MA’RUZA. Funksiya hosilasi va uning tatbiqlari.

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1. Hosila tushunchasiga olib keladigan masalalar.

Hosila tushunchasiga olib keladigan masalalar jumlasiga qattiq jismni to‘g‘ri chiziqli harakatini, yuqoriga vertikal holda otilgan jismning harakatini yoki dvigatel silindridagi porshen harakatini tekshirish kabi masalalarni kiritish mumkin. Bunday harakatlarni tekshirganda jismning konkret o‘lchamlarini va shaklini e‘tiborga olmay, uni harakat qiluvchi moddiy nuqta shaklida tasavvur qilamiz. Biz bitta masalani olib qaraymiz.

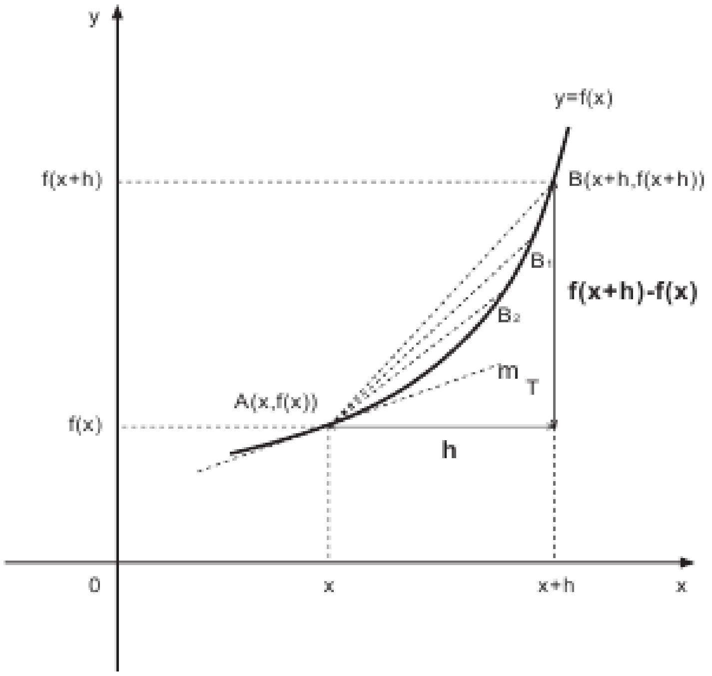
Harakat tezligi masalasi. Aytaylik, M moddiy nuqtaning to‘g‘ri chiziqli harakat qonuniga ko‘ra uning t=t_0 paytdagi tezligini (oniy tezligini) topish talab qilinsin. Nuqtaning t_0 *ва* $t_0+\Delta t$ $(\Delta t \neq 0)$ vaqtlar orasidagi bosib o‘tgan yo‘li $\Delta S = f(t_0+\Delta t) - f(t_0)$ bo‘ladi. Uning shu vaqtdagi o‘rtacha tezligi $\frac{\Delta S}{\Delta t} = \frac{f(t_0+\Delta t)-f(t_0)}{\Delta t}$ ga teng.

Ma‘lumki, Δt qanchalik kichik bo‘lsa, $\frac{\Delta S}{\Delta t}$ o'rtacha tezlik nuqtaning t_0 paytdagi tezligiga shunchalik yaqin bo‘ladi. Shuning uchun nuqtaning t_0 paytdagi tezligi quyidagi limitdan iborat. $V(t_0) = \lim\limits_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$

2. Fuksiya hosilasi.

Hosila ta’rifi.

Faraz qilaylik biz $y = f(x)$ chiziqning $A(x, f(x))$ nuqtasidagi urinmasini topmoqchimiz. m_T - A nuqtada chiziqqa o’tkazilgan urinmaning burchak koeffisienti bo’lsin. A nuqtaga o’tkazilgan urinmaning ikkinchi $B(x+h, f(x+h))$ nuqtasini olaylik.



Hamda AB vatarning gradientini m_{AB} deb qaraylik. Yetalicha kichik h uchun

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{f(x+h) - f(x)}{h}$$

Agar biz B nuqtani A ga yaqinlashtirsak $B_1, B_2, B_3 \ldots$ nuqtalar ketma-ketligi hosil bo'ladi. Bu nuqtalarga mos $AB_1, AB_2, AB_3 \ldots$vatarlarni chiziqning A nuqtasidagi urinmasiga qadar yaqinlashtiraylik.

$$f'(x) = \lim_{B_n \to A} m_{AB_n} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (*)^1$$

(*) tenglikka funksiyaning x nuqtadagi hosilasi deyiladi.

