

Or $M = ZQ$

Remember, quantity of electricity $Q = It$

Therefore, $M = ZIt$

Where I = current in amperes (A)

Z = Electrochemical equivalent kgC^{-1} or gC^{-1}

Also, the mass of the substance deposited during electrolysis depend on the density (ρ) of the substance, the surface area (A) of the cathode and the thickness (d) of the substance deposited on the cathode.

$\frac{\text{mass}}{\text{volume}}$ $\therefore$ Mass = density × area × thickness

$$M = \rho \times A \times d$$

$$M = ZIt$$

$$\therefore \quad \rho \times A \times d = ZIt$$

Faraday's second law of electrolysis states that when the same current passes through different electrolytes for the same time, the masses of the elements deposited or librated are proportional to their chemical equivalent.

$$\frac{Mass\ of\ elment\ A}{Mass\ of\ elment\ B} = \frac{Chemical\ equivalent\ of\ A}{Chemical\ equivalent\ of\ B}$$

Electrochemical equivalent $(e.c.e)$ of a substance is defined as the mass of the substance deposited during electrolysis by one coulomb of electricity. (One ampere of current flowing for one second). Copper has an $(e.c.e)$ of $0.00033gC^{-1}$ because when a current of $1A$ flows through a copper solution for 1sec, it deposits $0.00033g$ of copper.

$$e.c.e, \quad Z = \frac{M}{Q} = \frac{M}{It} = \frac{\rho A d}{It}$$

Example 1

Copper of mass $0.33g$ is liberated by passing 100 coulombs of electric charge through a copper voltameter. Calculate the mass of copper deposited when a current of $2A$ is passed through the voltameter for 5minutes *NECO* 2007[E7]

Solution

$e.c.e, Z = 033g/1000C = 3.30 \times 10^{-4} g/C$

Current $I = 2A$; time $= 5min = 5 \times 60 = 300s$;

Mass deposited $M = ZIt = 3.30 \times 10^{-4} \times 2 \times 300 = 0.198g$

Example 2

A charge of one coulomb liberate $0.0033g$ of copper in an electrolytic process. How long will it take a current of $2A$ to liberate $1.98g$ of copper in such a process?
A. $5minutes$ B. $30minutes$ C. $50minutes$ D. $60minutes$ E. $120minutes$

JAMB 1985[35]

Solution

$e.c.e, Z = 0.0033gC^{-1}$; current $I = 2A$; mass liberated $M = 1.98g$

From $M = ZIt$, time $t = \dfrac{M}{ZI} = \dfrac{1.98}{0.0033 \times 2} = 300s = 5min$

Example 3

In an electrolysis experiment, a cathode of, mass $5g$ is found to weigh $5.01g$ after a current of $5A$ flows for 50 second. What is the electrochemical equivalent of the deposited substance? A. $0.00004g/C$ B. $0.00002g/C$ C. $0.02500g/C$ D. $0.05000g/C$ E. $0.00001g/C$

JAMB 1979[30]

Solution

Mass deposited $M = 5.01g - 5g = 0.01g$; current $I = 5A$; time $t = 50s$

From $M = ZIt$, electrochemical equivalent $Z = \dfrac{M}{It}$

$$Z = \frac{0.01}{5 \times 50} = 0.0004g/C \quad \text{or} \quad 4 \times 10^{-5} gC^{-1}$$

Example 4

Copper of thickness d is plated on the cathode of a copper voltameter. If the total surface area of the cathode is $60cm^2$ and a steady current of $5.0A$ is maintained in the voltmeter for 1 hour, calculate the value of d. [density of copper $= 8.9 \times 10^3 kgm^{-3}$] [electrochemical equivalent of copper $= 3.3 \times 10^{-7} kgC^{-1}$]

WAEC 2007[ES]

Solution

Area, $A = 60cm^2 = 60 \times 10^{-4} m^2$; current, $I = 5A$;
Time, $t = 1hour = 3600s$; $e.c.e, Z = 3.3 \times 10^{-7} kgC^{-1}$
Density $\rho = 8.9 \times 10^3 kgm^{-3}$; thickness d =?

From $\rho \times A \times d = ZIt$

Thickness $d = \dfrac{ZIt}{\rho \times A}$

$$d = \frac{3.3 \times 10^{-7} \times 5 \times 3600}{8.9 \times 10^3 \times 60 \times 10^{-4}} = \frac{5.94 \times 10^{-3}}{53.4} = 1.11 \times 10^{-4} m$$

$$= 1.11cm$$

Example 5

When a current of $1.0A$ flows in an electrolyte for $3.0h$, a mass of $1.0g$ is deposited. What mass will be deposited in $10.0h$ if a current of $3.0A$ flows through the electrolyte?

NECO 2002[46]

Solution

Firstly: $I = 1.0A$; $t = 3$ hours $= 10800s$; $M = 1.0g$

Secondly: $t = 10$ hours $= 36000s$, $I = 3.0A$

Firstly: $Z = \frac{M}{It} = \frac{1}{1 \times 10800} = 9.26 \times 10^{-5} gC^{-1}$

Secondly: $M = ZIt = 9.26 \times 10^{-5} \times 36000 \times 3 = 10g$

Example 6

During the electrolysis of copper (ii) tetraoxosulphate (vi) solution, an ammeter shows a steady current reading of $1.0A$ for 30 minutes while $6.6 \times 10^{-4} kg$ of copper is liberated. Calculate the error in the ammeter reading. (The electrochemical equivalent of copper is $3.30 \times 10^{-7} kgC^{-1}$) *WAEC* 1997[47]

Solution

Ammeter reading $= 1.0A$; time, $t = 30$ min $= 30 \times 60 = 1800s$;

mass $M = 6.6 \times 10^{-4} kg$; $Z = 3.30 \times 10^{-7} kgC^{-1}$

From $M = ZIt$, current used $I = \frac{M}{Zt} = \frac{6.6 \times 10^{-4}}{3.30 \times 10^{-7} \times 1800} = 1.11A$

Ammeter error = current used − ammeter reading $= 1.11 - 1.0 = 0.111A$

Example 7

What is the mass of hydrogen liberated in an acidulated water voltameter if a current of $6A$ flows through the voltmeter in 1 hour?

[electrochemical equivalent of silver $= 0.001118gC^{-1}$]

[equivalent weight of silver = 108]

Solution

Current $I = 6A$; time $t = 1$ hour $= 3600s$

Quantity of electricity used $Q = It = 6 \times 3600 = 21600C$

$1C$ of electricity releases $0.001118g$ of Ag

$21600C$ of electricity will release x of Ag

$x = 21600 \times 0.001118 = 24.15g$ of Ag

$$\frac{Mass\ of\ Ag}{Mass\ of\ H_2} = \frac{Chemical\ equivalent\ of\ A_g}{Chemical\ equivalent\ of\ H_2}$$

$$\frac{24.15}{mass\ of\ H_2} = \frac{108}{1}$$

$$Mass\ of\ H_2 = \frac{24.15}{108} = 0.224g$$

EXERCISE 4A

1. A current of $2A$ is passed through a copper voltameter for 5minutes. If the electrochemical equivalent of copper is $3.27 \times 10^{-7} kgC^{-1}$, determine the mass of the copper deposited. *WAEC* 2001[E3] Ans: $0.1962g$ or $1.96 \times 10^{-4} kg$

2. During the electrolysis of copper (ii) tetraoxosulphate (vi) solution, a steady current of $4.0 \times 10^2 A$ flowing for one hour liberated $0.48g$ of copper. Calculate the mass of copper liberated by one coulomb of charge.

WAEC 2000[E9] Ans: $3.33 \times 10^{-7} g$

3. In copper plating, a current of $0.5A$ is allowed for a cathode area of $100cm^2$. If this current is maintained constant for 100 minutes, the thickness of the copper deposited

will be approximately [the electrochemical equivalent of copper = $0.0003g/C$, density of copper = $10^4 g/cm^3$] A. $10^{-2}mm$ B. $10^{-3}mm$ C. $10^{-4}mm$ D. $10^{-5}mm$ E. $10^{-6}mm$ *JAMB* 1982[31] *Ans*: $10^{-5}mm$

4. The electrochemical equivalent of silver is $0.0012g/C$. If $36.0g$ of silver is to be deposited by electrolysis on a surface by passing a steady current for 5.0 minutes, the current must be A. $6000A$ B. $100A$ C. $10A$ D. $1A$

JAMB 1989[44] Ans: $100A$

5. The electrochemical equivalent of platinum is $5.0 \times 10^{-7}kgC^{-1}$. To plate-out $1.0kg$ of platinum, a current of $100A$ must be passed through an appropriate vessels for A. 5.6 hours B. 56 hours C. 1.4×10^4 hours D. 2.0×10^4 hours

JAMB 1991[44] Ans: 5.6 hours

6. The electrochemical equivalent of a metal is $1.3 \times 10^{-7}kgC^{-1}$. The mass of the metal which 2.0×10^4C of electricity will deposit from a suitable electrolyte is A. $6.5 \times 10^{-2}kg$ B. $2.6 \times 10^{-2}kg$ C. $6.5 \times 10^{-3}kg$ D. $2.6 \times 10^{-3}kg$

JAMB 1998[47] Ans: $2.6 \times 10^{-3}kg$

7. A current of $0.5A$ flowing for 3hours deposits $2g$ of a metal during electrolysis. The quantity of the same metal that would be deposited by a current of $1.5A$ flowing in 1hour is A. $10g$ B. $18g$ C. $2g$ D. $6g$ *JAMB* 2002[40] Ans: $2g$

8. 2.1g of silver is deposited when a current of $0.5A$ is passed through a voltameter for 60 minutes. Calculate the mass of silver that will be deposited when a current of $1.0A$ is passed through it for 15 minutes. *NECO* 2008[E14] Ans: 1.05g

9. In the calibration of an ammeter using faraday's law of electrolysis, the ammeter reading is kept constant at $1.20A$. If $0.990g$ of copper is deposited in 40 minutes, the correction to be applied to the ammeter is A. $0.03A$ B. $0.04A$ C. 0.05 D. $0.06A$ [$e.c.e$ of copper $= 3.3 \times 10^{-4}gC^{-1}$] *JAMB* 2003[39] Ans: $0.05A$

10. During electrolysis, $2.0g$ of a metal is deposited using a current of $0.5A$ in 3 hours. The mass of the same metal which can be deposited using a current of $1.5A$ in 1 hour is A. $3.0g$ B. $2.0g$ C. $0.5g$ D. $1.0g$ *JAMB* 2006[49] Ans: $2.0g$

11. A cathode of mass $5g$ weighs $5.01g$ after a current of $5A$ had been passed through the voltameter for 50 seconds. The electrochemical equivalent of the metal is

NECO 2007[43] Ans: $4.0 \times 10^{-5}g/C$

12. $118.8cm^2$ surface of the copper cathode of a voltameter is to be coated with $10^{-6}m$ thick copper of density $9 \times 10^3 kg10^{-3}$. How long will the process run with $10A$ constant current? A. 15.0 min B. 5.4 min C. 20.0 min D. 10.8 min [$e.c.e$ of copper $= 3.3 \times 10^{-7}kgC^{-1}$] *JAMB* 2007[41] Ans: 32.4s or 0.54 min

13. In an electrolysis experiment, the ammeter records a steady current of $1A$. The mass of copper deposited in 30 minutes is $0.66g$. Calculate the error in the ammeter reading.[Electrochemical equivalent of copper $= 0.00033gC^{-1}$]

WAEC 2008[E2] Ans: $0.11A$

14. A steady current of 1.2A is passed through a copper voltameter for 30s. Calculated the mass of the copper deposited. [Electrochemical equivalent of copper $3.3 \times 10^{-4}gC^{-1}$] NECO 2009[45] Ans: $1.19 \times 10^{-2}g$

15. An electric charge of 9.6×10^4C liberates 1 mole of a substance containing 6.0×10^{23} atoms. Determine the value of the electronic charge.

WAEC 2009[E6] Ans: 1.60×10^{-19}C

MAGNETIC FIELD

Magnetic field is a region surrounding a magnet within which a force is exerted on any other magnetic substance. When a charge q moves with a velocity V in a magnetic field B, the magnetic force F is given by

$$F = qVB\ \mathrm{Sin}\theta$$

Where F = magnetic force in Newton (N)
V = Velocity in m/s
B = Magnetic field in Tesla (T) or Weber/m^2
θ = Angle between V and B

Note: **1.** If V and B are in the same direction or parallel to each other, $\theta = 0°$ or $180°$;
Therefore, $F = 0$
2. If V and B are perpendicular or at right angle to each other then $\theta = 90$,
Therefore, F is maximum.

The force F on a conductor of length l carrying a current I in a magnetic field B is given by;

$$F = BIl\ \mathrm{Sin}\theta$$

F is zero when B and I are parallel to each other. F is maximum when B and I are perpendicular.

Example 1
A particle of charge $5C$ moves perpendicularly to a magnetic field of magnitude $0.01T$. If the velocity of the charge is $1.5ms^{-1}$, calculate the magnitude of the force exerted on the particle. *WAEC* 1993[41]

Solution
Charge $q = 5C$; magnetic field $B = 0.01T$ $\theta = 90$ (perpendicularly)
Velocity $V = 1.5ms^{-1}$; magnetic force $F =?$

$$F = qVB\sin\theta = 5 \times 1.5 \times 0.01 \times \sin 90 = 0.075N$$

Example 2
A proton moving with a speed of $1.0 \times 10^6 ms^{-1}$ through a magnetic field of $1.0T$ experiences a magnetic force of magnitude $8.0 \times 10^{-14}N$. The angle between the proton's velocity and the field is A. 45° B. 30° C. 60° D. 90°
(proton charge = $1.6 \times 10^{-19}C$) *JAMB* 2005[6]
Solution
$V = 1.0 \times 10^6 ms^{-1}$; $B = 1.0T$; $F = 8.0 \times 10^{-14}N$, $q = 1.6 \times 10^{-19}C$

$$F = qVB\sin\theta$$

$$\sin\theta = \frac{F}{qVB} = \frac{8.0 \times 10^{-14}}{1.6 \times 10^{-19} \times 1.0 \times 10^6 \times 1} = \frac{8.0 \times 10^{-14}}{1.6 \times 10^{-13}}$$

$$\sin\theta = 0.5 \qquad \therefore \theta = \sin^{-1}0.5 = 30°$$

Example 3
An electron enters a magnetic field of flux density $1.5T$ with a speed of $2.0 \times 10^7 ms^{-1}$ at an angle of 30° to the field. Calculate the magnitude of the force on the electron.[Electron charge, $e = 1.6 \times 10^{19}C$] *NECO* 2006[Eu]

Solution

Flux density (magnetic field), $B = 1.5T$; $\theta = 30$; $V = 2.0 \times 10^7 ms^{-1}$; $q = 1.6 \times 10^{-19}C$

Magnetic force $F = qVB\sin\theta = 1.6 \times 10^{-19} \times 2.0 \times 10^7 \times 1.5 \times \sin30^\circ N$

$= 2.40 \times 10^{-12}$

Example 4

A conductor of length $2m$ carries a current of $0.8A$ while kept in a magnetic field of magnetic flux density $0.5T$. The maximum force acting on it is

A. $8.0N$ B. $3.2N$ C. $0.8N$ D. $0.2N$ *JAMB* 2000[50]

Solution

Length $l = 2m$; current $I = 0.8A$; magnetic flux density $B = 0.5T$.

Magnetic force $F = BIl = 0.5 \times 0.8 \times 2 = 0.8N$

EXERCISE 4B

1. A particle of charge q and mass m moving with a velocity V enters a uniform magnetic field of strength B in the direction of the field. The force on the particle is
A. qVB B. $mqVB$ C. qVb/m D. mVB/q E. 0 *WAEC* 1994[44] Ans: 0
2. A charge of $1.6 \times 10^{-19}C$ enters a magnetic field of flux density $2.0T$ with a velocity of $2.5 \times 10^7 ms^{-1}$ at an angle of 30° with the field. Calculate the magnitude of the force exerted on the charge by the field. *WAEC* 2003[E14] Ans: $4 \times 10^{-12}N$
3. If a charged ion goes through combined electric and magnetic fields, the resultant emergent velocity of the ion is A. E/B B. EB C. B/E D. $E - B$
JAMB 2000[32] Ans: A
4. The diagram below shows a closed square box of side $0.5m$ in a uniform electric field E in the direction shown by the arrows. What is the flux ϕ for the box
A. $0.5E$ B. $2.0E$ C. $0.2E$ D. $0.0E$ *JAMB* 2001[30] Ans: A

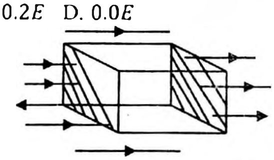


Fig 4.1

5. The force on a charge moving with velocity V in a magnetic field B is half of the maximum force when the angle between V and B is A. 90 B. 45 C. 30 D. 0
JAMB 2003[43] Ans: 45°
6. Two long parallel wires X and Y carry current $3A$ and $5A$ respectively. If the force experienced per unit length by X is $5 \times 10^{-5}N$, the force per unit length experienced by Y is A. $5 \times 10^{-5}N$ B. $5 \times 10^{-4}N$ C. $3 \times 10^{-6}N$ D. $3 \times 10^{-5}N$
JAMB 2007[38] Ans: D
7. A particle of charge q and mass m moving with a velocity V enters a uniform magnetic field B in the direction of the field. The force on the particle is
A. BqV B. BqV/m C. $BqVm$ D. mVB/q E. mV/Bq *NECO* 2000[45] Ans: 0
8. Calculate the magnitude of the force due to a magnetic field of $0.40T$ on an electron moving perpendicular to the field with a speed of $4.00 \times 10^6 ms^{-1}$.
$[e = 1.60 \times 10^{-19}C]$ *NECO* 2005[48] Ans: $2.56 \times 10^{-13}N$
9. A charge of $1.6 \times 10^{-19}C$ moving with a velocity of $2.5 \times 10^7 ms^{-1}$ enters a magnetic field of flux density $2.0T$ at an angle of 30° with the field. Calculate the magnitude of the force on the charge. *NECO* 2008[45] Ans: $4.0 \times 10^{-12}N$

10. A band of 500 rectangular loops of wire of area 20cm by 20cm, encloses a region of magnetic field which changes from 1.0T to 0.4T within 5 seconds, calculate the induced e.m.f..

A. 5.60V B. 24.00V C. 0.24V D. 2.40V JAMB 2009[42] Ans: D

Hint: $\frac{NAB_1 - NAB_2}{t}$

ELECTROMAGNETIC FIELD

Electromagnetic field is a field arising from the combined interaction of magnetic field and electric field. ***Electromagnetic induction*** is the generation of electric current or voltage in a conductor as a result of a relative motion between a conductor and a magnetic field.

Induced electromotive force $(e.m.f)$, E, in a straight conductor of length l, moving with a velocity V, perpendicular to a field of magnetic induction, B is given by;

$$E = BlV$$

Also the $e.m.f$ generated by an $a.c$ generator is given by

$$E = NAB\omega$$

$$E = NAB\omega\sin\theta$$

Where B = Magnetic field strength (T or weber$/m^2$)

l = Length of conductor (m)

V = Velocity (ms^{-1})

N = Number of turns

A = Area of coil (m^2)

ω = Angular velocity (rad s^{-1})

θ = Angle of inclination in degrees

Example 1

A circuit has an area of $0.4m^2$ and consist of 50 loops of wire. If the loops are twisted and allowed to rotate at a constant angular velocity of 10 rad s^{-1} in a uniform magnetic field of $0.4T$, the amplitude of the induced voltage is A. $8V$ B. $16V$

C. $20V$ D. $80V$ *JAMB* 2003[41]

Solution

Number of turns $N = 50$; area of coil $A = 0.4m^2$;

Angular velocity $\omega = 10$ rad s^{-1} ; magnitude of magnetic field $B = 0.4T$

Induced $e.m.f$ or voltage $E = NAB\omega$

$E = 50 \times 0.4 \times 10 \times 0.4 = 80V$

TRANSFORMERS

A transformer is a device for changing the voltage of an $a.c$ supply. ***A step up*** transformer converts low voltage to high voltage. ***A step down*** transformer converts high voltage to low voltage.

The induced $e.m.f$ in the secondary coil and in the primary coil of a transformer are related to the number of turns in the coil by the equation.

$$\frac{Secondary\ e.m.f\ (\boldsymbol{E_s})}{Primary\ e.m.f\ (\boldsymbol{E_p})} = \frac{Number\ of\ turns\ in\ secondary\ coil\ (\boldsymbol{N_s})}{Number\ of\ turn\ in\ primary\ coil\ (\boldsymbol{N_p})}$$

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \quad \cdots\cdots\cdots\cdots (1)$$

Also, power in secondary coil = power in primary coil.

That is, $P_s = P_p$

Or $I_s \times E_s = I_p \times E_p$ (from $P = IV$)

Rearranging, $\frac{E_s}{E_p} = \frac{I_p}{I_s}$ .... (2)

From equation (1) and (2) above, we obtain

$$\frac{N_s}{N_P} = \frac{I_P}{I_s}$$

Where I_P = current in primary coil

I_s = current in secondary coil

From, efficiency $= \frac{\text{power output}}{\text{power input}} \times 100$, we obtain

$$\text{Transformer efficiency}, \epsilon = \frac{\text{power in secondary coil}}{\text{power in primary coil}} \times 100$$

$$\epsilon = \frac{P_s}{P_P} \times 100$$

or $$\epsilon = \frac{I_s \times E_s}{I_P \times E_P} \times 100$$

or $$\epsilon = \frac{I_s \times N_s}{I_P \times N_P} \times 100$$

$$Turns\ ratio = \frac{N_s}{N_P}$$

If $N_s/N_p > 1$, it's a step up transformer.

If $N_s/N_p < 1$, it's a step down transformer

Example 2

A transformer is required to give $120V$ from a $240V$ mains supply. If the primary has 5500 turns, how many turns has the secondary? *NECO* 2003[51]

Solution

$E_s = 120V$, $E_P = 240V$, $N_P = 5500$, $N_s = ?$

$\frac{E_s}{E_p} = \frac{N_s}{N_p}$ $\therefore$ $N_s = \frac{E_s \times N_p}{E_p} = \frac{120 \times 5500}{240} = 2750$

Example 3

A transformer with 5500 turn in its primary is used between a $240V$ $a.c$ supply and a $120V$ kettle. Calculate the number of turns in the secondary. *WAEC* 1988[42]

Solution

$N_p = 5500$; $E_s = 120V$; $E_p = 240V$; $N_s = ?$

From, $\frac{E_s}{E_p} = \frac{N_s}{N_p}$, $N_s = \frac{E_s \times N_p}{E_p} = \frac{120 \times 5500}{240} = 2750$

Example 4

The current in the primary coil of a transformer is $2.5A$. If the coil has 50 turns and the secondary 250 turns, calculate the current in the secondary coil. (Neglect energy losses in the transformer). *WAEC* 1997[49]

Solution

$I_p = 2.5A$, $N_p = 50$, $N_s = 250$, $I_s = ?$

From $\frac{N_s}{N_p} = \frac{I_p}{I_s}$, current in secondary coil is, $I_s = \frac{50 \times 2.5}{250} = 0.5A$

Example 5

A 95% efficient transformer is used to operate a lamp rated $60W$, $220V$ from a $4400V$ $a.c$ supply. Calculate the

(i) ratio of the number of turns in the primary coil to the number of turns in the secondary coil of the transformer.

(ii) current taken from the mains circuit. *WAEC* 2007[E14]

Solution

Efficiency $\in = 95\%$, $P_s = 60W$, $E_p = 4400$, $E_s = 220V$

(i) $\frac{N_p}{N_s} = \frac{E_p}{E_s}$ $\therefore \frac{N_p}{N_s} = \frac{4400}{220} = 20$

(ii) $\in = \frac{P_s}{P_p} \times 100 = \frac{P_s}{I_p \times E_p} \times 100$

Main current $I_p = \frac{P_s \times 100}{\in \times E_p} = \frac{60 \times 100}{95 \times 4400} = \frac{6000}{418000} = 0.01435\,A$

$I_p = 0.014A$ Or $14mA$

Example 6

A transformer supplies $12V$ when connected to a $240V$ mains. If the transformer takes $0.120A$ from the mains when used to light a $12V$, $24W$ lamp, what is its efficiency? *NECO* 2002[50]

Solution

$E_s = 12V$, $E_p = 240V$, $I_p = 0.120A$ $P_s = 24W$, $\in = ?$

$$\text{Efficiency}, \in = \frac{P_s}{I_p \times E_p} \times 100 = \frac{24}{0.120 \times 240} \times 100 = \frac{2400}{28.8} = 83.33\%$$

Example 7

A transformer which can produce $8V$ from a $240V$ $a.c$ supply has an efficiency of 80%. If the current in the secondary coil is $15A$, calculate the current in the primary coil. A. $0.625A$ B. $1.600A$ C. $2.500A$ D. $6.250A$ *JAMB* 1998[46]

Solution

Efficiency $\in = 80\%$, $E_s = 8V$, $E_p = 240V$, $I_s = 15A$, $I_p = ?$

$\in = \frac{I_s \times E_s}{I_p \times E_p} \times 100$

Current in primary coil, $I_p = \frac{I_s \times E_s \times 100}{\in \times E_p} = \frac{15 \times 8 \times 100}{240 \times 80} = \frac{12000}{19200} = 0.625A$

Example 8

A $40KW$ electric cable is used to transmit electricity through a resistor of resistance 2.0Ω at $800V$. The power loss as internal energy is A. $5.0 \times 10^3 W$ B. $4.0 \times 10^3 W$ C. $5.0 \times 10^2 W$ D. $4.0 \times 10^2 W$ *JAMB* 2007[42]

Solution

Power transmitted, $P = 40KW = 40{,}000W$; resistance $R = 2.0\Omega$,

voltage $V = 800V$

From, $P = IV$, current $I = \frac{P}{V} = \frac{40000}{800} = 50A$

Power loss, $I^2R = 50^2 \times 2 = 500W = 5.0 \times 10^3 W$

EXERCISE 4C

1. The magnetic flux in a coil having 200 turns changes at the time rate of 0.08 $Wb\ s^{-1}$. The induced $e.m.f$ in the coil is A. 1.6V B. 16.0V C. 25.0V D. 250.0V
$JAMB$ 2000[43] Ans: 16.0V

2. A transformer with 5500 turns in its primary winding is used between a 240 $a.c$ supply and 120V kettle. Calculate the number of turns in the secondary winding.
$WAEC$ 1994[51] Ans: 2750

3. A house is supplied with a 240 $a.c$ mains. To operate a door bell rated at 8V, a transformer is used. If the number of turns in the primary coil of the transformer is 900 calculate the number of turns in the secondary coil of the transformer.
WAEC 1991[49]Ans: 30

4. If a transformer is used to light a lamp rated at 60W, 220V from a 4400V $a.c$ supply, calculate the
   (i) ratio of the number of turns of primary coil to the secondary coil in the transformer.
   (ii) current taken from the mains circuit if the efficiency of the transformer is 95%
$WAEC$ 1992[E3] (i)20: 1 (ii)0.014A

5. A transformer has 400 turns and 200 turns in the primary and secondary winding respectively. If the current in the primary and secondary winding are 3A and 5A respectively, calculate the efficiency of the transformer.
$WAEC$ 2002[42] Ans: 83.33%

6. A voltage and current in the primary of a transformer are 200V and 2A respectively. If the transformer is used to lighten 12V, 30W bulbs, calculate its efficiency.
$WAEC$ 1999[36] Ans: 75% Hint: $P_s = 10 \times 30W = 300W$

7. A transformer is rated 240V. If the primary coil is 4000 turns and the secondary voltage is 12V, determine the number of turns in the secondary coil. A. 250 B. 100 C. 150 D. 200 $JAMB$ 2005[1] Ans: 200

8. A transformer has 300 turns of wire in the primary coil and 30 turns in the secondary coil. If the input voltage is 100 volts, the output voltage is A. 5 volts B. 10 volts C. 15 volts D. 20 volts E. 25 volts $JAMB$ 1981[47] Ans: 10 volts

9. A transformer has a primary coil with 500 turns and a secondary coil with 2500 turns. When the voltage input to the primary coil is 120V, the output is
A. 6000V B. 600V C. 240V D. 60V E. 24V $JAMB$ 1985[46] Ans: 600V

10. The primary winding of a transformer has 400 turns and its secondary has 100 turns. If a source of $e.m.f$ of 12V is applied to the primary, the secondary $e.m.f$ will be A. 3V B. 6V C. 24V D. 48V $JAMB$ 1995[47] Ans: 3V

11. The primary coil of a transformer has N turns and is connected to a 120V $a.c$ power line. If the secondary coil has 1,000 turns and a terminal voltage of 1,200 volts, what is the value of N? A. 120 B. 100 C. 1000 D. 1200
$JAMB$ 2001[43] Ans: 100

12. A voltage of 240V is connected to the primary coil of a transformer. Calculate the ratio of the primary turns to the secondary turns if the voltage available at the secondary coil is 15V. $NECO$ 2005[50] Ans: 1: 16 or 0.06

13. A step down transformer has 300 turns in the primary coil and 30 turns in the secondary coil. If the input voltage is 100 volts, what is the output voltage?

