amaliy mashg'ulot. Boshlang'ich funksiya. Aniqmas integral. Aniq integral, uning tadbiqlari

1-misol. $\int\limits_{1}^{3} x^2\,dx$ integralni hisoblang: $\int\limits_{1}^{3} x^2\,dx = \dfrac{x^3}{3}\Big|_{1}^{3} = \dfrac{3^3}{3} - \dfrac{1^3}{3} = 9 - \dfrac{1}{3} = 8\dfrac{1}{3}$.

2-misol. $\int\limits_{0}^{\frac{\pi}{4}} \cos 2x\,dx$.

$$\int\limits_{0}^{\frac{\pi}{4}} \cos 2x\,dx = \dfrac{1}{2}\sin 2x\Big|_{0}^{\frac{\pi}{4}} = \dfrac{1}{2}\left(\sin 2\cdot\dfrac{\pi}{4} - \sin 2\cdot 0\right) = \dfrac{1}{2}\left(\sin\dfrac{\pi}{2} - \sin 0\right) = \dfrac{1}{2}(1-0) = \dfrac{1}{2}$$

Nyuton – Leybnits formulasi.

Agar [a,b] oraliqda $f(x)$ ning biror boshlang'ich funksiyasi $F(x)$ bo'lsa, quyidagi Nyuton-Leybnits formulasi o'rinlidir: $\int\limits_{a}^{b} f(x)\,dx = F(b) - F(a) = F(x)\,|_{a}^{b}$ [1]

1-misol. 1) $\int\limits_{0}^{1} x\,dx = \dfrac{1}{2} x^2\Big|_{0}^{1} = \dfrac{1}{2}(1^2 - 0^2) = \dfrac{1}{2};$

2) $\int\limits_{2}^{3} x\,dx = \dfrac{1}{2} x^2\Big|_{2}^{3} = \dfrac{1}{2}(3^2 - 2^2) = \dfrac{1}{2}\cdot 5 = 2\dfrac{1}{2}.$

2-misol.

$$\int\limits_{-1}^{2} (x^2 + 2x + 1)\,dx = \left(\dfrac{1}{3} x^3 + x^2 + x\right)\Big|_{-1}^{2} =$$

$$= \left[\dfrac{1}{3}\cdot 2^3 + 2^2 + 2\right] - \left[\dfrac{1}{3}(-1)^3 + (-1)^2 + (-1)\right] = 9$$

3-misol. $\int\limits_{0}^{4} \sqrt{x}\,dx = \int\limits_{0}^{4} x^{\frac{1}{2}}\,dx = \dfrac{2}{3} x^{\frac{3}{2}}\Big|_{0}^{4} = \dfrac{2}{3}(4^{\frac{3}{2}} - 0^{\frac{3}{2}}) = 5\dfrac{1}{3}$

4-misol. $\int\limits_{-1}^{1} e^x\,dx = e^x\Big|_{-1}^{1} = e^1 - e^{-1} = e - \dfrac{1}{e} = \dfrac{e^2 - 1}{e}$

5-misol. $\int\limits_{1}^{e} \dfrac{dx}{x} = \ln x\Big|_{1}^{e} = \ln e - \ln 1 = 1 - 0 = 1$

6-misol. $\int\limits_{0}^{1} \dfrac{dx}{x+2} = \ln(x+2)\Big|_{0}^{1} = \ln(1+2) - \ln(0+2) = \ln 3 - \ln 2 = \ln\dfrac{3}{2} = 0{,}4055$

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[1] J.H. Heinbockel. Introduction to Calculus. Volime 1.2012. 160-200 betlarning mazmum mohiyatidan foydalanildi.

8-misol.
$$\int_1^3 e^{2x}dx = \frac{1}{2}e^{2x}\Big|_1^3 = \frac{1}{2}(e^{2\cdot3} - e^{2\cdot1}) = \frac{e^2}{2}(e-1)$$

9-misol.
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx = \sin x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$$

1-topshiriq. Aniq integrallarni hisoblang.

1. $\displaystyle\int_1^3 x^3\,dx$

2. $\displaystyle\int_1^2 \left(x^2 + \frac{1}{x^4}\right)dx$

3. $\displaystyle\int_1^4 \sqrt{x}\,dx$

4. $\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}}$

5. $\displaystyle\int_a^{a\sqrt{3}} \frac{dx}{a^2 + x^2}$

6. $\displaystyle\int_0^3 e^{\frac{x}{3}}\,dx$

7. $\displaystyle\int_0^1 \frac{dx}{\sqrt{x^2+1}}$

8. $\displaystyle\int_0^{\frac{\pi}{4}} \sin 4x\,dx$

9. $\displaystyle\int_4^9 \frac{dx}{\sqrt{x}-1}$

10. $\displaystyle\int_0^1 \frac{dx}{e^x+1}$

11. $\displaystyle\int_0^{\frac{a}{2}} \sqrt{\frac{x}{a-x}}\,dx$

12. $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\cos^2 x\,dx$

13. $\displaystyle\int_0^1 \ln(x+1)\,dx$

14. $\displaystyle\int_{1}^{3} x^3 dx$

15. $\displaystyle\int_{1}^{2}\left(x^2 + \frac{1}{x^4} \right)dx$

16. $\displaystyle\int_{1}^{4} \sqrt{x}\,dx$

$\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{4 - x^2}}$