Namunaviy misollar.

1. $f(x) = x^2$ funksiya limitini hisoblang.

Yechish.

Agar $f(x) = x^2$ bo'lsa u holda $f(x+h) = (x+h)^2$ bo'ladi. Bundan

[1] Jane S Paterson, Dorothy A Watson "SQA Advanced Higher Mathematics" Unit1 43-44226 betlarning mazmun mohiyatidan foydalanildi.

$$f'(x) == \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^2-x^2}{h} =$$
$$= \lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h} = \lim_{h\to 0}\frac{h(2x+h)}{h} = 2x$$

Misollar:

Quyidagi funksiyalarning hosilalarini (*) formulasidan foydalanib toping.

1. $f(x)=x^2$
2. $f(x)=3x^2$
3. $f(x)=\sqrt{x}$

y=f(x) funksiya (a,b) intervalda aniqlangan bo‘lsin (a,b) intervalga tegishli x_0 va $x_0+\Delta x$ nuqtalarni olamiz.

Argument biror (musbat yoki manfiy - bari bir) Δx orttirmasini olsin, u vaqtda y funksiya biror Δy orttirmani oladi. Shunday qilib argumentning x_0 qiymatida $y_0=f(x_0)$ ga, argumentning $x_0+\Delta x$ qiymatda $y_0+\Delta y = f(x_0+\Delta x)$ga ega bo‘lamiz. Funksiya orttirmasi Δy ni topamiz.

$$\Delta y = f(x_0+\Delta x)-f(x_0) \qquad (1)$$

Funksiya orttirmasini argument orttirmasiga nisbatini tuzamiz.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (2)$$

Bu – nisbatning $\Delta x \to 0$ dagi limitini topamiz.

Agar bu limit mavjud bo‘lsa, u berilgan f(x) funksiyaning x_0 nuqtadagi hosilasi deyiladi va $f'(x_0)$ bilan belgilanadi. Shunday qilib, ta'rifga ko‘ra $f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ yoki

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \qquad (3)$$

Demak, berilgan y=f(x) funksiyaning argument x bo‘yicha hosilasi deb, argument orttirmasi Δx ixtiyoriy ravishda nolga intilganda funksiya orttirmasi Δy ning argument orttirmasi Δx ga nisbatining limitiga aytiladi.

Umumiy holda x ning har bir qiymati uchun $f'(x)$ hosila ma'lum qiymatga ega, ya’ni hosila ham x ning funksiyasi bo‘lishini qayd qilamiz. Hosilada $f'(x)$ belgi bilan birga boshqacha belgilar ham ishlatiladi. [2] $y';\, y'_x,\, \frac{dy}{dx}$

[2] Canuto, C., Tabacco, A. Mathematical Analysis I,168 226 betlarning mazmun mohiyatidan foydalanildi.

Hosilaning x=a dagi konkret qiymati $f'(a)$ yoki $y'\big|_{x=a}$ bilan belgilanadi.

Funksiya hosilasini hosila ta'rifiga ko‘ra hisoblashni ko`ramiz.

Misol: $y = x^2$ funksiya berilgan: uning:

1) ixtiyoriy x nuqtadagi va 2) x=5 nuqtadagi hosilasi y' topilsin.

Yechish:

1) argumentning x ga teng qiymatida $y = x^2$ ga teng. Argument $x + \Delta x$ qiymatida $y + \Delta y = (x + \Delta x)^2$ ga ega bo‘lamiz.

$\Delta y = (x + \Delta x)^2 - x^2 = 2x(\Delta x) + (x)^2,$ $\dfrac{\Delta y}{\Delta x}$ nisbatni tuzamiz.

$\dfrac{\Delta y}{\Delta x} = \dfrac{2x + \Delta x(\Delta x)^2}{\Delta x} = 2x + \Delta x$ Limitga o‘tib, berilgan funksiyadan hosila topamiz.

