Chiziqli algebraik tenglamalar sistemasi yechish usullari

1.3.1. Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \tag{1.6}$$

ko'rinishdagi sistemaga n *noma'lumli* m *ta chiqziqli algebraik tenglamalar sistemasi* deyiladi, bu yerda $a_{11}, a_{12}, \ldots, a_{mn}$ *–sistema koeffitsiyentlari,* $x_1, x_2, \ldots, x_n$ *– noma'lumlar,* $b_1, b_2, \ldots, b_m$ *–ozod hadlar.*

1). Ciziqli tenglamalar sistemasi yechishning *matritsalar usuli*da (1.6) sistemaning yechimi

$$\boxed{X = A^{-1}B.} \tag{1.7}$$

formula bilan topiladi.

2 – misol. Tenglamalar sistemasini matritsalar usuli bilan yeching:

$$\begin{cases} 3x_1 - x_2 + x_3 = 4, \\ 2x_1 + x_2 - 2x_3 = 2, \\ x_1 - 3x_2 + x_3 = 6. \end{cases}$$

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & -3 & 1 \end{pmatrix}, \quad \Delta = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -2 \\ 1 & -3 & 1 \end{vmatrix} = 3 + 2 - 6 - 1 - 18 + 2 = -18.$$

Demak, sistema maxsusmas.

Sistema determinantining algebraik to'ldiruvchilarini topamiz:

$$A_{11} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} = -5; \qquad A_{21} = -\begin{vmatrix} -1 & 1 \\ -3 & 1 \end{vmatrix} = -2; \qquad A_{31} = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} = 1;$$

$$A_{12} = -\begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = -4 \qquad A_{22} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2; \qquad A_{32} = -\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix} = 8;$$

$$A_{13} = \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} = -7; \qquad A_{23} = -\begin{vmatrix} 3 & -1 \\ 1 & -3 \end{vmatrix} = 8; \qquad A_{33} = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = -5.$$

U holda

$$A^{-1} = -\frac{1}{18}\begin{pmatrix} -5 & -2 & 1 \\ -4 & 2 & 8 \\ -7 & 8 & 5 \end{pmatrix}.$$

Tenglamaning yechimini (1.7) formula bilan topamiz:

$$X = A^{-1}B = -\frac{1}{18}\begin{pmatrix} -5 & -2 & 1 \\ -4 & 2 & 8 \\ -7 & 8 & 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix} = -\frac{1}{18}\begin{pmatrix} -20-4+6 \\ -16+4+48 \\ -28+16+30 \end{pmatrix} = -\frac{1}{18}\begin{pmatrix} -18 \\ 36 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}.$$

Demak, $x_1 = 1$, $x_2 = -2$, $x_3 = -1$.

2) (1.6) sistema yechimini

$$\boxed{x_i = \frac{\Delta x_i}{\Delta} \quad \left(i = \overline{1,n}\right)} \tag{1.8}$$

formulalar orqali topish mumkin. Bu formulalarga *Kramer formulalari* deyiladi. Bunda Δx_i determinant Δ determinantdan x_i noma'lumlar oldidagi koeffitsiyentlarni ozod hadlar bilan almashtirish orqali hosil qilinadi.

3 – misol. Tenglamalar sistemasini Kramer formulalari bilan yeching:

$$\begin{cases} 2x_1 + x_2 + 3x_3 = -1, \\ x_1 + 2x_2 - x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1. \end{cases}$$

- Δ va Δx_i determinantlarni hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 8 - 3 + 12 - 18 + 8 - 2 = 5;$$

$$\Delta x_1 = \begin{vmatrix} -1 & 1 & 3 \\ 0 & 2 & -1 \\ 1 & 4 & 2 \end{vmatrix} = -15; \quad \Delta x_2 = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 0 & -1 \\ 3 & 1 & 2 \end{vmatrix} = 10; \quad \Delta x_3 = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ 3 & 4 & 1 \end{vmatrix} = 5.$$

Tenglamaning yechimini (1.8) formulalar bilan topamiz:

$$x_1 = \frac{\Delta x_1}{\Delta} = \frac{-15}{5} = -3; \quad x_2 = \frac{\Delta x_2}{\Delta} = \frac{10}{5} = 2; \quad x_3 = \frac{\Delta x_3}{\Delta} = \frac{5}{5} = 1.$$

Agar (1.6) sistema maxsus bo‘lsa:

– $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ lardan birortasi noldan farqli bo‘lganda sistema yechimga ega bo‘lmaydi;

– $\Delta x_1 = \Delta x_2 = \ldots = \Delta x_n = 0$ bo‘lganda sistema cheksiz ko‘p yechimga ega bo‘ladi yoki birgalikda bo‘lmaydi.