mavzu. Ehtimollar nazariyasi elementlari.

Elementar hodisalar fazosi – ehtimolliklar nazariyasi uchun asosiy tushuncha bo‘lib, unga ta’rif berilmaydi. Formal nuqtai nazardan bu ixtiyoriy to‘plam hisoblanib, uning elementlari o‘rganilayotgan tajribaning “bo‘linmaydigan” va bir vaqtda ro‘y bermaydigan natijalairdan iborat bo‘ladi.

Elementar hodisalar fazosi Ω harfi bilan belgilanib, uning elementlari (elementar hodisalar) A, B, C,… lardan tashkil topgan to’plamni χ harf bilan belgilanadi.

We assume a finite *universe* or *sample space* Ω and a set $\mathcal{X}$ of subsets $A, B, C, \ldots$ of Ω, called *events*. We assume $\mathcal{X}$ is closed under finite

Faraz qilaylik χ to'plam chekli yig'indiga nisbatan (agar $A_1, A_2, \ldots A_n$ - hodisalar bo'lsa, u holda $\bigcup_{i=1}^{n} A_i$ - hodisa), kesishmaga nisbatan (agar $A_1, A_2, \ldots A_n$ - hodisalar bo'lsa, u holda $\bigcap_{i=1}^{n} A_i$ - hodisa) va to'ldiruvchiga (Ω ning A tegishli bo'lmagan elementlaridan tashkil topgan to'plam bo'lib, bu to'plam A^c orqali belgilanadi) nisbatan yopiq bo'lsin. Agar A va B lar hodisalar bo'lsa, $A \cap B = AB$ sifatida A va B hodisalar bir vaqtda yuz beradigan hodisani, $A \cup B$ sifatida A yoki B hodisalardan biri yuz beradigan hodisani A^c sifatida esa A hodisa yuz bermaydigan hodisani tushunamiz.

We assume a finite *universe* or *sample space* Ω and a set $\mathcal{X}$ of subsets $A, B, C, \ldots$ of Ω, called *events*. We assume $\mathcal{X}$ is closed under finite unions (if $A_1, A_2, \ldots A_n$ are events, so is $\cup_{i=1}^{n} A_i$), finite intersections (if $A_1, \ldots, A_n$ are events, so is $\cap_{i=1}^{n} A_i$), and complementation (if A is an event so is the set of elements of Ω that are not in A, which we write A^c). If A and B are events, we interpret $A \cap B = AB$ as the event "A and B both occur," $A \cup B$ as the event "A or B occurs," and A^c as the event "A does not occur."

Ehtimollar nazariyasi “Tasodifiy tajribalar”, yani natijasini oldindan aytib bo`lmaydigan tajribalardagi qonuniyatlarni o`rganuvchi matematik fandir. Bunda shunday tajribalar qaraladiki,ularni o`zgarmas (yani bir xil) shartlar kompleksida xech bo`lmaganda nazariy ravishda ixtiyoriy sonda takrorlash mumkin, deb hisoblanadi.

Bunday tajribalar har birining natijasi **tasodifiy hodisa** ro`y berishidan iboratdir. Biz kuzatadigan hodisalarni quyidagi uch turga ajratish mumkin: muqarrar, ro`y bermaydigan va tasodifiy hodisalar.

Muqarrar hodisa deb tayin shartlar to`plami S bajarilganda albatta ro`y beradigan hodisaga aytiladi.

Mumkin bo`lmagan hodisa deb shartlar to`plami S bajarilganda mutloqo ro`y bermaydigan hodisaga aytiladi.

Tasodifiy xodisa deb shartlar to`plami S bajarilganda ro`y berishi ham, ro`y bermasligi ham mumkin bo`lgan xodisaga aytiladi.

Masalan, tanga tashlaganda, u gerbli tomoni, yoki raqamli tomoni bilan tushishi mumkin. Shu sababli “tanga tashalganda gerbli tomoni bilan tushdi” xodisasi tasodifiydir.

Masalan,

1) Tangani ikki marta tashlash (yoki ikkita tangani birdaniga tashlash) tajribasi uchun Ω ={GG, GR, RG, RR}.