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

Demak, $y = x^2$ funksiyaning ixtiyoriy nuqtadagi hosilasi $y' = 2x$

2) x=5 da $y'\big|_{x=5} = 2 \cdot 5 = 10$

3. Differensiallash, uning asosiy qoidalari va formulalari.

Berilgan f(x) funksiyadan hosila topish amali shu funksiyani differensiallash deyiladi.

Differensiallashning asosiy qoidalari.

1. O‘zgarmas miqdorning hosilasi nolga teng, ya‘ni agar y=c bo‘lsa (c=const) y'=0 bo‘ladi.

2. O‘zgarmas ko‘paytuvchini hosila ishorasidan tashqariga chiqarish mumkin: y=cu(x) bo‘lsa y'=cu'(x) bo‘ladi.

3. Chekli sondagi differensiallanuvchi funksiyalar yig‘indisining hosilasi shu funksiyalar hosilalarining yig‘indisiga teng:

$$y = U(x) + V(x) + W(x); \quad y' = U'(x) + V'(x) + W'(x)$$

4. Ikkita differensiallanuvchi funksiyalar ko‘paytmasining hosilasi birinchi funksiya hosilasining ikkinchi funksiya bilan ko‘paytmasi hamda birinchi funksiyaning ikkinchi funksiya hosilasi bilan ko‘paytmasining yig‘indisiga teng:

y=uϑ bo‘lsa $y' = u'\vartheta + u\vartheta'$.

5. Ikkita differensiallanuvchi funksiyalar bo‘linmasining hosilasi (kasrda ifodalanib) bo‘linuvchi funksiya hosilasini bo‘luvchi funksiya bilan ko‘paytmasi hamda bo‘linuvchi funksiyani bo‘luvchi funksiya hosilasi bilan ko‘paytmasining ayirmasini bo‘luvchi(maxrajdagi) funksiya kvadratining nisbatiga teng: [3]

$y = \dfrac{u}{\vartheta}$ bo‘lsa $y' = \dfrac{u'\vartheta - u\vartheta'}{\vartheta^2}$

[3] Canuto, C., Tabacco, A. Mathematical Analysis I,172 226 betlarning mazmun mohiyatidan foydalanildi.

6. Aytaylik, y=F(u) murakkab funksiya bo‘lsin ya’ni y=F(u), $u=\varphi(x)$ yoki $y=F[\varphi(x)]$, u - o‘zgaruvchi, oraliq argumenti deyiladi. y=F(u) va $u=\varphi(x)$ differensiallanuvchi funksiyalar bo‘lsin.

Murakkab funksiyaning differensiallash qoidasini keltirib chiqaramiz.

Teorema: Murakkab F(u) funksiyaning erkli o‘zgaruvchi x bo‘yicha hosilasi bu funksiya oraliq argumenti bo‘yicha hosilasini oraliq argumentining erkli o‘zgaruvchi x bo‘yicha hosilasining ko‘paytmasiga teng, ya’ni

$$y'_x = F'_u(u)\cdot u'_x(x)\ldots\ldots(1)$$

Misol: $y=\left(x^5+4x^4+3x^2+2\right)^5$ funksiyaning hosilasini toping.

Yechish: berilgan funksiyani murakkab funksiya deb qaraymiz ya’ni $y=u^5;\quad u=x^5+4x^4+3x^2+2$ (1) formulaga asosan

$$y'_x=y'_u\cdot u'_x=\left((x^5+4x^4+3x^2+2)^5\right)'=5\left(x^5+4x^4+3x^2+2\right)^4\cdot\left(5x^4+16x^3+6x\right);$$

Differensiallashning asosiy formulalari jadvali:

1) y=const ; $y'=0$

2) $y=x^{\alpha};\quad y=\alpha\, x^{\alpha-1}$

3) $y=\sqrt{x};\ \ y'=\dfrac{1}{2\sqrt{x}}$

4) $y=\dfrac{1}{x};\qquad y=-\dfrac{1}{x^2}$

5) $y=a^x;\ y'=a^x\ln a$

6) $y=e^x;\ \ y'=e^x$

7) $y=\log_a x;\quad y'=\dfrac{1}{x}\log_a e$

8) $y=\ln x;\quad y'=\dfrac{1}{x}$

9) $y=\sin x;\quad y'=\cos x$

10) $y=\cos x;\quad y'=-\sin x$

11) $y=tgx;\quad y'=\dfrac{1}{\cos^2 x}$

12) $y=ctgx;\quad y'=-\dfrac{1}{\sin^2 x}$

isollar.

