#1. [3 points] Using the probability table below for the random variables X and Y, derive the following values.

(a) $\mathbf{Pr}(Y \neq 1)$

(b) $\mathbf{Pr}(X = 1 \cap Y = 0)$

(c) $\mathbf{Pr}(X = 1 \mid Y = 0)$

(d) Are X and Y independent? and explain why.

	$X = 0$	$X = 1$
$Y = 0$	0.4	0.1
$Y = 1$	0.4	0.1

#2. [3 points]

There are two bags. The first bag contains four mangos and two apples; the second bag contains four mangos and four apples.
We also have a biased coin, which shows "heads" with probability 0.6 and "tails" with probability 0.4. If the coin shows "heads". we pick a fruit at random from bag 1; otherwise we pick a fruit at random from bag 2.
Your friend flips the coin (you cannot see the result), picks a fruit at random from the corresponding bag, and presents you a mango.
What is the probability that the mango was picked from bag 2?
Hint: Use Bayes' theorem.

#3. [4 points]
Isidor has been gifted a coin from his grandfather. Let *p* be the probability that on any given flip of the coin is lands with heads up. Before conducting any flips of the coin, Isidor has no clue whatsoever as to the value of *p*.

(a) What is a reasonable prior distribution Isidor can use to model his current cluelessness about p? *[Hint: Keep it simple and uninformative.]*

(b) Isidor now conducts 10 procedurally identical flips of the coin. On each flip, the coin has probability p of landing heads up. And when it does, Isidor records a 1. Otherwise he records a 0. The result is $\{1, 0, 1, 1, 1, 1, 1, 0, 1, 1\}$. Let the ith flip be denoted X_i. Given this setup, a reasonable likelihood model for the flips is: $X_1, \ldots, X_{10}$ are independent (given p) and identically distributed under a *Bernoulli distribution with parameter* p, specifically where $\mathbf{Pr}(X_i = 1 \mid p) = p$ and $\mathbf{Pr}(X_i = 0 \mid p) = 1 - p$. Using this likelihood, and the prior in (a), what value of p gives the most likely model?