Bu yerda GG– tangani ikki marta ham “gerb” tomoni bilan tushish hodisasi, RG– birinchi marta “raqam” tomoni, ikkinchi marta esa “gerb” tomoni bilan tushish hodisasi va qolgan GR, RR hodisalar shularga oʻxshash hodisalar boʻladi. Bu holda $|\Omega| = 4$ va GR, RG hodisalar bir-biridan farq qiladi.

2) Tajriba o‘yin kubigini 2 marta tashlashdan iborat bo‘lsin. Bu holda elementar hodisalar ushbu ko‘rinishga ega: $A_{i,j}= (i, j)$ $i,j=1,2,...,6$.

Bunda $A_{i,j}$ hodisa kubikni birinchi tashlashda i raqamli yoq, ikkinchi tashlashda j raqamli yoq tushganligini bildiradi.

Bu tajribada elementar hodisalar fazosi $\Omega =\{ A_{i,j}, i,j=1,2,...,6\}$. Elementar hodisalar soni $|\Omega| = 36$.

Agar tajriba natijasida *A* ga kirgan ω elementar hodisalarning birortasi ro`y bersa, *A* hodisa ro`y bergan deyiladi.

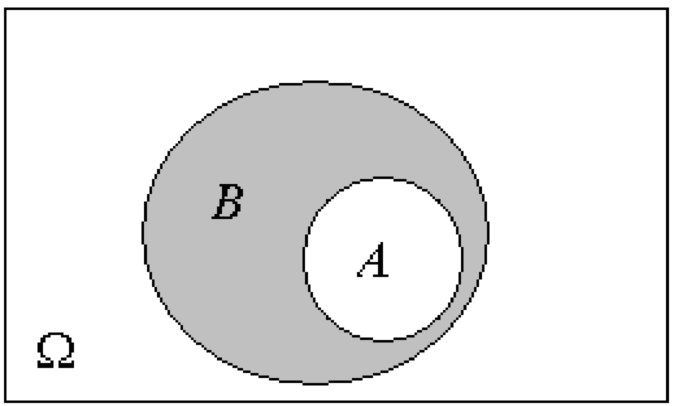
Agar shu elementar hodisalardan birortasi ham ro`y bermasa, *A* hodisa ro`y bermaydi, unda *A* xodisaga teskari xodisa ro`y bergan deyiladi.

($\overline{A}$ orqali belgilanadi)

A va $\overline{A}$ o`zaro qarama-qarshi hodisalar deyiladi.

Muqarrar hodisani Ω bilan belgilaymiz. Mumkin bo`lmagan xodisani $\varnothing$ bilan belgilaymiz.

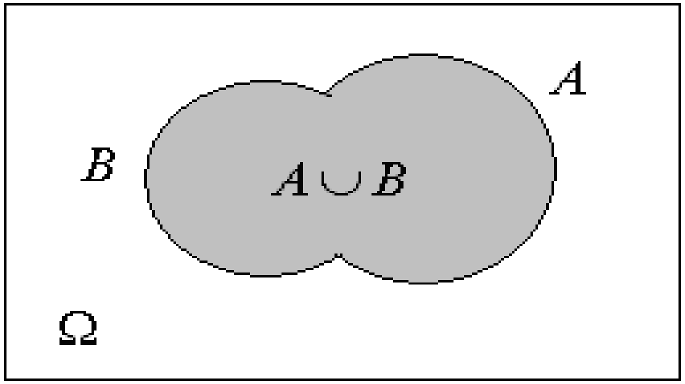
1. Agar A hodisani tashkil etgan elementar hodisalar B hodisaga ham tegishli bo‘lsa, *A hodisa B hodisani ergashtiradi* deyiladi va $A \subset B$ kabi belgilanadi (1-rasm).



