

Additional Mathematics Notes:

Sequences and Series (WASSCE Edition)

Prepared for: Abdul Sammed Takiyudeen

1. Introduction to Sequences and Series

A sequence is an ordered list of numbers following a definite rule or pattern. Each number in a sequence is called a term. A series is the sum of the terms of a sequence. Common types include Arithmetic Progressions (A.P.) and Geometric Progressions (G.P.).

2. Arithmetic Progression (A.P.)

An arithmetic progression (A.P.) is a sequence where the difference between any two consecutive terms is constant. Common difference = d . Formula for nth term: $a_n = a + (n - 1)d$ Sum of n terms: $S_n = n/2[2a + (n - 1)d]$ or $S_n = n/2(a + l)$ Example: Find the 10th term of 5, 9, 13, 17, ... Solution: $a = 5$, $d = 4$, $n = 10 \rightarrow a_{10} = 5 + (10 - 1) \times 4 = 41$

3. Geometric Progression (G.P.)

A geometric progression (G.P.) is a sequence where each term is obtained by multiplying the previous term by a fixed number (r). Formula for nth term: $a_n = ar^{n-1}$ Sum of n terms: $S_n = a(r^n - 1)/(r - 1)$ Sum to infinity (if $|r| < 1$): $S_\infty = a/(1 - r)$ Example: Find S_5 for 3, 6, 12, 24, ... Solution: $a = 3$, $r = 2$, $n = 5 \rightarrow S_5 = 3(2^5 - 1)/(2 - 1) = 93$

4. Arithmetic Mean and Geometric Mean

Arithmetic Mean (A.M.): $A = (a + b)/2$ Example: Between 10 and 26 $\rightarrow A = 18$ Geometric Mean (G.M.): $G = \sqrt{ab}$ Example: Between 4 and 9 $\rightarrow G = 6$

5. Applications

Example 1: Ama saves ₦50 monthly, increasing by ₦20 each month. Find total in 12 months: $S_{12} = 6[100 + 220] = ₦1,920$ Example 2: Population = 2000, increases 5% yearly. After 4 years: $a = 2000 \times (1.05)^4 = 2315.2$

Summary of Key Formulas

Type	Formula	Meaning
nth term of A.P.	$a_n = a + (n - 1)d$	Find any term
Sum of A.P.	$S_n = n/2[2a + (n - 1)d]$	Add terms
nth term of G.P.	$a_n = ar^{n-1}$	Multiply by r
Sum of G.P.	$S_n = a(r^n - 1)/(r - 1)$	Total up to n
Infinite G.P.	$S_\infty = a/(1 - r)$	If $ r < 1$

A.M.	$A = (a + b)/2$	Average in A.P.
G.M.	$G = \sqrt{ab}$	Average in G.P.

Practice Questions (With Answers)

1. Find the 15th term of 7, 10, 13, 16, ... → $a_{15} = 49$
2. Sum of first 12 terms of 4, 9, 14, 19, ... → $S_{12} = 378$
3. 8th term of 2, 6, 18, 54, ... → $a_8 = 4374$
4. Sum to infinity of 10, 5, 2.5, ... → $S_\infty = 20$
5. Between which two numbers is 24 the A.M.? → 18 and 30

Final Note: Mastering Sequences and Series strengthens logical reasoning and forms the base for higher mathematical studies.