1) $f(x)=(x^3+4x+7)^4$ funksiyaning hosilasini toping.

Yechish: Bu yerda $y(u)=u^4$ va $u(x)=x^3+4x+7$ U holda

$$f(x)=(u^4)'\cdot(x^3+4x+7)'=4u^3(3x^2+4)=4(x^3+4x+7)^3(3x^2+4)$$

2) $(x^2+x)'=(x^2)'+(x)'=2x+1$

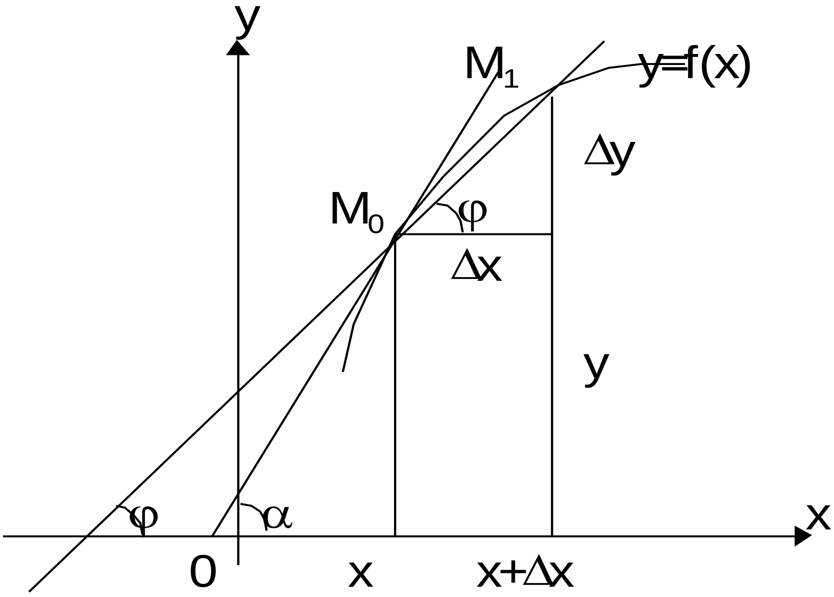
3) $(2x\sin x)'=(2x)'\sin x+2x(\sin x)'=2(x)'\sin x+2x\cos x=$
$2\sin x+2x\cos x=2(\sin x+x\cos x)$

4) $y=\sin 3x$ $\quad y'-?$ $\qquad y'=(\sin 3x)'=3\cos 3x$

1. Hosilaning geometrik va mexanik ma‘nosi.

Bizga berilgan u=f(x) funksiya x nuqta va uning atrofida aniqlangan bo’lsin. Argument x ning biror qiymatida u=f(x) funksiya aniq qiymatga ega bo’ladi, biz uni M_0(x, u) deb belgilaylik.

Argumentga Δx orttirma beramiz va natija funksiyaning u+Δu=f(x+Δx) orttirilgan qiymati to'g'ri keladi. Bu nuqtani M_1(x+Δx, u+Δu) deb belgilaymiz va M_0 kesuvchi o'tkazib uning OX o'qining musbat yo'nalishi bilan tashkil etgan burchagini φ bilan belgilaymiz.



Endi $\frac{\Delta y}{\Delta x}$ nisbatni qaraymiz. Rasmdan ko'rinadiki, $\frac{\Delta y}{\Delta x} = tg\varphi$ (1) ga teng.

Agar Δx→0 ga, u holda M_1 nuqta egri chiziq bo'yicha harakatlanib, M_0 nuqtaga yaqinlasha boradi. M_0M_1 kesuvchi ham Δx→0 da o'z holatini o'zgartira boradi, xususan φ burchak ham o'zgaradi va natijada φ burchak α burchakka intiladi. M_0M_1 kesuvchi esa M_0 nuqtadan o'tuvchi urinma holatiga intiladi. Urinmaning burchak koeffitsienti quyidagicha topiladi

$$tg\alpha = \lim_{\Delta x \to 0} tg\varphi = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x) \qquad (2)$$

Demak, $f'(x) = tg\alpha$, ya'ni, argument x ning berilgan qiymatida $f'(x)$ hosilaning qiymati f(x) funksiyaning grafigiga uning M_0(x, u) nuqtasidagi urinmaning OX o'qining musbat yo'nalishi bilan hosil qilgan burchak tangensiga teng.