1-rasm

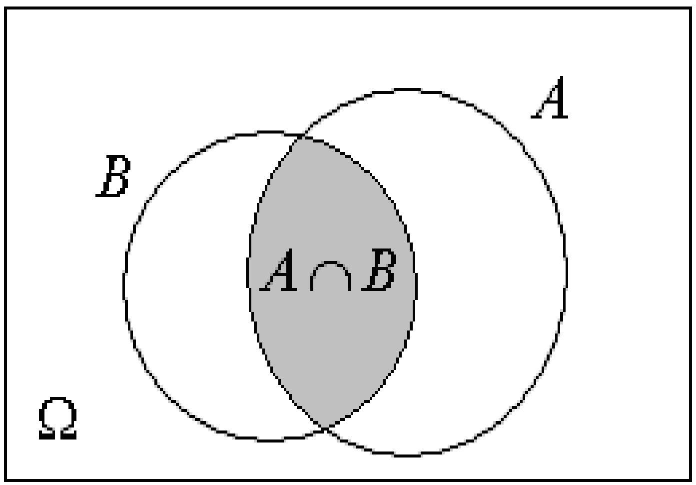
1. Agar $A \subset B$ va $B \subset A$, ya'ni A hodisa B ni ergashtirsa, va aksincha, B hodisa A ni ergashtirsa, A va B hodisalar *teng* deyiladi va $A = B$ kabi belgilanadi.

2. A va B tasodifiy hodisalarning *yig'indisi* deb, shunday C hodisaga aytiladiki, bu hodisa A va B hodisalarning kamida bittasi ro'y berganda ro'y beradi va $C = A \cup B$ (yoki $C = A + B$) kabi belgilanadi (2-rasm).

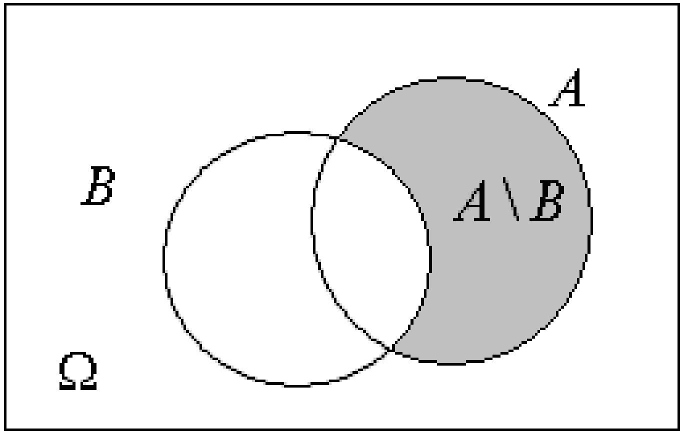


2-rasm.

1. A va B tasodifiy hodisalarning *ko'paytmasi* deb, shunday C hodisaga aytiladiki, bu hodisa A va B hodisalar bir paytda ro'y berganda ro'y beradi va $C = A \cap B$ $(ёки \quad C = A \cdot B)$ kabi belgilanadi (3-rasm).

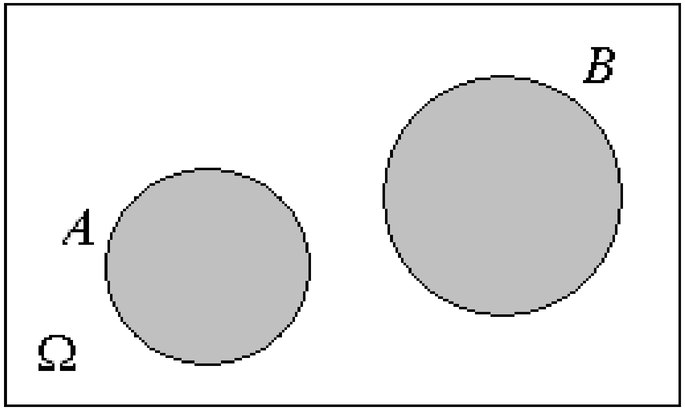


1. A va B tasodifiy hodisalarning *ayirmasi* deb, shunday C hodisaga aytiladiki, u A hodisa ro‘y berib, B hodisa ro‘y bermaganda ro‘y beradi va $C = A \setminus B$ (*ёки* $C = A - B$) kabi belgilanadi (4-rasm).