NECO 2007[48] Ans: $10V$

14. A transformer is required to supply $12V_{rms}$ to operate a toy train set from a $240V_{r.m.s}$ If the number of turns in the secondary coil is 100, calculate the number of turns required in the primary coil. *WAEC* 2008[43] Ans: 2000

15. A step down transformer has a power output of $50W$ and efficiency of 80%. If the mains supply voltage is $200V$, calculate the primary current of the transformer. A. $0.31A$ B. $3.20A$ C. $3.40A$ D. $5.00A$

JAMB 2008[43] *Ans*: $0.31A$ % *Hint*: $\epsilon = \frac{P_s}{I_p \times E_p} \times 100$

16. A transformer has an efficiency of 92.5%. The ratio of the numbers of turns in the primary coil to that in the secondary coil is 128:45. If the current passing through the secondary coil is 9.0A, calculate the current passing through the primary coil.

NECO 2009[49] Ans: 3.42A

17. A transformer with a primary coil of 800 turns and a secondary coil of 50 turns has its primary coil connected to a 240V a.c mains. If the current passing through the primary coil is 0.5A, calculate the

(i) potential difference across the secondary ends.

(ii) current passing through the secondary coil assuming no power losses.

(iii) Power in the secondary coil if 10% of that in the primary coil is lost.

NECO 2009[E14] Ans: (i) 15V (ii) 8.0A (iii) 108W

5

SIMPLE A.C CIRCUIT

An alternating current ($a.c$) changes its direction periodically. $A.C$ are produced by alternating voltages. Alternating currents and alternating voltages are represented as follows.

$$I = I_o \text{Sin} 2\pi f t \text{ or } I = I_o \text{Sin}\omega t$$

$$V = V_o \text{Sin} 2\pi f t \text{ or } V = V_o \text{Sin}\omega t$$

Where I = Instantaneous current (A)
I_o = Peak or maximum current (A)
f = Frequency (Hz)
t = Time (s)
$\omega t = \theta$ = Phase angle of current or voltage
V = Instantaneous voltage (V)
V_o = Peak or maximum voltage (V)

PEAK AND ROOT MEAN SQUARE ($r.m.s$) VALUES

The peak value of an $a.c$ is the maximum value of the current recorded in an $a.c$ cycle. The $r.m.s$ current is defined as the steady or direct current ($d.c$) which produces the same heating effect per second in a given resistor.

Root mean square current, $I_{r.m.s} = \frac{I_o}{\sqrt{2}} = 0.71 I_o$

$$\text{Peak current, } I_o = \sqrt{2} \times I_{r.m.s} = 1.414 I_{r.m.s}$$

Similarly, Root mean square voltage, $V_{r.m.s} = \frac{V_o}{\sqrt{2}} = 0.71 V_o$ or

$$\text{Peak voltage } V_o = \sqrt{2} \times V_{r.m.s} = 1.414 V_{r.m.s}$$

Example 1

The current, I in an $a.c$ circuit is given by the equation: $I = 30\sin\ 100\pi t$ where t is the time in seconds. Deduce the following from this equation.

(i) Frequency of the current (ii) Peak value of the current
(iii) $r.m.s$ value of the current

Solution

The equation $I = 30\text{Sin}\ 100\pi t$ can be compared with

$$I = I_o \sin 2\pi f t$$

(i) $100\pi t = 2\pi f t$

$100 = 2f$

$$f = \frac{100}{2} = 50 H_z$$

(ii) Peak current $I_o = 30A$

(iii) $r.m.s$ current, $I_{r.m.s} = \frac{I_o}{\sqrt{2}} = \frac{30}{\sqrt{2}}$

Multiply the denominator and numerator by $\sqrt{2}$

$$\frac{\sqrt{2} \times 30}{\sqrt{2} \times \sqrt{2}} = \frac{30\sqrt{2}}{\sqrt{4}} = \frac{30\sqrt{2}}{2} = 15\sqrt{2}$$

$$I_{r.m.s} = 15\sqrt{2} A \text{ or } 21.21$$

Example 2

Calculate the peak voltage of a mains supply of $r.m.s$ value $220V$. *NECO 2000*[44]

Solution

$V_{r.m.s} = 220V$

Peak voltage, $V_o = 1.414 \times V_{r.m.s} = 1.414 \times 220 = 311V$

Example 3

The current through a resistor in an $a.c$ circuit is given as $2\sin\omega t$. Determine the $d.c$ equivalent of the current. A. $\frac{1}{\sqrt{2}}A$ B. $2\sqrt{2}A$ C. $2A$ D. $\sqrt{2}A$

JAMB 2001[40]

Solution

From the given equation $I = 2\sin\omega t$, peak current $I_o = 2A$

The direct current $(d.c)$ equivalent of $a.c$ is the root mean square current $I_{r.m.s}$

$$I_{r.m.s} = \frac{I_o}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

Multiply denominator and numerator by $\sqrt{2}$

$$I_{r.ms} = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}A$$

Example 4

If an $a.c$ is represented by $I = I_o\sin\omega t$. Calculate the instantaneous value of such a current, if in a circuit it has $r.m.s$ value of $15.0A$ when its phase angle is $30°$.

WAEC 1989[E3]

Solution

$I_{r.m.s} = 15.0A$, phase angle $\omega t = 30$

From $I_{r.m.s} = \frac{I_o}{\sqrt{2}}$, peak current $I_o = \sqrt{2} \times I_{r.m.s} = \sqrt{2} \times 15 = 21.21$

Instantaneous current, $I = I_o\sin\omega t$

$$= 21.21 \times \sin 30 = 21.21 \times 0.5 = 10.61A$$

Example 5

In an $a.c$ circuit the peak value of the potential difference is $180V$. What is the instantaneous potential difference when it has reach $\frac{1}{8}$th of a cycle? *WAEC 1989*[51]

Solution

Peak voltage $V_o = 180V$, instantaneous potential difference, $V = ?$

A cycle is $360° \therefore \frac{1}{8}$th of a cycle $= \frac{1}{8} \times 360 = 45°$

$\therefore$ Phase angle $\omega t = 45°$

$$V = V_o\sin\omega t$$

$$V = 180 \times \text{Sin}45$$

From trigonometry, $\sin 45 = \frac{\sqrt{2}}{2}$ $\therefore$ $V = 180 \times \frac{\sqrt{2}}{2} = 90\sqrt{2}V$

Example 6

The instantaneous value of the induced $e.m.f$ as a function of time is $\varepsilon = \varepsilon_o\sin\omega t$ where ε_o is the peak value of the $e.m.f$. The instantaneous value of the $e.m.f$, one quarter of the period is A. $\frac{\varepsilon_o}{4}$ B. ε_o C. 0 D. $\frac{\varepsilon_o}{2}$ *JAMB 2007*[49]

Solution

$\varepsilon = \varepsilon_o \sin\omega t$ $\theta = \omega t$, substitute to obtain

$\varepsilon = \varepsilon_o \sin\theta$

One period = 360°; $\therefore$ $\frac{1}{4} \times 360 = 90°$

Substitute $\theta = 90°$ into equation

$\varepsilon = \varepsilon_o \sin 90 = \varepsilon_o \times 1 = \varepsilon_o$ Ans: B

REACTANCE AND IMPEDANCE

Reactance is defined as the opposition offered to the passage of an $a.c$ by either the inductor or capacitor or both. Reactance due to an inductor is called ***inductive reactance***. Reactance due to a capacitor is called ***capacitive reactance***.

Impedance is the overall opposition offered to the passage of an $a.c$ mixed circuit containing a resistor any one or both of an inductor and a capacitor. Reactance and impedance are both measured in ohms (Ω).

$$\text{Also, reactance} = \frac{\text{amplitude of voltage across inductor or capacitor}}{\text{amplitude of current through inductor or capacitor}}$$

A.C Circuit Containing Only Resistor

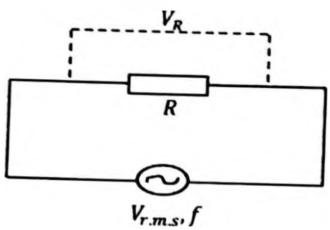


Fig 5.1

Ohm's law ($V = IR$) can be applied to the $a.c$ circuit with only resistance.
Substitute $V = V_o \sin\omega t$ and $I = I_o \sin\omega t$

$$\text{Resistance, } R = \frac{V}{I} = \frac{V_o \sin\omega t}{I_o \sin\omega t} = \frac{V_o}{I_o}$$

Substitute $V_o = \sqrt{2} \times V_{r.m.s}$ and $I_o = \sqrt{2} \times I_{r.m.s}$

$$\text{Resistance, } R = \frac{V_o}{I_o} = \frac{\sqrt{2} \times V_{r.m.s}}{\sqrt{2} \times I_{r.m.s}} = \frac{V_{r.m.s}}{I_{r.m.s}}$$

Example 7

A voltage supply of $12V_{r.m.s}$ and frequency $90Hz$ is connected to a 4Ω resistor. Calculate the peak value of the current. *NECO* 2004[50]

Solution

$V_{r.ms} = 12V$, resistance $R = 4\Omega$

From $R = \frac{V_{r.m.s}}{I_{r.m.s}}$, $I_{r.m.s} = \frac{V_{r.m.s}}{R} = \frac{12}{4} = 3A$

Peak current, $I_o = \sqrt{2} \times I_{r.m.s} = \sqrt{2} \times 3 = 4.2A$

Example 8

Calculate the amount of heat generated in an external load of resistance 8Ω if an $a.c$ of peak value $5A$ is passed through it for $100s$. *NECO 2003*[46]

Solution

Resistance $R = 8\Omega$, time $t = 100s$, peak current $I_o = 5A$

$$I_{r.m.s} = \frac{I_o}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54\ A$$

Heat energy $H = I^2Rt = 3.54^2 \times 8 \times 100 = 10025.28 = 1.0 \times 10^4 J$

A.C circuit containing only inductor.

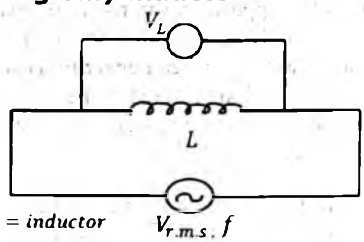


Fig 5.2

From ohm's law ($V = IR$), voltage across inductor is $V_L = I \times X_L$

Inductive reactance $X_L = \frac{V_o}{I_o} = \frac{V_{r.m.s}}{I_{r.m.s}} = \omega L = 2\pi f L$

From $V_L = I \times X_L$ and $X_L = 2\pi f L$, inductance L of an inductor is given by:

$$L = \frac{V_L}{2\pi f I} \quad or \quad L = \frac{V_L}{\omega I}$$

Example 9

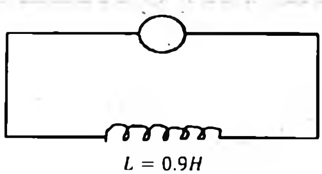


Fig 5.3

If the frequency of the $e.m.f$ source in the $a.c$ circuit illustrated above is $\frac{500}{\pi} Hz$. What is the reactance of the inductor? *WAEC 1991*[41]

Solution

Frequency $f = \frac{500}{\pi} Hz$, inductance $L = 0.9H$

Inductive reactance $X_L = 2\pi f L$

$$= \frac{2\pi \times 500 \times 0.9}{\pi} = 2 \times 500 \times 0.90 = 900\Omega$$

Example 10

Calculate the inductance L, of the coil in the circuit diagram shown below.

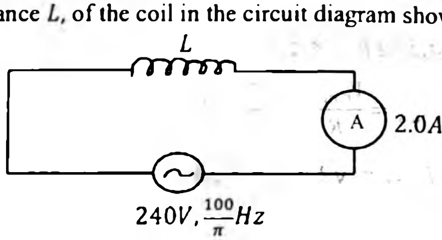


Fig 5.4

WAEC 2002[44]

Solution

Current, $I = 2.0A$, voltage, $V = 240V$, frequency, $f = \frac{100}{\pi}$

Inductance, $L = \frac{V}{2\pi fI} = \frac{240}{2\pi \times \frac{100}{\pi} \times 2} = \frac{240}{400} = 0.6H$

Example 11

A $220V$, $60Hz$ $a.c$ supply is connected to an inductor of $3.5H$. What is the current passing through the inductor? ($\pi = \frac{22}{7}$) *NECO* 2001

Solution

From $V = IX_L$, current through inductor, $I = \frac{V}{X_L}$

$$I = \frac{V}{2\pi fL} = \frac{220}{3 \times 3.142 \times 60 \times 3.5} = \frac{220}{1319.62} = 0.167A$$

Example 12

At what frequency would a $10H$ inductor have a reactance of 2000Ω?

A. $\frac{\pi}{200}Hz$ B. $\frac{\pi}{100}Hz$ C. $\frac{100}{\pi}Hz$ D. $100\pi Hz$ *JAMB* 1999[39]

Solution

Frequency $f =?$ inductance $L = 10H$, inductive reactance, $X_L = 2000\Omega$

From $X_L = 2\pi fL$, frequency $f = \frac{X_L}{2\pi L}$

$$f = \frac{2000}{2\pi \times 10} = \frac{100}{\pi}Hz$$

Example 13

Calculate the energy stored in an inductor of inductance $0.2H$ through which a current of $5.0A$ flows. *NECO* 2006[49]

Solution

Current, $I = 5.0A$; inductance, $L = 0.2H$

Energy, $E = \frac{1}{2}LI^2 = \frac{0.2 \times 5.0^2}{2} = 2.5J$

A.C Circuit Containing Only Capacitor.

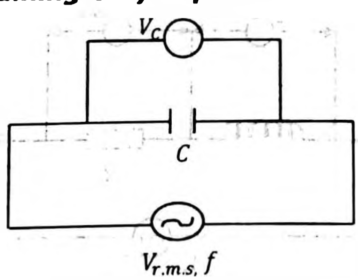


Fig 5.5

From ohm's law ($V = IR$), voltage across capacitor, C, is $V_c = I \times X_c$

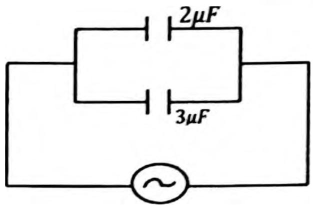
Capacitive reactance $X_c = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

From $V_c = I \times X_c$ and $X_c = \frac{1}{2\pi fC}$, capacitance, C, of a capacitor is given by:

$$C = \frac{I}{2\pi f V_C} \quad or \quad C = \frac{I}{\omega V_c}$$

Example 14

If the frequency of the $a.c$ circuit illustrate below is $\frac{500}{\pi} Hz$. What would be the reactance in the circuit?



WAEC 1992[52]

Fig 5.6

Solution

Frequency $f = \frac{500}{\pi} Hz$; effective capacitance. $C = 2\mu F + 3\mu F = 5\mu F = 5 \times 10^{-6} F$

Capacitive reactance $X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi \times \frac{500}{\pi} \times 5 \times 10^{-6}} = \frac{1}{0.005} = 200\Omega$

Example 15

At what frequency would a capacitor of $2.5\mu F$ used in a radio circuit have a reactance of 250Ω? A. $\frac{800}{\pi} Hz$ B. $200\pi Hz$ C. $2000\pi Hz$ D. $\frac{\pi}{800} Hz$ JAMB 2002[50]

Solution

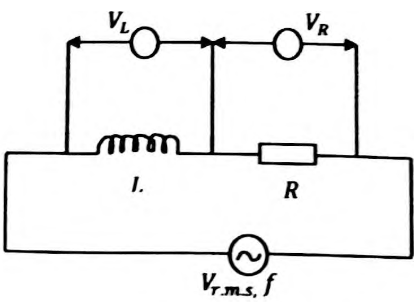
Capacitance $C = 2.5\mu F = 2.5 \times 10^{-6} F$, capacitor reactance $X_c = 250\Omega$

$$X_c = \frac{1}{2\pi f C} \quad \therefore \quad f = \frac{1}{2\pi C X_c}$$

$$f = \frac{1}{2\pi \times 2.5 \times 10^{-6} \times 250} = \frac{1}{\pi \times 1250 \times 10^{-6}} = \frac{1 \times 10^6}{\pi \times 1250} = \frac{1000000}{\pi \times 1250}$$

$$= \frac{800}{\pi} Hz$$

A C Circuit Containing Only Inductor And Resistor.



$L - R$ Series Circuit

Fig 5.7

For $L - R$ circuit, the impedance, $Z_{LR} = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{V_o}{I_o}$

$$Z_{LR} = \sqrt{R^2 + X_L^2}$$

$$\text{Current in } L - R \text{ circuit}, I_o = \frac{V_o}{\sqrt{R^2 + X_L^2}}$$

The applied voltage V also lead on the current I by an angle θ given by:

$$\text{Tan } \theta = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

Effective or total circuit voltage, $V = \sqrt{V_R^2 + V_L^2}$

Example 16

For the diagram shown below,

(i) Calculate the inductive reactance of the circuit.

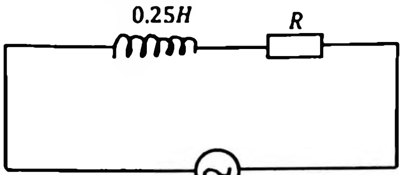


Fig 5.8

$50v, \frac{100}{\pi} H_z$

(ii) If the current in the resistor R is $0.05A$, calculate the potential difference across the inductor. *WAEC* 1995[41]

Solution

Inductance $L = 0.25H$, frequency $f = \frac{100}{\pi} H_z$, current $I = 0.05A$

(i) Inductive reactance $X_L = 2\pi f L$

$$= 2\pi \times \frac{100}{\pi} \times 0.25 = 50\Omega$$

(ii) Potential difference across inductor, $V_L = IX_L = 0.05 \times 50 = 2.5V$

Example 17

A circuit consist of a 10.0Ω resistor and a $0.8H$ inductor connected in series to a $240V$, $60Hz$ $a.c$ source.

(i) Draw the circuit diagram of the arrangement

Calculate its:

(i) Inductive reactance

(ii) Inductive reactance

(iii) Impedance circuit of the circuit

(iv) Capacitance of a capacitor that must be connected in series with these components to obtain maximum current $[\pi = 3.142]$ *NECO* 2005[E14]

Solution

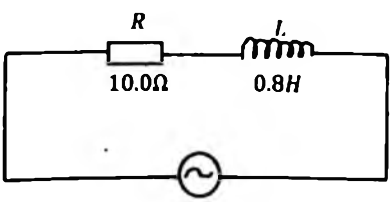


Fig 5.9

$240V, 60H_z$

$R = 10\Omega,\ L = 0.8H,\ V_{r.m.s} = 240V,\ f = 60H_z$

(ii) Inductive reactance $X_L = 2\pi fL = 2\pi \times 60 \times 0.8 = 301.63\Omega$

(iii) Impedance $Z_{LR} = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 301.63^2} = \sqrt{91080.66} = 301.8\Omega$

(v) Maximum current is obtained at resonance when $X_L = X_c$

$$301.63 = X_C$$

$$301.63 = \frac{1}{2\pi fC}$$

$$\text{Capacitance, } C = \frac{1}{2 \times 3.142 \times 60 \times 301.63} = 8.8 \times 10^{-6}F \ or \ 8.8\mu F$$

Example 18

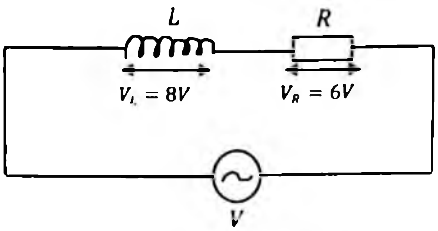


Fig 5.10

In the series $a.c$ circuit shown above, the potential difference across the inductor is $8V_{r.m.s}$ and that across the resistor is $6V_{r.m.s}$. The voltage is

A. $2V$ B. $10V$ C. $14V$ D. $48V$ *JAMB* 1999[45]

Solution

$$V_L = 8V, \qquad V_R = 6V$$

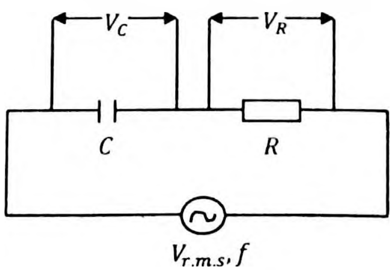
For a $L - R$ circuit, $V_{r.m.s} = \sqrt{V_R^2 + V_L^2}$

$$= \sqrt{8^2 + 6^2}$$

$$= \sqrt{64 + 36}$$

$$V_{r.m.s} = \sqrt{100} = 10V$$

A.C Circuit Containing Only Resistor and Capacitor.



R.C Series circuit

Fig 5.11

For $R.C$ circuit, the impedance $Z_{RC} = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{V_o}{I_o}$

$$Z_{RC} = \sqrt{R^2 + X_C^2}$$

$$\text{Current in the } R.C \text{ current, } I_o = \frac{V_o}{\sqrt{R^2 + X_C^2}}$$

The current I also leads on the applied voltage V by an angle θ given by

$$\text{Tan } \theta = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

Effective or total circuit voltage, $V = \sqrt{V_R^2 + V_C^2}$

Example 19

An $a.c$ source is connected in series to a capacitor of capacity reactance of $10^3\sqrt{3}$ and a resistor of resistance $10^3\Omega$. The impedance of the circuit is

A. $2000\sqrt{3}\Omega$ B. 2000Ω C. $1000\sqrt{3}$ D. 1000Ω

Solution

Capacitive reactance $X_C = 10^3\sqrt{3}$, resistance $R = 10^3\Omega$

Impedance $Z_{RC} = \sqrt{R^2 + X_C^2}$

$$= \sqrt{(10^3)^2 + \left(10^3\sqrt{3}\right)^2}$$

$$= \sqrt{10^6 + (10^3)^2\left(\sqrt{3}\right)^2}$$

$$= \sqrt{10^6 + 10^6 \times 3}$$

$$= \sqrt{10^6 + 3 \times 10^6}$$

$$= \sqrt{4 \times 10^6}$$

$$= 2000\Omega$$

Example 20

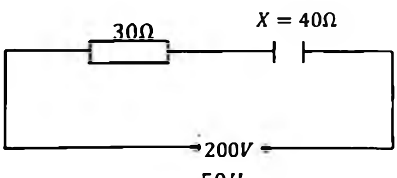


Fig 5.12

In the $a.c$ circuit above, the current valuo is A. $6.67A$ B. $4.00A$ C. $3.00A$ D. $0.58A$ *JAMB* 2000[46]

Solution

Resistance $R = 30\Omega$, $V_{r.m.s} = 200V$,

Frequency $f = 50H_z$ Capacitor reactance $X_C = 40\Omega$

For $R.C$ circuit, current $I = \dfrac{V}{\sqrt{R^2 + X_C^2}} = \dfrac{200}{\sqrt{30^2 + 40^2}} = \dfrac{200}{\sqrt{2500}} = \dfrac{200}{50} = 4.00A$

Example 21

The resistance in a series $R.C$ circuit is 5Ω. If the impedance of the circuit is 13Ω, calculate the reactance of the capacitor. *WAEC* 1996[54]

Solution

$R = 5\Omega$, $Z = 13\Omega$, capacitor reactance, $X_C = ?$

For $R.C$, $Z_{RC} = \sqrt{R^2 + X_C^2}$

$$13 = \sqrt{5^2 + X_C^2}$$

$$13^2 = 5^2 + X_C^2$$

$$169 - 25 = X_C^2$$

$$144 = X_C^2 \qquad \therefore \quad X_C = \sqrt{144} = 12\Omega$$

Example 22

A $2.0\mu F$ capacitor is connected in series with a 300Ω resistor across an $a.c$ source of potential difference $50V_{r.m.s}$ and frequency $\frac{314}{2\pi}Hz$. Calculate the $r.m.s$ value of the current in the circuit. *NECO 2006*[51]

Solution

Capacitance, $C = 2.0\mu F = 2 \times 10^{-6}F$, resistance, $R = 300\Omega$,

$V_{r.m.s} = 50V$, frequency, $f = \frac{314}{2\pi}Hz$

$$\text{Capacitive reactance } X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times \frac{314}{2\pi} \times 2 \times 10^{-6}} = \frac{1}{0.000628} = 1592.36\Omega$$

$$\text{From } Z_{RC} = \frac{V_{r.m.s}}{I_{r.m.s}},$$

$$I_{r.m.s} = \frac{V_{r.m.s}}{Z_{RC}} = \frac{V_{r.m.s}}{\sqrt{R^2 + X_C^2}} = \frac{50}{\sqrt{300^2 + 1592.36^2}} = \frac{50}{\sqrt{2625599.821}}$$

$$= \frac{50}{1620.37} = 0.031A$$

Example 23

A $40\mu F$ capacitor in series with a 40Ω resistor is connected to a $100V$, $50Hz$ $a.c$ supply.

(i) Draw a circuit diagram of the arrangement

(ii) Calculate the

I. Impedance in the circuit;

II. Current in the circuit;

III. Potential difference across the capacitor. *WAEC 2008*[E14]

Solution

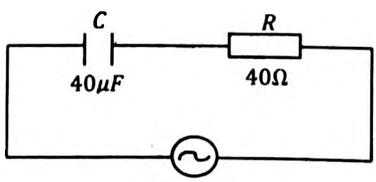


Fig 5.13 $100V, 50Hz$

Capacitance $C = 40\mu F = 40 \times 10^{-6}F$; resistance $R = 40\Omega$;

Frequency $f = 50Hz$ $V_{r.m.s} = 100V$;

(i) Impedance $Z_{RC} = \sqrt{R^2 + X_C^2}$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 40 \times 10^{-6}} = 79.58\Omega$$

$$\therefore \quad Z_{RC} = \sqrt{40^2 + 79.58^2} = 89.07\Omega$$

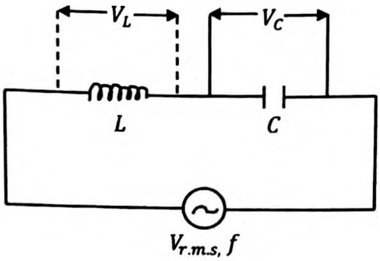
(ii) $$\text{current}, I = \frac{V_{r.m.s}}{\sqrt{R^2 + X_C^2}} = \frac{V_{r.m.s}}{Z}$$

$$\therefore \quad I = \frac{100}{89.07} = 1.12A$$

(iii) Potential difference across capacitor $V = I \times X_C$

$$\therefore \qquad V = 1.12 \times 79.58 = 89.35V$$

A.C Circuit Containing Only Inductor and Capacitor



L.C Series Circuit

Fig 5.14

For *L.C* circuit, the impedance, $Z_{LC} = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{V_o}{I_o}$

$$Z_{LC} = \sqrt{(X_L - X_C)^2} = X_L - X_C$$

Current in L − C circuit, $\quad I_o = \frac{V_o}{\sqrt{(X_L - X_C)^2}} = \frac{V_o}{X_L - X_C}$

In the *L.C* circuit, the inductor voltage V_L and the capacitor voltage V_C are out of phase by180°.

Effective voltage $V = \sqrt{(V_L - V_C)^2} = V_L - V_C$

Example 24

The frequency of the *a.c* circuit below is $\frac{500}{\pi} H_z$. What is the reactance in the circuit

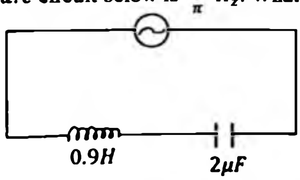


Fig 5.15

NECO 2002[52]

Solution

Inductance $L = 0.9H$, capacitance $C = 2\mu F = 2 \times 10^{-6}F$

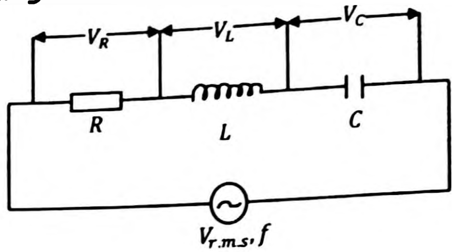
Inductor reactance $X_L = 2\pi f L = 2\pi \times 500/\pi \times 0.9 = 900\Omega$

Capacitor reactance, $\quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times \frac{500}{\pi} \times 2 \times 10^{-6}} = \frac{1}{0.002} = 500\Omega$

Circuit reactance, $Z_{LC} = \sqrt{X_L^2 + X_C^2} = \sqrt{900^2 + 500^2} = \sqrt{1060000}$

$$= 1029.56 \simeq 1030\Omega$$

A.C Circuit Containing Resistor, Inductor and Capacitor.



