Exercise 5

1. Find T_8, T_9 and T_{10} for each of the following sequences:
   (a) $5, 8, 11, 14, \ldots$
   (b) $3, 5, 7, 9, \ldots$
   (c) $-1, 2, 5, 8, \ldots$
   (d) $4, -1, -6, -11, \ldots$
   (e) $2, 3\frac{1}{2}, 5, 6\frac{1}{2}, \ldots$

2. The first term of a linear sequence is 3 and the 8th term is 31. Find the common difference.

3. The 10th term of an arithmetic progression is 68 and the common difference is 7, find the first term of the sequence.

4. The first term of a linear sequence is 5 and the common difference is –3, find the 15th term of the sequence.

5. Find the sum of the first twelve terms of the sequence 2, 5, 8, 11, ...

6. The 8th term of a linear sequence is 18 and the 12th term is 26. Find the first term, the common difference and the 20th term.

7. Find the sum of the first eight terms of a linear sequence whose first term is 6 and whose last term is 46.

8. The sum of the 4th and 6th terms of an arithmetical progression is 42. The sum of the 3rd and 9th terms of the progression is 52. Find the first term, the common difference and the sum of the first ten terms of the progression.

9. The sum of the first ten terms of a linear sequence is –60 and the sum of the first fifteen terms of the sequence is –165. Find the 18th term of the sequence.

10. The sum of the 6th and 8th terms of an arithmetic progression is 142. If the fourth term is 49, find the first term, the common difference and the sum of the first seven terms of the progression.

11. Show that the sum of n terms of the progression
$\log x, \log x^2, \log x^3, \log x^4, \ldots$ is

$$n\left(\frac{n+1}{2}\right)\log x.$$

12. The 5th term of an arithmetic progression is three times the first term. Find the sum of the first eight terms of the progression given that a quarter of the fifth term is 9.

13. Given that 4, p, q, 13 are consecutive terms of an arithmetic progression, find the values of p and q.

14. Find T_4, T_7 and T_9 for each of the following geometric progressions:
    (a) $3, 6, 12, 24, \ldots$
    (b) $5, 2\frac{1}{2}, 1\frac{1}{4}, \frac{5}{8}, \ldots$
    (c) $5, -10, 20, -40, 80, \ldots$
    (d) $1, -1\frac{1}{2}, \frac{1}{4} -\frac{1}{8}, \ldots$

15. Find the 8th term of an exponential sequence whose first term is 3 and whose common ratio is 2.

16. The 2nd term of an exponential sequence is 9 while the 4th term is 81. Find the common ratio, the first term and the sum of the first five terms of the sequence.

17. If T_n is the nth term of an exponential sequence, show that $T_1 T_n = T_2 T_{n-1}$.

18. Find the sum to infinity of the series:

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$$

19. Write each of the following decimals as proper fractions in their lowest terms:

(a) 0.621 (b) 0.324

(c) 0.723 (d) 0.426

20. If $3, p, q, 24$ are consecutive terms of an exponential sequence, find the values of p and q.

21. The second and fifth terms of a Geometric Progression (G.P.) are 1 and $\frac{1}{8}$ respectively. Find the:

(a) common ratio;

(b) first term;

(c) eighth term.

(WAEC)

22. (a) The nth term U_n of a sequence is given by $U_n = 3 \times 2^{n-1}$.

(i) Write down the first four terms of the sequence.

(ii) Calculate the least value of n for which $U_n > 9000$.

(b) The fifth, ninth and sixteenth terms of a linear sequence (a.p.) are consecutive terms of an exponential sequence (g.p.).

(i) Find the common difference of the linear sequence in terms of the first term.

(ii) Show that the twenty-first, thirty-seventh and sixty-fifth terms of the linear sequence are consecutive terms of an exponential sequence whose common ratio is $\frac{7}{4}$.

(WAEC)

23. The sum of the first n terms of a sequence is $20 - \frac{10}{2^{n-1}}$; $n \geq 1$. Find the:

(a) sum of the first five terms;

(b) fifth term; of the sequence.

24. (a) The sum of the first n term of a series is given by:

$$S_n = 5n^2 - 2n$$

A sequence $U_1, U_2, U_3 \ldots$ is defined by

$$U_r = S_r - S_{r-1}$$

Express U_r in terms of r in its simplest form and show that the sequence is linear (a.p).

(b) Find:

(i) the sum S_n of the of the first n terms of the sequence whose rth term is 4×2^{-r}:

(ii) the value of n for which the difference between S_n and 4 is less than 10^{-4}.

(WAEC)

25. (a) The sum S_n of the first n terms of a sequence is given by $S_n = n(n+2)$. Find:

(i) the sum of the twentieth, twenty-first and twenty-second terms of the sequence;

(ii) the first three terms of the sequence.

(b) The sum of the first twenty-one terms of a linear sequence (A.P.) is 28, and the sum of the first twenty-eight terms is 21. Find which term of the sequence is 0 and also the sum of the terms preceding it.

(WAEC)

26. A sequence of numbers, $U_1, U_2, U_3 \ldots$ satisfies the relation $(3n-2)\,U_{n+1} = (3n+1)\,U_n$ for all positive integers n. If $U_1 = 1$, find:
(i) U_3 and U_4;
(ii) an expression for the nth term U_n;
(ii) the sum of the first n terms of the sequence. (WAEC)

27. If S_n is the sum of the first n terms of the sequence $1, (1+X), (1+X)^2 \ldots (1+X)^{n-1}$ show that

$$S_n = n + \frac{1}{2}\,n(n-1)X + \frac{1}{6}\,n(n-1)(n-2)X^2,$$

neglecting all terms in X^3 and higher powers of X.
If $n = 20$ and $X = 0.01$, calculate the approximate value of S_n. (WAEC)

28. The first and second terms of an exponential sequence (GP) are respectively the first and third terms of a linear sequence (AP). The fourth term of the linear sequence is 10 and the sum of its first five terms is 60. Find:
(a) the first five terms of the linear sequence and the sum of the first n terms;
(b) the sum S_n of the first n terms of the exponential sequence;
(c) the limit of S_n for large value of n. (WAEC)

29. The sequence of numbers
$U_1, U_2, U_3 \ldots U_n \ldots$
satisfies the relation

$$U_n = U_{n-1} + \left(\frac{1}{2}\right)^{n-1}$$

for $n > 1$. If $U_1 = 1$, find:
(i) the values of U_2 and U_3;
(ii) an expression for U_n as a function