4-rasm

1. Agar $A \cap B = \varnothing$ bo‘lsa, A va B hodisalar *birgalikda bo‘lmagan hodisalar* deyiladi (5-rasm).



5-rasm

Agar $A_i A_j = \varnothing \ (i \neq j)$ *ва* $A_1 + A_2 + \ldots + A_n = \Omega$ bo‘lsa, u holda $A_1, A_2, \ldots, A_n$ lar *hodisalar to‘la guruxini* tashkil etadi deyiladi.

Murakkab hodisaning ehtimolliklarini oddiy hodisalarning ehtimolliklarini hisoblash orqali topiladi.

Birgalikda bo'lmagan hodisalarning ehtimolliklarini qo'shish qoidasi. Ikkita birgalikda bo'lmagan hodisadan istalgan birining ro'y berish ehtimolligi bu hodisalar ehtimolliklarining yig'indisiga teng:

$$P(A+B)=P(A)+P(B)$$

Natija: Har ikkitasi birgalikda bo'lmagan bir nechta hodisalardan istalgan birining ro'y berish ehtimolligi bu hodisalar ehtimolligining yig'indisiga teng.

$$P(A_1+A_2+...+A_n)=P(A_1)+P(A_2)+...+P(A_n)$$

Birgalikda bo'lgan hodisalar ehtimolliklarini qo'shish qoidasi. *Ikkita birgalikda bo'lgan hodisadan kamida bittasining (hech bo'lmaganda birining)* ro'y berish ehtimolligi bu hodisalar ehtimolliklari yig'indisidan ularning birgalikda ro'y berish ehtimolligini ayrilganiga teng:

$$P(A+B)=P(A)+P(B)-P(AB) \qquad (1)$$

Bu qoida istalgan chekli sondagi birgalikda bo'lgan hodisalar uchun umumlashtirilishi mumkin. Masalan, uchta birgalikda bo'lgan hodisa uchun:

$$P(A+B+C)=P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$$

Natija. Qarama- qarshi hodisalarning ehtimolliklari yig'indisi birga teng:

$$P(A)+P(\overline{A})=1$$

Bundan

$$P(\overline{A})=1-P(A)$$

1-masala: Idishda 40 ta shar bor, ulardan 15tasi oq rangda, 5 tasi yashil rangda, 20 tasi sariq rangda. Rangli shar chiqish ehtimolligini toping.

Yechilishi: Rangli shar chiqishi bu yashil shar yoki sariq shar chiqishini bildiradi.

$A=\{$ yashil shar chiqish hodisasi $\}$

$B=\{$ sariq shar chiqish hodisasi $\}$

Ularning mos ravishda ehtimolliklari $P(A)=\frac{5}{40}=\frac{1}{8}$ $P(B)=\frac{20}{40}=\frac{1}{2}$

A va B hodisalar birgalikda emas (bir rangli shar chiqishi boshqa rangli shar chiqishini yo'qqa chiqaradi).

Izlanayotgan ehtimollik quyidagiga teng

$$P(A+B)=P(A)+P(B)=\frac{1}{8}+\frac{1}{2}=\frac{5}{8}$$

2-masala. Ikkita ovchi bir paytda bir- biriga bog'liq bo'lmagan holda tulkiga qarata o'q uzishdi. Ovchilardan hech bo'lmaganda biri o'qni nishonga tekkizsa, tulki otib olingan bo'ladi. Birinchi ovchining nishonga urish ehtimolligi 0,8 ga, ikkinchisiniki 0,6 ga teng bo'lsa, hech bo'lmaganda bitta ovchining nishonga tekkazish ehtimolligini toping.

Yechilishi: **1-usul.**

A = { birinchi ovchining nishonga tekkazish hodisasi}.

B= { ikkinchi ovchining nishonga tekkazish hodisasi}.

$C=A+B$={hech bo'lmaganda bitta ovchining nishonga tekkazish hodisasi}.

U holda

$$P(C) = P(A) + P(B) - P(AB)$$

AB ikkala ovchi nishonga tekkazadi.

A va B hodisalar bog'liq bo'lmagan hodisalar. Shuning uchun

$$P(C) = P(A) + P(B) - P(A) \cdot P(B) = 0.8 + 0.6 + 0.8 \cdot 0.6 = 0.92$$

2-usul.