$R.L.C$ Series Circuit

Fig 5.16

For RLC circuit, the impedance, $Z = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{V_o}{I_o}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Current in RLC circuit, $I_o = \frac{V_o}{\sqrt{R^2 + (X_L - X_C)^2}}$

Phase angle is given by; $Tan\ \theta = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$

Effective voltage, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Example 25

An $a.c$ circuit of $e.m.f$ $12V$ has a resistor of resistance 8Ω connected in series to an inductor of inductive reactance 16Ω and a capacitor of capacitive reactance 10Ω. The current flow in the circuit is. $1.4A$ B. $14.0A$ C. $1.2A$ D. $12.0A$ *JAMB* 2004[29]

Solution

$V_{r.m.s} = 12V,\ R = 8\Omega,\ X_L = 16\Omega,\ X_C = 10\Omega$

Curent in RLC circuit $I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

$$I = \frac{12}{\sqrt{8^2 + (16 - 10)^2}} = \frac{12}{\sqrt{8^2 + 6^2}} = \frac{12}{\sqrt{64 + 36}} = \frac{12}{\sqrt{100}} = \frac{12}{10} = 1.2A$$

Example 26

A source of $e.m.f$ of $240V$ and frequency $50Hz$ is connected to a series arrangement of a resistor, an inductor and a capacitor. When the current in the capacitor is $10A$, the potential difference across the resistor is $140V$ and that across the inductor is $50V$. Calculate the

(i) Potential difference across the capacitor
(ii) Capacitance of the capacitor
(iii) Inductance of the inductor

WAEC 2002[E14]

Solution

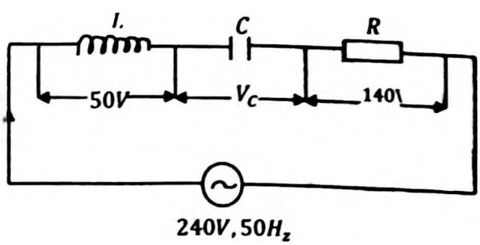


Fig 5.17

Potential difference across inductor $V_L = 50V$
Potential difference across resistor $V_R = 140V$
Potential difference across capacitor $V_C = ?$
Potential difference across RLC circuit $V = 240V$
Frequency $f = 50Hz$;
Current through capacitor $I = 10A$

(i) $$V^2 = V_R^2 + (V_L - V_C)^2$$

$$240^2 = 140^2 - (50 - V_C)^2$$

$$38000 = (50 - V_C)^2$$

$$\sqrt{38000} = 50 - V_C$$

$$\pm 194.9 = 50 - V_C$$

$$V_C = 50 + 194.9 \text{ or } 50 - 194.9$$

$$V_C = 244.9V \text{ or } -144.9V$$

Potential difference across capacitor $V_C = 245V$

(ii) Capacitance $C = \dfrac{I}{2\pi f V_C} = \dfrac{10}{2\pi \times 50 \times 245} = \dfrac{10}{76937.60}$

$$C = 1.299 \times 10^{-4} \simeq 130 \times 10^{-6}F \text{ or } 130\mu F$$

From $V_L = IX_L$ and $X_L = 2\pi f L$

$$\text{inductance}, L = \frac{V_L}{2\pi f I} = \frac{50}{2\pi \times 50 \times 10} = \frac{1}{2\pi \times 10} = 0.0159H$$

Example 27
A series circuit consisting of a 100Ω resistor, a coil of $0.10H$ inductance and a $20\mu F$ capacitor, is connected across a $110V, 60Hz$ power source.
(i) Draw the circuit diagram of the arrangement. Calculate the
(ii) Inductive resistance (iii) Capacitive reactance (iv) Impedance of the circuit
(v) Current in the circuit (vi) Power loss *NECO* 2002^{E14}
Solution

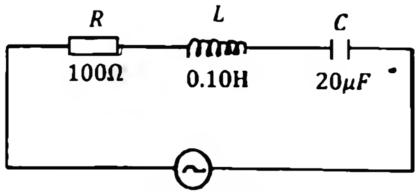


Fig 5.18

$R = 100\Omega$, $L = 0.10H$, $C = 20\mu F = 20 \times 10^{-6}F$, $V_{r.m.s} = 110V$, $f = 60H_z$

(ii) Inductive reactance $X_L = 2\pi f L = 2\pi \times 60 \times 0.10 = 37.70\Omega$

(iii) Capacitive reactance $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 60 \times 20 \times 10^{-6}} = 132.63\Omega$

(iv) Impedance for RLC circuit, $Z_{RLC} = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z_{RLC} = \sqrt{100^2 + (37.70 - 132.63)^2}$$

$$= \sqrt{100^2 + (-94.93)^2}$$

$$= \sqrt{10000 + 9011.70}$$

$$= \sqrt{19011.7} = 137.88\Omega$$

(vi) Power loss, $P = I^2R = 0.8^2 \times 100 = 64W$

$$or \quad P = 1Vcos\theta = IV \times \frac{R}{Z} = \frac{0.8 \times 110 \times 100}{137.88} = 63.8 \simeq 64W.$$

RESONANCE IN A.C CIRCUIT

A resonance circuit is an RLC circuit through which maximum alternating currents flows. The frequency at which resonance occurs is called resonance frequency (f_o) and it occurs when $X_L = X_C$. Also, at resonance $V_L = V_C$

Resonance frequency, $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

Maximum current at resonance $I = V/R$

Example 28

In a series $L - C$ circuit, the inductance and the capacitance are $5H$ and $2\mu F$ respectively. Calculate the resonance frequency of the circuit. *NECO* 2000[51]

Solution

Inductance $L = 5H$, capacitance $C = 2\mu F = 2 \times 10^{-6}F$

Resonance frequency, $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi \times \sqrt{5 \times 2 \times 10^{-6}}}$

$$f_o = \frac{1}{2\pi \times \sqrt{1 \times 10^{-5}}} = \frac{1}{2\pi \times 3.16 \times 10^{-3}} = \frac{1}{1.987 \times 10^{-2}} = 50.3Hz$$

Example 29

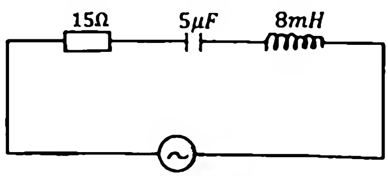


Fig 5.19 $E = \varepsilon_o sin\omega t$

In the $a.c$ circuit diagram above, the resonance frequency is

A. $\frac{5000}{\pi Hz}$ B. $\frac{2500}{\pi Hz}$ C. $\frac{5000}{\pi Hz}$ D. $\frac{2500}{\pi Hz}$ *JAMB* 1998[45]

Solution

Inductance $L = 8mH = 8 \times 10^{-3}H$, capacitance $C = 5\mu F = 5 \times 10^{-6}F$

Resonance frequency $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi \times \sqrt{8 \times 10^{-3} \times 5 \times 10^{-6}}}$

$$f_o = \frac{1}{2\pi\sqrt{4 \times 10^{-8}}} = \frac{1}{2\pi \times 0.0002} = \frac{10000}{4\pi} = \frac{2500}{\pi}Hz$$

Example 30

A capacitor of $20 \times 10^{-12}F$ and an inductor are joined in series. The value of the inductance that will give the circuit a resonant frequency of $200KHz$ is

A. $\frac{1}{16}H$ B. $\frac{1}{8}H$ C. $\frac{1}{64}H$ D. $\frac{1}{32}H$ *JAMB* 2001[49]

Solution
$C = 20 \times 10^{-12}F$; Resonance frequency, $f_o = 200KHz = 200000Hz$;
Inductance $L = ?$

$$\text{At resonance } X_L = X_C \quad or \quad 2\pi fL = \frac{1}{2\pi fC}$$

$$\text{Rearrange and substitute, } 2\pi \times 2 \times 10^5 \times L = \frac{1}{2\pi \times 2 \times 10^5 \times 20 \times 10^{-12}}$$

$$L = \frac{1}{2\pi \times 2 \times 10^5 \times 2\pi \times 2 \times 10^5 \times 20 \times 10^{-12}} = \frac{1}{32}H$$

Example 31
Given that the voltage amplitude of the $a.c$ source shown in figure 5.19 is $100V$
(i) Determine the resonant frequency of the source.
(ii) Calculate the maximum current passing through the resistor.

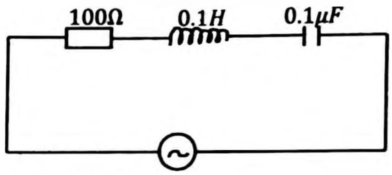


Fig 5.20

Solution
$R = 100\Omega$; $L = 0.1H$; $C = 0.1\mu F = 0.1 \times 10^{-6}F$; $V_{r.m.s} = 100V$

$$\text{(i) Resonant frequency, } f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 0.1 \times 10^{-6}}}$$

$$f_o = \frac{1}{2\pi \times 0.0001} = 1591.55Hz$$

$$\text{(ii) Maximum current } I_{r.m.s} = \frac{V_{r.m.s}}{Z} = \frac{V_{r.m.s}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

If current is maximum, $X_L = X_C$ or $X_L - X_C = 0$

$$\therefore I_{r.m.s} = \frac{V_{r.m.s}}{R} = \frac{100}{100} = 1A$$

POWER IN A.C CIRCUIT

Average power in $a.c$ circuit $P = IV\cos\theta$ or $P = I^2R$
Where I = root mean square current, $I_{r.m.s}$
V = root mean square voltage, $V_{r.m.s}$
θ = phase angle, angle of lag or lead between I and V

$$\cos\theta = \text{power factor} = \frac{\text{resistance}}{\text{impedance}} = \frac{R}{Z}$$

Example 32
The power dissipated in an $a.c$ circuit with an $r.m.s.$ current of $5A$, $r.m.s.$ voltage of $10V$ and a phase angle of $60°$ is A. $25W$ B. $70W$ C. $120W$ D. $125W$

JAMB 1997[46]

Solution

$I_{r.m.s} = 5A,\ V_{r.m.s} = 10V,$ phase angle $\theta = 60°$

Power $P = 1V\cos\theta = 5 \times 10 \times \cos 60 = 25W$

Example 33

When an $a.c$ given by $1 = 10\sin(120\pi)t$ passing through a 12Ω resistor, the power dissipated in the resistor is A. $1200W$ B. $600W$ C. $120W$ D. $30W$ *JAMB* 2003[45]

Solution

$I = 10\sin(120\pi)t$ From equation, peak current $I_o = 10A,$ resistance $R = 12\Omega$

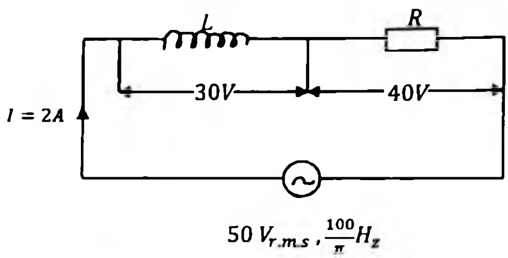
Root mean square current, $V_{r.m.s} = \frac{I_o}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07A$

$$\text{Power, } P = I_{r.ms}^2 R = 7.07^2 \times 12 = 599.8 = 600W$$

Example 34

The diagram below illustrate an $a.c$ source of $50V\ (r.m.s),\ \frac{100}{\pi}Hz$ connected in series with an inductor of inductance L and a resistor of resistance R. The current in the circuit is $2A$ and the potential difference across L and R are $30V$ and $40V$ respectively. Calculate

(i) The power factor of the circuit
(ii) The average power dissipated in the circuit



WAEC 1998[49]

Fig 5.21

Solution

$V_{r.m.s} = 50V;$ $f = \frac{100}{\pi}Hz;$ $I = 2A;$ $V_L = 30V;$ $V_R = 40V$

(i) Power factor, $\cos\theta = \frac{\text{resistance}}{\text{impedance}} = \frac{R}{Z}$

From, $V = IR$ for a resistor, $R = \frac{V_R}{I} = \frac{40}{2} = 20\Omega$

From, $V_L = IX_L$ for an inductor, $X_L = \frac{V_L}{I} = \frac{30}{2} = 15\Omega$

For $L - R$ circuit, $Z = \sqrt{R^2 + X_L}$

$$= \sqrt{20^2 + 15^2}$$

$$= \sqrt{400 + 225} = \sqrt{625} = 25\Omega$$

Power factor, $\cos\theta = \frac{R}{Z} = \frac{20}{25} = 0.8$

(ii) Average power $P = I^2R = 2^2 \times 20 = 80W$

EXERCISE 5.

1. Calculate the peak voltage of a mains supply of $r.m.s$ value of $220V$
$WAEC$ 1994[51] Ans: $311V$

2. In an $a.c$ circuit the peak value of the potential difference is $180V$. What is the instantaneous potential difference when the phase angle is 45°?
$WAEC$ 1994[56] Ans: $90\sqrt{2}$

3. In $A.C$ circuit theory, the root mean square $(r.m.s)$ current, $I_{r.m.s}$, and the peak (maximum) current I_o are related by A. $I_o = I_{r.m.s} / \sqrt{2}$ B. $I_{r.m.s} = I_o/\sqrt{2}$ C. $I_{r.m.s} = 1/I_o\sqrt{2}$ D. $I_o = 1/I_{r.m.s}\sqrt{2}$ $JAMB$ 1992[46] Ans: B

4. The voltage of the domestic electric supply is represented by the equation $V = 311\sin 314.2t$. Determine the frequency of the $a.c$ supply. A. $50.0Hz$ B. $100.0Hz$ C. $311.0Hz$ D. $314.2Hz$ $(\pi = 3.142)$ $JAMB$ 1997[47] Ans: $50.0Hz$

5. When a certain $a.c$ supply is connected to a lamp, it lights with the same brightness as it does with a $12V$ $d.c$ battery. The $r.m.s$ value of the $a.c$ supply is A. $8.5V$ B. $17.0V$ C. $3.5V$ D. $12.0V$ $JAMB$ 2006[47] Ans: $12.0V$

6. Calculate the peak voltage that could be obtained from a 220 $V_{r.m.s}$ main supply.
$NECO$ 2005[51] Ans: $311V$

7.

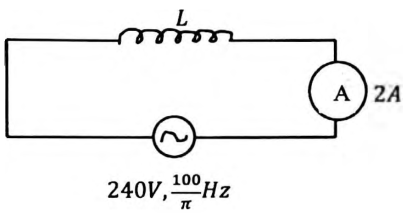


Fig 5.22

Calculate the inductance L of the coil in the circuit shown above.
$WAEC$ 1995[38] Ans: $0.6H$

8. An inductor of inductance $10H$ carries a current of $0.2A$. Calculate the energy stored in the inductor. $WAEC$ 2002[44] Ans: $0.2J$

9. Calculate the inductance of an inductor whose reactance is one ohm at $50Hz$.
$WAEC$ 2002[45] Ans: $3.18 \times 10^{-3}H$

10. A $2H$ inductor has negligible resistance and is connected to a $\frac{50}{\pi}Hz$ $a.c$ supply. The reactance of the inductor is A. 200Ω B. 50Ω C. $\frac{100}{\pi}\Omega$ D. $\frac{25}{\pi}\Omega$
$JAMB$ 2000[36] Ans: 200Ω

11. The energy stored in an inductor of inductance $5mH$ when a current of $6A$ flows through it is A. $1.8 \times 10^{-2}J$ B. $9.0 \times 10^{-3}J$ C. $1.4 \times 10^{-2}J$ D. $9.0 \times 10^{-2}J$ $JAMB$ 2004[27] Ans: $9.0 \times 10^{-2}J$

12. If two inductors of inductances $3H$ and $6H$ are arranged in series, the total inductance is A. $18.0H$ B. $9.0H$ C. $2.0H$ D. $0.5H$ $JAMB$ 2005[15] Ans: $9.0H$

13. Calculate the energy stored in an inductor of inductance $0.5H$ when a current of $3A$ flows through it. $NECO$ 2004[49] Ans: $2.25J$

14. A direct current of $5A$ flows through a $0.2H$ inductor. Calculate the energy stored in the inductor. $WAEC$ 2008[45] Ans: $2.5J$

15. Calculate the energy stored in an inductor of inductance $0.5H$ when a current of $5A$ flows through it. $NECO$ 2008[47] Ans: $6.25J$

16. Calculate the energy stored in an inductor of inductance $0.5H$ when a current of $5A$ flows through it. $NECO$ 2008[47] Ans: $6.25J$

17.

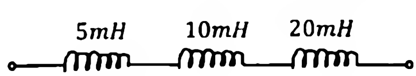


Fig 5.23

Given three inductors of inductances $5mH$, $10mH$ and $20mH$ connected in series, the effective inductance is A. $0.35mH$ B. $3.50mH$ C. $2.90mH$ D. $35.00mH$ *JAMB* 2008[44] Ans: $35.00mH$

18. A capacitor of capacitance $25\mu F$ is connected to an $a.c$ power source of frequency $\frac{200}{\pi}Hz$. Calculate the reactance of the capacitor.
WAEC 1995[43] Ans: 100Ω

19. A capacitor of capacitance $25\mu F$ is connected to an $a.c$ power source of frequency $\frac{200}{\pi}Hz$. Calculate the reactance of the capacitor. *WAEC* 1997[40] Ans: 100Ω

20.

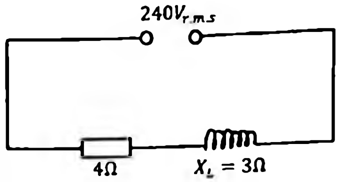


Fig 5.24

In the diagram above, determine the $r.m.s$ current. A. $31A$ B. $48A$ C. $60A$ D. $80A$
JAMB 2001[38] Ans: $48A$

21.

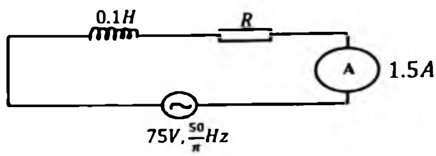


Fig 5.25

From the diagram above, the inductive reactance and the resistance R are respectively
A. 25Ω and 51Ω B. 20Ω and 50Ω C. 10Ω and 50Ω D. 50Ω and 45Ω
JAMB 2007[39] Ans: 10Ω and 50Ω

22. Calculate the reactance of the inductor in the circuit diagram shown. ($\pi = \frac{22}{7}$)

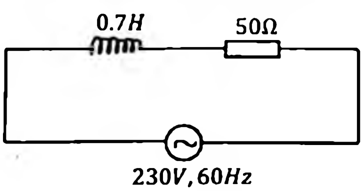


Fig 5.26 *WAEC* 2001[42] Ans: 264Ω

23. In a series $R - C$ circuit, the resistance of the resistor is 4Ω and the capacitive reactance is 3Ω. Calculate the impedance of the circuit. *WAEC* 1993[35] Ans: 5Ω

24. Calculate the following in the series circuit shown below
(i) Reactance of the capacitor.
(ii) Impedance of the circuit
(iii) Current through the circuit
(iv) Voltage across the capacitor
(v) Average power used in the circuit

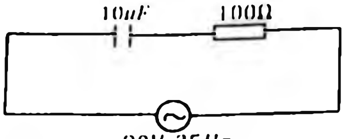


Fig 5.27 $90V, 25Hz$

$WAEC$ 1988[E4] (i) 636.62Ω (ii) 1185.4Ω (iii) $0.076A$ (iv) $48.33V$ (v) $5.8W$

25. The resistance of a series $R-C$ circuit is 5Ω. If the impedance of the circuit is 13Ω, calculate the reactance of the capacitor. $NECO$ 2003[50] Ans: 12Ω

26.

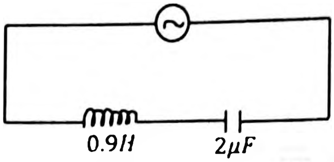


Fig 5.28

If the frequency of the $a.c$ circuit illustrated is $\frac{500}{\pi}Hz$ what would be the reactance in the circuit? $WAEC$ 1990[43] Ans: 400Ω

27. A source of $e.m.f$ $240V$ and frequency $50Hz$ is connected to a resistor, an inductor and a capacitor in series. When the current in the capacitor is $10A$, the potential difference across the resistor is $140V$ and that across the inductor is $50V$. Draw the vector diagram of the potential differences across the inductor, capacitor and the resistor. Calculate the

(i) Potential difference across the capacitor.

(ii) Capacitance of the capacitor.

(iii) Inductance of the inductor. $WAEC$ 1991[E3] Ans:(i) $245V$ (ii) $130\mu F$ (iii) $0.0159H$

28. In the diagram below the resistor has a resistance of 8Ω while the reactance of the inductor and the capacitor are 10Ω and 16Ω respectively. Calculate the current in the circuit.

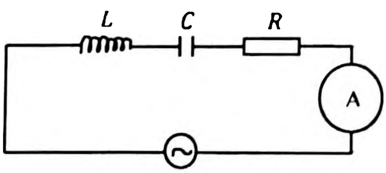


Fig 5.29 $130V, 50Hz$ $WAEC$ 1996[53] Ans: $13.0A$

29. A series RLC circuit comprises a $100\text{-}\Omega$ resistor, a $3\text{-}H$ inductor and a $4\text{-}\mu F$ capacitor. The $a.c$ source of the circuit has an $e.m.f$ of $100V$ and frequency of $\frac{160}{\pi}Hz$.

(i) Draw the circuit diagram of the arrangement. Calculate the

(ii) Capacitive reactance

(iii) Inductive inductance

(iv) Impedance of the circuit

(v) Current in the circuit

(vi) Average power dissipated in the circuit. $WAEC$ 1999[E4]

Ans: (i) 781.125Ω (ii) 960Ω (iii) 204.82Ω (iv) $0.488A$ (vi) $23.8W$

30. In a series $L-C$ circuit, the inductance and the capacitance are $0.5H$ and $20\mu F$ respectively. Calculate the resonant frequency of the circuit.

$WAEC$ 1998[48] Ans: $50.31Hz$

31. In a series $R.L.C$ circuit at resonance, the voltages across the resistor and the inductor are $30V$ and $40V$ respectively. What is the voltage across the capacitor?

A. $30V$ B. $40V$ C. $50V$ D. $70V$ $JAMB$ 1999[48] Ans: $40V$

32. The resonance frequency of a series RLC circuit is expressed as
A. $\frac{1}{2\pi\sqrt{LC}}$ B. $2\pi\sqrt{LC}$ C. $\frac{2\pi\sqrt{L}}{C}$ D. $\frac{C}{2\pi\sqrt{L}}$ E. $2\pi LC$ $NECO$ 2007[49] Ans: A

33. In a purely resistive $a.c$ circuit, the current, $I = I_o \sin\omega t$ and the voltage, $V = V_o \sin\omega t$. Calculate the instantaneous power dissipated in the circuit in time t.
A. I_oV_o B. $2I_oV_o$ C. $\frac{I_oV_o}{2}$ D. $I_oV_o\sin^2\omega t$ E. $I_o{}^2V_o{}^2$ $WAEC$ 1993[37] Ans: $I_oV_o\sin^2\omega t$

34. In the circuit diagram below, calculate the energy stored in the inductor at resonance.

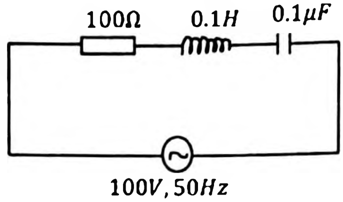


Fig 5.30 $WAEC$ 2001[41] $Ans: 0.05J$

35. An alternating current with a peak value of $5A$ passes through a resistor of resistance 10.0Ω. Calculate the rate at which energy is dissipated in the resistor.
$WAEC$ 1997[36] Ans: $250W$

36. What is the direct current equivalent of an alternating current $5\sin\omega t$?
$NECO$ 2007[50] Ans: $3.54A$

37. A $120V$, $60W$ lamp is to be operated on $220V$ $a.c$ supply mains. Calculate the value of non-inductive resistance that would be required to ensure that the lamp is run on correct value A, 200Ω B. 300Ω C. 500Ω D. 100Ω $JAMB$ 2007[46] Ans: 200Ω

38. Calculate the average power dissipated in an a.c circuit with an $r.m.s$ current of $5A$, $r.m.s$ voltage of $10V$ and phase angle of 60°. $NECO$ 2008[49] Ans: $25W$

39.

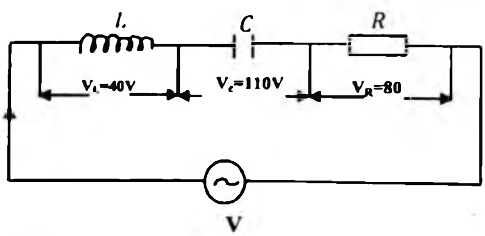


Fig. 5.31

From the diagram above, if the potential difference across the resistor, capacitor and conductor are 80V, 110V and 40V respectively, the effective potential difference is
A. 116.3V B. 50.0V C. 230.0V D. 106.3V JAMB 2009[44] Ans: D

40. An inductor of inductance 1.0H is connected in series with a capacitor of capacitance $2.0\mu F$ in an $a.c.$ circuit. Calculate the value of frequency that will make the circuit to resonate. WAEC 2009[44] Ans: 112.5Hz

41. An ammeter connected to an $a.c.$ circuit records 5.5A. What is the peak current in the circuit? WAEC 2009[45] Ans: 7.8A

42. A source of e.m.f. 110V and frequency 60Hz is connected to a resistor, an inductor and a capacitor in series. When the current in the capacitor is 2A, the potential difference across the resistor is 80V and that across the inductor is 40V.

Draw the vector diagram of the potential difference across the inductor, the capacitor and the resistor. Calculate the:

(i) Potential difference across the capacitor;
(ii) Capacitance of the capacitor;
(iii) Inductance of the inductor $[\pi = 3.14]$

WAEC 2009[E.14] Ans: (i) 115.5V (ii) 45.9×10^{-6}F or 46μF (iii) 0.05 H

6

ENERGY QUANTIZATION AND WAVE-PARTICLE PARADOX

ENERGY QUANTIZATION

According to the quantum theory suggested by Max Planck, heat or electromagnetic radiation is emitted in fixed, discrete or separate amounts known as quanta.

The energy E of a photon or quantum is given by:

$$E = hf$$

$$E = \frac{hc}{\lambda}$$

Where h = Planck's constant = $6.6 \times 10^{-34} Js$

c = Velocity of electromagnetic waves in free space = $3.0 \times 10^{8} ms^{-1}$

λ = Wavelength of electromagnetic wave (m)

f = Frequency of electromagnetic wave (Hz)

Electrons in an atom have definite energy levels with definite values like E_o, E_1, E_2, E_3...e.t.c. The energy of the electron could change from one level to another, for example, the energy of the electron could increase form lowest energy level or ground state E_o to E_1 or decrease from E_3 to a lower energy level E_2 or E_1

Energy change, $\Delta E = E_n - E_o = hf$

Or $\Delta E = E_n - E_o = \frac{hc}{\lambda}$

Where E_n = Energy in excited state, $n = 1, 2, 3$ -------

E_o = Energy in ground state.

The greater the energy change, the greater will be the frequency of the emitted radiation.

Example 1

An electron makes a transition from a certain energy level E_k to the ground state E_o. If the frequency of emission is $8.0 \times 10^{14} H_z$, the energy emitted is

A. $8.25 \times 10^{-19} J$ B. $5.28 \times 10^{-19} J$ C. $5.28 \times 10^{19} J$ D. $8.25 \times 10^{19} J$

$[h = 6.6 \times 10^{-34} Js]$ *JAMB* 2003[48]

Solution

Frequency $f = 8.0 \times 10^{14} Hz$, $h = 6.6 \times 10^{-34} Js$

Energy emitted $E = hf$

$E = 6.6 \times 10^{-34} \times 8.0 \times 10^{14} = 5.28 \times 10^{-19} J$

Example 2

Calculate the energy carried by an X-ray of wavelength $6.0 \times 10^{-10} m$. [Plank's constant = $6.6 \times 10^{-34} Js$, velocity of light = $3.0 \times 10^{8} ms^{-1}$]

WAEC 1995[58]

Solution

Wavelength $\lambda = 6.0 \times 10^{-10} m$, velocity of light $c = 3.0 \times 10^{8} ms^{-1}$

$h = 6.6 \times 10^{-34} Js$

Energy $E = hf$ or $E = \frac{hc}{\lambda}$

$$E = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^{8}}{6.0 \times 10^{-10}} = 3.3 \times 10^{-16} J$$

Example 3
An electron makes a transition from $-3.4eV$ energy level to $-13.6eV$ energy level. Calculate the

(i) value of the loss of energy due to transition in joules.
(ii) frequency of the emitted radiation
$[1eV = 1.6 \times 10^{-19} J;\ h = 6.6 \times 10^{-34} Js]$ *NECO* 2003[E15]

Solution
Initial energy level $E_i = -3.4eV$; final energy level $E_f = -13.6eV$,
$1eV = 1.6 \times 10^{-9} J$, $h = 6.6 \times 10^{-34} Js$

(i) Loss of energy,

$$\begin{aligned} \Delta E &= E_i - E_f \\ &= -3.4 - (-13.6) \\ &= -3.4 + 13.6 \\ &= 10.2eV \\ &= 10.2 \times 1.6 \times 10^{-19} \\ &= 1.63 \times 10^{-18} J \end{aligned}$$

(ii) From $E = hf$, frequency $f = \frac{E}{h} = \frac{1.63 \times 10^{-18}}{6.6 \times 10^{-34}} = 2.5 \times 10^{15} Hz$

Example 4
An atom radiates $1.5 \times 10^{-19} J$ when an electron jumps from one level to another. What is the wavelength of the emitted
radiation?[Planck's constant $= 6.6 \times 10^{-34} Js$][speed of light in vacuum $= 3.0 \times 10^{8} ms^{-1}$] *NECO* 2005[58]

Solution
Energy radiated, $E = 1.5 \times 10^{-19} J$, $h = 6.6 \times 10^{-34} Js$, $c = 3 \times 10^{8} ms^{-1}$

$$E = \frac{hc}{\lambda} \quad \therefore \quad \text{wavelength, } \lambda = \frac{hC}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.5 \times 10^{-19}} = 1.32 \times 10^{-6} m$$

Example 5
A photon of wavelength λ_o is emitted when an electron in an atom makes a transition from a level of energy $2E_k$ to that of energy E_k. If the electron transits from $\frac{4}{3}E_k$ to E_k level, determine the wavelength of the photon that would be emitted. *WAEC* 2001[44]

Solution

Energy charge, $\Delta E = E_1 - E_2 = \frac{hc}{\lambda}$

1st instance $\Delta E = 2E_k - E_k = E_k$

$$\therefore \quad E_k = \frac{hc}{\lambda_o} \quad \cdots\cdots\cdots\cdots (1)$$

2nd instance $\Delta E = \frac{4}{3}E_k - E_k = \frac{1}{3}E_k$

$$\therefore \quad \frac{E_k}{3} = \frac{hc}{\lambda} \quad \cdots\cdots\cdots\cdots (2)$$

Making the constants hc subjects of equation 1 and 2 we have

1. $hc = \lambda_o E_k$
2. $hc = \frac{\lambda E_k}{3}$

Equate hc in both equations to obtain

$\lambda_o E_k = \dfrac{\lambda E_k}{3}$

New wavelength, $\lambda = \dfrac{3\lambda_o E_k}{E_k} = 3\lambda_o$

Example 6

In a model of the hydrogen atom, the energy levels W_n are given by the formula $W_n = -\frac{R}{n^2}$ where n is an integer and R is a constant. Determine the energy released in the transition from $n = 3$ to $n = 2$ *WAEC 2002*[45]

Solution

Energy released $E = W_3 - W_2$

$$W_3 = -\frac{R}{(3)^2} = -\frac{R}{9}, \qquad W_2 = -\frac{R}{(2)^2} = -\frac{R}{4}$$

$$W_3 - W_2 = \frac{-R}{9} - \left(-\frac{R}{4}\right)$$

$$= -\frac{R}{9} + \frac{R}{4}$$

$$= \frac{R}{4} - \frac{R}{9}$$

$$= \frac{9R - 4R}{36} = \frac{5R}{36}$$

Electron Volt and Kinetic Energy

In atomic theory, the energy of an electron is expressed in joules (J) or electron volt (eV). An electron volt is the energy acquired by an electron in falling freely through a potential difference of 1 volt. Therefore, $1eV = 1.6 \times 10^{-19}J$.

