MA’RUZA
MATRITSALAR VA ULAR USTIDA AMALLAR. TESKARI MATRITSA.

Ma’ruza rejasi

1. Matritsalar ustida chiziqli amallar (qo’shish, ayirish, songa ko’paytirish);
2. Matritsaning xususiy hollari (satr-matritsa, ustun-matritsa, birlik matritsa, diagonal-matritsa, nol-matritsa, kvadrat matritsa, simmetrik matritsa);
3. Chiziqli almashtirishlar. Ularning matritsali shakli;
4. Teskari matritsa. Matritsalarni ko’paytirish;
5. Chiziqli tenglamalar sistemasini yechishda va chiziqli almashtirishlarda teskari matrtsaning tatbiqi.

Tayanch so’z va iboralar: satr matrisa, ustun matrisa, kvadratik matrisa, simmetrik matritsa, matritsaning minori, matritsaning to’ldiruvchisi, matritsaning rangi, matritsalar yig’indisi, matritsaning songa ko’paytmasi, matritsalar ko’paytmasi,teskari matritsa, chiziqli almashtirish, teskari almashtirish, transponirlangan matritsa.

1. Matritsalar va ular ustida amallar

m ta satr va n ta ustundan iborat

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots\dots & \dots\dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}=\left(a_{ij}\right), \qquad \left(i=\overline{1,m};\quad j=\overline{1,n}\right)$$

ko’rinishdagi jadvalga ($m\times n$)- *o’lchovli to’gri burchakli matritsa* yoki ($m\times n$) – *matritsa* deyiladi. Faqat nollardan iborat bo’lgan matritsa *nol-matritsa* deyiladi va u ko’pincha Q harfi bilan belgilanadi..
$m = n$ bo’lsa, A matritsa n – *tartibli kvadrat matritsa* deyiladi. Kvadrat matritsaning determinanti noldan farqli, ya’ni $\det A \neq 0$ bo’lsa, u *xosmas (maxsusmas)*, $\det A= 0$ da esa *xos (maxsus)* matritsa deyiladi. Kvadrat matritsa uchun *diagonal, skalyar, birlik (*u ko’pincha E harfi bilan belgilanadi) *matritsa* tushunchalari mavjud, ularni 3 – tartibli matritsa misolida keltiramiz:

$$\begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}; \qquad \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}; \qquad E=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

A matritsada satrlarni mos ustunlar bilan almashtirishdan hosil bo’lgan A^{T} matritsa A ga *transponirlangan* matritsa deyiladi. Agar $A = A^{\mathrm{T}}$ bo’lsa, A – *simmetrik matritsa* deyiladi. Matritsa bitta satrdan iborat bo’lsa *satr- matritsa*, bitta ustundan iborat bo’lsa *ustun- matritsa(yoki vektor ham)* deyiladi. Ustun-matritsaning transponirlangani satr-matritsa bo’ladi, va aksincha.

Mos elementlari teng bo’lgan bir xil o’lchamli matriyalar *teng matritsalar* deyiladi. Bir xil o’lchamli matritsalarni qo’shish (ayirish) mumkin. Bunning uchun ularning mos (bir xil o’rindagi) elementlarini qo’shish (ayirish) kerak. Istalgan matritsani songa ko’paytirish mumkin. Buning uchun uning barcha elementlarini shu songa ko’paytirish kerak.

1-misol.

$$A=\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix},\qquad B=\begin{pmatrix}-1 & 1 & 2\\ 2 & 3 & -4\end{pmatrix}$$

matritsalar berilgan. $C = 3A + 2B$ va C^T matritsalarni toping.

►

$$C=3\begin{pmatrix}1 & 2 & 3\\ 0 & 1 & 2\end{pmatrix}+2\begin{pmatrix}-1 & 1 & 2\\ 2 & 3 & -4\end{pmatrix}=\begin{pmatrix}3 & 6 & 9\\ 0 & 3 & 6\end{pmatrix}+\begin{pmatrix}-2 & 2 & 4\\ 4 & 6 & -8\end{pmatrix}=\begin{pmatrix}1 & 8 & 13\\ 4 & 9 & -2\end{pmatrix};$$

$$C^T=\begin{pmatrix}1 & 4\\ 8 & 9\\ 13 & -2\end{pmatrix};$$ ◄

Agar A matritsaning satrlar soni B matritsaning ustunlar soniga teng bo'lsa, A ni B ga ko'paytirish mumkin: $(m\times k)$- o'lchamli $A=(a_{ij})$ matritsani $(k\times n)$- o'lchamli $B=(b_{ij})$ matritsaga ko'paytirishdan $(m\times n)$ - o'lchamli $C=(c_{ij})=AB$ matritsa hosil bo'ladi. Ko'paytirish «satrni ustunga» qoidasi bo'yicha bajariladi: $C=(c_{ij})$ matrisaning c_{ij} elementi A ning i - satr elementlarini B ning j - ustuni mos elementlariga ko'paytirib qo'shishdan hosil bo'ladi:

$$C_{ij}=a_{i1}b_{1j}+a_{i2}b_{2i}+...+a_{ik}b_{kj},\qquad \left(i=\overline{1,m};\quad j=\overline{1,n}\right).$$