1. Geometrik ma'nosi.

Faraz qilaylik bizga $y = f(x)$ funksiya grafiga va unga tegishli bo'lgan $P_0(x_0, f(x_0))$ nuqta berilgan bo'lsin.

$f'(x_0)$ - f funksiyaning grafigiga $P_0(x_0, f(x_0))$ nuqtada o'tkazilgan urinmaning burchak koeffisientiga teng. Bundan foydalanib biz urinma tenglamasini keltirib chiqaramiz. Faraz qilaylik urinma tenglamasi

$y = kx + l$

ko'rinishida bo'lsin. Bu yerda $k = f'(x_0)$

$P_0(x_0, f(x_0))$ nuqta bu to'g'ri chiziqqa tegishli ekanidan $f(x_0) = f'(x_0)x_0 + l$

$l = f(x_0) - f'(x_0)x_0$

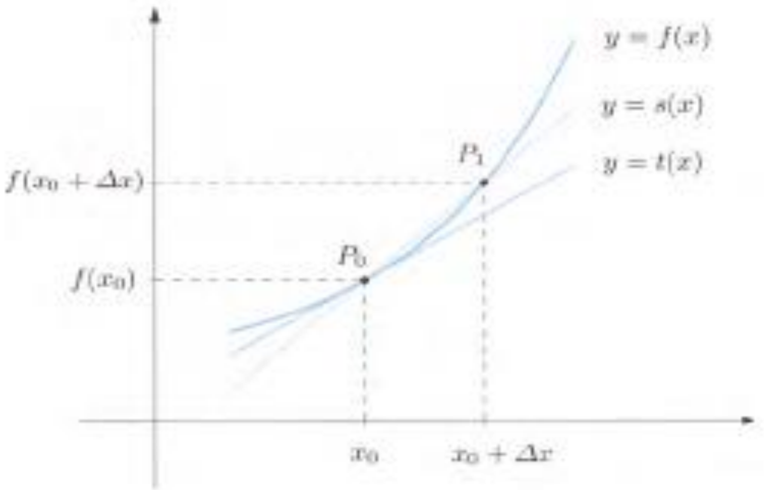
Bundan

$$y = t(x) = f(x_0) + f'(x_0)(x - x_0), \quad x \in R$$

2. Fizik ma’nosi

$$v(t_0) = s'(t_0) = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \qquad (**)$$

(**) formula , $s = s(t)$ qonun bo’yicha harakatlanayotgan M jismning t_0 vaqtdagi oniy tezligini ifodalaydi.[4]



3. **Hosilaning geometrik va fizik ma’nolari.**

Hosilaning fizik ma’nosi. Hosila tushunchasiga olib keladigan ikkinchi masalada harakat qonuni *s=s(t)* funksiya bilan tavsiflanadigan to‘g‘ri chiziq bo‘ylab harakatlanayotgan moddiy nuqtaning *t* vaqt momentidagi oniy tezligi $v_{oniy} = \lim\limits_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ ekanligini ko‘rgan edik. Bundan hosilaning fizik (mexanik) ma’nosi kelib chiqadi.

s=s(t) funksiya bilan tavsiflanadigan to‘g‘ri chiziqli harakatda *t* vaqt momentidagi harakat tezligining son qiymati hosilaga teng: $v_{oniy} = s'(t)$.

Hosilaning mexanik ma’nosini qisqacha quyidagicha ham aytish mumkin: yo‘ldan vaqt bo‘yicha olingan hosila tezlikka teng.

Hosila tushunchasi nafaqat to‘g‘ri chiziqli harakatning oniy tezligini, balki boshqa jarayonlarning ham oniy tezligini aniqlashga imkon beradi. Masalan, faraz qilaylik *y=Q(T)* jismni *T* tempyeraturaga qadar qizdirish uchun uzatilayotgan issiqlik miqdorining o‘zgarishini tavsiflovchi funksiya bo‘lsin. U holda jismning issiqlik sig‘imi issiqlik miqdoridan tempyeratura bo‘yicha olingan hosilaga teng bo‘ladi:

$$C = \frac{dQ}{dT} = \lim_{\Delta T \to 0} \frac{\Delta Q}{\Delta T}.$$

Umuman olganda, hosilani *f(x)* funksiya bilan tavsiflanadigan jarayon oniy tezligining *matematik modeli* deb aytish mumkin.