Emitted electrons are accelerated through a potential V and acquire a potential energy $\boldsymbol{eV}$. The potential energy is converted into kinetic energy and the electrons finally acquired a velocity $\boldsymbol{v}$. This process is expressed as follows.

$$eV = \frac{1}{2}m_e v^2$$

$$eV = E_K$$

$$eV = hf$$

Where e = electronic charge = $1.6 \times 10^{-19}C$

V = accelerating voltage or potential difference (V)

m_e = mass of electron $9.1 \times 10^{-31}kg$

v = velocity of electron ms^{-1}

E_K = kinetic energy of electron (J)

h = Planck's constant = $6.63 \times 10^{-34}Js$

f = frequency of electron (Hz)

Example 7

An electron of charge $1.60 \times 10^{-19}C$ is accelerated under a potential difference of 1.0×10^5V. Calculate the energy of the electron in joules. *WAEC 2008*[E7]

Solution

Electron charge $e = 1.60 \times 10^{-19}C$; accelerating p.d, $V = 1.0 \times 10^5V$

Kinetic Energy $E_K = eV$

$= 1.60 \times 10^{-19} \times 1.0 \times 10^5$

$= 1.60 \times 10^{-14} J$

Example 8

A $500KV$ is applied across an X-ray tube, calculate the maximum velocity of the electrons produced. [$m_e = 9.1 \times 10^{-31} kg, e = 1.6 \times 10^{-19} C$] *WAEC 2006*[47]

Solution

Potential difference $V = 500KV = 500000V = 5 \times 10^5 V$

Mass of electron $m_e = 9.1 \times 10^{-31} kg$; $e = 1.6 \times 10^{-19} C$

$$eV = \frac{1}{2} m_e v^2$$

$$2eV = m_e v^2$$

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 5 \times 10^5}{9.1 \times 10^{-31}}} = \sqrt{1.76 \times 10^{17}}$$

$$= 4.2 \times 10^8 ms^{-1}$$

Example 9

If the frequency of an emitted X-ray is $1.6 \times 10^{16} Hz$, the accelerating potential is

A. $6.6V$ B. $66.3V$ C. $663.0V$ D. $6630.0V$

[$e = 1.6 \times 10^{-19} C$, $h = 6.63 \times 10^{-34} Js$] *JAMB 2002*[44]

Solution

$f = 1.6 \times 10^{16} Hz$, $e = 1.6 \times 10^{-19} C$, $h = 6.63 \times 10^{-34} Js$

From $eV = hf$, accelerating potential, $V = \frac{hf}{e}$

$$\therefore \quad V = \frac{6.63 \times 10^{-34} \times 1.6 \times 10^{16}}{1.6 \times 10^{-19}} = 66.3V$$

Example 10

The potential difference between the cathode and target of an X-ray tube is $5.00 \times 10^4 V$ and the current in the tube is $2.00 \times 10^{-2} A$. Given that only one percent of the. total energy supplied is emitted as X-radiation, determine the

(i) maximum frequency of the emitted radiation.

(ii) rate at which heat is removed from the target in order to keep it at a steady temperature.

[Planck's constant, $h = 6.63 \times 10^{-34}$ Js; electronic charge $e = 1.60 \times 10^{-19}$ C]

WAEC 2004[E15]

Solution

$h = 6.63 \times 10^{-34} Js$; $e = 1.60 \times 10^{-19} C$;

$V = 5.00 \times 10^4 V$; Current, $I = 2.0 \times 10^{-2} A$

(i) From $hf = eV$, maximum frequency, $f = \frac{eV}{h}$

$$f = \frac{1\% \ of \ eV}{h} = \frac{0.01 \times 1.60 \times 10^{-19} \times 5.00 \times 10^4}{6.63 \times 10^{-34}} = 1.207 \times 10^7 Hz$$

(ii) Rate of heat energy removal = power, $P = IV(100 - 1\%)$ or 99% of heat has to be removed.

$P = 99\% \times I \times V$

$= 0.99 \times 2.00 \times 10^{-2} \times 5.00 \times 10^4$

$= 990 \ W$ Or $990 \ Js^{-1}$

Photoelectric Effect: Einstein Equation

Electrons are ejected from a metal surface when electromagnetic radiation of *sufficient* frequency falls on a metal. A metal emits electrons provided that the wavelength of the radiation is *less than* a certain value. If the wavelength is too long, the electrons will not be emitted no matter the intensity and duration of the radiation. For example, when ultraviolet radiation of a particular frequency and wavelength falls on zinc, electrons are emitted by the zinc atom.

Therefore, Photo electricity is the emission of electrons from a metal plate when it is illuminated by light. The maximum kinetic energy of the emitted electrons depends *only* on the frequency or wavelength of the incident light and not the intensity of the light beam.

The minimum energy needed to pull out an electron is called the work function(W_o) of the metal. Threshold frequency (f_o) or wavelength (λ_o) is the minimum frequency or wavelength that must be exceeded for electron emission to occur.

When a photon or quantum of light energy, E or hf, is incident on a metal, part of this energy known as the work function (W_o) liberates the electron from the metal. The remaining energy gives the liberated electron a kinetic energy of $\frac{1}{2}mv^2$. This process is represented by the Einstein equation.

Energy of photon = Work function + kinetic energy

$$E = W_o + E_K$$

$$hf = hf_o + \frac{1}{2}m_e v^2$$

$$hf = hf_o + eV$$

$$hf = \frac{hc}{\lambda_o} + eV$$

Where $W_o = hf_o = \frac{hc}{\lambda_o}$ = work function (J)

λ_o, f_o = Threshold wavelength (m), threshold frequency (Hz)

$E_K = \frac{1}{2}m_e v^2 = eV$ = maximum kinetic energy (J)

Depending on the question given, any, or a combination of the above equations can be applied.

Example 11

The work function of a metal is $4.65eV$ and the metal is illuminated with a radiation of $6.86eV$. What is the kinetic energy of the electrons ejected from the surface of the metal?

WAEC 1991[59]

Solution

Work function $W_o = 4.65eV$, energy of radiation, $E(hf) = 6.86eV$

$E = W_o + E_K$

Kinetic energy, $E_K = E - W_o$

$= 6.86 - 4.65$

$= 2.21eV$

Example 12

The work function of a metal is $8.6 \times 10^{-19}J$. Calculate the wavelength of its threshold frequency.

[speed of light in vacuum $= 3 \times 10^8\ ms^{-1}$, Planck's constant $= 6.6 \times 10^{-34}Js$]

WAEC 1994[60]

Solution

$W_o = 8.6 \times 10^{-19}J$, $c = 3 \times 10^8 ms^{-1}$, $h = 6.6 \times 10^{-34}Js$

Work function $W_o = hf_o$ or $W_o = \frac{hc}{\lambda_o}$

$$\text{Threshold wavelength, } \lambda_o = \frac{hc}{W_o} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{8.6 \times 10^{-19}} = 2.3 \times 10^{-7} m$$

Example 13
Calculate the frequency of the photon whose energy is required to eject a surface electron with a kinetic energy of $1.9 \times 10^{-16} eV$. If the work function is $1.33 \times 10^{-16} eV$.
[$1eV = 1.6 \times 10^{-19} J$, Planck's constant, $h = 6.60 \times 10^{-34} Js$] *NECO* 2002[E10]

Solution
Kinetic energy, $E_K = 1.9 \times 10^{-16} eV = 1.9 \times 10^{-16} \times 1.6 \times 10^{-9} = 3.04 \times 10^{-35} J$

Work function, $W_o = 1.33 \times 10^{-16} eV = 1.33 \times 10^{-16} \times 1.6 \times 10^{-19} = 2.13 \times 10^{-35} J$

$$hf = E_K + W_o$$

$$\text{Frequency, } f = \frac{E_K + W_o}{h} = \frac{3.04 \times 10^{-35} + 2.13 \times 10^{-35}}{6.60 \times 10^{-34}} = 7.83 \times 10^{-2} Hz$$

Example 14
Caesium has a work function of $3 \times 10^{-19} J$. The maximum energy of liberated electrons when it is illuminated by light of frequency $6.7 \times 10^{14} Hz$ is
A. $1.42 \times 10^{-19} J$ B. $3.00 \times 10^{-19} J$ C. $4.42 \times 10^{-19} J$ D. $7.42 \times 10^{-19} J$
[$h = 6.6 \times 10^{-34} Js$] *JAMB* 2008[47]

Solution
$W_o = 3 \times 10^{-19} J$; $f = 6.7 \times 10^{14} Hz$; $h = 6.6 \times 10^{-34} Js$

$$hf = E_K + W_o$$

Maximum kinetic energy, $E_K = hf - W_o$

$$E_K = 6.6 \times 10^{-34} \times 6.7 \times 10^{14} - 3 \times 10^{-19}$$
$$= 4.422 \times 10^{-19} - 3 \times 10^{-19}$$
$$= 1.422 \times 10^{-19} J$$

Example 15
A photo emissive surface has a threshold frequency of $4.0 \times 10^{14} Hz$. If the surface is illuminated by light of frequency $5.0 \times 10^{15} Hz$, Calculate the
(i) Threshold wavelength
(ii) Work function
(iii) Kinetic energy of the emitted photo electrons
[$c = 3.0 \times 10^8 ms^{-1}, h = 6.63 \times 10^{-34} Js$] *WAEC* 2008[E15]

Solution
Threshold frequency $f_o = 4.02 \times 10^{14} Hz$
Radiation frequency $f = 5.0 \times 10^{15} Hz$
$c = 3.0 \times 10^8 ms^{-1}$; $h = 6.63 \times 10^{-34} Js$

(i) From $c = \lambda f$, threshold wavelength $\lambda_o = \frac{c}{f_o}$

$$\lambda_o = \frac{3.0 \times 10^8}{4.02 \times 10^4} = 7.46 \times 10^{-7} m$$

(ii) Work function $W_o = hf_o$

$$= 6.63 \times 10^{-34} \times 4.02 \times 10^{14}$$
$$= 2.67 \times 10^{-19} J$$

(iii) Kinetic energy $E_K = hf - W_o$

$$= 6.63 \times 10^{-34} \times 5.0 \times 10^{15} - 2.67 \times 10^{-19}$$

$$= 3.315 \times 10^{-18} - 2.67 \times 10^{-19}$$

$$= 3.048 \times 10^{-18} J$$

Example 16

The graph below represents the result of a certain photoelectric experiment where the stopping potential V_s is plotted against the frequency f

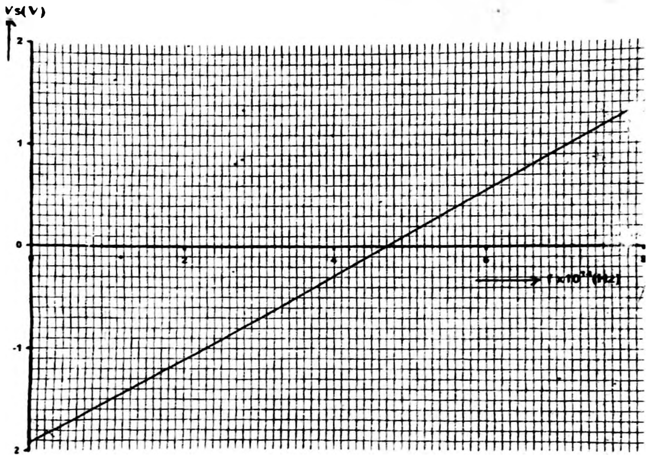
Determine from the graph, the

(i) threshold frequency

(ii) threshold wavelength

(iii) work function of the material

(iv) value of Planck's constant [$c = 3.0 \times 10^8 ms^{-1}$, $e = 1.6 \times 10^{-19} C$] NECO 2006[E15]



Solution

(i) Threshold frequency = intercept on f-axis

$\therefore \quad f_o = 4.7 \times 10^{14} Hz$

(ii) From $c = \lambda f$, threshold wavelength is

$$\lambda_o = \frac{c}{f_o} = \frac{3.0 \times 10^8}{4.7 \times 10^{14}} = 6.38 \times 10^{-7} m$$

(iii) Work function $W_o = e \times$ intercept on V_s-axis

$$= 1.6 \times 10^{-19} \times 1.9 = 3.04 \times 10^{-19} J$$

(iv) Planck's constant $(h) = e \times slope$

$$\text{From graph, slope} = \frac{\Delta V_s}{\Delta f} = \frac{0.8 - 0}{6.6 - 4.7} = \frac{0.8}{1.9 \times 10^{14}} = 4.2 \times 10^{-15}$$

$\therefore \quad h = 1.6 \times 10^{-19} \times 4.2 \times 10^{-15} = 6.7 \times 10^{-34} Js$

Stopping Potential

The stopping potential of a photoelectron emission is given by;

$$eV_s = E_K$$

$$eV_s = hf - W_o$$

Therefore, stopping potential $V_s = \frac{hf - W_o}{e} = \frac{E_K}{e}$

Where V_s = stopping potential in volt (V)
e = electronic charge = $1.6 \times 10^{-19}C$
E_K = maximum kinetic energy (J)
W_o = work function (J)

Example 17

The maximum kinetic energy of the photoelectrons emitted from a metal surface is $0.34eV$. If the work function of the metal surface is $1.83eV$, find the stopping potential.
A. $2.17V$ B. $1.49V$ C. $1.09V$ D. $0.34V$ *JAMB* 2003[44]

Solution

Maximum kinetic energy $E_K = 0.34eV$; $W_o = 1.83eV$

$$hf = W_o + E_K$$
$$= 1.83 + 0.34$$
$$= 2.17eV$$

$$\text{Stopping potential, } V_s = \frac{hf - W_o}{e} = \frac{(2.17 - 1.83)eV}{e} = \frac{0.34eV}{e} = 0.34V$$

Alternatively, we could have solved it quickly by using, $eV_s = E_K$

$$\text{Stopping potential, } V_s = \frac{E_K}{e} = \frac{0.34eV}{e} = 0.34V$$

Example 18

Light of wavelength $5.00 \times 10^{-7}m$ is incident on a material of work function $1.90eV$. Calculate the

(i) Photon energy
(ii) Kinetic energy of the most energetic photo electron
(iii) Stopping potential

[Planck's constant $h = 6.6 \times 10^{-34}Js$; $c = 3.0 \times 10^{8}ms^{-1}$, $1eV = 1.6 \times 10^{-19}J$]

WAEC 2001[E1]

Solution

$\lambda = 5.00 \times 10^{-7}m$; $W_o = 1.90eV = 1.90 \times 1.6 \times 10^{-19} = 3.04 \times 10^{-19}J$;
$h = 6.6 \times 10^{-34}Js$; $c = 3.0 \times 10^{8}ms^{-1}$

(i) Photo energy, $E = hf = \frac{hc}{\lambda}$

$$E = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^{8}}{5.00 \times 10^{-7}} = 3.96 \times 10^{-19}J$$

(ii) Maximum kinetic energy $E_K = hf - W_o$

$$= 3.96 \times 10^{-19} - 3.04 \times 10^{-19}$$
$$= 9.2 \times 10^{-20}J$$

(iii) Stopping potential $V_s = \frac{hf - W_o}{e} = \frac{9.2 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.575J$

WAVE – PARTICLE DUALITY

Matter appears to have a dual nature because in one instant it behaves like a wave, while in another it behaves like a stream of particle. According to De Broglie, the wavelength, λ, of a particle wave is given by;

$$\lambda = \frac{h}{P} \qquad or \qquad \lambda = \frac{h}{mv}$$

Where h = Planck's constant = $6.63 \times 10^{-34} Js$
m = mass of wave particle (kg)
v = velocity of wave particle (ms^{-1})
P = momentum of wave particle $(kgms^{-1})$

An electron of mass, m_e, with accelerating voltage, V, has a kinetic energy, $\frac{1}{2}m_e v^2$ and momentum, $P = m_e v$

Therefore, $eV = \frac{1}{2}m_e v^2$

Combining this equation and De Broglie's, the following equations can be derived

I. $$eV = \frac{1}{2}m_e v^2$$

or $$2eV = m_e v^2$$

Rearranging, $$\frac{2eV}{v} = m_e v$$

Substitute $m_e v = \frac{2eV}{v}$ into $\lambda = \frac{h}{m_e v}$

$$\lambda = \frac{h}{\frac{2eV}{v}}$$

$$\therefore \quad \lambda = \frac{hv}{2eV}$$

II. $$eV = \frac{1}{2}m_e v^2$$

$$v^2 = \frac{2eV}{m_e}$$

$$v = \sqrt{\frac{2eV}{m_e}}$$

Substitute the above equation into $P = m_e v$

$$P = m_e v$$

$$P = m_e \sqrt{\frac{2eV}{m_e}}$$

$$P = \sqrt{\frac{2eVm_e^2}{m_e}} = \sqrt{2eVm_e}$$

Substitute $P = \sqrt{2eVm_e}$ into $\lambda = \frac{h}{P}$

$$\lambda = \frac{h}{P}$$

$$\lambda = \frac{h}{\sqrt{2eVm_e}}$$

All symbols and constants maintain their usual meanings and values.

Example 19

Calculate the wavelength of an electron of mass $9.1 \times 10^{-31} kg$ moving with a velocity of $2.0 \times 10^6 ms^{-1}$. [plank's constant = $6.63 \times 10^{-34} Js$]

Solution

Mass of electron $m_e = 9.1 \times 10^{-31} kg$; $v = 2.0 \times 10^6 ms^{-1}$; $h = 6.63 \times 10^{-34} Js$

From De Broglie, $\lambda = \frac{h}{m_e v} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-34} \times 2.0 \times 10^6} = 3.64 \times 10^{-10} m$

Example 20

If electrons are accelerated from rest through a potential difference of $10KV$; what is the wavelength of the associated electrons?

A. $1.22 \times 10^{-11} m$ B. $3.87 \times 10^{-10} m$ C. $2.27 \times 10^{11} m$ D. $2.27 \times 10^{14} m$

[$m_e = 9.1 \times 10^{-31} kg$, $e = 1.6 \times 10^{-19} C$, $h = 6.6 \times 10^{-34} Js$] *JAMB* 2008[45]

Solution

Accelerating potential difference $V = 10KV = 10000V = 1 \times 10^4 V$

Mass of electron, $m_e = 9.1 \times 10^{-31} kg$ wavelength, $\lambda = ?$

Electronic charge, $e = 1.6 \times 10^{-19} C$

Planck's constant, $h = 6.6 \times 10^{-34} Js$

$$\lambda = \frac{h}{\sqrt{2eVm_e}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times 1 \times 10^4 \times 9.1 \times 10^{-31}}} = 1.22 \times 10^{-11} m$$

Example 21

The mass and wavelength of a moving electron are $9.0 \times 10^{-31} kg$ and $1.0 \times 10^{-10} m$ respectively. Calculate the kinetic energy of the electron. [$h = 6.6 \times 10^{-34} Js$]

WAEC 2001[E8]

Solution

$m_e = 9.0 \times 10^{-31} kg$; $\lambda = 1.0 \times 10^{-10} m$; $h = 6.6 \times 10^{-34} Js$

From, $\lambda = \frac{h}{m_e v}$, velocity $v = \frac{h}{m_e \lambda}$

Kinetic energy $KE = \frac{1}{2} m_e v^2$

Substitute $v = \frac{h}{m_e \lambda}$ into $KE = \frac{1}{2} m_e v^2$

$$KE = \frac{1}{2} \times m_e \times \left(\frac{h}{m_e \lambda}\right)^2$$

$$= \frac{1}{2} \times \frac{m_e}{m_e^2} \times \frac{h^2}{\lambda^2}$$

$$KE = \frac{h^2}{2m_e\lambda^2} = \frac{(6.6 \times 10^{-34})^2}{2 \times 9.0 \times 10^{-31} \times (1.0 \times 10^{-10})^2} = \frac{4.356 \times 10^{-67}}{1.8 \times 10^{-50}}$$

$$= 2.42 \times 10^{-17} J$$

Example 22

Calculate the minimum wavelength of X-rays when a voltage of $60KV$ is applied to an X-ray tube. $[e = 1.60 \times 10^{-19}C;\ h = 6.6 \times 10^{-34}Js;\ c = 3.0 \times 10^8 ms^{-1}]$

NECO 2000[E15]

Solution

$V = 60KV = 6 \times 10^4 V$; $e = 1.60 \times 10^{-19}C$;

$h = 6.6 \times 10^{-34}Js$ Velocity $V = 3.0 \times 10^8 ms^{-1}$

$$\lambda = \frac{hv}{2eV} = \frac{6.6 \times 10^{-34} \times 3.0 \times 10^8}{2 \times 1.60 \times 10^{-19} \times 6 \times 10^4} = 1.03 \times 10^{-11} m$$

Heisenberg Uncertainty Principle

According to Heisenberg, it is not possible to get an exact measurement of both the position and momentum of an atomic particle at the same time. If the electron position x is measured to a high degree of accuracy, then the simultaneous measurement of the momentum will be to a very low degree of accuracy.

The Heisenberg uncertainty principle states that if Δx is the uncertainty in measuring x, and ΔP is the uncertainty in measuring P, then their product is equal to or greater than h, the Planck's constant.

That is, $\Delta x.\Delta P \geq h$

Therefore, $\Delta x \geq \frac{h}{\Delta P}$ or $\Delta P \geq \frac{h}{\Delta x}$

The uncertainty principle can be applied to the measurement of the energy E of an object at a particular time t. If the uncertainty in E is ΔE and the uncertainty in t is Δt, the principle is represented as follows

$$\Delta E.\Delta t \geq h \quad or$$

$$\Delta E.\Delta t \geq \frac{h}{2\pi}$$

In all the above equations, $\Delta x, \Delta P, \Delta E$ and Δt are the uncertainties in the position, momentum, energy and time respectively.

Remember that momentum, $(P) = \text{mass}(m) \times \text{velocity } (v)$.

Example 23

If the uncertainty in the measurement of the position of a particle is $5 \times 10^{-10}m$, the uncertainty in the momentum of the particle is A. $1.32 \times 10^{-44}Ns$ B. $3.30 \times 10^{-44}Ns$ C. $1.32 \times 10^{-24}Ns$ D. $3.30 \times 10^{-24}Ns$

$[h = 6.6 \times 10^{-34}Js]$ *JAMB* 2003[42]

Solution

Uncertainty in position $\Delta x = 5 \times 10^{-10}m$

Planck's constant $h = 6.6 \times 10^{-34}Js$

From uncertainty principle $\Delta x.\Delta P \geq h$

Uncertainty in momentum, $\Delta P \geq \frac{h}{\Delta x} \geq \frac{6.6 \times 10^{-34}}{5 \times 10^{-10}} \geq 1.32 \times 10^{-24} Ns$

Example 24
The uncertainty in determining the duration during which an electron remains in a particular energy level before returning to the ground state is $2.0 \times 10^{-9} s$. Calculate the uncertainty in determining its energy at that level.
$\left[\text{Take } \frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34} Js\right]$ *WAEC* 2006[E8]
Solution

Uncertainty in time, $\Delta t = 2.0 \times 10^{-9} s$; $\frac{h}{2\pi} = 1.054 \times 10^{-34} J$

From uncertainty principle, $\Delta E.\Delta t \geq \frac{h}{2\pi}$

Uncertainty in energy level, $\Delta E \geq \frac{h}{2\pi} \times \frac{1}{\Delta t} \geq 1.054 \times 10^{-34} \times \frac{1}{2.0 \times 10^{-9}}$

$\geq 5.27 \times 10^{-26} J$

Example 25
The uncertainty in the velocity v of a moving electron of mass $10^{-30} kg$ is $3 \times 10^{6} ms^{-1}$. Calculate the uncertainty of the simultaneous measurement of its position X.
$[h = 6.62 \times 10^{-34} Js]$ *NECO* 2005[E10]
Solution
Mass of electron, $m = 10^{-30} kg$; Velocity of electron, $v = 3 \times 10^{6} ms^{-1}$

Uncertainty in momentum, $\Delta P = \Delta(m \times v)$
$\Delta x.\Delta P \geq h$

$\Delta x \geq \frac{h}{\Delta P}$ or $\Delta x \geq \frac{h}{\Delta(m \times v)}$ $\therefore$ Uncertainty in momentum, $\Delta x \geq \frac{6.62 \times 10^{-34}}{10^{-30} \times 3 \times 10^{6}}$

$\Delta x \geq 2.21 \times 10^{-10} m$

EXERCISES
1. An electron jumps from one energy level to another in atom radiating 4.5×10^{-19}joules. If Planck's constant is $6.6 \times 10^{-34} Js$, what is the wavelength of the radiation?[Take velocity of light $= 3 \times 10^{8} ms^{-1}$] *WAEC* 1989[57] Ans: $4.4 \times 10^{-7} m$
2. An atom radiates $1.5 \times 10^{-19} J$ of energy when an electron jumps from one energy level to another. What is the wavelength of the emitted radiation?
[Plank's constant $= 6.6 \times 10^{-34} Js$; speed of light in air $= 3 \times 10^{8} ms^{-1}$]
WAEC 1997[39] Ans: $1.32 \times 10^{-6} m$
3.