Hech bo'lmaganda bitta ovchining nishonga tekkazish hodisasi va nishonga tegmaganlik hodisasi bir-biriga qarama-qarshi hodisalardir. Shuning uchun

$$P(\overline{A}) = 1 - P(A) = 1 - 0.8 = 0.2 \qquad P(\overline{B}) = 1 - P(B) = 1 - 0.6 = 0.4$$

$$P(C) = P(A+B) = 1 - P(\overline{A}) \cdot P(\overline{B}) = 1 - 0.2 \cdot 0.4 = 1 - 0.08 = 0.92$$

A $(A \in X)$ hodisaning ehtimolligi deb $0 \leq P[A] \leq 1$ tengsizlikni qanoatlantiradigan P[A] haqiqiy songa aytiladi, muqarrar hodisaning ehtimoli 1 ga teng, ya'ni $P[\Omega] = 1$, shuningdek, $A_i \in X$ va $\{A_i\}$ lar erkli $(A_i \cap A_j = \emptyset$ bu yerda $i \neq j)$ hodisalar bo'lib, $A = \cup_{i=1}^{n} A_i$ bo'lsa, u holda $P[A] = \Sigma_{i=1}^{n} P[A_i]$ bo'ladi, ya'ni erkli hodisalarning ehtimoliklari yig'indisiga teng.

De Morgan qonunlari

Biz ikkita A va B hodisalar uchun

$(A \cup B)^c = A^c \cap B^c$

va $(A \cap B)^c = A^c \cup B^c$

larni yozishimiz mumkin.

The *probability* of an event $A \in \mathcal{X}$ is a real number P[A] such that $0 \leq$ P[A] ≤ 1. We assume that P[Ω] $= 1$, which says that with probability 1 *some* outcome occurs, and we also assume that if $A = \cup_{i=1}^{n} A_i$, where $A_i \in \mathcal{X}$ and the $\{A_i\}$ are disjoint (that is, $A_i \cap A_j = \emptyset$ for all $i \neq j$), then P[A] $= \sum_{i=1}^{n} P[A_i]$, which says that probabilities are additive over finite disjoint unions.[1]

5.3 De Morgan's Laws

Show that for any two events A and B, we have

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c.$$

Bular De Morgan qonunlari deyiladi. Bu formulalarni so’zlarda ifodalaymiz.

Agar “A hodisaning ro’y berishini” p, “B hodisaning ro’y berishini” q desak, u holda

Inkor(p yoki q) $\Leftrightarrow$ (inkor p va inkor q),

Inkor(p va q) $\Leftrightarrow$ (inkor p yoki inkor q).

Bu formulalar De Morgan qonunlari deb yuritiladi.

These are called *De Morgan's laws*. Express the meaning of these formulas in words.

Show that if we write p for proposition “event A occurs” and q for “event B occurs,” then

$$\text{not } (p \text{ or } q) \Leftrightarrow (\text{ not } p \text{ and not } q),$$

$$\text{not } (p \text{ and } q) \Leftrightarrow (\text{ not } p \text{ or not } q).$$

The formulas are also De Morgan’s laws. Give examples of both rules.

Takroriy ehtimollik

Faraz qilaylik Ω – hodisalar fazosi bo’lib, undagi hodisalar soni n ta, A hodisa esa bu elementlardan m tasini o’z ichiga olsin. U holda, A hodisaning ehtimolligi m/n ga teng. Bu ehtimollikni boshqacha ko’rinishda ham tariflash mumkin.

Faraz qilaylik, tajriba n ta alohida bir- biriga o’xshash natijalarga ega bo’lsin. Shuningdek A - natijalarning qism to’plami bo’lsin va A ning elementlar sonini n(A) harf bilan belgilanadi. U holda, A hodisaning ehtimolligi P(A)=n(A)/n ga teng bo’ladi.

Suppose the sample space Ω consists of a finite number n of equally probable elements. Suppose the event A contains m of these elements. Then the *probability of the event A* is m/n.

A second definition: Suppose an experiment has n distinct outcomes, all of which are equally likely. Let A be a subset of the outcomes, and $n(A)$ the number of elements of A. We define the *probability* of A as $\mathrm{P}[A] = n(A)/n$.