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[4] Claudio Canuto, Anita Tabacco “ Mathematical analysis I” 168-169 226 betlarning mazmun mohiyatidan foydalanildi.

4. Hosila hisoblash qoidalari

Quyida keltirilgan teoremalar isbotida hosila topish algoritmidan, limitga ega bo‘lgan funksiyalar ustida arifmetik amallar haqidagi teoremalardan foydalanamiz. Shuningdek $\Delta u=u(x+\Delta x)-u(x)$ va $\Delta v=v(x+\Delta x)-v(x)$ ekanligini hisobga olgan holda, $u(x+\Delta x)=u(x)+\Delta u$, $v(x+\Delta x)=v(x)+\Delta v$ tengliklardan foydalanamiz.

$u(x)$ va $v(x)$ funksiyalar (a,b) intervalda aniqlangan bo‘lsin.

Yig‘indining hosilasi.

1-teorema. Agar $u(x)$ va $v(x)$ funksiyalarning $x\in(a,b)$ nuqtada hosilalari mavjud bo‘lsa, u holda $f(x)=u(x)+v(x)$ funksiyaning ham x nuqtada hosilasi mavjud va

$$f'(x)=u'(x)+v'(x) \qquad (4.1)$$

tenglik o‘rinli bo‘ladi.

Isboti. 1^0. $f(x)=u(x)+v(x)$.

2^0. $f(x+\Delta x)= u(x+\Delta x)+ v(x+\Delta x)= u(x)+\Delta u+ v(x)+\Delta v$.

3^0. $\Delta y= f(x+\Delta x)- f(x)= \Delta u+\Delta v$.

4^0. $\dfrac{\Delta y}{\Delta x}=\dfrac{\Delta u+\Delta v}{\Delta x}=\dfrac{\Delta u}{\Delta x}+\dfrac{\Delta v}{\Delta x}$.

5^0. $\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta u+\Delta v}{\Delta x}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta u}{\Delta x}+\lim\limits_{\Delta x\to 0}\dfrac{\Delta v}{\Delta x}=u'(x)+v'(x)$.

Shunday qilib, (4.1) tenglik o‘rinli ekan. Isbot tugadi.

Misol. $(x^2+1/x)'=(x^2)'+(1/x)'=2x-1/x^2$.

Matematik induksiya metodidan foydalanib, quyidagi natijani isbotlash mumkin:

Natija. Agar $u_1(x), u_2(x), \ldots ,u_n(x)$ funksiyalarning x nuqtada hosilalari mavjud bo‘lsa, u holda $f(x)= u_1(x)+ u_2(x+ \ldots+u_n(x)$ funksiyaning ham x nuqtada hosilasi mavjud va quyidagi formula o‘rinli bo‘ladi:

$f'(x)=(u_1(x)+ u_2(x+ \ldots+u_n(x))'= u'_1(x)+ u'_2(x+ \ldots+u'_n(x)$.

Ko‘paytmaning hosilasi.

2-teorema. Agar $u(x)$ va $v(x)$ funksiyalar $x\in(a,b)$ nuqtada hosilaga ega bo‘lsa, u holda ularning $f(x)=u(x)\cdot v(x)$ ko‘paytmasi ham $x\in(a,b)$ nuqtada hosilaga ega va

$$f'(x)=u'(x)v(x)+u(x)v'(x) \qquad (4.2)$$

tenglik o‘rinli bo‘ladi.

Isboti. 1^0. $f(x)=u(x)\cdot v(x)$.

2^0. $f(x+\Delta x)=u(x+\Delta x)\cdot v(x+\Delta x)=(u(x)+\Delta u)\cdot(v(x)+\Delta v)=$

$=u(x)v(x)+\Delta uv(x)+\Delta vu(x)+ \Delta u\Delta v$.

3^0. $\Delta y= f(x+\Delta x)- f(x)= \Delta uv(x)+\Delta vu(x)+\Delta u\Delta v$.

4^0. $\dfrac{\Delta y}{\Delta x}=\dfrac{\Delta uv(x)+\Delta vu(x)+\Delta u\Delta x}{\Delta x}=\dfrac{\Delta u}{\Delta x}v(x)+\dfrac{\Delta v}{\Delta x}u(x)+\dfrac{\Delta u}{\Delta x}\Delta v$.