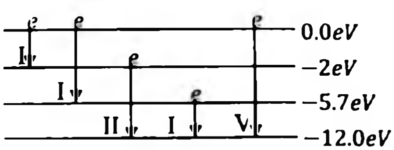


Fig 6.2

The diagram above illustrates the energy transition of five electrons of an atom.
(i) Which of the transition will produce the emission of longest wavelength?
(ii) Which of the transitions will produce emission of highest frequency?
WAEC 1999[42,43] Ans: (i) I (ii) *V*

4. In which of the following transitions is the largest quantum of energy liberated by an hydrogen atom when the electron changes energy levels?
[n is the quantum number] A. $n = 2$ to $n = 1$ B. $n = 1$ to $n = 2$
C. $n = 2$ to $n = 3$ D. $n = 3$ to $n = 2$ *WAEC* 2001[43] Ans: A

5. The energy associated with the photon of a radio transmission at $3 \times 10^5 Hz$ is
A. $1.30 \times 10^{-29} J$ B. $2.00 \times 10^{-29} J$ C. $1.30 \times 10^{-28} J$ D. $2.00 \times 10^{-28} J$
$[h = 6.63 \times 10^{-34} Js]$ *JAMB* 2003[42] Ans: $1.32 \times 10^{-24} Ns$

6. The energy associated with the emitted photon when a mercury atom changes from one state to another is $3.3eV$. Calculate the frequency of the photon.
A. $8.0 \times 10^{14} Hz$ B. $3.1 \times 10^{52} Hz$ C. $1.3 \times 10^{-15} Hz$ D. $3.2 \times 10^{-53} Hz$
$[e = 1.6 \times 10^{-19} C\,;\ h = 6.6 \times 10^{-34} Js]$ *JAMB* 2006[8] Ans: $8.0 \times 10^{14} Hz$

7. The energy E of a photon and its wavelength are related by $E\lambda = X$. The numerical value of X is A. 6.60×10^{-26} B. 1.99×10^{-25} C. 1.99×10^{-27}
D. 6.60×10^{-28} $[h = 6.63 \times 10^{-34} Js;\ c = 3 \times 10^8 ms^{-1}]$
JAMB 2007[26] Ans: 1.99×10^{-25}

8. An electron makes a transition from $-1.60eV$ energy level to $-10.4eV$ energy level. Calculate the energy loss due to transition. *NECO* 2004[56] Ans: $8.8eV$

9. An electron of charge $1.6 \times 10^{-19} C$ is accelerated in vacuum from rest at zero volt towards a plate of $40KV$. Calculate the kinetic energy of the electron.
WAEC 1988[58] Ans: $6.4 \times 10^{-15} J$

10. An electron of charge $1.6 \times 10^{-19} C$ is accelerated in vacuum from rest at zero volt towards a plate of $40KV$. Calculate the kinetic energy of the electron.
WAEC 1991[57] Ans: $6.4 \times 10^{-15} J$

11. An electron is accelerated from rest through a potential difference of $70KV$ in a vacuum. Calculate the maximum speed acquired by the electron. [electronic charge $= -1.6 \times 10^{-19} C$; mass of an electron $= 9.1 \times 10^{-31} kg$]
WAEC 1998[53] Ans: $1.57 \times 10^8 ms^{-1}$

12. A metal is illustrated with a radiation of energy $6.88eV$. If the kinetic energy of the emitted electrons is $1.50eV$, calculate the work function of the metal.
WAEC 1990[56] Ans: $5.3eV$

13. A metal has a work function of $4.375eV$. Calculate its threshold frequency.
$[h = 6.6 \times 10^{-34} Js, 1eV = 1.6 \times 10^{-19} J]$ *WAEC* 1999[45] Ans: $1.06 \times 10^{15} Hz$

14. If light with photon energy $2eV$ is incident suitably on the surface of a metal with work function $3eV$, then
A. No electron will be emitted
B. The few electrons emitted will have a maximum kinetic energy of 1eV
C. The few electrons emitted will have a maximum kinetic energy of 3eV.
D. Many electrons will be emitted *JAMB* 1994[50] Ans: A

15. A light of energy $5eV$ falls on a metal and the electrons with a maximum kinetic energy of $2eV$ are ejected. The work function of the metal is
A. $0.4eV$ B. $2.5eV$ C. $3.0eV$ D. $7.0eV$ *JAMB* 1995[50] Ans: $3.0eV$

16. The work function of a metal is $2.7eV$. Which of the following pairs correspond to the threshold frequency and wavelength of the metal respectively? $[h = 6.6 \times 10^{-34} Js$, $c = 3.0 \times 10^8 ms^{-1}, 1eV = 1.6 \times 10^{-19} J\,]$
A. $4.6 \times 10^{-26} Hz, 6.5 \times 10^{34} m$
B. $4.6 \times 10^{-7} Hz, 6.5 \times 10^{14} m$
C. $6.5 \times 10^{14} Hz, 4.6 \times 10^{-7} m$
D. $6.5 \times 10^{14} Hz, 4.6 \times 10^{7} m$
E. $6.5 \times 10^{34} Hz, 4.6 \times 10^{-26} m$ *NECO* 2006[55] Ans: C

17. A light of wavelength $5.0 \times 10^{-7} m$ is incident on a metal resulting in photoemission of electrons. If the work function of the metal is $3.04 \times 10^{-19} J$, Calculate the
(i) frequency of the light

(ii) energy of the incident photon

(iii) maximum kinetic energy of the photoelectrons

[speed of light $= 3.00 \times 10^8 ms^{-1}$; Planck's constant $= 6.6 \times 10^{-34} Js$]

WAEC 1998[E4] Ans: $(i) 6.0 \times 10^{14} Hz$ $(ii) 3.96 \times 10^{-19} J$ $(iii) 9.2 \times 10^{20} J$

18. Light of energy $5eV$ falls on a metal of work function $3eV$ and electrons are liberated. The stopping potential is A. $15.0V$ B. $8.0V$ C. $2.0V$ D. $1.7V$

JAMB 2000[41] Ans: $2.0V$

19. Light of energy $5.0eV$ falls on a metal of work function $3.0eV$ and electrons are emitted, determine the stopping potential. [Electronic charge, $e = 1.60 \times 10^{-19}$]

WAEC 2005[48] Ans: $2.0V$

20. An electron of mass $9.1 \times 10^{-31} kg$ moves with a velocity of $4.2 \times 10^7 ms^{-1}$ between the cathode and anode of an x – ray tube. Calculate the wavelength. [Take Planck's constant $h = 6.6 \times 10^{-34} Js$] *WAEC* 2000[E10] Ans: $1.73 \times 10^{-11} m$

21. A photon of wavelength $6.0 \times 10^{-7} m$ behaves like a particle of a certain mass. The value of that mass is

A. $1.1 \times 10^{-35} kg$ B. $3.5 \times 10^{-36} kg$ C. $2.2 \times 10^{-27} kg$ D. $2.2 \times 10^{-35} kg$

[$h = 6.63 \times 10^{-34} Js$, $c = 3 \times 10^8 ms^{-1}$] *JAMB* 2007[24] Ans: $3.5 \times 10^{-36} kg$

22. If Δx is the uncertainty in the measurement of the position along the x-axis and ΔP_x is the uncertainty in the measurement of the linear momentum along the x-axis then the uncertainty principle relation is given as

A. $\Delta P_x \Delta x \geq h$ B. $\Delta P_x \Delta x = h$ C. $\Delta P_x \Delta x = 2\pi$ D. $\Delta P_x \Delta x = 2\pi/h$ E. $\Delta P_x \Delta x = 1$

NECO 2007[60] Ans: *A*

23. What is the uncertainty in the measurement of time, if the uncertainty in measuring the energy of an electron of mass $10^{-30} kg$ is $2.45 \times 10^{-19} J$?

[$h = 6.6 \times 10^{-34} J$] *NECO* 2004[60] Ans: $2.69 \times 10^{-15} s$

24. The uncertainty in the energy of a particle is $1.0 \times 10^{-10} J$ determines its uncertainty in the time measurement.[Planck's constant $= 6.63 \times 10^{-24} s$]

NECO 2008[60] *Ans*: $6.63 \times 10^{-24} s$

The table below shows the energy distribution for various levels of an atom. Use it to answers question 25 and 26.

Energy level (n)	1	2	3	4
Energy (eV)	-13.6	-3.39	-1.51	-0.85

[$h = 6.6 \times 10^{-34} Js$; $e = 1.6 \times 10^{-19} C$; $c = 3.0 \times 10^8 ms^{-1}$]

25. Calculate the first excitation energy of the atom.

WAEC 2009[46] Ans: 1.60×10^{-18}

26. If the atom de-excites from $n = 2$, what is the wavelength of the emitted radiation?

WAEC 2009[47] Ans: 1.2×10^{-7} m

27. An electron of mass 9.1×10^{-31} kg moves with a speed of $10^7 ms^{-1}$. Calculate the wavelength of the associated wave. [$h = 6.6 \times 10^{-34}$ Js]

WAEC 2009[50] Ans: 7.25×10^{-11} m

28. An electron jumps from one energy level of -1.6 *eV* to one of -10.4 *eV* in an atom. Calculate the energy and wavelength of the emitted radiation.

[$h = 6.6 \times 10^{-34}$ Js; $eV = 1.6 \times 10^{-19}$ J; $c = 3.0 \times 10^8 ms^{-1}$]

WAEC 2009[E15] Ans: $\Delta E = 8.8\, eV$ or 1.41×10^{-8} J, $\lambda = 1.4 \times 10^{-7} m$

29. The work function of a metal is 1.06×10^{-18} J. What is the threshold wavelength of the metal? [Planck's constant $h = 6.6 \times 10^{-18}$ J, $c = 3.0 \times 10 ms^{-1}$]

NECO 2009[56] Ans: $1.9 \times 10^{-7} m$

30. An electron makes a transition from an energy level of $-1.51eV$ to that of $-13.6eV$ in an atom. Calculate the

(i) loss of energy due to the transition in joules,

(ii) frequency of the emitted radiation.

[$1eV = 1.6 \times 10^{-19}$ J, $h = 6.6 \times 10^{-34}$ Js]

NECO 2009[E15] Ans: (i) 1.93×10^{-18} J (ii) 2.92×10^{15} Hz

There is no democracy in physics. We can't say that some second-rate guy has as much right to an opinion as Fermi.

Luis Alvarez (1911 - 1988)
U.S. physicist.
Referring to Enrico Fermi, who achieved the first controlled nuclear reaction.

7

RADIOACTIVITY AND NUCLEAR REACTION

RADIOACTIVITY

Radioactivity is defined as the spontaneous disintegration of an unstable atomic nuclei with the emission of α, β or γ – *radiations* and release of energy. Every radioactive element has its unique decay constant and half life.

The disintegration or decay of radioactive element or atom is a random process that is totally independent of chemical combination, temperature and pressure. It occurs when an unstable nucleus usually with atomic number greater than 83 undergoes natural decay in a bid to achieve a stable condition.

Fig 7.1 shows a typical decay curve for a disintegrating radioactive atom obtained by plotting the number of atoms present against half-life. If the initial number of atom is N_1 at time T_1 and falls to $\frac{N_1}{2}$ at time T_2, then the half-life (T) is given by; $T = T_2 - T_1$. In the same manner, the half-life is also given by; $T = T_3 - T_2$, if the number of atom reduces from $\frac{N_1}{2}$ to $\frac{N_1}{4}$.

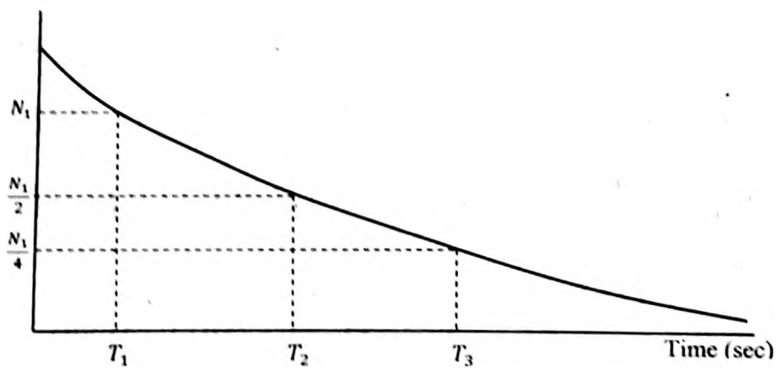


Fig 7.1 Half life and Decay

Half Life: Equations of Radioactive Disintegration

The half life of a radioactive element is the time taken for *half the atoms* of the element to decay. Half life could also be defined as the time taken for a given mass of radioactive substance to disintegrate to *half its initial mass*.

Assume that a radioactive element with half life of 5seconds contains 192 atoms initially.

After the first 5seconds (1 half-life), 96 atoms would have decayed and 96 atoms will be left.

After 10 seconds (2 half-lives), 144 atoms would have decayed and 48 atoms will remain.

After 15 seconds (3 half-lives), 168 atoms would have decayed and 24 atoms will remain.

After 20 seconds (4 half-lives), 180 atoms would have disintegrated and 12 atoms will be left.

After 25 seconds (5 half-lives), 186 atoms would have disintegrated and 6 atoms will remain.

The following new equations can be used in calculations involving half-life. It is fundamentally different from the conventional method you are familiar with.

1. *Halflife,* $T = \frac{t}{\log_2 R} = \frac{t \times \log 2}{\log R}$ 7.1

$$\left(\text{Where } R = 2^n = \frac{N_1}{N_2} \text{ and } n = \frac{t}{T}\right)$$

2. Number of atoms decayed, $N_d = N_1 - N_2$ 7.2

$$= N_1\left(\frac{R-1}{R}\right)$$
$$= N_2(R-1)$$

3. Fraction remaining undecayed, $f_r = \frac{N_2}{N_1} = \frac{1}{R}$ 7.3

4. Fraction of atoms decayed, $f_d = \frac{N_d}{N_1} = \frac{R-1}{R}$ 7.4

Where
T = half life of radioactive element
t = time taken for radioactive element to decay
n = number of half lives
N_1 = initial mass or initial number of atoms present/initial count rate
N_2 = final mass or final number of atom remaining undecayed/final count rate
N_d = number of atom or mass of atom that has decayed or disintegrated.
R = disintegrating ratio
f_r = fraction of initial number of atoms remaining undecayed.
f_d = fraction of initial number of atoms that has decayed

Though the term, " disintegrating ratio" is a newly coined expression, it is NOT a new or additional concept in physics. It does not contradict any term or concept in radioactivity. It is simply a coined name for an established relationship $\left(\frac{N_o}{N} = 2^n\right)$ and its modification, $R = \frac{N_1}{N_2} = 2^n$ and simplified application in solving radioactive decay problems.

For purpose of identification and reference, equation 7.1 should henceforth be called, ***Zhepwo radioactive equation***. Other derived equations (equations 7.2, 7.3 & 7.4) should be referred to as ***Zhepwo derivative(s)***, so as to differentiate between the two groups of equations.

Details of how these equations are derived; their relationship with the decay constant and decay curve, and proof of their applicability to WAEC, JAMB and NECO questions and examples and exercises from fifteen physics textbooks are contained in a ground-breaking book by Solomon Dauda Yakwo: ***Can These New Equations Significantly Simplify Half-Life Calculation?*** (ISBN 978-978-49081-5-3)

A good knowledge of the theory of logarithm is necessary for using some of the above equations nevertheless, like the multiplication table, it is advisable to commit the following to memory.

If $R = 2^n$	*Then*	$\log_2 R = n$
$\therefore$ *If* $2 = 2^1$	*Then*	$\log_2 2 = 1$
$4 = 2^2$		$\log_2 4 = 2$
$8 = 2^3$		$\log_2 8 = 3$
$16 = 2^4$		$\log_2 16 = 4$
$32 = 2^5$		$\log_2 32 = 5$
$64 = 2^6$		$\log_2 64 = 6$
$128 = 2^7$		$\log_2 128 = 7$
$256 = 2^8$		$\log_2 256 = 8$
$512 = 2^9$		$\log_2 512 = 9$
$1024 = 2^{10}$		$\log_2 1024 = 10$

Example 1
In 24 days, a radioactive isotope decrease in mass from $64g$ to $2g$. What is the half life of the radioactive material? *WAEC* 1994[58]

Solution: Conventional method

$64g \longrightarrow 32g = 1$ half-life

$32g \longrightarrow 16g = 2$ half-life

$16 \longrightarrow 8g = 3$ half-life

$8g \longrightarrow 4g = 4$ half-life

$4g \longrightarrow 2g = 5$ half-life

If 5 half-life is equal to 24 days

Then 1 half-life will be T

$\therefore T \times 5$ half-life $=1$ half-life $\times 24$ days

$$T = \frac{1 \text{ half life} \times 24 \text{ days}}{5 \text{ half life}} = 4.8 days$$

Example 1
In 24 days, a radioactive isotope decrease in mass from $64g$ to $2g$. What is the half life of the radioactive material? *WAEC* 1994[58]

Solution: Zhepwo method
Decay time, t =24 days; Half-life, T = ?
initial mass, $N_1 = 64g$;
Final mass remaining, $N_2 = 2g$

$$R = \frac{N_1}{N_2} = \frac{64}{2} = 32$$

$$T = \frac{t}{\log_2 R} = \frac{24}{\log_2 32} = \frac{24}{5} = 4.8 \text{ days}$$

Example 2
The half life of a radioactive material is 6 hours. What quantity of $1kg$ of the material would decay in 24 hours? *WAEC* 1997[52]

Solution: Conventional method

After 6hrs, $\frac{1}{2}$ kg decays, $\frac{1}{2}$ kg remains

After 6hrs, $\frac{1}{2}$ of $\frac{1}{2}$ kg decays, $\frac{1}{4}$ kg remains

After 6hrs, $\frac{1}{2}$ of $\frac{1}{4}$ kg decays, $\frac{1}{8}$ kg remains

After 6hrs, $\frac{1}{2}$ of $\frac{1}{8}$ kg decays, $\frac{1}{16}$ kg remains

Therefore, the material decayed would be:

Example 2
The half life of a radioactive material is 6 hours. What quantity of $1kg$ of the material would decay in 24 hours? *WAEC* 1997[52]

Solution: Zhepwo method
Half life $T = 6$ h; initial mass $N_1 = 1kg$;
mass of material decayed, $N_d = ?$

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{24}{6} = 4$$

$$R = 2^n = 2^4 = 16$$

$$N_d = N_1\left(\frac{R-1}{R}\right) = 1kg\left(\frac{16-1}{16}\right)$$

$1\text{kg} - \frac{1}{16}kg = \frac{15}{16}\text{kg}$

$$= \frac{15}{16}\,kg$$

Example 3

A radioactive sample initially contains N atoms. After three half-lives the number of atoms that have disintegrated is $A.\frac{N}{8}$ $B.\frac{3N}{8}$ $C.\frac{5N}{8}$ $D.\frac{7N}{8}$

JAMB 1985[50]

Solution: Conventional method

After 1 half-life, $\frac{N}{2}$ decay, $\frac{N}{2}$ remain

After 2 half-life, $\frac{1}{2}$ of $\frac{N}{2}$ decay, $\frac{N}{4}$ remain

After 3 half-life, $\frac{1}{2}$ of $\frac{N}{4}$ decay, $\frac{N}{8}$ remain

Number of atom disintegrated = sum of disintegrated atoms or fractions

$$= \frac{N}{2} + \left(\frac{1}{2} \times \frac{N}{2}\right) + \left(\frac{1}{2} \times \frac{N}{4}\right)$$

$$= \frac{N}{2} + \frac{N}{4} + \frac{N}{8} = \frac{7N}{8}$$

Example 3

A radioactive sample initially contains N atoms. After three half-lives the number of atoms that have disintegrated is

$A.\frac{N}{8}$ $B.\frac{3N}{8}$ $C.\frac{5N}{8}$ $D.\frac{7N}{8}$

JAMB 1985[50]

Solution: Zhepwo method

Initial number of atoms $N_1 = N$;

number of half lives, $n = 3$;

Numbers of atoms decayed, $N_d = ?$

$$R = 2^n = 2^3 = 8$$

$$N_d = N_1\left(\frac{R-1}{R}\right)$$

$$= N\left(\frac{8-1}{8}\right) = N \times \frac{7}{8} = \frac{7}{8}N$$

Example 4

After three half -lives, the fraction of a radioactive material that has decayed is

A. $\frac{1}{8}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{7}{8}$

JAMB 1992[49]

Solution: Conventional method

1 half-life $\Longrightarrow$ $\frac{1}{2}$ decays, $\frac{1}{2}$ remains

2 half-life $\Longrightarrow$ $\frac{1}{4}$ decays, $\frac{1}{4}$ remains

3 half-life $\Longrightarrow$ $\frac{1}{8}$ decays, $\frac{1}{8}$ remains

Fraction decayed = original fraction − remaining fraction

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

Note that original fraction is 1 or $\frac{1}{1}$

Example 4

After three half -lives, the fraction of a radioactive material that has decayed is

A. $\frac{1}{8}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{7}{8}$

JAMB 1992[49]

Solution: Zhepwo method

Number of half-lives, $n = 3$;

fraction decayed $f_d = ?$

Disintegrating ratio, $R = 2^n = 2^3 = 8$

$$f_d = \frac{R-1}{R} = \frac{8-1}{8} = \frac{7}{8}$$

Example 5

The half - life of a radioactive element is 24 hours, calculate the fraction of the original element that would have disintegrated in 96 hours.*NECO* 2006[58]

Example 5

The half - life of a radioactive element is 24 hours, calculate the fraction of the original element that would have disintegrated in 96 hours.*NECO* 2006[58]

Solution: Conventional method

After 24hrs, $\frac{1}{2}$ disintegrate, $\frac{1}{2}$ remain

After another 24hrs, $\frac{1}{4}$ decay, $\frac{1}{4}$ remain

After a further 24hrs, $\frac{1}{8}$ decay, $\frac{1}{8}$ remain

After another 24hrs, $\frac{1}{16}$ decay, $\frac{1}{16}$ remain

Fraction disintegrated = sum of decayed fractions

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$$

Solution: Zhepwo method

Half life, $T = 24h$; decay time, $t = 96h$;

fraction disintegrated, f_d =?

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{96h}{24h} = 4$$

$$R = 2^n = 2^4 = 16$$

$$f_d = \frac{R-1}{R} = \frac{16-1}{16} = \frac{15}{16}$$

Example 6

A radioactive isotope has a half life of 8 days. What fraction of the atoms will remain after 72 days? *NECO*2004[59]

Solution: Conventional method

Let n be the original number of nuclei.

After 8 days, $\frac{n}{2}$ disintegrate, $\frac{n}{2}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{2}$ decay, $\frac{n}{4}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{4}$ decay, $\frac{n}{8}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{8}$ decay, $\frac{n}{16}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{16}$ decay, $\frac{n}{32}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{32}$ decay, $\frac{n}{64}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{64}$ decay, $\frac{n}{128}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{128}$ decay, $\frac{n}{256}$ remain

After 8days, $\frac{1}{2}$ of $\frac{n}{256}$ decay, $\frac{n}{512}$ remain

$\therefore$ After 72 days, fraction of atoms that will remain is $\frac{1}{512}$ or 1.953×10^{-3}

Example 6

A radioactive isotope has a half life of 8 days. What fraction of the atoms will remain after 72 days? *NECO*2004[59]

Solution: Zhepwo method

half-life, T = 8days;

decay time, t = 72 days;

fraction remaining f_r =?

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{72}{8} = 9$$

$$R = 2^n = 2^9 = 512$$

Fraction remaining, $f_r = \frac{1}{R} = \frac{1}{512}$

$= 1.953 \times 10^{-3}$

Example 7

A radioactive substance has a half life of 20 hours. What fraction of the original radioactive nuclei will remain after 80 hours? *WAEC* 1990[55]

Solution: Conventional method

Let n be the original number of nuclei.

After 20 hours, $\frac{n}{2}$ disintegrates and $\frac{n}{2}$ remains.

After a further 20 hours, ½ of $\frac{n}{2}$

Example 7

A radioactive substance has a half life of 20 hours. What fraction of the original radioactive nuclei will remain after 80 hours? *WAEC* 1990[55]

Solution: Zhepwo method

Half life, $T = 20hrs$;

decay time, t = 80hrs

Number of half lives, $n = \frac{t}{T} = \frac{80}{20} = 4$

disintegrates and $\frac{n}{4}$ remains.

After another 20 hours, ½ of $\frac{n}{4}$ disintegrates and $\frac{n}{8}$ remains.

After another 20 hours, ½ of $\frac{n}{8}$ disintegrates and $\frac{n}{16}$ remains.

Therefore, after 80 hours, fraction of the original number remaining would be $\frac{1}{16}$

Disintegrating ratio, $R = 2^n = 2^4 = 16$

Fraction remaining, $f_r = \frac{1}{R} = \frac{1}{16}$

Alternative direct method

$T = 20;\ t = 80$

Fraction remaining, $f_r = \frac{1}{2^{t/T}} = \frac{1}{2^{80/20}}$

$= \frac{1}{2^4} = \frac{1}{16}$

Example 8

Two radioactive elements A and B has half-lives of 100 and 50 years respectively. Samples of A and B initially contain equal number of atoms. What is the ratio of remaining atoms of A to that of B after 200 years? *WAEC* 1988[60]

Solution: Conventional method

Let n be the original number of nuclei

Sample A: half-life = 100yrs

After 100 years, $\frac{n}{2}$ disintegrates and $\frac{n}{2}$ remains.

After another 100 years, $\frac{n}{4}$ disintegrates and $\frac{n}{4}$ remains.

Therefore, after 200 years, fraction of the original number remaining would be $\frac{1}{4}$

Sample B: half-life = 50yrs

After 50yrs, $\frac{n}{2}$ disintegrate, $\frac{n}{2}$ remain

After 50yrs, $\frac{n}{4}$ decay, $\frac{n}{4}$ remain

After 50yrs, $\frac{n}{8}$ decay, $\frac{n}{8}$ remain

After 50yrs, $\frac{n}{16}$ decay, $\frac{n}{16}$ remain

$\therefore$ After 200yrs, fraction of atoms that will remain is $\frac{1}{16}$

Ratio of A: B $= \frac{1}{4} : \frac{1}{16}$ (multiply both sides by 16)

Ratio of A: B $= 16\left(\frac{1}{4}\right) : 16\left(\frac{1}{16}\right) = 4:1$

Example 8

Two radioactive elements A and B has half-lives of 100 and 50 years respectively. Samples of A and B initially contain equal number of atoms. What is the ratio of remaining atoms of A to that of B after 200 years? *WAEC* 1988[60]

Solution: Zhepwo method

	A	B
Half life T	100	50
Decay time, t	200	200
Number of half lives $n = \frac{t}{T}$	2	4
Disintegrating ratio $R = 2^n$	4	16

Atoms remaining $N_2 = \frac{N_1}{R}$ (initial no of atoms $N_1 = 1$)	$\frac{1}{4}$		$\frac{1}{16}$
Multiply by 16	$16 \times \frac{1}{4}$		$16 \times \frac{1}{16}$
Ratio of A to B	4	:	1

Example 9

A radioactive substance has a half life of 80 days. If the initial number of atoms in the sample is 6.00×10^{10}, how many atoms would remain at the end of 320 days? A. 3.75×10^9 B. 7.50×10^9 C. 3.00×10^{10} D. 5.63×10^{10}

JAMB 1998[48]

Solution: Conventional method

After 80 days, $\frac{1}{2}(6 \times 10^{10}$ atoms) decay, 3×10^{10} atoms remain

After 80 days, $\frac{1}{2}(3 \times 10^{10}$ atoms) decay, 1.5×10^{10} atoms remain

After 80 days, $\frac{1}{2}(1.5 \times 10^{10}$ atoms) decay, 7.5×10^9 atoms remain

After 80 days, $\frac{1}{2}(7.5 \times 10^9$ atoms) decay, 3.75×10^9 atoms remain

$\therefore$ after a total of (80 + 80 + 80 + 80) Or 320 days 3.75×10^9 atoms remain.

Example 9

A radioactive substance has a half life of 80 days. If the initial number of atoms in the sample is 6.00×10^{10}, how many atoms would remain at the end of 320 days? A. 3.75×10^9 B. 7.50×10^9 C. 3.00×10^{10} D. 5.63×10^{10}

JAMB 1998[48]

Solution: Zhepwo method

Half life $T = 80$ days;
initial no. of atoms $N_1 = 6.00 \times 10^{10}$
decay time, $t = 320$ days;
Final number of atoms remaining $N_2 = ?$

$$T = \frac{t}{n} \quad \therefore n = \frac{t}{T} = \frac{320}{80} = 4$$

Disintegrating ratio, $R = 2^n = 2^4 = 16$

$$R = \frac{N_1}{N_2} \quad \therefore \quad N_2 = \frac{N_1}{R} = \frac{6.00 \times 10^{10}}{16}$$

$$= 3.75 \times 10^9 \text{atoms}$$

Alternative direct method

$T = 80; \quad t = 320; \quad N_1 = 6.00 \times 10^{10}$

$$N_2 = \frac{N_1}{2^{t/T}}$$

$$= \frac{6.00 \times 10^{10}}{2^{320/80}} = \frac{6.00 \times 10^{10}}{2^4}$$

$$= \frac{6.00 \times 10^{10}}{16} = 3.75 \times 10^9 \text{ atoms}$$

Example 10

The percentage of the original nuclei of a sample of a radioactive substance left after 5 half lives is A. 1% B. 8% C. 5% D. 3% *JAMB* 2002[45]

Solution: Conventional method

Percentage of original left

$$= \frac{\text{amount left (fraction remaining)}}{\text{original amount}} \times 100$$

Example 10

The percentage of the original nuclei of a sample of a radioactive substance left after 5 half lives is A. 1% B. 8% C. 5% D. 3% *JAMB* 2002[45]

Solution: Zhepwo method

Percentage of original left

$$= \frac{\text{amount left (fraction remaining)}}{\text{original amount}} \times 100$$

Number of half lives, $n = 5$

1 half-life $\Rightarrow$ $\frac{1}{2}$ decays, $\frac{1}{2}$ remains

2 half-life $\Rightarrow$ $\frac{1}{4}$ decays, $\frac{1}{4}$ remains

3 half-life $\Rightarrow$ $\frac{1}{8}$ decays, $\frac{1}{8}$ remains

4 half-life $\Rightarrow$ $\frac{1}{16}$ decays, $\frac{1}{16}$ remains

5 half-life $\Rightarrow$ $\frac{1}{32}$ decays, $\frac{1}{32}$ remains

$\therefore$ after 5 half-lives, $\frac{1}{32}$ of the original nuclei is left. Note that initial fraction present is 1 or $\frac{1}{1}$

Percentage of original left

$$= \frac{1/32}{1} \times 100$$

$$= \frac{1}{32} \times 100 = 3.125 \simeq 3\%$$

Original amount or fraction $N_1 = 1$

Disintegrating ratio, $R = 2^n = 2^5 = 32$

Fraction remaining, $f_r = \frac{1}{R} = \frac{1}{32}$

$$\therefore \text{ percentage left } = \frac{f_r}{N_1} \times 100$$

$$= \frac{1/32}{1} \times 100 = \frac{1}{32} \times 100$$

$$= 3.125 \simeq 3\%$$

Example 11
A radioactive substance of mass $768g$ has a half life of 3years. After how many years does this substance leave only $6g$ undecayed? *WAEC* 2006[48]
Solution: Conventional method

After 3 years, $\frac{768g}{2} = 384g$

After 3 years, $\frac{384g}{2} = 192g$

After 3 years, $\frac{192g}{2} = 96g$

After 3 years, $\frac{96g}{2} = 48g$

After 3 years, $\frac{48g}{2} = 24g$

After 3 years, $\frac{24g}{2} = 12g$

After 3 years, $\frac{12g}{2} = 6g$

Thus making a total of (3+3+3+3+3+3+3)yrs or 21yrs

Example 11
A radioactive substance of mass $768g$ has a half life of 3years. After how many years does this substance leave only $6g$ undecayed? *WAEC* 2006[48]
Solution: Zhepwo method

Half life $T = 3$years;
initial mass present, $N_1 = 768g$;
final mass remaining, $N_2 = 6g$;
decay time, $t = ?$

Disintegrating ratio, $R = \frac{N_1}{N_2} = \frac{768}{6}$

$$= 128$$

$$T = \frac{t}{log_2 R} \quad \therefore t = T \times log_2 R$$

$$t = 3 \times log_2 128 = 3 \times 7 = 21 \text{ years}$$

Example 12
An element whose half-life is 10 days is of mass $12g$. Calculate the time during which $11.25g$ of the element would have

Example 12
An element whose half-life is 10 days is of mass $12g$. Calculate the time during which $11.25g$ of the element would have

decayed? *NECO* 2005[57]

Solution: Conventional method

10 days ⟹ 6g decay, 6g remain

10 days ⟹ 3g decay, 3g remain

10 days ⟹ 1.5g decay, 1.5g remain

10 days ⟹ 0.75g decay, 0.75g remain

∴ after 40 days, (6 + 3 + 1.5 + 0.75)g or 11.25g would have decayed

NECO 2005[57]

decayed?

