Amaliy mashguloti. Aniqmas integral va uning xossalari.

Ta'rif. Agar *F(x)* funksiya biror oraliqda *f(x)* funksiyaning boshlang'ich funksiyasi bo'lsa, u holda *F(x)+C* (bu yerda *C* – ihtiyoriy doimiy) funksiyalar to'plami shu kesmada *f(x)* funksiyaning aniqmas integrali deyiladi va $\int f(x)dx = F(x) + C$ kabi belgilanadi.

Bu yerda *f(x)* – integral ostidagi funksiya, *f(x)dx* integral ostidagi ifoda,

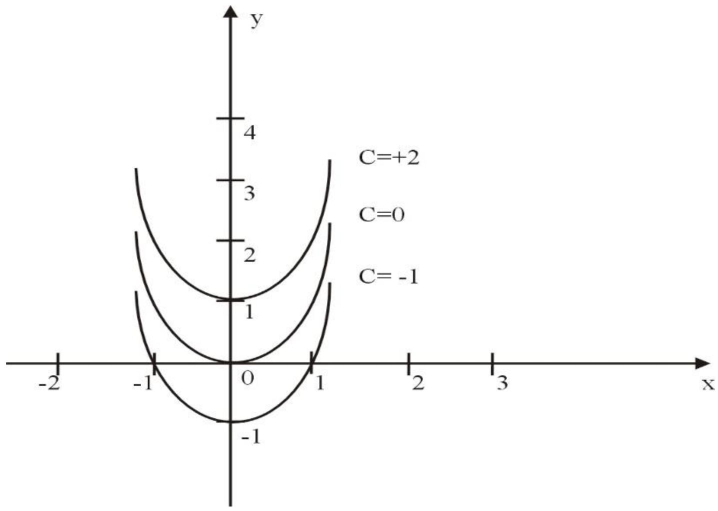
$\int$ – integral belgisi deyiladi.

Aniqmas integralni topish jarayoni yoki berilgan funksiyaning boshlang'ich funksiyasini topish jarayoni **integrallash** deyiladi.

1-misol: $\int \cos x dx = \sin \tilde{o} + C$, chunki $(\sin x)' = \cos x$

2-misol: $\int 3x^2 dx = x^3 + C$, chunki $(x^3)' = 3x^2$.

Boshlang'ich funksiyalarning grafigi integral egri chizig'i deyiladi, shuning uchun aniqmas integral geometrik jihatdan ihtiyoriy *C* o'zgarmasga bog'liq bo'lgan hamma egri chiziqlar to'plamini ifodalaydi.



Aniqmas integralning xossalari:

1) Aniqmas integralning hosilasi integral ostidagi funksiyaga teng, ya'ni

$$\left(\int f(x)dx' = f(x)\right)$$

2) Aniqmas integralning differensiali integral belgisi ostidagi ifodaga teng, ya'ni

$$d\left(f(x)dx\right) = f(x)dx$$

3) Biror funksiyaning hosilasidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int F'(x)dx = F(x) + C$$

4) Biror funksiyaning differentsialidan olingan aniqmas integral shu funksiya bilan ihtiyoriy o'zgarmasning yig'indisiga teng, ya'ni

$$\int dF(x) = F(x) + C$$

5) **If** $\int f(x)dx = F(x) + C$, then

$$\int \alpha f(x)dx = \alpha\int f(x)dx = \alpha[F(x)+C] = \alpha F(x) + K$$

for all constants α. Here $k = \alpha C$ is just some new constant of integration. This property is read, “The integral of a constant times a function equals the constant times the integral of the function.”

Agar $\int f(x)dx = F(x) + C$ b’lsa, u holda barcha o’zgarmas α lar uchun

$\int \alpha f(x)dx = \alpha\int f(x)dx = \alpha[F(x)+C] = \alpha F(x) + K$ bo’ladi. Bu yerda $k = \alpha C$ - integraldagi yangi o’zgarmas sondir. Bu xossa quyidagichadir: “funktsiyani o’zgarmas songa ko’paytmasining integrali o’zgarmas sonni shu funktsiya integraliga ko’paytmasiga teng”[1].

6) Chekli sondagi funksiyalarning algerbaik yig‘indisidan olingan aniqmas integral shu funksiyalarning har biridan olingan aniqmas integrallarning algebraik yig‘indisiga teng, ya’ni

$$\int (f_1(x) + f_2(x) + f_3(x))dx = \int f_1(x)dx + \int f_2(x)dx + \int f_3(x)dx$$

7) Agar $F(x)$ funksiya $f(x)$ uchun boshlang‘ich funksiya bo‘lsa, ya’ni

$\int f(x)dx = F(x) + C$ bo‘lsa u holda $\int f(u)du = F(u) + C$

tenglik to‘g‘ri bo‘ladi, bu yerda $u = u(x)$ x ning differensiallanuvchi funksiyasi. Bu xossa integrallash formulalarining invariantligi deyiladi.

3. Asosiy integrallash jadvali:

1) $\int 0 \cdot dx = C$

2) $\int 1 \cdot dx = x + C$

3) $\int x^{\alpha}dx = \dfrac{x^{\alpha+1}}{\alpha+1} + C \quad (\alpha \neq -1)$

4) $\int \dfrac{1}{x}dx = \ln|x| + C$

5) $\int \dfrac{1}{1+x^2}dx = \mathrm{arctg}x + C$

6) $\int \dfrac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$

7) $\int a^x dx = \dfrac{a^x}{\ln a} + C$

8) $\int \sin x dx = -\cos x + C$

9) $\int \cos x dx = \sin x + C$

10) $\int \dfrac{1}{\cos^2 x}dx = \mathrm{tg}x + C$

11) $\int \dfrac{1}{\sin^2 x}dx = -\mathrm{ctg}x + C$

The following integrals occur quite often and should be memorized.

