# Teskari matrisa, Matrisa rangi.

Namunaviy mimsollar

$A$ kvadrat matritsa uchun $AA^{-1} = A^{-1}A = E$ tenglik bajarilsa, $A^{-1}$ matritsa $A$ matritsaga *teskari matritsa* deyiladi.

Har qanday maxsusmas $A$ matritsa uchun $A^{-1}$ matritsa mavjud va yagona boladi.

$A$ matritsaning teskari matritsasi

$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}.$$

formula bilan aniqlanadi.

misol. $A$ matritsaga teskari matritsani toping:

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & 3 \\ 1 & 2 & -1 \end{pmatrix}.$$

- Matritsaning determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = -16 \neq 0 .$$

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$$A_{11}=\begin{vmatrix}1 & 3\\ 2 & -1\end{vmatrix}=-7;\qquad A_{21}=-\begin{vmatrix}-1 & 0\\ 2 & -1\end{vmatrix}=-1;\qquad A_{31}=\begin{vmatrix}-1 & 0\\ 1 & 3\end{vmatrix}=-3;$$

$$A_{12}=-\begin{vmatrix}-1 & 3\\ 1 & -1\end{vmatrix}=2;\qquad A_{22}=\begin{vmatrix}2 & 0\\ 1 & -1\end{vmatrix}=-2;\qquad A_{32}=-\begin{vmatrix}2 & 0\\ -1 & 3\end{vmatrix}=-6;$$

$$A_{13}=\begin{vmatrix}-1 & 1\\ 1 & 2\end{vmatrix}=-3;\qquad A_{23}=-\begin{vmatrix}2 & -1\\ 1 & 2\end{vmatrix}=-5;\qquad A_{33}=\begin{vmatrix}2 & -1\\ -1 & 1\end{vmatrix}=1.$$

Teskari matritsani formuladan topamiz:

$$A^{-1}=-\frac{1}{16}\begin{pmatrix}-7 & -1 & -3\\ 2 & -2 & -6\\ -3 & -5 & 1\end{pmatrix}=\begin{pmatrix}\frac{7}{16} & \frac{1}{16} & \frac{3}{16}\\ -\frac{1}{8} & \frac{1}{8} & \frac{3}{8}\\ \frac{3}{16} & \frac{5}{16} & -\frac{1}{16}\end{pmatrix}$$

$m \times n$ o‘lchamli $A$ matritsadan $k\ (k \le \min(m;n))$ ta satr va $k$ ta ustunni ajratib, hosil qilingan $k-$tartibli kvadrat matritsaning determinantiga *$A$ matritsaning $k-$tartibli minori* deyiladi.

$A$ matritsa noldan farqli minorlarining yuqori tartibiga *$A$ matritsaning rangi* deyiladi va $r(A)$ (yoki $rangA$) bilan belgilanadi. Bunda $A \ne Q$ uchun $1 \le r(A) \le \min(m;n)$, $A = Q$ uchun $r(A) = 0$.

$r(A)$ni ta’rif asosida topish usuli *minorlar ajratish usuli* deb ataladi.

Matritsalar ustida bajariladigan quyidagi almashtirishlarga *elementar almashtirishlar* deyiladi:

a) faqat nollardan iborat satrni (ustunni) o‘chirish;

b) ikkita satrning (ustunning) o‘rinlarini almashtirish;

c) biror satrning (ustunning) barcha elementlarini noldan farqli songa ko‘paytirish;

d) biror satrning (ustunning) barcha elementlarini noldan farqli songa ko‘paytirib, boshqa satrning (ustunning) mos elementlariga qo‘shish.

Elementar almashtirishlar natijasida matritsaning rangi o‘zgarmaydi.

misol. Matritsaning rangini elementar almashtirishlar usuli bilan toping:

$$A = \begin{pmatrix} 0 & 5 & -10 & 0 \\ -1 & -4 & 5 & -3 \\ 3 & 1 & 7 & 9 \\ 1 & -7 & 17 & 3 \end{pmatrix}.$$

Matritsani kanonik ko‘rinishga keltiramiz.

Buning uchun elementar almashtirishlarni bajaramiz:

– avval matritsaning $1-$va $4-$satrlarining o‘rinlarini almashtiramiz, keyin $2-$satr elementlariga $1-$satrning mos elementlarini qo‘shamiz va $3-$satr elementlariga $(-3)$ga ko‘paytirilgan $1-$satrning mos elementlarini qo‘shamiz;

– hosil bo‘lgan matrisaning $2,3$ va $4-$ satr elementlarini mos ravishda $(-11)$, 22 va 5 ga bo‘lamiz, keyin $(-1)$ga ko‘paytirilgan $2-$satr elementlarini $3$ va $4-$satrning mos elementlariga qo‘shamiz;

– hosil bo‘lgan matritsaning $2,3$ va $4-$ ustun elementlariga mos ravishda 7, $(-17)$ va $(-3)$ ga ko‘paytirilgan $1-$ustun elementlarini qo‘shamiz, keyin $3-$ ustun elementlariga 2 ga ko‘paytirilgan $2-$ustun elementlarini qo‘shamiz.

Bajarilgan elementar almashtirishlarni sxema tarzida keltiramiz:

$$A=\begin{pmatrix}0&5&-10&0\\-1&-4&5&-3\\3&1&7&9\\1&-7&17&3\end{pmatrix}\sim\begin{array}{c}\\1\\-3\\ \\ \end{array}\begin{pmatrix}1&-7&17&3\\-1&-4&5&-3\\3&1&7&9\\0&5&-10&0\end{pmatrix}\sim$$

$$\sim\begin{array}{r}\\ :(-11)\\ :22\\ :5\end{array}\begin{pmatrix}1&-7&17&3\\0&-11&22&0\\0&22&-44&0\\0&5&-10&0\end{pmatrix}\sim\begin{array}{c}\\ \\ -1\\ -1\end{array}\begin{pmatrix}1&-7&17&3\\0&1&-2&0\\0&1&-2&0\\0&1&-2&0\end{pmatrix}\sim$$

$$\sim\begin{array}{c}\begin{matrix}\ &7&-17&-3\end{matrix}\\ \begin{pmatrix}1&-7&17&3\\0&1&-2&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\end{array}\sim\begin{array}{c}\begin{matrix}\ &\ &2&\ \end{matrix}\\ \begin{pmatrix}1&0&0&0\\0&1&-2&0\\0&0&0&0\\0&0&0&0\end{pmatrix}\end{array}\sim\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}.$$

Demak, $r(A)=2$.