Solution: Zhepwo method

Half life $T = 10$ days;
initial mass present $N_1 = 12g$;
Mass of element decayed, $N_d = 11.25g$

$N_d = N_1 - N_2$,

final mass remaining, $N_2 = N_1 - N_d$

$N_2 = 12 - 11.25 = 0.75g$

$$R = \frac{N_1}{N_2} = \frac{12}{0.75} = 16$$

$$T = \frac{t}{\log_2 R} \quad \therefore \; t = T\log_2 R$$

$$\therefore \; t = 10 \times \log_2 16$$
$$= 10 \times 4 = 40\ days$$

Example 13

A radioactive element decrease in mass from $100g$ to $15g$ in 6 days. What is the half life of the radioactive material?

Solution

Initial mass present, $N_1 = 100g$; final mass remaining, $N_2 = 15g$;
Decay time, $t = 6$ days

$$\text{Disintegrating ratio, } R = \frac{N_1}{N_2} = \frac{100}{15} = 6.667$$

$$\text{Half life,} \quad T = \frac{t}{\log_2 R} = \frac{6\ days}{\log_2 6.667}$$

While 4, 8, 16, 32 can easily be expressed as $2^2, 2^8, 2^4, 2^5$ respectively, it is very clear that expressing 6.667 as 2 raised to the power of a given number will be a very, very difficult task. So, when you encounter such a situation, you are advised to use the variant of the ***Zhepwo radioactive equation***, equation **7.1** on page 118

$$T = \frac{t \times \log 2}{\log R} = \frac{6\ days \times \log 2}{\log 6.667} = \frac{6\ days \times 0.30103}{0.824} = 2.2 days$$

$\log 2$ and $\log 6.667$ can be obtained or solved directly from your scientific calculator

Example 14

The time it will take a certain radioactive material with a half-life of 50 days to reduce to 1/32 of its original number is
A. 300 days B. 150 days C. 200 days
D. 250 days *JAMB* 2005[9]

Solution: Conventional method

1 half-life, 50 days ⟹ reduces to $\frac{1}{2}$

2 half-life, 100 days ⟹ reduces to $\frac{1}{4}$

3 half-life, 150 days ⟹ reduces to $\frac{1}{8}$

Example 14

The time it will take a certain radioactive material with a half-life of 50 days to reduce to 1/32 of its original number is
A. 300 days B. 150 days C. 200 days
D. 250 days *JAMB* 2005[9]

Solution: Zhepwo method

Half life $T = 50$ days; decay time $t = ?$

fraction remaining $f_r = \frac{1}{32}$;

$$f_r = \frac{1}{R} \quad \therefore \quad \frac{1}{32} = \frac{1}{R}$$

4 half-life, 200 days $\Longrightarrow$ reduces to $\frac{1}{16}$

5 half-life, 250 days $\Longrightarrow$ reduces to $\frac{1}{32}$

$\therefore$ time taken to reduce to $\frac{1}{32}$ of original number is 250 days

Rearranging, $R = 32$

$$T = \frac{t}{log_2 R} \quad \therefore \quad t = T \times log_2 R$$

$$t = 50 \times log_2 32$$

$$= 50 \times 5 = 250 \text{ days}$$

Example 15

A radioactive substance has a half-life of 3 minutes. After 9 minutes, the count rate was observed to be 200, what was the count rate at zero time? *NECO* 2000[58]

Solution: Conventional method

Note that count rate at zero time is the same as initial count rate.

9 min $\Longrightarrow$ count rate 200

6 min $\Longrightarrow$ count rate $(200 \times 2) = 400$

3 min $\Longrightarrow$ count rate $(400 \times 2) = 800$

0 min $\Longrightarrow$ count rate $(800 \times 2) = 1600$

$\therefore$ count rate at zero time is 1600

Example 15

A radioactive substance has a half-life of 3 minutes. After 9 minutes, the count rate was observed to be 200, what was the count rate at zero time? *NECO* 2000[58]

Solution: Zhepwo method

Final count rate $N_2 = 200$;
half life $T = 3$min; decay time $t = 9$ min
Initial count rate $N_1 =?$
Note that count rate at zero time is the same as initial count rate

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{9}{3} = 3$$

Disintegrating ratio $R = 2^n = 2^3 = 8$

$$R = \frac{N_1}{N_2} \quad \therefore \quad N_1 = N_2 \times R$$

$$N_1 = 200 \times 8 = 1600$$

Example 16

The count rate of a radioactive material is 800 count/min. If the half life of the material is 4 days, what would the count rate be16 days later? A. 200 count/min B. 100 count/min C. 50 count/min D. 25 count/min *JAMB* 2003[40]

Solution: Conventional method

Initial count rate = 800 count/min,
half-life = 4 days

after 4days $\Longrightarrow$ $\frac{1}{2}(800) = 400$ count/min

after 8days $\Longrightarrow$ $\frac{1}{2}(400) = 200$ count/min

after 12days $\Longrightarrow$ $\frac{1}{2}(200) = 100$ count/min

after 4days $\Longrightarrow$ $\frac{1}{2}(100) = 50$ count/min

$\therefore$ the count rate 16 days later is 50 count/min

Example 16

The count rate of a radioactive material is 800 count/min. If the half life of the material is 4 days, what would the count rate be16 days later? A. 200 count/min B. 100 count/min C. 50 count/min D. 25 count/min *JAMB* 2003[40]

Solution: Zhepwo method

Initial count rate, $N_1 = 800$;
Half life, $T = 4$ days;
Final count rate $N_2 =?$
decay time, $t = 16$ days

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{16}{4} = 4$$

Disintegrating ratio, $R = 2^n = 2^4 = 16$

$$R = \frac{N_1}{N_2} \quad \therefore \quad N_2 = \frac{N_1}{R} = \frac{800}{16}$$

$$= 50 \text{ count/min}$$

Alternatively,

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{800}{2^{16/4}} = \frac{800}{2^4} = \frac{800}{16}$$

$$= 50 count/min$$

Example 17
The half life of a radioactive source is 1 minute. If a rate meter connected to the source registers $200\mu A$ at a given time, what would be its reading after 3 minutes? *WAEC* 1998[57]

Solution: Conventional method

A rate meter measures the count rate of radioactive substance.

Half-life = 1min

After 1 min, rate meter reads $\frac{1}{2}(200\mu) = 100\mu$

After 2 min, rate meter reads $\frac{1}{2}(100\mu) = 50\mu$

After 3 min, rate meter reads $\frac{1}{2}(50\mu) = 25\mu$

$\therefore$ rate meter reading after 3min is 25μ

Example 17
The half life of a radioactive source is 1 minute. If a rate meter connected to the source registers $200\mu A$ at a given time, what would be its reading after 3 minutes? *WAEC* 1998[57]

Solution: Zhepwo method

A rate meter measures the count rate of radioactive substance.

Half life $T = 1$min; decay time$t = 3min$
initial count rate $N_1 = 200\mu A$;
final count rate $N_2 =?$

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{3}{1} = 3$$

Disintegrating ratio, $R = 2^n = 2^3 = 8$

$$R = \frac{N_1}{N_2} \quad \therefore \quad N_2 = \frac{N_1}{R} = \frac{200\mu A}{8} = 25\mu A$$

Alternative Zhepwo method

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{200}{2^{3/1}} = \frac{200\mu A}{2^3} = \frac{200\mu A}{8}$$
$$= 25\mu A$$

Example 18
In 90 seconds, the mass of a radioactive element reduces to $\frac{1}{32}$ of its original values. Determine the half-life of the element. *WAEC* 1998[53]

Solution: Conventional method

Let n be the original mass of radioactive element

After 1 half-life element reduces to $\frac{n}{2}$

After 2 half-life element reduces to $\frac{n}{4}$

After 3 half-life element reduces to $\frac{n}{8}$

After 4 half-life element reduces to $\frac{n}{16}$

After 5 half-life element reduces to $\frac{n}{32}$

If 5 half-life takes 90 sec

Then 1 half-life takes T

$$\text{Hence } T = \frac{1 \text{ halflife} \times 90 \text{ sec}}{5 \text{ halflife}} = \frac{90\text{sec}}{5}$$
$$= 18\text{sec}$$

$\therefore$ half-life = 18sec

Example 18
In 90 seconds, the mass of a radioactive element reduces to $\frac{1}{32}$ of its original values. Determine the half-life of the element. *WAEC* 1998[53]

Solution: Zhepwo method

Decay time $t = 90s$;

fraction of initial mass remaining, $f_r = \frac{1}{32}$

$$f_r = \frac{1}{R} \quad \therefore \quad \frac{1}{32} = \frac{1}{R}$$

Rearranging, disintegrating ratio, $R = 32$

$$\text{Half life}, T = \frac{t}{\log_2 R} = \frac{90}{\log_2 32} = \frac{90}{5}$$
$$= 18s$$

Example 19
A radioactive substance has a half-life of 3 days. If a mass of $1.55g$ of this substance is left after decaying for 15 days, determine the original value of the mass. *WAEC* 2004[49]

Solution: Conventional method

Half-life = 3 days

15 days = $\frac{15}{3}$ or 5 half-life, 1.55g remain

12 days = $\frac{12}{3}$ or 4 half-life, 2 × 1.55g = 3.1g remain

9 days = $\frac{9}{3}$ or 3 half-life, 2 × 3.1g = 6.2g remain

6 days = $\frac{6}{3}$ or 2 half-life, 2 × 6.2g = 12.4g remain

3 days = $\frac{3}{3}$ or 1 half-life, 2 × 12.4g = 24.8g remain

0 day = $\frac{0}{3}$ or 0 half-life, 2 × 24.8g = 49.6g remain

∴ original value of the mass = 49.6g

Example 19
A radioactive substance has a half-life of 3 days. If a mass of $1.55g$ of this substance is left after decaying for 15 days, determine the original value of the mass. *WAEC* 2004[49]

Solution: Zhepwo method

Half life, $T = 3$days;

decay time $t = 15$ days;

final mass remaining, $N_2 = 1.55g$

$$T = \frac{t}{n} \quad \therefore \quad n = \frac{t}{T} = \frac{15}{3} = 5$$

Disintegrating ratio, $R = 2^n = 2^5 = 32$

$$R = \frac{N_1}{N_2} \quad \therefore \quad N_1 = RN_2$$

∴ original mass, $N_1 = 32 \times 1.55$
$= 49.6g$

Alternative Zhepwo method

$$N_1 = 2^{t/T} \times N_2 = 2^{15/3} \times 1.55$$
$$= 2^5 \times 1.55 = 32 \times 1.55$$
$$= 49.6g$$

Example 20
In 90 seconds, the mass of a radioactive element reduces to 1/16 of its original value. Determine the half-life of the element. *NECO* 2003[56]

Solution: Conventional method

Let n be the original mass of radioactive element

After 1 half-life element reduces to $\frac{n}{2}$

After 2 half-life element reduces to $\frac{n}{4}$

After 3 half-life element reduces to $\frac{n}{8}$

After 4 half-life element reduces to $\frac{n}{16}$

If 4 half-life takes 90 sec

Then 1 half-life takes T

$$\text{Hence } T = \frac{1 \text{ halflife } \times 90 \text{ sec}}{4 \text{ halflife}}$$

$$= \frac{90\text{sec}}{4} = 22.5\text{sec}$$

Example 20
In 90 seconds, the mass of a radioactive element reduces to 1/16 of its original value. Determine the half-life of the element. *NECO* 2003[56]

Solution: Zhepwo method

Decay time $t = 90s$;

fraction of mass remaining $f_r = \frac{1}{16}$;

$$f_r = \frac{1}{R} \quad \therefore \quad \frac{1}{16} = \frac{1}{R}$$

Rearranging, disintegrating ratio, $R = 32$

$$\text{Half life, } T = \frac{t}{\log_2 R} = \frac{90}{\log_2 16} = \frac{90}{4}$$
$$= 22.5s$$

∴ half-life = 22.5sec

Decay Constant

The decay constant of a radioactive atom is the instantaneous rate of decay per unit atom of the radioactive element.

For a radioactive atom, the number of atoms disintegrating per second, $\frac{dN}{dt}$, is directly proportional to the number, N, of atoms present at that instant.

That is, $$\frac{dN}{dt} = -\lambda N \quad \cdots\cdots 7.5$$

Where, λ is a constant peculiar to the radioactive atom called the *decay constant*. The negative sign shows that N becomes smaller as time, t, increases.

Rearranging 7.5, we obtain decay constant;

$$\lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right) \quad \cdots\cdots 7.6$$

$$\therefore \lambda = \frac{\text{number of disintegrating atoms per second}}{\text{number of atoms present at that instant}}$$

By integrating equation 7.5, the following is obtained

$$N = N_o e^{-\lambda t} \quad \cdots\cdots 7.7$$

Where N_o = number of atoms present at time $t = 0$

The value of half-life T, can be derived considering that, at half-life the number of decaying atoms is half its initial value.

$$\text{That is,} \quad N = \frac{N_o}{2} \quad \text{and } t = T$$

$$\therefore \quad eqn\ 7.7,\ N = N_o e^{-\lambda t} \quad becomes$$

$$\frac{N_o}{2} = N_o e^{-\lambda T}$$

Dividing both side by N_o we obtain;

$$\frac{1}{2} = e^{-\lambda T}$$

Taking logarithm of both side to base e we have;

$$\log_e \frac{1}{2} = \log_e e^{-\lambda t}$$

Applying subtraction law and power law in the theory of logarithm, we get

$$\log_e 1 - \log_e 2 = -\lambda T \log_e e$$

Remembering that $\log_e 1 = 0$ and $\log_e e = 1$, the equation becomes

$$-\log_e 2 = -\lambda T$$

$$\ln 2 = \lambda T$$

$$0.693 = \lambda T$$

$$\therefore \text{Decay constant, } \lambda = \frac{0.693}{T}$$

$$\text{or Half-life, } T = \frac{0.693}{\lambda} \quad \cdots\cdots 7.8$$

Decay constant is measured in per second, s^{-1} while half-life is measured in second, s The value of ln 2 can be obtained from your scientific calculator. **Don't bother yourself with the process of deriving the decay constant.** Equation 7.8 and equation 7.5 are all you need, so commit them to memory. $\lambda = \frac{0.693}{T}$ and $\frac{dN}{dt} = -\lambda N$

Example 21
A piece of radioactive material contains 10^{20}atoms. If the half-life of the material is 20 seconds, the number of disintegrations in the first second is
A. 3.47×10^{18} B. 6.93×10^{20} C. 3.47×10^{20} D. 6.93×10^{18}
JAMB 2009[47]

Solution
Number of atoms initially present, $N = 10^{20}$, half-life, $T = 20s$

$$\therefore \quad \lambda = \frac{0.693}{T} = \frac{0.693}{20} = 0.03465$$

The number of atoms disintegrating the 1st second, $\frac{dN}{dt} = -\lambda N$

$$\frac{dN}{dt} = 0.03465 \times 10^{20} \cong 3.47 \times 10^{18}$$

Example 22
A radioactive element has a decay constant of $0.077s^{-1}$. Calculate its half-life.
WAEC 2008[50]

Solution
Decay constant $\lambda = 0.077s^{-1}$ $\quad \therefore$ Half life, $T = \frac{0.693}{\lambda} = \frac{0.693}{0.077} = 9s$

Example 23
The half-life of a radioactive substance is 2 seconds. Calculate the decay constant.
NECO 2000[60]

Solution
Half life, $T = 2s$ $\quad \therefore$ Decay constant, $\lambda = \frac{0.693}{T} = \frac{0.693}{2} = 0.347s^{-1}$

Example 24
A radioactive element decays to one eight of its original quantity in 9 seconds. Calculate its decay constant.
WAEC 1998[57]

Solution: Conventional method

Let n be the original mass of radioactive element

After 1 half-life element reduces to $\frac{n}{2}$

After 2 half-life element reduces to $\frac{n}{4}$

After 3 half-life element reduces to $\frac{n}{8}$

If 3 half-life takes 9 sec

Then 1 half-life takes T

Hence $\text{T} = \frac{1 \text{ halflife} \times 9 \text{ sec}}{3 \text{ halflife}}$

Example 24
A radioactive element decays to one eight of its original quantity in 9 seconds. Calculate its decay constant.
WAEC 1998[57]

Solution: Zhepwo method

Fraction remaining, $f_r = \frac{1}{8}$;

decay time $t = 9s$

$f_r = \frac{1}{R}$

Rearranging, disintegration ratio $R = 8$

Half life, $T = \frac{t}{\log_2 R} = \frac{9}{\log_2 8} = \frac{9}{3} = 3$

Decay constant, $\lambda = \frac{0.693}{T} = \frac{0.693}{3}$

$= 0.231s^{-1}$

$$= \frac{9\text{sec}}{3} = 3\text{sec}$$

$$\text{Decay constant}, \lambda = \frac{0.693}{T} = \frac{0.693}{3}$$
$$= 0.231s^{-1}$$

Relationship between Decay Constant(λ) and Disintegrating Ratio(R)

Disintegrating ratio(R) is related to the established equation for decay constant (λ) as follows.

$$\lambda = 2.303\frac{logR}{t} \quad \cdots\cdots\cdots\cdots \quad 7.9$$

$$\text{Where } R = \frac{N_1}{N_2}$$

So, equation 7.9 can be used to find the decay constant even when the half-life is not given, as long as the initial number of atoms(N_1), final number of atoms(N_2) and decay time(t) are given.

Example 25

A radioactive element decays to one eight of its original quantity in 9 seconds. Calculate its decay constant.

WAEC 1998[57]

Solution: Conventional method

Let n be the original mass of radioactive element

After 1 half-life element reduces to $\frac{n}{2}$

After 2 half-life element reduces to $\frac{n}{4}$

After 3 half-life element reduces to $\frac{n}{8}$

If 3 half-life takes 9 sec

Then 1 half-life takes T

$$\text{Hence } T = \frac{1 \text{ halflife } \times 9 \text{ sec}}{3 \text{ halflife}} = \frac{9\text{sec}}{3}$$
$$= 3\text{sec}$$

$$\text{Decay constant}, \lambda = \frac{0.693}{T} = \frac{0.693}{3}$$
$$= 0.231s^{-1}$$

Example 25

A radioactive element decays to one eight of its original quantity in 9 seconds. Calculate its decay constant.

WAEC 1998[57]

Solution: Zhepwo method

Fraction remaining, $f_r = \frac{1}{8}$;

decay time $t = 9s$

$$f_r = \frac{1}{R} \quad \therefore \quad \frac{1}{8} = \frac{1}{R}$$

Rearranging, disintegrating ratio $R = 8$

$$\text{decay constant}, \quad \lambda = 2.303\frac{logR}{t}$$
$$= 2.303\frac{\log 8}{9}$$
$$= \frac{2.303 \times 0.90309}{9} = 0.231s^{-1}$$

Example 26

If $\frac{15}{16}$th of the radioactive atoms of an element decay in 8min, find the decay constant of the element.

Solution: Conventional method

Example 26

If $\frac{15}{16}$th of the radioactive atoms of an element decay in 8min, find the decay constant of the element.

Solution: Zhepwo method

If $\frac{15}{16}$ atoms have decayed, then the fraction of atom remaining is

$1 - \frac{15}{16} = \frac{1}{16}$

After 1 half-life, $\frac{1}{2}$ of atoms remain

After 2 half-life, $\frac{1}{4}$ of atoms remain

After 3 half-life, $\frac{1}{8}$ of atoms remain

After 4 half-life, $\frac{1}{16}$ of atoms remain

$\therefore$ $\frac{1}{16}$ of atoms remain after 4 half-life.

This means that after 4 half-life, the number of atoms left is $\frac{1}{16}$th of the original.

If 4 half-life takes 8 min

Then 1 half-life takes T

$$\text{Hence T} = \frac{1\text{ halflife} \times 8\text{ min}}{4\text{ halflife}} = \frac{8\text{min}}{4}$$

$$= 2\text{min}$$

$$\text{Decay constant}, \lambda = \frac{0.693}{T} = \frac{0.693}{2\text{min}}$$

$$= 0.347\text{min}^{-1}$$

Fraction decayed, $f_d = \frac{15}{16}$;

decay time $t = 8$min

$$f_d = \frac{R-1}{R}$$

$$\frac{15}{16} = \frac{R-1}{R}$$

$$16R - 16 = 15R$$

$$16R - 15R = 16$$

$$R = 16$$

$$\text{decay constant}, \lambda = 2.303\frac{logR}{t}$$

$$= \frac{2.303 \times \log 16}{8\text{min}}$$

$$= \frac{2.303 \times 1.204}{8} = 0.347\text{min}^{-1}$$

Example 27
In 24 days, a radioactive isotope decreases in mass from 128g to 2g. What is the decay constant of the radioactive material?
Solution: Conventional method

$$128\text{g} \Rightarrow 64\text{g} \longrightarrow 1 \text{ half life}$$

$$64\text{g} \Rightarrow 32\text{g} \longrightarrow 2 \text{ half life}$$

$$32\text{g} \Rightarrow 16\text{g} \longrightarrow 3 \text{ half life}$$

$$16\text{g} \Rightarrow 8\text{g} \longrightarrow 4 \text{ half life}$$

$$8\text{g} \Rightarrow 4\text{g} \longrightarrow 5 \text{ half life}$$

$$4\text{g} \Rightarrow 2\text{g} \longrightarrow 6 \text{ half life}$$

If 6 half-life is equal to 24 days

Then 1 half-life takes T

$\therefore$ T × 6 half life = 1half life × 24days

$$\text{Hence,} \quad T = \frac{1\text{ half life} \times 24\text{days}}{6\text{ halflife}}$$

Example 27
In 24 days, a radioactive isotope decreases in mass from 128g to 2g. What is the decay constant of the radioactive material?
Solution: Zhepwo method

Decay time $t = 24$days;

initial mass, $N_1 = 128$ g;

final mass $N_2 = 2$g

$$R = \frac{N_1}{N_2} = \frac{128\text{g}}{2\text{g}} = 64$$

$$\text{decay constant}, \lambda = 2.303\frac{\log R}{t}$$

$$= \frac{2.303 \times \log 64}{24\text{days}}$$

$$= \frac{2.303 \times 1.806}{24}$$

$$= 0.1733\text{day}^{-1}$$

$$= \frac{24\text{days}}{6} = 4\text{days}$$

$$\text{Decay constant}, \lambda = \frac{0.693}{T} = \frac{0.693}{4 \text{ days}}$$

$$= 0.1733 day^{-1}$$

Disintegrating Ratio and Graphical Solutions

The disintegration or decay of a radioactive substance can be represent by a decay curve when the number of atoms remaining is plotted against the time t. The concept of disintegrating ratio could also be used to solve graphical problems as the following examples illustrates. Answers that should normally be obtained from the plotted graph can be obtained accurately by calculation.

Example 28

The half-life of uranium X_1 is 24 days. Calculate the mass remaining unchanged of 0.64g of the substance after

(i) 24 days;
(ii) 48 days
(iii) 72 days
(iv) 96 days
(v) 120 days

Plot your answer on a graph and hence determine:

(vi) the mass remaining unchanged after 84 days
(vii) after how many days there will be exactly 0.25g unchanged.

(A.E.B) *Chapter 47, page 579*. exercise 4(d) *ABBOT A.F (1999). Physics. Fifth Edition. Heinemann*

Solution: Zhepwo method

Half-life, $T = 24$ days ; initial mass present, $N_1 = 0.64g$

final mass remaining, $N_2 = ?$

We are asked to find N_2 for decay time, t = 24, 48, 72, 96, and 120 days.

From disintegrating ratio, $\frac{N_1}{N_2} = 2^n$

we substitute, $n = \frac{t}{T}$ to obtain $\frac{N_1}{N_2} = 2^{t/T}$

$$\therefore \quad N_2 = \frac{N_1}{2^{t/T}}$$

(i) $t = 24$ days $T = 24$ days; $N_1 = 0.64g$;

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{24/24}} = \frac{0.64g}{2^1} = \frac{0.64g}{2} = 0.32g$$

(ii) $t = 48$ days; $T = 24$ days; $N_1 = 0.64g$

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{48/24}} = \frac{0.64g}{2^2} = \frac{0.64g}{4} = 0.16g$$

(iii) $t = 72$ days; $T = 24$ days; $N_1 = 0.64g$

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{72/24}} = \frac{0.64g}{2^3} = \frac{0.64g}{8} = 0.08g$$

(iv) $t = 96$ days; $T = 24$ days; $N_1 = 0.64g$

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{96/24}} = \frac{0.64g}{2^4} = \frac{0.64g}{16} = 0.04g$$

(v) $t = 120$ days; $T = 24$ days; $N_1 = 0.64g$

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{120/24}} = \frac{0.64g}{2^5} = \frac{0.64g}{32} = 0.02g$$

Solution: Conventional method

(i) After **24 days**, $\frac{1}{2}$(0.64g) decay, $\frac{1}{2}$(0.64g) i.e. **0.32g** remain

(ii) After 24 days, 0.32g decay, 0.32g remain
After **48 days**, $\frac{1}{2}$(0.32g) decay, $\frac{1}{2}$(0.32g) i.e. **0.16g** remain

(iii) After 24 days, 0.32g decay, 0.32g remain
After 48 days, 0.16g decay, 0.16g remain

After **72 days**, $\frac{1}{2}$(0.16g) decay, $\frac{1}{2}$(0.16g) i.e. **0.08g** remain

(iv) After 24 days, 0.32g decay, 0.32g remain
After 48 days, 0.16g decay, 0.16g remain

After 72days, 0.08g decay, 0.08g remain
After **96 days**, $\frac{1}{2}$(0.08g) decay, $\frac{1}{2}$(0.08g) i.e. **0.04g** remain

(v) After 24 days, 0.32g decay, 0.32g remain
After 48 days, 0.16g decay, 0.16g remain

After 72days, 0.08g decay, 0.08g remain
After 96days, 0.04g decay, 0.04g remain

After **120 days**, $\frac{1}{2}$(0.04g) decay, $\frac{1}{2}$(0.04g) i.e. **0.02g** remain

Though the answers to (vi) and (vii) are to be determined from the graph, equations could also be used as shown below. Students are advised to plot the graph and compare their answers with those obtained through calculation.