5^0. $\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}=(\lim\limits_{\Delta x\to 0}\dfrac{\Delta u}{\Delta x})\cdot v(x)+(\lim\limits_{\Delta x\to 0}\dfrac{\Delta v}{\Delta x})\cdot u(x)+\lim\limits_{\Delta x\to 0}\dfrac{\Delta u}{\Delta x}\cdot\lim\limits_{\Delta x\to 0}\Delta v=$

$=u'(x)\cdot v(x)+u(x)\cdot v'(x)++u'(x)\cdot\lim\limits_{\Delta x\to 0}\Delta v$.

Bunda $v(x)$ funksiyaning uzluksizligini e’tiborga olsak $\lim\limits_{\Delta x\to 0}\Delta v=0$ va natijada (4.2) formulaga ega bo‘lamiz.

1-natija. Quyidagi $(Cu(x))'=C\cdot u'(x)$ formula o‘rinli.

Isboti. Ikkinchi teoremaga ko‘ra $(Cu(x))'=C'\cdot u(x)+C\cdot u'(x)$. Ammo $C'=0$, demak $(Cu(x))'=C\cdot u'(x)$.

Misollar. 1. $(6x^2)'=6(x^2)'=6\cdot 2x=12x$.
2. $(x^4)'=((x^2)(x^2))'=(x^2)'(x^2)+(x^2)(x^2)'=2x(x^2)+(x^2)\cdot 2x=4x^3$.
3. $(0{,}25x4-3x2)'=(0{,}25x^4)'+(3x^2)'=0{,}25\cdot 4x^3+3\cdot 2x=x^3+6x$.

2-natija. Agar $u_1(x), u_2(x), \ldots ,u_n(x)$ funksiyalar x nuqtada hosilaga ega bo‘lsa, u holda ularning ko‘paytmasi $f(x)= u_1(x)\cdot u_2(x)\cdot \ldots\cdot u_n(x)$ ham x nuqtada hosilaga ega va quyidagi formula o‘rinli bo‘ladi:
$f'(x)= (u_1(x)\cdot u_2(x)\cdot \ldots\cdot u_n(x))'= u'_1(x)\cdot u_2(x)\cdot \ldots\cdot u_n(x)+ u_1(x)\cdot u'_2(x)\cdot \ldots\cdot u_n(x)+\ldots+ u_1(x)\cdot u_2(x)\cdot \ldots\cdot u'_n(x)$.

Bo‘linmaning hosilasi.

3-teorema. Agar $u(x)$ va $v(x)$ funksiyalar $x\in(a,b)$ nuqtada hosilaga ega, $v(x)\neq 0$ bo‘lsa, u holda ularning $f(x)=u(x)/v(x)$ bo‘linmasi $x\in(a,b)$ nuqtada hosilaga ega va

$$f'(x)=\frac{u'(x)v(x)-u(x)v'(x)}{v^2(x)} \qquad (4.3)$$

formula o‘rinli bo‘ladi.

Isboti. 1^0. $f(x)=\dfrac{u(x)}{v(x)}$.

2^0. $f(x+\Delta x)=\dfrac{u(x+\Delta x)}{v(x+\Delta x)}=\dfrac{u(x)+\Delta u}{v(x)+\Delta v}$.

3^0. $\Delta y= f(x+\Delta x)- f(x)=\dfrac{u(x)+\Delta u}{v(x)+\Delta v}-\dfrac{u(x)}{v(x)}=\dfrac{\Delta u\cdot v(x)-\Delta v\cdot u(x)}{(v(x)+\Delta v)v(x)}$

4^0. $\dfrac{\Delta y}{\Delta x}=\dfrac{\Delta u\cdot v(x)-\Delta v\cdot u(x)}{(v(x)+\Delta v)v(x)\Delta x}=\left(\dfrac{\Delta u}{\Delta x}v(x)-u(x)\dfrac{\Delta v}{\Delta x}\right)\cdot\dfrac{1}{v^2(x)+v(x)\Delta v}$

5^0. $\Delta x\to 0$ da limitga o‘tamiz, limitga ega funksiyalarning xossalari va 2-teorema isbotidagi kabi $\lim\limits_{\Delta x\to 0}\Delta v=0$ tenglikdan foydalansak

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\left(\frac{\Delta u}{\Delta x}v(x)-u(x)\frac{\Delta v}{\Delta x}\right)\cdot\frac{1}{v^2(x)+v(x)\Delta v}=\frac{u'(x)v(x)-u(x)v'(x)}{v^2(x)}$$

natijaga yerishamiz, ya’ni (4.3) formula o‘rinli ekan.