[1] J.H.Heinbockel. Introduction to Calculus Volume 1, 181 226 betlarning mazmun mohiyatidan foydalanildi.

If $\frac{d}{dx}x=1$, then $\int 1dx=x+C$ or $\int dx=x+C$.

If $\frac{d}{dx}x^2=2x$, then $\int 2xdx=x^2+C$ or $\int d(x^2)=x^2+C$.

If $\frac{d}{dx}x^3=3x^2$, then $\int 3x^2dx=x^3+C$ or $\int d(x^3)=x^3+C$

If $\frac{d}{dx}x^n=nx^{n-1}$, then $\int nx^{n-1}dx=x^n+C$ or $\int d(x^n)=x^n+C$.

If $\frac{d}{dx}(\frac{u^{m+1}}{m+1})=u^m$, then $\int u^m du=\frac{u^{m+1}}{m+1}+C$ or $\int d(\frac{u^{m+1}}{m+1})=\frac{u^{m+1}}{m+1}+C$.

If $\frac{d}{dt}\sin t=\cos t$, then $\int \cos t dx=\sin t+C$ or $\int d(\sin t)=\sin t+C$.

If $\frac{d}{dt}\cos t=-\sin t$, then $\int \sin t dx=-\cos t+C$ or $-\int d(\cos t)=-\cos t+C$.

Quyidagi integrallar ko'p qo'llanilgani uchun eslab qolish lozim:

Agar $\frac{d}{dx}x=1$ bo'lsa, u holda $\int 1dx=x+C$ yoki $\int dx=x+C$ bo'ladi.

Agar $\frac{d}{dx}x^2=2x$ bo'lsa, u holda $\int 2xdx=x^2+C$ yoki $\int d(x^2)=x^2+C$ bo'ladi.

Agar $\frac{d}{dx}x^3=3x^2$ bo'lsa, u holda $\int 3x^2dx=x^3+C$ yoki $\int d(x^3)=x^3+C$ bo'ladi.

Agar $\frac{d}{dx}x^n=nx^{n-1}$ bo'lsa, u holda $\int nx^{n-1}dx=x^n+C$ yoki $\int d(x^n)=x^n+C$ bo'ladi.

Agar $\frac{d}{dx}(\frac{u^{m+1}}{m+1})=u^m$ bo'lsa, u holda $\int u^m du=\frac{u^{m+1}}{m+1}+C$ yoki $\int d(\frac{u^{m+1}}{m+1})=\frac{u^{m+1}}{m+1}+C$ bo'ladi.

Agar $\frac{d}{dt}\sin t=\cos t$ bo'lsa, u holda $\int \cos t dx=\sin t+C$ yoki $\int d(\sin t)=\sin t+C$ bo'ladi.[2]

Agar $\frac{d}{dt}\cos t=-\sin t$ bo'lsa, u holda $\int \sin t dx=-\cos t+C$ yoki $-\int d(\cos t)=-\cos t+C$ bo'ladi.

1-misol. $\int 5dx.$

Yechilishi: 1-xossaga asosan o'zgarmas ko'paytuvchi 5 ni integral ishorasi tashqarisiga chiqaramiz va formulani qo'llab quyidagini hosil qilamiz:

$$\int 5dx=5\int dx=5x+C.$$

Tekshirish. $d(5x+C)=5dx.$ Integral ostidagi ifodani hosil qildik, demak, integral to'g'ri olingan.

2-misol. $\int x^3dx=\frac{x^{3+1}}{3+1}+C=\frac{1}{4}x^4+C.$ Tekshirish: $d\left(\frac{1}{4}x^4+C\right)=\frac{1}{4}\cdot 4x^2dx=x^3dx.$

[2] J.H.Heinbockel. Introduction to Calculus Volume 1, p.181 226 betlarning mazmun mohiyatidan foydalanildi.

Tekshirish: $d\left(\frac{4}{3}x^3 - 2x^2 + 12x + C\right) = (4x^2 - 4x + 12)dx = 4(x^2 - x + 3)dx.$

3-misol. To find the integral given by $I = \int (3x+7)^2\,dx$ you would make a substitution $u = 3x + 7$ with $du = 3dx$ and then perform the necessary scaling to write

$$I = \frac{1}{3}\int (3x+7)^2\,3dx = \frac{1}{3}\int u^2 du = \frac{1}{3}\int u^2 du = \frac{1}{3}\frac{u^3}{3} + C = \frac{1}{3}\frac{(3x+7)^3}{3} + C = \frac{1}{9}(3x+7)^3 + C$$

Quyidagi $I = \int (3x+7)^2\,dx$ integralni hisoblash uchun $u = 3x + 7$ ni $du = 3dx$ ga almashtirishingiz va o'rniga qo'yib yozishingiz kerak:

$$I = \frac{1}{3}\int (3x+7)^2\,3dx = \frac{1}{3}\int u^2 du = \frac{1}{3}\int u^2 du = \frac{1}{3}\frac{u^3}{3} + C = \frac{1}{3}\frac{(3x+7)^3}{3} + C = \frac{1}{9}(3x+7)^3 + C$$ [3]

[3] **J.H.Heinbockel. Introduction to Calculus Volume 1, p.184, example 3-3**