(vi) "*...mass remaining unchanged after 84 days...*"
$\therefore$ $t = 84$ *days;* $T = 24$ *days* $N_1 = 0.64$

$$N_2 = \frac{N_1}{2^{t/T}} = \frac{0.64g}{2^{84/24}} = \frac{0.64g}{2^{3.5}} = \frac{0.64g}{11.314} = 0.056g$$

(vi) From, *...after how many days there will be exactly 0.25g unchanged...* we obtain:
Final mass remaining, $N_2 = 0.25g$; decay time t =?
$T = 24$ days $N_1 = 0.64g$

Disintegrating ratio $R = \frac{N_1}{N_2} = \frac{0.64g}{0.25g} = 2.56$

$$T = \frac{t \times \log 2}{\log R} \quad \therefore t = \frac{T \times \log R}{\log 2} = \frac{24 \text{ days} \times \log 2.56}{\log 2} = \frac{24 \text{ days} \times 0.40824}{0.301}$$
$$= 32.5 \text{ days}$$

Transformation of Elements through Radioactivity

In natural radioactivity, a radioactive element undergoes decay when it emits α - particle; β-particle or γ- rays. If a radioactive element X with mass number A and atomic number Z emits an alpha particle (Helium nucleus, ${}_2^4He$) to form another element Y, the nuclear equation is given by

$$ {}_Z^AX \longrightarrow {}_2^4He + {}_{Z-2}^{A-4}Y $$

Similarly, if a radioactive element emits a beta particle (electron, ${}_{-1}^{0}e$) and forms another element Y, the nuclear equation is given by

$$ {}_Z^AX \longrightarrow {}_{-1}^{0}e + {}_{Z+1}^{A}Y $$

Emission of gamma rays (γ) results in no loss of mass number and atomic. In artificial radioactivity, high energy α- particle (${}_2^4He$) and neutron (${}_0^1n$) are used to bombard element thereby changing them from one form to another. Example of α-particle bombardment is;

$$ {}_7^{14}N + {}_2^4He \longrightarrow {}_8^{17}O + {}_1^1H $$

Example of neutron bombardment is given by

$$ {}_{12}^{24}Mg + {}_0^1n \longrightarrow {}_{11}^{24}Na + {}_1^1H $$

Note: ${}_1^1H$ represents a proton
${}_1^2H$ represents a deuterium
${}_1^3H$ represent a tritium
${}_0^1n$ represents a neutron
${}_{-1}^{0}e$ represents s beta particle
${}_2^4He$ represents an alpha particle

Example 29

A certain radioisotope of ${}_{92}^{235}U$ emits four alpha particles and three beta particles. The mass number and the atomic number of the resulting element respectively are

A. 219 and 87 B. 84 and 223 C. 223 and 87 D. 219 and 81 *JAMB* 1994[48]

Solution

α-particle $= {}_2^4He \quad \therefore 4\alpha = 4{}_2^4He$
β-particle $= {}_{-1}^{0}e \quad \therefore 3\beta = 3{}_{-1}^{0}e$

Let A and Z be the mass number and atomic number respectively of the resulting element Y.

$$ {}_{92}^{235}U \longrightarrow 4{}_2^4He + 3{}_{-1}^{0}e + {}_Z^AY $$

Writing a balanced equation for mass number we obtain

$$235 = (4 \times 4) + (3 \times 0) + A$$
$$235 = 16 + 0 + A$$
$$235 = 16 + A$$
$$235 - 16 = A$$

Mass number, $A = 219$

Writing a balanced equation for atomic numbers we obtain

$$92 = (4 \times 2) + (3 \times -1) + Z$$
$$92 = 8 + (-3) + Z$$
$$92 = 8 - 3 + Z$$
$$92 = 5 + Z$$
$$92 - 5 = Z$$

Atomic number $Z = 87$

Ans $=$ 219 and 87 or ${}_{87}^{219}Y$

Example 30
A nuclide X is produced by bombarding a nitrogen (N) nucleus with an alpha (α) particle with the release of heavy hydrogen (D) nucleus as shown by the following nuclear equation.

$$\alpha + {}^{14}_{7}N \longrightarrow {}^{P}_{q}X + {}^{2}_{1}D$$

Determine the values of P and q in the equation *WAEC* 2002[50]

Solution

$$\alpha\text{-particle} = {}^{4}_{2}He$$

$$\alpha + {}^{14}_{7}N \longrightarrow {}^{P}_{q}X + {}^{2}_{1}D$$
$${}^{4}_{2}He + {}^{14}_{7}N \longrightarrow {}^{P}_{q}X + {}^{2}_{1}D$$

Writing a balanced equation for mass ($nucleon$) number, we obtain

$$4 + 14 = P + 2$$
$$18 = P + 2$$
$$P = 18 - 2$$
$$P = 16$$

Writing a balanced equation for atomic ($proton$) number we obtain,

$$2 + 7 = q + 1$$
$$9 = q + 1$$
$$q = 9 - 1$$
$$q = 8$$

Ans: $P = 16,\ q = 8$ Or ${}^{16}_{8}X$

Example 31

$${}^{14}_{7}N + {}^{4}_{2}He \longrightarrow {}^{17}_{0}O + X$$

In the equation above, the particle X is
A. a proton B. a neutron C. an α-particle D. a β-particle *JAMB* 2008[46]

Solution
Let A represent mass number and Z atomic number

$${}^{14}_{7}N + {}^{4}_{2}He \longrightarrow {}^{17}_{8}O + {}^{A}_{Z}X$$

Writing a balanced equation for mass numbers we obtain

$$14 + 4 = 17 + A$$
$$18 = 17 + A$$
$$A = 18 - 17$$
$$A = 1$$

Writing a balance equation for atomic numbers we obtain

$$7 + 2 = 8 + Z$$
$$9 = 8 + Z$$
$$Z = 9 - 8$$
$$Z = 1$$

$\therefore$ ${}^{A}_{Z}X$ become ${}^{1}_{1}X$ which is a proton (${}^{1}_{1}H$) Ans: *A* (a proton)

NUCLEAR REACTION

Nuclear reaction is divided into nuclear fission and nuclear fusion.

Nuclear fission is the splitting up of a heavy atomic nucleus by neutron bombardment. It results in the formation of two approximately equal lighter nuclei, more neutrons and the release of very large amount of energy. Uranium 235 undergoes neutron bombardment as shown below to form Barium and krypton nuclei.

$$ {}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2{}^{1}_{0}n + \text{energy} $$

Nuclear fusion is the combination, under extremely high temperature and pressure, of light nuclei to form heavier nucleus with release of huge amount of energy. Deuterium and tritium nuclei are fused to form helium nucleus as shown below.

$$ {}^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + {}^{1}_{0}n + \text{energy} $$

Nuclear reaction (fission and fusion) occur with a loss in mass and the release of huge amount of energy. The loss in mass, also known as mass defect, is given by

$Mass\ defect$ = mass of original substance in $a.m.u$ – mass of products in $a.m.u$

Note: a.m.u. means atomic mass unit and is denoted by the symbol, u.

The difference in mass that happens in a nuclear reaction is a measure of the binding energy of the particular nucleus or nuclei.

Binding energy is the quantity of energy that must be put into a nucleus in order to break it into its constituent particles. Therefore, binding energy of a nucleus is proportional to the difference between the total mass of the individual *nucleons* and the mass of the *nucleus*. Usually, the total mass of the stable nucleus or nuclide is less than the sum of the masses of its constituent nucleons.

The energy released during nuclear reaction (binding energy) is given by Einstein's energy equation.

$$ E = mc^2 $$

Where c = velocity of electromagnetic wave (light wave) = $3 \times 10^8 ms^{-1}$

m = Mass defect in kilogram where $1a.m.u = 1.66 \times 10^{-27} kg$

E = Nuclear energy

The unit of energy in nuclear reaction can be expressed in unified atomic mass unit (u) or the electron volt (eV) or joules (J), as follows

$1eV = 1.6 \times 10^{-19} J$

$1MeV = 1.6 \times 10^{-13} J$

$1u = 931MeV = 1.490 \times 10^{-10} J$

Example 32

In a nuclear reaction the mass defect is $2.0 \times 10^{-6} g$. Calculate the energy released, given that the velocity of light is $3.0 \times 10^8 ms^{-1}$. *WAEC 1997*[57]

Solution

Mass defect $m = 2.0 \times 10^{-6} g = 2 \times 10^{-9} kg$

Velocity of light $c = 3.0 \times 10^8 ms^{-1}$

Energy released $E = mc^2$

$= 2.0 \times 10^{-9} \times (3 \times 10^8)^2$

$= 2.0 \times 10^{-9} \times 9 \times 10^{16}$

$= 1.8 \times 10^8 J$

Example 33

In a thermonuclear reaction, the total initial mass is $5.02 \times 10^{-27}kg$ and the total final mass is $5.01 \times 10^{-27}kg$. The energy released in the process is A. $9.0 \times 10^{-10}J$ B. $9.0 \times 10^{-11}J$ C. $9.0 \times 10^{-12}J$ D. $9.0 \times 10^{-13}J$ $[c = 3 \times 10^{8}ms^{-1}]$

JAMB 2000[42]

Solution

Initial mass= $5.02 \times 10^{-27}kg$; final mass= $5.01 \times 10^{-27}kg$; $c = 3 \times 10^{8}ms^{-1}$

Mass defect = initial mass− final mass

$m = 5.02 \times 10^{-27} - 5.01 \times 10^{-27}$

$m = 1.0 \times 10^{-29}kg$

Energy release $E = mc^2$

$$= 1.0 \times 10^{-29} \times (3 \times 10^{8})^2$$
$$= 1.0 \times 10^{-29} \times 9 \times 10^{16}$$
$$E = 9.0 \times 10^{-13}J$$

Example 34

In the fusion of hydrogen isotopes into helium, the decrease in mass is about 0.65%. Calculate the energy obtainable when $1.0g$. of hydrogen is used.

$[c = 3.0 \times 10^{8}ms^{-1}]$ *NECO 2006*[E10]

Solution

$$1.0g = 0.001kg; \quad c = 3.0 \times 10^{8}ms^{-1}$$

Mass defect, $m = \dfrac{0.65}{100} \times 0.001$

$$m = 6.5 \times 10^{6}kg$$

Energy obtained $E = mc^2$

$$= 6.5 \times 10^{-6} \times (3 \times 10^{8})^2 = 5.85 \times 10^{11}J$$

Example 35

A possible fusion reaction is ${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + {}_1^1H + Q$, where Q is the energy released as a result of the reaction. If $Q = 4.03MeV$, Calculate the atomic mass of ${}_1^3H$ in atomic mass units. $[{}_1^2H = 2.01410u;\ {}_1^1H = 1.00783u;\ 1u = 931MeV]$ *WAEC 2000*[E15]

Solution

${}_1^2H + {}_1^2H \longrightarrow {}_1^3H + {}_1^1H + Q$

Let x be the atomic mass of ${}_1^3H$ in u.

$1u = 931MeV$

Therefore, $4.03MeV = \dfrac{4.03u}{931} = 0.004329u$

Substitute given values into the nuclear equation to obtain:

$$2.01410 + 2.01410 \longrightarrow x + 1.00783 + 0.004329$$
$$4.0282 = x + 1.01216$$
$$x = 4.0282 - 1.01216 = 3.01604u$$

Example 36

The radioactive nuclei ${}_{84}^{210}Po$ emits an α-particle to produce ${}_{82}^{206}Pb$. Calculate the energy, in MeV, released in each disintegration.

(Take the masses of ${}_{84}^{210}Po =$ $209.936730u$; ${}_{82}^{206}Pb = 205.929421u$; ${}_2^4He = 4.001504u$, and that $1u = 931MeV$)

WAEC 2006[E15]

Solution

α-particle $= {}_2^4He$

$$^{210}_{84}Po \longrightarrow {}^{4}_{2}He + {}^{206}_{82}Pb + \text{Energy } (Q)$$

$1u = 931MeV$

Substituting given values into the nuclear equation, we obtain:

$$(209.936730)931 = (4.001504 + 205.929421)931 + Q$$

$$195451.0956 = 209.930925 \times 931 + Q$$

$$195451.0956 = 195445.6912 + Q$$

$$Q = 195451.0956 - 195445.6912$$

$$Q = 5.404MeV$$

Example 37

The binding energy of helium ${}^{4}_{2}He$ is A. $2.017u$ B. $0.033u$ C. $4.033u$ D. $0.330u$ [atomic mass of proton $= 1.00783u$, atomic mass of neutron $= 1.00867u$]

JAMB 2004[21]

Solution

Atomic mass of proton = $1.00783u$

Atomic mass of neutron = $1.00867u$

Atomic mass of ${}^{4}_{2}He$ nucleus = $4u$

Number of protons = 2

Number of neutrons = $4 - 2 = 2$

Mass of protons = $2(1.00783u) = 2.01566u$

Mass of neutrons = $2(1.00867u) = 2.01734u$

Total mass neutron = $2.01566u + 2.01734u = 4.033u$

Binding energy = difference between mass of nucleon and that of nucleus

= mass of nucleon – mass of nucleus

= $4.033u - 4u$

= $0.033u$

Example 38

The mass of a proton is $1.0074u$ and that of a neutron is $1.0089u$. Determine the energy evolved in stabilizing the nucleus of nitrogen of a mass number 14 with 7 protons and 7 neutrons. [$speed\ of\ light = 3.0 \times 10^{8} ms^{-1}$; $1u = 1.67 \times 10^{-27} kg$]

WAEC 2005[49]

Solution

mass of proton = $1.0074u$; mass of neutron = $1.0089u$ mass of nucleus = $14u$;
speed of light $c = 3 \times 10^{8} ms^{-1}$ number of proton = 7; number of neutron = 7
$1u = 1.67 \times 10^{-27} kg$

Mass of protons = $7(1.0074u) = 7.0518u$

Mass of neutrons = $7(1.0089u) = 7.0623u$

Total mass of nucleon = mass of proton + mass of neutron

= $7.0518 + 7.0623 = 14.1141u$

Mass of nucleon in $kg = 14.1141 \times 1.67 \times 10^{-27} kg$

$= 2.3570547 \times 10^{-26} kg$

Mass of nucleus in $kg = 14 \times 1.67 \times 10^{-27} kg$

$= 2.338 \times 10^{-26} kg$

Mass defect = mass of nuclcon – mass of nucleus

$m = 2.3570547 \times 10^{-26} - 2.338 \times 10^{-26}$

$m = 1.90547 \times 10^{-28} kg$

Energy evolved $E = mc^2$

$= 1.90547 \times 10^{-28} \times (3 \times 10^8)^2$

$= 1.714923 \times 10^{-11} J \quad \simeq 1.715 \times 10^{-11} J$

Example 39

Calculate, in joules, the binding energy for ^9_4Be.

[Atomic mass of $^9_4Be = 9.01219u$; mass of proton $= 1.00783u$, mass of neutron $= 1.00867u$, unified atomic mass unit, $u = 931 MeV$, $1eV = 1.6 \times 10^{-19} J$]

NECO 2007^{E15}

Solution

Mass of ^9_4Be = mass of nucleus = $9.01219u$

Mass of proton = $1.00783u$; $u = 931 MeV$

Mass of neutron = $1.00867u$; $1eV = 1.6 \times 10^{-19} J$

Number of protons = 4

Number of neutrons = $9 - 4 = 5$

Mass of protons = $4(1.00783u) = 4.03132u$

Mass of neutrons = $5(1.00867u) = 5.04335u$

Mass of nucleon = mass of proton + mass of neutron

$= 4.03132u + 5.04335u$

$= 9.07467u$

Mass defect (binding energy) = mass of nucleon – mass of nucleus

$= 9.07467u - 9.01219u$

$= 0.06248u$

Binding energy $= 0.06248 \times 931 \times 10^6 \times 1.6 \times 10^{-19} J = 9.3 \times 10^{-12} J$

Example 40

Deuteron and tritium fused to form a helium nucleus according to the equation.

$$^2_1H + ^3_1H \longrightarrow ^4_2He + ^1_0n + Q$$

Calculate, in joules, the energy released.

($^3_1H = 3.01605u$; $^2_1H = 2.01410u$; $^4_2He = 4.00260u$;)

($^1_0n = 1.00867$; $1u = 931 MeV$; $1eV = 1.6 \times 10^{-19} J$) *NECO* 2004^{E15}

Solution

Total mass of reactants $= 2.01410u + 3.01605u$

$= 5.03015u$

Total mass of products $= 4.00260u + 1.00867u$

$= 5.01127u$

Mass defect = mass of reactant – mass of product

$= 5.03015 - 5.01127$

$= 0.01888u$

$$\text{Energy released} = 0.01888 \times 931MeV$$

$$= 17.57728MeV$$

$$= 17.57728 \times 10^6 \times 1.6 \times 10^{-19}$$

$$Q = 2.812 \times 10^{-12}J$$

EXERCISE 7.

1. An element whose half life is 3 years has N atoms. How many atoms would have decayed after 9 years? $WAEC$ 1992[56] Ans: $\frac{7}{8}N$atoms

2. The half life of a radioactive element is 9 days. What fraction of atoms has decayed in 36 days? A. $\frac{1}{16}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{15}{16}$ $JAMB$ 1995[49] Ans: $\frac{15}{16}$

3. A radioactive element has a half life of 4 days. The fraction that has decayed in 16 day is A. $\frac{3}{4}$ B. $\frac{1}{16}$ C. $\frac{15}{16}$ D. $\frac{1}{4}$ $JAMB$ 2006[4] Ans: $\frac{15}{16}$

4. A radioactive substance has a half life of 20 hours. What fraction of the original radioactive nuclide will remain after 80 hours? $WAEC$ 1993[55] Ans: $\frac{1}{16}$

5. A sample of radioactive material has a half life of 35 days. Calculate the fraction of the original quantity that that will remain after 105 days.

$WAEC$ 2001[E15] Ans: $\frac{1}{8}$

6. A radioactive substance has a half life of 20 days. What fraction of the original radioactive nuclei will remain after 80 days?

A. $\frac{1}{4}$ B. $\frac{1}{8}$ C. $\frac{1}{16}$ D. $\frac{1}{32}$ $JAMB$ 2007[2] Ans: $\frac{1}{16}$

7. A radioactive substance has a half life of 2 years. If the initial mass is $40g$, which of the following rows correctly give the mass of substance left at the times stated?

	2years	3years	4years	5years
A	$20g$		$10g$	
B	$30g$	$20g$	$10g$	$0g$
C	$30g$	$20g$	$15g$	
D	$20g$		$10g$	$0g$

$JAMB$ 1989[46] Ans: A

8. A radioactive nuclide of mass $6.0g$ has a half life of 8 days. Calculate the time during which $5.25g$ of the nuclide would have decayed. $WAEC$ 1995[54] Ans: 24days

9. The half life of a radioactive substance is 14 days. If $48g$ of this substance is stored, after how many days will $1.5g$ of the original substance remain?

$WAEC$ 2000[48] Ans: 70days

10. $4g$ of a radioactive material of half life 10 days is spilled on a laboratory floor. How long would it take to disintegrate $3.5g$ of the material?

A. $1\frac{1}{4}$ days B. $8\frac{3}{4}$ days C. 30 days D. 80 days. $JAMB$ 1991[47] Ans: 30 days

11. The count rate of an alpha particle source is 400 per minute. If the half life of the source is 5 days, what would be the count rate per minute after 15 days?

$WAEC$ 1996[60] Ans: 50

12. The half-life of a radioactive substance is 3 years. How long, in years, will it take 384 of the substance to decay to a mass of 6g? $NECO$ 2008[57] Ans: 18 years

13. A substance has a half-life of 3 seconds. After 6 seconds, the count rate was observed to be 400. What was its count rate at zero time?

A. 200 B. 1200 C. 1600 D. 2400 $JAMB$ 1990[48] Ans: 1600

14. If the fraction of the atoms of a radioactive material left after 120 years is $\frac{1}{64}$, what is the half life of the material? A. 2 years B. 10 years C. 20 years D. 24 years. *JAMB* 2000[37] Ans: 20 years

15. In 50 days, a radioactive isotope decreases in mass from $33g$ to $1g$. What is the half life of the radioactive material? *NECO* 2001 Ans: 10 days

16. In 24 days, a radioactive isotope decreases in mass from $128g$ to $2g$. What is the half life of the radioactive material? *NECO* 2001 Ans: 4 days

17. The half life of a radioactive substance is 2 seconds. Calculate the decay constant. *WAEC* 1991[50] Ans: $0.3465s^{-1}$

18. What is the decay constant of a radioactive element whose half life is3 seconds? *WAEC* 1996[55] Ans: $0.231\ sec^{-1}$

19. The decay constant of the isotope of a nuclide is $1.36 \times 10^{-4}s^{-1}$. Calculate its half-life. *NECO* 2008[E10] Ans: $5.096 \times 10^{3}s$

20. The half life of a radioactive element is 5 seconds. Calculate its decay constant. *WAEC* 1999[47] Ans: $0.1386s^{-1}$

21. A radioactive element has a decay constant of $0.077s^{-1}$. Calculate its half life. *WAEC* 2003[47] Ans: 9 seconds

22. Which of the following representation is correct for an atom X with 36 electrons and 40 neutrons? A. $^{40}_{36}X$ B. $^{36}_{40}X$ C. $^{76}_{40}X$ D. $^{76}_{36}X$ E. $^{40}_{76}X$ *NECO* 2008[56] Ans: *D*

23. A radioisotope has a decay constant of $10^{-7}s^{-1}$. The average life of the radioisotope is A. $6.93 \times 10^{8}s$ B. $1.00 \times 10^{-4}s$ C. $1.00 \times 10^{7}s$ D. 6.93×10^{7} *JAMB* 2004[19] Ans: $6.93 \times 10^{6}s$

24. Eight α-particles and six β-particle are emitted from an atom of $^{238}_{92}U$ before it achieves stability. What is the nucleon number of the final product in the chain reaction? *WAEC* 1993[58] Ans: 206

25. Uranium of atomic number 92 and mass number 238 emits an alpha particle from its nucleus. The new nucleus formed has, respectively, atomic number and mass number. *WAEC* 2005[50] Ans: 90 and 234

26. Eight alpha decays and six beta decays are necessary before an atom of $^{238}_{92}U$ achieves stability. The final product in the chain has an atomic number of
A. 70 B. 78 C. 82 D. 90 *JAMB* 1989[49] Ans: 82

27. An element X of atomic number 88 and mass number 226 decays to form an element Z by emitting two beta particles and an alpha particle. Z is represented by
A. $^{222}_{82}Z$ B. $^{222}_{88}Z$ C. $^{226}_{86}Z$ D. $^{226}_{80}Z$ *JAMB* 1993[49] Ans: *B*

28. The nucleon number and the proton number of an neutral atom of an element are 23 and 11 respectively. How many neutrons are present in the atom? *WAEC* 2001[46] Ans: 12

29. $^{23}_{11}Na + X \longrightarrow {}^{20}_{9}F + {}^{4}_{2}He$ What particle is X in the react on above?
A. Beta B. Gamma C. Alpha D. Neutron. *JAMB* 2002[49] Ans: Neutron

30. In a nuclear fusion experiment the loss of mass amounts to $1.0 \times 10^{-6}kg$. The amount of energy obtained from the fusion is
A. $3.0 \times 10^{-4}J$ B. $3.0 \times 10^{-1}J$ C. $9.0 \times 10^{4}J$ D. $9.0 \times 10^{10}J$
[speed of light $= 3.0 \times 10^{8}ms^{-1}$] *JAMB* 1998[49] Ans: $9.0 \times 10^{10}J$

30. The amount of energy released when $0.5kg$ of uranium is burnt completely is
A. $4.5 \times 10^{16}J$ B. $1.5 \times 10^{16}J$ C. $1.5 \times 10^{8}J$ D. $4.5 \times 10^{8}J$
$(c = 3 \times 10^{8}ms^{-1})$ *JAMB* 2005[12] Ans: $4.5 \times 10^{6}J$

32. A substance of mass $2.0 \times 10^{-5}kg$ undergoes a fission process which decreases its mass by 0.3%. What amount of energy is released in the process? $(c = 3.0 \times 10^{8}ms^{-1})$ *NECO* 2008[59] Ans: $5.4 \times 10^{9}J$

33. A material of mass $1.0 \times 10^{-3}kg$ undergoes a fission process which decreases its mass by 0.02 percent. Calculate the amount of energy released in the process. $[c = 3 \times 10^{8}ms^{-1}]$ *WAEC* 2000[49] Ans: $1.8 \times 10^{10}J$

34. In a fission process, the decrease in mass is 0.01%. How much energy could be obtained from the fission of 1.0g of the material A. $9.0 \times 10^{9} J$ B. $9.0 \times 10^{10} J$ C. $6.3 \times 10^{11} J$ D. $9.0 \times 10^{11} J$ $[c = 3.0 \times 10^{8} ms^{-1}]$
JAMB 2003[46] Ans: $9.0 \times 10^{9} J$

35. In a nuclear fusion process, four protons each of mass M_p were fused to produce a nucleus X of mass M_X. Which of the following equation is correct?
A. $4M_p > M_X$ B. $4M_p = M_X$ C. $4M_p < M_X$ D. $M_p = M_X$ *JAMB 1991*[48] Ans: A

36. Calculate in joules, the binding energy for $^{59}_{27}C_o$.
[Atomic mass of $^{59}_{27}C_o = 58.9332u$] [Mass of proton = $1.00783u$]
[Mass of neutron = $1.00867u$] [Unified atomic mass unit u = $931 MeV$]
[$1eV = 1.6 \times 10^{-19} J$] *WAEC 1999*[E5] Ans: $8.28 \times 10^{-11} J$

37. Calculate, in joules, the binding energy for $^{7}_{3}L_i$
[Atomic mass of $^{7}_{3}L_i = 7.01600u$] [Mass of neutron = $1.00867u$] [Mass of proton = $1.00783u$] [Unified atomic mass unit, $u = 931 MeV$] [$1eV = 1.6 \times 10^{-19} J$]
NECO 2002[E15] Ans: $6.28 \times 10^{-12} J$

38. A nucleus has a proton number of 84. It emits an α-particle and then a β-particle to achieve stability. What is the proton number of the product?
WAEC 1997[56] Ans: 83

39. $^{23}_{11}Na$ + proton $\longrightarrow$ $^{p}_{q}X$ + alpha particle. What are the values of P and q respectively in the equation above?
A. 10 and 20 B. 12 and 24 C. 20 and 10 D. 24 and 12 *JAMB 1999*[44] Ans: C

40. If the decay constant of a radioactive substance is $0.231 s^{-1}$, the half-life is
A. 3.00s B. 0.12s C. 0.33s D. 1.50s JAMB 2009[45] Ans: A

41. A piece of radioactive material contains 10^{20} atoms. If the half-life of the material is 20 seconds, the number of disintegrations in the first second is
A. 3.47×10^{18} B. 6.93×10^{20} C. 3.47×10^{20} D. 6.93×10^{18}
JAMB 2009[47] Ans: A

42. The half-life of a radioactive substance is 5hours. If 5g of the substance is left after 20hours, determine the original mass of the substance. NECO 2009[58] Ans: 80g

43. Determine the value of x from the nuclear reaction below.

$$^{22}_{86}Rn \longrightarrow {}^{x}_{88}Ra \longrightarrow 2\beta + \text{energy}$$ NECO 2009[59] Ans: 222

44. Calculate the decay constant of a radioactive substance which has a half-life of 25 days. NECO 2009[E9] Ans: $2.77 \times 10^{-2} day^{-1}$

Table 1: Quantities, Formulae and Units

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Speed	$\frac{\text{distance}}{\text{time}}$	$v = \frac{s}{t}$	metre per second	m/s	LT^{-1}
Velocity	$\frac{\text{displacement}}{\text{time}}$	$v = \frac{s}{t}$	metre per second	m/s	LT^{-1}
Acceleration	$\frac{\text{change in velocity}}{\text{time}}$	$a = \frac{v-u}{t}$	meter per second squared	m/s^2	LT^{-2}

Equations of motion, rectilinear acceleration:

$$v = u + at, s = ut + \frac{1}{2}at^2, v^2 = u^2 + 2as$$

Equations of motion, gravitational acceleration:

$$v = u \pm gt, h = ut \pm \frac{1}{2}gt^2, v^2 = u^2 \pm 2gh$$

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Force	mass × acceleration	$F = ma$	newton	N	MLT^{-2}
Friction coefficient	$\frac{\text{frictional force}}{\text{normal reaction}}$	$\mu = \frac{F}{R}$ $\mu = \tan\theta$			
Tensile stress	$\frac{\text{force}}{\text{area}}$	$\frac{F}{A}$ or $\frac{F}{\pi r^2}$	newton per meter squared	N/m^2	$ML^{-1}T^{-2}$
Tensile stress	$\frac{\text{change in length}}{\text{original length}}$	$\frac{e}{l}$			
Elastic force	force constant × extension	$F = ke$	newton	N	MLT^{-2}
Young modulus	$\frac{\text{stress}}{\text{strain}}$	$Y = \frac{F/A}{e/l}$	newton per meter squared	N/m^2	$ML^{-1}T^{-2}$
Work done in elastic spring/string	average force × extension	$W = \frac{1}{2}Fe$ $= \frac{1}{2}ke^2$	joule	J	ML^2T^{-2}
Linear expansivity	$\frac{\text{increase in length}}{\text{original length} \times \text{temperature rise}}$	$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$	Per Kelvin or Per celsius	K^{-1} or $°C^{-1}$	
Area expansivity	$\frac{\text{increase in area}}{\text{original area} \times \text{temperature rise}}$	$\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)}$ $= 2\alpha$	Per Kelvin or Per celsius	K^{-1} or $°C^{-1}$	
Cubic expansivity	$\frac{\text{increase in volume}}{\text{original volume} \times \text{temperature rise}}$	$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)}$ $= 3\alpha$	Per Kelvin or Per celsius	K^{-1} or $°C^{-1}$	
Apparent volume expansivity	$\gamma_a = \frac{\text{mass(volume) of liquid expelled}}{\text{mass(volume) of liq remaining} \times \text{temp rise}}$ $= \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{temperature rise}}$		Per Kelvin or Per celsius	K^{-1} or $°C^{-1}$	