Endi tajribani bir necha marta takrorlaylik va har bitta natija takrorlanmaydigan bo'lsin. Olaylik, A – har bir sinovda amalga oshadigan yoki oshmaydigan hodisa bo'lsin. $n_t(A)$ orqali t – tajribagacha A hodisa amalga oshgan natijalar soni bo'lsin. U holda, A hodisaning nisbiy chastotasi deb, $n_t(A)/t$ songa, A hodisaning ehtimolligi $\lim_{t\to\infty} n_t(A)/t$ songa aytiladi.

Agar A va B hodisalar uchun P(A) ning qiymati B hodisaning amalga oshish yoki oshmaslikka bog'liq bo'lmasa va P(B) ning qiymati A hodisaning amalga oshish yoki oshmaslikka bog'liq bo'lmasa, u holda A va B hodisalar bog'liqmas deyiladi.

Agar A va B bog'liqmas hodisalar bo'lsa, u holda P(AB)= P(A)P(B) tenglik o'rinli.

A third definition: Suppose an experiment can be repeated any number of times, each outcome being independent of the ones before and after it. Let A be an event that either does or does not occur for each outcome. Let $n_t(A)$ be the number of times A occurred on all the tries up to and including the t^{th} try. We define the *relative frequency* of A as $n_t(A)/t$, and we define the *probability of A* as

$$\mathrm{P}[A] = \lim_{t\to\infty} \frac{n_t(A)}{t}.$$

We say two events A and B are *independent* if $\mathrm{P}[A]$ does not depend on whether B occurs or not and, conversely, $\mathrm{P}[B]$ does not depend on whether A occurs or not. If events A and B are independent, the probability that both occur is the product of the probabilities that either occurs: that is,

$$\mathrm{P}[A \text{ and } B] = \mathrm{P}[A] \times \mathrm{P}[B].$$

Glossary	
The space of events	**Elementar hodisalar fazosi.** Elementar hodisalar fazosi Ω harfi bilan belgilanib, uning elementlari (elementar hodisalar) A, B, C,… lardan tashkil topgan to'plamni χ harf bilan belgilanadi.
An inevitable event	**Muqarrar hodisa** deb tayin shartlar to`plami S bajarilganda albatta ro`y beradigan hodisaga aytiladi.
The same events	Agar $A \subset B$ va $B \subset A$, ya'ni A hodisa B ni ergashtirsa, va aksincha, B hodisa A ni ergashtirsa, A va B hodisalar *teng* deyiladi va $A = B$ kabi belgilanadi.

Addition of events	A va B tasodifiy hodisalarning *yig'indisi* deb, shunday C hodisaga aytiladiki, bu hodisa A va B hodisalarning kamida bittasi ro'y berganda ro'y beradi va $C = A \cup B$ (yoki $C = A + B$) kabi belgilanadi
The product of events	A va B tasodifiy hodisalarning *ko'paytmasi* deb, shunday C hodisaga aytiladiki, bu hodisa A va B hodisalar bir paytda ro'y berganda ro'y beradi va $C = A \cap B$ (*ёки* $C = A \cdot B$) kabi belgilanadi
Disjoint events	Agar $A \cap B = \varnothing$ bo'lsa, A va B hodisalar *birgalikda bo'lmagan hodisalar* deyiladi

The probability of an event	A ($A \in X$) hodisaning ehtimolligi deb $0 \leq P[A] \leq 1$ tengsizlikni qanoatlantiradigan P[A] haqiqiy songa aytiladi
relative frequency	U holda, A hodisaning nisbiy chastotasi deb, $n_t(A)/t$ songa,
The probability of repeated event	A hodisaning ehtimolligi $\lim_{t\to\propto} n_t(A)/t$ songa aytiladi.
Independent events	**Bog'liqmas hodisalar.** Agar A va B hodisalar uchun P(A) ning qiymati B hodisaning amalga oshish yoki oshmaslikka bog'liq bo'lmasa va P(B) ning qiymati A hodisaning amalga oshish yoki oshmaslikka bog'liq bo'lmasa, u holda A va B hodisalar bog'liqmas deyiladi.
To occur	**Yuz bermoq**
Event	**Hodisa**