Misol. Ushbu $f(x)=\dfrac{3x+7}{5x-4}$ funksiyaning hosilasini toping.

Yechish. $\left(\dfrac{3x+7}{5x-4}\right)'=\dfrac{(3x+7)'\cdot(5x-4)-(3x+7)\cdot(5x-4)'}{(5x-4)^2}=\dfrac{3(5x-4)-5(3x+7)}{(5x-4)^2}=-\dfrac{47}{(5x-4)^2}$

Shunday qilib biz ushbu paragrafda hosilani hisoblashning quyidagi qoidalarini keltirib chiqardik:

1. Ikkita, umuman chekli sondagi funksiyalar yig‘indisining hosilasi hosilalar yig‘indisiga teng.

2. O‘zgarmas ko‘paytuvchini hosila belgisi oldiga chiqarish mumkin.
3. Ikkita $u(x)$ va $v(x)$ funksiyalar ko‘paytmasining hosilasi $u'v+uv'$ ga teng.
4. Ikkita $u(x)$ va $v(x)$ funksiyalar bo‘linmasining hosilasi $(u'v-uv')/v^2$ ga teng.

1- va 2-teorema natijalaridan foydalangan holda quyidagi qoidaning ham o‘rinli ekanligini ko‘rish qiyin emas:

5. Chekli sondagi differensiallanuvchi funksiyalar chiziqli kombinatsiyasining hosilasi hosilalarning aynan shunday chiziqli kombinatsiyasiga teng, ya’ni agar $f(x)=c_1u_1(x)+c_2u_2(x)+...+c_nu_n(x)$ bo‘lsa, u holda $f'(x)=c_1u'_1(x)+c_2u'_2(x)+...+c_nu'_n(x)$.

Bu qoidaning isbotini o‘quvchilarga havola qilamiz.

Eslatma. Yuqoridagi teoremalar funksiyalar yig‘indisi, ko‘paytmasi, bo‘linmasining hosilaga ega bo‘lishining yetarli shartlarini ifodalaydi. Demak, ikki funksiya yig‘indisi, ayirmasi, ko‘paytmasi va nisbatidan iborat bo‘lgan funksiyaning hosilaga ega bo‘lishidan bu funksiyalarning har biri hosilaga ega bo‘lishi har doim kelib chiqavyermaydi. Masalan, $u(x)=|x|$, $v(x)=|x|$ deb, ularning ko‘paytmasini tuzsak, $y=x^2$ ko‘rinishdagi funksiya hosil bo‘ladi. Bu funksiyaning $\forall x\in(-\infty;+\infty)$ nuqtada, xususan, $x=0$ nuqtada hosilasi mavjud. Ammo, ma’lumki $y=|x|$ funksiyaning $x=0$ nuqtada hosilasi mavjud emas.

Asosiy trigonometrik jadvallar[5]

$$\mathrm{D}\, x^\alpha = \alpha x^{\alpha-1} \qquad (\forall \alpha \in \mathbb{R})$$

$$\mathrm{D}\, \sin x = \cos x$$

$$\mathrm{D}\, \cos x = -\sin x$$

$$\mathrm{D}\, \tan x = 1+\tan^2 x = \frac{1}{\cos^2 x}$$

$$\mathrm{D}\, \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

[5] Claudio Canuto, Anita Tabacco “ Mathematical analysis I” 175 226 betlarning mazmun mohiyatidan foydalanildi.

$$\mathrm{D}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\mathrm{D}\arctan x = \frac{1}{1+x^2}$$

$$\mathrm{D}\,a^x = (\log a)\,a^x \qquad \text{in particular,} \qquad \mathrm{D}\,\mathrm{e}^x = \mathrm{e}^x$$

$$\mathrm{D}\log_a |x| = \frac{1}{(\log a)\,x} \qquad \text{in particular,} \qquad \mathrm{D}\log|x| = \frac{1}{x}$$