Matritsalarni ko'paytirish amali uchun o'rin almashtirish (kommutativlik) qonuni o'rinli emas: $AB\neq BA$. Matritsalarni ko'paytirish amalining xossalari:

1) $A(CB) = (AB)C$; 2) $(A + B)C = AC + BC$; 3) $(\lambda A)B = \lambda(AB)$;
4) $AE = EA = A$; 5) $AQ = QA = Q$; 6) $(AB)^T = B^T A^T$;
7) $det(AB) = detA\cdot detB$.

2 – misol.

$$A=\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix},\qquad B=\begin{pmatrix}-2 & 3\\ 1 & 2\end{pmatrix}$$

matritsalar berilgan. AB va BA matritsalarni toping.

► "Satrni ustunga" qoidasi bo'yicha ko'paytiramiz:

$$AB=\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}-2 & 3 & 0\\ 1 & 2 & -1\end{pmatrix}=\begin{pmatrix}1\cdot(-2)+2\cdot1 & 1\cdot3+2\cdot2 & 1\cdot0+2\cdot(-1)\\ 0\cdot(-2)+1\cdot1 & 0\cdot3+1\cdot2 & 0\cdot0+1\cdot(-1)\\ 1\cdot(-2)+0\cdot1 & 1\cdot3+0\cdot2 & 1\cdot0+0\cdot(-1)\end{pmatrix}=$$

$$=\begin{pmatrix}1 & 7 & -2\\ 1 & 2 & -1\\ -2 & 3 & 0\end{pmatrix}.$$

(3×2) – matritsani (2×3) – matritsaga ko'paytirib, 3 - tartibli kvadrat matritsa hosil qildik. BA matritsani hisoblab ko'ramiz:

$$B\cdot A=\begin{pmatrix}-2 & 3 & 0\\ 1 & 2 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}-2+0+0 & -4+3+0\\ 1+0-1 & 2+2-0\end{pmatrix}=\begin{pmatrix}-2 & -1\\ 0 & 4\end{pmatrix}.$$

Demak, $AB\neq BA$. ◄

3-misol. A matritsaga bog'liq $f(A)$ matritsaviy ko'phadning qiymatini toping.

$$f(A) = A^2 - 5A + 6E; \quad A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}.$$

► $$f(A) = A^2 - 5A + 6E = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} - 5 \cdot \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} + 6 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 4-1 & -2-3 \\ 2+3 & -1+9 \end{pmatrix} - \begin{pmatrix} 10 & -5 \\ 5 & 15 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 5 & 8 \end{pmatrix} - \begin{pmatrix} 10 & -5 \\ 5 & 15 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} =$$

$$= \begin{pmatrix} 3-10+6 & -5-10+0 \\ 5-5+0 & 8-15+6 \end{pmatrix} = \begin{pmatrix} -1 & -15 \\ 0 & -1 \end{pmatrix}.$$

Javobi: $$f(A) = \begin{pmatrix} -1 & -15 \\ 0 & -1 \end{pmatrix}.$$ ◄

2. Teskari matritsa

Agar A xosmas kvadrat matritsa (ya'ni $\Delta = detA \neq 0$) bo'lsa, u holda shunday A^{-1} matritsa mavjudki, uning uchun

$$A \cdot A^{-1} = A^{-1} \cdot A = E$$

tenglik o'rinli bo'ladi, bu yerda E – birlik matritsa. A^{-1} *matritsa A ga teskari matritsa* deyiladi. Teskari matritsaning xossalari:

1. $\det A^{-1} = \dfrac{1}{\det A}$. 2. $(AB)^{-1} = B^{-1} \cdot A^{-1}$.

3. $(A^{-1})^T = (A^T)^{-1}$. 4. $(A^V)^T \cdot A = A \cdot (A^V)^T = \det A \cdot E$,

A^V matritsa $detA$ determinant elementlarining algebraik to'ldiruvchilaridan tuzilgan matritsa bo'lib, A ga *biriktirilgan* matritsa deyiladi. Oxirgi xossadan

$$A^{-1} = \frac{1}{\det A}(A^V)^T,$$

yoki

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad \text{bo'lsa,} \quad A^{-1} = \frac{1}{\det A}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}. \qquad (1)$$

Bu - teskari matritsani topish formulasidir.