Real cubic expansivity	Actual increase in volume per unit volume per degree rise in temperature, when vessel expansion is considered	$\gamma_r = \gamma_a + \gamma$	Per Kelvin or Per celsius	K^{-1} or $^{o}C^{-1}$	
Work	force × distance	$W = F \times s$ $= mg \times s$	Joule or newton meter	J or Nm	ML^2T^{-2}
Work done (horizontal)	force × cosine of angle × distance	$= F\cos\theta \times s$ $= mg\cos\theta \times s$	Joule or newton meter	J or Nm	ML^2T^{-2}
Work done (vertical)	force × sine of angle × distance	$F\sin\theta \times s$ $mg\sin\theta \times s$	Joule or newton meter	J or Nm	ML^2T^{-2}
Kinetic energy	energy due to motion	$KE = \frac{1}{2}mv^2$	Joule or newton meter	J or Nm	ML^2T^{-2}
Potential energy	energy by virtue of position or height	$PE = mgh$	Joule or newton meter	J or Nm	ML^2T^{-2}
Power	$\frac{\text{work done(energy expended)}}{\text{time}}$	$P = \frac{F \times s}{t}$ $= \frac{mg \times s}{t}$ $= F \times s$	Watt or joule per second	W Or Js^{-1}	ML^2T^{-3}
Efficiency (ϵ)	$\frac{\text{power output}}{\text{power input}} \times 100$	$\frac{p_o}{p_i} \times 100$			

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Electric current	$\frac{\text{quantity of charge}}{\text{time}}$	$I = \frac{Q}{t}$	ampere	A	
Potential difference	current × resistance	$V = IR$	volt	V	
Resistance	$\frac{\text{potential difference}}{\text{current}}$	$R = \frac{V}{I}$	ohm	Ω	
Resistance in parallel		$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	ohm	Ω	
Two resistor in parallel		$R = \frac{R_1R_2}{R_1 + R_2}$	ohm	Ω	
Resistance in series		$R = R_1 + R_2$	ohm	Ω	
E.m.f.(cell) in series		$E = E_1 + E_2$	volt	V	
E.m.f.(cell) in parallel		$E = E_1 = E_2$	volt	V	
E.m.f.	Potential difference between cell terminals at open circuit	$E = I(R + r)$ $E = V + v$	volt	V	

	× 100	$= \frac{R}{R+r} \times 100$			
Electrical energy	Charge quantity × pot. difference	$W = QV$ $= IVt$ $= I^2Rt = \frac{V^2t}{R}$	Joule	J	
Electrical power	$\frac{\text{electrical energy transfered}}{\text{time taken}}$	$P = \frac{W}{t} = \frac{QV}{t}$ $= I^2R = \frac{V^2}{R}$	watt	W	
Electrical heat energy		$H = I^2Rt$ $= Pt$ $= \frac{V^2t}{R} = IVt$	Joule	J	

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Resultant of two perpendicular vectors		$R = \sqrt{F_1^2 + F_2^2}$ $\theta = \tan^{-1}\left[\frac{F_1}{F_2}\right]$			
Resultant of two non-perpendicular vectors		$R = \sqrt{F_1^2 + F_1^2 - 2F_1F_2\cos\theta}$ $\alpha = \sin^{-1}\left(\frac{F_1 \sin\theta}{R}\right)$			
Resolved horizontal component of force	force × cosine of angle given	$F\cos\theta$	newton	N	MLT^{-2}
Resolved vertical component of force	force × cosine of angle given	$F\sin\theta$	newton	N	MLT^{-2}
Projectile time of flight	time taken to return to same projection level	$T = \frac{2U\sin\theta}{g}$	second	s	T
Projectile maximum height	highest vertical distance from projection plane	$H = \frac{U^2\sin^2\theta}{2g}$	meter	m	L
Projectile range	horizontal distance from projection point to projection plane	$R = \frac{U^2\sin 2\theta}{g}$	meter	m	L
Equations of motion under gravity: $v = u \pm gt,\ h = ut \pm \frac{1}{2}gt^2,\ v^2 = u^2 \pm 2gh$					
Moment of force	force × perpendicular	$F \times s$	newton meter	Nm	ML^2T^{-2}
Density	$\frac{\text{mass}}{\text{volume}}$	$\rho = \frac{m}{v}$	kilogram per meter cubed	kg/m^3	ML^{-3}
Relative density	$\frac{\text{mass of substance}}{\text{mass of equal vol } H_2O}$				
Upthrust	density of fluid × volume × g	$U_F = \rho \times v \times g$	newton	N	MLT^{-2}
Centripetal acceleration		$a = \frac{V^2}{r}$	meter per second squared	m/s^2	LT^{-2}
	mass × centripetal	mV^2			

Linear acceleration	× radius; angular acceleration × amplitude	$= \omega^2 A$ $a = \alpha r$	per second squared	m/s^2	LT^{-2}
Period	$\frac{\text{time taken}}{\text{number of oscillations}}$	$T = \frac{t}{n} = \frac{2\pi}{\omega} = \frac{1}{f}$	second	s	T
Frequency	$\frac{\text{number of oscillations}}{\text{time taken}}$	$f = \frac{n}{t} = \frac{\omega}{2\pi} = \frac{1}{T}$	Per second or Hertz	s^{-1} or Hertz	T^{-1}
Period of simple pendulum	Time for one complete cycle	$T = 2\pi\sqrt{\frac{l}{g}}$ $T_1 = T_2\sqrt{\frac{l_1}{l_2}}$	second	s	T
Energy of simple harmonic motion	$T = 2\pi\sqrt{\frac{m}{k}}$; $F = ke$; $T = 2\pi\sqrt{\frac{e}{g}}$; $\omega = \sqrt{\frac{k}{m}}$; $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$; $\omega = \frac{1}{2}kA^2$; $\omega = \frac{1}{2}m\omega^2A^2$				
Momentum	Mass × velocity	$m \times v$	Newton second	Ns or kgm/s	MLT^{-1}
Impulse	Force × time or Mass × acceleration	$I = F \times t$ $= m \times (v - u)$	Newton second	Ns or kgm/s	MLT^{-1}
Force	Mass × acceleration	$F = ma$ $F = \frac{m(v-u)}{t}$	newton	N	MLT^{-2}
Mechanical Advantage	$\frac{\text{load}}{\text{effort}}$, $\frac{\text{output force}}{\text{input force}}$	$M.A. = \frac{L}{E}$			
Velocity Ratio	$\frac{\text{distance moved by effort}}{\text{distance moved by load}}$	$V.R = \frac{e}{l} = \frac{1}{\sin\theta}$ $= \frac{2\pi r}{p} = \frac{R}{r}$			
Efficiency	$\frac{\text{work output}}{\text{work input}} \times 100\%$	$\varepsilon = \frac{M.A}{V.R} \times 100\%$ $\varepsilon = \frac{L/E}{e/l} \times 100\%$			

$$\text{Celsius}(^\circ C) = \frac{5(^\circ F - 32)}{9}, \quad \text{fahrenheit}(^\circ F) = \frac{9^\circ C}{5} + 32, \quad K = {}^\circ C + 273$$

bsolute emperature	temperature measured from absolute zero	$T = 273 + {}^\circ C$	kelvin	K	
esistance ermometer mperature		$\theta = \frac{R - R_0}{R_{100} - R_0} \times 100^\circ C$	celsius	°C	
eat capacity	heat required to raise temperature of substance by 1°C	$C = mc$	joules per kelvin	JK^{-1} or $J^\circ C^{-1}$	
ecific heat pacity	heat required to raise temperature of unit mass of substance by 1°C	$c = \frac{Q}{m(\theta_2 - \theta_1)}$	joule per kilogram per kelvin	$Jkg^{-1}K^{-1}$ or $Jkg^{-1}{}^\circ C^{-1}$	

Specific latent heat of vapourization	heat required to change unit mass of substance at boiling point to vapour without temperature change	$L = \frac{\quad}{m}$ $= \frac{IVt}{m}$	joule per kilogram	Jkg^{-1}	
Relative humidity		$= \frac{m}{M} \times 100\%$ $= \frac{V.P}{S.V.P} \times 100\%$ $= \frac{S.V.P\ at\ dew\ point}{S.V.P\ at\ air\ temperature} \times 100\%$			
Pressure	force per unit area	$p = \rho g h$	newton per meter squared	N/m^2	$ML^{-1}T^{-2}$
Work done by expanding gas	Pressure × change in volume	$W = P(V_2 - V_1)$	joules	J	ML^2T^{-2}

Boyle's law: $P_1V_1 = P_2V_2$, Charles law: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$, Pressure law: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

General gas law: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

Cubic expansivity	$\frac{\text{change in volume from 0°C}}{\text{vol at 0°C × change in temp.}}$	$\gamma = \frac{V_\theta - V_0}{V_0 \times \theta}$ $\gamma = \frac{l_\theta - l_0}{l_0 \times \theta}$	per kelvin or per celsius	K^{-1} or $°C^{-1}$	
Pressure expansivity	$\frac{\text{change in pressure from 0°C}}{\text{vol at 0°C × change in temp.}}$	$\beta = \frac{P_\theta - P_0}{P_0 \times \theta}$	per kelvin or per celsius	K^{-1} or $°C^{-1}$	
Pressure	$\frac{\text{force}}{\text{area}}$	$p = \frac{F}{A}$, $p = \rho g h$	newton per meter squared	N/m^2	$ML^{-1}T^{-2}$

Pascal principle: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$, Hare'sapparatus: $\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$

Frequency	$\frac{\text{number of cycles}}{\text{time taken}}$ $\frac{\text{number of wavelength}}{\text{time taken}}$	$f = \frac{1}{T}$	hertz	Hz	T^{-1}
Wavelength	Distance between successive crest or trough		meter	m	M
Wave velocity	Wavelength × freq $\frac{\text{wavelength}}{\text{period}}$	$v = \frac{\lambda}{T} = \lambda f$	meter per second	m/s	LT^{-1}

Mathematical wave equation; $Y = A\sin\frac{2\pi x}{\lambda}$, $Y = A\sin\frac{2\pi}{\lambda}(x - vt)$, $Y = A\sin 2\pi\left(\frac{x}{\lambda} - ft\right)$

Magnification	$\frac{\text{image distance}}{\text{object distance}}$, $\frac{\text{image height}}{}$	$M = \frac{v}{u}$			

Equations are more important to me, because politics is for the present, but an equation is something for eternity.

Albert Einstein (1879 - 1955)
German-born U.S. physicist.

The aim of research is the discovery of the equations which subsist between the elements of phenomena.

Ernst Mach (1838 - 1916)
Austrian physicist and philosopher.

Image formed by inclined mirror		$n = \frac{360}{\theta} - 1$			
Mirror or lens formula		$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ $f = \frac{uv}{u+v}$			
Refractive index	$\frac{\text{sin of angle of incidence}}{\text{sin of angle of refraction}}$ $\frac{\text{speed in air}}{\text{speed in glass}}$ $\frac{\text{wavelength in air}}{\text{wavelength in glass}}$ $\frac{\text{real depth}}{\text{apparent depth}}$	$\cap = \frac{\sin i}{\sin r}$ ${}_a\cap_g = \frac{V_a}{V_g}$ ${}_a\cap_g = \frac{\lambda_a}{\lambda_g}$ $\cap = \frac{R}{A}$ ${}_a\cap_g = \frac{\sin\frac{1}{2}(A+D_{min})}{\sin\frac{1}{2}A}$			
Critical angle	Angle of incidence when angle of refraction is 90°	$\cap = \frac{1}{\sin C}$			
Wave speed	Wavelength × freq $\frac{\text{wavelength}}{\text{period}}$	$V = \frac{\lambda}{T}$ $V = \lambda f$ $V = \sqrt{\theta}$	meter per second	m/s	LT^{-1}
Velocity of sound		$V = \frac{2x}{t}$	meter per second	m/s	LT^{-1}
Vibrating string & vibration in open open	Fundamental freq First overtone Second overtone Third overtone	$f_0 = \frac{v}{2l}$ $f_1 = 2f_0 = \frac{v}{l}$ $f_2 = 3f_0 = \frac{3v}{2l}$ $f_3 = 4f_0 = \frac{2v}{l}$			

Vibration in closed pipe: $f_0 = \frac{v}{4l}$, $f_1 = 3f_0 = \frac{3v}{4l}$, $f_2 = 5f_0 = \frac{5v}{4l}$, $f_3 = 7f_0 = \frac{7v}{4l}$

Sonometer: velocity of wave along string, $v = \sqrt{\frac{T}{M}}$, frequency $f = \frac{K}{l}\sqrt{\frac{T}{M}}$

fundamental frequency, $f_0 = \frac{1}{2l}\sqrt{\frac{T}{M}}$, first overtone, $2f_0 = \frac{1}{l}\sqrt{\frac{T}{M}}$

second overtone, $f_2 = 3f_0 = \frac{3}{2l}\sqrt{\frac{T}{M}}$, third overtone, $f_3 = 4f_0 = \frac{2}{l}\sqrt{\frac{T}{M}}$

Velocity of sound in resonance tube		$v = 2f(l_2 - l_1)$	meter per second	m/s	LT^{-1}
Beat frequency		$f_b = f_2 - f_1$	per second or hertz	s^{-1}	T^{-1}

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Gravitational force	force of attraction between two bodies	$F = \frac{Gm_1m_2}{r^2}$	newton	N	MLT^{-2}
Gravitational potential	work done by gravitational field in taking unit mass from infinity to a point	$V = -\frac{GM}{r}$	joules per kilogra m	Jkg^{-1}	L^2T^{-2}
Escape velocity	maximum velocity with which object escape from gravitational field	$V_e = \sqrt{2gR}$ $= \sqrt{\frac{2GM}{R}}$	meter per second	m/s	LT^{-1}
Electric force	force between two charges	$F = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q_1q_2}{r^2}$	newton	N	MLT^{-2}
Electric field intensity	electric force per unit charge	$E = \frac{q}{4\pi\varepsilon_o r^2}$	newton per coulomb	NC^{-1}	
Electric potential	work done by electric field to bring unit positive charge from infinity	$V = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r}$	volt	V	
Capacitance	$\frac{\text{charge on plate}}{\text{p.d. between plates}}$	$C = \frac{Q}{V}$ $C = \frac{\varepsilon A}{d}$	farad	F	
Capacitance in series		$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	farad	F	
Capacitance in parallel		$C = C_1 + C_2 + C_3$	farad	F	

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Energy stored in capacitor		$W = \frac{1}{2}qV = \frac{1}{2}CV^2$	joule		
Resistivity	resistance per unit length per unit cross sectional area	$\rho = \frac{RA}{l} = \frac{R\pi r^2}{l}$	ohm-meter	Ωm	
Electrical Conductivity	reciprocal of resistivity	$\sigma = \frac{1}{\rho} = \frac{l}{RA}$	per ohm-meter	$(\Omega m)^{-1}$	
Shunt resistant		$R = \frac{I_g \times r}{I - I_g}$	ohm	Ω	
Multiplier resistant		$R = \frac{V}{I_g} - r$	ohm	Ω	
Electrochemical equivalent	mass deposited per coulomb during electrolysis	$Z = \frac{M}{It} = \frac{\rho A d}{It}$	kilogram per coulomb	kg/C	
Magnetic force		$F = qVB\text{Sin}\theta$ $F = BIl\text{Sin}\theta$	newton	N	MLT^{-2}
Induced e.m.f. in electromagnetic field		$E = BlV$ $= NAB\omega\sin\theta$	volt	V	
Transformer equations		$\frac{E_s}{E_p} = \frac{N_s}{N_p}$ $\frac{E_s}{E_p} = \frac{I_p}{I_s}$			
Transformer efficiency		$\epsilon = \frac{I_s \times E_s}{I_p \times E_p} \times 100$ $= \frac{I_s \times N_s}{I_p \times N_p} \times 100$			
Alternating current		$I = I_o \text{Sin} 2\pi f t$	ampere	A	
Alternating voltage		$V = V_o \text{Sin} 2\pi f t$	volt	V	
Root mean square current		$I_{r.m.s} = \frac{I_o}{\sqrt{2}}$	ampere	A	
Root mean square voltage		$V_{r.m.s} = \frac{V_o}{\sqrt{2}}$	volt	V	
A.C. resistance		$R = \frac{V_{r.m.s}}{I_{r.m.s}}$	ohm	Ω	
Inductive reactance		$X_L = \omega L = 2\pi f L$	ohm		
Inductance		$L = \frac{V_L}{2\pi f I} = \frac{V_L}{\omega I}$	henry	H	

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Voltage across inductor		$V_L = I \times X_L$	volt	V	
Voltage across capacitor		$V_c = I \times X_c$	volt	V	
Capacitive reactance		$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$	ohm	Ω	
Capacitance		$C = \frac{I}{2\pi f V_C} = \frac{I}{\omega V_c}$	farad	F	
L-R circuit impedance		$Z_{LR} = \sqrt{R^2 + X_L^2}$	ohm	Ω	
L-R circuit current		$I_o = \frac{V_o}{\sqrt{R^2 + X_L^2}}$	ampere	A	
L-R circuit total voltage		$V = \sqrt{V_R^2 + V_L^2}$	volt	V	
R-C circuit impedance		$Z_{RC} = \sqrt{R^2 + X_C^2}$	ohm	Ω	
R-C circuit current		$I_o = \frac{V_o}{\sqrt{R^2 + X_C^2}}$	ampere	A	
R-C circuit total voltage		$V = \sqrt{V_R^2 + V_C^2}$	volt	V	
L-C circuit impedance		$Z_{LC} = X_L - X_C$	ohm	Ω	
L-C circuit current		$I_o = \frac{V_o}{X_L - X_C}$	ampere	A	
L-C circuit total voltage		$V = V_L - V_C$	volt	V	
R-L-C circuit impedance		$\sqrt{R^2 + (X_L - X_C)^2}$	ohm	Ω	
R-L-C circuit current		$\frac{V_o}{\sqrt{R^2 + (X_L - X_C)^2}}$	ampere	A	
R-L-C circuit total voltage		$\sqrt{V_R^2 + (V_L - V_C)^2}$	volt	V	
Resonance frequency		$f_o = \frac{1}{2\pi\sqrt{LC}}$	hertz	Hz	
Power in A.C circuit		$P = IV\cos\theta$ $\left(\cos\theta = \frac{R}{Z}\right)$	watt	W	
Power factor	Cosine of phase angle	$\cos\theta = \frac{resistance}{impedance} = \frac{R}{Z}$			

QUANTITY	DEFINITION	FORMULA	S.I. UNIT	SYMBOL	DIMENSION
Energy of photon		$E = hf$ $E = \frac{hc}{\lambda}$	joule	J	
Electron volt/ kinetic energy		$eV = \frac{1}{2}m_e v^2$ $eV = hf$			
Energy of photon (Einstein equation)	work function + kinetic energy	$E = W_o + \frac{1}{2}m_e v^2$ $hf = hf_o + ev$ $hf = \frac{hc}{\lambda_o} + ev$	joule	J	
Stopping potential		$V_s = \frac{hf - W_o}{e}$	volt	V	
Wave-particle duality equation (De Broglie)		$\lambda = \frac{h}{P}$ $\lambda = \frac{h}{mv}$ $\lambda = \frac{hv}{2eV}$ $\lambda = \frac{h}{\sqrt{2eVm_e}}$			
Heisenberg principle		$\Delta x \geq \frac{h}{\Delta P}$ $\Delta P \geq \frac{h}{\Delta x}$ $\Delta x \geq \frac{h}{\Delta(m \times v)}$ $\Delta E.\Delta t \geq \frac{h}{2\pi}$ $\Delta E.\Delta t \geq h$			
Disintegration ratio		$R = 2^n = \frac{N_1}{N_2}$ $(n = t/T)$			
Zhepwo half life equation		$T = \frac{t}{\log_2 R}$ $= \frac{t}{log_2\left(\frac{N_1}{N_2}\right)}$ $= \frac{t\ log2}{logR}$			
Number of atoms decayed		$N_d = N_1 - N_2$ $= N_1\left(\frac{R-1}{R}\right)$ $= N_2(R-1)$			

Fraction of atom undecayed		$f_r = \frac{N_2}{N_1}$ $= \frac{1}{R}$			
Fraction of atoms decayed		$f_d = \frac{N_d}{N_1}$ $= \frac{R-1}{R}$			
Decay constant		$\lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right)$ $= \frac{0.693}{T}$ $= 2.303\frac{\log R}{t}$			

Table 2: Greek Alphabet

Upper Case	Lower Case	Name	Upper Case	Lower Case	Name	Upper Case	Lower Case	Name	Upper Case	Lower Case	Name
A	α	alpha	H	η	eta	N	ν	nu	T	τ	tau
B	β	beta	Θ	θ	theta	Ξ	ξ	xi	Υ	υ	upsilon
Γ	γ	gamma	I	ι	iota	O	o	omicron	Φ	ϕ	phi
Δ	δ	delta	K	κ	kappa	Π	π	pi	X	χ	chi
E	ϵ	epsilon	Λ	λ	lambda	P	ρ	rho	Ψ	ψ	psi
Z	ζ	zeta	M	μ	mu	Σ	σ	sigma	Ω	ω	omega

Table 3: Physical Constants

Constant	Symbol	Value
Speed of light in vacuum	c	$3.0 \times 10^{8}\text{ms}^{-1}$
Gravitational constant	G	$6.67 \times 10^{-11}\text{Nm}^{2}\text{kg}^{-2}$
Electron rest mass	m_e	$9.11 \times 10^{-31}\text{kg}$
Proton rest mass	m_p	$1.67 \times 10^{-27}\text{kg}$
Neutron rest mass	m_n	$1.67 \times 10^{-27}\text{kg}$
Atomic mass unit	u	$1.66 \times 10^{-27}\text{kg}$
Avogadro constant	N_A	$6.023 \times 10^{23}\text{mol}^{-1}$
Planck constant	h	$6.63 \times 10^{-34}\text{Js}$
Faraday constant	F	$9.65 \times 10^{4}\text{Cmol}^{-1}$
Electron charge	e	$1.60 \times 10^{-19}\text{C}$
Electron-volt	eV	$1.6 \times 10^{-19}\text{J}$
Electron charge-to-mass ratio	e/m_e	$1.76 \times 10^{11}\text{Ckg}^{-1}$
Proton charge-to-mass ratio	e/m_p	$9.6 \times 10^{7}\text{Ckg}^{-1}$
Permittivity of vacuum	ε_o	$8.85 \times 10^{-12}\text{Fm}^{-1}$
Molar gas constant	R	$8.31447\text{JK}^{-1}\text{mol}^{-1}$
Acceleration due to gravity	g	9.807ms^{-2}
Standard atmospheric pressure	P	$1.01325 \times 10^{5}\text{Nm}^{-2}$

Table 4: Fundamental Quantities and Units

Quantity	Symbol	S.I Unit	Symbol
Length	l	meter	m
mass	m	kilogramme	kg
Time	t	second	s
Thermodynamic temperature	θ, T	kelvin	K
Luminous intensity	I_v	candela	cd
Electric current	I	ampere	A
Amount of substance	n	mole	mol

Table 5: Submultiples and Multiples of Metric Prefixes

Submultiples	Prefix	Symbol	Multiples	Prefix	Symbol
10^{-1}	deci	d	10^{1}	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

UNIT CONVERSION FACTORS

MASS
1 kg = 1000 g
1 g = 10^{-3} kg
1 mg = 10^{-3} g = 10^{-6} kg
1 μg = 10^{-6} g = 10^{-9} kg

LENGTH
1 m = 100 cm = 10^2cm
1 m = 1000 mm = 10^3 mm
1 m = 1000000 μm = 10^6 μm
1 m = 1000000000 nm = 10^9 nm
1 m = 3.281 ft
1 m = 39.37 in
1 cm = 0.3937 in
1 in = 2.540 cm
1 ft = 30.48 cm
1 yd = 91.44 cm
1 mi = 5280 ft
1 mi = 1.609 km
1 light year = 9.461×10^{15} m

ACCELERATION
1 ms^{-2} = 100 cm^{-2}
1 ms^{-2} = 3.281 ft/s^2
1 cm^{-2} = 0.01 ms^{-2}
1 cm^{-2} = 0.03281 ft/s^2

SPEED
1 km/h = 0.2778 ms^{-1}
1 mile/h = 0.447 ms^{-1}
1 mile/h = 1.609 km/h
1 ms^{-1} = 3.281 ft/s
1 mile/min = 60 mile/h = 88 ft/s
1 in/s = 2.54 cm/s

FORCE
1 N = 1 $kgms^{-2}$
1 N = 10^5 dyn
1 N = 0.2248 lb
1 $lbft^2s^{-1}$ = 0.0421 kgm^2s^{-2}
1N = 23.75 $lbft^2s^{-1}$

ENERGY
1 J = 1 Nm
1 kWh = 3.6×10^6 J
1 eV = 1.602×10^{-19} J

POWER
1 W = 1 Js^{-1}
1 hp = 746W

TIME
1 min = 60 s
1 millisecond = 1 ms = 10^{-3} s
1 microsecond = 1 μs = 10^{-6} s
1 nanosecond = 1 ns = 10^{-9} s
1 h = 3600 s
1 d = 86400 s
1 y = 365.24 d
1 y = 3.156×10^7 s

AREA
1 m^2 = 10000 cm^2 = 10^4 cm^2
1 m^2 = 1000000 mm^2 = 10^6 mm^2
1 mm^2 = 10^{-6} m^2
1 cm^2 = 10^{-4} m^2
1 m^2 = 10.76 ft^2
1 cm^2 = 0.155 in^2

VOLUME
1 m^3 = 10^6 cm^3
1 m^3 = 10^9 mm^3
1 cm^3 = 10^{-6} m^3
1 mm^3 = 10^{-9} m^3
1 litre = 1000 cm^3
1 litre = 10^{-3} m^3
1 m^3 = 35.31 ft^3
1 m^3 = 61.02×10^3 in^3

ANGLE
1° = 0.01745 rad
1° = π/180 rad
1 revolution = 360° = 2π rad
1 rad = 57.30° = 180°/π
1 rev/min(rpm) = 0.1047 rad/s
1 rpm = 0.0167 rev/s

PRESSURE
1 Pa = 1 Nm^{-2}
1 bar = 10^5 Pa
1 mmHg = 133.3 Nm^{-2}
1 bar = 10^5 Nm^{-2}
1 mmH_2O = 9.807 Nm^{-2}
1 atm = 101.325 KNm^{-2}

MASS-ENERGY EQUIVALENCE
1 kg $\leftrightarrow$ 8.988×10^{16} J
1 u $\leftrightarrow$ 931.5 MeV
1eV $\leftrightarrow$ 1.074×10^{-9} u

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By academic freedom I understand the right to search for truth and to publish and teach what one holds to be true. This right implies also a duty: one must not conceal any part of what one has recognized to be true.

Albert Einstein (1879 - 1955)
German-born U.S. physicist.

ALL-INCLUSIVE CALCULATIONS IN PHYSICS

NEW EDITION

- This book is creatively unique, highly simplified and very comprehensive in its conception, content and arrangement. If calculations in physics is one major reason students find physics difficult and perform poorly in examinations, then this book is the complete, definitive and timely solution.
- Every concept with calculations in the WAEC, NECO and JAMB syllabuses is contained in this book and as much as possible, the arrangement is in sequence and in line with the Senior Secondary School Physics Curriculum.
- For every chapter, concept, topic and subtopic, ***ALL*** the calculation questions in WAEC/SSCE (1988-2009), NECO/SSCE (2000-2009) and JAMB/UME (1978-2009) are included either as *typical* examples or as exercises (with answers).
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ABOUT THE AUTHOR

A graduate of education and engineering, Solomon Dauda Yakwo is a dynamic and fledgling Educator, Author and Innovator. He has taught physics at the secondary school level for almost a decade and is the author of *"What They Don't Teach You in School"* and *"Can These New Equations Significantly Simplify Half-life Calculations?"*

The author has done a good deal of work in compiling this book for WAEC, NECO, JAMB, NABTEB, and NDA examinations. This book may also meet the requirements for the Screening Tests now conducted for admission to most universities. The work is adequate and commendable. In summary, I can say the contents are relevant and adequate to the tests and examination needs of the senior secondary school students in physics.

Engr. M.D. Abdullahi, BSc(Physics), MSc(Eng), FIET, FNSE, FAEng, MFR.
Chairman,
National Business and Technical Examination Board (NABTEB)

This book seems to be the best in its class. Thus, I strongly advise that you should have this book and at least one theory physics textbook. **However, if you can't afford many but one, let that one be THIS ONE:** *ALL-INCLUSIVE CALCULATIONS IN PHYSICS*

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